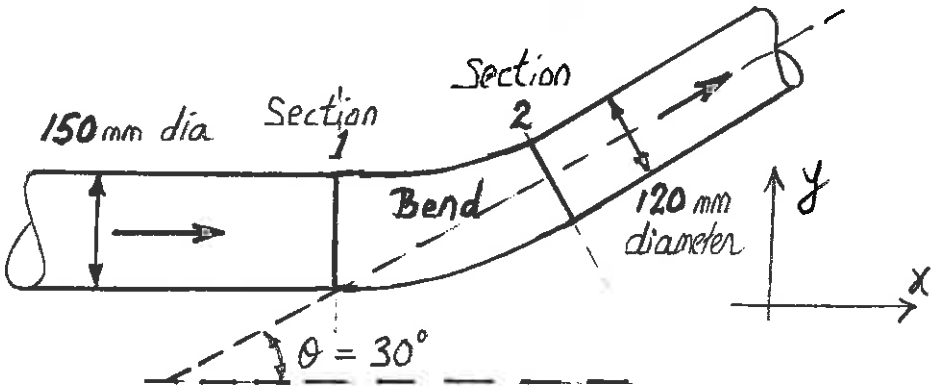
21st Feb 2019 | CA-1 MA 3006 Fluid Mechanics | S/No :

Full Name : ______________________________ **Time : 40 minutes**

Qu 1

Water flows through a reducing pipe bend as shown. The flow rate is $0.02 m^3/s$. Pressure at section 1 and 2 is 280 kPa and 260 kPa respectively. Determine the resultant force and direction to hold the pipe bend stationary.
Weight of nozzle and water inside the nozzle maybe neglected. There is no height difference between 1 and 2.
(Write down the governing equations) (10 marks)

This is internal flow
Hence, there is pressure
P_1 & P_2 acting on CV.



1) Apply BE to determine V_1 and V_2, P_1 & P_2 are given

2) Find $\dot{m}$ kg/s.

3). Apply momentum equation in x and y direction

$$(\Sigma F)_x = \dot{m}(V_{2x} - V_{1x}).$$

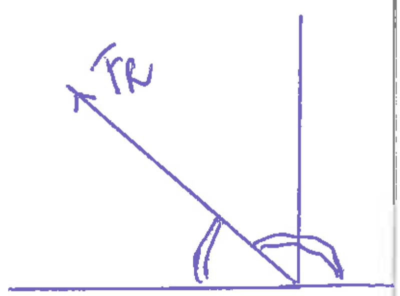
$$F_x + P_1 A_1 - P_2 A_2 \cos 30 = \dot{m}(V_2 \cos 30 - V_1).$$

Note : P_2 acting in opposite direction at angle 30°

$$(\Sigma F)_y = \dot{m}(V_{2y} - V_{1y}).$$

$$F_y - P_2 A_2 \sin 30° = \dot{m}(V_{2y}) \qquad V_{1y} = 0.$$

Resultant force $F_R = \sqrt{F_x^2 + F_y^2}$



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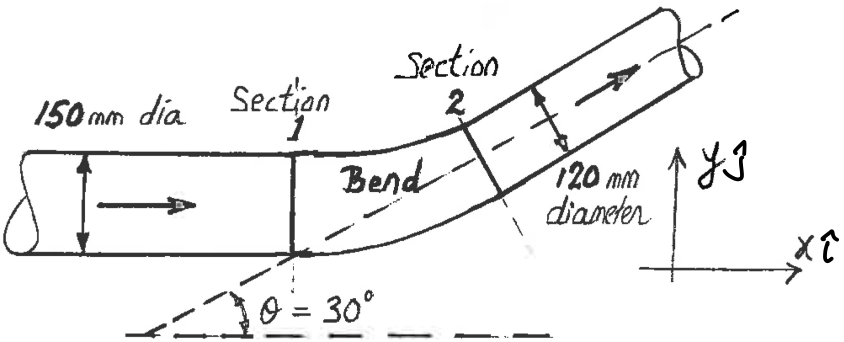


At 1:
$P_1 = 280 kPa$
$V_1 =$
$A_1 = \frac{\pi}{4}(150m)^2 = 0.0177 m^2$

$Q = 0.02 m^3/s$
$\dot{m} = 20 kg/s$

At 2:
$P_2 = 260 kPa$
$V_2 =$
$A_2 = \frac{\pi}{4}(120m) = 0.01131 m^2$

$\dot{m}_1 = \dot{m}_2$
Continuity $\Rightarrow Q_1 = Q_2$

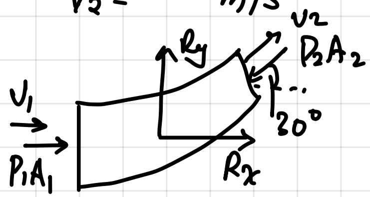
$V_1 =$ m/s
$V_2 =$ m/s

$$P_1 + \cancel{\tfrac{1}{2}\rho g h_1} + \tfrac{1}{2}\rho V_1^2 = P_2 + \cancel{\tfrac{1}{2}\rho g h_2} + \tfrac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \tfrac{1}{2}\rho(V_2^2 - V_1^2)$$

$$\frac{2(P_1 - P_2)}{\rho} = V_2^2 - V_1^2$$

$$\sum \vec{F} = \sum \dot{m}(\vec{V}_2 - \vec{V}_1)$$



$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1}{A_2} V_1$$

$$[R_x + P_1A_1 - P_2A_2\cos 30°]\hat{i} + [R_y - P_2A_2\sin 30°]\hat{j} = 20[(V_2\cos 30° - V_1)\hat{i} + (V_2\sin 30°)\hat{j}]$$

$$\frac{2(P_1 - P_2)}{\rho} = \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]V_1^2$$

In $\hat{j}$:
$R_y = (260k)(0.01131)\sin 30° + 20(8.22\sin 30°)$
$R_y = 1552.5 N$

$V_1 = 5.25 m/s$
$V_2 = 8.22 m/s$

In $\hat{i}$:
$R_x = -(280k)(0.0177) + (260k)(0.0113)\sin 30°) + 20[(8.22\cos 30° - 5.25)]$

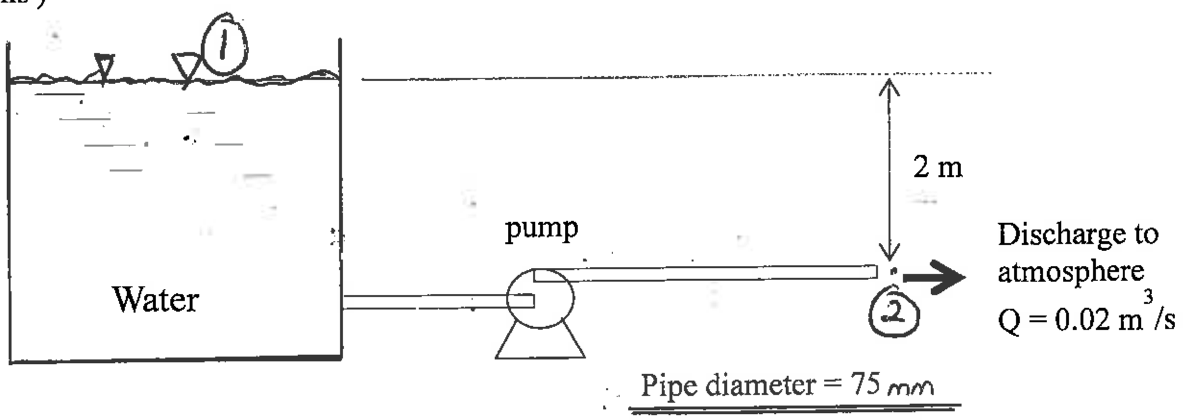
$R_x = -4956 + 1170.3 + 37.37 = -3448.33 N$

Qu.2

Water is delivered from an open tank through a pump and piping system with diameter of 75mm.. Water exits the pipe to the atmosphere at $0.02 m^3/s$. The total loss in the piping system is, $h_L = 6.5V^2/2g$ where V is the velocity of water in the pipe. What is the power required by the pump ?

It is proposed to double the flow rate to $Q = 0.04\ m^3/s$ by sealing the water tank completely and pressurizing the air space. **Determine the air pressure required.** (The pump remains in the piping system and piping system loss is, $h_L = 6.5V^2/2g$.)

(10 marks)



(i) Apply energy equation from 1 ⟶ 2

$$\frac{7.5(4.527^2)}{19.6} + 2$$

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 - h_L + h_p = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + z_2$$

$$h_L \downarrow 6.5\frac{V_2^2}{2g}$$

$$h_p = 9.842 m$$

$Q = 0.02 m^3/s$, hence $V_2 = 4.527 m/s$ hence $h_p = \underline{\quad} \checkmark$

power $= \rho g Q \underline{h_p}$ W

(ii). $Q = 0.04 m^3/s$, hence, new V_2

Same pump head applies

P_1 is not zero, P_1 is now pressurized, (find P_1).

Apply energy equation from 1 ⟶ 2 to find P_1

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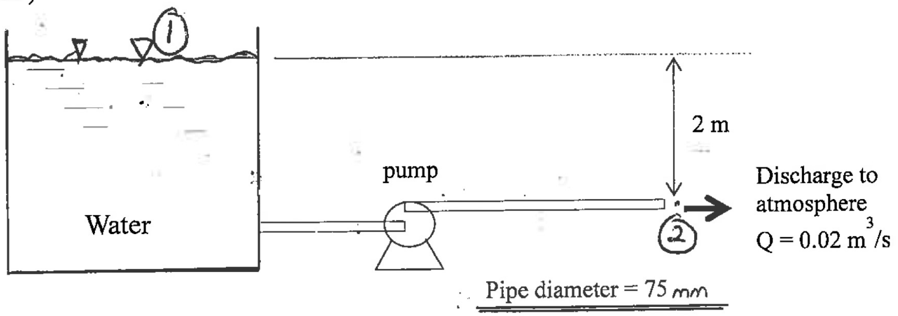


$\check{e} = \frac{P}{\rho} + \frac{1}{2}v^2 + gz$

$\check{U}$ => internal energy

h_L

$\frac{U_{out} - U_{in}}{m} = g h_L$

Energy equation:

$$\Sigma \dot{W}_{in} + \Sigma \dot{m}(\check{e} + \check{u})_{in} = \Sigma \cancel{\dot{W}_{out}}^{0} + \Sigma \dot{m}(\check{e} + \check{u})_{out}$$

$$\dot{w}_{in} = \dot{m}[\check{U}_{out} - \check{U}_{in}] + \dot{m}[\frac{P_2 - P_1}{\rho} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1)]$$

(i) $\check{U}_{out} - \check{U}_{in} = \frac{6.5V_2^2}{2}$

$P_2 - P_1 = 0$, $V_1 = 0$, $z_2 - z_1 = 2m$

$V_2 = \frac{Q}{A_2} = \frac{0.02}{\frac{\pi}{4}(75m)^2} = 4.527 m/s$

$\check{u}_{out} - \check{u}_{in} = 6.8$

$$\dot{w}_{in} = 20(6.8)^{(9.8)} + 20[\frac{1}{2}(4.527)^2 + (9.8)(2)]$$

$$\dot{w}_{in} = 1332.8 + 596.94 = 1929.74\ W$$

(ii) $V_{2new} = 9.054 m/s$

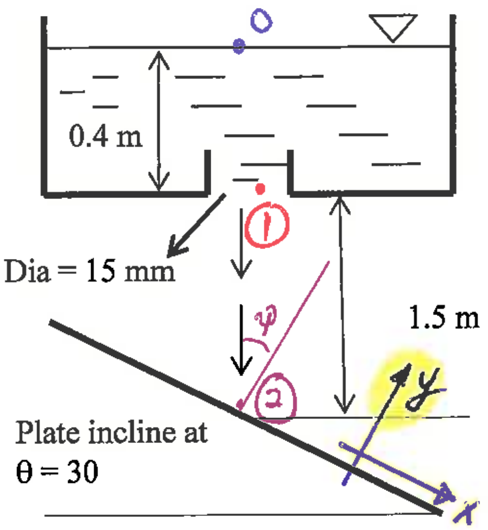
$$\dot{w}_{in} = [\frac{6.5(9.054)^2}{2}] + [\frac{\Delta P}{1000} + \frac{1}{2}(9.054)^2 + g(2)] = \frac{1929.74}{20}$$

$$266.42 + \frac{\Delta P}{1000} + 60.59 = 96.49$$

$\Delta P = -230.52\ kPa$ => $P_1 = 230.52 kPa$

Name : ______________________________ **Time : 40 minutes**

Qu 1 : A jet of water leaves a 15 mm diameter opening at the bottom of the tank and strikes an inclined plate and the water split equally with half of the water flows upwards and the other half downwards. Find the resultant force *F* to hold the plate stationary. Neglect mass of plate and assume water level in tank remains constant. (*10 marks*)
Hint : Find velocity at exit 15 mm diameter opening.



(1) Find velocity at exit point 1. Apply B.E between (0) → (1)

(2) With V_1, find. Q flow rate and hence $\dot{m}$ kg/s.

(3) Jet exit (1) will strike plate at point (2). Find V_2.

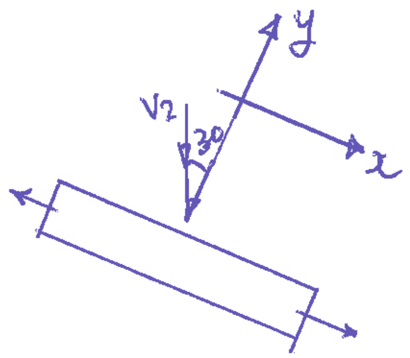
Momentum along the plate (x-direction) = 0.

Apply momentum equation along y-direction (perpendicular to plate)

$$\Sigma F_y = \dot{m}(V_{2y} - V_{1y})$$

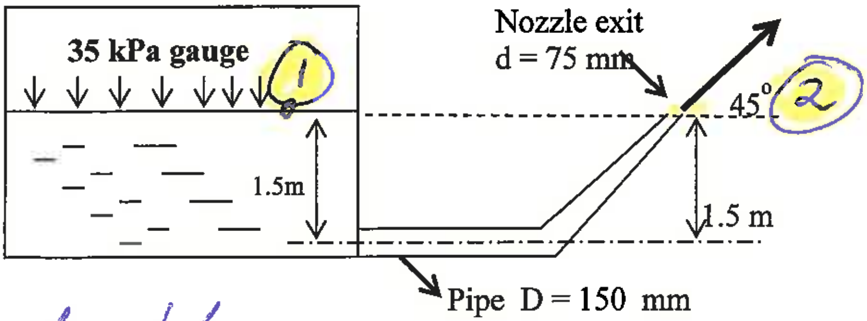
$$\Sigma F_y = (\Sigma \dot{m} V)_{out} - (\Sigma \dot{m} V)_{in}$$

$$F_y = \dot{m}(0 - V_2 \cos 30)$$



Qu 2 : Water is discharged from a pressurised tank (35 KPa) thru a pipe and a nozzle. Total loss for the pipe and nozzle is : $h_L = 1.4\ V_2^2/2g$, where V_2 is the velocity at nozzle exit. a) Find the discharge flow rate. b) if a pump is added to triple the discharge flow rate, find the fluid power required by the pump. Pressure in the tank and h_L remains unchanged ($h_L = 1.4\ V_2^2/2g$)

(10 marks)



(a) Apply energy equation between

① → nozzle exit ②.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 - \text{losses} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$P_2 \to 0.$

$P_1 = 35$ kPa.

$\text{losses} = 1.4 \frac{V_2^2}{2g}.$

V_2 can be found, hence flow rate $Q = A_2 V_2$.

(2). triple flow rate with pump.

$Q * 3$, hence find new velocity V_2

Apply energy equation from ① → 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 - \text{losses} + h_p = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$P_1 = 35$ kPa.

$\left(\frac{1.4 V_2^2}{2g}\right)$

Find h_p ____ = m.

$\text{Power} = \rho g Q h_p$ (W)