NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2021-22

MA3002 – SOLID MECHANICS AND VIBRATION

April/May 2022 | Time Allowed: $2^1/_2$ hours

INSTRUCTIONS

1. This paper contains **SECTION A & SECTION B** and comprises **SIX (6)** pages.
2. **COMPULSORY** to answer **ALL** questions in both sections.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN BOOK** examination. One double-sided A4 size reference sheet of paper is allowed.

SECTION A

1 (a) Figure 1 shows a rigid bar ABC pin-jointed to a support at A and connected to two springs of equal stiffness k at B. The other ends of the springs are roller supported at D and E. The portions AB and BC are of lengths $2L$ and L, respectively. The free length of each spring is equal to l_o. The horizontal distance between point A and D is a (as shown). Under the action of horizontal force P applied at C, the springs are in stretched condition and the bar is inclined at an angle θ as shown. Ignore the weights of all components and any friction at the joints and contacts. Using the principle of virtual work ($\delta W = \delta U$), obtain an expression for P in terms of θ and possibly other parameters such as L, k and a.

(12 marks)

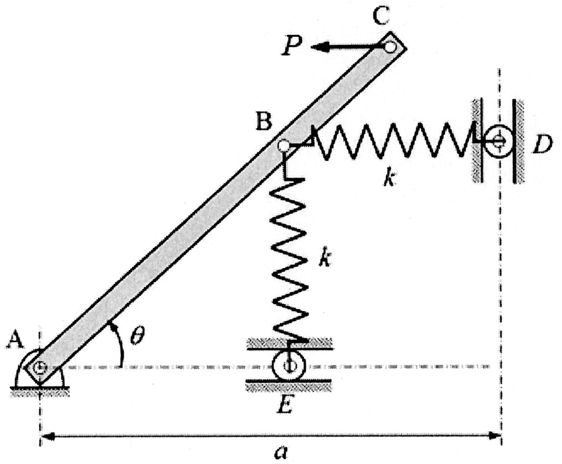

Figure 1

Note: Question 1 continues on page 2.

(b) Figure 2 shows a weightless beam AB of length L and bending rigidity EI with its left end fixed to a rigid wall at A and right end supported by a weightless spring of stiffness k at B. The top end of the spring is roller supported at C. The beam carries a uniformly distributed load of intensity w and a concentrated force P as shown. Consider only bending effects in the beam.

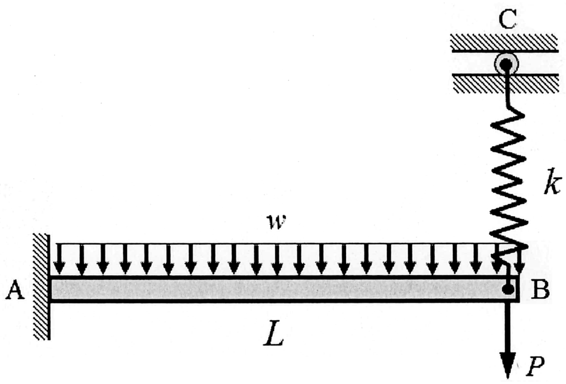

Figure 2

(i) Mark all unknown reactions on a neat sketch and thereby determine the degree of indeterminacy of this structure.

(3 marks)

(ii) Determine the reaction force at C by unit load method.

(10 marks)

2 (a) A steel plate of length 600 mm, width 150 mm, and thickness 1 mm, has a central crack of 60 mm length and carries a tensile load of P. The geometry correction, Y, is given below in Figure 2.1:

$$Y = \left(\frac{W}{\pi a} tan \frac{\pi a}{W}\right)^{1/2}$$

where a is the half crack length and W is the plate width.

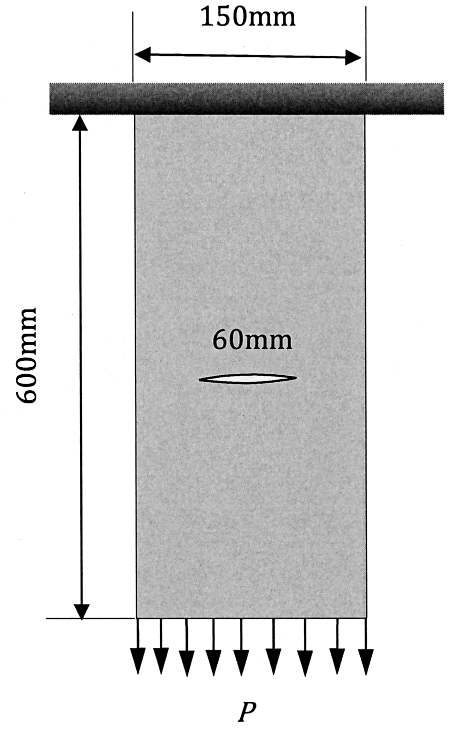

Figure 2.1

The static properties of the plate are a yield strength of 1200 MPa and a fracture toughness K_{IC} of 50 MPa $\sqrt{m}$.

(i) What is the geometry correction factor, Y?

(4 marks)

(ii) What is the maximum load, P, that the plate can be subjected to before fracture?

(6 marks)

Note: Question 2 continues on page 4.

(b) An engine component is subjected to 25 cycles of tensile loading per day with a stress amplitude of $S_a = 138\ MPa$ and a mean stress of $S_m = 0\ MPa$. The S-N curve of the material for fully reversed cycle is given in Figure 2.2. The ultimate tensile strength is 460 MPa and the endurance limit is 70 MPa.

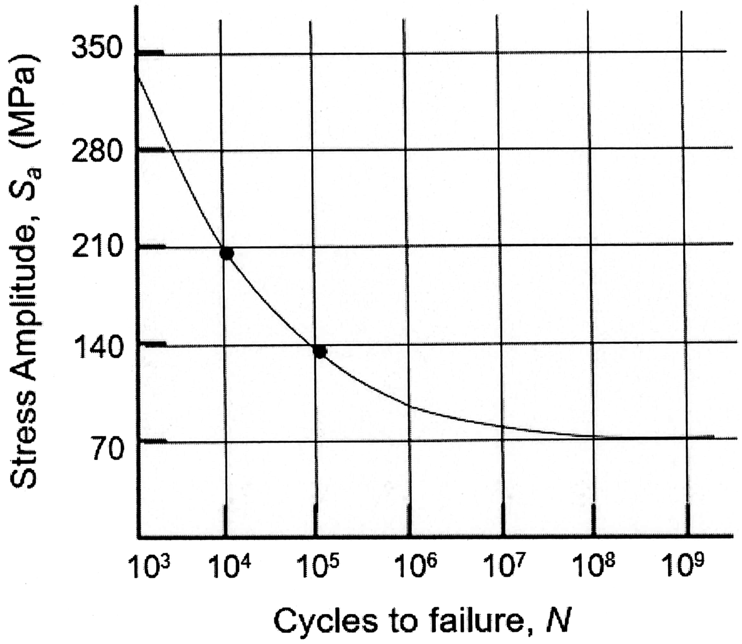

Figure 2.2 S-N Curve

(i) Based on the S-N curve, determine the maximum number of years that the engine component can be used?

(5 marks)

(c) The engine component with the same loading given in 2(b) was found with an initial edge crack size of $a = 4\ mm$. The geometry correction factor, $Y = 1.12$. The fracture toughness K_{IC} of 28 MPa $\sqrt{m}$. Use the Paris Law equation of

$$da/dN \text{ (m/cycle)} = 2.6\times10^{-12}\ \Delta K^{3.5}$$

where ΔK is in MPa $\sqrt{m}$ unit.

(i) What is the maximum edge crack size before fracture if the applied stress remains at 138 MPa?

(3 marks)

(ii) How many cycles will it take to grow from the initial crack to failure?

(7 marks)

SECTION B

3. A vehicle travels on a rough road with a constant speed v. The road has a sinusoidal profile $y = Y \sin \omega t$ with a period L, where Y, ω and t are the amplitude, excitation frequency and time respectively. The vehicle can be modeled as a mass m with stiffness k and damping constant c. Let x be the vertical degree of freedom of the vehicle, as measured from the static equilibrium position.

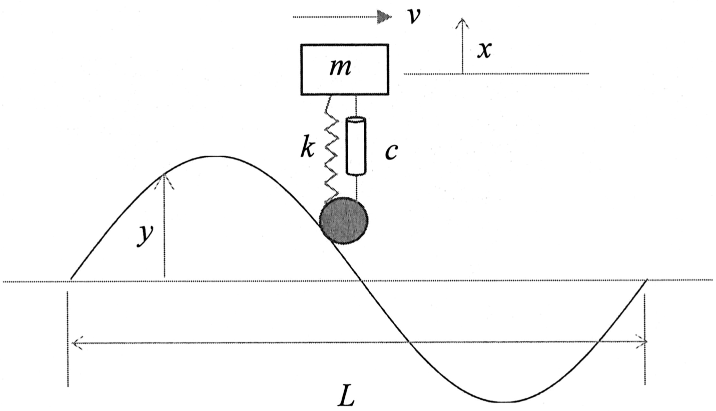

Figure: 3: A vehicle moving on a rough surface with a sinusoidal profile.

(a) Draw the relevant free body diagram and identify the forces acting on it.

(5 marks)

(b) Derive the equation of steady-state motion of the mass m in the form

$$m\ddot{x} + c\dot{x} + kx = A\sin(\omega t + \alpha),$$

where A and α must be stated explicitly in terms of k, c, ω and Y. Do not solve the equation.

(10 marks)

(c) It can be shown that the ratio $\bar{X}$ of the steady-state amplitude X of the motion to the amplitude Y of the road profile can be written as:

$$\bar{X} = \frac{X}{Y} = \sqrt{\frac{1 + (2\zeta\bar{\omega})^2}{(1 - \bar{\omega}^2)^2 + (2\zeta\bar{\omega})^2}},$$

where $\bar{\omega} = \omega / \omega_n$, $\omega_n = \sqrt{k/m}$ and ζ is the damping ratio. For $\zeta = 0.5$, determine the value of $\bar{\omega}$ that will yield the largest amplitude $\bar{X}$.

(6 marks)

(d) For the numerical value of $\bar{\omega}$ determined in (c), calculate the vehicle speed that will result in the largest $\bar{X}$, given that $m = 1500$ kg, $k = 100$ kN/m, and $L = 25$m.

(4 marks)

4. A car can be simplified as a two-degree-of freedom system for the purpose of determining its natural frequencies and the normal modes of vibration.

(a) If the degrees of freedom are chosen as the displacement x of the center of mass G and the rotation θ as shown in Fig. 4(a), the equations of motion in matrix form are:

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & k_2 d_2 - k_1 d_1 \\ k_2 d_2 - k_1 d_1 & k_1 d_1^2 + k_2 d_2^2 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix},$$

where m is the mass, J the mass moment of inertia about G, k_1, k_2 the spring stiffnesses, and d_1, d_2 distances to G from the left and right ends, respectively. For $m = 1200$ kg, $J = 1875$ kg.m^2, $k_1 = 40$ kN/m, $k_2 = 36$ kN/m, $d_1 = 1.8$ m and $d_2 = 1.4$ m, determine numerically the natural frequencies and the corresponding normal modes of vibration.

(10 marks)

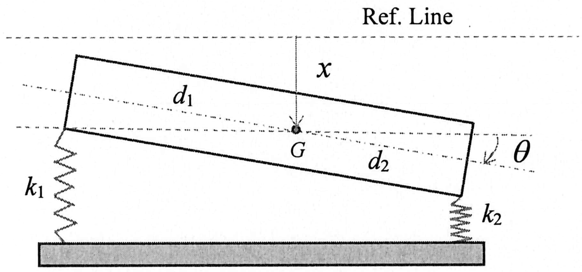

Figure: 4(a): A system with x and θ chosen as the degrees of freedom.

(b) If the degrees of freedom are chosen as x_A and θ (see Fig. 4(b)), where x_A is the displacement of the point A such that $k_1 d_3 = k_2 d_4$ and A is at a distance of e from G, derive the matrix equations of motion symbolically in terms of m, k_1, k_2, d_3, d_4, e and J_A. Note that d_3 and d_4 are the distances between the left end and A, and between the right end and A, respectively. Also, J_A is the mass moment of inertia of the system with respect to A. Hint: $J_A = J + me^2$ and $x = x_A + e\theta$.

(10 marks)

(c) Comment on the differences between the equations in (a) and (b).

(5 marks)

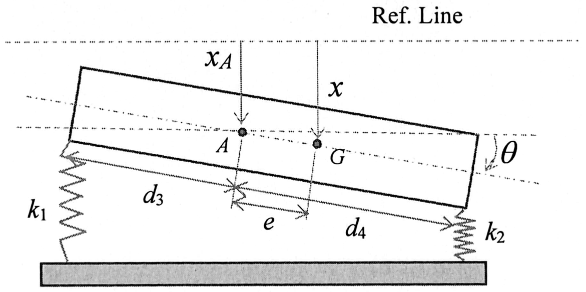

Figure: 4(b): A system with x_A and θ chosen as the degrees of freedom, such that $k_1 d_3 = k_2 d_4$.

END OF PAPER

MA3002 SOLID MECHANICS & VIBRATION

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.