NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2023-2024

MA3002– SOLID MECHANICS AND VIBRATION

Nov/Dec 2023 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is Restricted Open Book Examination. You are allowed to bring into the examination hall one double-sided A4-size reference sheet with texts handwritten or typed on the A4 paper or one restricted material as instructed by the examiner(s) without any attachments (e.g. sticky notes, post-it notes, gluing or stapling of additional papers).

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1. Figure 1 shows a diving board composed of a weightless beam AB with a length of $2L$. The beam is firmly clamped to a rigid wall at end A and supported by a frictionless roller at its midpoint, C. Take the flexural rigidity of the beam as EI. A penguin with weight W stands at the right end B of the beam.

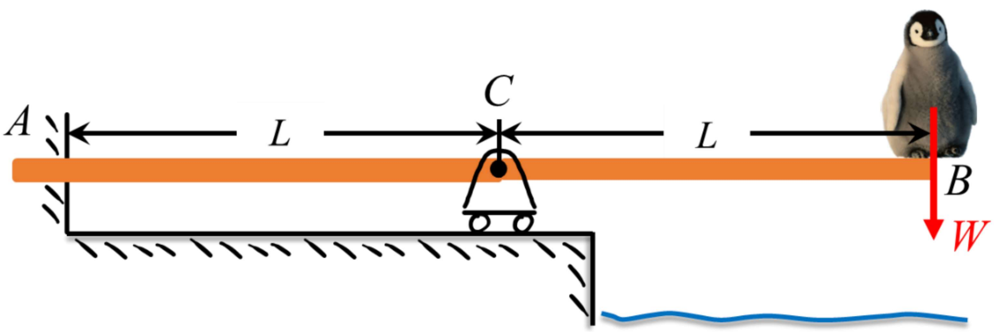

Figure 1

(a) Draw a free body diagram of the cantilever beam AB, and then use the unit load method to determine the reaction force at point C, denoted as R_c.

(10 marks)

(b) Use the unit load method to determine the vertical deflection at point B, denoted as Δ_B.

(10 marks)

(c) Write down the maximum vertical deflection Δ_{max} at B when a penguin of weight W drops from a height of H and impacts the end of the beam. Determine the impact factor by taking $H = 1\text{m}$, W=300N, L=1m, $E = 200\,\frac{\text{kN}}{\text{mm}^2}$, $I = 10^6\ \text{mm}^4$.

(5 marks)

2 (a) A motorcycle component is made of Aluminum alloy. Experimental result shows that when this material contains an internal crack of 4 mm length, it would fracture at a tensile stress of 300 MPa. The correction factor, Y, of the crack is 1.3.

(i) Determine the fracture toughness of the material using Linear Elastic Fracture Mechanics (LEFM).

(3 marks)

(ii) Determine the failure stress of the component if the internal crack is 9 mm long.

(3 marks)

(iii) This component is subjected to 10,000 cycles of tensile loading with a maximum stress of $S_{max} = 130\ MPa$ and a minimum stress of $S_{min} = 0\ MPa$. Use the Paris Law equation of

$$da/dN \text{ (m/cycle)} = 4.2\times10^{-12}\ \Delta K^4$$

where ΔK is in MPa √m unit. Assuming the initial crack size is 4 mm, determine the final crack size after the 10,000 cycles.

(10 marks)

(b) A car shaft is subjected to a cyclic bending moment of 1,000 Nm. The shaft is made of high strength carburized steel. The bending stress of a circular shaft is given as

$$\sigma = \frac{My}{I} \quad \text{where} \quad I = \frac{\pi r^4}{4}$$

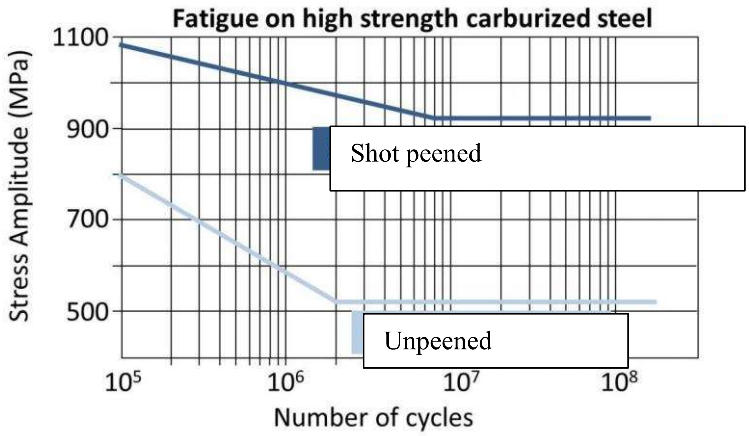

Figure 2

The SN curve for the material is given in Figure 2. The shaft is designed for infinite life.

(i) Determine the endurance limit (maximum stress amplitude for infinite life) and the diameter of the shaft using the unpeened data.
(ii) Determine the diameter of the shaft using the shot peened data.
(iii) Determine the weight saving achieved using the shot peened process.

(9 marks)

3 (a) Figure 3 shows a table-top model of the suspension system of a car in static equilibrium configuration. The wheel has a mass of M and is suspended using a spring of stiffness k and a damper of damping constant c as shown. The axle AB carries the wheel at B and is pin-jointed to a rigid support at A. The axle is rigid and its mass can be ignored in comparison with the wheel mass M. The mass moment of inertia of the wheel about axis CD (that passes through the centre of gravity of the wheel) is I. Treat the suspension system as a 1-DOF vibrating system.

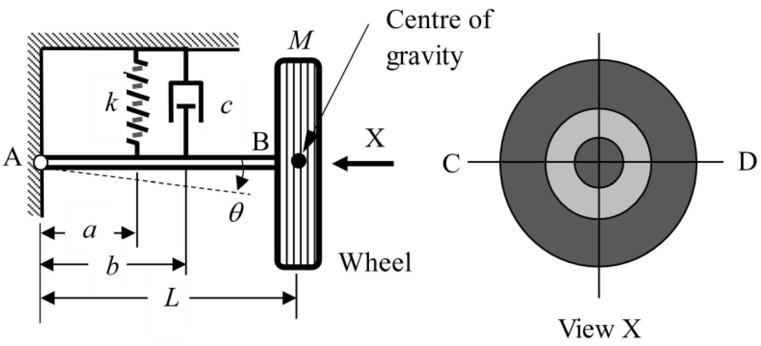

Figure 3

(i) Draw a neat free body diagram of the wheel-axle assembly.

(4 marks)

(ii) Apply Newton's 2nd law to derive the equation of motion of the system for small vibratory displacement θ. From the equation of motion, identify and write down the expressions for the effective inertia, effective damping constant and effective stiffness.

(8 marks)

(iii) Determine the damped natural frequency of the system (in Hz) for L = 0.3 m, a = 0.12 m, b = 0.18 m, k = 25,111 N/m, c = 546 Ns/m, M = 25 kg and I = 0.25 kgm^2.

(8 marks)

(b) Figure 4 shows a mass m initially held by hand, and a spring of stiffness k and damper of damping constant c are now connected to the mass at A and B, respectively. The top ends of the spring and damper are connected to a rigid support. The spring is as such in unstrained condition when the mass is held by hand. Now, the mass is released suddenly by pulling the hand away. The weight of the mass causes a sudden downward loading and this sets the system into vertical oscillations.

NOTE: Question 3 continues on page 4.
Figure 4 appears on page 4.

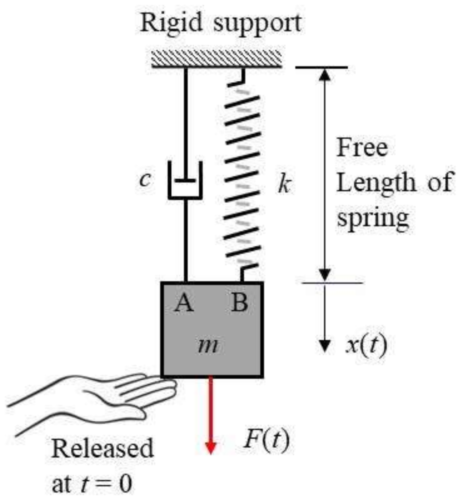

Figure 4

(i) Write down the equation of motion for the mass and the displacement response $x(t)$ of the mass. You need not derive them.

(2 marks)

(ii) Sketch the displacement response of the system $x(t)$ showing clearly how the displacement of the mass varies with time for $t > 0$. No derivation or calculation is needed.

(3 marks)

4. Figure 5 shows a 2-DOF system wherein a rigid block of mass M is roller supported at its bottom side and carries a smaller sized roller-supported block of mass m on its top side. The horizontal motions of the blocks are restrained by the three springs the stiffness of which are as shown. The mass M includes the masses of the two restraint plates shown. Assume all the parts are rigid except the springs. Assume the rollers are of negligible mass in comparison with mass M and m. Ignore friction at all joints and contacting surfaces.

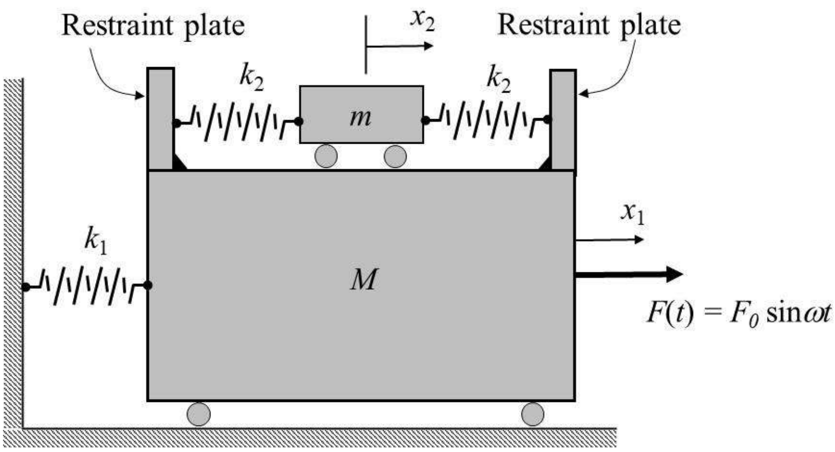

Figure 5

(a) Draw neat free body diagrams of both rigid blocks indicating clearly all the forces acting on them.

(6 marks)

(b) Derive the equations of motion for the system in terms of the coordinates x_1 and x_2, and write them in matrix form.

(7 marks)

(c) Determine the natural frequencies of the system (in Hertz) for $M = 10$ kg, $m = 1$ kg, $k_1 = 10$ N/m and $k_2 = 20$ N/m.

(6 marks)

(d) Taking the same numerical values as in part (c) and taking $F_0 = 5$ N, $\omega = 5$ rad/s, determine the amplitude of steady-state vibration of the smaller block.

(6 marks)

END OF PAPER