23/24 S1

1. a) power joint → torque joint → link

pitch angle of incremental encoder in joint 2: 350 pulses per rotation

$$\measuredangle = \frac{360^\circ}{350} = 1.0286^\circ \ \#$$

b) repeatability error at the link joint 2: $\dfrac{1.0286^\circ}{600} = 0.0017143^\circ \ \#$

(OR) 350 pulse per rotation of power joint = $\frac{1}{600}$ rotation of torque joint = $\frac{360^\circ}{600}$

$$\frac{360^\circ}{600} \div 350 = 0.0017143^\circ \ \#$$

c) $x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$

$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$

or $\theta_1 + \theta_2 + 15^\circ$?

desired output at $\theta_2 = 15^\circ$: $x = 20\cos\theta_1 + 15\cos(\theta_1 + 15^\circ)$ or encoder count ?

$y = 20\sin\theta_1 + 15\sin(\theta_1 + 15^\circ)$ #

d) $\Delta x = -L_1 \Delta\theta_1 \sin\theta_1 - L_2(\Delta\theta_1 + \Delta\theta_2)\sin(\theta_1 + \theta_2)$

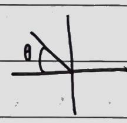
$\Delta y = L_1 \Delta\theta_1 \cos\theta_1 + L_2(\Delta\theta_1 + \Delta\theta_2)\cos(\theta_1 + \theta_2)$

$|\Delta x| = L_1 \Delta\theta_1 \sin\theta_1 + L_2(\Delta\theta_1 + \Delta\theta_2)\sin(\theta_1 + \theta_2)$

$\max|\Delta x| = L_1 \Delta\theta_1 + L_2(\Delta\theta_1 + \Delta\theta_2)$

$= \left[20 \times \frac{360}{700 \times 250} + 15 \times \left(\frac{360}{700 \times 250} + 0.0017143\right)\right] \times \frac{\pi}{180}$ ✓

$= \cancel{0.00817\text{cm}}$ ✗ 0.00171 cm ✓



2. a) $(x-25)^2 + y^2 = 25^2$ # ✓

b) $A(0,0)$, $C(0.3790, 4.3412)$ ⇒ $x = 0.3790 = 25 - 25\cos\theta$ ⇒ $\theta_c = 0.17435$ rad

$y = 4.3412 = 25\sin\theta$

$\theta_c = 0.17435$ rad

$\theta_c = \frac{1}{2}\alpha t^2 = 0.17435 = \frac{1}{2}(\alpha)(1.5)^2 \Rightarrow \alpha = 0.155 \text{ rad s}^{-2}$

$\omega = 0.155t + C, \quad C=0 \because \omega(0)=0$

$\theta = 0.0775t^2 + C, \quad c=0 \because \theta(0)=0$

~~X~~ Subst. into $x = 25 - 25\cos\theta$

$$\Rightarrow x = 25 - 25\cos(0.0775t^2) \quad \#$$

c) $49.6210 = 25 - 25\cos\theta$
$4.3412 = 25\sin\theta$ $\Rightarrow$ $\theta = 2.9672$ rad

$\theta_D = 2.9672$ rad, $\theta_c = 0.17435$ rad (from (b))

$\omega = 0.155(1.5) = 0.2325 \text{ rad s}^{-1}$ (from (b) — constant circular velocity)

$$\frac{2.9672 - 0.17435}{0.2325} = 12.01\text{s} \ \#$$

3. Let coord sys $x_1y_1z_1$ be attached to the rotating base.
then ${}^0P_p = R_{01}\,{}^1P_p$

$$R_{01} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad {}^1P_1 = \begin{pmatrix} 0 \\ b+K_1+d_1t+e \\ a-c+d_2t \end{pmatrix}$$

$$\frac{d}{dt}(dt) = \frac{d}{dt}(d)\cdot t + \frac{d}{dt}(t) * d$$

$${}^0\dot{P}_p = \dot{R}_{01}\,{}^1P_p + R_{01}\,{}^1\dot{P}_p$$

$$\dot{R}_{01} = \begin{bmatrix} -\dot\theta\sin\theta & -\dot\theta\cos\theta & 0 \\ \dot\theta\cos\theta & -\dot\theta\sin\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} = \dot\theta \begin{bmatrix} -\sin\theta & -\cos\theta & 0 \\ \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad {}^1\dot{P}_p = \begin{pmatrix} 0 \\ \dot d_1 \\ \dot d_2 \end{pmatrix}$$

$$\therefore {}^0\dot{P}_p = \dot\theta \begin{bmatrix} -\sin\theta & -\cos\theta & 0 \\ \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} 0 \\ b+K_1+d_1t+e \\ a-c+d_2t+K_2 \end{pmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ \dot d_1 \\ \dot d_2 \end{pmatrix}$$

$$= \begin{pmatrix} -\cos\theta(b+K_1+\dot d_1t+e) \\ -\sin\theta(b+K_1+\dot d_1t+e) \\ 0 \end{pmatrix}\dot\theta + \begin{pmatrix} -\dot d_1\sin\theta \\ \dot d_1\cos\theta \\ \dot d_2 \end{pmatrix} \quad \#$$

3. (Cont.) $\quad {}^0\dot{P}_p = \dot{R}_{01}\,{}^1P_p + R_{01}\,{}^1\dot{P}_p \Rightarrow {}^0\ddot{P}_p = \ddot{R}_{01}\,{}^1P_p + \dot{R}_{01}\,{}^1\dot{P}_p + \dot{R}_{01}\,{}^1\dot{P}_p + R_{01}\,{}^1\ddot{P}_p$

$$= \ddot{R}_{01}\,{}^1P_p + 2\times\dot{R}_{01}\,{}^1\dot{P}_p + R_{01}\,{}^1\ddot{P}_p$$

$$\dot{R}_{01} = \dot{\theta}\begin{bmatrix}-\sin\theta & -\cos\theta & 0\\ \cos\theta & -\sin\theta & 0\\ 0 & 0 & 0\end{bmatrix} \Rightarrow \ddot{R}_{01} = \overset{0}{\cancel{\ddot{\theta}}}\begin{bmatrix}-\sin\theta & -\cos\theta & 0\\ \cos\theta & -\sin\theta & 0\\ 0 & 0 & 0\end{bmatrix} + \dot{\theta}^2\begin{bmatrix}-\cos\theta & \sin\theta & 0\\ -\sin\theta & -\cos\theta & 0\\ 0 & 0 & 0\end{bmatrix}$$

direct trf

$${}^1\dot{P}_p = \begin{pmatrix}0\\ \dot{d}_1\\ \dot{d}_2\end{pmatrix} \Rightarrow {}^1\ddot{P}_p = \begin{pmatrix}0\\0\\0\end{pmatrix}$$ and $\ddot{\theta}=0$ since $\dot{\theta} = \text{const.}$

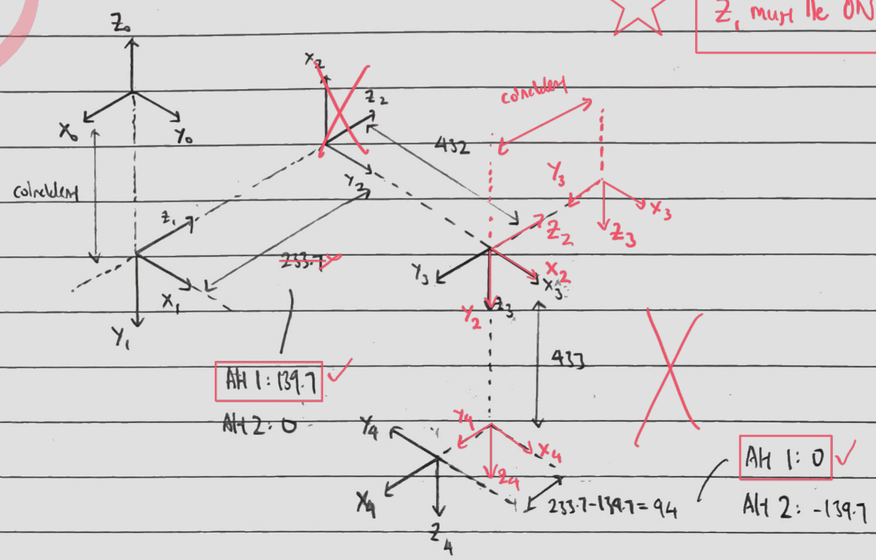
$$\therefore {}^0\ddot{P}_p = \begin{bmatrix}-\cancel{\ddot{\theta}\sin\theta} - \dot{\theta}^2\cos\theta & -\cancel{\ddot{\theta}\cos\theta} + \dot{\theta}^2\sin\theta & 0\\ \cancel{\ddot{\theta}\cos\theta} - \dot{\theta}^2\sin\theta & -\cancel{\ddot{\theta}\sin\theta} - \dot{\theta}^2\cos\theta & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\ b+K_1+d_1t+e\\ a-c+d_2t+K_2\end{bmatrix}$$

$$+\,2\dot{\theta}\begin{bmatrix}-\sin\theta & -\cos\theta & 0\\ \cos\theta & -\sin\theta & 0\\ 0 & 0 & 0\end{bmatrix}\begin{pmatrix}0\\ \dot{d}_1\\ \dot{d}_2\end{pmatrix} \quad \#$$

REDO

4. a)

$\boxed{Z_1 \text{ must be ON joint 2 axis}}$



b)

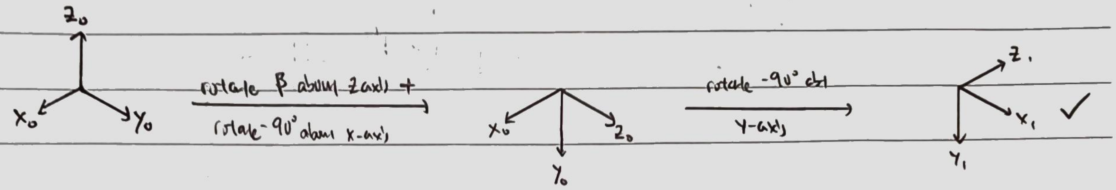
Joint	θ	d	a	α	$\cos\alpha$	$\sin\alpha$
	$x_{i-1}\to x_i$ along z_{i-1}		$z_{i-1}\to z_i$ along x_i			
1	90	0	0	-90	0	-1
2	-90	233.7	0	0	1	0
3	90	0	432	-90	0	-1
4	90	433	94	0	1	0

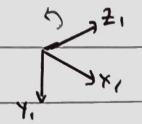
c) $$H_{i-1,i} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_{02} = H_{01} H_{12}$$

$$= \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 2337 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \#$$

d) coord sys. 1 is coincident with coord sys. 0 (base)





$R_{02} = R_{01} R_{12}$

$R_{20} = R_{21} \cdot R_{10} = R_{12}{}^T R_{01}{}^T$

this is 1 to base. for base to 1

$$\therefore R_{01} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) \\ 0 & \sin(-90) & \cos(-90) \end{bmatrix} \begin{bmatrix} \cos(-90) & 0 & \sin(-90) \\ 0 & 1 & 0 \\ -\sin(-90) & 0 & \cos(-90) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \sim \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -\sin\beta & 0 & -\cos\beta \\ \cos\beta & 0 & -\sin\beta \\ 0 & -1 & 0 \end{bmatrix} \quad \#$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2023-2024

MA4825 – ROBOTICS

November / December 2023 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is an **OPEN-BOOK** examination.

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1. A planar robot arm with two links is shown in Figure 1. Link 1's length is 20.0 cm while link 2's length is 15.0 cm. Inside the robot's joint 1, the speed reduction ratio of torque joint is 700 while the power joint's incremental encoder will output 250 pulses after a full rotation. Similarly, inside the robot's joint 2, the speed reduction ratio of torque joint is 600 while the power joint's incremental encoder will output 350 pulses after a full rotation.

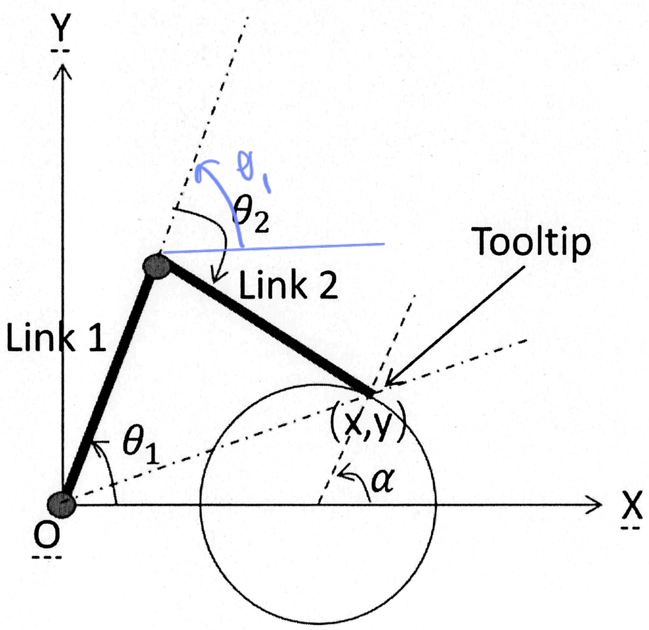


Figure 1

NOTE: Question 1 continues on page 2.

(a) What is the pitch angle of the incremental encoder inside the robot's joint 2?

(5 marks)

(b) What is the repeatability error at the link joint of the robot's joint 2 if the position control of the power joint is 100% accurate?

(5 marks)

(c) The power joint inside the robot's joint 2 is under the control of a position feedback loop. If the link joint inside the robot's joint 2 makes an angular displacement of 15.0 degrees, what should be the desired output from the position control loop?

(5 marks)

(d) What is the accuracy of the X coordinate of the tooltip which is placed at the end of the robot's link 2?

(10 marks)

2. A robotic arm is moving its gripper along a circular path/trajectory as shown in Figure 2. Initially, the gripper is at rest at point A. After the motion, the gripper will stop at point B. Assume that the coordinates of point A are (0.0, 0.0) (cm) while the coordinates of point B are (50.0, 0.0) (cm). The gripper's orientation v_g at point A is +90.0° while the gripper's orientation v_g at point B is -90.0°. Both orientations are tangential to the circular path/trajectory.

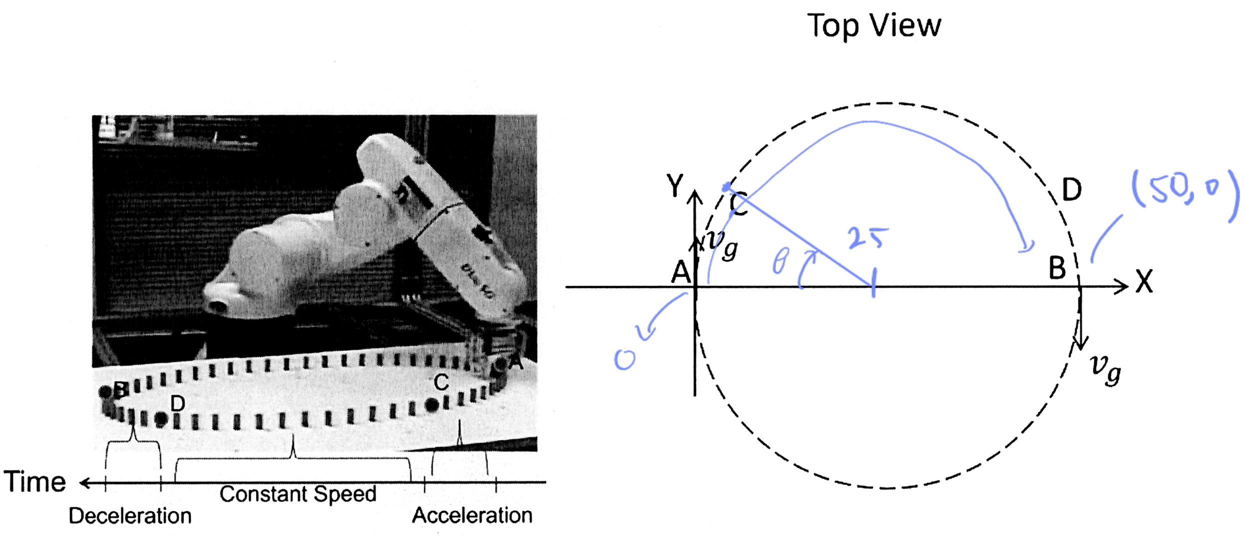


Figure 2

(a) What is the equation of the path to be followed by the gripper's position?

(5 marks)

NOTE: Question 2 continues on page 3.

(b) The robot moves its gripper's position from point A to point C by undergoing a circular motion with a constant tangential acceleration. Point C's coordinates are (0.3790, 4.3412) (cm). If the duration of this motion is 1.5 seconds, what is the equation of X coordinate's trajectory between point A and point C?

(10 marks)

(c) The robot continues to move its gripper's position from point C to point D by undergoing a circular motion with constant circular velocity. If point D's coordinates are (49.6210, 4.3412) (cm), what is the duration of motion from point C to point D?

(10 marks)

3. The robot in Figure 3 consists of one rotating u-bar (link 1) and two translational links (links 2 and 3). The angular displacement of the rotational joint is θ, and the displacements of the translational joints are d_1 and d_2. Link 1 is rotating with a constant angular velocity of $\dot{\theta}$ about the vertical axis; link 2 is translating horizontally at constant velocity of $\dot{d}_1$; link 3 is an L-bar translating vertically at constant velocity of $\dot{d}_2$. Note that the configuration shown in the figure is the initial position of the system where $\theta = 0,\ d_1 = K_1$, and $d_2 = K_2$. Find the absolute velocity and acceleration of the end point of the robot in terms of $\theta,\ d_1$and d_2.

(25 marks)

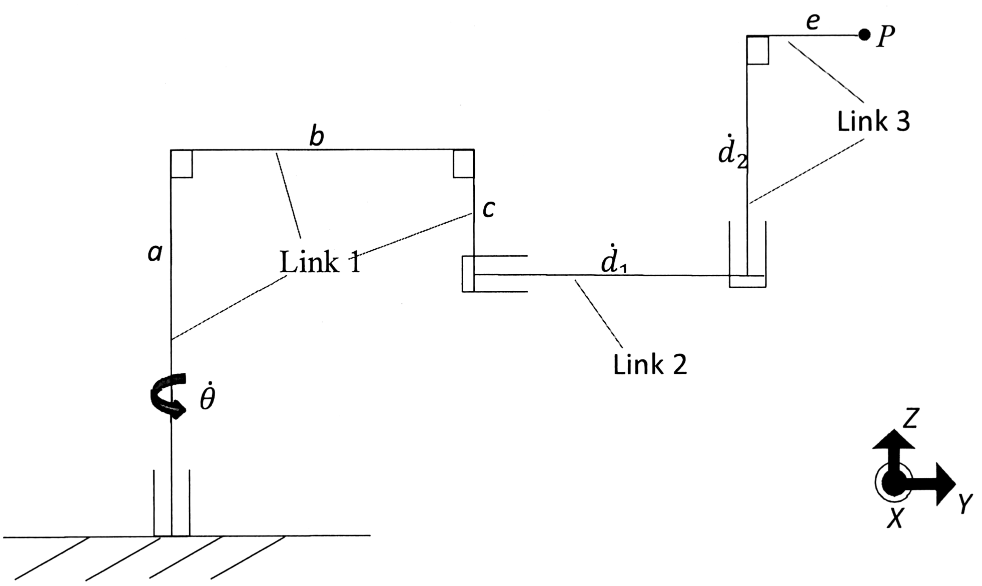


Figure 3

4. An illustration of an industrial robot used for pick-and-place is shown in Figure 4(a) without its end effector. The arm has four joints: $\boldsymbol{q_1} = \{\theta_1, \theta_2, \theta_3, \theta_4\}^T$.

(a) Given the base coordinate system in Figure 4(b), draw the coordinate systems i (i=1 to 3) of the robot up to the wrist joint. (Note: Sketch a skeletal diagram in isometric view & provide the positions and directions of the coordinate systems in your answer.)

(10 marks)

(b) Tabulate the Denavit-Hartenberg (D-H) parameters for the position of the robot shown in Figure 4(b) for joints 1 to 4, according to the coordinate systems specified in part (a).

(6 marks)

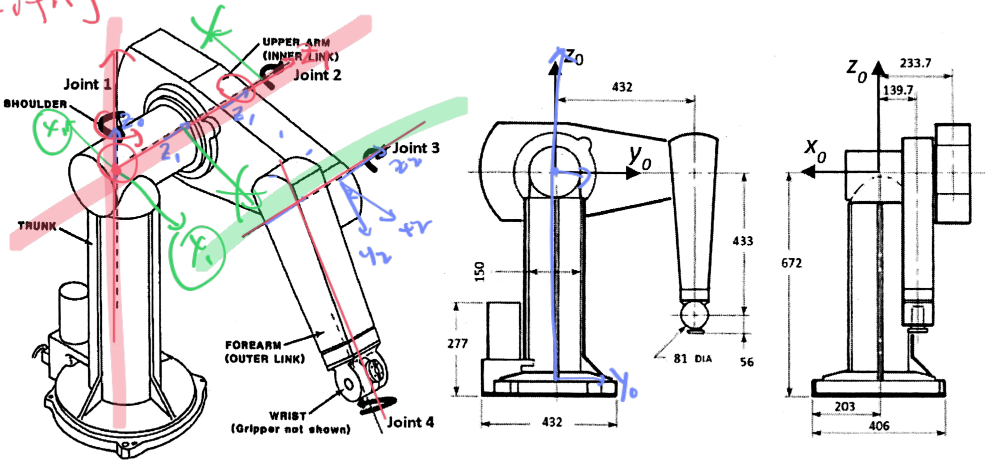
(c) Find the generic D-H matrix $\boldsymbol{H}_{02}$ for the robot where $\boldsymbol{H}_{02}$ is a 4x4 homogeneous matrix transforming a vector from frame 2 (coordinates x_2, y_2, z_2) to the base coordinates (x_0, y_0, z_0). Do not multiply the individual matrices.

(4 marks)

(d) Describe a possible sequence of rotation that aligns the base coordinate system to coordinate system 1. Derive the rotational matrix.

(5 marks)

See Tut 2 Ans



(a) (b)

Figure 4

END OF PAPER