NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2022-2023

MA3002– SOLID MECHANICS AND VIBRATION

November/December 2022 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.

2. Answer **ALL** questions.

3. All questions carry equal marks.

4. This is a **RESTRICTED OPEN-BOOK** examination. One double-sided A4-size reference sheet with texts handwritten or typed on the A4 paper (no sticky notes/post-it notes on the reference sheet) is allowed.

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1. The L-shaped beam ACB (shown in Figure 1) is made from two segments: segment AC of length 2L and CB of length L. The flexural stiffness *EI* is constant for both segments. If the beam is subjected to concentrated load *F* at the center point of segment AC and bending moment *M* at point B, answer the following questions:

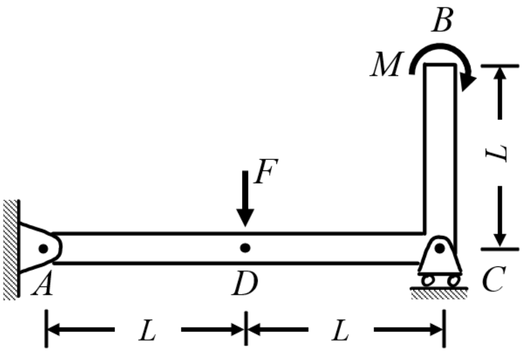

Figure 1

(a) Determine the reaction force at point C.

(8 marks)

(b) Write down the bending moment expression for the Real Load along segment BC.

(8 marks)

(c) Remove the bending moment at point B (i.e., set $M = 0$). Use the Unit Load Method to determine the rotation at point B (in rad) with the following data: L = 2 m; $EI = 10^6$ Nm^2; F = 1000 N.

(9 marks)

Note: Question 2 continues on page 2.

2 (a) An aluminium rectangular block with an elliptical surface crack (shown in Figure 2) is subjected to fatigue load with a mean stress of 120 MPa and stress amplitude of 100 MPa. The initial half crack length, a_0 = 2mm. Take K_{IC} = 80 MPa$\sqrt{m}$, $C = 1.7\times10^{-11}$ and m = 3.9. Assume the aspect ratio of the elliptical crack remains constant during crack growth.

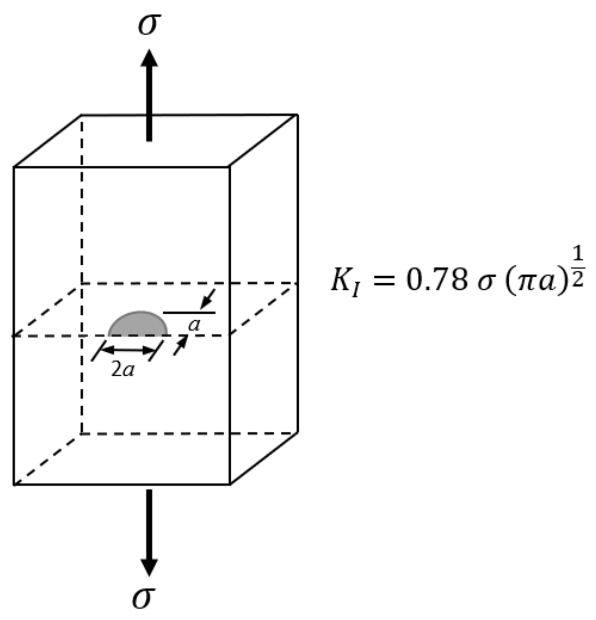

Figure 2

(i) Calculate the maximum stress, σ_{max}, and the stress range, S_R, of the fatigue cycle.

(4 marks)

(ii) Using Linear Elastic Fracture Mechanics, determine the maximum crack length, $2a$,

(5 marks)

(iii) Determine the number of fatigue cycle(s) to fracture.

(6 marks)

(b) A steel railway track is subjected to a cyclic stress of 20 MPa to 300 MPa. The steel has an Ultimate Tensile Strength of 600 MPa and a fatigue endurance limit in a fully reversed loading of 250 MPa.

(i) Calculate the mean stress, S_m, and the stress amplitude, S_a, of the fatigue cycle.

(4 marks)

(ii) Calculate the factor of safety (against fatigue failure) using Goodman rule.

(6 marks)

3 (a) Figure 3 shows the static equilibrium configuration of a 1-DOF vibrating system wherein a rigid rod AB of mass M and length L is hinged at point A and is carrying a rigid sphere mass m at point C. The mass is connected to a vertical spring of stiffness k and the rod is connected to a viscous damper of damping constant, c. Treat the sphere as point mass.

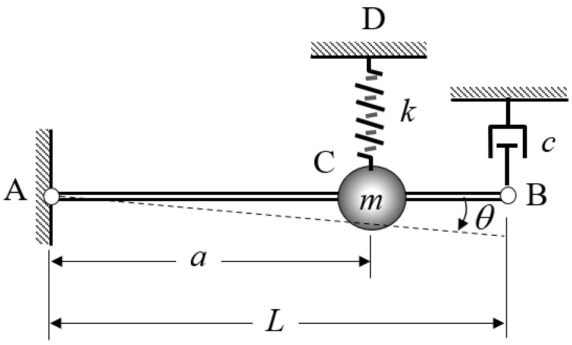

Figure 3

(i) With the help of a neat free body diagram and Newton's 2nd law, derive the equation of motion for the free vibration of the system considering small vibratory displacement θ. From the equation of motion, identify and write down the expressions for effective inertia, effective damping constant and effective stiffness.

(10 marks)

(ii) Determine the damped natural frequency of the system (in Hz) for L = 1 m, a = 0.75 m, k = 5000 N/m, M = 2 kg, m = 1 kg and c = 50 Ns/m.

(7 marks)

(b) Figure 4(a) shows a motor of mass m operating at a speed of 1200 rpm supported on four identical rubber pads. Take the effective stiffness of all four springs put together as k. In starting, the motor passes through a resonant frequency at 600 rpm. Ignore the damping effects of rubber pads.

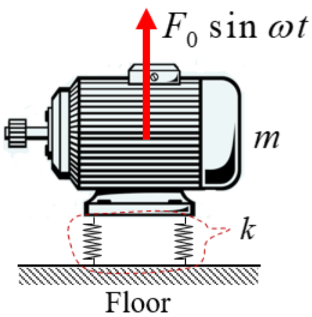

Figure 4(a)

NOTE: Question 3 continues on page 4.

It is intended to reduce the amplitude of force transmitted to the floor by mounting the motor on a heavy mass M which in turn is supported on the floor, as shown in Figure 4(b), by the same four rubber pads mentioned above. It is given that M is 6 times of the mass of the motor m.

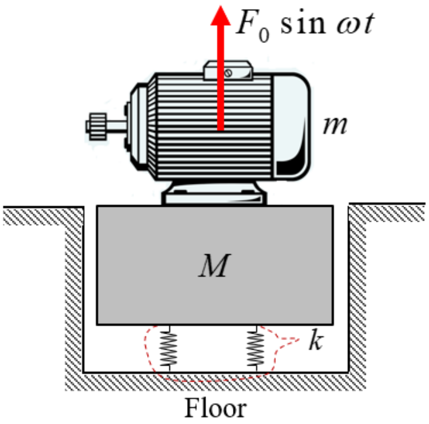

Figure 4(b)

Determine the percentage reduction in the amplitude of force transmitted to the floor. For this calculation, numerical values of k, m and F_0 are not needed and hence are not given.

(8 marks)

4. Figure 5 shows a 2-DOF system wherein a rigid rod ABC of length L and mass M is hinged at B and connected to two springs at points A and C. All the three springs shown have the same stiffness constant k. The rigid block has a mass of m and is subjected to a harmonic excitation force $F_0 \sin\omega t$ as shown. The angular displacement θ of the rod and the linear displacement x of the block are measured with respect to their respective static equilibrium positions. Consider small displacements so that linear vibration theory can be applied.

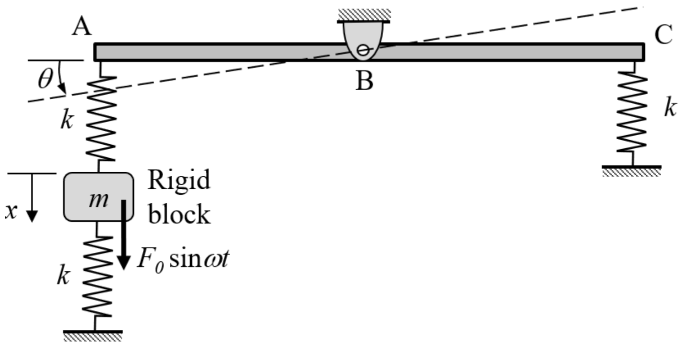

Figure 5

(a) Draw neat free body diagrams of the rigid block and rigid rod indicating all the forces acting on them clearly. Static forces need not be shown. Derive the equations of motion for the system in terms of the coordinates θ and x, and write them in matrix form.

(10 marks)

NOTE: Question 4 continues on page 5.

(b) Determine the natural frequencies of the system (in Hertz) for L = 2 m, M = 3 kg, m = 1 kg, k = 10 N/m and F_0 = 1 N.

(8 marks)

(c) For the same numerical values as in part (b), determine the amplitudes of steady-state vibration of the rigid rod and rigid block taking ω = 5 rad/s.

(7 marks)

END OF PAPER