NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2022-23

MA3002 – SOLID MECHANICS AND VIBRATION

April/May 2023 Time Allowed: $2^{1}/_{2}$ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN-BOOK** examination. One double-sided A4-size reference sheet with texts handwritten or typed on the A4 paper (no sticky notes/post-it notes on the reference sheet) is allowed.

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1. A mechanism consists of a beam ABC as shown in Figure 1. It has a quarter-circular part AB of radius r and a straight part BC of length r. The beam is fixed at A, supported on a roller at B, and subjected to a uniformly distributed load w from B to C.

(a) Determine the degree of indeterminacy of the beam.

(2 marks)

(b) If the beam material is linear elastic with flexural rigidity *EI*, determine the reaction at the roller support in terms of w and r. Consider only the bending effect.

(8 marks)

(c) Determine all the possible reaction forces/moment at A in terms of w and r.

(3 marks)

(d) Derive an expression for the maximum bending moment on the beam under this loading in terms of w and r.

(4 marks)

(e) Derive an expression for the vertical deflection at C in terms of w, r and *EI*. Again, consider only the bending effect.

(8 marks)

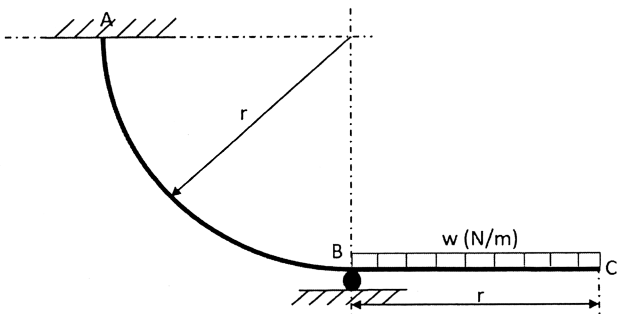

Figure 1. A mechanism consisting of a curved beam AB and a straight beam BC, fixed at A and roller-supported at B. The straight part is subjected to a uniformly distributed load w.

2 (a) A cylindrical steel rod is subjected to a cyclic tensile fatigue test (fully reverse). The S-N curve equation is given below:

$$S_a = 1500\, N^{-0.2},$$

where S_a is the stress amplitude (in MPa) and N is the number of fatigue cycles to failure. The rod is subjected to 10,000 cycles at the stress amplitude of $S_a = 210$ MPa and 4,000 cycles at the stress amplitude of $S_a = 260$ MPa. Determine the cumulative fatigue damage of the rod based on Miner's rule. Would this rod be expected to fail?

(7 marks)

(b) A solar panel is erected on the top of a roof. The width of the panel, w, is 100 cm. During inspection, double edge cracks were found as shown in Figure 2. The size of the edge crack, a, is 1cm. The stress intensity factor of the crack is defined as

$$K_I = \sigma(\pi a)^{\frac{1}{2}}\left(\frac{W}{\pi a}\tan\frac{\pi a}{W} + \frac{0.2W}{\pi a}\sin\frac{\pi a}{W}\right)^{\frac{1}{2}}.$$

This panel is subjected to one cycle of tensile loading per day with a maximum stress of $S_{max} = 30$ MPa and a minimum stress of $S_{min} = 0$ MPa. The fracture toughness K_{IC} is 8 MPa $\sqrt{m}$. Use the Paris Law equation of

$$da/dN \text{ (m/cycle)} = 3.2\times10^{-10}\, \Delta K^4,$$

where ΔK is in MPa $\sqrt{m}$ unit.

(i) What is the geometric correction factor, Y, of the double edge crack with $a = 1$cm?

(3 marks)

(ii) What is the maximum edge crack size before fracture if the applied maximum stress remains at 30 MPa?

(5 marks)

(iii) How many cycles will it take to grow from the initial crack to failure assuming Y remains constant?

(7 marks)

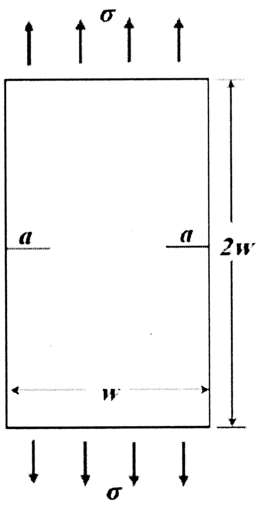

Figure 2: Double edge cracks.

NOTE: Question 2 continues on page 3.

(c) From a fatigue design perspective, describe the advantage of fail-safe concept over safe-life concept.

(3 marks)

3. A mechanical system consists of a uniform rigid bar of mass $2m$, length l and a block of mass m. The bar is pinned at O and is attached to two springs of stiffnesses k and $2k$ in the middle. The block is connected to the right end of the bar via a light rigid strut and is also attached to a spring of stiffness k and a dashpot of constant c. Assume that the system is in static equilibrium when the bar is horizontal.

(a) Draw the free body diagrams of the bar and the block separately and identify all forces acting on them, including the force T in the strut.

(5 marks)

(b) Develop the two equations of motion using the free bodies in Part (a). Combine them into a single equation by eliminating T.

(10 marks)

(c) Determine the constant c in terms of m and k if the system is critically damped.

(4 marks)

(d) What is the general solution to the combined equation of motion in Part (b) for the critically damped system?

(6 marks)

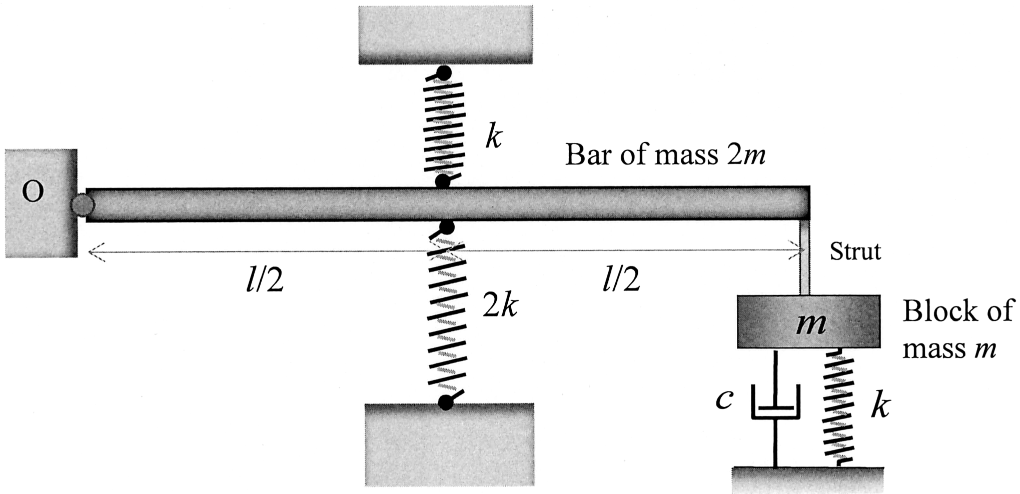

Figure 3: A mechanical system consisting of a uniform rigid bar and a block connected via a light rigid strut. Three springs and a dashpot are also connected as shown.

4. A one-degree-of-freedom (1-DOF) system consisting of a mass M and a spring of stiffness K is subjected to a harmonic force $F_o \sin \omega t$, where F_o is the amplitude, ω is the driving frequency and t is the time, as shown in Figure 4(a). The corresponding displacement X is measured from the static equilibrium position (SEP). Figure 4(b) shows the addition of an absorber mass m and a spring of stiffness k, resulting in a 2-DOF system. The displacements measured from the SEPs are respectively denoted by X and x for the main mass and the absorber mass, respectively. The force $F_o \sin \omega t$ is applied to the main mass only.

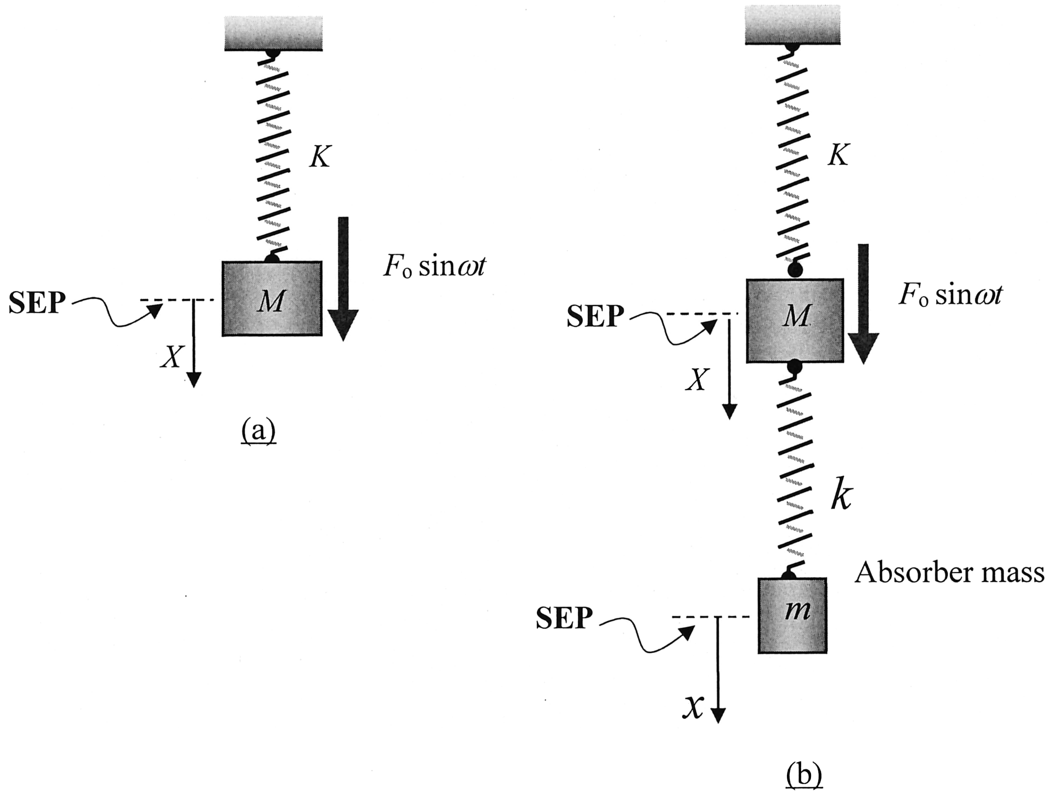

Figure 4:(a) A 1-DOF system subjected to a harmonic force. (b) A 2-DOF system with an absorber mass m added. The harmonic force is applied to the main mass only.

(a) For the 1-DOF system, (i) draw the free body diagram, (ii) develop the equation of motion and (iii) obtain the solution in the form of $X = X_o \sin \omega t$, where X_o is the amplitude which must be determined explicitly in terms of the given parameters.

(6 marks)

(b) What driving frequency will cause resonance in the 1-DOF system?

(4 marks)

(c) For the 2-DOF system, (i) draw the free body diagrams, (ii) develop the equations of motion and (iii) obtain the solutions in the form of $X = X_o \sin \omega t$ and $x = x_o \sin \omega t$ where X_o and x_o are respectively the amplitudes of motion of the main and absorber masses. Obtain X_o explicitly (do not determine x_o).

(10 marks)

NOTE: Question 4 continues on page 5.

(d) For the 2-DOF system, when the driving frequency equals the natural frequency of the original 1-DOF system in Fig. 4(a), determine X_o of the main mass. Hence, what ratio should k/m of the absorber be if X_o is to be zero at this frequency?

(5 marks)

END OF PAPER

MA3002 SOLID MECHANICS & VIBRATION

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.