



From Predictive to Prescriptive

Come la Data Science diventa Decision Making

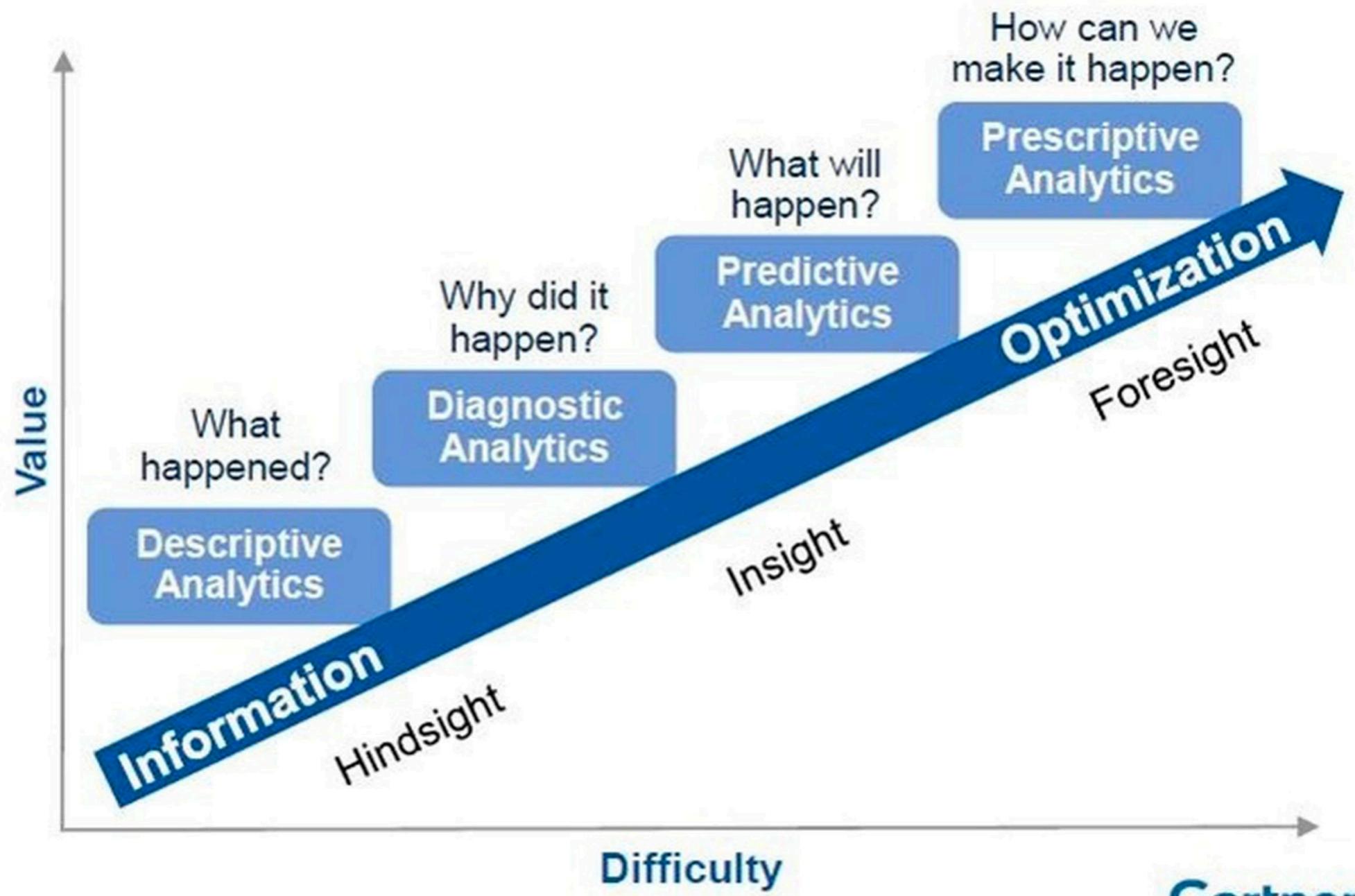
Gabriele Gabrielli

22 febbraio 2024

What is Prescriptive analytics?

A Business Point of View

*Gartner's
Analytics
Continuum*

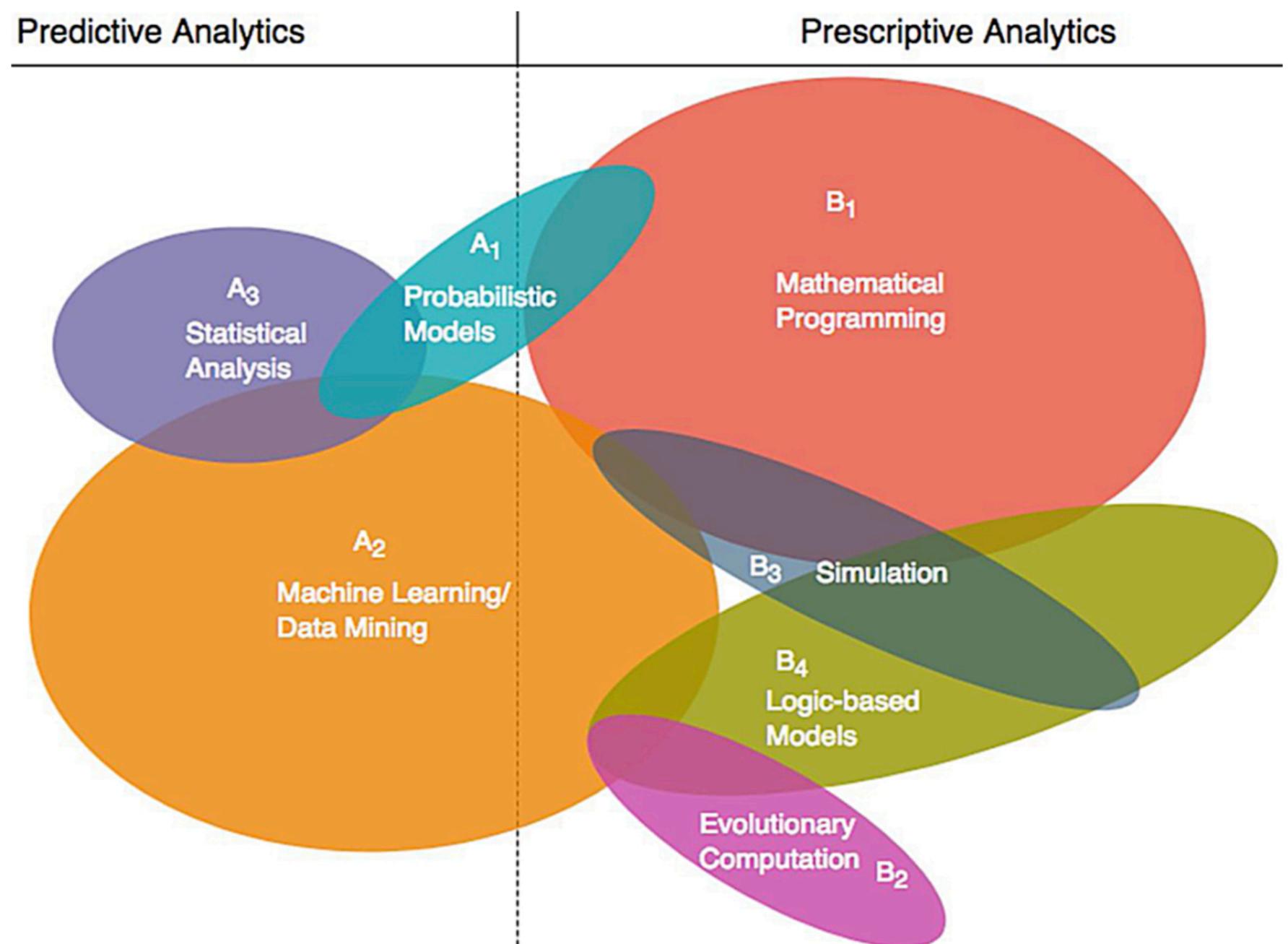


Gartner.

What is Prescriptive analytics?

A Technical Point of View

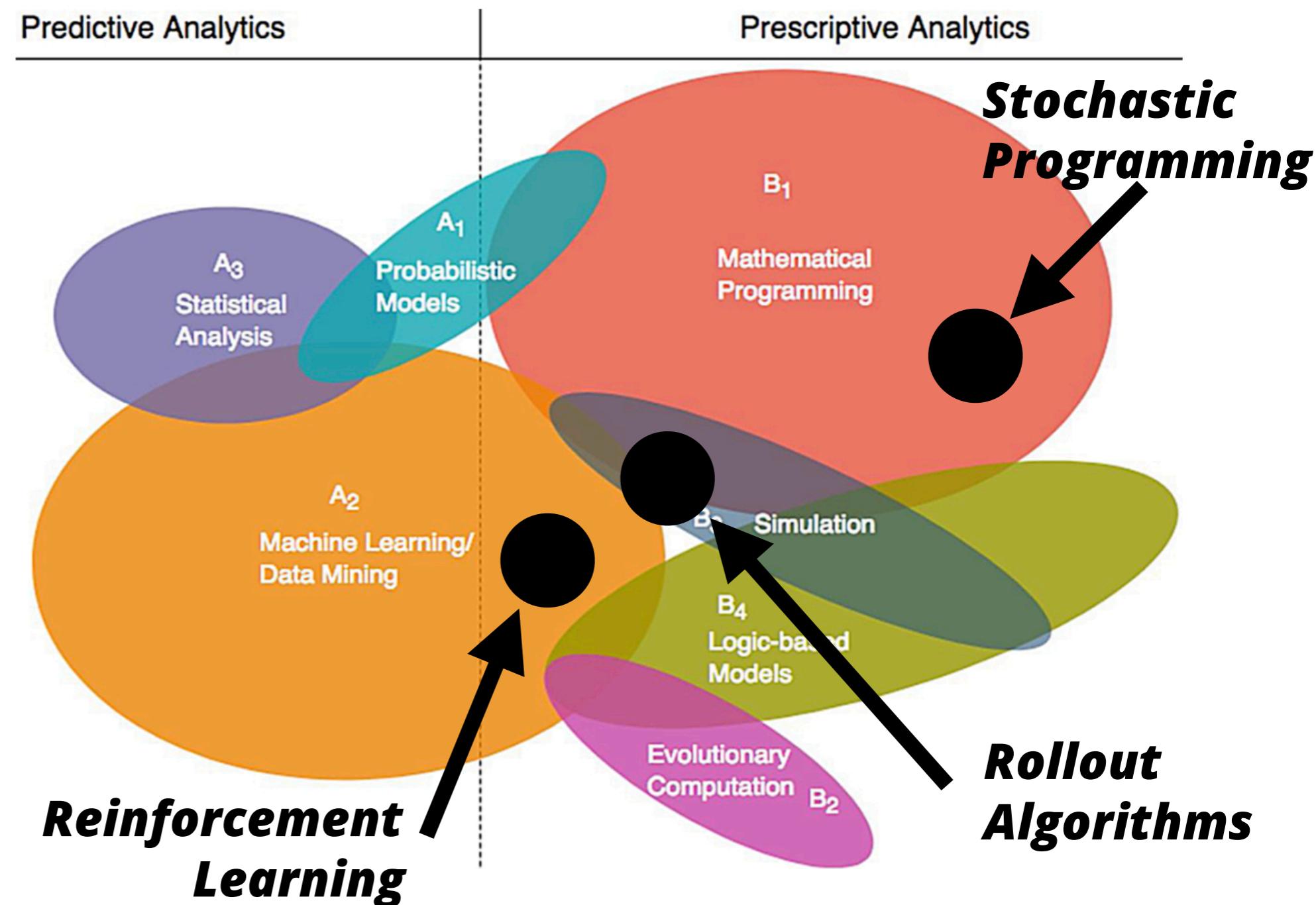
*Literature
Review and
Research
Challenges*



What is Prescriptive analytics?

A Technical Point of View

*Literature
Review and
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Challenges*



Making Decisions from Data

Generic overview



Making Decisions from Data

Generic overview



- Data have noise

Making Decisions from Data

Generic overview



- ▶ Data have noise
- ▶ Must assume that the future will behave like the past

Making Decisions from Data

Generic overview

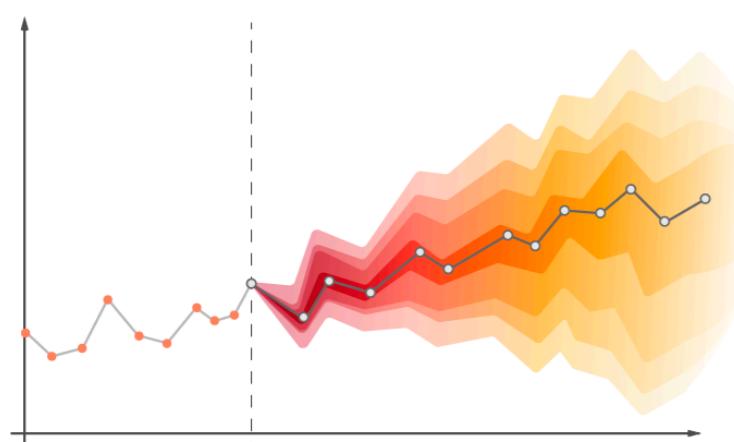


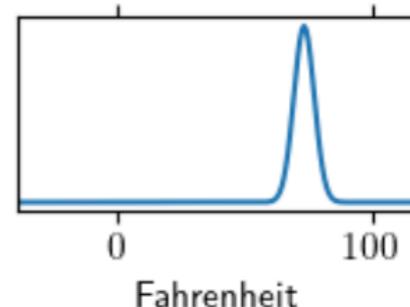
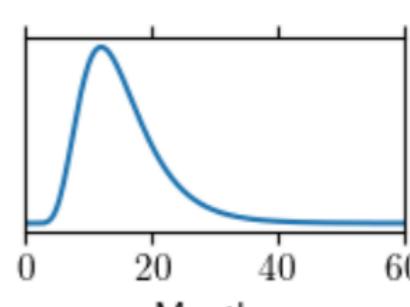
- Data have noise
- Must assume that the future will behave like the past
- Are we sure about the predictions?

Uncertainty-aware predictions

Probabilistic prediction and forecasting (distribution based)

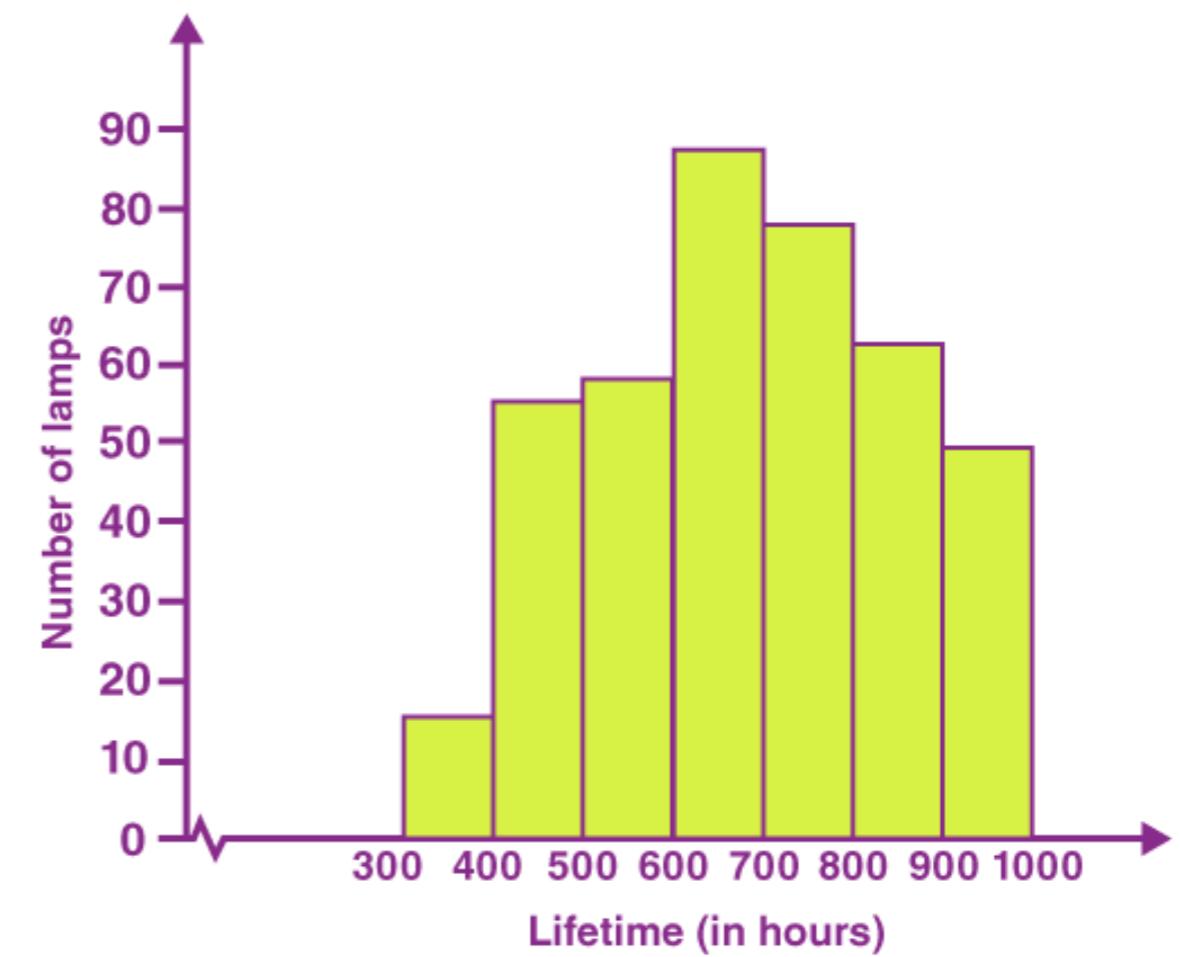
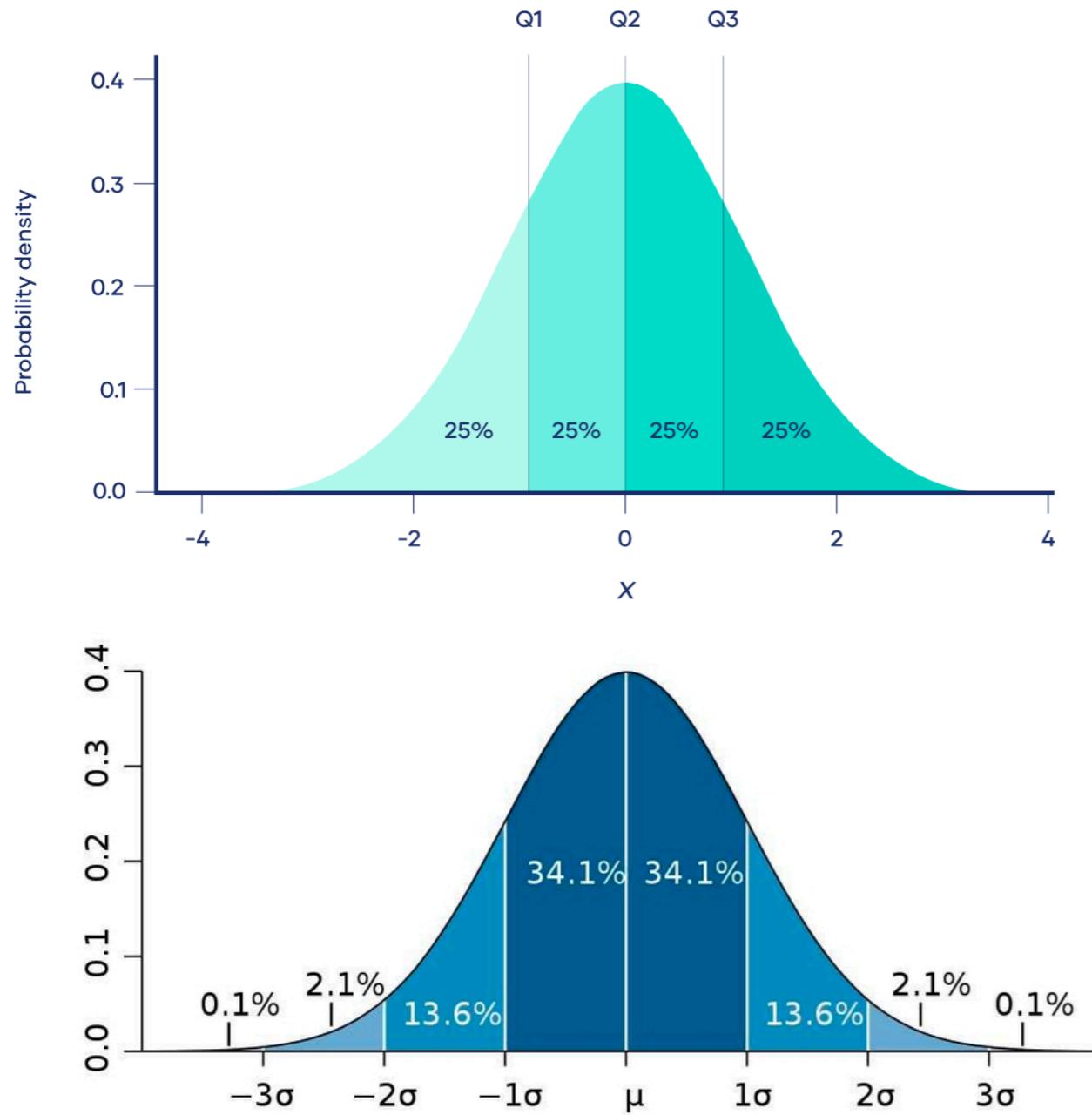
NGBoost



Question	Point Prediction (No uncertainty estimate)	Probabilistic Prediction (Uncertainty is implicit)
What will be the temperature at noon tomorrow?	73.4 Fahrenheit	
How long will this patient live?	11.3 months	

Uncertainty-aware predictions

Quantile Regression and Histogram models (non-parametric)



Making Decisions from Data

Generic overview



Making Decisions from Data

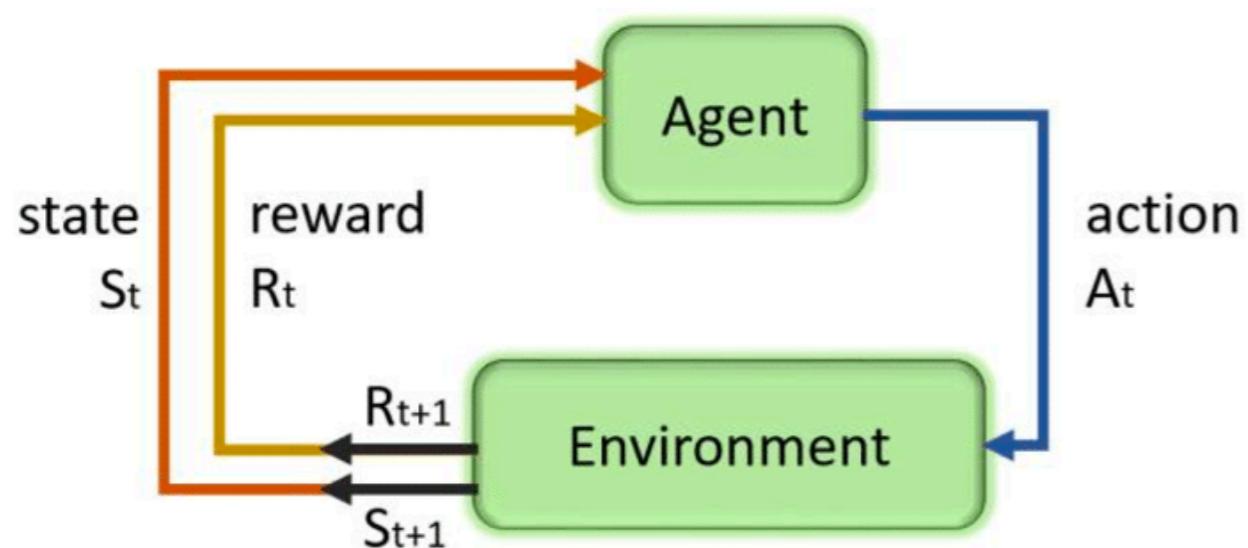
Generic overview



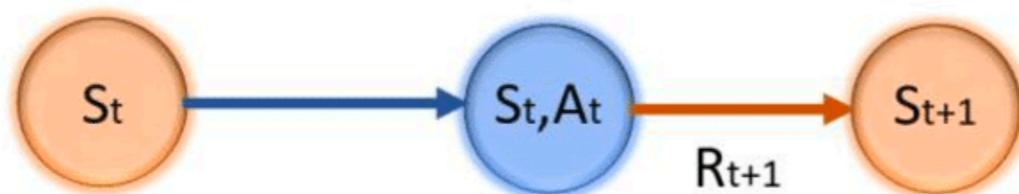
Hands-on:
Sequential Decisions

Modelling Sequential Decisions

Markov Decision Models



Looking for a *Policy*:
given a State
returns an Action



Modelling Sequential Decisions

Reinforcement Learning

PROS

- Sometimes it works

CONS

- Very strict Markov's assumption
- Training requires a huge amount of computation in practice
- Rely only on the reward: cannot embed external information

There are less fashionable alternatives



Modelling Sequential Decisions

*The Universal Framework for Sequential Decisions
by W. Powell, Princeton University*



The Universal Framework for Sequential Decisions

Warren B. Powell, Princeton University

$$\max_{\pi} \mathbb{E} \left\{ \sum_{t=0}^T C(S_t, X^\pi(S_t)) | S_0 \right\}$$

where $S_{t+1} = S^M(S_t, X^\pi(S_t), W_{t+1})$

and given $(S_0, W_1, W_2, \dots, W_t, \dots)$

Policy

The four classes of policies (PFAs, CFAs, VFAs and DLAs) are *universal*.

Any sequential decision problem will use

one of these four classes (or a hybrid), including whatever you might be doing now.

Policy Search

Lookahead Approximations

Policy Function Approximation (PFA)

$$X^{PFA}(S_t | \theta) = \begin{cases} \text{If this then do} \\ \sum_{f \in F} \theta_f \phi_f(S_t) \\ \text{Neural network} \end{cases}$$

Cost Function Approximation (CFA)

$$X^{CFA}(S_t | \theta) = \begin{cases} \operatorname{argmax}_x c_t x_t + \sum_f \theta_f \phi_f(S_t) \\ \operatorname{argmax}_x (\mu_{tx} + \theta^{IE} \bar{\sigma}_{tx}) \end{cases}$$

Value Function Approximation (VFA)

$$\begin{aligned} X^{VFA}(S_t | \theta) &= \operatorname{argmax}_x (C(S_t, x) + \mathbb{E}[V_{t+1}(S_{t+1}) | S_t, x_t]) \\ &= \operatorname{argmax}_x (C(S_t, x) + \tilde{V}_t^x(S_t^x)) \\ &= \operatorname{argmax} Q(S_t, x) \end{aligned}$$

Direct Lookahead (DLA)

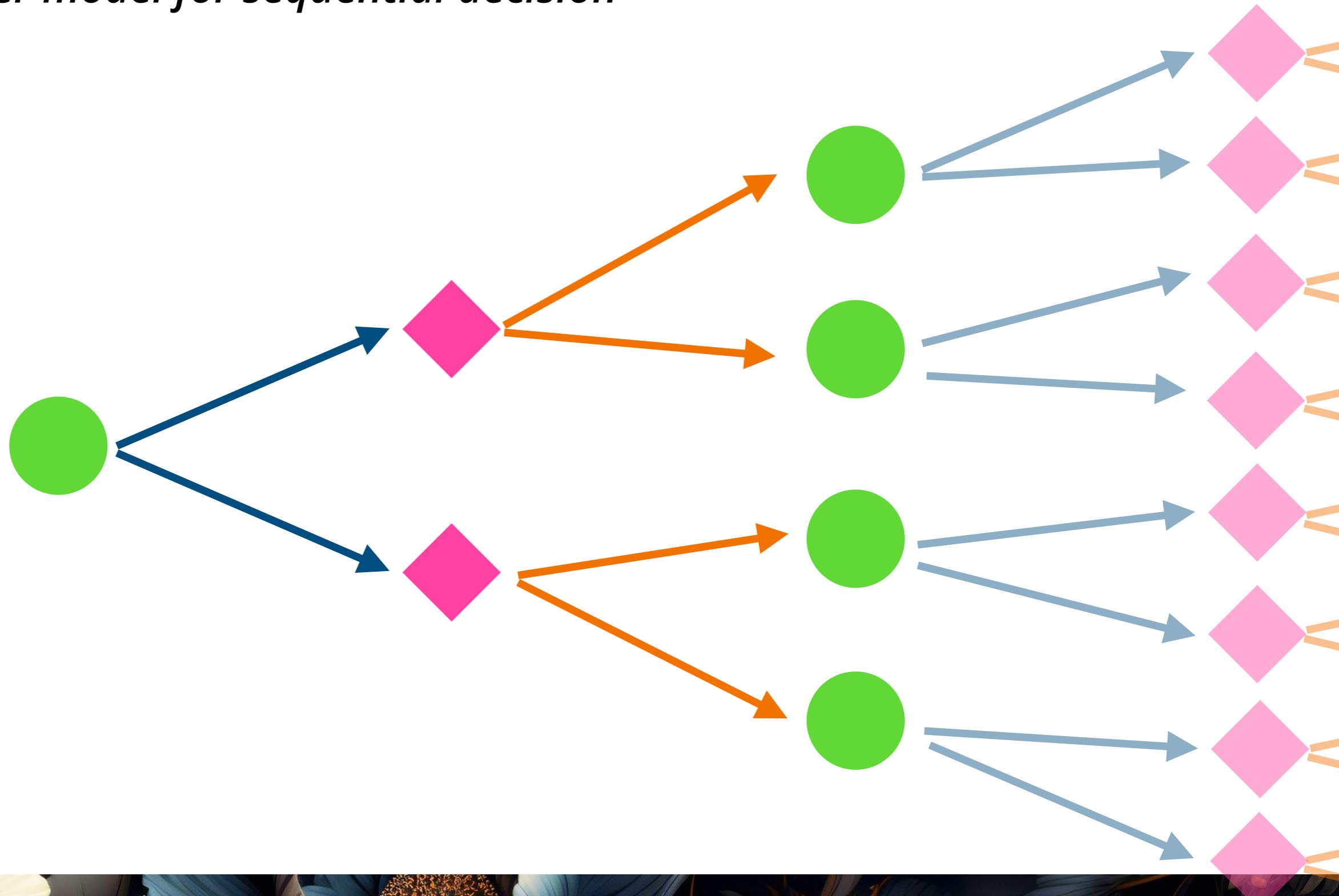
$$X^{DLA}(S_t | \theta) = \operatorname{argmax}_x \left(c_t x_t + \sum_{t'=t+1}^{t+H} \tilde{c}_{tt'} \tilde{x}_{tt'} \right)$$

The optimal policy (if we could solve it) is given by

$$X^*(S_t) = \operatorname{argmax}_x \left(C(S_t, x) + \mathbb{E} \left\{ \max_{\pi} \mathbb{E} \left\{ \sum_{t'=t+1}^{t+H} C(S_{t'}, X^\pi(S_{t'})) | S_{t+1} \right\} | S_t, x_t \right\} \right)$$

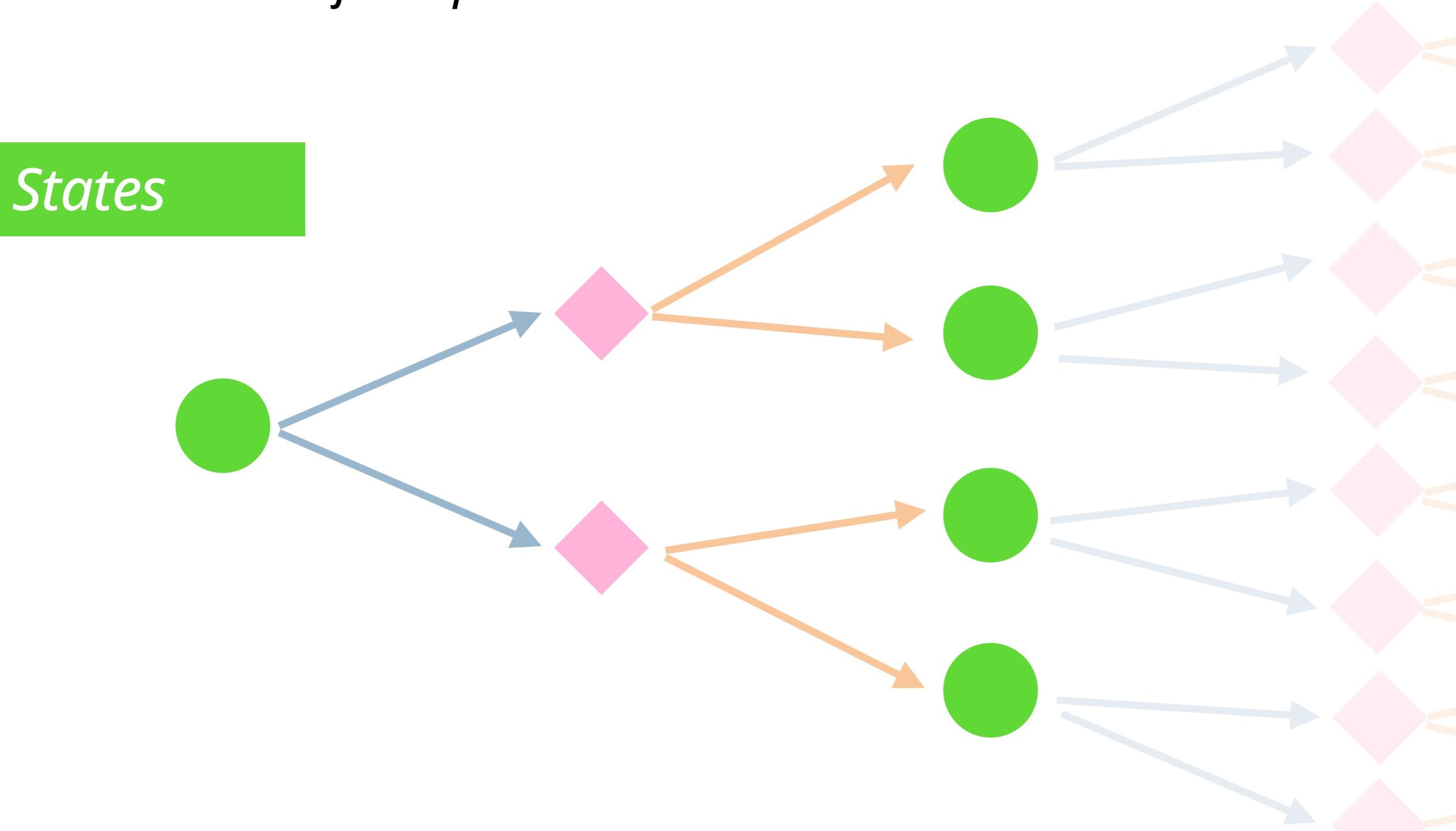
The Universal Framework: the model

A richer model for sequential decision



The Universal Framework: the model

A richer model for sequential decision

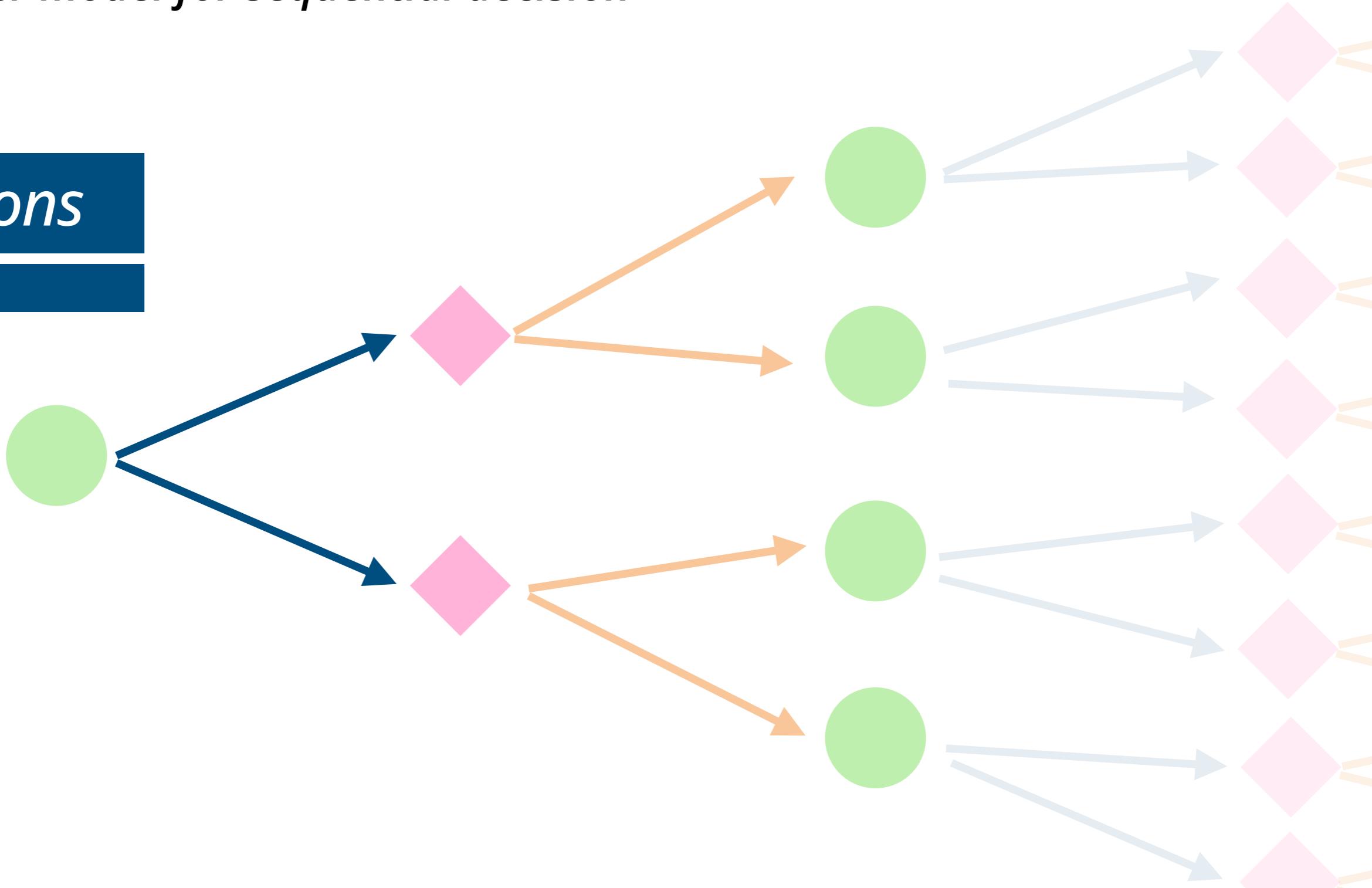


The Universal Framework: the model

A richer model for sequential decision

Decisions

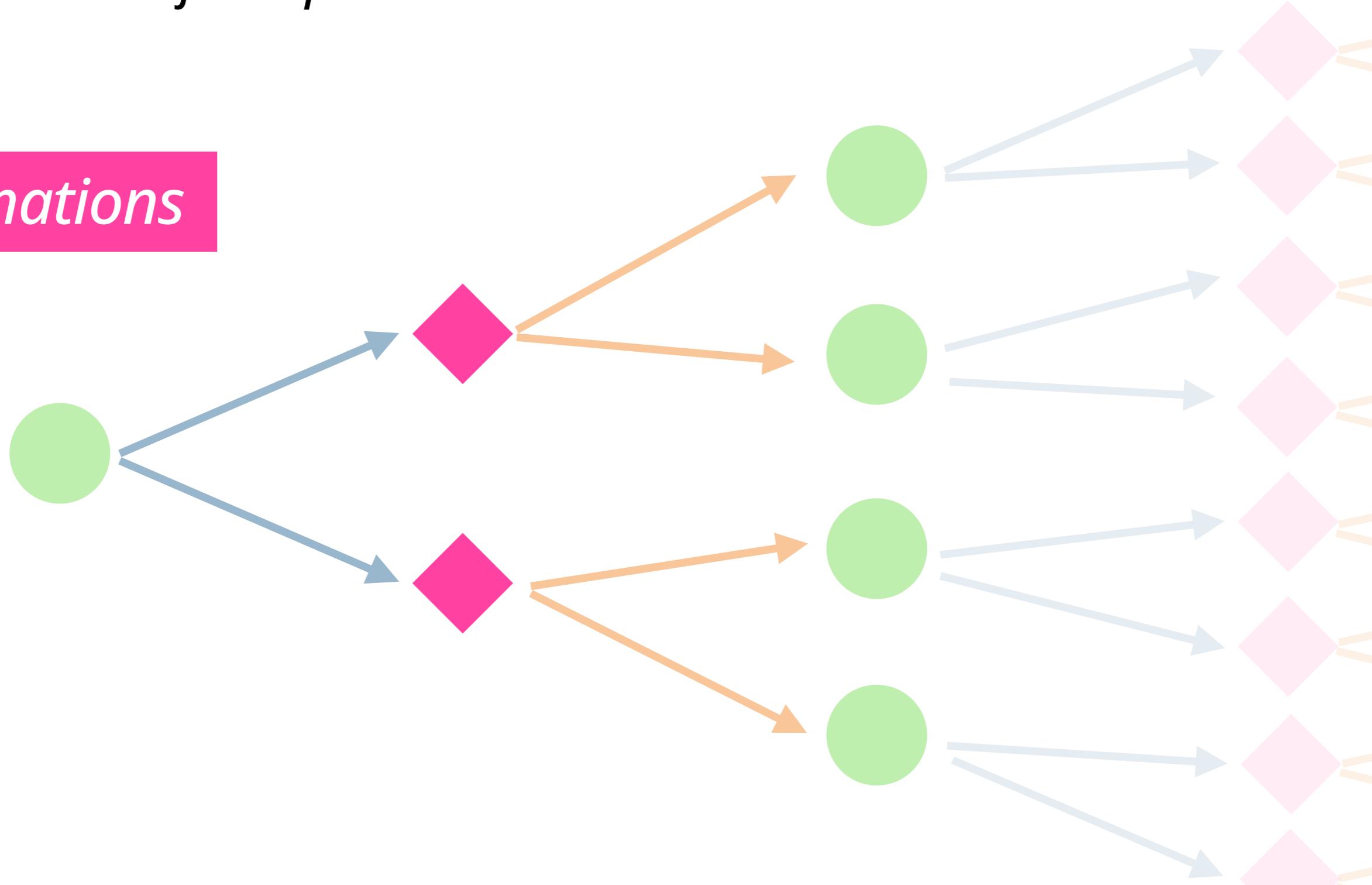
First Signal



The Universal Framework: the model

A richer model for sequential decision

Informations

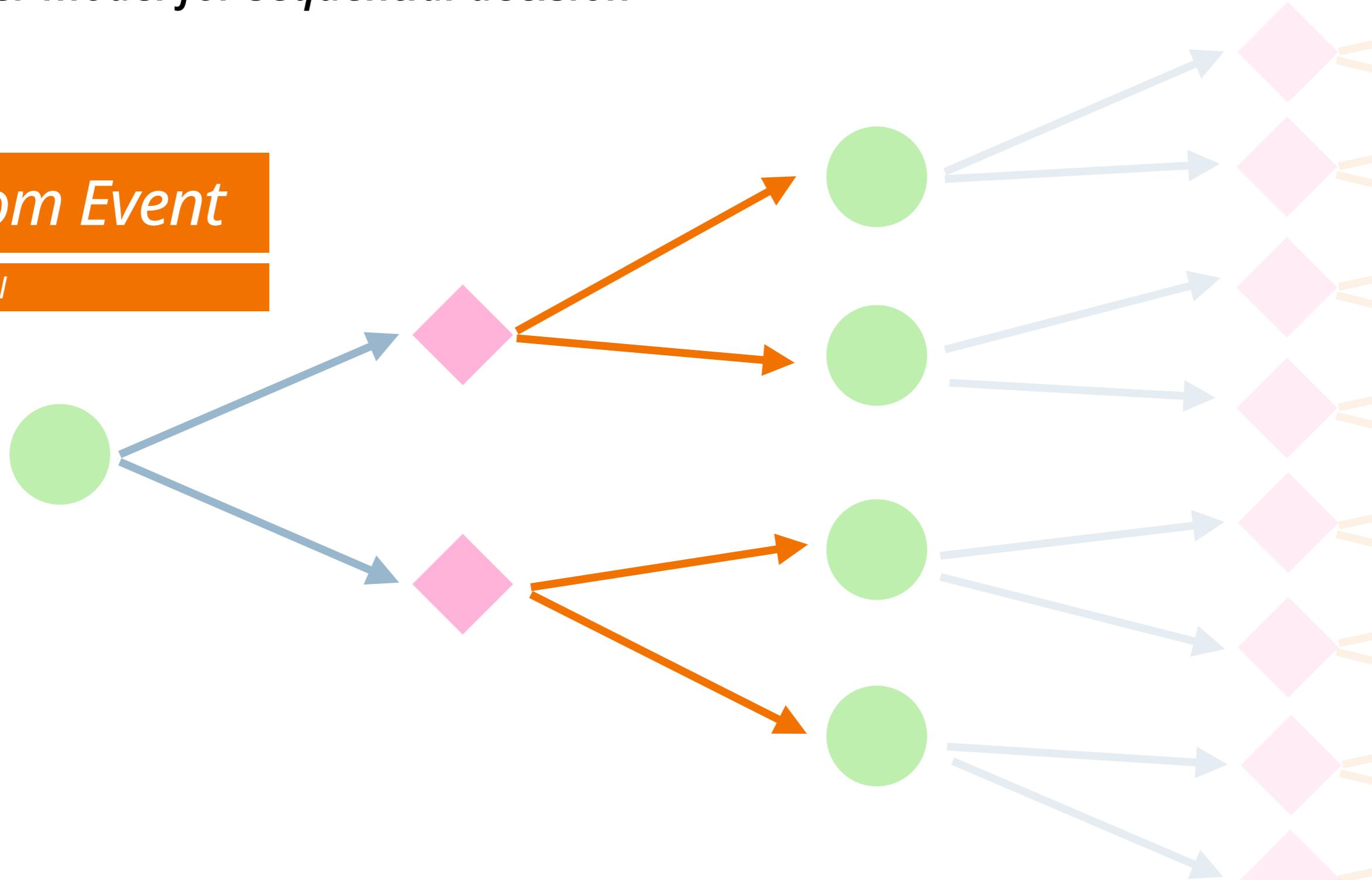


The Universal Framework: the model

A richer model for sequential decision

Random Event

Second Signal



The Universal Framework: the policy

How to solve any kind of sequential decisions problem

Type of policy	Policy search	Lookahead	Function approximation	Imbedded optimization
Policy Function Approximation	x		x	
Cost Function Approximation	x		x	x
Value Function Approximation		x	x	x
Direct Lookahead		x		x

PFA - Policy Function Approximation

What is PFA:

Learn a function (input the state and return an action) that approximates the optimal policy.

How to get a PFA:

- Lookup tables or ITTT
- Runtime: evaluate the actions and choose the best one
- Any kind of technique of Supervised Learning, as Neural Networks

Performances:

- Easy but very myopic.



CFA - Cost Function Approximation

What is CFA:

Solve an optimization problem to minimize the cost (or maximize the reward).

How to get a CFA:

- Multistage Stochastic Programming

Performance:

- The selected action minimizes long-term costs.
- The solutions are very close to the optimal policy.
- Computational intensive - cannot handle large state space.
- Not easy to model but can include many extra sources of information.



VFA - Cost Function Approximation

What is VFA:

For each state (or couple of state-actions), find a value function that approximates the total sum of rewards of costs.

How to get a VFA:

- Reinforcement Learning (DP, MC, TD)

Performance:

- The solutions are close to the optimal policy.
- Tradeoff between training time, size of the state space and performance.
- Can include many extra sources of information with planning.



DLA - Direct LookAhead

What is DLA:

Run simulations to identify the best action for each state.

How to get a DLA:

- Rollout algorithms

Performance:

- Simple, need a probabilistic model for the environment.
- It needs a large number of simulations, but computationally fast.





Take-home message

- ▶ *Get more from your data: go prescriptive!*
- ▶ *Always consider uncertainty*
- ▶ *Each problem requires its own method*



Repository

github.com/gabryallaseconda/from-prescriptive-to-prescriptive-speech

Contacts

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Credits

- ▶ Stochastic Dynamic Programming course by *Justin Goodson*
- ▶ Various material by *Warren Powell*, CASTLE at Princeton Univ.
- ▶ Gartner's 2017 Planning Guide for Data and Analytics
- ▶ Prescriptive analytics: Literature review and research challenges
- ▶ Reinforcement Learning: An Introduction by R. Sutton, A Barto

Images credits:

- ▶ Theme image: Laxmonaut on AlphaCoders
- ▶ Prescriptive analytics: Literature review and research challenges
- ▶ NGBoost <https://stanfordmlgroup.github.io/projects/ngboost/>
- ▶ Wikipedia
- ▶ Scribbr (quantile plot)
- ▶ Byju's (histogram plot)
- ▶ Towards using multi-agents systems for assisting undergraduate STEM students learning (MDP diagram)