

# Some toy models for GNN-based epidemiological estimation

# Outline

1. Modeling and toy models
2. Dynamical systems
3. Toy model 1
4. Epidemiological modeling, SIR model
5. Toy model 2
6. Mobility data - Origin Destination Survey
7. Metapopulation approach - Eulerian version
8. Toy model 3

# Outline

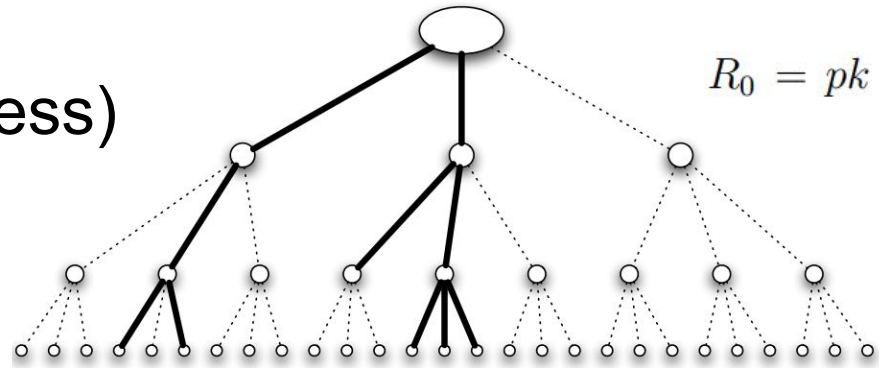
1. Modeling and toy models
  - a. Representation of the reality, tools for testing
2. Dynamical systems
  - a. Rules/equations, initial condition, spatial conditions (neighborhood and boundaries), updating
3. Toy model 1
  - a. (slide 4)
4. Epidemiological modeling, SIR model
  - a. Compartmental models, transitions
5. Toy model 2
  - a. (slide 5)
6. Mobility data - Origin Destination Survey
  - a. Actual data
7. Metapopulation approach - Eulerian version
  - a. Equations
8. Toy model 2
  - a. (slide 6)

# Toy model 1 (Branching process)

Given a graph,

Given an initial infected node,

For each time-step, for each infected node, each neighbor could be infected with a probability “p”.



Could a GNN estimate the infected time series?

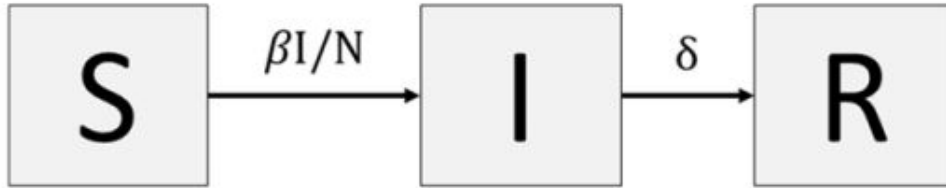
(<https://www.cs.cornell.edu/home/kleinber/networks-book/networks-book-ch21.pdf>)

# Toy model 2 - preliminary discussion

Given the number of cases for one city for some timestamps,

For each time-step, a SIR dynamic:

## A. Classical SIR model



Equation

$$\frac{dS}{dt} = -\frac{\beta IS}{N}$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \delta I$$

$$\frac{dR}{dt} = \delta I$$

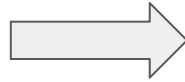
# Toy model 2 - Here we go

Given a mobility graph, [person-to-person network: Papaco's Master]

Given the number of cases for each node for some timestamps,

For each time-step, for each node, a SIR metapopulation dynamic:

$$\frac{dS}{dt} = -\frac{\beta IS}{N}$$



What is NewCases\_i(t)?

Could a GNN estimate it?

[Jessica's Ph.D. - intraurban/Lag,

Duarte's Master - inter/Eul,

Gabriel's monografie]

Handwritten notes on a piece of paper showing mathematical derivations for a metapopulation model. The notes include a table for mobility matrix elements, a legend for variables, and three equations for the rate of change of  $S_i$ .

	$I^0$	$I^D$
$S_i^0$	from no cases	from no cases
$S_i^D$	from no cases	from no cases

Legend:

- $i$ : referenc
- $j$ : Origin
- $k$ : Origin

Equations:

- $$1) -\beta \frac{m_{ii}}{N_i} \left( \frac{m_{ii}}{N_i} \right) I_i, \quad j=i, \quad k=j=i$$
- $$2) -\beta \frac{m_{ii}}{N_i} \sum_{k \neq i} \left( \frac{m_{ki}}{N_k} \right) I_k, \quad j=i$$
- $$3) -\beta \sum_{j \neq i} \left( \frac{m_{ji}}{N_j} \right) I_j, \quad k=i$$

So  
S.

road no lake	Chignon unsee lake
road to sea	unconform surface to sea

= reference

j' New

k Origem

$$1) -\beta \frac{m_{ii}}{N_i} \int_i \left( \frac{m_{ii}}{N_i} \right) I_i \quad ; \quad j=i$$

$$k=j=i$$

$$2) -\beta \frac{m_{ii}}{N_i} \int_i \frac{\sum_{\substack{k=L \\ k \neq i}}^V \left( \frac{m_{ki}}{N_k} \right) I_k}{\sum_{\substack{k=L \\ k \neq i}}^V \left( \frac{m_{ki}}{N_k} \right) N_k} \quad , \quad j=i$$

$$3) -\beta \sum_{\substack{j=L \\ j \neq i}}^V \left( \frac{m_{ij}}{N_i} \right) \int_i \frac{m_{ij}}{N_i} I_i \quad , \quad k=i$$

$$4) -\beta \sum_{\substack{j=L \\ j \neq i}}^V \left( \frac{m_{ij}}{N_i} \right) I_i \frac{\sum_{\substack{k=L \\ k \neq i}}^V \left( \frac{m_{ki}}{N_k} \right) I_k}{\sum_{\substack{k=L \\ k \neq i}}^V \left( \frac{m_{ki}}{N_k} \right) N_k} :$$

$$L) -\beta \frac{m_{ii}}{N_i} \frac{\int_i \left( \frac{m_{ii}}{N_i} \right) \mathbf{I}_i}{\left( \frac{m_{ii}}{N_i} \right) N_i}$$

$$; j = i$$

$$k = j = i$$



$$2) -\beta \frac{m_{ii}}{\mu_i} \int_i \frac{\sum_{\substack{K=L \\ K \neq i}}^U \left( \frac{m_{K,i}}{N_K} \right) I_K}{\quad}, \quad j=i$$

$$\sum_{\substack{K=L \\ K \neq i}}^U \left( \frac{m_{K,i}}{N_K} \right) N_K$$

$$3) - \beta \sum_{j=1}^V \left( \frac{m_{ij}}{N_i} \right) \int_i \frac{m_{ij}}{N_i} I_i, \quad k=i$$

$$j=1$$

$$j \neq i$$

$$\frac{m_{ij}}{N_i} N_i$$

$$4) -\beta \sum_{\substack{j=L \\ j \neq i}}^V \left( \frac{m_{ji}}{N_i} \right) I_i - \frac{\sum_{\substack{k=L \\ k \neq i}}^V \left( \frac{m_{ki}}{N_k} \right) I_k}{\sum_{\substack{k=L \\ k \neq i}}^V \left( \frac{m_{ki}}{N_k} \right) N_k} :$$

$$1) -\beta \frac{m_{ii}}{N_i} \int_i \left( \frac{m_{ii}}{N_i} \right) I_i \quad ; \quad j=i$$

$$\frac{\left( \frac{m_{ii}}{N_i} \right) N_i}{k=j=i}$$

$$2) -\beta \frac{m_{ii}}{N_i} \int_i \frac{\sum_{\substack{k=L \\ k \neq i}}^V \left( \frac{m_{ki}}{N_k} \right) I_k}{\sum_{\substack{k=L \\ k \neq i}}^V \left( \frac{m_{ki}}{N_k} \right) N_k} \quad ; \quad j=i$$

$$3) -\beta \left( \frac{m_{ij}}{N_i} \right) \int_i \frac{\frac{m_{ij}}{N_i} I_i}{\frac{m_{ij}}{N_i} N_i} \quad , \quad k=i$$

$$j=1$$

$$j \neq i$$

$$4) -\beta \sum_{\substack{j=L \\ j \neq i}}^V \left( \frac{m_{ij}}{N_i} \right) \int_i \frac{\sum_{\substack{k=L \\ k \neq i}}^V \left( \frac{m_{ki}}{N_k} \right) I_k}{\sum_{\substack{k=L \\ k \neq i}}^V \left( \frac{m_{ki}}{N_k} \right) N_k} \quad ;$$

# Roadmap

1. A graph (C3, networkx)
2. An (edge-independent) SIR model in each node:
  - a.  $B_1 > B_2, G_3 < G_2$
3. Plotting:
  - a. Time-series ( $S_i, I_i$ )
  - b. Time-series-image: colour graph ( $I_i$ )
4. Edge-dependence:
  - a. 1-2 10, 2-1 8, 2-3 0, 3-2 0, 1-3 5, 3-1 3