High Energy Description of Processes with Multiple Hard Jets

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High Energy Jets (HEJ) is a new framework for approximating the all-order perturbative corrections to multi-jet processes, with a focus on the hard, wide-angle QCD emissions, which underpins the perturbative description of hard jets. In this contribution we review the basic concepts of *HEJ*, and present some new predictions for observables in dijet-production, and for W-boson production in association with at least 3 jets.

1. INTRODUCTION

Since the production cross-section at the LHC for particles charged under QCD generally will be larger than that for colourless particles, many of the discovery channels used in the search for new physics involves the detection of hard, hadronic jets. The large mass hierarchy between any (necessarily heavy, in order to avoid existing exclusion limits) produced new particle and those of the decay products means often many such jets should be produced in the decay of a new state. The finger prints of any such new physics will, however, have to be found amongst a large contribution to the same signal from multi-jet processes within the Standard Model. Therefore, a detailed understanding of the Standard Model processes will assist in the search for new physics. Examples of Standard Model processes acting as background to many searches are e.g. W, Z+jets (especially with 3,4 jets or more).

However, even the nature of some Standard Model processes is best studied in events with jets. For example, the CP-structure of the induced Higgs boson couplings to gluons through a top-loop could be measured by a study of the azimuthal angle between the two jets in events with a Higgs boson in association with dijets[1,2].

In both examples, hard radiative corrections will be sizeable at the LHC, by which we mean that the exclusive $(n + 1)$-jet rate is a significant component of the inclusive n-jet rate. Therefore,

a tree-level description of the n-jet rate will be insufficient for a satisfactory description of the final state (even if dressed with a parton shower). The reason for the increased importance in many situations of hard, perturbative corrections at the LHC over the situation at previously, lower energy colliders is very simple. Two effects act to suppress hard corrections: the increasing powers of the perturbative coupling, and the necessary increase in the light-cone momentum fraction of the partons extracted from the proton beyond that necessary for the final state without the additional hard jet. The suppression from this last kinematic effect is caused by the decrease in the parton density functions (pdf) as the light-cone momentum fraction x is increased. However, for processes with at least two particles in the final state, there is a fine trade-off between the suppression from the pdf and the phase space for additional emission (even when this is hard in transverse momentum), as the rapidity span between the two particles is increased. At previous, lower-energy colliders, this balance was tipped more towards a suppression than will be the case at the LHC. At previous colliders, the "significant" rapidity separation of the two objects which is necessary for the opening of phase space for additional radiation would already bring the light-cone momentum fractions into the region of extremely fast falling pdfs as $x \to 1$, thus effectively vetoing additional emissions. However, the situation is different for the LHC processes discussed

above, since a significant rapidity separation will be directly imposed in the analysis of the *CP*-properties of the Higgs boson couplings, and in the case of *W*-boson production with at least 3 jets, two jets will naturally be produced with a size-able separation in rapidity[3].

The considerations above are just simple kinematical observations, which hold true for any multi-particle process, and any reasonable theoretical description thereof. However, the amount of hard emissions in the span between the two extremal (in rapidity) particles will differ between:-

> **Processes:** according to whether there is a colour octet channel between the two particles (i.e. the possibility of a gluon exchange)[4]. This induces a difference in the radiation pattern between e.g. the weak-boson- and gluon-fusion channel in hjj.

> **Theoretical models:** A *fixed order calculation* like NLO will obviously be able to generate just one hard jet beyond that of the LO process, irrespectively of the length of the rapidity span between the two extremal jets and the connected growth of the phase space for additional emissions.

> A *parton shower* description will generally underestimate the amount of hard radiation (and thus the number of jets) in-between the two extremal jets, although the description can be improved by a few fixed orders through a CKKW-style matching[5,6,7], or to full NLO accuracy[8,9], so far for processes of low multiplicity only.

In Fig. 1 we show the predictions obtained in three different approaches for the average number of hard jets ($p_\perp > 40$ GeV) in events with a Higgs boson and at least two jets at a 10 TeV pp-machine, as a function of the rapidity difference Δy_{ab} between the most forward and most backward hard jet. The bands describe the uncertainty of each prediction as estimated by varying the renormalisation and factorisation scales by a factor of two. The scale choice is identical in the calculations of MCFM[10] and High Energy Jets[12,13,14,15] (described in the next section),
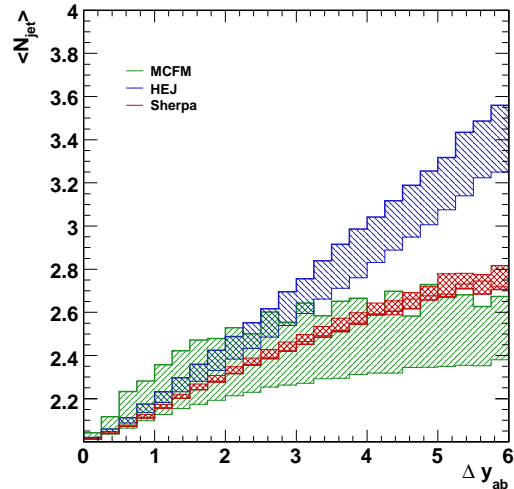

Figure 1. The average number of hard jets ($p_\perp > 40$ GeV) in events with a Higgs boson and at least two jets at a 10 TeV pp-machine, as a function of the rapidity difference Δy_{ab} between the most forward and most backward hard jet. The predictions from three different models are compared: Fixed order NLO by MCFM[10] (green), the SHERPA shower Monte Carlo[11], including CKKW-matching with processes for a Higgs-boson and up to 4 final state partons (red), and finally the all-order framework of *High Energy Jets*[12,13,14,15] (blue). See Ref.[3] for more details.

whereas the SHERPA[11] setup chooses scales according to the CKKW-procedure (with a much smaller range induced by varying the scale by a factor of two). See Ref.[3] for more details on the comparison. All models obviously agree on the increasing importance of hard radiative corrections with increasing rapidity span[1]. At smaller rapidity spans ($\Delta y_{ab} < 2$), where the phase space for many hard emissions is suppressed, the predictions agree between each other. However, as

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[1]We note in passing that a large rapidity span (3-4 units) between two jets is often *required* in the analysis of hjj, also in analysis of the gluon fusion (GF) channel[1].

the rapidity span is increased (into the region of interest of at least 3-4 units), differences start to emerge, since the models which can reach higher multiplicities start becoming sensitive to these. An obvious observation is that with an average number of jets of around 2.5 in the region of interest, the exclusive two and three jet rates in the NLO calculation are equally large. It would seem mandatory to care about the description of further hard radiation, if one is concerned with any observable which depends on the configuration of the final state. Even the "total cross section" can depend crucially on the jet configuration after acceptance cuts have been imposed (see Ref.[3] for a discussion on the estimated effect of jet vetos as obtained in the three models).

A reliable understanding and description of the final state in terms of jets is obviously desirable, in order to e.g. form observables for the extraction of the CP-properties in hjj which are stable against the higher order corrections[2], or to help in further discriminating Standard Model contributions to the search channels for new physics, by isolating the regions in phase space where the amount of hard radiative corrections occur for the SM process (like illustrated above in the example of GF contribution to hjj).

2. HIGH ENERGY JETS (HEJ)

The all-order perturbative framework of *High Energy Jets* (*HEJ*) developed in Ref.[12,13,14,15] is addressing the short-comings in the description of multiple hard, perturbative corrections in both the (low) fixed-order and in the parton shower formulation. The perturbative description obtained with *HEJ* reproduces the correct, all-order, full QCD, limit for both real and virtual corrections to the hard perturbative matrix element for the hard, wide-angle emissions which underpins the perturbative description of the formation of additional jets. The central parts of the formalism were presented in Ref.[14,15] and discussed further in Ref.[16]. In the following, we will first give just a brief overview of the underlying formalism. Since two comparative studies of results obtained with (CKKW-matched) shower, NLO and HEJ for hjj and Wjj were already reported in Ref.[3],

we will here discuss new results obtained from the application of HEJ to the LHC processes of dijets and $W+3$ jets. These will form parts of forthcoming publications.

2.1. Dominance of the t-channel poles

The limit of pure N-jet amplitudes for large invariant mass between each jet of similar transverse momentum is described by the FKL-amplitudes[17,18], which are at the foundation of the BFKL framework[19]. The physical picture arising from the FKL amplitudes is one of effective vertices connected by t-channel propagators. The reduction of the formalism to the two-dimensional BFKL integral equation relies on many kinematical approximations, which are extended to all of phase space. Using an explicit (or so-called iterative) solution to the BFKL equation[20], it is however straightforward to show that despite the logarithmic accuracy, the perturbative expansion of the (B)FKL solution does not give a satisfactory description of the results obtained order by order with the true perturbative series from QCD[13].

High Energy Jets[14,15] inherits the idea of effective vertices connected by t-channel currents in order to reproduce the correct limit of N-jet amplitudes, but goes beyond controlling just the logarithmic accuracy of the FKL formalism. The kinematic building blocks of the FKL formalism depend on transverse momenta only, as a result of the kinematic limits applied in order to separate the amplitude into effective vertices separated by t-channel exchanges[21]. We will discuss how to obtain such separation without resorting to kinematic approximations.

The $2 \rightarrow 2$ scattering $qQ \rightarrow qQ$ obviously proceeds through just a t-channel exchange of the gluon current generated by a quark. A careful analysis[14,15] of the helicity structure in $qg \rightarrow qg$ and $gg \rightarrow gg$-scattering reveals that all the amplitudes, where the helicity of the gluon is unchanged[2], factorise again into two currents, contracted over a t-channel pole. This allows for a definition of the current exchanged in the t-channel also for scatterings involving gluons. The

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[2] All helicity-flip amplitudes are systematically suppressed by a factor of $\hat{s}$.

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emission of additional gluons is performed by gauge-invariant[3], effective vertices. The virtual corrections are approximated with the *Lipatov ansatz* for the *t*-channel gluon propagators (see Ref.[14] for more details). The end result is a formalism which provides a good approximation order-by-order to the full QCD results, while being sufficiently fast to evaluate that all-order results for the amplitudes can be explicitly constructed and integrated over the *n*-body phase spaces (with an upper limit on *n* sufficiently high to guarantee convergence).

2.2. Matching to Fixed Order

In the cases of low jet multiplicity (up to 4), where the exact tree-level amplitudes are known, the formalism is matched to this accuracy by mapping the generated *n*-jet, *m*-parton configuration into a configuration of *n* on-shell final-state partons, for which the tree-level amplitudes can be evaluated. The all-order event weight is then multiplied by the ratio of the full and the approximate tree-level amplitude. The low-multiplicity, tree-level amplitudes are evaluated using MadGraph[22].

3. RESULTS

In the following, we will presents results obtained for processes of pure jets (inclusive dijets) and for W-production in association with at least 3 jets. We will see that the increasing relevance of hard, perturbative corrections with increasing rapidity span (as indicated in Fig. 1) is completely general for all the processes under consideration. The exact rate of the increase depends on the jet cuts and definition, and the pdfs (i.e. whether it is a processes dominated by a gg (as in the case of dijets) or qg (W+jets) initial state), with gg-dominated processes being less dominated by hard, radiative corrections because of the steeply falling gluon pdf. In the case of pure jets, we will discuss how the effect of the evolution of the amount of real radiation with the rapidity span can be measured directly with the data of

the first year of running with the LHC by measuring $d\sigma/d\phi_{fb}$ (where ϕ_{fb} is the azimuthal angle between the forward/backward hard jet) in bins of the rapidity difference between the forward/backward hard jet.

While an increasing rapidity span clearly forces the increasing relevance of hard, perturbative corrections, the importance of resumming such corrections to get a stable, perturbative description of the final state is obviously not limited to the study of increasing rapidity spans. In the case of W+3 jets we will see that the tail of the H_T-distribution also attracts large contributions from hard, perturbative corrections.

3.1. Pure Jets

In this study, we present results for dijet-production at a 7 TeV pp-collider. The jet-algorithm is anti-kt, with an R-parameter of 0.6, and the transverse momentum of the jets are required to be harder than 75GeV, with an absolute rapidity less than 2.5. Earlier analyses of results obtained using the less accurate BFKL formalism[23] already indicated there should be a strong dependence between the average number of jets and the rapidity span between the most forward/backward hard jet. In Fig. 2 (top) we show the prediction obtained using *HEJ* for the correlation between the average number of hard (all above 75GeV in transverse momentum) jets and the rapidity span between the most forward/backward hard jet.

Clearly, the increasing importance of hard, radiative corrections will impact many observables and event shapes. In Fig. 2 (bottom) we show the results for $1/\sigma \ d\sigma/d\phi_{fb}$ (where ϕ_{fb} is the azimuthal angle between the forward/backward hard jet) for three bins of the rapidity difference y_{fb} between the forward/backward jet. For increasing rapidity spans, the increasing amount of hard radiative corrections leads to a distribution which is less peaked at the situation of back-to-back jets.

3.2. W+3 Jets

The formalism of *HEJ* supersedes the less accurate BFKL description of W+jets implemented in Ref.[24]: not only does *HEJ* include matching

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[3]by which we of course mean fully gauge invariant, not just up to sub-asymptotic terms as it is often meant in the BFKL literature.
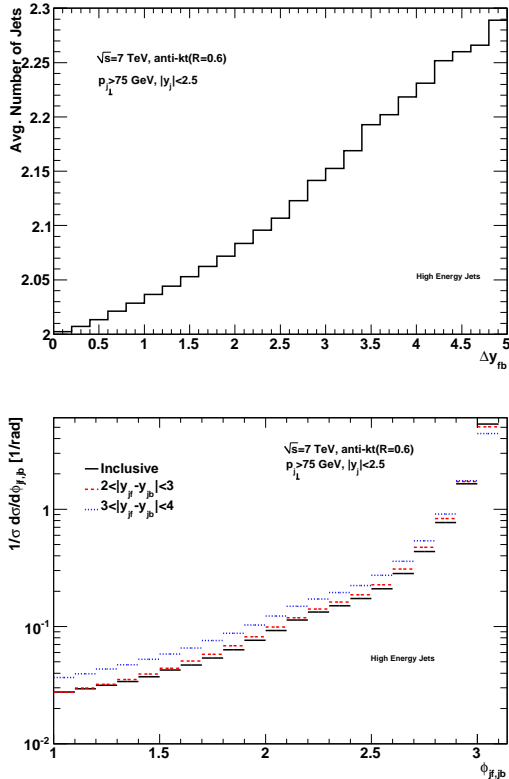
Figure 2. Top: The average number of hard jets ($p_\perp > 75$GeV) in dijet events at the LHC (7 TeV). Bottom: The increasing weight of hard, radiative corrections for increasing rapidity spans between the forward/backward hard jet leads to the distribution on the azimuthal angle between these jets to be less peaked at the back-to-back configuration.

to fixed order results, but the matching corrections are much smaller than those which would arise in a pure BFKL approach.

In this section we report results obtained for the process of W-production in association with at least three jets, using the cuts of Ref.[25] (and mentioned on the plots in Fig. 3). Ultimately, a comparative study between the NLO results and those of *HEJ* is desirable, which would require

also similar choice of renormalisation and factorisation scales etc.

In Fig. 3(top) we report the average number of jets (with transverse momentum above 25GeV and rapidities less than 2.5) vs. the rapidity difference between the most forward/backward hard jet. Again, we see a strong correlation, indicative of the increasing phase space for hard emissions. In fact, for W+3j-production, $\mathrm{d}\sigma/\mathrm{d}y_{fb}$ peaks at rapidities of 1-2 units, as illustrated in Fig. 3(middle). This would increase to even larger rapidities, if the rapidity of each jet was allowed to be larger than the 2.5 units allowed in the initial analyses at the LHC.

Finally, in Fig. 3(bottom) we show the average number of jets vs. the scalar sum of transverse momenta H_T. There is a strong correlation between H_T and the average number of hard jets. The tail is dominated by radiative corrections, so a stable description of the final state in terms of the number of hard jets is clearly necessary, in order to reach a stable description of H_T within the SM, and thus assist in the discrimination between that and any sign of new physics.

4. CONCLUSIONS

We have briefly discussed the new all-order framework of *High Energy Jets*, and illustrated clear similarities between the jet radiation pattern in processes of pure jets, and jet production in association with a W or H-boson. A thorough understanding of the increasing relevance of higher-order hard, perturbative corrections and jet production will clearly be important for LHC analyses, and should assist both the analysis of new SM processes, and the search for new physics.
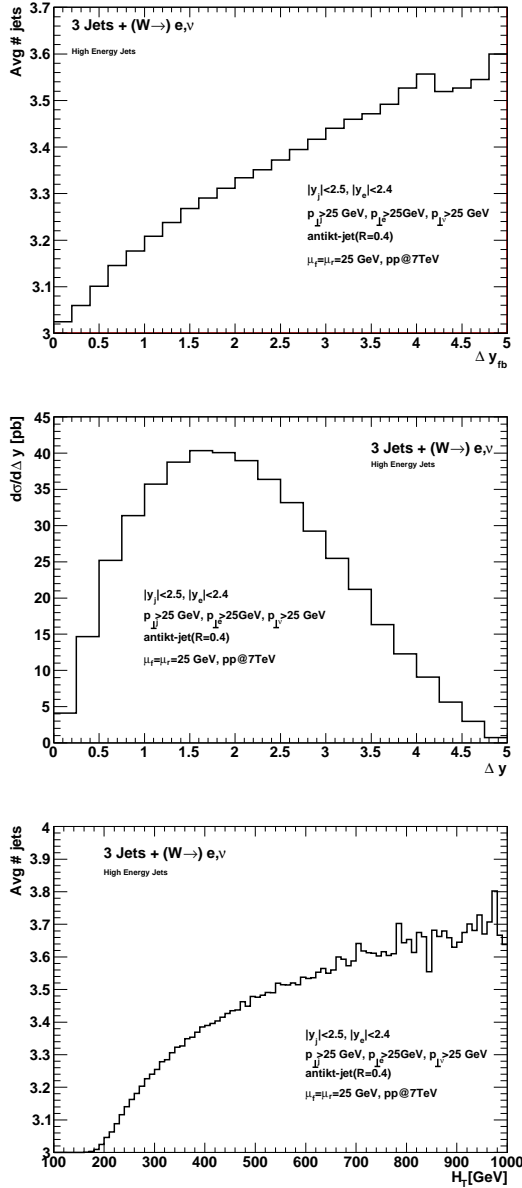
Figure 3. Results obtained with *HEJ* for *W*-production in association with at least three jets.

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