MA3006 sem1 17/18

1(a)(i) $\dot{m} = \rho AV = 1000 \times \pi(0.02)^2 \times 12 = 4.8\pi \text{ kg/s}$

$V_{1x} = 12 \text{m/s} \quad V_{1y} = 0 \text{m/s}$

$V_{2x} = -12 \text{m/s} \quad V_{2y} = 0 \text{m/s} \quad \dot{m}_1 = \dot{m}_2 = 4.8\pi$

$\Sigma F_y = \Sigma \dot{m}(V_{out})_y - \dot{m}(V_{in})_y$

$F_y - W = 0 \Rightarrow F_y = W = 200 \times 9.81 = 1962 N$

$\Sigma F_x = \Sigma \dot{m}(V_{out})_x - \dot{m}(V_{in})_x$

$F_x = \dot{m}_2 V_{2x} - \dot{m}_1 V_{1x}$

$= 4.8\pi(-12-12) = -361.9 N$

$F_R = \sqrt{(-361.9)^2 + (1962)^2} = 1995 N$ (✓)

1(a)(ii) Relative velocity $= 12 - V$ & let $W = 12 - V$

$\dot{m} = \rho WA = 1000 \times \pi(0.02)^2 (W) = 1.2566 W \text{ kg/s}$

$F_x = \dot{m}(W_{in} - W_{out}) = 1.2566 W(W - (-W)) = 2.5132 W^2$

$a = \frac{F_{net}}{m} = \frac{2.5132 W^2}{200} = 0.012566 W^2 = \frac{dV}{dt}$

$t = \int_0^3 \frac{1}{a} dV = \int_0^3 \frac{1}{0.012566 W^2} dV$

$= \int_0^3 \frac{1}{0.012566(12-V)^2} dV = 2.21s$

just use GC!

$Q_{actual} = C_0 A_2$

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2(a) $F_L = \phi(1$

pt 0: pt @ atmosphere

1(b) $\beta = \frac{d_3}{D_3} = \frac{0.5}{0.8} = 0.625$

$Q_{Actual} = C_0 Q_{ideal} = A_0 V_0$

$Q_{actual} = C_0 A_0 \sqrt{\frac{2(P_3 - P_0)}{\rho(1-\beta^4)}} = 0.62 \times \pi (0.25)^2 \sqrt{\frac{2(100\times10^3)}{1000(1-0.625^4)}} = 1.87 m^3/s$

$V_{actual} = \frac{1.87}{\pi(0.4)^2} = 3.721 m/s$ (velocity @ pipe)

$\dot{m}_3 = 1870 kg/s$

$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

$\dot{m}_1 = \rho A_1 V_1 = 1000 \times \pi (0.2)^2 \times 8 = \cancel{502.7 kg/s}$ $1005.3 kg/s$

$\dot{m}_2 = 864.7 kg/s$

$V_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{864.7}{1000 \times \pi (0.15)^2} = 12.23 m/s$

(−Hpower) only missing power output

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} + \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2} - h_L = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + \cancel{Z_3} + H_{power}$

$\frac{300\times10^3}{9810} + \frac{8^2}{19.62} + \frac{400\times10^3}{9810} + \frac{12.23^2}{19.62} - h_L = \frac{200\times10^3}{9810} + \frac{3.721^2}{19.62}$

$h_L = 61.15 m$

$P_{loss} = \rho g Q h_L = 9810 \times 1.87 \times 61.15 = 1122 kW$

cannot just use BE because all have diff mass flow rate

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \frac{P_2}{\rho g} + \frac{V_2^2}{2g} - \frac{P_3}{\rho g} + \frac{V_3^2}{2g} = H_{power\ output} + H_{power\ loss}$ $\frac{50\times10^3}{9810\times1.87}$

$H_{power\ output} = \frac{(300+400-200)\times10^3}{9810} + \frac{(8^2+12.23^2-3.721^2)}{19.62} - \frac{50\times10^3}{9810\times1.87} = 58.4m$

$P = \rho g Q H_{power\ output} = 9810 \times 1.87 \times 58.4 =$

Only can use energy eqn & separate them differently! & need to consider each of their $\dot{m}$

☆ $\sum_{in} \dot{m}(\frac{p}{\rho} + \frac{V^2}{2}) - \sum_{out} \dot{m}(\frac{p}{\rho} + \frac{V^2}{2}) = W_{out} + \text{Power loss}$

$1005.3\left(\frac{300\times10^3}{1000} + \frac{8^2}{2}\right) + 864.7\left(\frac{400\times10^3}{1000} + \frac{12.236^2}{2}\right) - 1870\left(\frac{200\times10^3}{1000} + \frac{3.721^2}{2}\right)$

$= W_{out} + 50\times10^3 \Rightarrow W_{out} = 307481 W$

~~Wout~~ output $= 0.8 \times 307481 = 246 kW$

2(a) $F_L = \phi(V, l, S, \alpha, c, \rho, \mu)$

use ρ, V, l as parameters
5 π grps
$\pi_1: F_L$, $\pi_2: S$, $\pi_3: \alpha$ no units
$\pi_4: c$, $\pi_5: \mu$

$F_L = kg\left(\frac{m}{s^2}\right)$ $S = m$

$c = \frac{m}{s}$

$\mu = kg\left(\frac{m}{s^2}\right)\left(\frac{s}{m^2}\right) = \frac{kg}{ms}$

$\pi_1 = F_L(\rho)^a(V)^b(l)^c$

$= \frac{kg\,m}{s^2}\left(\frac{kg}{m^3}\right)^a\left(\frac{m}{s}\right)^b(m)^c$

kg: $1+a=0 \Rightarrow a=-1$
m: $1-3a+b+c=0 \Rightarrow 1+3-2+c=0 \Rightarrow c=-2$
s: $-2-b=0 \Rightarrow b=-2$

$\pi_1 = \frac{F_L}{\rho V^2 l^2}$

$\pi_2 = \frac{S}{l}$ & $\pi_3 = \frac{c}{V}$ & $\pi_4 = \alpha$

$\pi_5 = \mu(\rho)^a(V)^b(l)^c$

OR $\overset{\text{dimensionless}}{Re} = \frac{\rho V d}{\mu}$ & l & d has the same parameter

$= \frac{kg}{ms}\left(\frac{kg}{m^3}\right)^a\left(\frac{m}{s}\right)^b(m)^c$

kg: $1+a=0 \Rightarrow a=-1$
m: $-1-3a+b+c=0 \Rightarrow c=-1$
s: $-1-b=0 \Rightarrow b=-1$

$\pi_5 = \frac{\mu}{\rho V l}$

$\frac{F_L}{\rho V^2 l^2} = \phi\left(\frac{S}{l}, \frac{c}{V}, \alpha, \frac{\mu}{\rho V l}\right)$

2(b) $\frac{\mu_p}{\rho_p l_p V_p} = \frac{\mu_m}{\rho_m l_m V_m}$ $l_p = 20$ & $l_m = 1$ $\mu_m = 1.82 \times 10^{-5}$

$\mu_p = 1.57 \times 10^{-5}$, $\rho_p = 1.514$ $\rho_m = 1.204$

$$\frac{1.57 \times 10^{-5}}{1.514 \times 20 V_p} = \frac{1.82 \times 10^{-5}}{1.204 V_m}$$

$$\frac{V_p}{V_m} = 0.0343$$

$$\frac{(F_L)_p}{\rho_p V_p^2 l_p^2} = \frac{(F_L)_m}{\rho_m V_m^2 l_m^2} \Rightarrow (F_L)_p = (F_L)_m \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{l_p}{l_m}\right)^2$$

$$(F_L)_p = 1.25 \times \left(\frac{1.514}{1.204}\right)(0.0343)^2(20)^2 = 0.74\,kN$$

2(c) For laminar flow,

$$V_{avg} = \frac{V_c}{2} = \frac{\Delta P D^2}{32 \mu l}$$

$Q = AV$; $V = V_{avg}$

$$Q = \frac{\pi}{4} D^2 \cdot \frac{\Delta P D^2}{32 \mu l} = \frac{\pi D^4 \Delta P}{128 \mu l} \quad \text{(Shown)}$$

$$Re = \frac{\rho V d}{\mu} \Rightarrow V = \frac{Re\,\mu}{\rho d} = \frac{800(0.00025)}{1050 \times 0.01} = 0.0190\,m/s$$

$$V_c = 2(0.0190) = 0.0380\,m/s$$

$$\frac{\Delta P}{l} = \frac{16 \mu V_c}{D^2} = \frac{16(0.00025)(0.0380)}{0.01^2} = 1.52\,N/m^3$$

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3(a) $\frac{P_1}{\rho g}+\cancel{\frac{V_1^2}{2g}}+\cancel{Z_1}-h_L=\frac{P_2}{\rho g}+\cancel{\frac{V_2^2}{2g}}+\cancel{Z_2}$ & $h_L=f\frac{l}{d}\frac{V^2}{2g}$

$V_1=V_2$
$Z_1=Z_2=0$

$h_L=\frac{P_1-P_2}{\rho g}=\frac{\Delta P}{\rho g}=f\frac{l}{d}\frac{V^2}{2g}$

$\Delta P=f\frac{l}{d}\frac{1}{2}\rho V^2 \Rightarrow V=\sqrt{\frac{2\Delta P}{f}\frac{d}{l}\frac{1}{\rho}}$ (shown)

Since laminar flow,

$f=\frac{64}{Re}=\frac{64\mu}{\rho Vd}=\frac{64(0.001)}{1000V(0.01)}=\frac{4}{625V}$

$500=\frac{4}{625\cancel{V}}\times\frac{10}{0.01}\times 500V^{\cancel{2}} \Rightarrow 500=3200V \Rightarrow V=0.15625m/s$

$Q=AV=\pi(0.005)^2\times 0.15625=1.23\times10^{-5}m^3/s$

3(b) $\frac{P_A}{\rho g}+\cancel{\frac{V_A^2}{2g}}+\cancel{Z_A}-h_L=\cancel{\frac{P_B}{\rho g}}+\frac{V_B^2}{2g}+\cancel{Z_B}$

V_A @ tank $\Rightarrow V_A=0$
V_B @ pipe $\Rightarrow V_B=V_1$

~~$P_A+2\rho g h_L=2.5\frac{1}{2}\rho V_1^2$~~

$\frac{P_A}{\rho g}=3.5\frac{V_1^2}{2g}-2 \Rightarrow P_A=3.5(1000)V_1^2-2(9810)$

$V_1=\sqrt{\frac{120\times10^3+2(9810)}{3.5(1000)}}=6.32\ m/s$

3(b)(i) $\frac{P_0}{\rho g}+\cancel{\frac{V_0^2}{2g}}+Z_0-h_L=\frac{V_2^2}{2g}$

V_0@ tank $\Rightarrow V_0=0$
$V_1\neq V_2$ (diameter is diff)
$Q_1=Q_2$

$\frac{120\times10^3}{9810}+2-2.5\frac{V_1^2}{2g}=\frac{V_2^2}{2g}$ ⇒ outlet's velocity head

$V_1=\frac{Q}{A_1}=\frac{Q}{\pi(0.05)^2}=127.32Q$

$14.232-\frac{2.5}{19.62}(127.32Q)^2-\frac{1}{19.62}(2037.18Q)^2=0$

$V_2=\frac{Q}{A_2}=\frac{Q}{\pi(0.0125)^2}=2037.18Q$

$213589Q^2=14.232 \Rightarrow Q=0.00816m^3/s$

$V_2=2037.18(0.00816)=16.629m/s$

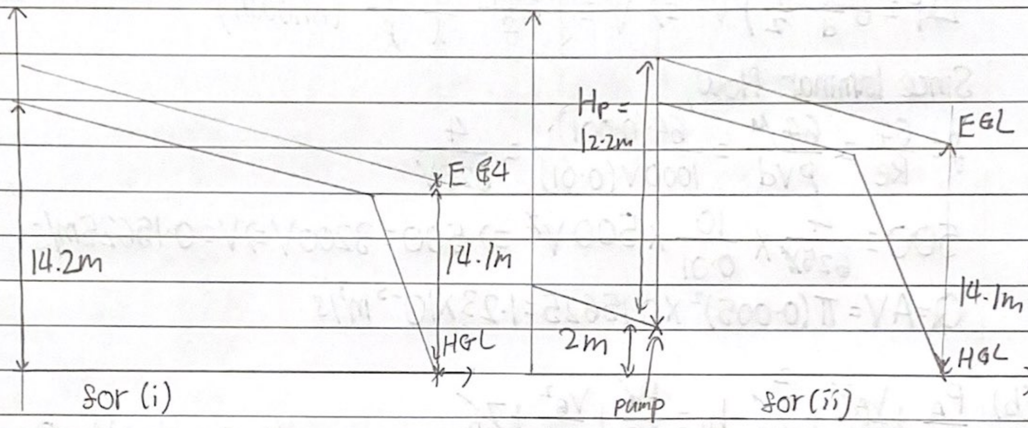
velocity head @ outlet $=\frac{16.629^2}{19.62}=14.1m$

3(b)(ii) Power input $=\frac{\rho gQH_P}{\eta}=\frac{9810(0.00816)H_P}{\eta}$ ⇒ ?

3(b)(ii) $H_p + \cancel{z_2} = 2.5\frac{V_1^2}{2g} + \frac{V_2^2}{2g} - 2 = 213589(0.00816)^2 - 2 = 12.22m$

Power input $= \frac{\rho g Q H_p}{\eta} = \frac{9810(0.00816)(12.22)}{0.7} = 1398\cancel{k}W$

3(b)(iii)



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4(a) $E=(Z_2-Z_1)+f_1\frac{l_1}{d_1}\frac{V_1^2}{2g}+f_2\frac{l_2}{d_2}\frac{V_2^2}{2g}$ $V_1 \neq V_2$ (diameter diff)

$V_1=\frac{Q}{\pi(0.1525)^2}=13.687Q$ & $V_2=\frac{Q}{\pi(0.125)^2}=20.372Q$ $V=\frac{Q}{A}$

$E=(Z_2-Z_1)+\frac{f_1 l_1}{d_1}\frac{1}{2g}\left(\frac{Q}{A_1}\right)^2+\frac{f_2 l_2}{d_2}\frac{1}{2g}\left(\frac{Q}{A_2}\right)^2$ $A=\frac{\pi}{4}d^2$

$\frac{1}{A}=\frac{4}{\pi d^2}$

$E=(Z_2-Z_1)+\frac{f_1 l_1}{d_1}\frac{1}{2g}\frac{Q^2}{(\frac{\pi}{4}d_1^2)^2}+\frac{f_2 l_2}{d_2}\frac{1}{2g}\frac{Q^2}{(\frac{\pi}{4}d_2^2)^2}$

$E=(Z_2-Z_1)+\frac{1}{2g(\frac{\pi}{4})^2}\left[\frac{f_1 l_1}{d_1^5}+\frac{f_2 l_2}{d_2^5}\right]Q^2 \Rightarrow E=(Z_2-Z_1)+0.08262\left[\frac{f_1 l_1}{d_1^5}+\frac{f_2 l_2}{d_2^5}\right]Q^2$ (shown)

4(b)(i) $E=6+0.08262(1822.7)Q^2 \Rightarrow E=6+150.6Q^2$

Operating flow rate = pump chara & system chara intersection

Power flow = $\rho g Q h_p$

Input power required = $\frac{\rho g Q h_p}{\eta}$ ← don't have graph but must write down the steps!

4(b)(ii) Cavitation will likely occur b4 the pump.

There is no energy input frm suction surface up to the point b4 the pump ~~HGL is~~ Since velocity is constant, HGL likely to dip below the datum line as energy is used to gain height & overcome friction

4(b)(iii) $\frac{P_s-P_v}{\rho g}+\frac{V_s^2}{2g}+Z_s=\frac{P_1-P_v}{\rho g}+\cancel{\frac{V_1^2}{2g}}+Z_1-h_L$ V_1 @ tank $\Rightarrow V_1=0$

$\frac{P_s-P_v}{\rho g}+\frac{V_s^2}{2g}=\frac{P_1-P_v}{\rho g}+Z_1-Z_s-h_L$

$NPSH_A=\frac{100\times10^3-2750}{1020(9.81)}+(-1)-0.08262\left(\frac{f_1 l_1}{d_1^5}\right)Q^2$

$NPSH_A > NPSH_R \Rightarrow$ cavitation will not occur

$NPSH_A < NPSH_R \Rightarrow$ cavitation will occur

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4(b)(iv) limiting suction level is at the pt where $NPSH_A = NPSH_R$

If $H_p > NPSH_A \Rightarrow$ pump is suitable

If $H_p < NPSH_A \Rightarrow$ pump is not suitable