

Introduction to Event Generators

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Topics of the lectures

- ① Lecture 1: *The Monte Carlo Principle*
- ② Lecture 2: *Parton level event generation*
- ③ Lecture 3: *Dressing the Partons*
- ④ Lecture 4: *Modelling beyond Perturbation Theory*

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Menu of lecture 3

- Prelude: Orientation
- Why we need parton showers
- An analogy
- The parton shower as a theory instrument
- Improving the accuracy
- New showers

Prelude: Orientation

Event generator paradigm

Divide event into stages, separated by different scales.

- **Signal/background:**

Exact matrix elements.

- **QCD-Bremsstrahlung:**

Parton showers (also in **initial state**).

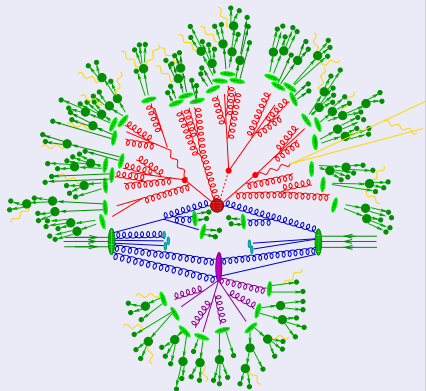
- **Multiple interactions:**

Beyond factorisation: Modelling.

- **Hadronisation:**

Non-perturbative QCD: Modelling.

Sketch of an event



Motivation: Why parton showers?

Common wisdom

- Well-known: Accelerated charges radiate
- QED: Electrons (charged) emit photons
Photons split into electron-positron pairs
- QCD: Quarks (coloured) emit gluons
Gluons split into quark pairs
- Difference: Gluons are coloured (photons are not charged)
Hence: Gluons emit gluons!
- Cascade of emissions: Parton shower

Some more refined reasons

- Experimental definition of jets based on hadrons.
- But: Hadronisation through phenomenological models
(need to be tuned to data).
- Wanted: Universality of hadronisation parameters
(independence of hard process important).
- Link to fragmentation needed: Model softer radiation
(inner jet evolution).
- Similar to PDFs (factorisation) just the other way around
(fragmentation functions at low scale,
parton shower connects high with low scale).

An analogy: Radioactive decays

The form of the solution

- Consider the radioactive decay of an unstable isotope with half-life τ .

(Ignore factors of $\ln 2$.)

- “Survival” probability after time t is given by

$$\mathcal{S}(t) = \mathcal{P}_{\text{nodec}}(t) = \exp[-t/\tau].$$

(Note “unitarity relation”: $\mathcal{P}_{\text{dec}}(t) = 1 - \mathcal{P}_{\text{nodec}}(t)$.)

- Probability for an isotope to decay at time t :

$$\frac{d\mathcal{P}_{\text{dec}}(t)}{dt} = -\frac{d\mathcal{P}_{\text{nodec}}(t)}{dt} = \frac{1}{\tau} \exp(-t/\tau).$$

- Now: Connect half-life with width $\Gamma = 1/\tau$.
- Probability for the isotope to decay at any fixed time t is determined by Γ .

Adding a non-trivial time dependence

- Rewrite Γt in the exponential as $\int_0^t dt' \Gamma$.

(This allows to make life more interesting, see below.)

- Allows to have a time-dependent decay probability $\Gamma(t')$.
- Then decay-probability at a given time t is given by

$$\frac{d\mathcal{P}_{\text{dec}}(t)}{dt} = \Gamma(t) \exp \left[- \int_0^t dt' \Gamma(t') \right] = \Gamma(t) \mathcal{P}_{\text{nodec}}(t).$$

(Unitarity strikes again: $d\mathcal{P}_{\text{dec}}(t)/dt = -d\mathcal{P}_{\text{nodec}}(t)/dt$.)

- Interpretation of l.h.s.:
 - First term is for the actual decay to happen.
 - Second term is to ensure that no decay before t
 \implies Conservation of probabilities.
 The exponential is called the **Sudakov form factor**.

A detour: The Altarelli-Parisi equation

The form of the equation for one parton type q

- AP describes the scaling behaviour of the parton distribution function: (which depends on Bjorken-parameter and scale Q^2)

$$\frac{dq(x, Q^2)}{d \ln Q^2} = \int_x^1 \frac{dy}{y} [\alpha_s(Q^2) P_q(x/y)] q(y, Q^2)$$

- Here the term in square brackets determines the probability that the parton emits another parton at scale Q^2 and Bjorken-parameter y . (after the splitting, $x \rightarrow yx + (1-y)x$.)
- Driving term: Splitting function $P_q(x)$.
Important property: Universal, process independent.

Splitting functions and large logarithms

$e^+e^- \rightarrow \text{jets}$

- Differential cross section:

$$\frac{d\sigma_{ee \rightarrow 3j}}{dx_1 dx_2} = \sigma_{ee \rightarrow 2j} \frac{C_F \alpha_s}{\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Singular for $x_{1,2} \rightarrow 1$.

- Rewrite with opening angle θ_{qg}
and gluon energy fraction $x_3 = 2E_g/E_{\text{c.m.}}$:

$$\frac{d\sigma_{ee \rightarrow 3j}}{d \cos \theta_{qg} dx_3} = \sigma_{ee \rightarrow 2j} \frac{C_F \alpha_s}{\pi} \left[\frac{2}{\sin^2 \theta_{qg}} \frac{1 + (1-x_3)^2}{x_3} - x_3 \right]$$

Singular for $x_3 \rightarrow 0$ (“soft”), $\sin \theta_{qg} \rightarrow 0$ (“collinear”).

Collinear singularities

- Use

$$\frac{2d \cos \theta_{q\bar{g}}}{\sin^2 \theta_{q\bar{g}}} = \frac{d \cos \theta_{qg}}{1 - \cos \theta_{qg}} + \frac{d \cos \theta_{q\bar{g}}}{1 + \cos \theta_{q\bar{g}}} = \frac{d \cos \theta_{qg}}{1 - \cos \theta_{qg}} + \frac{d \cos \theta_{\bar{q}g}}{1 - \cos \theta_{\bar{q}g}} \approx \frac{d\theta_{qg}^2}{\theta_{qg}^2} + \frac{d\theta_{\bar{q}g}^2}{\theta_{\bar{q}g}^2}$$

- Independent evolution of two jets (q and \bar{q}):

$$d\sigma_{ee \rightarrow 3j} \approx \sigma_{ee \rightarrow 2j} \sum_{j \in \{q, \bar{q}\}} \frac{C_F \alpha_s}{2\pi} \frac{d\theta_{jg}^2}{\theta_{jg}^2} P(z) ,$$

where $P(z) = \frac{1+(1-z)^2}{z}$ (DGLAP splitting function)

Expressing the collinear variable

- Same form for any $t \propto \theta^2$:
- Transverse momentum $k_{\perp}^2 \approx z^2(1-z)^2 E^2 \theta^2$
- Invariant mass $q^2 \approx z(1-z) E^2 \theta^2$

$$\frac{d\theta^2}{\theta^2} \approx \frac{dk_{\perp}^2}{k_{\perp}^2} \approx \frac{dq^2}{q^2}$$

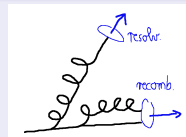
- Parametrisation-independent observation:
(Logarithmically) divergent expression for $t \rightarrow 0$.
- Practical solution: Cut-off t_0 .
 \implies Divergence will manifest itself as $\log t_0$.
- Similar for $P(z)$: Divergence for $z \rightarrow 0$ cured by cut-off.

Parton resolution

- What is a parton?
Collinear pair/soft parton recombine!
- Introduce resolution criterion $k_{\perp} > Q_0$.
- Combine virtual contributions with unresolvable emissions:
Cancels infrared divergences \Rightarrow Finite at $\mathcal{O}(\alpha_s)$

(Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)

- Unitarity: Probabilities add up to one
 $\mathcal{P}(\text{resolved}) + \mathcal{P}(\text{unresolved}) = 1$.



The Sudakov form factor

- Diff. probability for emission between q^2 and $q^2 + dq^2$:

$$d\mathcal{P} = \frac{\alpha_s}{2\pi} \frac{dq^2}{q^2} \int_{z_{\min}}^{z_{\max}} dz P(z) =: dq^2 \Gamma(q^2).$$

- $\Gamma(q^2)$ often dubbed “integrated splitting” function.

(Terms like $1/q^2$ may be pulled out in literature.)

- No-emission prob. $P_{\text{no dec}}$ given by Sudakov form factor Δ .
- From radioactive example: Evolution equation for Δ

$$-\frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2) \frac{d\mathcal{P}}{dq^2} = \Delta(Q^2, q^2) \Gamma(q^2)$$

$$\Rightarrow \Delta(Q^2, q^2) = \exp \left[- \int_{q^2}^{Q^2} dk^2 \Gamma(k^2) \right].$$

The Sudakov form factor (cont'd)

- Remember: Sudakov form factor describes probabilities for (no) branchings.
- It has been derived here by analysing the structure of gluon radiation off a $q\bar{q}$ pair in the (collinear) approximation of large logarithms.

(In the splitting function we only took terms $\propto 1/z$ into account.)

- It can be shown that this structure **factorises to all orders**:

(c.f. proof of the AP equation)

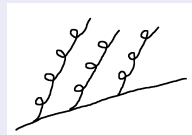
$$d\sigma_{N+1} \approx \frac{dk^2}{k^2} \frac{d\phi}{2\pi} dz \alpha_s P(z) d\sigma_N$$

- This allows the **resummation** of all large logs.

Many emissions

- Iterate emissions (jets)

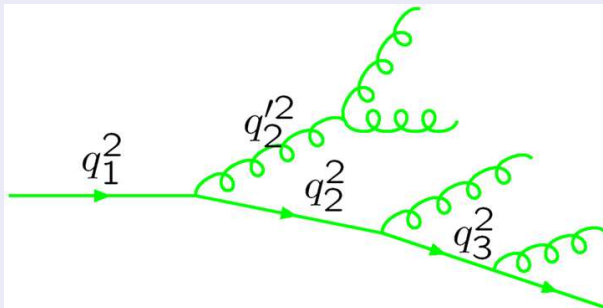
Maximal result for $t_1 > t_2 > \dots t_n$:



$$d\sigma \propto \sigma_0 \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \dots \int_{Q_0^2}^{t_{n-1}} \frac{dt_n}{t_n} \propto \log^n \frac{Q^2}{Q_0^2}$$

- How about Q^2 ? **Process-dependent!**

Ordering the emissions : Radiation pattern

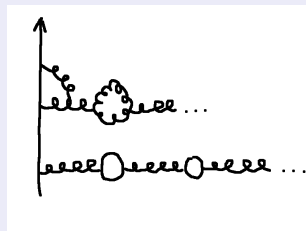


$$q_1^2 > q_2^2 > q_3^2, q_1^2 > q_2'^2$$

Improvement: Inclusion of quantum effects

Running coupling

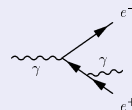
- Effect of summing up higher orders (loops): $\alpha_s \rightarrow \alpha_s(k_\perp^2)$



- Much faster parton proliferation, especially for small k_\perp^2 .
- Must avoid Landau pole: $k_\perp^2 > Q_0^2 \gg \Lambda_{\text{QCD}}^2$
 $\implies Q_0^2 = \text{physical parameter.}$

Soft logarithms : Angular ordering

- Soft limit for single emission also universal
- Problem: Soft gluons come from all over (not collinear!)
Quantum interference? Still independent evolution?
- Answer: Not quite independent.
 - Assume photon into e^+e^- at θ_{ee} and photon off electron at θ
 - Energy imbalance at vertex: $k_\perp^\gamma \sim zp\theta$, hence $\Delta E \sim k_\perp^2/zp \sim zp\theta^2$.
 - Time for photon emission: $\Delta t \sim 1/\Delta E$.
 - ee-separation: $\Delta b \sim \theta_e e \Delta t > \Lambda/\theta \sim 1/(zp\theta)$
 - Thus: $\theta_{ee}/(zp\theta^2) > 1/(zp\theta) \implies \theta_{ee} > \theta$
- Thus: Angular ordering takes care of soft limit.



Soft logarithms : Angular ordering

G. Marchesini and B.R. Webber, Nucl. Phys. B 238 (1984) 1.

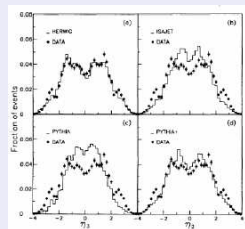
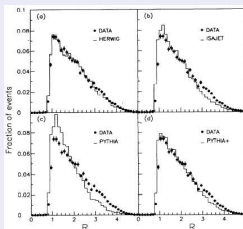
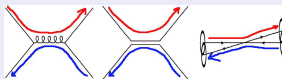


Gluons at large angle from combined colour charge!

Soft logarithms : Angular ordering

Experimental manifestation:

ΔR of 2nd & 3rd jet in multi-jet events in pp-collisions



Aside: Using the Sudakov form factor analytically

Resummed jet rates in $e^+e^- \rightarrow \text{hadrons}$

S.Catani *et al.* Phys. Lett. **B269** (1991) 432

- Use Durham jet measure (k_\perp -type):

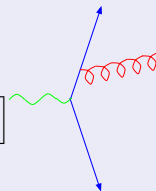
$$k_{\perp,ij}^2 = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij}) > Q_{\text{jet}}^2.$$

- Remember prob. interpretation of Sudakov form factor.
- Then:

$$\mathcal{R}_2(Q_{\text{jet}}) = [\Delta_q(E_{\text{c.m.}}, Q_{\text{jet}})]^2$$

$$\mathcal{R}_3(Q_{\text{jet}}) = 2\Delta_q(E_{\text{c.m.}}, Q_{\text{jet}})$$

$$\cdot \int dq \left[\Gamma_q(q) \frac{\Delta_q(E_{\text{c.m.}}, Q_{\text{jet}})}{\Delta_q(q, Q_{\text{jet}})} \Delta_q(q, Q_{\text{jet}}) \Delta_g(q, Q_{\text{jet}}) \right]$$



Aside: Dipole shower(s)

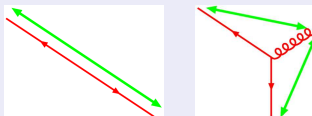
First implemented in Ariadne ([L.Lonnblad, Comput. Phys. Commun. 71, 15 \(1992\)](#)).

Upshot

- Essentially the same as parton shower (benefit: particles always on-shell)

$$d\sigma = \sigma_0 \frac{C_F \alpha_s(k_\perp^2)}{2\pi} \frac{dk_\perp^2}{k_\perp^2} dy.$$

- Always colour-connected partners (**recoil of emission**)
 \Rightarrow emission: 1 dipole \rightarrow 2 dipoles.



Features of dipole showers

- Quantum coherence on similar grounds for angular and k_T -ordering, typical ordering in dipole showers by k_\perp .
- Many new shower formulations in past few years, many (nearly all) based on dipoles in one way or the other.
- Seemingly closer link to NLO calculations: Use subtraction kernels like antennae or Catani-Seymour kernels.
- Typically: First emission fully accounted for.

Survey of existing showering tools

Tools	evolution	AO/Coherence
Ariadne	k_{\perp} -ordered	by construction
Herwig	angular ordering	by construction
Herwig++	improved angular ordering	by construction
Pythia	old: virtuality ordered new: k_{\perp} -ordered	by hand by construction
Sherpa	virtuality ordered (like old Pythia) new: k_{\perp} -ordering	by hand by construction
Vincia	k_{\perp} -ordered	by construction

Summary of lecture 3

- Parton showers as simulation tools.
- Discussed theoretical background: Universal approximation to full matrix elements in the collinear limit.
- Highlighted some systematic improvements.
- Hinted at close relation to resummation.