NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2018-2019

MA3002– SOLID MECHANICS AND VIBRATION

April/May 2019 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SEVEN (7)** pages.

2. Answer **ALL** questions.

3. All questions carry equal marks.

4. This is a **RESTRICTED OPEN-BOOK** examination. One double-sided A4 size reference sheet of paper is allowed.

1(a) A spring loaded lever mechanism has a horizontal force, F, applied at B, and is in equilibrium with the spring attached as shown in Figure 1. The rigid bar, ACB, has an angle, θ, relative to the horizontal plane at A. The spring CD, is connected to the rigid bar at C from a roller support at D. The linear spring has a stiffness of k. When the load, F, is removed, the spring will return the mechanism back to its initial position on the vertical plane where $\theta = 90°$.

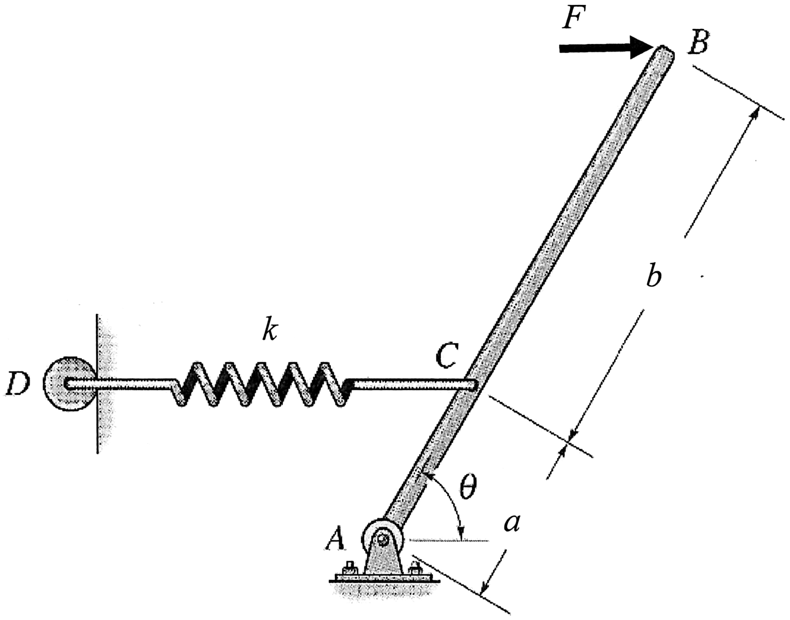

Figure 1

Note: Question 1 continues on page 2.

(i) Determine the relationship between the applied force (F) with respect to the spring stiffness (k), angle (θ), the bar dimension, a, and b. Use the Principle of Virtual Work (PVW) method. Draw and specify your datum. Neglect friction and self-weight of the bars.

(4 marks)

(ii) Calculate the applied force F in Newton (N), if the angle, $\theta = 45°$, the spring stiffness, $k = 50$ N/m. The bar dimension, $a = 1.0$ m, and $b = 2.0$ m respectively.

(2 marks)

(b) A welded beam ABC, has a horizontal segment of length, L, connected to a curved segment with radius R. The beam is fully fixed at point C, and has a vertical load P, applied at point A as shown in Figure 2. The deflection and rotation at any point of the beam can be calculated using the Unit Load and Unit Moment method.

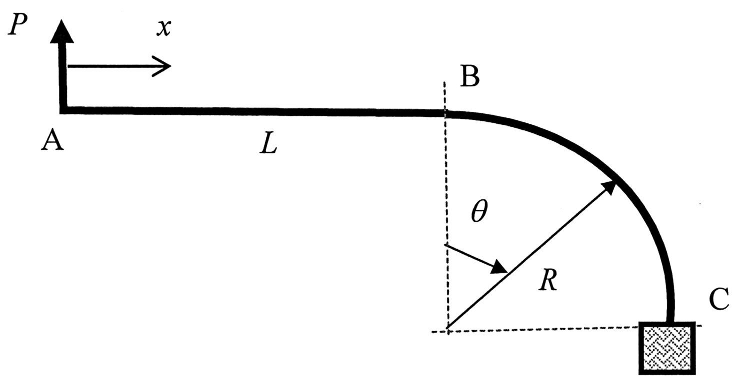

Figure 2

(i) Use the Unit Load method to determine the Vertical Deflection at A. Draw the Real Load diagram and Virtual Load diagram and write down the bending moment expressions for AB and BC segments respectively. Show your datum and moment sign convention.

(6 marks)

(ii) Determine the Vertical Deflection expression at point A using the Unit Load method. Consider only bending effects where the flexural rigidity of the beam is given by EI.

(6 marks)

(iii) Calculate the Vertical Deflection at A, given, $P = 100$ N; $L = 0.75$ m; $R = 0.5$ m, and $EI = 21{,}000$ Nm ^{2}.

(2 marks)

(iv) Determine the Rotation expression at point B, using the Unit Moment method. Consider only bending effects where the flexural rigidity of the beam is EI. Calculate the angle of rotation at B, given, $P = 100$ N; $L = 0.75$ m; $R = 0.5$ m, and $EI = 21{,}000$ Nm ^{2}.

(5 marks)

2(a) A Centre-Crack in an infinite width plate has a tensile stress normal to the crack indicated by σ, as shown in Figure 3. The crack length is $2a$, but the half crack length, a, will be used for Fracture and Fatigue calculations.

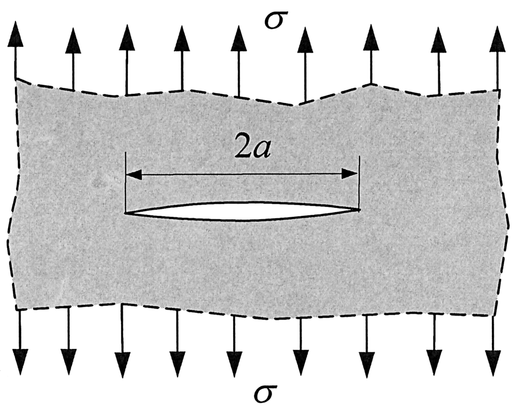

Figure 3

(i) Calculate the Stress Intensity Factor, K_I, if the half crack length, a = 0.01m, and the applied tensile stress, σ = 150MN/m^2.

(2 marks)

(ii) Calculate the critical half crack length, a_c, due to brittle fracture, if the plane strain fracture toughness, K_{IC} = 51 MN/m$^{3/2}$.

(2 marks)

(iii) Calculate the crack propagation life, N_f, if the half crack length propagates from an initial half crack length of a_o= 0.01m to a final half crack length (a_f) due to brittle fracture. The fatigue stress level is from σ_{min} = 0 MN/m^2 to σ_{max} = 150 MN/m^{2}with a fatigue stress range of S_R = 150 MN/m^2.

The crack propagation life, N_f, expression is given in the equation below, where C = 2.4 x 10^{-12} (m/cycle); m = 3.0; a_o is an initial half crack length; and a_f is the final half crack length at brittle fracture.

$$N_f = \frac{2}{C(YS_R)^m \pi^{m/2}(2-m)}\left(a_f^{1-m/2} - a_0^{1-m/2}\right)$$

(5 marks)

(iv) Calculate a new initial half crack length, a_o, if the crack propagation life, N_f= 500,000 cycles. The fatigue stress range, S_R = 150 MN/m^2, remains the same.

(5 marks)

Note: Question no. 2 continues on page 4.

(v) Calculate the allowable fatigue stress range, S_R, to give a fatigue design life of 1 million cycles (1,000,000 cycles), for the initial half crack length of $a_o = 0.01$ m to a final half crack length (a_f) due to brittle fracture.

(5 marks)

(b) A structure is subjected to three blocks of repeated variable amplitude fatigue stress loading. The structure is operated for 8 hours per day. Table 1 provides the stress range (S_R), number of stress range cycles per day (n_i) and the S-N curve fatigue life cycles (N_i).

Table 1

S_R (Stress Range)	n_i (Load cycles)	N_i (Fatigue life)
400 MPa	2	50,000
300 MPa	8	400,000
200 MPa	20	2,000,000

(i) Calculate the cumulative damage summation ratio, D, using Miner's Law, with the data given in Table 1.

(2 marks)

(ii) Calculate the fatigue life of the structure as a number of years of service, before failure occurs when the cumulative damage ratio, $D = 1.0$.

(2 marks)

(iii) If the structure has to operate longer for 12 hours per day, what will be the expected fatigue life as a number of years of service? State your assumptions.

(2 marks)

3(a) Figure 4(a) shows a springboard in a swimming pool, which has a mass of 12 kg but remains straight with negligible deflection due to its self-weight. When the tip of the spring board is deflected downward by 0.1m and released, the spring board executes damped oscillations as shown in Figure 4(b).

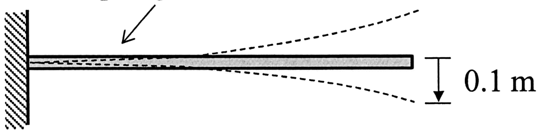

Figure 4(a)

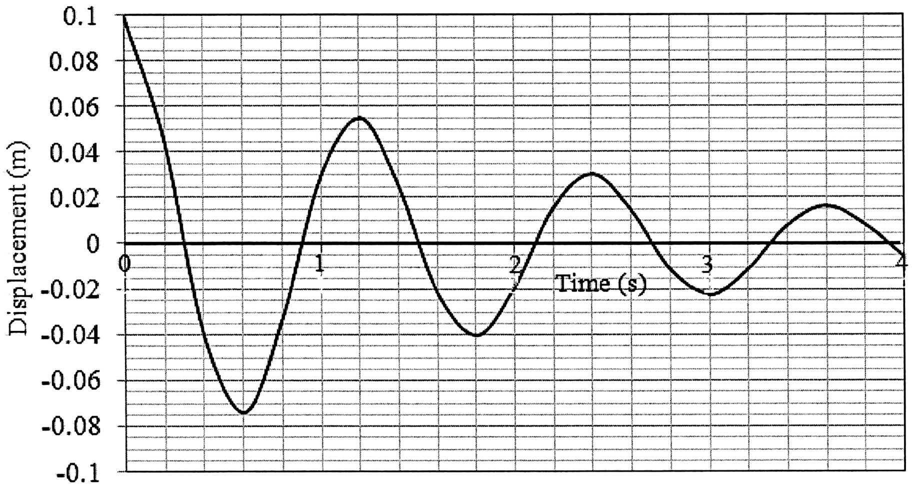

Figure 4(b)

(i) Determine the damping ratio (ζ) and natural frequency (ω_n) extracting necessary data for this calculation from the graph in Figure 4(b).

(4 marks)

(ii) The springboard is to be modelled as a spring-mass-damper system by lumping the effective mass of the springboard at its tip. Take the effective mass of the springboard as $1/4^{th}$ of its total mass. Using your answers in part (i), determine the spring stiffness and damping constant for the spring-mass-damper system, and thereby write the equation of motion for the free vibration of the system.

(3 marks)

Note: Question 3 continues on page 6.

(iii) A diver of mass 70 kg drops from a height of 0.4 m above the tip of the springboard. As soon as he hits the tip of the springboard, he clings on to the tip of the springboard as shown in Figure 4(c). This results in damped oscillations of the springboard with the diver clinging on at the tip. Modelling the problem as a spring-mass-damper system, derive the equation of motion and identify the initial conditions. (You are not required to obtain the solution to the equation of motion.) Also determine the period of damped oscillations of the springboard with the diver clinging on at the tip.

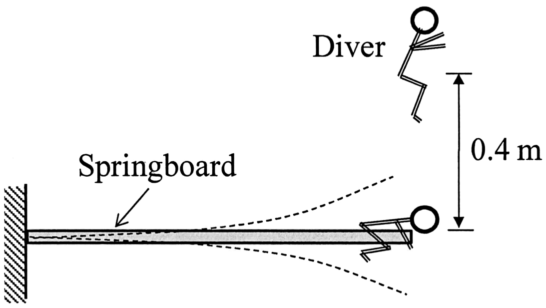

Figure 4(c)

(8 marks)

(b) Figure 5 shows a T-shaped frame hinged at A and connected to two springs of equal stiffness k and a damper of damping coefficient c as illustrated. The T-shaped frame is made of two uniform rigid bars of equal length L and equal mass M. In the position shown, the springs are free (unstrained).

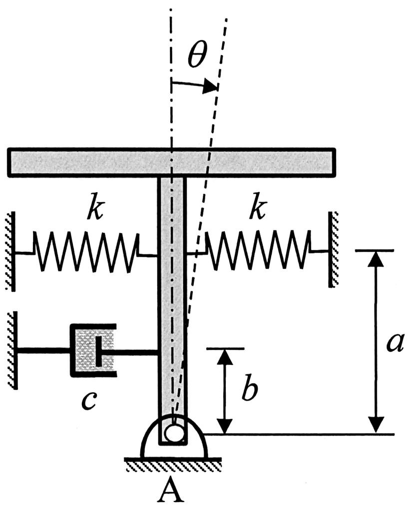

Figure 5

Derive the equation of motion for the system in terms of the coordinate θ shown in Figure 5, and thereby determine the natural frequency of the system (ω_n) in terms of a, k, M and L. Show your derivations clearly by drawing appropriate free body diagrams.

(10 marks)

4. Figure 6 shows a vibration absorber mounted on a reciprocating engine. The vibration absorber consists of a mass m_2 and two springs of same stiffness k_2. The casing of the vibration absorber is rigidly fixed to the engine as shown. The reciprocating engine is modelled as spring-mass system with a mass of M supported on two springs of total stiffness of k_1 as shown. The engine has an unbalanced mass of m_1. The total mass M includes m_1. The rotational speed of the engine is ω rad/s. The displacements x_1 and x_2 shown are measured from the respective static equilibrium positions of masses.

Note that the casing of the vibration absorber is rigidly fixed to the engine. Hence, at any instant of time, if engine mass M moves by a distance x_1, the casing of the vibration absorber also moves by the same distance x_1.

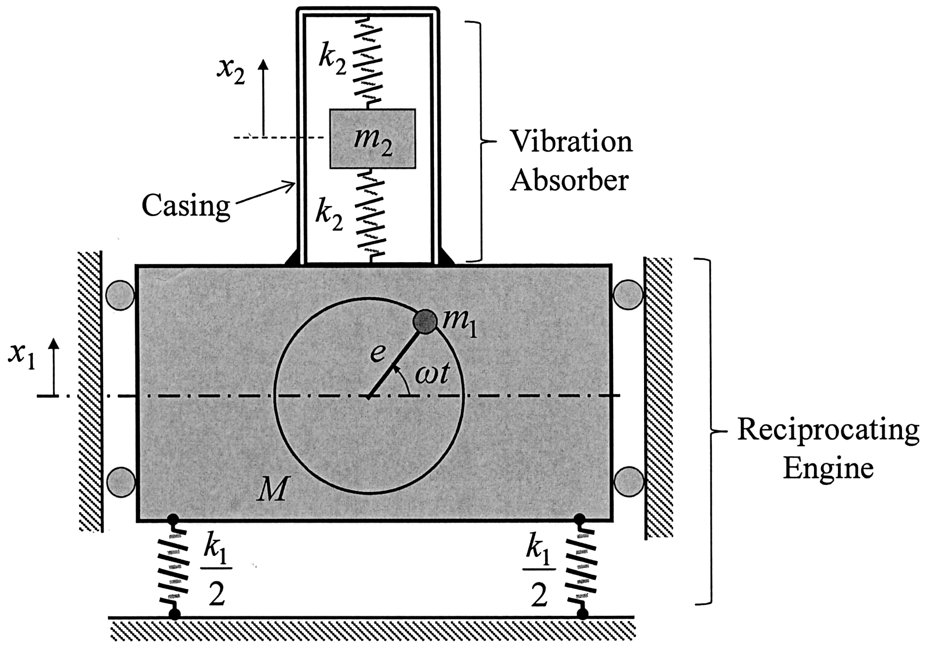

Figure 6

(a) Derive the equations of motion for this 2-DOF system by drawing neat free body diagrams of mass M and m_2, and applying Newton's 2nd law.

(10 marks)

(b) Taking $M = 200$ kg, $k_1 = 10$ kN/m, $m_1 = 1$ kg, $e = 0.1$ m, $m_2 = 2$ kg and $k_2 = 100$ N/m, determine the natural frequencies (ω_1, ω_2) of the system. You may not need all the numerical values given for your calculation.

(6 marks)

(c) Using the same numerical values given in part (b), determine the vibration amplitudes of engine mass (M) and absorber mass (m_1) at the excitation frequency $\omega = 7.5$ rad/s.

(6 marks)

(d) Using the same numerical values given in part (b), determine the values of excitation frequency (ω) at which the vibration amplitude of the engine mass (M) becomes zero.

(3 marks)

End of Paper

MA3002 SOLID MECHANICS & VIBRATION

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.