MA4825 Robotics Assignment

Assignment (Due 16/11/24@2359hrs, Saturday)

Instructions:

1. Answer all questions.
2. Show all your workings and drawings clearly.
3. Submit online thru MA4825 ntulearn course channel> Assignment> MA4825 Assignment 2 submission.

Question 1

The robot in Figure 1 consists of a rotating links (link 1 and 3) and a translational links (links 2). The angular displacements of rotational joints are θ and β, and the displacements of the translational joints are d. Link 1 is rotating with a constant angular velocity of $\dot{\theta}$ about the Z-axis; link 2 is translating horizontally at constant velocity of $\dot{d}$; link 3 is rotating with a constant angular velocity of $\dot{\beta}$ about the X-axis. Note that the configuration shown in the figure is the initial position of the system where $\theta = \beta = 0°$ and $d = K_1$. Find the absolute velocity of the end point P of the robot in terms of θ, β, and d. (25 marks)

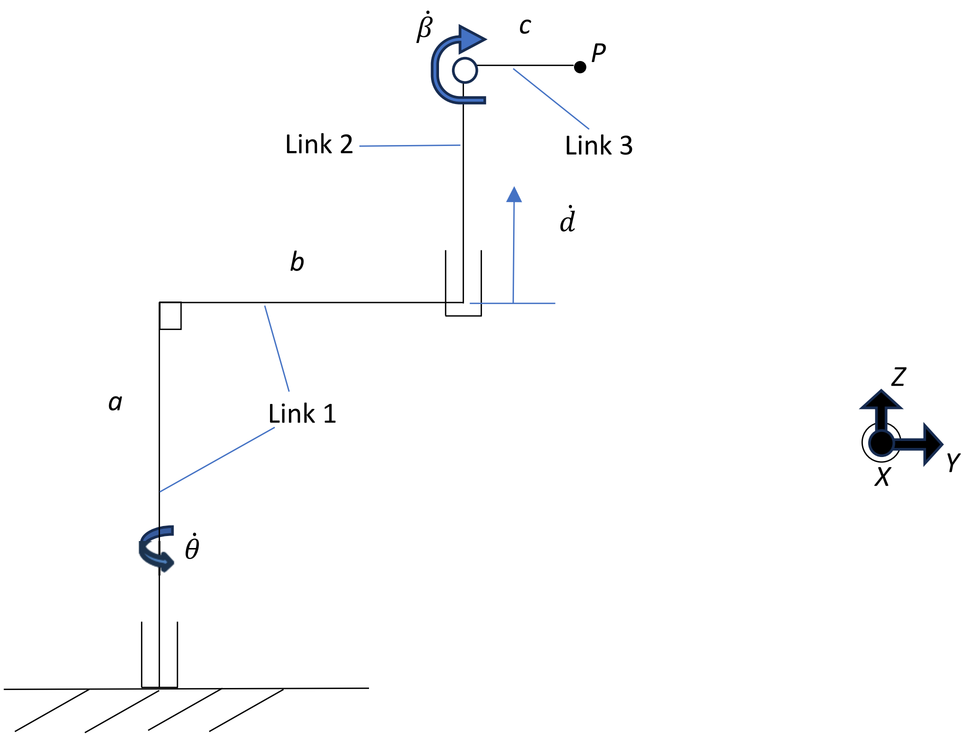

Figure 1

Question 1

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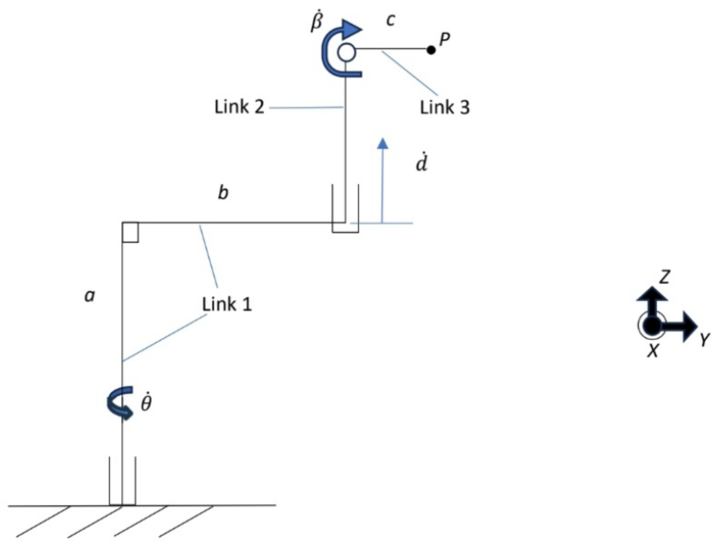


Figure 1

R_{01} : Z - axis

R_{12} : x - axis

$$R_{01} = \begin{bmatrix} \cos(\dot{\theta}t) & -\sin(\dot{\theta}t) & 0 \\ \sin(\dot{\theta}t) & \cos(\dot{\theta}t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\dot{\beta}t) & -\sin(\dot{\beta}t) \\ 0 & \sin(\dot{\beta}t) & \cos(\dot{\beta}t) \end{bmatrix}$$

$${}^{1}P_1 = \begin{bmatrix} 0 \\ b \\ a + \dot{d}t \end{bmatrix} \qquad {}^{2}P_2 = \begin{bmatrix} 0 \\ c \\ 0 \end{bmatrix}$$

$${}^{0}P_2 = R_{01}\,{}^{1}P_1 + R_{01} R_{12}\,{}^{2}P_2$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ b \\ a + \dot{d}t \end{bmatrix} +$$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} 0 \\ c \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -b\sin\theta \\ b\cos\theta \\ a + \dot{d}t \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ c \cdot \cos\beta \\ c \cdot \sin\beta \end{bmatrix}$$

$$= \begin{bmatrix} -b\sin\theta \\ b\cos\theta \\ a + \dot{d}t \end{bmatrix} + \begin{bmatrix} -c \cdot \sin\theta \cdot \cos\beta \\ c \cdot \cos\theta \cdot \cos\beta \\ c \cdot \sin\beta \end{bmatrix}$$

$$\Rightarrow \; {}^{0}P_{2} = \begin{bmatrix} -b\sin\theta - c\cdot\sin\theta\cdot\cos\beta \\ b\cos\theta + c\cdot\cos\theta\cdot\cos\beta \\ a + \dot{d}t + c\cdot\sin\beta \end{bmatrix}$$

$$\therefore \; {}^{0}\dot{P}_{2} = \begin{bmatrix} -b\dot{\theta}\cos\theta - c(\dot{\theta}\cos\theta\cos\beta - \dot{\beta}\sin\beta\sin\theta) \\ -b\dot{\theta}\sin\theta + c(-\dot{\theta}\sin\theta\cos\beta - \dot{\beta}\sin\beta\cos\theta) \\ \dot{d} + c\dot{\beta}\cos\beta \end{bmatrix}$$ # ANS

Question 2

An illustration of an industrial robot (ABB IRB 5720 robot) used for spotwelding is shown in Figure 2 without its end effector and in its initial position. The arm has six joints: $\boldsymbol{q} = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6\}^T$.

(a) Given the base coordinate system, draw and specify the right-handed coordinate systems i (i = 1 to 6) of the robot up to the wrist joint. (Note: Sketch a skeletal diagram in isometric view & provide the positions and directions of the coordinate systems in your answer, take note of the positive directions.)

(10 marks)

(b) Tabulate the Denavit-Hartenberg (D-H) parameters for the position of the robot shown in Figure ~~3~~ 2 for joints 1 to 6, according to the coordinate systems specified in part (a). Note that θ_i(i = 1 to 6) are the joint variables.

(6 marks)

(c) Find the generic D-H matrix $\boldsymbol{H}_{03}$ for the robot where $\cancel{\boldsymbol{H}_{14}}$ H_{03} is a 4 x 4 homogeneous matrix transforming a vector from frame 3 to frame 1.

(4 marks)

(d) Figure 3 shows a schematic of the robot from link 1 to link 2 moving θ_2 and θ_3 respectively from its initial position. Assume a point mass m_1 and m_2 acting at point b and c respectively. Derive the Lagrangian L of the system.

(5 marks)

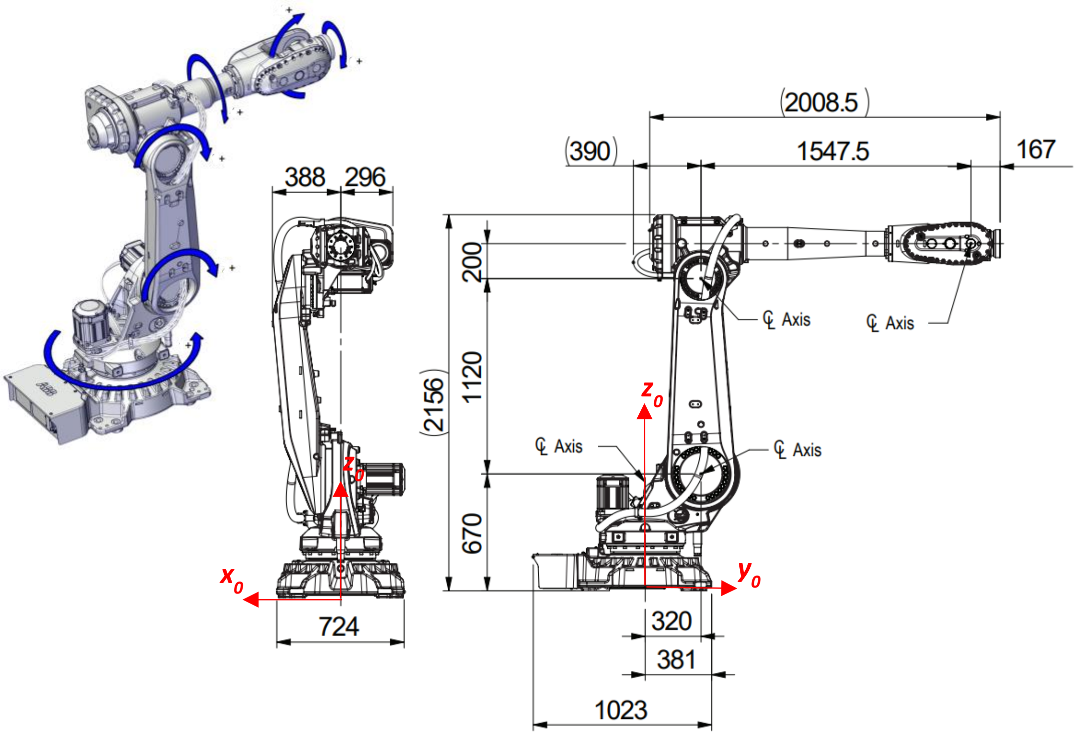


Figure 2

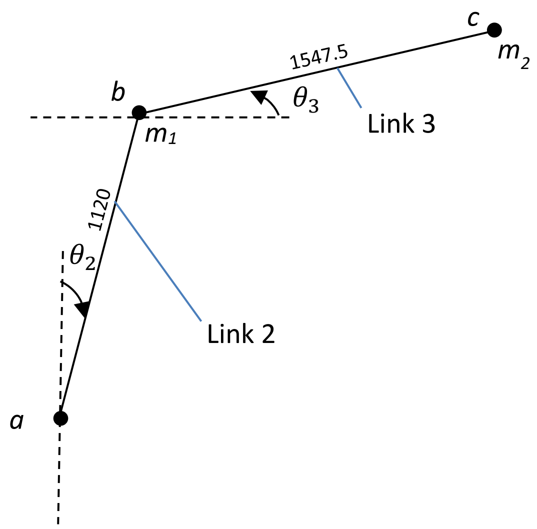
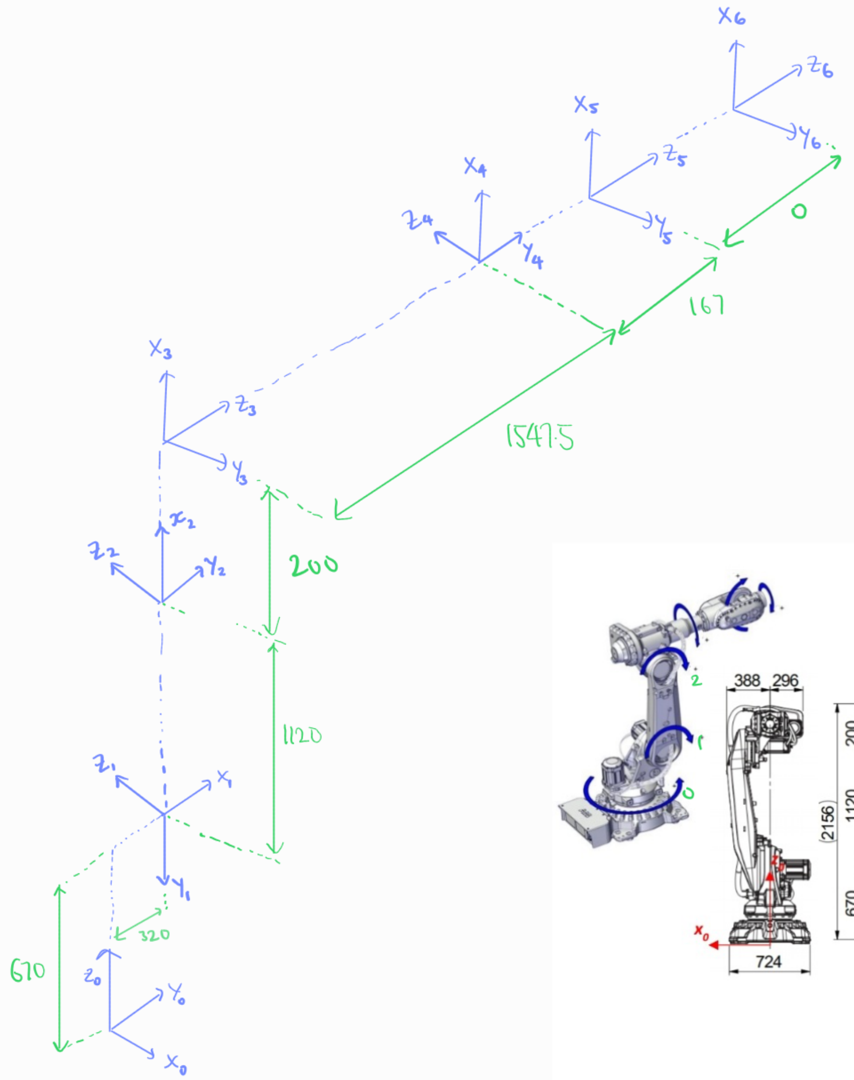


Figure 3

END

a)



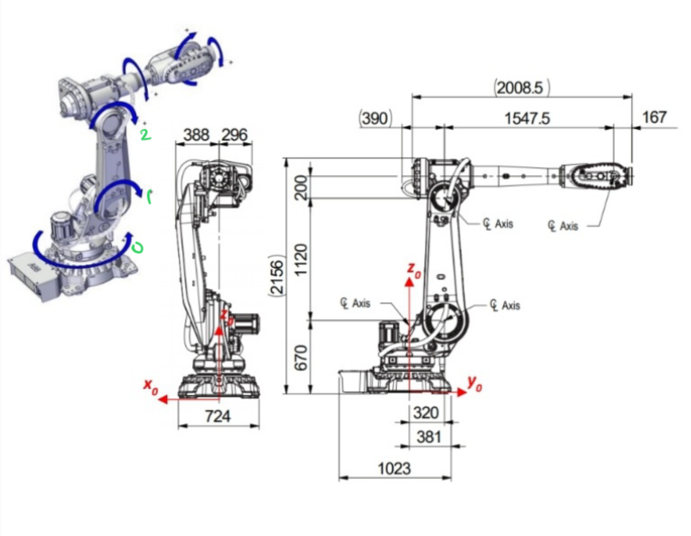


Figure 2

b)

Joint i	∡ btw x_{i-1} and x_i about z_{i-1} θ_i (deg)	dist btw origins in z_{i-1} dir d_i (mm)	∡ btw z_{i-1} and z_i about x_i α_i (deg)	dist btw origins in x_i dir a_i (mm)	$\cos\alpha_i$	$\sin\alpha_i$
1	90	670	-90	320	0	-1
2	-90	0	0	1120	1	0
3	0	0	-90	200	0	-1
4	0	1547.5	90	0	0	1
5	0	167	-90	0	0	-1
6	0	0	0	0	1	0

c) given $[H_{(i-1),i}] = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$H_{01} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 320 \\ 0 & -1 & 0 & 670 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

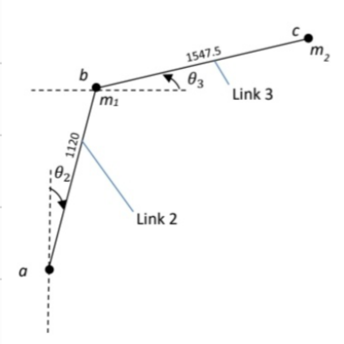
$$H_{12} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1120 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_{23} = \begin{bmatrix} 1 & 0 & -1 & 200 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore H_{03} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 320 \\ 0 & -1 & 0 & 670 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1120 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 200 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 320 \\ 1 & 0 & -1 & 1990 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{\# ANS}$$

d)



$$P_b = \begin{bmatrix} 1120\sin\theta_2 \\ 1120\cos\theta_2 \\ 0 \end{bmatrix}$$

$$V_b = \dot{P_b} = \begin{bmatrix} \dot\theta_2 \cdot 1120\cos\theta_2 \\ -\dot\theta_2 \cdot 1120\sin\theta_2 \\ 0 \end{bmatrix}$$

$$P_c = P_b + {}^bP_c$$

$$= \begin{bmatrix} 1120\sin\theta_2 \\ 1120\cos\theta_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1547.5\cos\theta_3 \\ 1547.5\sin\theta_3 \\ 0 \end{bmatrix}$$

$$V_c = \dot{P_c} = \begin{bmatrix} \dot\theta_2 \cdot 1120\cos\theta_2 - \dot\theta_3 \cdot 1547.5\sin\theta_3 \\ -\dot\theta_2 \cdot 1120\sin\theta_2 + \dot\theta_3 \cdot 1547.5\cos\theta_3 \\ 0 \end{bmatrix}$$

$$V_b^2 = (\dot\theta_2 \cdot 1120\cos\theta_2)^2 + (-\dot\theta_2 \cdot 1120\sin\theta_2)^2$$

$$V_c^2 = (\dot\theta_2 \cdot 1120\cos\theta_2 - \dot\theta_3 \cdot 1547.5\sin\theta_3)^2 + (-\dot\theta_2 \cdot 1120\sin\theta_2 + \theta_3 \cdot 1547.5\cos\theta_3)^2$$

$$KE = KE_1 + KE_2$$

$$= \frac{1}{2}m_1 V_b^2 + \frac{1}{2}m_2 V_c^2$$

$$= \frac{1}{2}m_1 \left[(\dot\theta_2 \cdot 1120\cos\theta_2)^2 + (-\dot\theta_2 \cdot 1120\sin\theta_2)^2\right]$$

$$+ \frac{1}{2}m_2 \left[(\dot\theta_2 \cdot 1120\cos\theta_2 - \dot\theta_3 \cdot 1547.5\sin\theta_3)^2 + (-\dot\theta_2 \cdot 1120\sin\theta_2 + \theta_3 \cdot 1547.5\cos\theta_3)^2\right]$$

$$PE = m_1 g (1120\cos\theta_2) + m_2 g (1120\cos\theta_2 + 1547.5\sin\theta_3)$$

$$\therefore L = KE - PE$$

$$= \frac{1}{2}m_1 \left[(\dot\theta_2 \cdot 1120\cos\theta_2)^2 + (-\dot\theta_2 \cdot 1120\sin\theta_2)^2\right] + \frac{1}{2}m_2 \left[(\dot\theta_2 \cdot 1120\cos\theta_2 - \dot\theta_3 \cdot 1547.5\sin\theta_3)^2 + (-\dot\theta_2 \cdot 1120\sin\theta_2 + \theta_3 \cdot 1547.5\cos\theta_3)^2\right]$$

$$- \left[m_1 g (1120\cos\theta_2) + m_2 g (1120\cos\theta_2 + 1547.5\sin\theta_3)\right]$$

ANS