NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2020-2021

MA3002 – SOLID MECHANICS AND VIBRATION

November/December 2020 | T[illegible]owed: 1 hour

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **F[illegible]4)** pages.

2. Answer **ALL** questions.

3. All questions carry equal marks.

4. This is an **OPEN BOOK E-EXAMINATIO[illegible]** [illegible] are allowed to refer to hard copies of lecture and tutorial materials.

---

1. The frame shown in Figure 1 is made of tw[illegible]ents: segment AB of length 5 m and BC of length 4 m. The flexural stiffness [illegible]0^5 Nm^2 is constant for both segments. If the frame is subjected to point load F[illegible] N and F_2 = 100 N as shown, determine the rotation at point C (in rad). Use un[illegible] method. Consider only bending effects.

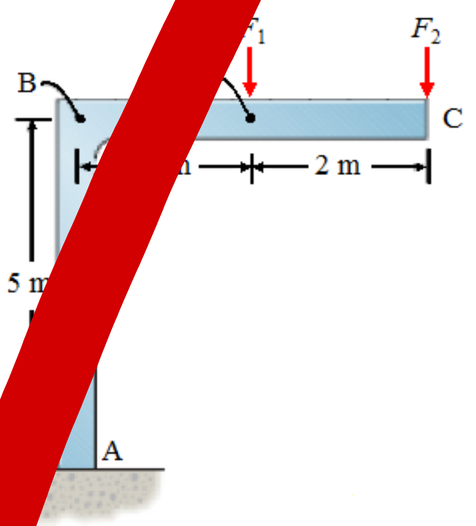


Figure 1

(25 marks)

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2020-2021

MA3002 – SOLID MECHANICS AND VIBRATION

November/December 2020 Time Allowed: 1 hour

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.

2. Answer **ALL** questions.

3. All questions carry equal marks.

4. This is an **OPEN BOOK E-EXAMINATION**. You are allowed to refer to hard copies of lecture and tutorial materials.

---

1. The frame shown in Figure 1 is made of two segments: segment AB of length 5 m and BC of length 4 m. The flexural stiffness $EI = 10^5$ Nm^2 is constant for both segments. If the frame is subjected to point load $F_1 = 100$ N and $F_2 = 100$ N as shown, determine the rotation at point C (in rad). Use unit load method. Consider only bending effects.

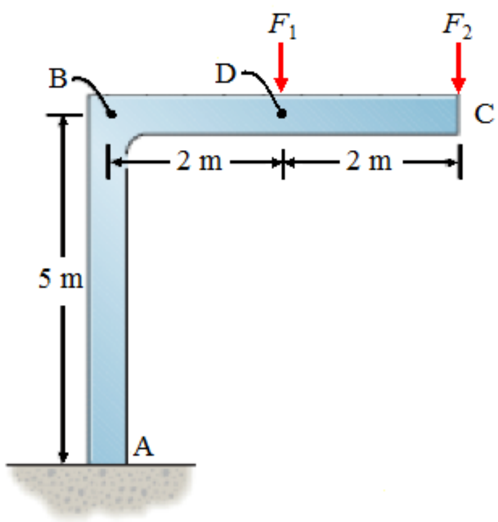


Figure 1

(25 marks)

2. The wall of the pressure vessel shown in Figure 2 is subjected to cyclic pressure loading. The stress engineer designs the wall to have a maximum stress of 1/4 of the yield strength of the material. Material data for the steel alloy is given in Table 1. Assume Y =1.0.

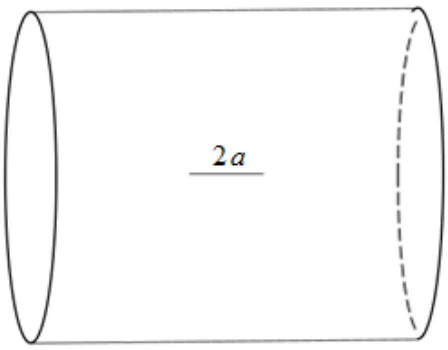


Figure 2

Table 1

Material	K_{IC} (MPa$\sqrt{m}$)	σ_{yield} (MPa)	$\sigma_{ultimate}$ (MPa)	Elongation limit (%)	C	m
Maraging Steel	82	1400	1600	12	1.5E-10	4.1

(a) Using Linear Elastic Fracture Mechanics, calculate the maximum defect size for the pressured vessel with the steel alloy.

(10 marks)

(b) The vessel is maintained with regular Nondestructive Inspection (NDI) using Fluorescent Penetrant Inspection (FPI). The FPI detection capability is 5 mm. Assuming the worst case that there is an initial crack size of 5 mm after inspection, determine the number of fatigue cycles to cause fracture for the steel.

(15 marks)

3. An electric motor of mass M = 30 kg is centrally mounted on a beam of negligible mass as shown in Figure 3. The static deflection (Δ_{st}) caused by the weight of the motor at the mid-section of the beam is 4 mm. If the motor is pushed downwards by 8 mm from the static equilibrium position, the amplitude of vertical oscillation of the motor is observed to dampen to 1 mm in 2 cycles.

(a) Determine the undamped and damped natural frequencies of the system in Hz.

(15 marks)

(b) Assuming that the motor has an unbalanced mass m with an eccentricity e, determine the magnification ratio MX/me at a rotational speed of 600 rpm.

(10 marks)

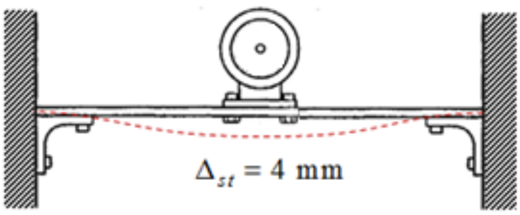


Figure 3

4. Figure 4 shows two rigid rods of equal length l and mass m hinged at points A and C and supported on two springs of equal stiffness k. In the configuration shown, the beams are in the horizontal position with the springs in the compressed state and the whole system is in static equilibrium. For free vibration analysis, the angular displacements θ_1 and θ_2 are measured with respect to the horizontal position (i.e., static equilibrium position) of the rods.

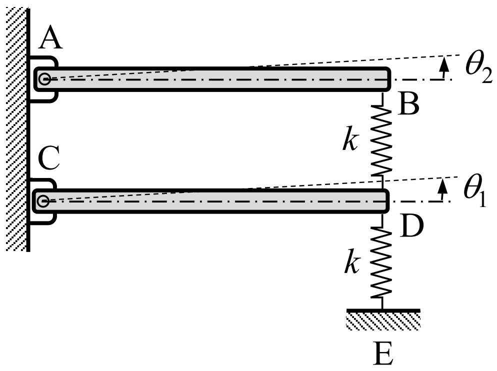


Figure 4

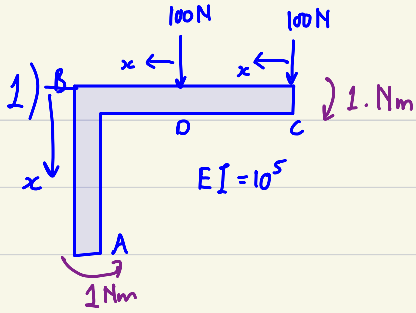
(a) Draw neat free body diagrams of rods AB and CD marking all the forces clearly. The static forces need not be included in the free body diagram for this problem. Explain the reason.

(10 marks)

(b) Applying Newton's 2nd law, derive the equations of motion for this 2-DOF system in terms of the coordinates θ_1 and θ_2 and write them in matrix form. You need not solve these equations.

(15 marks)

End of Paper

1) 

Cantilever: can provide counter-moment.

$\therefore$ no additional R_A needed. only $M_A = -1$ Nm

Real Load:

$M_{CD} = 100x$

$M_{DB} = 200 + 200(x)$

$M_{BA} = 600$

Virtual load:

$m_{CD} = m_{DB} = m_{BA} = 1$

$$1.(\theta_c) = \delta U^* = \int_0^2 \frac{100x}{EI}dx + \int_0^2 \frac{200+200x}{EI}dx + \int_0^5 \frac{600}{EI}dx$$

$$= \frac{1}{10^5}\left\{50\times2^2 + 200\times2 + 100\times2^2 + 600\times5\right\}$$

$$= 0.04 \text{ radians} = 2.29183^\circ \quad (\text{clockwise})$$

2a) $K_{IC} = Y\frac{\sigma_Y}{4}\sqrt{\pi a_c}$

$$a_c = \left(\frac{K_{IC}}{Y\frac{\sigma_Y}{4}}\right)^2\frac{1}{\pi}$$

$$= \left(\frac{82}{1.0\times350}\right)^2\frac{1}{\pi} = 17.4719647 \text{ mm}$$

crack size $= 2a_c = 34.944$ mm

2b) Paris Law:

$$N_f = \frac{2}{C(YS_R)^m\pi^{m/2}(2-m)}\left(a_f^{1-\frac{m}{2}} - a_0^{1-\frac{m}{2}}\right) \qquad a_0 = \frac{5}{2}\text{ mm}$$

$$= \frac{2}{1.5\times10^{-10}(1.0\times350)^{4.1}\pi^{2.05}(-2.1)}\left(0.017472^{-1.05} - 0.0025^{-1.05}\right)$$

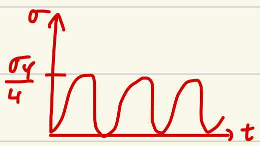
↑ in MPa

$= 10.584$ cycles.

$\therefore$ It takes 11 fatigue cycles to fracture the steel.

★ Pressure Vessel $\Rightarrow$ only tensile stresses.

Worst case scenario $\Rightarrow S_R = \frac{1}{4}\sigma_{Yield}$.



gives the largest possible S_R

3a) $k_{eff} = \frac{F}{\Delta_{st}} = \frac{30\times9.81}{0.004} = 73\,575$ (assume linear elasticity)

$\delta = \frac{1}{2}\ln 8 = 1.03972$

$$\zeta^2 = \frac{\delta^2}{4\pi^2+\delta^2} = 0.02665272$$

$$w_n = \sqrt{\frac{k_{eff}}{m_{eff}}} = 49.52272206 \text{ rad/s} \Rightarrow f_n = 7.88 \text{ Hz}$$

$$f_d = f_n\sqrt{1-\zeta^2} = 7.77 \text{ Hz}$$

3b) 600 rpm $= 20\pi$ rad/s

$$\frac{MX}{me} = \frac{(w/w_n)^2}{\sqrt{(1-w^2/w_n^2)^2 + (2\zeta w/w_n)^2}}, \quad \left(\frac{w}{w_n}\right)^2 = 1.60972$$

$$= \frac{1.60972}{\sqrt{(1-1.60972)^2 + 4\zeta^2\times1.60972}} = 2.18374 \text{ no units}$$

$\sin\theta \approx \tan\theta \approx \theta$
for small θ.

4b) EOM$_1$: $J_\theta \ddot{\theta}_1 + \sum k_\theta \theta = 0$

$$\frac{ml^2}{3}\ddot{\theta}_1 + k\left(l^2\theta_1\right) + k\left(l\theta_1 - l\theta_2\right)l = 0$$

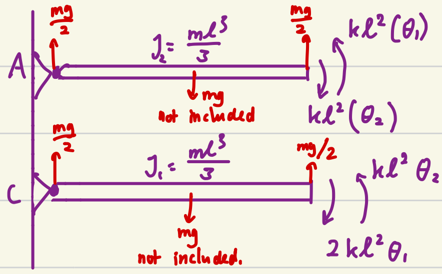
EOM$_2$: $\frac{ml^2}{3}\ddot{\theta}_2 + k\,l^2(\theta_2 - \theta_1) = 0$

$$\begin{bmatrix} \frac{ml^2}{3} & 0 \\ 0 & \frac{ml^2}{3} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} 2kl^2 & -kl^2 \\ -kl^2 & kl^2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$\ddot{\theta}_1 = -\omega^2\theta_1$ and $\ddot{\theta}_2 = -\omega^2\theta_2$:

$$l^2\begin{bmatrix} 2k - \frac{\omega^2 m}{3} & -k \\ -k & k - \frac{\omega^2 m}{3} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

4a)



The static forces are balanced by the static deflections in the springs. Hence they are not required in the dynamic FBD and they would be cancelled out in the Equation Of Motion.