MA4825 23/24 S2 (incomplete ; no Q4)

Q1. a) joint 1: power joint encoder : 200 pulses per rotation

torque joint : 1:600 ratio

$$\Rightarrow 1 \text{ pulse} = \frac{360°}{200 \times 600} = 0.003° \text{ resolution } \#$$

b) joint 2: $1 \text{ pulse} = \frac{360}{250 \times 500} = 0.00288° \text{ resolution } \#$

c) $x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$

$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$

assuming start pos $\theta_1 = \theta_1$, $\theta_2 = \theta_2$, after link 2 makes $10°$ angular displacement,

$x = 35\cos\theta_1 + 25\cos(\theta_1 + \theta_2 + 10°)$ # (cm)

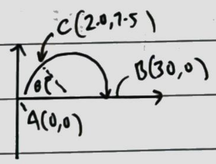
$y = 35\sin\theta_1 + 25\sin(\theta_1 + \theta_2 + 10°)$ # (cm)

d) $y = 35\sin\theta_1 + 25\sin(\theta_1 + \theta_2) \Rightarrow \Delta y = \Delta\theta_1 \cdot 35\cos\theta_1 + (\Delta\theta_1 + \Delta\theta_2) \cdot 25\cos(\theta_1 + \theta_2)$

$\max \Delta y = \Delta\theta_1 \cdot 35 + (\Delta\theta_1 + \Delta\theta_2) \cdot 25$

subst $\Delta\theta_1 = 0.003 \times \frac{\pi}{180}$, $\Delta\theta_2 = 0.00288 \times \frac{\pi}{180} \Rightarrow \Delta y = 0.00440\text{cm}$ #

Q2



a) eqn of circle path: $(x - 15)^2 + y^2 = 15^2$

parametric ($\curvearrowleft$): $x = 15 - 15\cos\theta$

$y = 15\sin\theta$

linear vel 2 cm s^{-1} : arc length $= \pi r = 15\pi$ cm

time taken $= \frac{15\pi}{2}$ s check: $v = r\omega$, $\omega = \frac{v}{r} = \frac{2}{15} \text{ rad s}^{-1}$ ✓

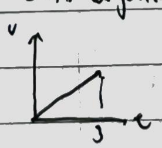
angular velocity $\dot{\theta} = \frac{\pi}{\left(\frac{15\pi}{2}\right)} = \frac{2}{15} \text{ rad s}^{-1}$

Integrating, $\theta = \frac{2}{15}t$ $(\theta(t=0) = 0)$

~~subst. into parametric eqn:~~ $y = 15\sin\left(\frac{2}{15}t\right)$ #

b) $\left.\begin{array}{l} 2 = 15 - 15\cos\theta \\ 7.5 = 15\sin\theta \end{array}\right\} \theta = \frac{\pi}{6}$

const tangential acceleration = const. angular acceleration



$\frac{\pi}{6} = \frac{1}{2}(\ddot{\theta})t^2 \Rightarrow t = 3,\ \ddot{\theta} = 0.11636 \text{ rad s}^{-2}$

integrating, $\theta(t) = 0.0582t^2$

$\therefore x = 15 - 15\cos(0.0582t^2)$

$y = 15\sin(0.0582t^2)$ #

2. c) at C, $\dot{\theta} = 0.11636 \times 3 = 0.34908 \text{ rad s}^{-1}$

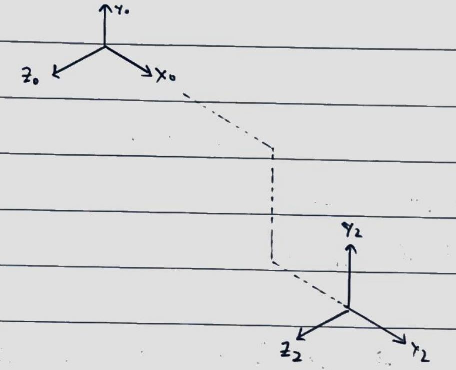
$$\left.\begin{aligned} 28 &= 15 - 15\cos\theta \\ 7.5 &= 15\sin\theta \end{aligned}\right\} \quad \theta = \frac{5\pi}{6}$$

$$\theta_{CO} = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$$

$$2\pi/3 \div 0.34908 = 6.0\,\text{s} \;\#$$

$\boxed{Q3}$ a). attach frame $x_1y_1z_1$ to point O, which rotates along shared x_0/x_1 axis

attach frame $x_2y_2z_2$ to point E, which rotates along the local z_2 axis



$$^0P_p = {^0P_{\vec{OE}}} + {^0P_{\vec{EP}}}$$
$$= R_{01}\,{^1P_{\vec{OE}}} + R_{02}\,{^2P_{\vec{EP}}}$$

$$R_{01} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad {^1P_{\vec{OE}}} = \begin{pmatrix} a + K_1 + \dot{s}_1 t \\ -b \\ 0 \end{pmatrix}$$

$$R_{12} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad {^2P_{\vec{EP}}} = \begin{pmatrix} d \\ c + K_2 - \dot{s}_2 t \\ 0 \end{pmatrix}$$

$$^0\dot{P}_p = \dot{R}_{01}\,{^1P_{\vec{OE}}} + R_{01}\,{^1\dot{P}_{\vec{OE}}} + \dot{R}_{02}\,{^2P_{\vec{EP}}} + R_{02}\,{^2\dot{P}_{\vec{EP}}}$$
$$= \quad '' \quad + \dot{R}_{01} R_{12}\,{^2P_{\vec{EP}}} + R_{01}\dot{R}_{12}\,{^2P_{\vec{EP}}} + R_{01}R_{12}\,{^2\dot{P}_{\vec{EP}}}$$

$$\dot{R}_{01} = \dot{\theta}\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\theta & -\cos\theta \\ 0 & \cos\theta & -\sin\theta \end{bmatrix} \qquad \dot{R}_{12} = \dot{\beta}\begin{bmatrix} -\sin\beta & -\cos\beta & 0 \\ \cos\beta & -\sin\beta & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad {^1\dot{P}_{\vec{OE}}} = \begin{pmatrix} \dot{s}_1 \\ 0 \\ 0 \end{pmatrix} \qquad {^2\dot{P}_{\vec{EP}}} = \begin{pmatrix} 0 \\ -\dot{s}_2 \\ 0 \end{pmatrix}$$

$$\therefore {^0\dot{P}_p} = \dot{\theta}\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\theta & -\cos\theta \\ 0 & \cos\theta & -\sin\theta \end{bmatrix}\begin{pmatrix} a + K_1 + \dot{s}_1 t \\ -b \\ 0 \end{pmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}\begin{pmatrix} \dot{s}_1 \\ 0 \\ 0 \end{pmatrix}$$

$$+ \dot{\theta}\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\theta & -\cos\theta \\ 0 & \cos\theta & -\sin\theta \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} {^2P_{\vec{EP}}} + \dot{\beta}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} -\sin\beta & -\cos\beta & 0 \\ \cos\beta & -\sin\beta & 0 \\ 0 & 0 & 0 \end{bmatrix} {^2P_{\vec{EP}}}$$

$$+ \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{pmatrix} 0 \\ -\dot{s}_2 \\ 0 \end{pmatrix}$$

✓ shd be ok

Q3 (Cont).

$$^0\dot{P}_0 = \dot\theta\begin{pmatrix}0\\ b\sin\theta\\ -b\cos\theta\end{pmatrix} + \begin{pmatrix}\dot s_1\\0\\0\end{pmatrix} + \dot\theta\begin{bmatrix}0 & 0 & 0\\ -s\theta s\beta & -s\theta c\beta & -c\theta\\ c\theta s\beta & c\theta c\beta & -s\theta\end{bmatrix}\begin{pmatrix}d\\ c+k_2+\dot s_2 t\\ 0\end{pmatrix}$$

$$+\dot\beta\begin{bmatrix}-s\beta & -c\beta & 0\\ c\theta c\beta & -c\theta s\beta & 0\\ s\theta c\beta & -s\theta s\beta & 0\end{bmatrix}\begin{pmatrix}d\\ c+k_2+\dot s_2 t\\ 0\end{pmatrix} + \begin{bmatrix}c\beta & -s\beta & 0\\ c\theta s\beta & c\theta c\beta & -s\theta\\ s\theta s\beta & s\theta c\beta & c\theta\end{bmatrix}\begin{pmatrix}0\\ \dot s_2\\ 0\end{pmatrix}$$

$$= \begin{pmatrix}\dot s_1\\ b\dot\theta\sin\theta\\ -b\dot\theta\cos\theta\end{pmatrix} + \dot\theta\begin{pmatrix}0\\ -d\sin\theta\sin\beta - (c+k_2+\dot s_2 t)(\sin\theta\cos\beta)\\ d\cos\theta\sin\beta + (c+k_2+\dot s_2 t)(\cos\theta\cos\beta)\end{pmatrix}$$

$+ \quad \ldots$ (too huge, just use prev step result)

b) absolute angular velocity of point E $= {}^0\omega_1 + {}^0\omega_2$

$$= \begin{pmatrix}\dot\theta\\0\\0\end{pmatrix} + R_{01}\begin{pmatrix}0\\0\\ \dot\beta\end{pmatrix}$$

$$= \begin{pmatrix}\dot\theta\\0\\0\end{pmatrix} + \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix}\begin{pmatrix}0\\0\\ \dot\beta\end{pmatrix}$$

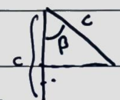
$$= \begin{pmatrix}\dot\theta\\ -\dot\beta\sin\theta\\ \dot\beta\cos\theta\end{pmatrix} \#$$

$$^2\dot P_p = R_{21}R_{10}\,{}^0\dot P_p$$
$$= R_{12}^T R_{01}^T\,{}^0\dot P_p \quad \ldots$$

c) KE sources: ~~1. link c.. ar $\dot\beta$~~
~~2. P a. $\dot\rho$~~
~~3. link & load on $\dot\theta$~~
~~4. P on $\dot\beta$~~

link 3 & load
link 4 & load

taking E as reference,
initial PE $= -cm_3 g$
final PE $= -c\cos\beta\, m_3 g$
$\Delta PE = c(1-\cos\beta)m_3 g$ #



① $-\cos\beta \cdot c m_3 g$

as $\beta\uparrow$, PE$\uparrow$

① $KE = I\omega^2 - \cancel{I_E}$
$= I_E\dot\beta^2$

② $KE = \left[I_p + m_4(c+s_2)^2\right]\dot\beta^2$
$+\frac{1}{2}m_4\dot s_2^{\,2}$

① $PE = [c - c\cos\beta]m_3 g$ ✗
② $PE = [c+s_2][1-\cos\beta]$ ✗

find init, datum
NOT final - initial

taking E as reference,
initial $PE = (-c+k_2)m_4 g$
final $PE = (-c - s_2)\cos\beta\, m_4 g$ ✓ #

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2023-2024

MA4825 – ROBOTICS

April/May 2024 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is an **OPEN-BOOK** examination.

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1. A planar robot arm with two links is shown in Figure 1. Link 1's length is 35.0 cm while link 2's length is 25.0 cm. Inside the robot's joint 1, the speed reduction ratio of torque joint is 600 while the power joint's incremental encoder will output 200 pulses after a full rotation. Similarly, inside the robot's joint 2, the speed reduction ratio of torque joint is 500 while the power joint's incremental encoder will output 250 pulses after a full rotation.

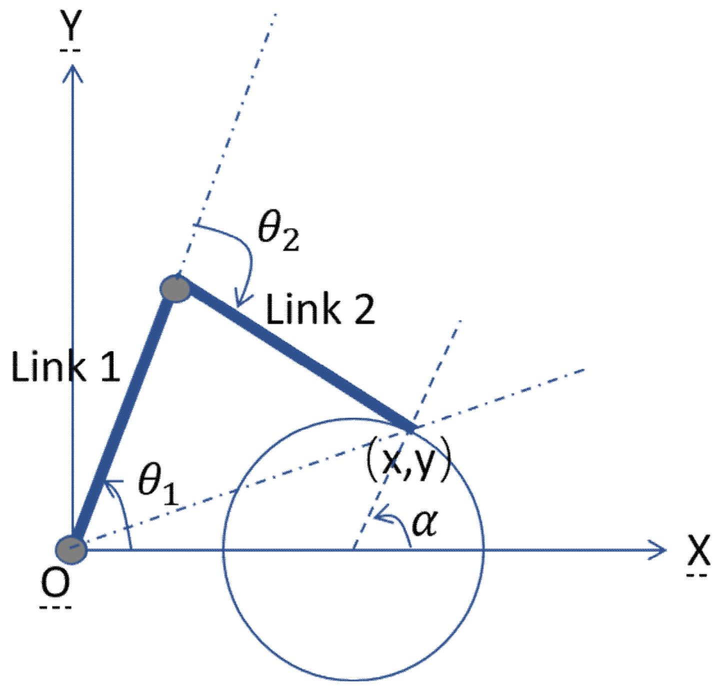

Figure 1

Note: Question 1 continues on page 2.

(a) What is the best accuracy of link 1's angular position?

(5 marks)

(b) What is the best accuracy of link 2's angular position?

(5 marks)

(c) The power joint inside the robot's joint 2 is under the control of a position feedback control loop. If the robot's link 2 makes an angular displacement of 10.0 degrees, what should be the desired output from its position control loop?

(5 marks)

(d) What is the accuracy of the Y coordinate of the tooltip which is placed at the end of the robot's link 2?

(10 marks)

2. A robotic arm is moving its gripper along a circular path/trajectory as shown in Figure 2. The circular path contains points A, C, D, and B. Assume that the coordinates of point A are (0.0, 0.0) (cm) while the coordinates of point B are (30.0, 0.0) (cm). The orientation of gripper at point A is +90.0° while the orientation of gripper at point B is -90.0°. Both orientations are tangential to the circular path/trajectory.

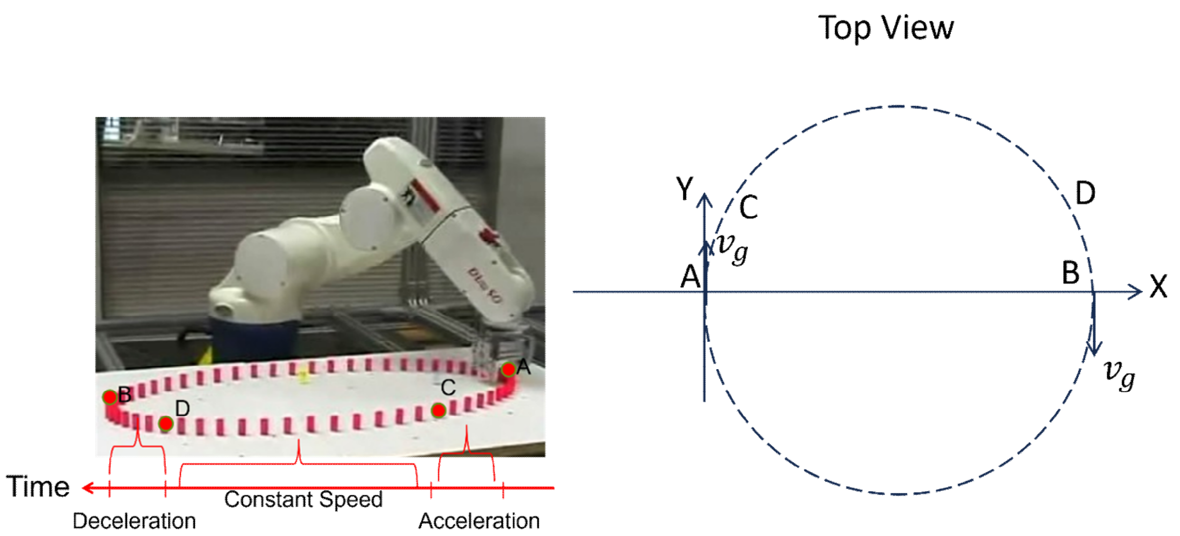

Figure 2

(a) If the gripper continually moves along the circular path with a linear velocity of 2.0 cm/s, what should be the time functions of the gripper's coordinates (x, y)?

(10 marks)

Note: Question 2 continues on page 3.

(b) Now, assume that the robot's gripper is initially at rest at point A. Then, the robot moves its gripper's position from point A to reach point C by undergoing a circular motion with a constant tangential acceleration. Point C's coordinates are (2.0, 7.5) (cm). If the duration of this motion is 3.0 seconds, what should be the time functions of the gripper's coordinates (x, y)?

(10 marks)

(c) When the gripper reaches point C, it continues the motion toward point D by undergoing a uniform circular motion with the initial circular velocity at point C. If point D's coordinates are (28.0, 7.5) (cm), what is the duration of motion from point C to point D?

(5 marks)

3. The robot in Figure 3(a) consists of two rotating L-bars (link 1 and 3) and two translational links (links 2 and 4). The angular displacements of rotational joints are θ and β, and the displacements of the translational joints are s_1 and s_2. Link 1 is rotating with a constant angular velocity of $\dot{\theta}$ about the X- axis; link 2 is translating horizontally at constant velocity of $\dot{s}_1$; link 3 is rotating with a constant angular velocity of $\dot{\beta}$ about the Z-axis and link 4 is translating vertically at a constant velocity of $\dot{s}_2$. Note that the configuration shown in the figure is the initial position of the system.

where $\theta = \beta = 0°$, $s_1 = K_1$, and $s_2 = -K_2$.

(a) Find the absolute velocity of the end point P of the robot in terms of θ, β, s_1 and s_2.

(9 marks)

(b) Find the absolute angular velocity of Point E and absolute velocity of the end point P expressed in the local coordinate system of link 3.

(8 marks)

(c) By making use of the results obtained above, express the steps to find the kinetic and potential energies of the system shown in Figure 3(b) for link 3 and link 4 only towards formulating Lagrangian. The moment of inertia at point E and P are I_E & I_P respectively. The telescopic link 4 and load are modelled as a mass m_4 acting at point P and the rotating link 3 and load are modelled as a mass m_3 acting at the corner point of the L-bar. (Note: Do not expand the expressions.)

(8 marks)

Note: Figure 3 appears on page 4.

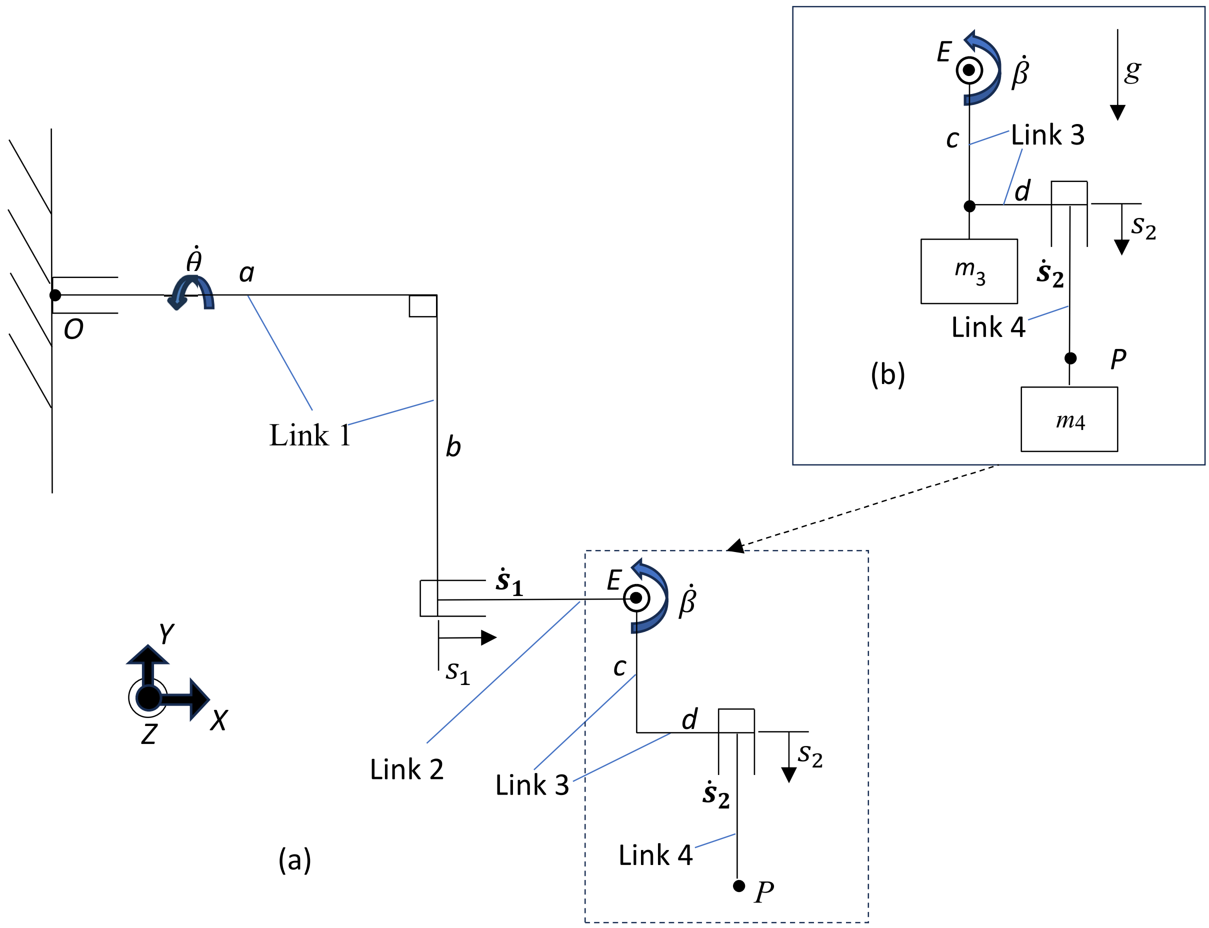

Figure 3

4. An illustration of an industrial robot (ABB IRB140 robot) used for spraying or welding is shown in Figure 4(a) without its end effector. The arm has six joints: $\boldsymbol{q} = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6\}^T$.

(a) Given the base coordinate system in Figure 4(b), draw and specify the right-handed coordinate systems i (i=1 to 6) of the robot up to the wrist joint. (Note: Sketch a skeletal diagram in isometric view & provide the positions and directions of the coordinate systems in your answer.)

(10 marks)

(b) Tabulate the Denavit-Hartenberg (D-H) parameters for the position of the robot shown in Figure 4 for joints 1 to 6, according to the coordinate systems specified in part (a). Note that θ_i(i=1 to 6) are the joint variables.

(6 marks)

Note: Question 4 continues on page 5.
Figure 4 appears on page 5.

(c) Find the generic D-H matrix $\boldsymbol{H}_{base}^{elbow}$ for the robot where $\boldsymbol{H}_{base}^{elbow}$ is a 4x4 homogeneous matrix transforming a vector from frame 3 (coordinates x_3, y_3, z_3) to the base coordinates (x_0, y_0, z_0). Do not multiply the individual matrices.

(5 marks)

(d) Describe a possible sequence of rotation that aligns the base coordinate system to coordinate system 6. Derive the rotational matrix.

(4 marks)

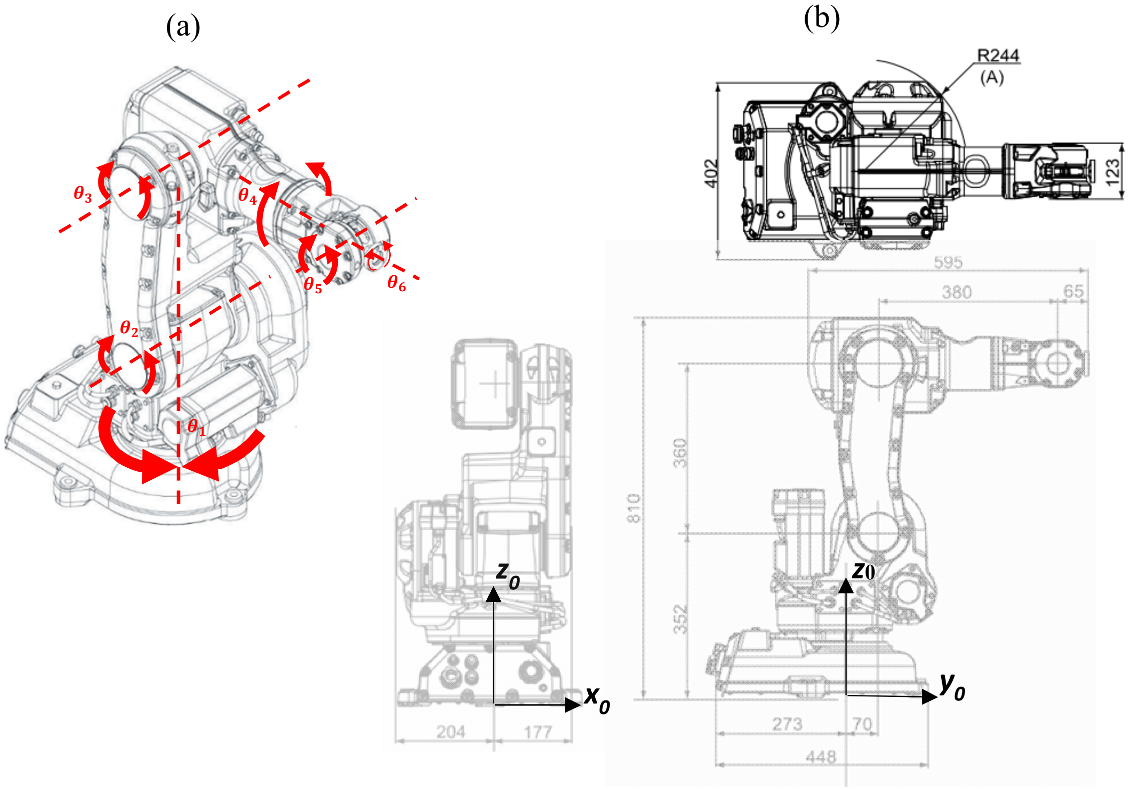

Figure 4

END OF PAPER