NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

MA3002–SOLID MECHANICS AND VIBRATION

April/May 2021 Time Allowed: [illegible]urs

INSTRUCTIONS

1. This paper contains **SECTION A & SECTION B** and comprises **FI**[illegible] pages.

2. **COMPULSORY** to answer **ALL** questions in both sections.

3. All questions carry equal marks.

4. This is a **RESTRICTED OPEN BOOK** examination. One [illegible]-sided A4 size reference sheet of paper is allowed.

Section A

1 (a) Figure 1.1 shows a rigid weightless bell-crank leve[illegible] hinged to a support at point C and pin-jointed to a spring of stiffness k at B[illegible]ther end of the spring is pin-jointed to another support at A. The spring sho[illegible]n the stretched condition under the action of a vertical downward force P appli[illegible]oint D. The arms BC and CD are of length L and $2L$, respectively, and are [illegible]ndicular to each other. The free (unstrained) length of the spring is L_o. The [illegible]ce between supports A and C is $2L$. Assume that there is no friction at the j[illegible]onnections. The system is in static equilibrium condition in the configuratio[illegible]n.

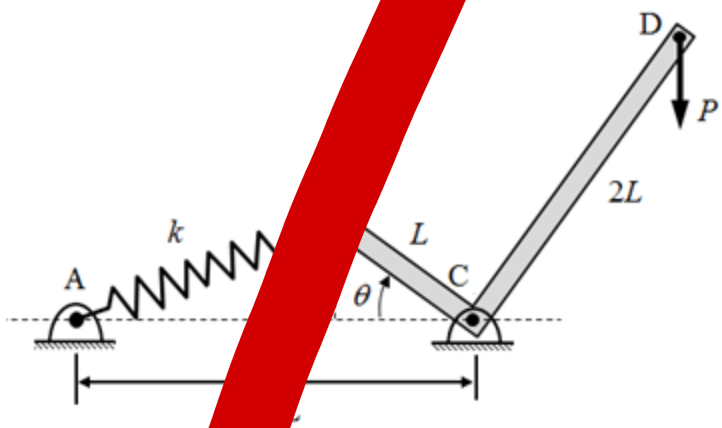


Figure 1.1

Using the Principle of V[illegible] Work (PVW), derive an expression for P as a function [illegible]f angle θ and possib[illegible]er system parameters such as k and L. You may draw [illegible]itional coordinate([illegible]ecessary to aid your derivation but such coordinates should [illegible]ally be elimina[illegible]om the final expression for P.

(10 marks)

Note: Questio[illegible]inue[illegible]age 2.

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

MA3002–SOLID MECHANICS AND VIBRATION

April/May 2021 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **SECTION A & SECTION B** and comprises **FIVE (5)** pages.
2. **COMPULSORY** to answer **ALL** questions in both sections.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN BOOK** examination. One double-sided A4 size reference sheet of paper is allowed.

Section A

1 (a) Figure 1.1 shows a rigid weightless bell-crank lever BCD hinged to a support at point C and pin-jointed to a spring of stiffness k at B. The other end of the spring is pin-jointed to another support at A. The spring shown is in the stretched condition under the action of a vertical downward force P applied at point D. The arms BC and CD are of length L and $2L$, respectively, and are perpendicular to each other. The free (unstrained) length of the spring is L_o. The distance between supports A and C is $2L$. Assume that there is no friction at the joints/connections. The system is in static equilibrium condition in the configuration shown.

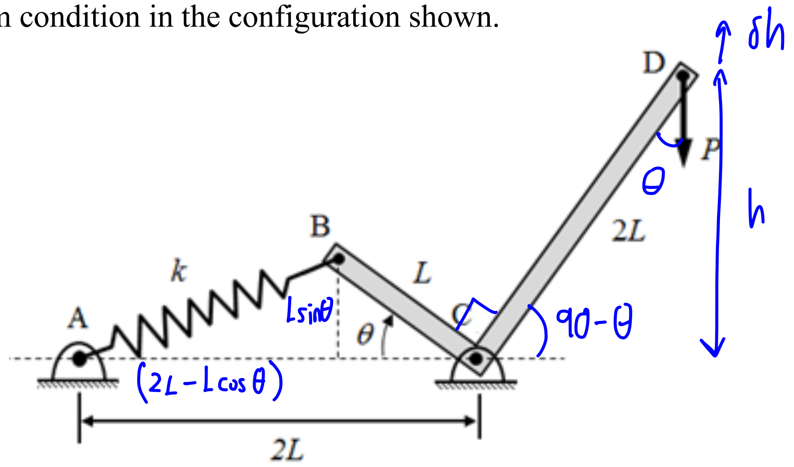


Figure 1.1

Using the Principle of Virtual Work (PVW), derive an expression for P as a function of angle θ and possibly other system parameters such as k and L. You may draw additional coordinate(s) as necessary to aid your derivation but such coordinates should eventually be eliminated from the final expression for P.

(10 marks)

Note: Question 1 continues on page 2.

(b) Figure 1.2 shows a weightless curved beam AB that forms a quadrant of a circle of radius R. The beam is rigidly fixed at end A and supported by a spring of stiffness k at end B. The free length of the spring is R. The top end of the spring is connected to a pin (at C) that is guided to move horizontally so that the spring remains vertical all the time. End B carries a vertical force P as shown. The bending rigidity of the beam is EI.

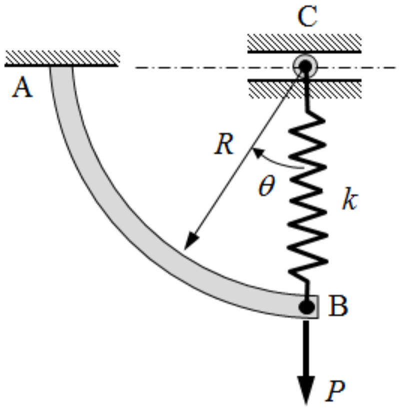


Figure 1.2

(i) Draw all reaction forces and moment on a neat sketch of the supports and thereby show that the degree of indeterminacy of this structure is equal to 1 (unity).

(2 marks)

(ii) Using unit load method, derive an expression for the support reaction at C in terms applied force P, beam radius R, spring stiffness k and bending rigidity EI.

(10 marks)

(iii) Using your answer in part (ii), derive an expression for P for the case where the spring stiffness is extremely large compared to the bending rigidity of the beam (i.e., $k >> EI$).

(3 marks)

2 (a) A steel plate is subjected to a fatigue load with a mean stress of 80 MPa and stress amplitude of 80 MPa. Take $Kc = 85\ \text{MPa}\sqrt{\text{m}}$, $C = 0.15 \times 10^{-11}$, $m = 4.1$, and assume $Y = 1.1$.

(i) Calculate the maximum stress and the stress ratio.

(4 marks)

(ii) Using Linear Elastic Fracture Mechanics, calculate the maximum length of the center through crack.

(4 marks)

(iii) Calculate the number of fatigue cycle to fracture. Assume an initial flaw size of 2mm.

(6 marks)

(b) A finite circular hole steel plate with 1 mm thickness shown in Figure 2.1 is subjected to uniaxial load, F. The material is made of 1020R steel with S_{ut} = 1 GPa.

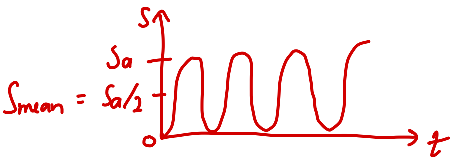
(i) Using the charts for theoretical stress concentration factor K_t (Figure 2.2) and notch sensitivity factor q (Figure 2.3), determine the fatigue strength reduction factor K_f .

$K_f = 2.08$

(5 marks)

(ii) The endurance limit for a flat plate specimen under uniaxial tensile load (zero-to-max) is 250 MPa. Determine the maximum tensile load (zero-to-max), F, for the finite hole plate with infinite fatigue life accounting for the stress concentration factor.

(6 marks)



Goodman criterion for ∞ life: (with K_f)

$$\frac{S_a}{S_e/K_f} + \frac{S_m}{S_u} \le 1$$

$$\therefore \frac{S_a}{250\,\text{MPa}/2.08} + \frac{S_a/2}{1\,\text{GPa}} \le 1$$

Solve for S_a.

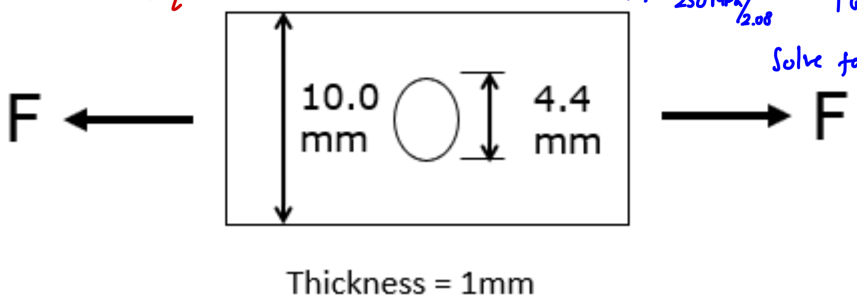


Figure 2.1

Note: Figure 2.2 & 2.3 appears on page 4.

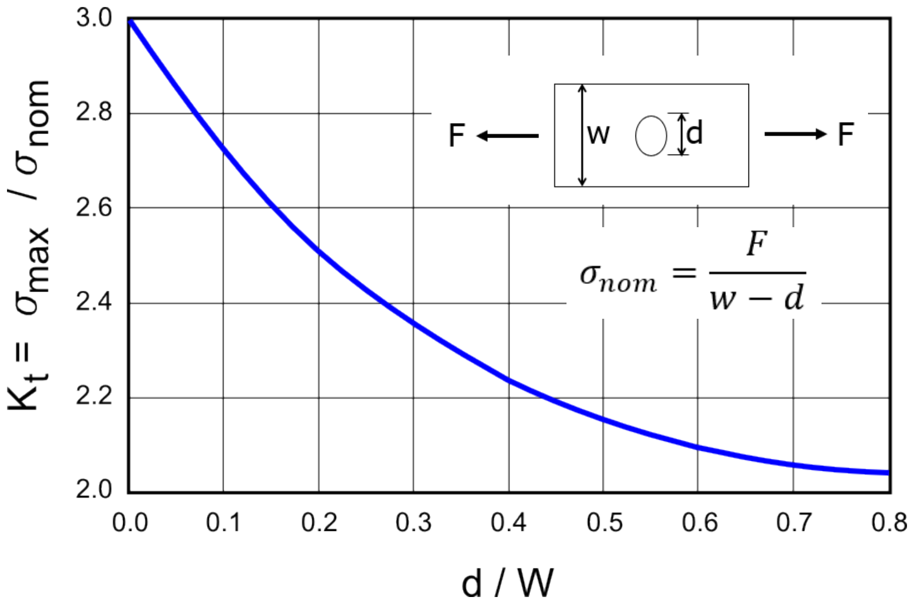


Figure 2.2 Stress Concentration factor where $\sigma_{nom} = \frac{F}{w-d}$

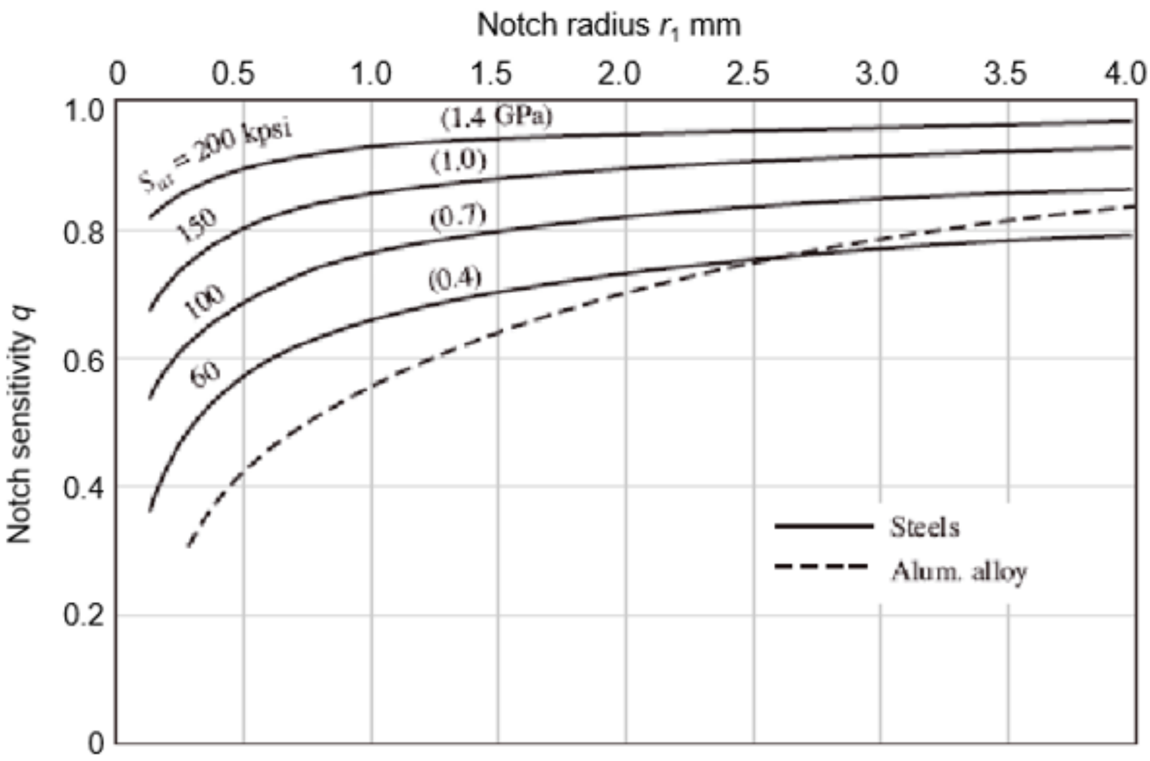


Figure 2.3 Notch sensitivity curves

Section B

3 (a) A uniform rigid rod of mass M and length L carries a concentrated mass m and is attached to a linear spring of stiffness k and a dashpot with damping constant c at its end. In addition, a second linear spring of stiffness k is attached to the rod at the distance of a from the pivot. Figure 3.1 shows the configuration at static equilibrium, with the rod horizontal. Derive the equation of motion for the system.

(15 marks)

(b) Determine the critical damping constant $(c_\theta)_c$ and the corresponding damping ratio ζ. If a is allowed to vary from 0 to L, what is the smallest ζ?

(10 marks)

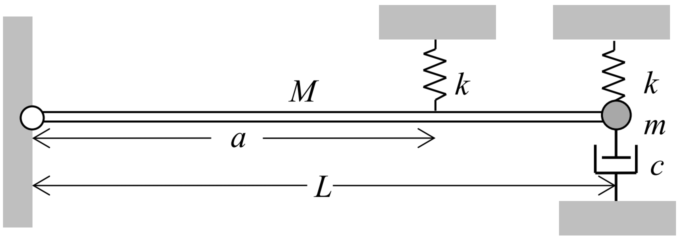


Figure 3.1

4 (a) A coupled pendulum consists of two masses m attached to light rods of lengths $2L$ and L, respectively. The rods are connected by a spring of stiffness k at a distance of d from their pivoting points O_1 and O_2, as shown in Fig. 4.1. Assuming that the angles of swing θ_1 and θ_2 are small, derive the equations of motion.

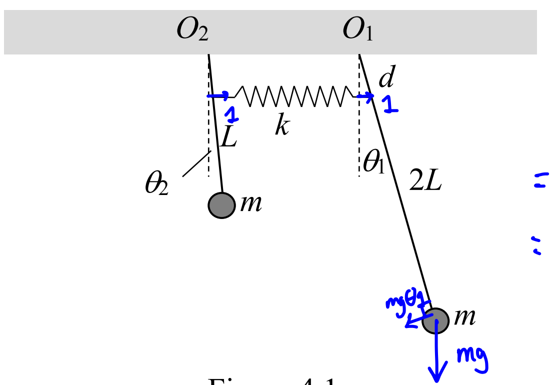
(8 marks)

(b) Assuming further that m = 1 kg, L = 1m, k = 1N/m, and a weak gravitational constant g = 1m/S^2, determine the natural frequencies of vibration in terms of d.

(12 marks)

(c) Calculate the natural frequencies when $d \to 0$ and $d \to L$, where L = 1m.

(5 marks)



$-kd\Delta$

$= -kd(d\theta_1 - d\theta_2)$

$= -kd^2(\theta_1 - \theta_2)$

$-kd^2(\theta_2 - \theta_1)$

Figure 4.1

END OF PAPER

1) PVW: $\delta U = \delta W$

$$P\,\delta h = ke\,\delta e$$

$h = 2L\cos\theta$ $\qquad\qquad$ $e = \sqrt{L^2\sin^2\theta + (2L - L\cos\theta)^2} - L_0$

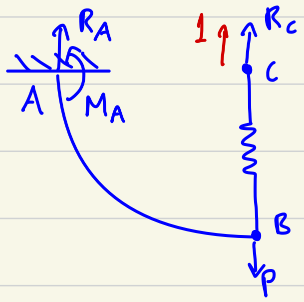
$\delta h = \frac{dh}{d\theta}\delta\theta$ $\qquad\qquad$ $\delta e = \frac{de}{d\theta}\delta\theta$

$= -2L\sin\theta\,\delta\theta$ $\qquad\qquad$ $= \dfrac{L^2 2\sin\theta\cos\theta + 2L\sin\theta(2L - L\cos\theta)}{\sqrt{L^2\sin^2\theta + (2L - L\cos\theta)^2}}\,\delta\theta$

$$(-P)(-2\cancel{L}\,\cancel{\sin\theta})\cancel{\delta\theta} = k\left(L^2\cancel{\sin 2\theta} + 4L^{\cancel{2}}\cancel{\sin\theta} - L^2\cancel{\sin 2\theta} - \frac{4L^{\cancel{2}}\cancel{\sin\theta}\,L_0}{\sqrt{L^2\sin^2 + (2L - L\cos\theta)^2}}\right)\cancel{\delta\theta}$$

$$P = 2kL\left(1 - \frac{L_0}{\sqrt{L^2\sin^2\theta + (2L - L\cos\theta)^2}}\right) //$$

1bi)



There are 3 unknowns, R_A, R_c and M_A

but only 2 equations: $\Sigma F_y = 0$ and $\Sigma M_o = 0$.

$\therefore$ indeterminacy $= 3 - 2 = 1$.

1bii) Real Load: $\qquad\qquad$ Virtual Load: $\qquad\qquad$ $F = kx$ $\qquad$ $\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$M_{AB} = (R_c - P)R\sin\theta$ $\qquad$ $m_{AB} = 1.R\sin\theta$ $\qquad$ $x = \frac{F}{k}$ $\qquad$ $\downarrow \int d\theta$

$\downarrow\Delta_B = \frac{R_c}{k}$ $\qquad$ $\frac{\theta}{2} - \frac{\sin 2\theta}{4}$

$$1.\Delta_B = \int_0^{\pi/\cancel{2}4} \frac{Mm}{EI}\,ds$$

$$= \frac{R^3}{EI}(R_c - P)\int_0^{\pi/\cancel{2}4} \sin^2\theta\,d\theta$$

$$= \frac{R^3}{EI}(R_c - P)\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/\cancel{2}4}$$

$$= \frac{R^3}{EI}(R_c - P)\frac{\pi}{\cancel{2}4}$$

For static equilibrium, $\Delta_B = \frac{R_c}{k}$

$$\frac{R_c}{k} = \frac{R^3\pi}{4\cancel{2}EI}(R_c - P)$$

$$R_c\left(\frac{1}{k} - \frac{R^3\pi}{4\cancel{2}EI}\right) = \frac{R^3\pi}{4\cancel{2}EI}(-P)$$

$$\therefore R_c = \frac{\frac{R^3\pi}{4\cancel{2}EI}P}{\frac{R^3\pi}{4\cancel{2}EI} - \frac{1}{k}} //$$

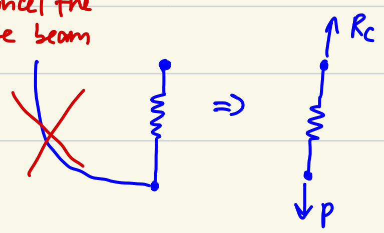
(all the support is given by spring)

1biii) for $k \gg EI$, $\frac{1}{k} \approx 0$.

$$R_c = P$$

This is consistent with the analogous FBD

cancel the curve beam



2ai) $\sigma_{max} = 80\text{ MPa} + 80\text{ MPa} = 160\text{ MPa}$ //

$R = \frac{min}{max} = \frac{80-80}{160} = 0$ //

2aii) $K_c = Y\sigma_f\sqrt{\pi \alpha_c}$

$$\alpha_c = \left(\frac{K_c}{Y\sigma_f}\right)^2 / \pi$$

$$= \left(\frac{85\times\cancel{10^6}}{1.1\times160\times\cancel{10^6}}\right)^2 \div \pi$$

$$= 74.244\text{ mm}$$

centre through crack $= 2\alpha = 148.5$ mm //

2aiii) $$N_f = \frac{2}{C(YS_R)^m \pi^{m/2}(2-m)}\left(a_f^{1-m/2} - a_0^{1-m/2}\right)$$

$$= \frac{2}{0.15\times10^{-11}\times(1.1\times160\times\cancel{10^6})^{4.1}\pi^{2.05}(-2.1)}\left(0.074244^{-1.05} - 0.001^{-1.05}\right)$$

$= 52749$ cycles (round down)

2bi) $\frac{d}{W} = \frac{4.4}{10} = 0.44$

$q \approx 0.9$

$K_t \approx 2.2$

$q = \frac{k_f - 1}{k_t - 1}$

$k_f = 1 + q(k_t - 1) = 1 + 0.9(2.2-1) \approx 2.08$ //

2bii) $S_e' = 250\text{ MPa}$

$$\frac{S_a}{250\text{MPa}/k_f} + \frac{0}{S_Y} \leq 1$$

$$S_a \leq \frac{250\text{ MPa}}{2.08}$$

$$\leq 120.1923\text{ MPa}$$

Any of the three criteria:
- Goodman, Gerber, Soderberg gives the same answer.

3a) EOM: $J_\theta \ddot{\theta} + C_\theta \dot{\theta} + k_\theta \theta = 0$

$$J_\theta = \frac{1}{3}ML^2 + mL^2$$

$$C_\theta = cL^2$$

$$k_\theta = kL^2 + ka^2$$

$\therefore$ EOM: $\left(\frac{1}{3}ML^2 + mL^2\right)\dot{\theta} + cL^2\dot{\theta} + (kL^2 + ka^2)\theta = 0$

3b) $(C_\theta)_c = 2\sqrt{J_\theta k_\theta}$

$$= 2\sqrt{\left(\frac{1}{3}ML^2 + mL^2\right)(kL^2 + ka^2)}$$

$$\zeta\downarrow = \frac{C_\theta}{(C_\theta)_c\uparrow} = \frac{cL^2}{2\sqrt{\left(\frac{1}{3}ML^2 + mL^2\right)(kL^2 + ka^2)}}$$

smallest $\zeta\downarrow = \dfrac{c\cancel{L^2}}{2\sqrt{\left(\frac{1}{3}M + m\right)\cancel{L^2}(k+k)\cancel{L^2}}}$

$$= \frac{c}{2\sqrt{2k\left(\frac{1}{3}M + m\right)}} \;//$$

4a) EOM_1 : $m(2L)^2\ddot{\theta}_1 = -mg2L\theta_1 - kd^2(\theta_1 - \theta_2)$

$$4L^2m\ddot{\theta}_1 + (mg2L + kd^2)\theta_1 - kd^2\theta_2 = 0$$

EOM_2 : $mL^2\ddot{\theta}_2 + (mgL + kd^2)\theta_2 - kd^2\theta_1 = 0$

4b) $\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} 2+d^2 & -d^2 \\ -d^2 & 1+d^2 \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

$\ddot{\theta} = -\omega^2\theta$, $\theta = \Theta \sin\omega t$

$$\begin{bmatrix} -4\omega^2+2+d^2 & -d^2 \\ -d^2 & -\omega^2+1+d^2 \end{bmatrix}\begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \vec{0}$$

for non-trivial soln, $\det[F(\omega)] = 0$

$121 - 80 = 41$

$$(-4\omega^2 + 2 + d^2)(-\omega^2 + 1 + d^2) - d^4 = 0$$

$$4\omega^4 - (4 + 4d^2 + 2 + d^2)\omega^2 + 2 + 3d^2 + \cancel{d^4} - \cancel{d^4} = 0$$

$$4\omega^4 - (6 + 5d^2)\omega^2 + 2 + 3d^2 = 0$$

$$\omega^2 = \frac{6 + 5d^2 \pm \sqrt{(6+5d^2)^2 - 16(2+3d^2)}}{8}$$

$\omega_1 = \dfrac{6 + 5d^2 + \sqrt{4 + 12d^2 + 25d^4}}{8}$, $\omega_2 = \dfrac{6 + 5d^2 - \sqrt{4 + 12d^2 + 25d^4}}{8}$

4c) $\left.\begin{aligned} \omega_1 &= \frac{6 + \sqrt{4}}{8} = 1 \\ \omega_2 &= \frac{6 - \sqrt{4}}{8} = \frac{1}{2} \end{aligned}\right\} d \to 0$ $\qquad$ $\left.\begin{aligned} \omega_1 &= \frac{6 + 5 + \sqrt{41}}{8} = 2.1754 \\ \omega_2 &= \frac{6 + 5 - \sqrt{41}}{8} = 0.57461 \end{aligned}\right\} d \to L$

☆ ω in rad/s