ON THE POMERANCHUK SINGULARITY IN ASYMPTOTICALLY FREE THEORIES

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We calculate the high-energy amplitude in the spontaneously broken Yang–Mills model within the leading logarithmic approximation, and find out that the vector meson is reggeised whereas the Pomeranchon is a fixed branch point located at the right hand from $j = 1$. The asymptotic freedom alters the situation in this model significally.

The hypothesis of the Pomeranchuk pole gives a possibility to build the theory of high energy hadron collisions [1]. Its verification in quantum field models is of great importance. The models based on the gauge vector fields allow to study the problem in perturbation theory. In the simplest gauge model-quantum electrodynamics (QED) the leading j-plane singularity turns out to be a fixed branch point at $j = 1 + \frac{11}{32}\pi\alpha^2$ if one takes into account only main logarithmic terms [2]. The violation of the Froissart bound here is a result of the fact that the photon is not reggeized [3]. More realistic models for strong interactions will probably be based on the Yang–Mills theory with the Higgs phenomenon [4]. Indeed, in some of these models the necessary criteria for vector meson reggeization are fulfulled [5] and in contrast to the case of QED [6] there is an asymptotic freedom providing the approximate Bjorken scaling [7].

In this letter we discuss the asymptotic behaviour of scattering amplitudes at large energies $\sqrt{s}$ in nonabelian gauge theories. The main results do not depend essentially on group properties of the theory, so we consider further the simplest model based on the isotriplet of vector fields with mass M resulting from nonvanishing vacuum expectation of the complex isospinor field [8]. In this model a scalar field with the isospin $T=0$ appears after the breakdown. Suppose the following conditions are satisfied:

$$s \gg t \sim m^2, \qquad g^2 \ln(s/m^2) \sim 1, \qquad g^2 \ll 1, \tag{1}$$

where g is the coupling constant of the theory.

We use a dispersion method to calculate asymptotic contributions of Feynman diagrams. It was applied earlier by one of the authors for finding the scattering amplitudes in the region (1) up to the sixth order of perturbation theory [9]. This result was generalized to other models [10].

We have calculated the amplitudes in the next order of perturbation theory. The final result is very simple, so it can be easily generalized to the arbitrary order. We present it in the generalized form below (see (4)).

It is necessary to know inelastic amplitudes $2 \to 2+n$ for finding the elastic scattering amplitude by the dispersion method. Main contribution to the s-channel imaginary part of the elastic amplitude arises from the multiregge kinematics: $s_i \gg m^2$, $t_i \sim m^2$. In this region our calculations give the following expressions for the inelastic amplitudes with vector initial particles in the main logarithmic approximation:

$$A_{2\to 2+n} = s\,\Gamma^{C_1}_{\mathrm{A}\to\mathrm{D}_0}\,\frac{(s_1/m^2)^{\alpha(t_1)-1}}{t_1 - m^2}\,\gamma^{\mathrm{D}_1}_{c_2 c_1}\,\frac{(s_2/m^2)^{\alpha(t_2)-1}}{t_2 - m^2}\,\gamma^{\mathrm{D}_2}_{c_3 c_2}\cdots\frac{(s_{n+1}/m^2)^{\alpha(t_{n+1})-1}}{t_{n+1} - m^2}\,\Gamma^{C_{n+1}}_{\mathrm{B}\to\mathrm{D}_{n+1}}, \tag{2}$$

where $s_i = (P_{\mathrm{D}_i} + P_{\mathrm{D}_{i-1}})^2$ is the squared sum of the energies of particles D_i and D_{i-1} in their c.m. system, $C = 1,2,3$ denotes isotopic states of the virtual vector meson i with the mass $\sqrt{t_i}$,

$$\alpha(t) = 1 + \frac{g^2}{(2\pi)^3}(t - m^2)\int\frac{\mathrm{d}^2k}{(k^2 - m^2)((q-k)^2 - m^2)}; \qquad t = q^2, \tag{2a}$$

is its Regge trajectory passing through the point $j=1$ at $t=m^2$ which is a consequence of the vector meson reggeization.

The vertex functions Γ and γ are equal

$$\Gamma^{C_1}_{A\to D_0} = g\frac{\sqrt{2}}{2}\delta_{A,C_1}\delta_{\lambda_A,3}\,; \qquad \gamma^{D_1}_{C_2C_3} = g\delta_{C_2C_1}\cdot m\,, \tag{3a}$$

for scalar particle production and

$$\Gamma^{C_1}_{A\to D_0} = \mathrm{i}\epsilon_{C_1D_0A}\sqrt{2}g\,a_{\lambda_A}\delta_{\lambda_A,\lambda_{D_0}}\,; \qquad a_{\lambda_A} = \begin{cases} 1, & \lambda_A = 1,2 \\ 1/2, & \lambda_A = 3 \end{cases}\,,$$

$$\gamma^{D_1}_{C_2C_1} = -\mathrm{i}\epsilon_{D_1C_2C_1}g\left[-(q_2+q_1)^{\perp}_{\mu} + P_{A_\mu}\left(\frac{P_{D_1}P_B}{P_AP_B} - \frac{(m^2-t_1)}{P_AP_{D_1}}\right) - P_{B_\mu}\left(\frac{P_{D_1}P_A}{P_BP_A} - \frac{(m^2-t_2)}{P_{D_1}P_B}\right)\right]e^{\mu}_{\lambda_{D_1}}\,, \tag{3b}$$

in the case of vector meson production. Here $D = 1,2,3$ is its isotopic state and $\lambda_A = 1,2,3$ is its s-channel polarization ($\lambda_A = 3$ correspond to a longitudinally polarized vector particle).

Using the expression (2) for inelastic amplitudes we can calculate with the help of unitarity conditions the contributions to the vector particles scattering elastic amplitude from t-channel states with all possible isospins $T = 0,1,2$ and signature:

$$A(s,t) = s\sum_T\int\frac{\mathrm{d}\omega}{\mathrm{i}}\left(\frac{s}{m^2}\right)^{\omega}\frac{[(-1)^T - \exp(-\mathrm{i}\pi\omega)]}{\sin\pi\omega}f^T_\omega(q^2)\gamma^{A'B'}_{AB}(T)\,, \tag{4}$$

where

$$f^T_\omega(q^2) = f^T_\omega(k,q-k)|_{k^2=(q-k)^2=m^2}\,;$$

$$\gamma^{A'B'}_{AB}(T) = \left\{\delta_{\lambda_A,\lambda_{A'}}[a^2_{\lambda_A}(2-\tfrac{1}{2}T(T+1)) + \tfrac{1}{4}\delta_{\lambda_A,3}]\right\}\left\{\begin{matrix}\text{same } A\to B\\ \text{with } A'\to B'\end{matrix}\right\}C^{Tm}_{A'A}C^{Tm}_{B'B}\,, \tag{4a}$$

$C^{Tm}_{A'A}$ are the Clebsch–Gordon coefficients and t-channel partial waves $f^T_\omega(k,q-k)$ satisfy the following equation:

$$(\omega - \alpha(k^2) - \alpha((q-k)^2) + 2)f^T_\omega(k,q-k) = \frac{g^2\omega}{A_T(q^2)} + \frac{g^2}{8\pi^3}\int\frac{\mathrm{d}^2k'f^T_\omega(k',q-k')}{(k'^2-m^2)((q-k')^2-m^2)}$$

$$\times\{A_T(q^2) - (2-\tfrac{1}{2}T(T+1))[(k^2-m^2)((q-k')^2-m^2) + (k'^2-m^2)((q-k)^2-m^2)][(k-k')^2-m^2]^{-1}\}\,,$$

$$A_T(q^2) = \frac{m^2}{2} + (2-\tfrac{1}{2}T(T+1))(q^2-\tfrac{3}{2}m^2)\,. \tag{5}$$

The solution of this equation for $T=1$

$$f^{T=1}_\omega(k,q-k) = \frac{\omega g^2}{\omega+1-\alpha(q^2)}\frac{1}{q^2-m^2}\,, \tag{6}$$

shows that the assumption is selfconsistent: we have a bootstrap scheme in which a multiregge equation gives in the j-plane only the same Regge pole.

For the cases $T=0$ and $T=2$ equation (5) leads to the j-plane cut singularities resulting from two reggeized vector meson exchange. In the vacuum channel $T=0$ the equation can be solved exactly for the region $k^2 \gg m^2$. The leading j-plane singularity turnes out to be a square root branchpoint at $j = 1 + (g^2/\pi^2)2\ln 2$. (In the general case of the gauge group SU(N), the branchpoint is located at $j = 1 + (g^2/\pi^2)N\ln 2$).

Thus we obtain that in the main logarithmic approximation in nonabelian gauge theories the total cross section violates the Froissart bound [11]. The mechanism of this violation differs from the case of QED [2]. In the nonabelian theory the cross section for production of a certain number of particles decreases rapidly with energy as a result of the multiregge form of inelastic amplitudes (2) and only due to increasing number of opening channels the total cross section increases as a power of s. The reason of the violation lies in the fact that the s-channel unitarity is not fulfilled in the main logarithmic approximation. (In equation (2) the contribution of the vacuum t-channel state should be taken into account at $g^2 \ln s/m^2 \gg 1$). We can satisfy the elastic unitarity conditions by s-channel eikonal iteration of the expression (4) [12] but we would like to consider another interesting possibility. The perturbation series for the amplitude $f_\omega^{T=0}(q^2)$ have a form $f_\omega^{T=0}(q^2) = g^2 \Sigma_n (g^2/\omega)^n \varphi_n(q^2)$ and the asymptotic behaviour of φ_n at $n \to \infty$ determines the leading singularity of $f_\omega(q^2)$. We get for $\varphi_{2n}(q^2)$

$$\varphi_{2n}(q^2) = \int \frac{\mathrm{d}^2 k}{(k^2 - m^2)((q-k)^2 - m^2)} |\varphi_n(k, q-k)|^2 ,$$

and reccurence relation for $\varphi_n(k, q-k)$ can be easily obtained from eq. (5). It turns out that the behaviour of $\varphi_{2n}(q^2)$ for $n \to \infty$ is determined by the region of large k^2 as a result of the fact that $\varphi_n(k, q-k)$ contains the logarithmically large terms $\sim(\ln k^2/m^2)^n$. Moreover, the recurrence relation for $\varphi_n(k, q-k)$ at large k^2 can be written in the form of a diffusion equation in which n is a time and $\ln k^2/m^2$ is a coordinate* so at large n we have $\ln k^2/m^2 \sim \sqrt{n}$. The above mentioned possibility consists in the observation that for large k^2 the coupling constant g in eq. (5) should be replaced by the effective charge $g(k^2)$ which vanishes when $k^2 \to \infty$. As a result of this replacement only Regge poles may appear to the right from the point $j = 1$. This question will be discussed elsewhere.

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