Exploring Quantum Process Calculi

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Qubits

Superposition

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No Cloning theorem

IqCCS

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Linear qCCS: Linearity

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In IqCCS, the programmer must **explicitly** describe what happens to each qubit

$$P' = c?q.X(q).c!q$$
 $Q' = c?q.H(q).c!q$

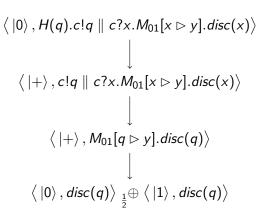
$$P'' = c?q.X(q).disc(q)$$
 $Q'' = c?q.H(q).discq$

Linear qCCS: Probabilistic Transition System

- Transition system made of **configurations**, of the form $\langle |\psi \rangle, P \rangle$
- Reduction system, without labels
- Probabilistic behaviour, a configuration can evolve in a distribution of configurations

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Linear qCCS: Bisimilarity via barbs and contexts

Saturated Bisimilarity

Two processes are **saturated bisimilar** if they express the same **observable behaviour** under any **context**

- Observable behaviour (barb): the capability of sending some value on a specific channel
- Context: a "program" with a hole, like $[-] \parallel R$. We compare P and Q "inside" this context, i.e. we study $P \parallel R$ and $Q \parallel R$.

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The two processes seen before

$$P' = c?q.X(q).c!q$$
 $Q' = c?q.H(q).c!q$

are not bisimilar, because there is a context R which tells them apart

$$R = c?q.M_{01}[q > y].(disc(q) || if x = 0 then a!0 else b!0)$$

Probabilistic Bisimilarity

Thanks to IqCCS, we can compare the existing bisimilarities. Consider the two configurations

$$\begin{split} \mathcal{C} &= \left\langle \left. \left| + \right\rangle, M_{01}[q \rhd x].c!q \right\rangle \to \left\langle \left. \left| 0 \right\rangle, c!q \right\rangle_{\frac{1}{2}} \oplus \left\langle \left. \left| 1 \right\rangle, c!q \right\rangle \\ \mathcal{C}' &= \left\langle \left. \left| 0 \right\rangle, M_{\pm}[q \rhd x].c!q \right\rangle \to \left\langle \left. \left| + \right\rangle, c!q \right\rangle_{\frac{1}{2}} \oplus \left\langle \left. \left| - \right\rangle, c!q \right\rangle \end{split}$$

According to the usual notion of **probabilistic bisimilarity**, two distributions are bisimilar if they assign the same probability to bisimilar processes. In our example, the two configurations are **not** bisimilar, because they evolve in two non-bisimilar distributions.

Indistinguishable distributions

According to quantum mechanics, it's impossible to distinguish the two sources S and S':

```
S emits a qubit |0\rangle or |1\rangle with the same probability S' emits a qubit |+\rangle or |-\rangle with the same probability
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Suppose we receive a qubit from either S or S', and measure it in the computational basis. If we measure a qubit from S, it would result in either $|0\rangle$ or $|1\rangle$. If we measure a qubit from S', a $|+\rangle$ qubit would decay in either $|0\rangle$ or $|1\rangle$, and a $|-\rangle$ qubit would decay in either $|0\rangle$ or $|1\rangle$ as well.

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Inadequacy of Probabilistic Bisimilarity

The usual notion of probabilistic bisimilarity is too fine, when comparing distributions of quantum configurations.

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We introduce an equivalence relation

$$\equiv \subseteq \mathfrak{D}(\mathit{Conf}) \times \mathfrak{D}(\mathit{Conf})$$

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$$\not\sim$$

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Minimal Quantum Process Algebra

Solution II: mQPA

We introduce a new calculus, equipped with a minimal set of features: communication, non-determinism and quantum measurement. In mQPA, the transitions are of the form

$$\rightarrow \subseteq S \times \mathfrak{D}(S)^{\mathcal{H}}$$

i.e. the probabilistic observable behaviour is **parametric** with respect to an input quantum state.

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In mQPA, the previous example can be rewritten as

$$Q = (s_{|0\rangle\langle 0|} \boxplus s')_{|+||+|} \boxplus (s_{|0\rangle\langle 0|} \boxplus s')$$

$$Q' = (s_{\mid + \downarrow + \mid} \boxplus s')_{\mid 0 \downarrow 0 \mid} \boxplus (s_{\mid + \downarrow + \mid} \boxplus s')$$

What have we seen

- **IqCCS**, an asynchronous linear calculus inspired by qCCS. It rephrases the syntax e semantics of previous calculi in a more standard formalism, and allows to compare different notions of bisimilarity.
- Even though quantum systems exhibit a probabilistic behaviour,
 probabilistic bisimilarity is not really well suited for the quantum setting.
- mQPA.

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- From weak transitions, it is possible to define reachability, temporal logics and model checking.
- Saturated bisimilarity can be cumbersome to prove, it will be interesting to explore how the existing proof techniques adapt to the quantum setting
- We will investigate the relation between mQPA semantics and the usual, configuration-based semantics, together with their respective bisimilarities.

Thank you for your attention!

