NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2018-2019

MA3002 – SOLID MECHANICS AND VIBRATION

November/December 2018 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SEVEN (7)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN BOOK** examination. One double-sided A4 size reference sheet of paper is allowed.

1(a) A pin-jointed three-bar mechanism carries a horizontal force, P, applied at C, and is in equilibrium with the spring attached as shown in Figure 1. The rigid bars each have a length of L. The rigid members AB and DC, has an angle, θ, relative to the vertical plane. The spring has a linear spring constant of k. When the load, P, is removed, the spring will return the mechanism back to its initial position where both members AB and DC are in the vertical position where $\theta = 0°$.

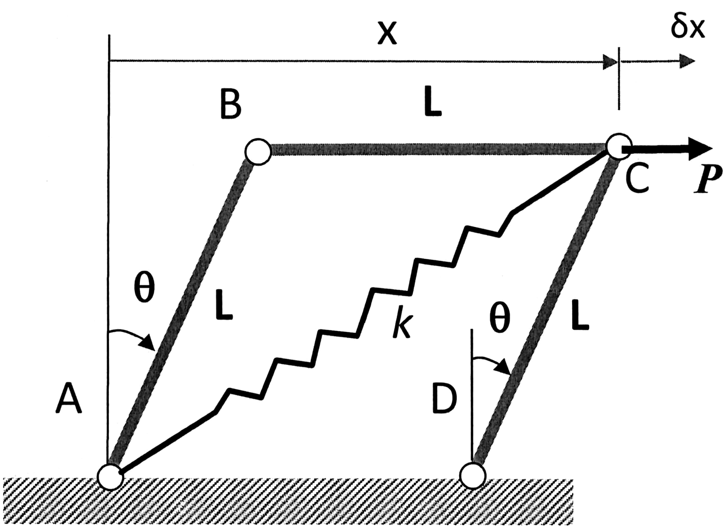

Figure 1

Note: Question 1 continues on page 2.

(i) Determine the relationship between the applied force (P) with respect to the spring stiffness (k), the bar length, L, and the angle theta (θ). Use the Principle of Virtual Work (PVW). Neglect friction and self-weight of the bars.

(6 marks)

(ii) Calculate the applied force P in newton (N), if the angle theta, θ is 30°, the spring stiffness, $k = 100$ kN/m. The length, $L = 1.0$ m.

(3 marks)

(b) A cantilever beam, AB with a length of L, is welded to a curved beam, BC at joint B, as shown in Figure 2. The curved beam of radius of R, has a concentrated load, **P**, applied at the free end at C. The vertical deformation at point C can be calculated using the Unit Load method by applying a unit load at point C.

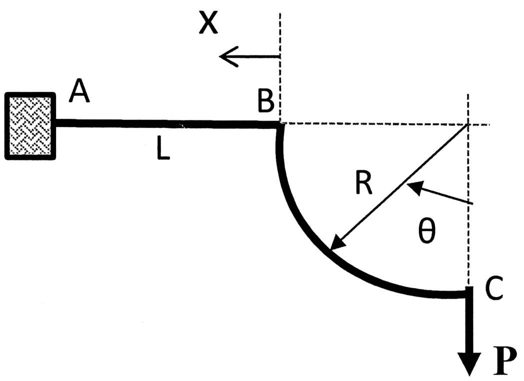

Figure 2

(i) Draw and illustrate the Real Load diagram and Virtual Load diagram using the datum shown in Figure 2. Write down the bending moment expressions for M_{CB}, and M_{BA} for the Real Load diagram and virtual moment expressions for m_{CB}, and m_{BA} for the Virtual Load diagram.

(6 marks)

(ii) Use the Unit Load method to derive an expression for the vertical displacement of the free end at point C. Consider only bending effects where the flexural rigidity is EI.

(8 marks)

(iii) Calculate the vertical deflection given, $P = 1000$ N; $R = L = 0.1$ m; $EI = 21{,}000$ Nm ^{2}.

(2 marks)

2(a) A cantilever beam plate structure is subjected to a tensile load, ***F***, and a bending load, **P**, applied at the free end as shown in Figure 3. After non-destructive test inspection, an edge crack of length, *a*, was found on the top surface at a distance of L, from the free end.

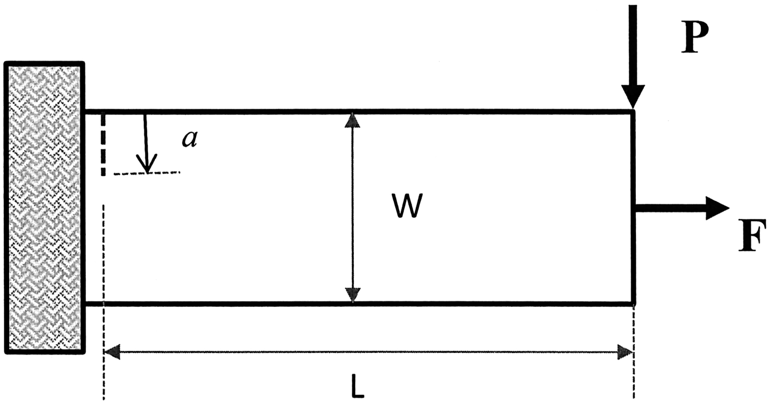

Figure 3: An edge crack in a finite width plate.

The stress intensity factor equations are given below.

$$K_I\,(tension) = \sigma(\pi a)^{\frac{1}{2}}\left[1.12 - 0.23\left(\frac{a}{W}\right) + 10.6\left(\frac{a}{W}\right)^2 - 21.7\left(\frac{a}{W}\right)^3 + 30.4\left(\frac{a}{W}\right)^4\right]$$

$$K_I\,(bending) = \sigma(\pi a)^{\frac{1}{2}}\left[1.122 - 1.4\left(\frac{a}{W}\right) + 7.33\left(\frac{a}{W}\right)^2 - 13.08\left(\frac{a}{W}\right)^3 + 14.0\left(\frac{a}{W}\right)^4\right]$$

(i) Calculate the Stress Intensity Factor in tension, *K (tension)*, if the edge crack length, a = 10 mm, F = 100 KN, W = 100 mm and plate thickness, t = 10.0 mm.

$$\sigma = \frac{F}{Area}$$

(4 marks)

(ii) Calculate the Stress Intensity Factor in bending, *K (bending)*, if the edge crack length, a = 10 mm, P = 10 KN, L = 0.2 m. W = 100 mm and plate thickness, t = 10.0 mm.

$$\sigma = 6M\,/\,t(W)^2$$

(4 marks)

(iii) Calculate the total applied stress intensity factor and check if it will fail by brittle fracture given the plane strain fracture toughness, K_{IC} = 50 MPa $(m)^{1/2}$.

(2 marks)

Note: Question 2 continues on page 4.

(iv) A cyclic tensile stress of 0 to 100 MN/m^2 was applied to the steel plate. Calculate the number of crack propagation cycles needed to grow the edge crack from an initial crack size of 10 mm to 12 mm using the given equation below,

$$N_f = \frac{2}{C(YS_R)^m \pi^{m/2}(2-m)}\left(a_f^{1-m/2} - a_0^{1-m/2}\right)$$

where, C = 0.24×10^{-11} (m/cycle), and m = 3.3

(7 marks)

(b) The fatigue design S-N curve of a material is given by the equation,

$$Sa = A\left(N\right)^{-b}$$

where the stress amplitude, Sa, is in units of MN/m^2, N is the cycles to failure, the constants, A = 300, and the exponent, b = 0.07.

(i) The fatigue limit of the materials can be assumed at 1,000,000 cycles. Calculate the fatigue limit Stress Amplitude, Sa, from the S-N curve equation.

(2 marks)

(ii) Show by calculation if the material will have finite life using Goodman and Gerber rules, if the material has an Ultimate Tensile Strength of Su = 500 MN/m^2, and a Mean Stress of Sm = 300 MN/m^2 and a Fatigue Limit Strength of Se = 250 MN/m^2.

$$\text{Goodman Rule: } \frac{S_a}{S_e} + \frac{S_m}{S_u} > 1$$

$$\text{Gerber Rule: } \frac{S_a}{S_e} + \left(\frac{S_m}{S_u}\right)^2 > 1$$

(6 marks)

3(a) Figure 4(a) shows a rider of mass m on a scooter travelling at an initial speed v before he stopped suddenly the rear wheel. This sudden braking caused a counter-clockwise tilt θ to the rider and the scooter.

To predict the transient of braking motion, Figure 4(b) shows a model consisting of an inverted pendulum rigidly fused to a rotating base. Mass of the scooter is negligible as compared to the rider while its length is L. The stopped rear wheel is modelled as a fixed pivot; whereas, the front wheel is modelled as a spring of spring constant k. This inverted pendulum has a mass m located at a height H from the base and a horizontal distance $L/2$ from the pivot.

(i) Write the equation of motion for this model in terms of a dynamic tilt θ.
(4 marks)

(ii) Determine the maximum spring force due to depression of the front spring given the initial dynamic tilt of zero radian and initial horizontal velocity v.
(4 marks)

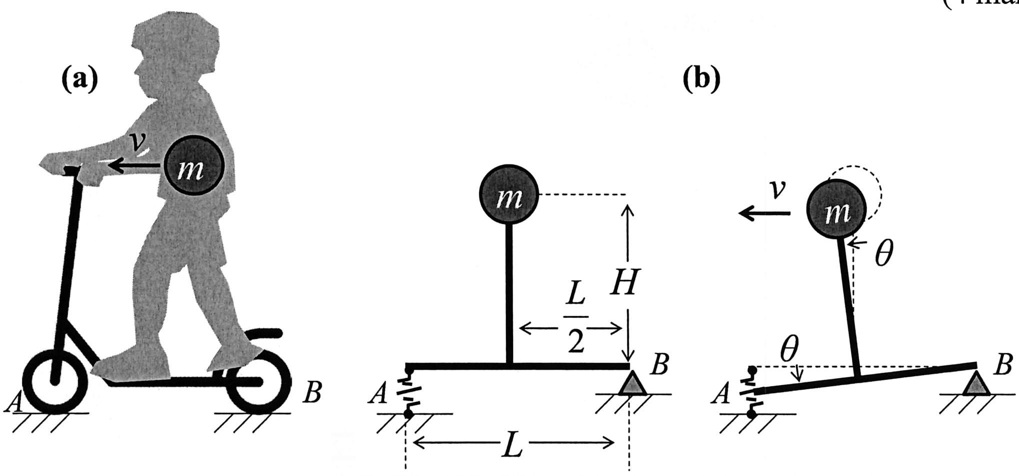

Figure 4

(b) Figure 5 shows a portable air conditioner of total mass M on wheels while its flexible ventilation hose was fixed to an exhaust hole. The flexible hose presents a horizontal spring constant of value k. It happened that the portable air-conditioner vibrated excessively because the fan was spoilt. The spoilt fan is subjected to an unbalance mass m located at an eccentricity radius e from the axle.

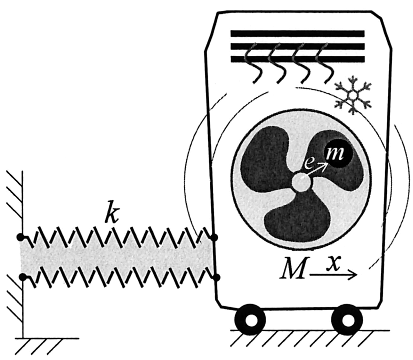

Figure 5

Note: Question 3 continues on page 6.

(i) Derive the expression of vibration amplitude in response to the excitation by rotating unbalance.

(4 marks)

(ii) Derive and plot the amplitude of spring force as a function of excitation frequency and determine the asymptotic limit of the spring force amplitude (at very high excitation frequency).

(3 marks)

(iii) Propose a method to reduce the vibration caused by rotating unbalance.

(3 marks)

(c) A double-decker bus of 15,000 kg self-weight can carry 98 passengers that contribute to an extra 6,000kg load. Suspensions for this bus were designed to be critically damped when this bus was not loaded at all.

(i) Find the damping ratio when the bus was fully loaded by the passengers.

(4 marks)

(ii) Describe and draw a damper design suitable for suspending this double decker bus that undergoes a substantial load change.

(3 marks)

4(a) A 1 kg piston was supported on a spring and immersed in a pot of oil. When displaced, it oscillated at a 2Hz frequency and the amplitude of oscillation decayed from 15mm to 5mm in a cycle.

(i) What is the logarithmic decrement?

(3 marks)

(ii) What is the damping ratio?

(3 marks)

(iii) What is the spring constant?

(3 marks)

Note: Question 4 continues on page 7.

(b) Figure 6 shows two identical compound pendulums each of mass m and length L being pivoted at different height. The pivot of the right pendulum is half span (length) higher than that of the left one. Meanwhile, the two pendulums were coupled by a spring of spring constant k, which connects the middle of the left pendulum and the tip of the right one.

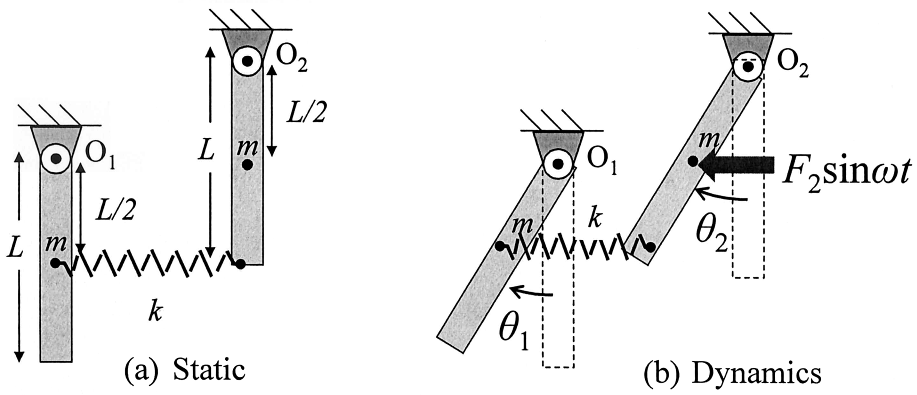

Figure 6

(i) The two coupled pendulum underwent forced vibrations when a harmonic force $F\sin\omega t$ was applied at the middle of the right pendulum. Derive the equations of motion and solve the amplitude of angular displacement θ_1 for the left pendulum

(8 marks)

(ii) The two coupled pendulum underwent free vibrations for a while after removal of the excitation force. Derive the expressions for the two natural frequencies and two mode shapes. Sketch the mode shapes for illustration.

(8 marks)

End of Paper

MA3002 SOLID MECHANICS & VIBRATION

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.