Ninth hands-on: Game Theory I

Algorithm Design (2021/2022)

Gabriele Pappalardo
Email: g.pappalardo4@studenti.unipi.it
Department of Computer Science

April 2022

1 Introduction

1.1 Problem 1: Diabolik

After Diabolik's capture, Inspector Ginko is forced to free him because the judge, scared of Eva Kant's revenge, has acquitted him with an excuse. Outraged by the incident, the mayor of Clerville decides to introduce a new legislation to make judges personally liable for their mistakes. The new legislation allows the accused to sue the judge and have him punished in case of error. Consulted on the subject, Ginko is perplexed and decides to ask you to provide him with a formal demonstration of the correctness/incorrectness of this law.

1.2 Problem 2: Investment Agency

An investment agency wants to collect a certain amount of money for a project. Aimed at convincing all the members of a group of N people to contribute to the fund, it proposes the following contract: each member can freely decide either to contribute with 100 euros or not to contribute (retaining money on its own wallet). Independently on this choice, after one year, the fund will be rewarded with an interest of 50% and uniformly redistributed among all the N members of the group. Describe the game and find the Nash equilibrium.

2 Solutions

2.1 Diabolik

Work in progress.

2.2 Investment Agency

This game can be modelled as follows:

- we have N *players*, which are the contributors. We will label each player with P_i for $1 \leq i \leq N$.
- each player P_i has his/her own *set of strategies* S_i, where $S_i = \{\text{Contribute}, \text{Not Contribute}\}$.
- with the functions $u : S \to \mathbb{Z}$ and $c : S \to \mathbb{Z}$ to denote *payoff* and *cost* for each player, defined below:

$$u(s_i) = \frac{(k \cdot 100) \cdot 0.5}{N} \quad \text{where } k = \#\text{contributing players, including } i$$

$$c(s_i) = \begin{cases} 100 & \text{if player } P_i \text{ Contibutes} \\ 0 & \text{otherwise} \end{cases}$$

The total incomes for a player P_i, denoted with g_i, are computed using the following equation:

$$g_i = u(s_i) - c(s_i)$$

We can have two specific situations. In the foremost, if the incomes $g_i \geq 0$ then the player benefits from the investments, otherwise, if the incoming are $g_i \leq 0$ the contributor loses money. The important question to ask ourselves is the following one: when should I contribute for the project? To answer that, let us build an incoming matrix, containing all the possible situations for the game. We restrict ourselves in the case of $N = 2$ players (this could be easily generalized). With C we indicate a *contributing* player, instead, with NC a non-contributing one.

		Contributor 2	
		C	NC
Contributor 1	C	$(-50, -50)$	$(-75, 25)$
	NC	$(25, -75)$	$(0, 0)$

Let us analyze the table:

1. if the two players decide to contribute, then they will gain 50 euros, but they will have a gain of -50 euros which do not cover the initial costs.
2. if one of the two decides to contribute, then the non-contributing player will benefit of 25 euros for the next year, instead the contributing one has lost 75 euros.
3. if none of them contribute, then they will not gain or lose nothing.

Surprisingly enough, the only one *Nash Equilibrium* is obtained when none of the players contributes.