2021/22 Sem 1

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1a) $\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$

$RF_x = 0.5(17.6839 - (-9.9471 \cos 60))$

$RF_x = 11.3287$ N

$\rho A V_1 = \dot{m}$

$V_1 = 9.9471$ m/s

$V_2 = 17.6839$ m/s

$\Sigma F_y = \dot{m}(V_{2y} - V_{1y})$

$RF_y = 0.5(0 - (-9.9471 \sin 60))$

$RF_y = 4.3072$ N

resultant $RF = \sqrt{RF_y^2 + RF_x^2} = 12.12$ N

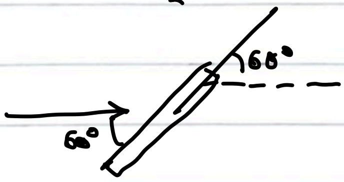
angle $= \tan^{-1}\left(\frac{4.3072}{11.3287}\right) = 20.8169°$ (from right horizontal)

b) $\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$

$RF_x = 0.5(0 - 17.6839)$

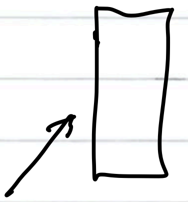
$RF_x = 8.842$ N

$V_{2x} = \frac{\dot{m}}{\rho A_2}$



c) $W_{2x} = 17.6839 - (-3) = 20.6839$ m/s

new relative $\dot{m} = 0.58482$ kg/s



$\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$

$RF_x = 0.58482(-3 - 17.6839)$

$RF_x = -12.097$ N

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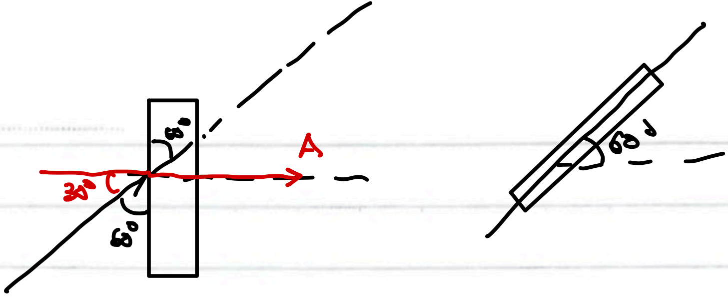
1d)



$\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$

$RF_x = 0.5(0 - 17.6839 \cos 30)$

$RF_x = -7.6576$ N

$\Sigma F_y = \dot{m}(V_{2y} - V_{1y})$

$RF_y = 0.5(17.6839 - 17.6839 \sin 30)$

$RF_y = 4.421$ N

resultant $= \sqrt{RF_x^2 + RF_y^2} = 8.842$ N

direction $= 0 - \tan^{-1}\left(\frac{4.421}{7.6576}\right) = -30°$

(wrt to axis A illustrated)

right side of axis perpendicular to plate

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2a) $$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + H_p - h_L = \frac{P_j}{\rho g} + \frac{V_j^2}{2g} + z_j$$

$$\frac{50}{\rho g} + \frac{3^2}{2g} - 3 + H_p - \frac{600}{\rho g} = \frac{P_j}{\rho g} + \frac{V_j^2}{2g} + z_j$$

$$\frac{P_j}{\rho g} + \frac{V_j^2}{2g} + z_j = H_p - 2.4944$$

$$Q_2 = \frac{\pi}{4}(60\times10^{-3})^2 \times 4 = 0.0113097 \ m^3/s$$

$$Q_1 = \frac{\pi}{4}(150\times10^{-3})^2 \times 3 = 0.053014 \ m^3/s$$

$$Q_3 = Q_1 - Q_2 = 0.041705 \ m^3/s \Longrightarrow 8.2969 \ m/s$$

$j \rightarrow 2$: $$\frac{P_j}{\rho g} + \frac{V_j^2}{2g} + z_j = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$H_p - 2.4944 = \frac{500\,000}{\rho g} + \frac{4^2}{2g} + 3$$

$$H_p = 57.2783$$

$j \rightarrow 3$: $$\frac{P_j}{\rho g} + \frac{V_j^2}{2g} + z_j = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + z_3$$

$$H_p - 2.4944 = \frac{300\,000}{\rho g} + \frac{8.2969^2}{2g} + 0.5$$

$$H_p = 37.084$$

combined H_p required $= 94.3623$ m

combined power required $= \rho g Q H_p$

actual power $= \dfrac{\rho g Q H_p}{\eta} = 70106.79$ W

$$\frac{N\,m^{-2}}{m} = kg\,m\,s^{-2}\,m^{-3}$$

6 variables

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2b) 3 units $\Rightarrow 3\,\pi$

(i) $\Delta p = \phi(\rho, \mu, D, \omega, Q)$

$\pi_1 = \Delta p\,(\rho^a D^b \omega^c)$ $\quad a = -1$

$\pi_1 = (ML^{-2}T^{-2})(ML^{-3})^a(L^b)(T^{-1})^c$ $\quad b = -1$

$= M^{1+a} L^{-2-3a+b} T^{-2-c}$ $\quad c = -2$

$\pi_1 = \frac{\Delta p}{\rho D^2 \omega^2}$ //

$\pi_2 = \mu(\rho^a D^b \omega^c)$ $\quad a = -1$

$\pi_2 = (ML^{-1}T^{-1})(ML^{-3})^a(L^b)(T^{-1})^c$ $\quad b = -2$

$\pi_2 = M^{1+a} L^{-1-3a+b} T^{-1-c}$ $\quad c = -1$

$\pi_2 = \frac{\mu}{\rho D^2 \omega}$ //

$\pi_3 = Q(\rho^a D^b \omega^c)$ $\quad a=0$

$= (L^3 T^{-1})(ML^{-3})^a(L^b)(T^{-1})^c$ $\quad b=-3 \quad c=1$

$\pi_3 = \frac{Q}{D^3\omega}$ //

ans: $\frac{\Delta p}{\rho D^2 \omega^2} = \phi\left(\frac{\mu}{\rho D^2 \omega}, \frac{Q}{D^3 \omega}\right)$ //

(ii) dynamic similarity $\Rightarrow$ reynold

larger = proto
smaller = model

$Q = AV = (D^2 V)$

$\frac{\pi_3}{\pi_2} : \frac{Q}{D^3 \cancel{\omega}} \times \frac{\rho \cancel{D^2} \cancel{\omega}}{\mu} = \frac{D^2 V}{D} \times \frac{\rho}{\mu} = \frac{\rho D V}{\mu}$

$Re_m = Re_p$

$\left(\frac{\cancel{\rho} D V}{\cancel{\mu}}\right)_m = \left(\frac{\cancel{\rho} D V}{\cancel{\mu}}\right)_p$

$\frac{D_m}{D_p} = \frac{V_p}{V_m}$

$\frac{200}{400} = \frac{V_p}{V_m}$

$\frac{V_p}{V_m} = \frac{1}{2}$

$\frac{Q_m}{Q_p} = \frac{A_m V_m}{A_p V_p} = \left(\frac{D_m}{D_p}\right)^2 \frac{V_m}{V_p}$

$\frac{Q_m}{0.25} = \left(\frac{200}{400}\right)^2 \times 2$

$Q_m = 0.125\ m^3/s$

$\frac{\pi_1}{(\pi_3)^2} : \frac{\Delta p}{\rho D^2 \cancel{\omega^2}} \times \left(\frac{D^3 \cancel{\omega}}{Q}\right)^2 = \frac{\Delta p D^4}{\rho Q} = \frac{\Delta p\ D^2}{\rho\ V}$

POP bazic™

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pressure coefficient similarity: $\left(\frac{\Delta p D^4}{\rho Q}\right)_m = \left(\frac{\Delta p D^4}{\rho Q}\right)_p$

$$\frac{(\Delta p)_m}{(\Delta p)_p} = \frac{Q_m}{Q_p}\left(\frac{D_p}{D_m}\right)^4$$

$$\frac{(\Delta p)_m}{3.5\times10^5} = \frac{1}{2}\left(\frac{400}{200}\right)^4$$

$$\Delta p_m = 2.8\times10^5 \text{ Pa}$$

(iii) $\pi_1 \times \pi_3 = \frac{\Delta p}{\rho D^2\omega^2} \times \frac{Q}{D^3\omega}$

power coeff $= \frac{Q\Delta p}{\rho D^5\omega^3} = \frac{W}{\rho D^5\omega^3}$

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3a)

(i) $Re = \frac{\rho V D}{\mu} = \frac{1200(1.2)(0.05)}{0.04} = 1800$ (<2100, laminar)

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} - h_L = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}$$

$$\frac{P_1 - P_2}{\rho g} = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

$$\frac{P_1 - P_2}{\rho g} = \frac{64}{1800}\left(\frac{10}{0.05}\right)\left(\frac{1.2^2}{2g}\right)$$

$$P_1 - P_2 = 5120 \text{ Pa}$$

$$\Delta\rho g h = P_1 - P_2$$

$$(1200 - 400) g h = 5120 \text{ Pa}$$

$$h = 0.652 \text{ m}$$

(ii) inviscid $\Rightarrow h_L = 0$

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}$$

$$P_1 = P_2$$

$$\Delta\rho g h = P_1 - P_2$$

$$h = 0$$

$\frac{\rho V D}{\mu}$ $\mu \frac{du}{dr}$

NO: (Q) find k_2, Q_1, Q_2, Q_3 DATE:

3b) let flow rate thru pipe 2 be Q

$Q_3 = 2Q \quad A_3V_3 = 2A_2V_2$

$Q_1 = 3Q \quad A_1V_1 = A_2V_2 + A_3V_3 = A_2V_2 + 2A_2V_2$

$$\cancel{\frac{P_A}{\rho g}} + \cancel{\frac{V_A^2}{2g}} + z_A - h_\ell = \cancel{\frac{P_B}{\rho g}} + \cancel{\frac{V_B^2}{2g}} + z_B$$

$$z_A - z_B = k_1Q_1^2 + k_2Q^2 + k_3Q_3^2$$

$$40 = 400(3Q)^2 + k_2Q^2 + 100(3Q)^2$$

$$40 = (500 + k_2)Q^2 = (500 + k_2)(A_2V_2)^2$$

parallel pipe head loss (pipe 2 h_ℓ = pipe 3 h_ℓ)

$$k_2Q_2^2 = k_3Q_3^2$$

$$k_2Q^2 = k_3(2Q)^2$$

$$\frac{k_2}{k_3} = \frac{4Q^2}{Q^2}$$

$$\frac{k_2}{100} = 4$$

$$k_2 = 400$$

$$40 = (500 + 400)Q^2$$

$$Q_2 = 0.2108 \text{ m}^3/\text{s}$$

$Q_3 = 2Q_2 = 0.4216$ m^3/s

$Q_1 = 3Q_2 = 0.6325$ m^3/s

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4a) $\cancel{\frac{P_A}{\rho g}} + \cancel{\frac{V_A^2}{2g}} + z_A + H_p - h_L = \cancel{\frac{P_c}{\rho g}} + \cancel{\frac{V_c^2}{2g}} + z_c$

$$H_p = \left(f_1 \frac{l_1}{D_1} + f_2 \frac{l_2}{D_2} + K_1 + K_2\right)\frac{V^2}{2g} + 100$$

$$H_p = 22\left(\frac{1}{2g}\right)\left(\frac{1}{\frac{\pi}{4}(0.2)^2}\right)^2 Q^2 + 100$$

$$H_p = 1136.119\, Q^2 + 100$$

$100 - 800 Q^2 = 1136.119 Q^2 + 100$

$Q = 0 \Rightarrow$ one pump not enough

series : $2(100 - 800Q^2) = 1136.119 Q^2 + 100$

$Q = 0.1912 \ m^3/s$

$H_p (Q = 0.1912) = 100 - 800(0.1912)^2$

$= 70.76 \ m$

parallel : $100 - 800\left(\frac{Q}{2}\right)^2 = 1136.119 Q^2 + 100$

$Q = 0$ (invalid)

$\therefore$ series arrangement optimal

$V = \frac{Q}{A} = 6.08609$

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4b) $NPSH_A (1^{st} \text{ pump}) = \frac{P_{atm} - P_v}{\rho g} - H_L + z_1 - z_2$

$= \frac{100\,000 - 2340}{\rho g} - \left(f_1 \frac{l_1}{D_1} \frac{V^2}{2g}\right) + 2$

$= \frac{100\,000 - 2340}{\rho g} - \left(0.02 \times \frac{10}{0.2} \times \frac{6.08609^2}{2g}\right) + 2$

$= 10.0673 \text{ m}$

suction ② pump 2 inlet

$\frac{P_s}{\rho g} = \frac{P}{\rho g} + 100 - 800(0.1912)^2$

$P_s = 1078180.972$

$NPSH_A (2^{nd} \text{ pump}) = \frac{P_s - P_v}{\rho g} + \frac{V_s^2}{2g}$

$= 111.55$

combined $NPSH_A = 111.55 - 10.0673 = 121.62 \text{ m}$

cavitation onset: $NPSH_A = NPSH_R$

$121.62 = A + 50(0.1912^2)$

$A = 119.792$