NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

MA3002– SOLID MECHANICS AND VIBRATION

November/December 2019 Time Allowed: 2[illegible]rs

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **EIGHT (8)** p[illegible]

2. Answer **ALL** questions.

3. All questions carry equal marks.

4. This is a **RESTRICTED-OPEN BOOK** examination. O[illegible]ble-sided A4 size reference sheet of paper is allowed.

1(a) A spring loaded mechanism is made up of rigid bars A[illegible] and CD pin jointed and subject to a vertical force, **P**, applied at point C. The m[illegible]ism is in equilibrium with the two springs shown in Figure 1. The rigid bar[illegible] uniform length of L. The mechanism has an angle, θ, relative to the horizo[illegible]urface. The two linear spring stiffness are indicated as k_1 and k_2. When the load[illegible]emoved, the springs will return the mechanism back to its initial position on the v[illegible] plane when $\theta = 90°$.

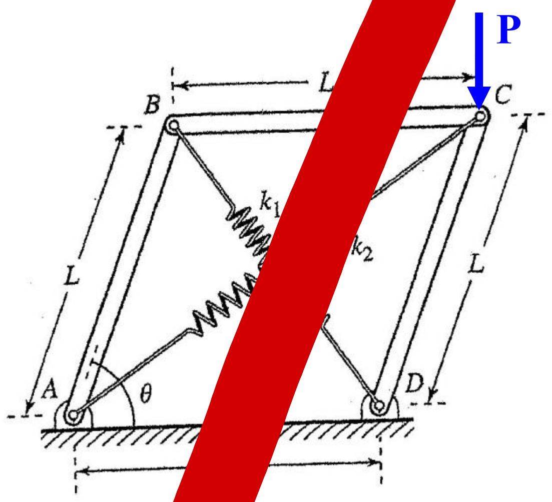

Figure 1

[illegible]etermine the relation[illegible]between the applied force (P) with respect to the spring [illegible]ffness k_1 and k_2, an[illegible]), and the rigid bar length L. Use the Principle of Virtual [illegible] (PVW) meth[illegible]raw and specify your datum. Neglect friction and self-w[illegible]of the bars.

(4 marks)

Note: Question 1[illegible]ues o[illegible]e 2.

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1(a) A spring loaded mechanism is made up of rigid bars AB, BC and CD pin jointed and subject to a vertical force, **P**, applied at point C. The mechanism is in equilibrium with the two springs shown in Figure 1. The rigid bars have uniform length of L. The mechanism has an angle, θ, relative to the horizontal surface. The two linear spring stiffness are indicated as k_1 and k_2. When the load, P, is removed, the springs will return the mechanism back to its initial position on the vertical plane when $\theta = 90°$.

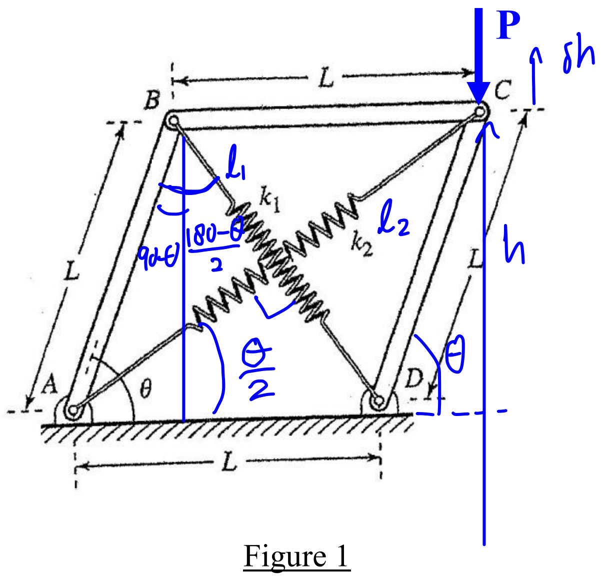


Figure 1

(i) Determine the relationship between the applied force (P) with respect to the spring stiffness k_1 and k_2, angle (θ), and the rigid bar length L. Use the Principle of Virtual Work (PVW) method. Draw and specify your datum. Neglect friction and self-weight of the bars.

(4 marks)

Note: Question 1 continues on page 2.

(ii) Calculate the applied force P in newton (N), if the bar dimension L = 1.0 m, the angle, $\theta = 60°$, the spring stiffness, $k_1 = 1000$ N/m and $k_2 = 1000$ N/m respectively.
(2 marks)

(b) A cantilevered curved beam (A-B-C-D) is subjected to a load, W, at the free end (D) as shown in Figure 2. Using the Unit Load method (consider only bending effects and state clearly your datum), determine the following:

(i) Vertical Displacement expression of the free end at D.
(ii) Horizontal Displacement expression of the free end at D.

Draw your REAL and VIRTUAL load diagrams and specify the relevant Bending Moment equations to be used. Calculate the Vertical and Horizontal Displacements in millimeters (mm) if the Point Load W = 100N, R = 1.0 m, and EI = 21,000 N/m^2.
(6 marks)

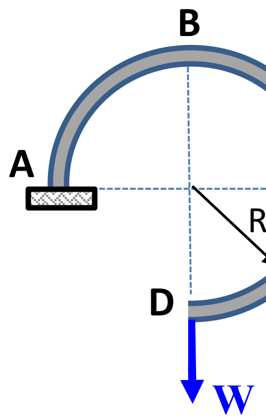


Figure 2

(c) A statically indeterminate structure is in a form of a square steel member is subjected to symmetrical Point Loads, P, as shown in Figure 3. A quarter symmetry segment is indicated by the position of A-B-C. The length of the square edge is given as L.

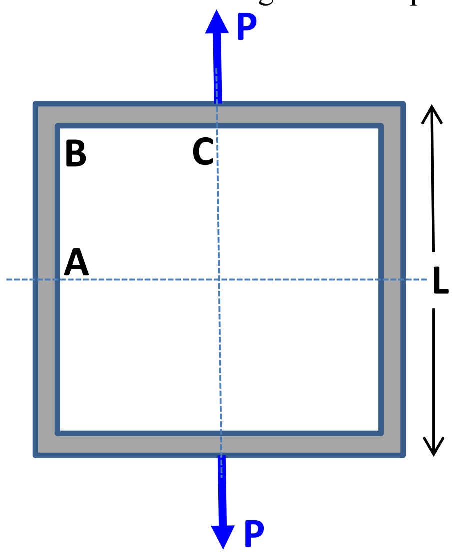


Figure 3

Note: Question 1 continues on page 3.

(i) Draw the free body diagram for a Quarter Symmetry segment A-B-C, and indicate the equilibrium forces and moments in a free body diagram.

(4 marks)

(ii) Show that this problem is Statically Indeterminate to what degree of Indeterminacy.

(2 marks)

(iii) Determine the Vertical Displacement expression at the Load Point, C, for the Quarter Symmetry segment A-B-C using the Unit Load and Unit Moment methods. Consider bending effects only. Neglect friction and self-weight of the bars.

(5 marks)

(iv) Calculate the Vertical Displacement at the Load Point, C, for the Quarter Symmetry segment ABC in millimeters (mm) if the Point Load P = 1000N, L = 1.0 m, and EI = 26,000 N/m^2

(2 marks)

2(a) A Plane Strain fracture toughness test was conducted using a Compact Tension (CT) Specimen (with a thickness of B = 50mm) and the test data is recorded in Table 1.

Table 1: Compact Tension (CT) Specimen Test Data

P_{max} = 170 kN
P_Q = 160 kN
B = 50mm (Thickness)
a = 50mm (Crack Length)
W = 100mm (Width)

$$K_Q = \frac{P_Q}{BW^{1/2}} f_2\left(\frac{a}{W}\right)$$

$$f_2\left(\frac{a}{W}\right) = 9.6$$

Yield Stress, σ_y = 1200 MN/m^2.

(i) Calculate the Qualifying Stress Intensity Factor, K_Q value from the test data given in Table 1.

(4 marks)

(ii) Determine if the CT specimen test result meet the Plane Strain Fracture Toughness three-pronged checking criteria. Conclude if, K_Q is a valid K_{IC} (plane strain fracture toughness) test result?

(4 marks)

Note: Question 2 continues on page 4.

(b) An edge crack in an infinite width plate has a flaw in the form of an edge crack of length, a, as shown in Figure 4. The geometric factor, Y = 1.12, can be used in calculating the Stress Intensity Factor, solution.

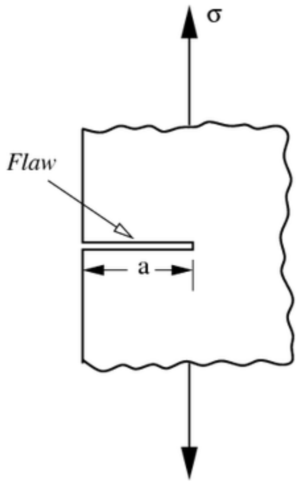


Figure 4

(i) Calculate the applied stress intensity factor, K_I, if the crack length, a = 1.0 mm (or 0.001m), and the applies stress, $\sigma = 250$ MN/m^2.

(2 marks)

(ii) Calculate the flaw size or critical crack length, a_{cr}, due to brittle fracture, if the plane strain fracture toughness of the material, $K_{IC} = 56$ MN/m$^{3/2}$.

(2 marks)

(iii) Calculate the crack propagation life, N_f, for a cyclic stress varying from zero to 250 MN/m^2 giving a stress range, S_R = 250 MN/m^2. The initial crack length, a_o = 1.0 mm (or 0.001m) and the final critical crack length a_f can be obtained from part (ii). The fatigue crack propagation life expression and the Paris Law constants are given below.

$$N_f = \frac{2}{C(YS_R)^m \pi^{m/2}(2-m)}\left(a_f^{1-m/2} - a_0^{1-m/2}\right)$$

where, $C = 4 \times 10^{-12}$ (m/cycle); $m = 3.3$

(6 marks)

Note: Question 2 continues on page 5.

(c) A steel specimen was tested under fatigue stress range conditions and the fatigue S-N curve test data is given below in Table 2.

Table 2: Fatigue S-N Curve Test Data

Test Specimen No.1
Stress Range : $S_R = 200$ MN/m^2.
Fatigue Life : $N_f = 900{,}000$ cycles

Test Specimen No.2
Stress Range : $S_R = 310$ MN/m^2.
Fatigue Life : $N_f = 12{,}000$ cycles

(i) The steel structure is subjected to two stress range level exposures every day and strain gage measurement show that there are 30 cycles of stress range of $S_R = 200$ MN/m^2 and 3 cycles of stress range of $S_R = 310$ MN/m^2, in one day of loading operation.

Using Miners Rule, calculate the number of years it will take for the structure to fail by fatigue when the Cumulative Damage Summation index is equal to D = 1.0. Use the Fatigue test data provided in Table 2.

(4 marks)

(ii) The S-N Curve equation can be curve-fitted to the fatigue test data when analyzed on a Log Stress versus Log Cycles plot.

Determine the S-N curve equation constants A and b using the fatigue test data from Table 2.

Note: A typical S-N Curve equation has two constants **A** and **–b**.

$$S_R = \mathbf{A} \,.\, (N_f)^{-\mathbf{b}}$$

(3 marks)

3(a) It is intended to analyse the vertical oscillations of a small bridge by modelling it as a spring-mass system as shown in Figure 5(a) where k_{eff} is the effective stiffness of the bridge and m_{eff} is the corresponding effective mass. When the bridge is deflected down by imposing a certain vertical displacement at O (the mid-section of the beam) and allowed to oscillate freely, the bridge executes natural oscillations in the vertical direction as indicated. The natural period of vertical oscillations is found to be 0.4 s. When a mass 8000 kg is placed at the mid-section of the bridge at O, the natural period of the oscillation increases to 0.42 s. Determine k_{eff} and m_{eff} of the spring-mass system. Ignore damping effects.

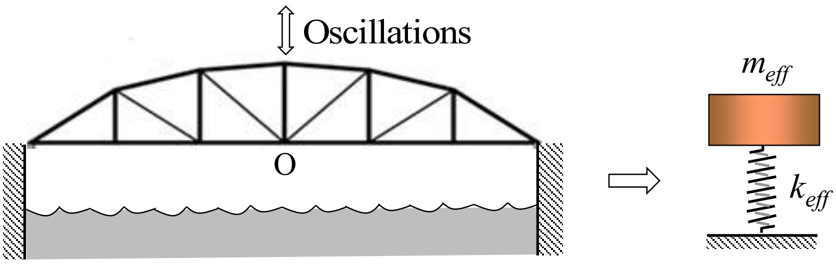


Figure 5(a)

(5 marks)

(b) When a chicken egg of mass m = 50 g is placed gently on a block of sponge material, it produces a static deflection of 5 mm as shown in Figure 5(b).

(i) Assuming that the sponge material behaves like a spring, determine the stiffness (k) of the sponge material.

(2 marks)

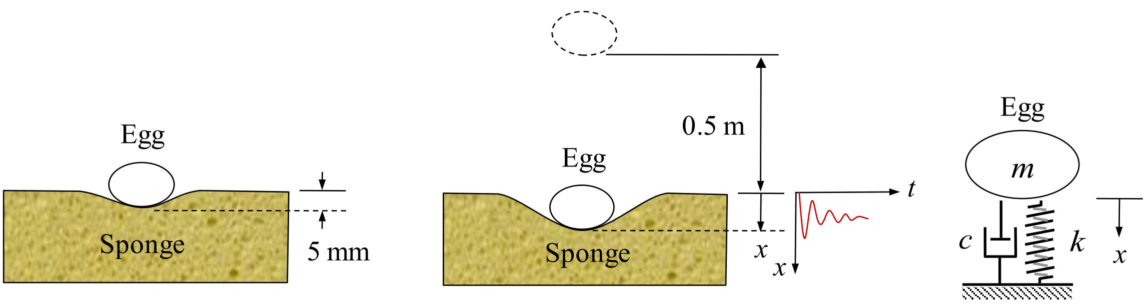


Figure 5(b) Figure 5(c) Spring 5(d)

Note: Question 3 continues on page 7.

(ii) When the egg is dropped from a height of 0.5 m above the surface of sponge block (as shown in Figure 5(c)), the motion of egg after landing on sponge block is under-damped with successive vibration amplitudes reducing to 0.1 times that of the previous. Assume the egg does not rebound or break during the drop. Ignore the mass of sponge material. Determine the damping ratio, damping coefficient, undamped natural frequency (rad/s) and damped natural frequency (rad/s).

(5 marks)

(iii) As the egg is falling from the height of 0.5 m, the time t is taken as 0 when the egg just touches the sponge material. Using an appropriate free body diagram, derive the equation of motion of egg for time $t > 0$ by modelling the sponge-egg combination as a spring-mass-damper system as shown in Figure 5(d), and determine the initial conditions $x(0)$ and $\dot{x}(0)$.

(4 marks)

(iv) Determine the solution to the equation of motion derived in part (iii).

(7 marks)

(v) What should be the value of damping coefficient of sponge material for which the motion of egg is critically damped? Assume that sponge material has the same stiffness as calculated in part (i).

(2 marks)

4. Figure 6 shows a 2-DOF vibrating system in its static equilibrium configuration. The system consists of a rigid beam ABC of length l and mass m_1, pin-jointed at A and supported by two springs of same stiffness k_1 at the mid-section of the beam at B. The beam carries a spring-mass system of mass m_2 and spring stiffness k_2 at C. Mass m_2 is subjected to a harmonic force excitation of amplitude F_2 at excitation frequency ω.

$J\ddot{\theta} + k\square + k\square = \square$

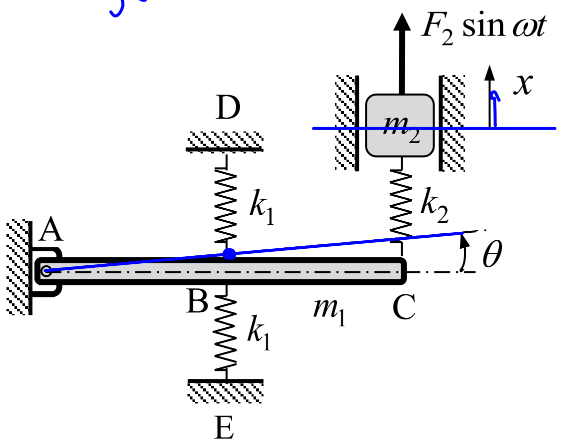


Figure 6

(a) Applying Newton's 2nd law, derive the equations of motion for this 2-DOF system in terms of the coordinates x and θ, and represent them in matrix form. Draw neat free body diagrams to aid your derivation. Static forces need not be shown on the free body diagrams as they are balanced.

(8 marks)

(b) Taking $l = 1$ m, $m_1 = 1$ kg, $k_1 = 1000$ N/m, $m_2 = 1$ kg, $k_2 = 1000$ N/m, determine the natural frequencies (in Hz) and the corresponding amplitude ratios of the system.

(10 marks)

(c) Taking $F_2 = 20$ N, determine the vibration amplitude of the beam (Θ) at the excitation frequency $\omega = 15$ rad/s.

(5 marks)

(d) What is the amplitude of force transmitted to each of the supports at E and D.

(2 marks)

End of Paper

$\sin^2\theta = 1 - \cos^2\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\frac{180-\theta}{2} - 90 + \theta = \frac{\theta}{2}$

$h = L\sin\theta$

1ai) PVW: $\delta W = \delta U$

$P\delta h = \sum F\delta e$

$= \sum ke\,\delta e$

$\delta h = \frac{dh}{d\theta}\delta\theta$

$= L\cos\theta\,\delta\theta$

$e_1 = \sqrt{2}L - \frac{L\sin\theta}{\cos\theta/2}$

$\delta e_1 = -\frac{d}{d\theta}\left(2L\sin\frac{\theta}{2}\right)\delta\theta$

$= -L\cos\frac{\theta}{2}\,\delta\theta$

$e_2 = \frac{L\sin\theta}{\sin\theta/2} - \sqrt{2}L$

$\delta e_2 = \frac{d}{d\theta}\left(2L\cos\frac{\theta}{2}\right)\delta\theta$

$= -L\sin\frac{\theta}{2}\,\delta\theta$

$-PL\cos\theta\,\cancel{\delta\theta} = k_1\left(\sqrt{2}L - 2L\sin\frac{\theta}{2}\right)\left(-L\cos\frac{\theta}{2}\right)\cancel{\delta\theta} + k_2\left(2L\cos\frac{\theta}{2} - \sqrt{2}L\right)\left(-L\sin\frac{\theta}{2}\right)\cancel{\delta\theta}$

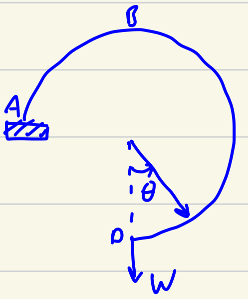
$= k_1\sqrt{2}L\left(-L\cos\frac{\theta}{2}\right) + k_2\sqrt{2}L\left(L\sin\frac{\theta}{2}\right) + (k_1 - k_2)(L^2\sin\theta)$

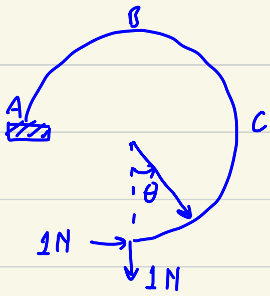
1aii) when $L = 1m$ and $\theta = 60°$, $k_1 = 1000$ and $k_2 = 1000$,

$-P\cos 60° = 1000\sqrt{2}\left(-\cos 30° + \sin 30°\right)$

$P = 1035.27618\ N$ //

2b) Real. Virtual





$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\sin\theta - \sin\theta\cos\theta$

$\left[\cos\theta - \frac{1}{2}\sin^2\theta\right]_0^{3\pi/2}$

-0.5

$M_{DA} = WR\sin\theta$

$m_{DA} = \sin\theta$

$m_{DA} = 1 - \cos\theta$

i) $1.(\Delta_D)_{vertical} = \int_0^{3\pi/2}\frac{Mm}{EI}ds$

$= \int_0^{3\pi/2}\frac{WR^2\sin^2\theta}{EI}d\theta$

$= \frac{100}{21000}\int_0^{3\pi/2}\frac{1-\cos 2\theta}{2}d\theta$

$= \frac{1}{210}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{3\pi/2}$

$= \frac{1}{210}\left(\frac{3\pi}{4}\right) = 11.22\ mm\ \downarrow$ //

ii) $1.(\Delta_D)_{horiz} = \int_0^{3\pi/2}\frac{Mm}{EI}ds =$

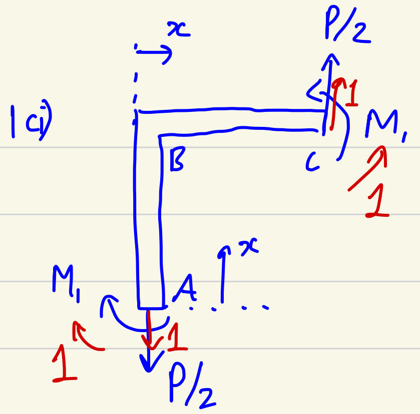
$= \frac{1}{210}\int_0^{3\pi/2}\sin\theta - \frac{1}{2}\sin 2\theta\,d\theta$

$= \frac{1}{210}\left[-\cos\theta + \frac{1}{4}\cos 2\theta\right]_0^{3\pi/2}$

$= \frac{1}{210}\left[\frac{1}{4}(-1) + 1 - \frac{1}{4}(1)\right]$

$= \frac{1}{210}\left(\frac{1}{2}\right) = 2.38\ mm \rightarrow$ //

1c)



Statistically indeterminate because M_1 is unknown.

1 degree of indeterminacy as there is only one unknown.

Finding M:

Real moments:

$M_{AB} = M_1 + \frac{PL}{4}$

$M_{BC} = M_1 - \frac{P}{2}x + \frac{PL}{4}$

Virtual moments:

$m_{AB} = 1$

$m_{BC} = 1$

$$1.(\Delta_c) = \frac{1}{EI}\int_0^{L/2} M_1 + \frac{PL}{4}\,dx + \frac{1}{EI}\int_0^{L/2} M_1 - \frac{P}{2}x + \frac{PL}{4}\,dx$$

$$= \frac{1}{EI}\left(M_1\frac{L}{2} + \frac{PL^2}{8}\right) + \frac{1}{EI}\left(M_1\frac{L}{2} - \frac{PL^2}{16} + \frac{PL^2}{8}\right)$$

$$= \frac{1}{EI}\left(M_1L + \frac{4PL^2}{16} - \frac{PL^2}{16}\right)$$

For $1.\Delta_c = 0$, $M_1L = -\frac{3PL^2}{16} \Rightarrow M_1 = -\frac{3PL}{16}$

Finding vertical displacement:

Real moments:

same

virtual load:

$m_{AB} = \frac{L}{2}$

$m_{BC} = \frac{L}{2} - x$

2a i) $K_Q = \frac{160\,000}{0.05 \times 0.1^{1/2}} \times 9.6 = 97.145 \text{ MPa}\sqrt{m}$

ii) Validity check:

1) $\frac{F_{max}}{F_Q} < 1.1$. $\frac{170}{160} = 1.0625 < 1.1$ ✓

$2.5\left(\frac{K_Q}{\sigma_Y}\right)^2 = 0.016384$.

2) $B = 0.05$, $a = 0.05$, $W - a = 0.05$, all of them > 0.016384. ✓

3) $0.45 < \frac{a}{W} = 0.5 < 0.55$ ✓

2b) i) $K_I = Y\sigma\sqrt{\pi a}$

$= 1.12 \times 250\text{M}\sqrt{\pi \times 0.001}$

$= 15.694 \text{ MPa}\sqrt{m}$

ii) $a_{cr} = \frac{1}{\pi}\left(\frac{K_{IC}}{Y\sigma}\right)^2$

$= 12.73 \text{ mm}$

iii) $N_f = \frac{2}{4\times10^{-12}(1.12\times250)^{3.3}\,\pi^{1.65}\,(-1.3)}\left(0.01273^{-0.65} - 0.001^{-0.65}\right)$

$= 35\,225.78$ cycles

2c i) one day: $\frac{30}{900\,000} + \frac{3}{12\,000}$

$\text{days}\left(\frac{3}{90\,000} + \frac{3}{12\,000}\right) = 1$ life

$\therefore$ days $= 3529.411765$

$= 9.67$ years.

2c ii) $\lg S_R/A = -b \lg N_f$

$200\,000\,000 = A(900\,000)^{-b}$ — ①

$310\,000\,000 = A(12\,000)^{-b}$ — ②

① ÷ ②:

$\frac{20}{31} = \left(\frac{900}{12}\right)^{-b}$

$-b = \frac{\lg(20/31)}{\lg(900/12)} = -0.1015069$

$A = 804.3162 \times 10^6$

$\therefore S_R = 804(N_f)^{-0.1015}$, where S_R is in MPa and N_f is in cycles.

3a) $\omega_{n_1} = \sqrt{\frac{k_{eff}}{m_{eff}}} = \frac{2\pi}{0.4}$ $\qquad k_{eff} = 15.708 m_{eff}$

$\omega_{n_2} = \sqrt{\frac{k_{eff}}{m_{eff}+8000}} = \frac{2\pi}{0.42}$ $\qquad k_{eff} = 14.96(m_{eff}+8000)$

$15.708 m_{eff} = 14.96 m_{eff} + 119679.7201$

$\therefore m_{eff} = 160\,000 \text{ kg}$

$k_{eff} = 2513.27 \text{ N/mm}$

3b) $k = \frac{F}{x} = \frac{0.05 \times 9.81}{0.005} = 98.1 \text{ N/m}$

3bii) $\delta = \ln 0.1$

$\zeta = \sqrt{\frac{\delta^2}{4\pi^2+\delta^2}} = 0.34409$

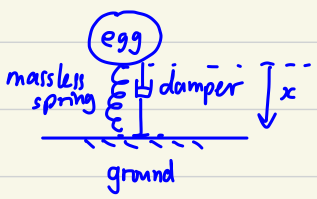
$c_c = 2\sqrt{km} = 2\sqrt{98.1 \times 0.05} = 4.429447 \text{ Ns/m}$

$c = c_c \times \zeta = 1.52413 \text{ Ns m}^{-1}$

$\omega_n = \sqrt{\frac{k}{m}} = 44.29 \text{ rad/s}$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 41.5897 \text{ rad/s}$

3biii) Drop test model:



EOM: $0.05\ddot{x} + 1.52413\dot{x} + 98.1x = 0.4905$

$x(0) = 0, \quad \dot{x}(0) = \sqrt{2gh} = 3.13209$

3biv) $\zeta < 1$: $x(t) = e^{-\zeta\omega_n t}(A\cos\omega_d t + B\sin\omega_d t) + \frac{mg}{k}$

at $x(0) = 0$,

$0 = (A) + \frac{mg}{k} \Rightarrow A = -\frac{mg}{k}$

at $\dot{x}(0) = \sqrt{2gh}$,

$\sqrt{2gh} = -\zeta\omega_n(A) + (\omega_d B)$

$B = \frac{\sqrt{2gh} + \zeta\omega_n\left(-\frac{mg}{k}\right)}{\omega_d}$

$m\ddot{x} + c\dot{x} + kx = \frac{mg}{k}$

Sub $x = C_1 e^{rt}$

$r^2 m + rc + k = \frac{mg}{k C_1 e^{rt}}$

$\therefore x(t) = e^{-15.24t}\left(-\frac{1}{200}\cos 41.6t + 0.73477\sin 41.6t\right) + \frac{1}{200}$

3bv) $c_c = 2\sqrt{km} = 4.43 \text{ Ns m}^{-1}$

for small θ, $\sin\theta \approx \theta$

4a) EOM_{m_2} : $m_2\ddot{x} + k_2(x - \ell \cancel{\sin}\theta) = F_2 \sin \omega t$

EOM_{m_1} : $\frac{1}{3}m_1\ell^2\ddot{\theta} + k_1\left(\frac{\ell}{2}\right)^2 \times 2 \times \cancel{\sin\theta}\,\theta + k_2(\ell \cancel{\sin\theta}\,\theta - x)\ell = 0$

$$\begin{bmatrix} \frac{m_1\ell^2}{3} & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} \frac{k_1\ell^2}{2} + k_2\ell^2 & -k_2\ell \\ -k_2\ell & k_2 \end{bmatrix} \begin{Bmatrix} \theta \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_2 \sin \omega t \end{Bmatrix}$$

$\ddot{\theta} = -\omega^2\theta$, $\ddot{x} = -\omega^2 x$

$$\begin{bmatrix} -\frac{\omega^2 m_1\ell^2}{3} + \frac{k_1\ell^2}{2} + k_2\ell^2 & -k_2\ell \\ -k_2\ell & -\omega^2 m_2 + k_2 \end{bmatrix} \begin{Bmatrix} \theta \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_2 \sin \omega t \end{Bmatrix}$$

4b) $$\begin{bmatrix} -\frac{\omega^2}{3} + 1500 & -1000 \\ -1000 & -\omega^2 + 1000 \end{bmatrix} \begin{Bmatrix} \Theta \\ X \end{Bmatrix} \cancel{\sin \omega t} = \begin{Bmatrix} 0 \\ F_2 \end{Bmatrix} \cancel{\sin \omega t}$$

to find ω_n, $\det[E(\omega)] = 0$

$\left(-\frac{\omega^2}{3} + 1500\right)\left(-\omega^2 + 1000\right) - 1000^2 = 0$

$\frac{\omega^4}{3} - 1833\omega^2 + 500\,000 = 0$

$\omega^2 = 5212.21445$ or 287.78555

$\omega_1 = 72.19567$ $\quad\omega_2 = 16.9642$

$f_1 = 11.5\ Hz$ $\quad f_2 = 2.7\ Hz$

$$\left(\frac{\Theta}{X}\right) = \frac{1000}{-\frac{\omega^2}{3} + 1500}$$

$\left(\frac{\Theta}{X}\right)^{(1)} = -4.2122\,m^{-1}$ at $\omega_1 = 72.19507\ rad/s$

$\left(\frac{\Theta}{X}\right)^{(2)} = 0.7122\,m^{-1}$ at $\omega_2 = 16.9642\ rad/s$

4c) Cramer's Rule :

$$\Theta = \frac{\det\begin{bmatrix} 0 & -1000 \\ 20 & -15^2 + 1000 \end{bmatrix}}{\det\begin{bmatrix} 1425 & -1000 \\ -1000 & 775 \end{bmatrix}} = \frac{-20 \times (-1000)}{1425 \times 775 - 1000^2} = 0.1916167665\ rad.$$

4d) $F_{E,0} = k_1 \times \frac{\ell}{2}\,\theta \sin \omega t$

$F_{o\,E,0} = k_1 \times \frac{\ell}{2}\,\Theta = 95.81\ N$ //