NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

MA3002 – SOLID MECHANICS AND VIBRATION

April/May 2017 Time Allowed: 2[illegible]

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages[illegible]

2. Answer **ALL** questions.

3. All questions carry equal marks.

4. This is a **RESTRICTED OPEN-BOOK** examination. O[illegible]ble-sided A4 size reference sheet is allowed.

1(a) Figure 1 shows a rigid uniform bar AB of length [illegible]ported on rigid rollers pin-joined at the bar ends A and B. A spring of stiffn[illegible]d free length l_0 connects the end A of the bar to a rigid wall at C. A load P a[illegible]he end B of the bar as shown. Ignore friction at all the joints and the weights [illegible]e parts.

Using the *principle of virtual work*, derive [illegible]ession for the force P in terms of y and other parameters.

(10 marks)

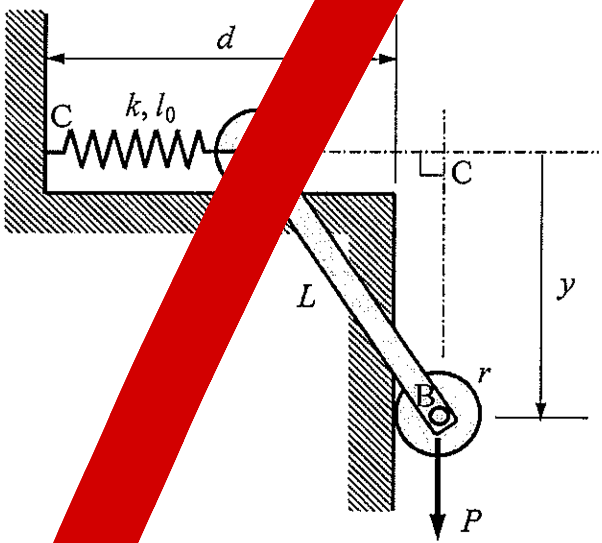

Figure 1

Note: Q[illegible]n 1 co[illegible] on page 2.

21

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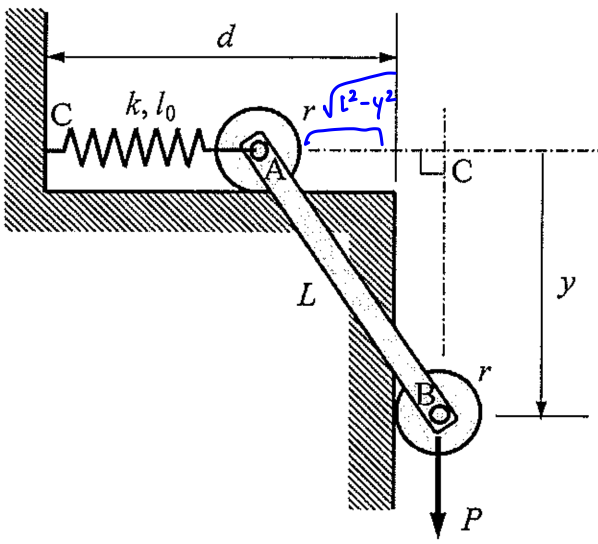
1(a) Figure 1 shows a rigid uniform bar AB of length L supported on rigid rollers pin-joined at the bar ends A and B. A spring of stiffness k and free length l_0 connects the end A of the bar to a rigid wall at C. A load P acts at the end B of the bar as shown. Ignore friction at all the joints and the weights of all the parts.

Using the *principle of virtual work*, derive an expression for the force P in terms of y and other parameters.

(10 marks)



$P\delta y = ke\,\delta e$

$e = d - \sqrt{L^2-y^2} - r - l_0$

$\delta e = \frac{de}{dy}\delta y$

$= \frac{2y}{\sqrt{L^2-y^2}}\delta y$

$P\cancel{\delta y} = k(d - \sqrt{L^2y^2} - l_0 - r)\left(\frac{2}{\sqrt{L^2-y^2}}\cancel{\delta y}\right)$

$P = 2k\left(\frac{d-l_0-r}{\sqrt{L^2-y^2}} - 1\right)$

Figure 1

Note: Question 1 continues on page 2.

21

(b) Figure 2 shows a "Z" shaped beam ABCD of with segments AB and CD having the same length L, and segment BC having a length $\sqrt{2}L$. All the three segments have the same flexural rigidity EI. The beam is fixed to a rigid wall at A and carries a vertical load P at D. Consider only bending effects. Assume linear elastic behavior of the structure.

(i) Using the *unit load method*, derive an expression for the vertical deflection at D in terms of P, L and EI.

(10 marks)

Real Load:

$M_{DC} = Px$

$M_{CB} = PL - \frac{P}{\sqrt{2}}x$

$M_{BA} = Px$

Virtual:

$m_{DC} = x$

$m_{CB} = L - \frac{x}{\sqrt{2}}$

$m_{BA} = x$

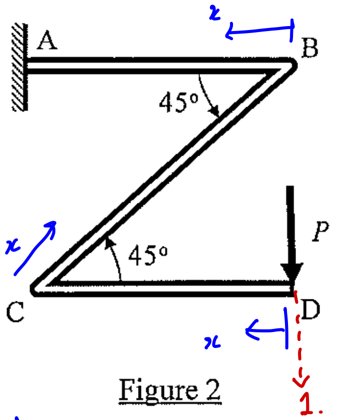


Figure 2

$$1 \cdot \Delta_D = \frac{1}{EI}\left\{\int_0^L Px^2 dx + \int_0^{\sqrt{2}L} PL^2 - \frac{PxL}{\sqrt{2}} - \frac{PxL}{\sqrt{2}} + \frac{Px^2}{2} dx + \int_0^L Px^2 dx\right\}$$

$$= \frac{1}{EI}\left\{2\left[\frac{Px^3}{3}\right]_0^L + \left[PL^2x - \frac{\sqrt{2}Px^2L}{2} + \frac{Px^3}{6}\right]_0^{\sqrt{2}L}\right.$$

$$= \frac{1}{EI}\left\{\frac{2}{3}PL^3 + \sqrt{2}PL^3 - \sqrt{2}PL^3 + \frac{\sqrt{2}PL^3}{3}\right\} = \frac{PL^3}{3EI}(2+\sqrt{2})$$

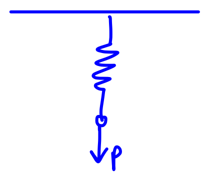
(ii) Now, instead of the static load P, consider an impact load due to a mass 10 kg falling from a height of 0.3 m directly above point D. Calculate the maximum vertical deflection at D caused by the impact load. List the usual simplifying assumptions made in such an analysis. Take $L = 1$ m and $EI = 10^4$ Nm^2 for the calculation.

(5 marks)

$F = kx$

$$k_{eff} = \frac{F}{x} = \frac{P}{\frac{PL^3}{3EI}(2+\sqrt{2})} = \frac{3EI}{L^3(2+\sqrt{2})}$$

Simplify into:



$$\Delta_{static} = \frac{F}{k} = \frac{9.81 \times 10}{\frac{3 \times 10^4}{2+\sqrt{2}}}$$

$=$

spring with constant k
whereby $k = k_{eff} = \frac{F}{\Delta_s}$

$$\max \Delta = \Delta_{static}\left(1 + \sqrt{1 + \frac{2h}{\Delta_s}}\right)$$

$=$

2(a) A beam of a rectangular cross section with width $W = 40$ mm and thickness $t = 15$ mm is subjected to a steady bending moment $M = 700$ Nm as shown in Figure 3(a). The top surface of the beam has a through edge crack of length $a = 10$ mm. The beam is made of a material with fracture toughness $K_c = 42$ MNm$^{-3/2}$ and yield stress $\sigma_Y = 300$ MPa.

The tensile stress at the top surface is given by

$$\sigma = \frac{6M}{W^2 t} = \frac{6 \times 700}{0.04^2 \times 0.015} = 175 \text{ MPa}$$

The stress intensity factor at the crack tip is given by

$Y = 1.44073619$

$$K = \sigma\sqrt{\pi a}\,\sec\beta\left(\frac{\tan\beta}{\beta}\right)^2\left[0.923 + 0.199(1-\sin\beta)\right]^4 \quad \text{where } \beta = \frac{\pi a}{2W} \text{ and } \sigma = \frac{6M}{W^2 t}$$

$K = 44.7$ MNm$^{-3/2}$ radian mode. $= 0.03927$

The beam can fail by yielding ($\sigma > \sigma_Y$) or by fracture ($K > K_c$). Ignore plastic zone correction for fracture calculations.

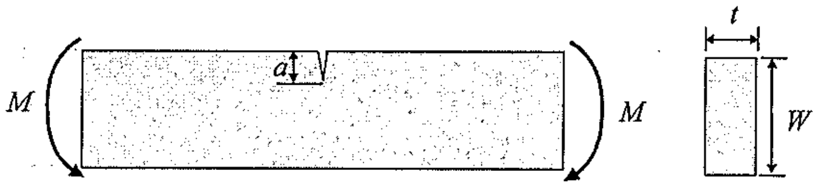


Figure 3(a)

(i) Determine safety factors against yield failure and fracture, and comment if the beam is safe under the given loading. Yield: $SF = \frac{300}{175} = 1.7143$

fracture: $SF = \frac{42}{44.7} < 1$ (5 marks)

$\therefore$ not safe

(ii) It is intended to remove a 6mm-thick layer of the material by grinding from the top surface of the beam as shown in Figure 3(b) so that the crack length reduces to 4 mm. Determine again the safety factors against yield failure and fracture, and comment if material removal helps in improving the safety factors.

new $W = 34$ mm

β and σ change.

K also change.

α also change

Lazy do.

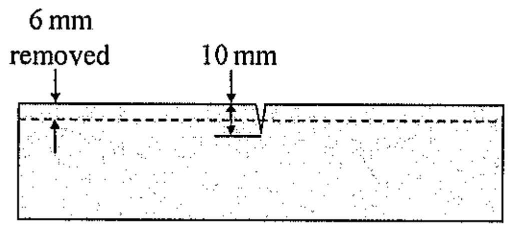


Figure 3(b)

(5 marks)

Note: Question 2 continues on page 4.

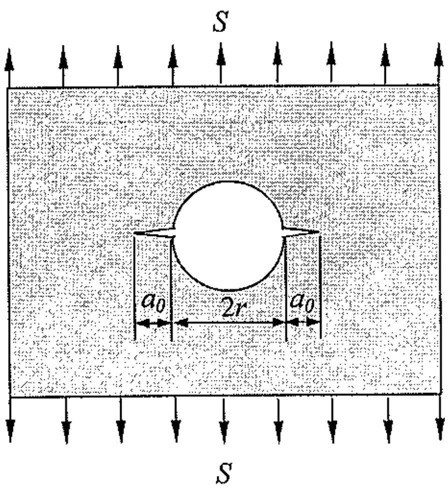
(b) Figure 3(c) shows a large plate in a ship hull with a central hole subjected to a fully reversed uniaxial sinusoidal loading with the stress S fluctuating between +200 MPa and −200 MPa. The frequency of loading is 3 cycles per minute. In a routine inspection of the ship hull, it is discovered that the hole has two edge cracks of same length $a_0 = 0.005$ m symmetrically located as shown. $K_c = Y\sigma_f\sqrt{\pi a_f}$

$a_f = \left(\frac{K_c}{Y\sigma_f}\right)^2\frac{1}{\pi} = \left(\frac{90}{1.24\times200}\right)^2\frac{1}{\pi} = 41.92$ mm

Determine the remaining fatigue life (in hours) of the plate. For this calculation, assume that the cracks grow symmetrically conforming to Paris' law. Because of symmetry, it is sufficient to consider only one of the two cracks. Take the following data for your calculations: $K_c = 90$ MNm$^{-3/2}$; radius of hole $r = 0.08$ m; Paris' law constants $m = 3$ and $C = 1.2\times10^{-11}$ (with $\frac{da}{dN}$ in m/cycle and ΔK in MNm$^{-3/2}$. Geometry factor $Y = 1.24$ (assumed to remain constant throughout the crack growth).

$S_R = 400$

after you find a_f can find N_f easily with Paris law.



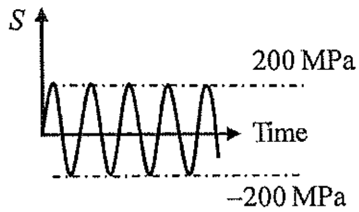
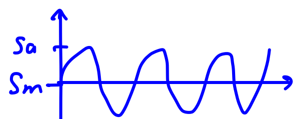


Figure 3(c)



(10 marks)

(c) Specimens made of a certain grade of steel material were subjected to a fully reversed uniaxial sinusoidal loading in an axial load fatigue testing machine. It was found that the material had a fatigue life of 755,243 cycles at a stress amplitude 240 MPa and a fatigue life of 214,286 cycles at a stress amplitude 300 MPa. It is intended to curve fit this fatigue test data so as to obtain an SN curve in the form $S = A + B/N$ where S is the stress amplitude, N is the life in number of cycles, and A and B are appropriate constants.

(i) Determine the values of constants A and B. simultaneous eq.

(3 marks)

(ii) Determine the endurance limit for the material.

(2 marks)

Goodman $\frac{S_a}{S_e} + \frac{\cancel{S_m}^{0}}{S_u} \leq 1$ Gerber $\frac{S_a}{S_e} + \left(\frac{\cancel{S_m}^{0}}{S_u}\right)^2 \leq 1$ Soderberg $\frac{S_a}{S_e} + \frac{\cancel{S_m}^{0}}{S_Y} \leq 1$ for no fatigue failure.

$\therefore$ All 3 reduces to $\frac{S_a}{S_e} \leq 1$

$\therefore S_a = S_e = ?$

OR can assume $N = 50\times10^6$ cycles as long life.

$S = A + \frac{B}{50\times10^6}$

3(a) Figure 4 shows a pendulum, which consists of a rod of a mass m and a length L, pivoted at the point O of the celling. A spring of axial stiffness k and negligible mass is fixed to the rod next to the free end A.

(i) Find the natural frequency of this pendulum system, as shown in Figure 4(i). (5 marks)

$J\ddot{\theta} + k_\theta \theta = 0 \quad \frac{1}{3}mL^2\ddot{\theta} + \frac{mgL}{2}\theta = 0 \quad \therefore W_n = \sqrt{\frac{k_\theta}{J}} = \sqrt{\frac{\cancel{m}gL/2}{\frac{1}{3}\cancel{m}L^{\cancel{2}}}} = \sqrt{\frac{3g}{2L}}$

(ii) Find the maximum angular velocity that the pendulum attains during its free swing after the release from an initial 30° angular position and a zero initial speed as shown in Figure 4(ii). GPE $\Rightarrow$ KE. (3 marks)

$mg\Delta h = \frac{1}{2}J_0 w^2 \Rightarrow mg\left(\frac{L}{2} - \frac{L}{2}\cos 30°\right) = \frac{1}{6}mL^2w^2 \Rightarrow w^2 = \frac{3g}{L}(1-\cos 30)$

$\frac{mg}{2}L(1-\cos 30°)$ $\quad w = \sqrt{\frac{3g}{L}(1-\cos 30)}$

(iii) Derive the expression for the pendulum's angular acceleration upon its contact at 0° onto an adjacent wall after its release from an initial 30° angular position and a zero initial speed as shown in Figure 4(iii). (7 marks)

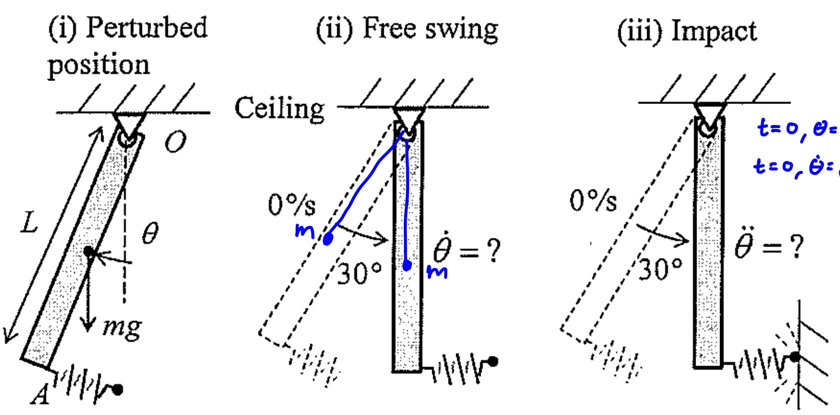


Figure 4

EOM:

$\frac{1}{3}mL^2\ddot{\theta} + \left(\frac{mgL}{2} + kL^2\right)\theta = 0$

$\therefore W_n = \sqrt{\frac{\frac{mgL}{2} + kL^2}{\frac{1}{3}mL^2}} = \sqrt{\frac{3}{2}\frac{g}{L} + 3\frac{k}{m}}$

$t=0, \theta=0$

$t=0, \dot{\theta} = \sqrt{\frac{3g}{L}(1-\cos 30)}$

no damping $\rightarrow$ harmonic

$\theta = \Theta \sin(w_n t + \phi)$

$0 = \Theta \sin(W_n \times 0 + \phi)$

$\therefore \phi = 0$

$\dot{\theta} = \sqrt{\frac{3g}{L}(1-\cos 30)} = w_n \Theta \cos(W_n \times 0)$

$\therefore \Theta = \frac{\sqrt{\frac{3g}{L}(1-\cos 30)}}{\sqrt{\frac{3}{2}\frac{g}{L} + 3\frac{k}{m}}}$

$\therefore \ddot{\theta} = -w_n^2\theta$

$= -\sqrt{\frac{3g}{L}(1-\cos 30)\left(\frac{3}{2}\frac{g}{L} + 3\frac{k}{m}\right)} \sin\left(\sqrt{\frac{3}{2}\frac{g}{L} + 3\frac{k}{m}}\, t\right)$

(b) A 500 kg engine is mounted on rubber pads of negligible mass. The static loading by the engine's weight causes a static displacement of 5mm to depress the rubber pads. Its free vibration is observed to have the amplitudes decay in a ratio of 3 to 1 in each consecutive cycle. Find the amplitude of resonant vibration and the force transmission to the ground when this mounted engine is excited into resonant vibration by a harmonic vertical force with 1000N amplitude. (10 marks)

$k_{eff} = \frac{F}{\Delta} = \frac{500 \times 9.81}{0.005} = 981000$

$\delta = \ln \frac{1}{3}$

$\zeta = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}} = 0.17223 = \frac{c}{c_c}$

$c_c = 2\sqrt{km}$

$= 44294.46$

$c = 7628.83$

$w_n = \sqrt{\frac{k}{m}} = 44.3$

$X = \frac{F_0}{\sqrt{(k - mw^2)^2 + (cw)^2}}$

$X = \frac{F_0}{cW} = 2.959\text{mm}$

$X_{res} = \frac{F_0}{k2\zeta} = 2.959\text{mm}$

$F_T = kX + mg$

$= 981000(X_{res} \sin W_n t) + 500 \times 9.81$

$= 2902 \sin 44.3t + 4905$

23