UNIVERSITÀ DI PISA

Tenth hands-on: Game Theory II

Algorithm Design (2021/2022)

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1 Introduction

1.1 Problem 1: Crossing Streets

Two players drive up to the same intersection at the same time. If both attempt to cross, the result is a fatal traffic accident. The game can be modeled by a payoff matrix where crossing successfully has a payoff of 1, not crossing pays 0, while an accident costs 100.

- Build the payoff matrix
- Find the Nash equilibria
- Find a mixed strategy Nash equilibrium
- Compute the expected payoff for one player (the game is symmetric)

1.2 Problem 2: Mixed strategy for Back Stravinsky Game

Find the mixed strategy and expected payoff for the Back Stravinsky game.

1.3 Problem 3: Children to Kindergartends

The Municipality of your city wants to implement an algorithm for the assignment of children to kindergartens that, on the one hand, takes into account the desiderata of families and, on the other hand, reduces 'city traffic caused by taking children to school. Every school has a maximum capacity limit that cannot be exceeded under any circumstances. As a form of welfare the Municipality has established the following two rules:

1. in case of a child already attending a certain school, the sibling is granted the same school;
2. families with only one parent have priority for schools close to the workplace.

Model the situation as a stable matching problem and describe the payoff functions of the players. Question: what happens to twin siblings?

D_1 / D_2	Cross	Stop
Cross	-100, -100	1, 0
Stop	0,1	0, 0

Table 1: Payoff Table of Crossing Streets

2 Solution

2.1 Solution for 'Crossing Streets'

The action set is build with two events $A = \{Cross, Stop\}$. With $D_i, i \in [1, 2]$ we define the i-th driver that is crossing the street. The payoff table is shown in Table 2.1.
As we can see from the table, we have two Nash equilibria, which are the profiles: $(Cross, Stop)$ and $(Stop, Cross)$.

To find the mixed strategies for this problem, we solve the following equations:

$$\begin{cases} \mu_{D_1}(Cross) = \mu_{D_1}(Stop) \\ \mu_{D_2}(Cross) = \mu_{D_2}(Stop) \end{cases} \tag{1}$$

$$\iff \begin{cases} -100q + 1(1-q) = 0q + 0(1-q) \\ -100p + 1(1-p) = 0p + 0(1-p) \end{cases} \tag{2}$$

$$\iff \begin{cases} q = \frac{1}{101} \\ p = \frac{1}{101} \end{cases} \tag{3}$$

Since the game is symmetric, the expected payoff is equal for both players:

$$\begin{aligned} E[\![\mu_{D_1}]\!] = E[\![\mu_{D_2}]\!] &= pq(-100) + p(1-q)1 + (1-p)q0 + (1-p)(1-q)0 \\ &= -100pq + p - qp = -101pq + p = p(1-101q) = p(1-101p) = 0 \end{aligned}$$

2.2 Solution for 'Mixed Strategy for Back Stravinsky Game'

TODO.

2.3 Solution for 'Children to Kindergartends'

TODO.