$$\frac{dS_i}{dt} = -\beta\left(\frac{M_{ii}}{N_i}\right)S_i \frac{\left(\frac{M_{ii}}{N_i}\right)I_i}{\left(\frac{M_{ii}}{N_i}\right)N_i} - \beta\left(\frac{M_{ii}}{N_i}\right)S_i \frac{\sum_{\substack{k=1\\k\neq i}}^{N}\left(\frac{M_{ki}}{N_k}\right)I_k}{\sum_{\substack{k=1\\k\neq i}}^{N}\left(\frac{M_{ki}}{N_k}\right)N_k} - \beta \sum_{\substack{j=1\\j\neq i}}^{N}\left(\frac{M_{ij}}{N_i}\right)S_i \frac{\left(\frac{M_{ij}}{N_i}\right)I_i}{\left(\frac{M_{ij}}{N_i}\right)N_i}$$

$$- \beta \sum_{\substack{j=1\\j\neq i}}^{N}\left(\frac{M_{ij}}{N_i}\right)S_i \frac{\sum_{\substack{k=1\\k\neq i}}^{N}\left(\frac{M_{kj}}{N_k}\right)I_k}{\sum_{\substack{k=1\\k\neq i}}^{N}\left(\frac{M_{ki}}{N_k}\right)N_k} =$$

$$= -\beta\frac{S_i}{N_i}\left[M_{ii}\frac{I_i}{N_i} + M_{ii}\sum_{\substack{k=1\\k\neq i}}^{N}\frac{I_k}{N_k} + \sum_{\substack{j=1\\j\neq i}}^{N} M_{ij}\frac{I_i}{N_i} + \sum_{\substack{j=1\\j\neq i}}^{N} M_{ij}\sum_{\substack{k=1\\k\neq i}}^{N}\frac{M_{kj}\cdot I_k}{M_{ki}\cdot N_k}\right] =$$

duas possibilidades:

$$(1) = -\beta\frac{S_i}{N_i}\left[\frac{I_i}{N_i}\left(M_{ii} + \sum_{\substack{j=1\\j\neq i}}^{N} M_{ij}\right) + M_{ii}\sum_{\substack{k=1\\k\neq i}}^{N}\frac{I_k}{N_k} + \sum_{\substack{j=1\\j\neq i}}^{N} M_{ij}\sum_{\substack{k=1\\k\neq i}}^{N}\frac{M_{kj}}{M_{ki}}\frac{I_k}{N_k}\right] =$$

$$(2) = -\beta\frac{S_i}{N_i}\left[M_{ii}\left(\frac{I_i}{N_i} + \sum_{\substack{k=1\\k\neq i}}^{N}\frac{I_k}{N_k}\right) + \sum_{\substack{j=1\\j\neq i}}^{N} M_{ij}\left(\frac{I_i}{N_i} + \sum_{\substack{k=1\\k\neq i}}^{N}\frac{M_{kj}}{M_{ki}}\cdot\frac{I_k}{N_k}\right)\right] =$$

considering:

$\frac{I_k}{N_k} = i_k$	$\frac{S_i}{N_i} = s_i$

$$= -\beta s_i\left[M_{ii}\left(i_i + \sum_{\substack{k=1\\k\neq i}}^{N} i_k\right) + \sum_{\substack{j=1\\j\neq i}} M_{ij}\left(i_i + \sum_{\substack{k=1\\k\neq i}}^{N}\frac{M_{kj}}{M_{ki}}i_k\right)\right]$$