NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2023-2024

MA3002– SOLID MECHANICS AND VIBRATION

April/May 2024 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is **RESTRICTED OPEN BOOK EXAMINATION**. You are allowed to bring into the examination hall one double-sided A4-size reference sheet with texts handwritten or typed on the A4 paper or one restricted material as instructed by the examiner(s) without any attachments (e.g. sticky notes, post-it notes, gluing or stapling of additional papers).

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1. A steel spring component, semi-circular in shape of radius R, is fixed at one end and the other end is constrained such that it can only move in the horizontal direction and under a force P, as shown in Figure 1. Assuming the material is linear elastic with flexural rigidity EI and consider bending actions only.

(a) State the degree of indeterminacy of the spring.

(4 marks)

(b) Determine the horizontal displacement at the point of application of the force P in terms of P, R and EI.

(13 marks)

(c) Derive the maximum bending moment on the spring under this loading in terms of P and R.

(8 marks)

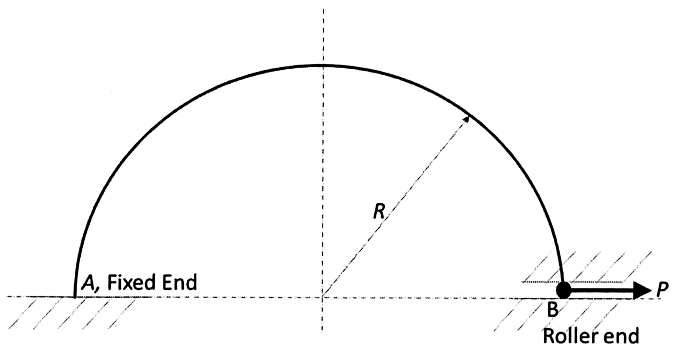

Figure 1: A semi-circular steel spring, fixed at one end and constrained to move only horizontally at the other end, is subjected to a horizontal force P.

2 (a) During the investigation of a railway accident, it was discovered from the fracture surface, the wheel shaft failed due to fatigue. The failure was caused by the crack propagation of an initial edge crack. It was recorded that the failed shaft was under a cyclic stress between 360 MPa and -360 MPa and had survived 18,000 cycles before a sudden rupture. Assume Y=1.12. The shaft is made of high strength steel with a fracture toughness of 65 MPa$\sqrt{\text{m}}$ with the crack length a satisfying the Paris Law

$$da/dN \text{ (m/cycle)} = 2.1\times10^{-12}\ \Delta K^{3.4}$$

where ΔK is in MPa $\sqrt{}$m unit.

(i) Determine the maximum crack length before fracture using Linear Elastic Fracture Mechanics (LEFM).

(5 marks)

(ii) The detection capability of the Nondestructive Inspection (NDI) is 0.5mm. Assuming the worst case that the initial edge crack size is 0.5mm, calculate the theoretical number of fatigue cycles to fracture. By comparing the difference between the actual number of cycles to failure (18,000 cycles) as observed in the field and your calculated solution, determine if the NDI inspection has been conducted correctly. Explain why.

(10 marks)

(b) A stepped rod with the dimension shown in Figure 2a is subjected to a cyclic axial load of an amplitude of P. The rod is made of quenched and tempered steel with an endurance limit of $S_e = 400$ MPa and ultimate tensile strength of 1600 MPa. Using the stress concentration factor K_t in Figure 2b and notch sensitivity factor q in Figure 2c, determine the maximum P allowed for the rod to achieve an infinite life.

(10 marks)

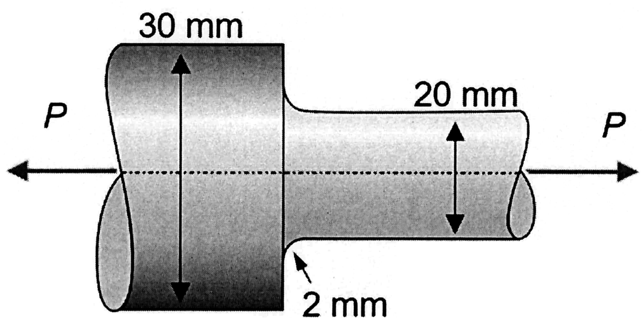

Figure 2a

Note: Figures 2b & 2c appear on page 3.

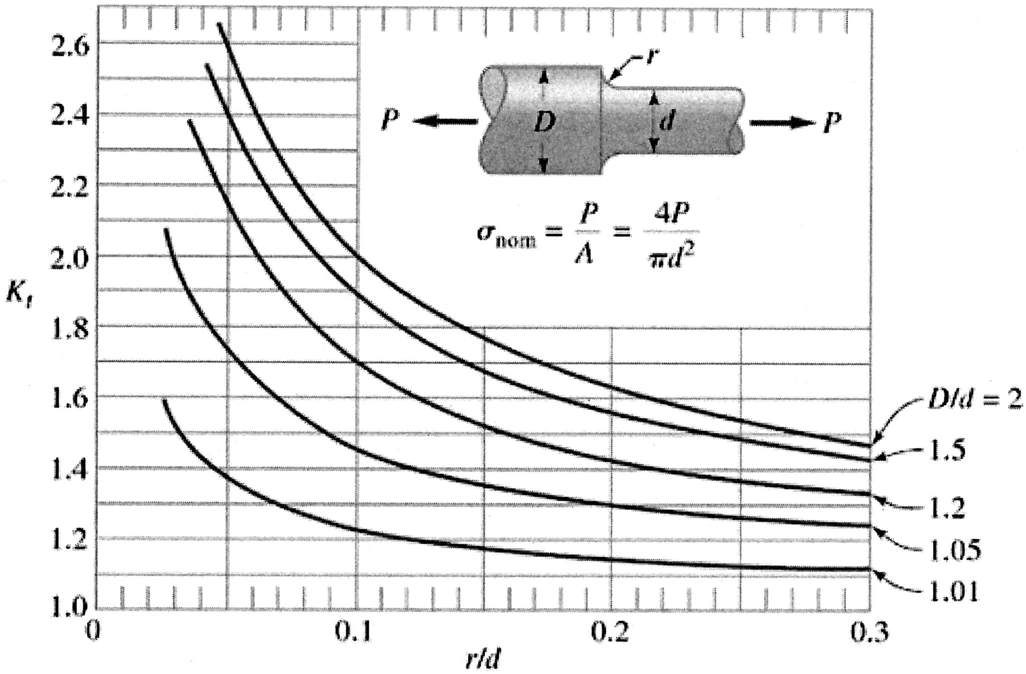

Figure 2b

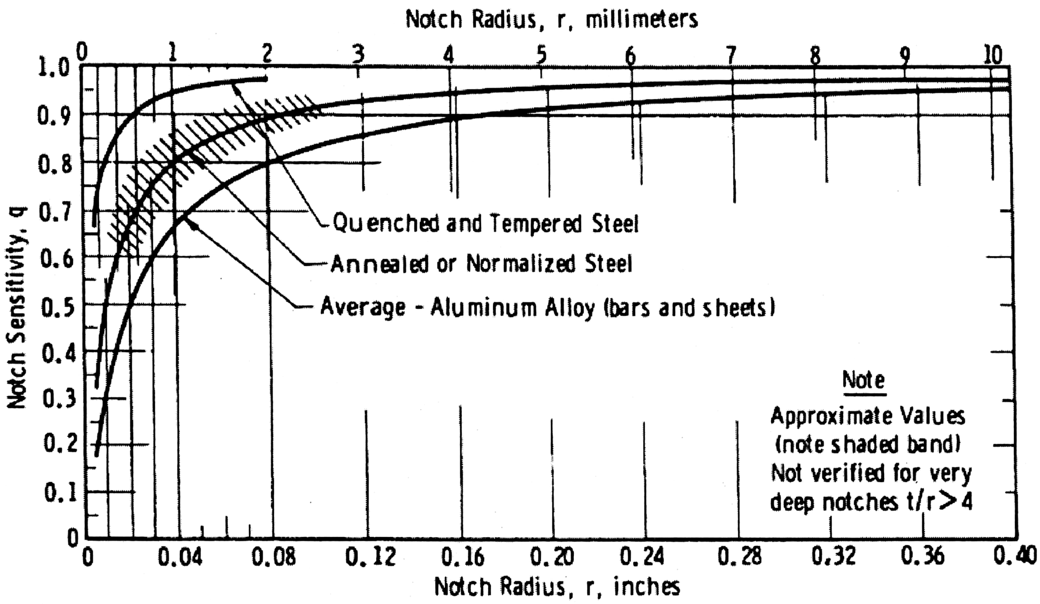

Figure 2c

3 (a) A uniform solid cylinder of radius r and mass m rolls without slipping inside a cylindrical surface of radius $R > r$, as shown in Figure 3. The center C of the cylinder can be viewed as oscillating with respect to the center O of the cylindrical surface. Denote the angle of oscillation by θ and the rolling angle by φ as shown.

(i) Show, with clear explanation, that the oscillating angular velocity $\dot{\theta}$ and the rolling angular velocity $\dot{\varphi}$ are related by the expression: $(R - r)\dot{\theta} = r\dot{\varphi}$.

(3 marks)

(ii) Draw the free body diagram of the cylinder, derive its equation of motion in terms of $\ddot{\theta}$ and θ. Assume small angles. Note that the mass moment of inertia of the cylinder with respect to its longitudinal axis is $J_C = mr^2/2$.

(8 marks)

(iii) Determine the natural frequency of oscillation of the cylinder.

(4 marks)

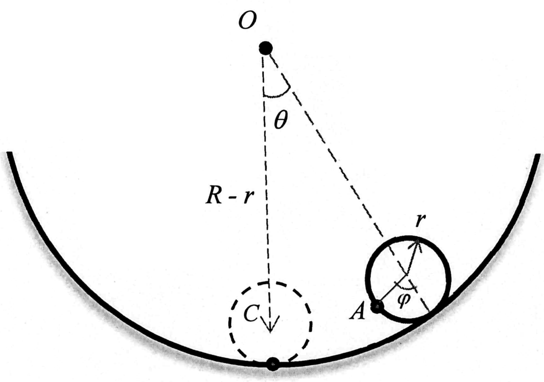

Figure 3: A uniform cylinder of radius r rolling without slipping on a cylindrical surface of radius R.

Note: Question 3 continues on page 5.

(b) An electric motor has a total mass M which contains an eccentric mass m with eccentricity e. The magnification ratio under steady-state motion is given by:

$$\frac{MX}{me} = \frac{\bar{\omega}^2}{\sqrt{\left(1-\bar{\omega}^2\right)^2 + (2\zeta\bar{\omega})^2}},$$

where X is the amplitude, $\bar{\omega} = \omega / \omega_n$ is the ratio of the driving frequency ω to the natural frequency ω_n, and ζ is the damping ratio. Consider two cases: (i) the motor is supported on rubber pads with the overall stiffness k, and (ii) an additional identical set of pads is attached to the original set in series. For Case (i), the motor is run at forced resonance $\omega_1 = \omega_{n1}$. For Case (ii), it is run at the same frequency as in Case (i), i.e., $\omega_2 = \omega_{n1} \neq \omega_{n2}$ The amplitudes for the two cases are related by $X_2 = X_1 / 1.6$. Determine the damping ratios ζ_1 and ζ_2 for the two cases.

(10 marks)

4. Two masses $m_1 = 2$ kg and $m_2 = 1$ kg are attached at the distances of $d_1 = 1$ m and $d_2 = 2$ m respectively from the fixed end of a cantilever, as shown in Figure 4. The cantilever is assumed to be of negligible mass and linearly elastic with constant flexural rigidity EI. Neglecting the deflection profile, it is also assumed to be horizontal when the masses are in static equilibrium on it.

(a) Determine the equations of motion for vertical free vibrations of the two-degree-of-freedom system. The x-y reference frame is as indicated in the figure.

Hint: For a force P acting on the cantilever at the position $x = d$ from the fixed end, the deflection equations for y are given by:

$$y = \tfrac{Px^2}{6EI}(3d - x) \qquad 0 \le x \le d,$$
$$y = \tfrac{Pd^2}{6EI}(3x - d) \qquad d \le x \le l,$$

where l is the cantilever length. For two forces P_1 and P_2 at the positions d_1 and d_2, the vertical deflection y_1 at the first location $x_1 = d_1$ is made up of the deflections due to P_1 and P_2, i.e., $y_1 = P_1 d_1^3 / 3EI + P_2 d_1^2 (3d_2 - d_1) / 6EI$, which equals $P_1 / 3EI + 5P_2 / 6EI$ for the values of d_1 and d_2 stated previously. Derive a similar expression for the deflection y_2 at the second location. Solve for P_1 and P_2 from $y_1 = P_1 / 3EI + 5P_2 / 6EI$ and the expression for y_2 in terms of y_1, y_2 and EI. Noting that the restoring forces are opposite to P_1 and P_2, write down the equation of motion for each mass in the simplest possible form.

(10 marks)

Note: Question 4 continues on page 6.
Figure 4 appears on page 6.

(b) Determine the natural frequencies in terms of EI and the associated mode shapes.

(15 marks)

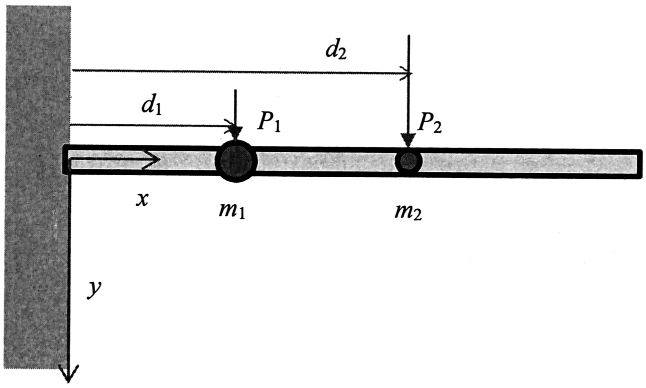

Figure 4: Two masses m_1 and m_2 at the locations d_1 and d_2 from the fixed end of a cantilever.

END OF PAPER

MA3002 SOLID MECHANICS & VIBRATION

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.