Some toy models for GNN-based epidemiological estimation

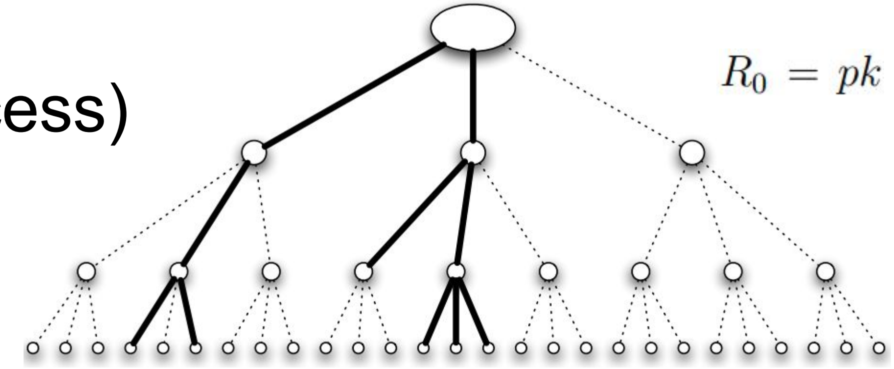
Outline

1. Modeling and toy models
2. Dynamical systems
3. Toy model 1
4. Epidemiological modeling, SIR model
5. Toy model 2
6. Mobility data - Origin Destination Survey
7. Metapopulation approach - Eulerian version
8. Toy model 3

Outline

1. Modeling and toy models
   a. Representation of the reality, tools for testing
2. Dynamical systems
   a. Rules/equations, initial condition, spatial conditions (neighborhood and boundaries), updating
3. Toy model 1
   a. (slide 4)
4. Epidemiological modeling, SIR model
   a. Compartimental models, transitions
5. Toy model 2
   a. (slide 5)
6. Mobility data - Origin Destination Survey
   a. Actual data
7. Metapopulation approach - Eulerian version
   a. Equations
8. Toy model 2
   a. (slide 6)

Toy model 1 (Branching process)



Given a graph,

Given an initial infected node,

For each time-step, for each infected node, each neighbor could be infected with a probability “p”.

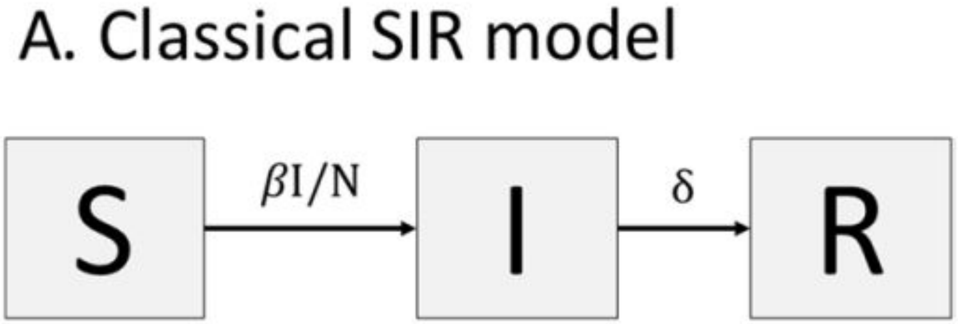
Could a GNN estimate the infected time series?

(https://www.cs.cornell.edu/home/kleinber/networks-book/networks-book-ch21.pdf)

Toy model 2 - preliminary discussion

Given the number of cases for one city for some timestamps,

For each time-step, a SIR dynamic:



Equation

$$\frac{dS}{dt} = -\frac{\beta IS}{N}$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \delta I$$

$$\frac{dR}{dt} = \delta I$$

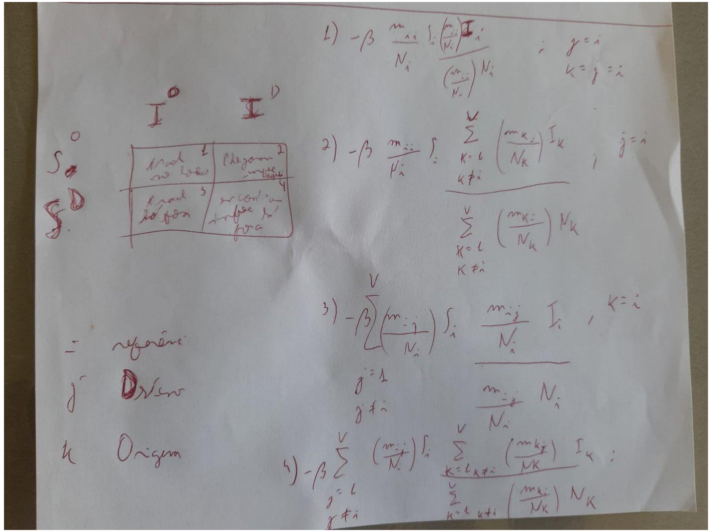
Toy model 2 - Here we go

Given a mobility graph, *[person-to-person network: Papaco's Master]*

Given the number of cases for each node for some timestamps,

For each time-step, for each node, a SIR metapopulation dynamic:

$$\frac{dS}{dt} = -\frac{\beta IS}{N}$$



What is NewCases_i(t)?

Could a GNN estimate it?

[Jessica's Ph.D. - intraurban/Lag,

Duarte's Master - inter/Eul,

Gabriel's monografie]

	I^O	I^D
S^O	1 Ainda no local	2 Chegaram infectados aqui
S^D	3 Ainda lá fora	4 encontram infectado lá fora

i referência

j Destino

k Origem

1) $$-\beta \frac{m_{ii}}{N_i} S_i \frac{\left(\frac{m_{ii}}{N_i}\right) I_i}{\left(\frac{m_{ii}}{N_i}\right) N_i} \quad ; \quad j=i, \; k=j=i$$

2) $$-\beta \frac{m_{ii}}{N_i} S_i \frac{\sum_{k=1,\, k\neq i}^{V} \left(\frac{m_{ki}}{N_k}\right) I_k}{\sum_{k=1,\, k\neq i}^{V} \left(\frac{m_{ki}}{N_k}\right) N_k} \quad , \quad j=i$$

3) $$-\beta \sum_{j=1,\, j\neq i}^{V} \left(\frac{m_{ij}}{N_i}\right) S_i \frac{\frac{m_{ij}}{N_i} I_i}{\frac{m_{ij}}{N_i} N_i} \quad , \quad k=i$$

4) $$-\beta \sum_{j=1,\, j\neq i}^{V} \left(\frac{m_{ij}}{N_i}\right) S_i \frac{\sum_{k=1,\, k\neq i}^{V} \left(\frac{m_{kj}}{N_k}\right) I_k}{\sum_{k=1,\, k\neq i}^{V} \left(\frac{m_{kj}}{N_k}\right) N_k} \quad ;$$

1) $-\beta \dfrac{m_{ii}}{N_i} \dfrac{S_i \left(\dfrac{m_{ii}}{N_i}\right)^{I_i}}{\left(\dfrac{m_{ii}}{N_i}\right)^{N_i}}$; $j = i$

$k = j = i$

2) $$-\beta \frac{m_{ii}}{\mu_i} S_i \frac{\sum\limits_{\substack{k=1 \\ k \neq i}}^{V} \left(\frac{m_{ki}}{N_k} \right) I_k}{\sum\limits_{\substack{k=1 \\ k \neq i}}^{V} \left(\frac{m_{ki}}{N_k} \right) N_k} , \quad j = i$$

$$3)\quad -\beta \sum_{\substack{j=1 \\ j\neq i}}^{V} \left(\frac{m_{ij}}{N_i}\right) S_i \, \frac{\frac{m_{ij}}{N_i} I_i}{\frac{m_{ij}}{N_i} N_i} \; , \quad k=i$$

$$4)\ -\beta \sum_{\substack{j=1 \\ j\neq i}}^{V} \left(\frac{m_{ij}}{N_i}\right) I_i \; \frac{\displaystyle\sum_{k=1,\, k\neq i}^{V} \left(\frac{m_{kj}}{N_k}\right) I_k}{\displaystyle\sum_{k=1,\, k\neq i}^{V} \left(\frac{m_{kj}}{N_k}\right) N_k} ;$$

1) $-\beta \dfrac{m_{ii}}{N_i} S_i \dfrac{\left(\frac{m_{ii}}{N_i}\right) I_i}{\left(\frac{m_{ii}}{N_i}\right) N_i}$; $j = i$, $k = j = i$

2) $-\beta \dfrac{m_{ii}}{N_i} S_i \dfrac{\sum_{k=1, k \neq i}^{V} \left(\frac{m_{ki}}{N_k}\right) I_k}{\sum_{k=1, k \neq i}^{V} \left(\frac{m_{ki}}{N_k}\right) N_k}$, $j = i$

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4) $-\beta \sum_{j=1, j \neq i}^{V} \left(\dfrac{m_{ij}}{N_i}\right) S_i \dfrac{\sum_{k=1, k \neq i}^{V} \left(\frac{m_{kj}}{N_k}\right) I_k}{\sum_{k=1, k \neq i}^{V} \left(\frac{m_{kj}}{N_k}\right) N_k}$;

Roadmap

1. A graph (C3, networkx)

2. An (edge-independent) SIR model in each node:
   a. B1 > B2, G3 < G2

3. Plotting:
   a. Time-series (S_i, I_i)
   b. Time-series-image: colour graph (I_i)

4. Edge-dependence:
   a. 1-2 10, 2-1 8, 2-3 0, 3-2 0, 1-3 5, 3-1 3