NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

MA3002 – SOLID MECHANICS AND VIBRATION

April/May 2018 Time Allowed: 2 ½ h[illegible]

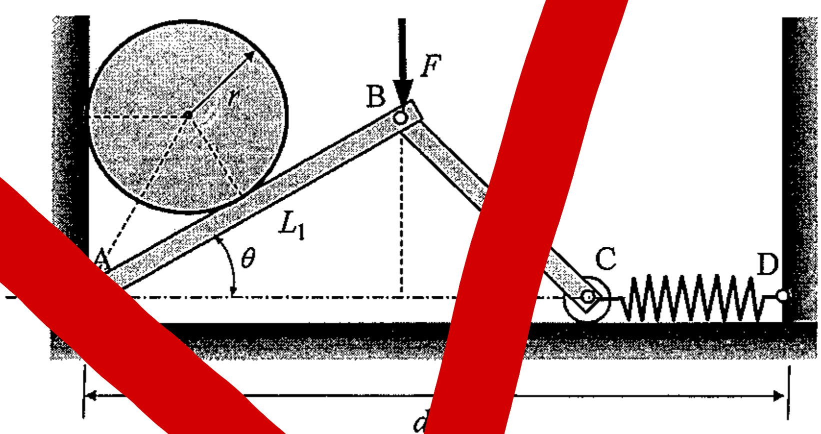
INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SEVEN (7)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN BOOK** examination. One doub[illegible]d A4 size reference sheet is allowed.

1(a) Figure 1 shows the equilibrium configuration of a system where [illegible]nder of mass M and radius r is carried by a pin-jointed structure consisting of tw[illegible] rigid weightless bars AB and BC, and a spring CD of stiffness k. The bars A[illegible]BC are of lengths L_1 and L_2, respectively. In the configuration shown, the spri[illegible]compressed by the combined action of the weight of the cylinder and the forc[illegible]plied at B. Assume the spring does not buckle. The free length of the spring ([illegible]e initial length of the spring before it is compressed) is l_0. Assume that all th[illegible]nbers in the structure except the spring are rigid. Ignore the effect of frict[illegible] joints and contacting surfaces.

Derive an expression for the force F in terms of θ an[illegible] system parameters using the *principle of virtual work*. Show all the coordinate[illegible] virtual displacements used for the purpose on a neat sketch of the system.

(13 marks)



Figu[illegible]

Note: Question 1 continues on page [illegible]

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1(a) Figure 1 shows the equilibrium configuration of a system where a cylinder of mass M and radius r is carried by a pin-jointed structure consisting of two thin rigid weightless bars AB and BC, and a spring CD of stiffness k. The bars AB and BC are of lengths L_1 and L_2, respectively. In the configuration shown, the spring is compressed by the combined action of the weight of the cylinder and the force F applied at B. Assume the spring does not buckle. The free length of the spring (i.e, the initial length of the spring before it is compressed) is l_0. Assume that all the members in the structure except the spring are rigid. Ignore the effect of friction at joints and contacting surfaces.

Derive an expression for the force F in terms of θ and other system parameters using the *principle of virtual work*. Show all the coordinates and virtual displacements used for the purpose on a neat sketch of the system.

(13 marks)

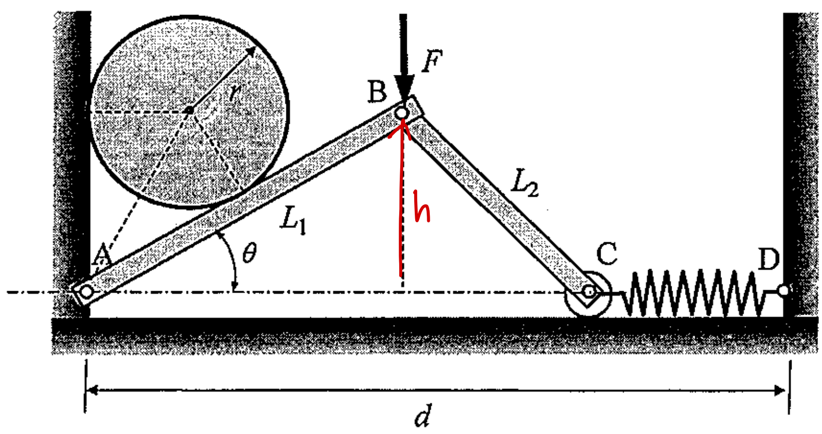

Figure 1

Note: Question 1 continues on page 2.

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(b) Figure 2 shows a statically indeterminate weightless beam structure ABC of flexural rigidity *EI*. The lengths of portion BC and AB are *L* and *L*cos30°, respectively. End A is rigidly fixed to a wall and end C is guided to move vertically. A vertical load *W* is applied at point B as shown. Consider only bending effects and ignore friction at the vertical guide.

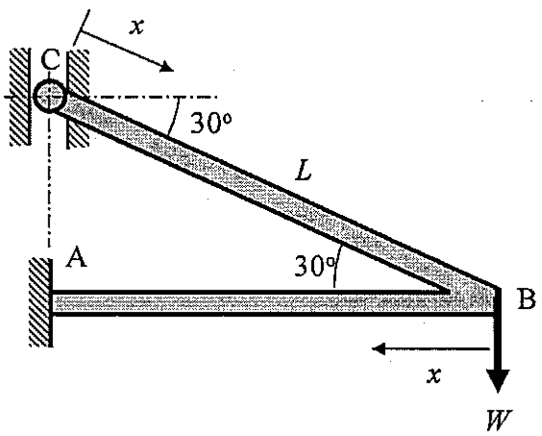

Figure 2

(i) Mark all unknown support reactions on a neat sketch and show that the degree of indeterminacy is 1.

(2 marks)

(ii) Determine the horizontal reaction force at C by *unit load method*.

(10 marks)

2(a) A steel tension member in a crane has a circular cross section with 20 mm diameter. The supply of steel tension members from a vendor has yield stress σ_Y = 800 MPa and fracture toughness K_{Ic} = 30 $MNm^{-3/2}$, and is believed to have central penny-shaped internal cracks in the cross section. The non-destructive test facility available in the laboratory is not capable of detecting cracks smaller than 3 mm diameter.

Assume the cracks are well separated and interaction between cracks is negligible, and hence, a single isolated crack as shown in Figure 3 can be considered for the analysis. For penny-shaped cracks, the geometry correction factor $Y = 2/\pi$. Ignore plastic zone correction. Assume linear elastic fracture mechanics holds.

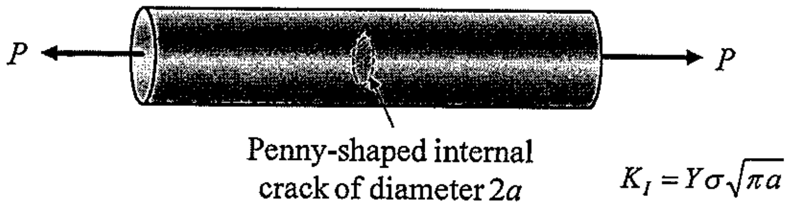

Figure 3

Note that the rod can fail by yielding ($\sigma > \sigma_Y$) or by fracture ($K_I > K_{Ic}$).

(i) Determine the maximum tensile load P that can be carried by the member purely based on yield failure (assuming no cracks). Also determine the maximum tensile load P that can be carried purely based on fracture consideration. Comment on how the presence of cracks affects the load carrying capacity of the rod.

(7 marks)

(ii) Determine the maximum crack size ($2a_c$) that can be permitted so that the maximum load carrying capacity with cracks is the same as that without cracks.

(4 marks)

Note: Question 2 continues on page 4.

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(b) A beam of width $W = 50$ mm and thickness $t = 50$ mm is subjected to 4-point bending as shown in Figure 4 where $b = 100$ mm. At the mid-section of the beam, there is a through edge crack of depth $a = 5$ mm on the bottom surface. The fracture toughness of the material is $K_{Ic} = 50$ MNm$^{-3/2}$. Assume linear elastic fracture mechanics holds.

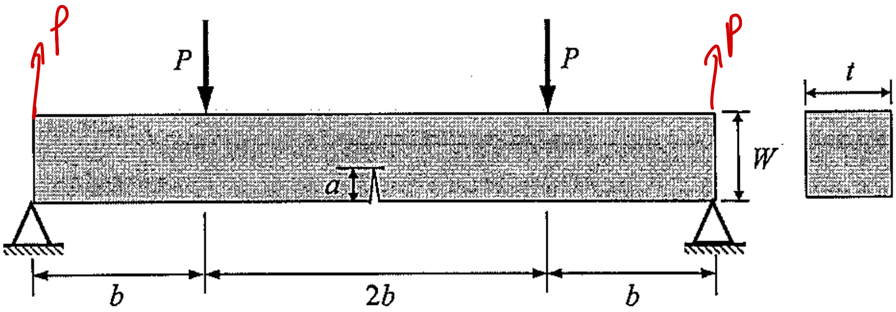

Figure 4

The formula for the stress intensity factor for the crack is as follows:

$$K_I = Y\sigma\sqrt{\pi a}$$

where $Y = 1.122 - 1.4\beta + 7.33\beta^2 - 13.08\beta^3 + 14.0\beta^4$ is the geometry correction factor, $\beta = a/W$, $\sigma = 6M/(tW^2)$ is the nominal bending stress and $M = Pb$ is the bending moment at the mid-section of the beam.

(i) Determine the maximum value of load P that the beam can carry before fracture.

(7 marks)

(ii) If the load P in Figure 4 fluctuates sinusoidally between 0 and 60 kN once every day, determine how many years it will take for the crack to grow in length by 20%. Assume the geometry correction factor (Y) to remain constant at the value you calculated in part (i). Use Paris law $da/dN = C(\Delta K)^m$ for this calculation where da/dN is in m/cycle and ΔK is in MNm$^{-3/2}$, $m = 3.5$ and $C = 0.25\text{x}10^{-11}$.

(7 marks)

3(a) Figure 5 shows a durian fruit of mass 2.5 kg hanging from a tree branch. Free oscillation of the branch with the durian on is different from that of the branch alone (after durian falling). It is observed that the period of free oscillation for the branch with the durian on is 1.5 s. After the durian fell, the branch alone oscillates freely faster at a period of 1.0 s.

(i) By neglecting the damping, estimate the effective mass and effective stiffness of the branch. Estimate also the static displacement Δ of the branch upon unloading (after durian falling).

(6 marks)

(ii) Explain why the effective lumped mass calculated above is smaller than the actual mass of the branch.

(3 marks)

(iii) Calculate the damping constant based on the damped free vibration of the branch with durian whose amplitude decays in a ratio of 2 to 1 in a consecutive cycle.

(2 marks)

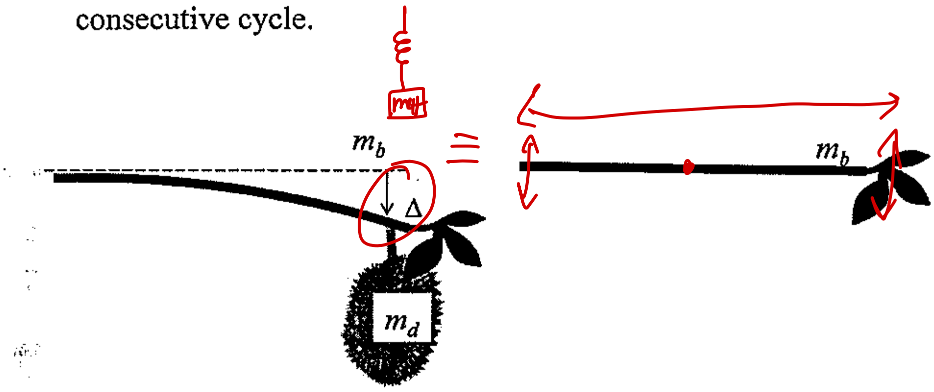

Figure 5

(iv) Estimate the damping ratio for the branch alone that undergoes damped free oscillation. (Use the solution obtained above in the Question 3a(i)-(iii).)

(3 marks)

(v) Identify and describe the possible damping mechanisms that dissipate the energy from this freely oscillating branch with the durian on.

(4 marks)

Note: Question 3 continues on page 6.

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(b) Figure 6 shows a boy sitting on a swing and being pumped (pushed) by a harmonic force $F_o \sin\omega t$ where F_o is the force amplitude and ω is the angular frequency of pumping in the horizontal direction. This forced oscillation can be modelled by a simple pendulum which consists of a lumped mass m swinging at a distance L from the pivot.

(i) Write the equation of motion for this pendulum under forced vibration.

(ii) Find the amplitude of angular swing based on the assumption of a small angle.

(7 marks)

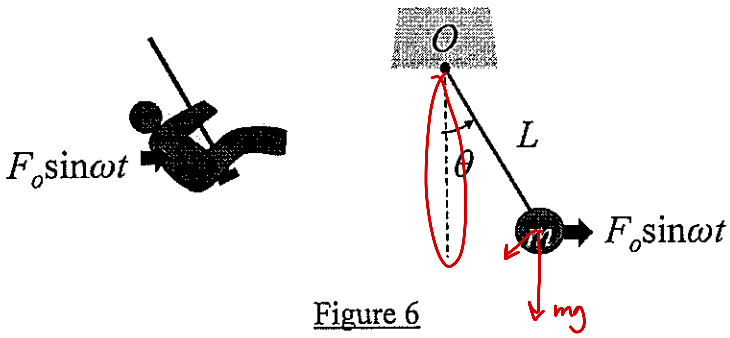

Figure 6

4(a) Figure 7 shows a ceramic vase of mass m being packed in a box with packaging foams of a total stiffness k. During the shipping by post, this box of vase is subjected to various handling conditions which include a base excitation (of conveyor belt) and a drop test.

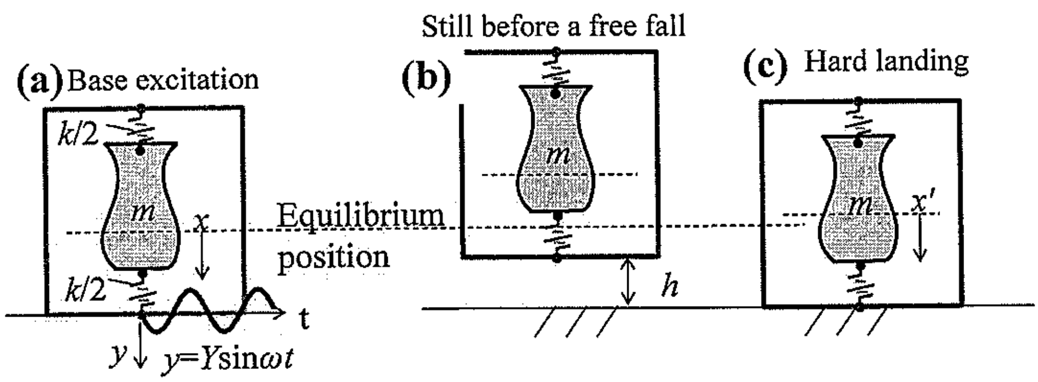

Figure 7

(i) Derive the expression of acceleration at which the vase undergoes due to base excitation $y = Y\sin\omega t$ that bounces the box where Y and ω are the amplitude and angular frequency respectively of the base motion.

(4 marks)

Note: Question 4 continues on page 7.

(ii) Derive the expression of the acceleration amplitude imparted to the vase during a drop test. The drop test has the box released from rest and falling through a height h before landing hard on the rigid ground.

(6 marks)

(b) Figure 8 shows the box of vase (as described in Question 4(a)) now being placed on a leverage plate with spring support. This plate of a moment of inertia J_o is pivoted at point O and supported by a spring of stiffness K at a distance b from the pivot O. The box of vase is located at a distance a from the pivot O.

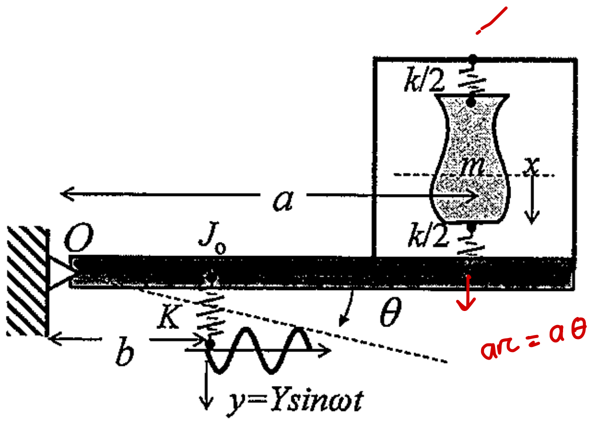

Figure 8

(i) Determine the amplitude of harmonic motion x of the vase due to the base excitation y acting at the lower spring support of the plate.

(7 marks)

(ii) Find the lowest natural frequency and the corresponding mode shape for this two-degree-of-freedom system undergoing free vibrations (upon removal of base excitation). Sketch the mode shape (i.e. the lever's angular displacement amplitude per unit vase's displacement amplitude). Given the leverage design having $J_o = ma^2$ and the base spring having the same stiffness as the foam, $K = k$.

(8 marks)

END OF PAPER

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1a) $\delta W^* = \delta U^*$ Work $= F \times d$ or Moment $\times \delta\theta$

$\tan\theta = \frac{opp}{adj}$

$h = L_1 \sin\theta$

$\Sigma P\delta x = \Sigma k e \delta e$

$\delta h = \frac{dh}{d\theta}\delta\theta = L_1\cos\theta\,\delta\theta$

$-F\delta h - M_c\delta\theta = ke\,\delta e$

$e = l_0 - \left(d - L_1\cos\theta - \sqrt{L_2^2 - L_1^2\sin^2\theta}\right)$

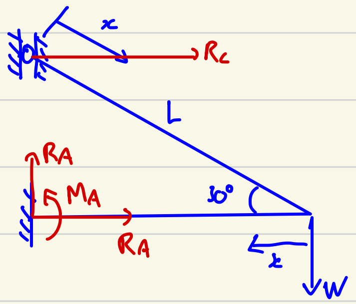
$-FL_1\cos\theta\,\cancel{\delta\theta} - Mg\cos\theta\left(\frac{r}{\tan\frac{90-\theta}{2}}\right)\cancel{\delta\theta}$

$\delta e = \frac{de}{d\theta}\delta\theta = -L_1\sin\theta + \frac{(-L_1^2\sin\theta\cos\theta)}{\sqrt{L^2 - L_1^2\sin^2\theta}}\,\delta\theta$

$= k\left(l_0 - d + L_1\cos\theta + \sqrt{L_2^2 - L_1^2\sin^2\theta}\right)\left(-L_1\sin\theta + \frac{-L_1^2\sin\theta\cos\theta}{\sqrt{L_2^2 - L_1^2\sin^2\theta}}\right)\cancel{\delta\theta}$

$$F = -\frac{Mg}{L_1}\left(\frac{r}{\tan\frac{90-\theta}{2}}\right) + k\left(l_0 - d_1 + L_1\cos\theta + \sqrt{L_2^2 - L_1^2\sin^2\theta}\right)\left(\tan\theta + \frac{L_1\sin\theta}{\sqrt{L_2^2 - L_1^2\sin^2\theta}}\right) //$$

1b)



i) There are four support reaction forces and only 3 equations to use, $\Sigma F_y = 0$, $\Sigma F_x = 0$, $\Sigma M_0 = 0$ for static equilibrium. Hence indeterminacy $= 4 - 3 = 1$ //

ii) Real Load:

$M_{CB} = (R_c \sin 30)x$

$M_{BA} = (R_c \sin 30)L + Wx$

Virtual Load:

$m_{cb} = (1\sin 30)x$

$m_{BA} = (1\sin 30)L$

To find R_c:

$$1.(\Delta_c) = \frac{1}{EI}\int_0^L M m_{CB}\,dx + \frac{1}{EI}\int_0^{L\cos\theta} M m_{BA}\,dx$$

$$= \frac{R_c}{EI}\int_0^L \frac{x}{2}\times\frac{x}{2}\,dx + \frac{1}{EI}\int_0^{L\cos\theta} \frac{R_c L}{2}\times\frac{L}{2} + \frac{Wx L}{2}\,dx$$

$$= \frac{R_c}{EI}\left[\frac{x^3}{12}\right]_0^L + \frac{1}{EI}\left[\frac{R_c L^2 x}{4} + \frac{WLx^2}{2}\right]_0^{L\cos\theta}$$

$$= \frac{R_c L^3}{12EI} + \frac{1}{\cancel{EI}}\left[\frac{R_c L^3\cos\theta}{4EI} + \frac{WL^3\cos^2\theta}{2EI}\right]$$

BC: $(\Delta_c) = 0$

$$\frac{R_c L^3}{12} + \frac{R_c L^3}{4}\left(\frac{\sqrt{3}}{2}\right) + \frac{WL^3}{2}\left(\frac{3}{4}\right) = 0$$

$$R_c\left(\frac{L^3}{12} + \frac{\sqrt{3}L^3}{8}\right) = -\frac{3WL^3}{8}$$

$$\therefore R_c = \frac{-3/8}{\frac{1}{12} + \frac{\sqrt{3}}{8}}\,W$$

2a) Yield failure :

$\sigma = \frac{P}{A} = \sigma_Y = 800 \times 10^6$

$P = 800 \times 10^6 \times (\pi \times 0.010^2) = 251.327 \text{ kN}$

Crack fracture:

$K_{Ic} = Y \sigma_f \sqrt{\pi \alpha_c}$ (Assume worst case scenario, a crack of 2.9mm diameter exists)

$\sigma_f = \frac{K_{Ic}}{Y\sqrt{\pi \alpha_c}}$

$= \frac{30 \times 10^6}{\frac{2}{\pi}\sqrt{\pi \times 0.0015}} = 686 \text{ MPa}$

$\therefore P = 215.66 \text{ kN}$

The presence of cracks will reduce the carrying capacity of the load.

at $\alpha_c = \left(\frac{K}{Y\sigma_f}\right)^2 \frac{1}{\pi} = \left(\frac{30 \times 10^6}{\frac{2}{\pi} \times 800 \times 10^6}\right)^2 \frac{1}{\pi} = 1.1\text{mm}, \therefore 2\alpha = 2.2\text{mm}.$

Cracks larger than 2.2mm.

2aii) Found above at 2.2089mm

2bi) $\beta = \frac{\alpha}{W} = \frac{5}{50} = \frac{1}{10}$ $M = P \times 2b - P \times b = Pb$

$Y = 1.04362$, $\sigma = \frac{6Pb}{tW^2} = 4800 P$

$K_I = Y\sigma\sqrt{\pi \alpha}$

$\sigma = \frac{K_I}{Y\sqrt{\pi\alpha}} = \frac{50 \times 10^6}{1.04362\sqrt{\pi \times 0.005}} = 4800P$

$\therefore P = 79\,639 \text{ N}$

2bii) Paris Law: $S_R = \frac{6(P_r)b}{tW^2} = \frac{6 \times 60\,000 \times 0.1}{0.05 \times 0.05^2} = 288$

$$N_f = \frac{2\left(0.006^{-0.75} - 0.005^{-0.75}\right)}{0.25 \times 10^{-11}(1.04362 \times 12)^{3.5}\pi^{1.75}(-1.5)} = 1038.8$$

$\therefore 2.84$ years.

3ai) $\omega_n = \sqrt{\frac{k_{eff}}{m_{eff}}}$

with durian: $\frac{2\pi}{1.5} = \sqrt{\frac{k_{eff}}{m_b + 2.5}}$ $\Rightarrow k_{eff} = 17.546 m_b + 43.865$

no durian: $\frac{2\pi}{1} = \sqrt{\frac{k_{eff}}{m_b}}$ $\Rightarrow k_{eff} = 39.4784 m_b$

$\therefore m_b = 2 \text{ kg}, \ k_{eff} = 78.957 \text{ N/m}$

$F = k\Delta, \ \Delta = \frac{F}{k} = \frac{2 + 2.5}{78.957} = 0.057\text{m}$

3aii) The lumped mass only takes into account the mass that affects oscillation, in this scenario, the mass closer to the pivot will contribute less to the lumped mass as its moment arm is shorter, and it moves less distance than Δ. In the formula $m_{eff}g = k_{eff}\Delta$, to fix Δ as the displacement read at the tip of the beam, m_{eff} must drop, (Δ is overestimated to characterise the motion of m_b) hence $m_{eff} < $ real mass.

3aiii) $\delta = \ln 2$

$$\zeta = \sqrt{\frac{(\ln 2)^2}{4\pi^2 + (\ln 2)^2}} = 0.10965$$

$$C_c = 2\sqrt{k_{eff}\, m_{eff}} = 37.7 \text{ Ns/m}$$

$$C = \zeta C_c = 4.133712 \text{ Ns/m}$$

3aiv) $C_c = 2\sqrt{k_{eff}\, m_{eff}} = 2\sqrt{78.957 \times 2.5} = 28.1$

$$\zeta = \frac{c}{C_c} = 0.147111$$

3av) Plastic deformation of the branch as it oscillates (energy lost as heat, sound and elongation, not returned to system)

Air resistance (energy lost as KE of air, sound and heat, not returned to system)

3bi) EOM: $mL^2\ddot{\theta} + (mg\sin\theta)L = F_0 \sin wt\,(L\cos\theta)$

$$mL^2\ddot{\theta} + mgL\,\theta = F_0 L \sin wt \cos\theta \qquad \text{for small } \theta,\ \sin\theta \approx \theta,\ \cos\theta \approx 1 - \frac{\theta^2}{2} \approx 1$$

$$mL\ddot{\theta} + mg\,\theta = F_0 \sin wt$$

ii) $\ddot{\theta} = -w^2\theta$, $\theta = \Theta \sin wt$

$$mL(-w^2)\Theta + mg\,\Theta = F_0$$

$$\Theta = \frac{F_0}{mg - mLw^2}$$

4ai) EOM: $m\ddot{x} + kx = kY\sin wt$

$$\ddot{x} = -w^2 x$$

$$-mw^2 x + kx = kY\sin wt$$

$$x = \frac{kY\sin wt}{k - mw^2}$$

$$\therefore\ \ddot{x} = w^2\left(\frac{kY\sin wt}{mw^2 - k}\right)$$

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh}$$

4aii) Drop test (no damping):

General solution:

$$x(t) = A\cos W_n t + B\sin W_n t + \frac{mg}{k} \qquad W_n = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k}{m}}$$

at $t=0,\ x=0 \Rightarrow A = -\frac{mg}{k}$

at $t=0,\ \dot{x} = V_0 = \sqrt{2gh} \Rightarrow BW_n = \sqrt{2gh}$

$$x(t) = -\frac{mg}{k}\cos W_n t + \frac{\sqrt{2gh}}{W_n}\sin W_n t + \frac{mg}{k}$$

$$\ddot{x}(t) = +W_n^2\left(\frac{mg}{k}\right)\cos W_n t - \sqrt{2gh}\,W_n \sin W_n t$$

$$= \frac{k}{m}\left(\frac{mg}{k}\right)\cos\sqrt{\frac{k}{m}}\,t - \sqrt{2gh}\sqrt{\frac{k}{m}}\sin\sqrt{\frac{k}{m}}\,t$$

$$= g\cos\sqrt{\frac{k}{m}}\,t - \sqrt{\frac{2ghk}{m}}\sin\sqrt{\frac{k}{m}}\,t$$

amplitude $= \sqrt{g^2 + \frac{2ghk}{m}}$ (R formula)

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\sum m\ddot{x} = \sum F$$

4b) EOM_1 (vane) : $m\ddot{x} = -k(x - a\theta)$

$$m\ddot{x} + kx - ka\theta = 0$$

$k(\Delta x)$ $\parallel$ $\underline{b} \times f$

$$\sum J\ddot{\theta} = \sum M$$

driving moment

EOM_2 (plate) : $J_o\ddot{\theta} = -k(a\theta - x)a + b \times K(Y\sin\omega t - b\theta)$

$$J_o\ddot{\theta} + ka^2\theta + b^2K\theta - kax = bKY\sin\omega t$$

$$\begin{bmatrix} m & 0 \\ 0 & J_o \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} k & -ka \\ -ka & ka^2 + b^2K \end{bmatrix}\begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ bKY\sin\omega t \end{Bmatrix}$$

$$\ddot{x} = -\omega^2 X\sin\omega t, \quad \ddot{\theta} = -\omega^2 \Theta \sin\omega t$$

$$\begin{bmatrix} k - m\omega^2 & -ka \\ -ka & ka^2 + b^2K - J_o\omega^2 \end{bmatrix}\begin{Bmatrix} X \\ \Theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ bKY \end{Bmatrix}$$

Cramer's Rule:

$$X = \frac{\det\begin{bmatrix} 0 & -ka \\ bkY & ka^2 + b^2k - J_o\omega^2 \end{bmatrix}}{\det\begin{bmatrix} k - m\omega^2 & -ka \\ -ka & ka^2 + b^2k - J_o\omega^2 \end{bmatrix}} = \frac{k^2abY}{(k - m\omega^2)(ka^2 + b^2K - J_o\omega^2) - k^2a^2}$$

4bii) $\det\begin{bmatrix} k - m\omega^2 & -ka \\ -ka & ka^2 + b^2K - J_o\omega^2 \end{bmatrix} = 0$

$$(k - m\omega^2)(ka^2 + kb^2 - J_o\omega^2) = k^2a^2$$

$$J_o m\omega^4 - (ka^2m + kb^2m + J_o k)\omega^2 + \cancel{k^2a^2} + k^2b^2 = \cancel{k^2a^2}$$

$$\min \omega^2 = \frac{-b - \sqrt{b^2 - 4AC}}{2a}$$

$$= \frac{ka^2m + kb^2m + J_o k - \sqrt{(ka^2m + kb^2m + J_o k)^2 - 4J_o m k^2b^2}}{2J_o m}$$

$f = \frac{\omega}{2\pi}$. If $J_o = ma^2$ and $K = k$,

$$\min \omega^2 = \frac{2ka^2m + kb^2m - \sqrt{(2ka^2m + kb^2m)^2 - 4m^2a^2k^2b^2}}{2ma^2}$$

$$= \frac{k}{m} + \frac{kb^2}{2ma^2} - \frac{1}{2m^2a^2}\sqrt{4k^2a^4m^2 + k^2b^4m^2}$$

$$= \frac{k}{m} + \frac{kb^2}{2ma^2} - \frac{k}{m}\sqrt{\cancel{k^2}1 + \cancel{k^2}\frac{b^4}{4a^4}}$$

$$= \frac{k}{m}\left[1 + \frac{b^2}{2a^2} - \sqrt{1 + \frac{b^4}{4a^4}}\right]$$

From eq 1: $\begin{bmatrix} k - m\omega^2 & -ka \end{bmatrix}\begin{Bmatrix} X \\ \Theta \end{Bmatrix} = 0$

$$\left\{k - k\left[1+\frac{b^2}{2a^2} - \sqrt{1+\frac{b^4}{4a^4}}\right]\right\}X - ka\,\Theta = 0$$

$\uparrow \sqrt{a^2+b^2}$ vs $b \downarrow$

$$\frac{\Theta}{X} = \frac{1-\left(1+\frac{b^2}{2a^2} - \sqrt{1+\frac{b^4}{4a^4}}\right)}{a} \quad (+ve)$$

for minimum ω,

$$\frac{\Theta}{X} = \frac{1-\left(1+\frac{b^2}{2a^2} + \sqrt{1+\frac{b^4}{4a^4}}\right)}{a} \quad (-ve)$$