NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

MA3002 – SOLID MECHANICS AND VIBRATION

April/May 2017 Time Allowed: 2½ hours

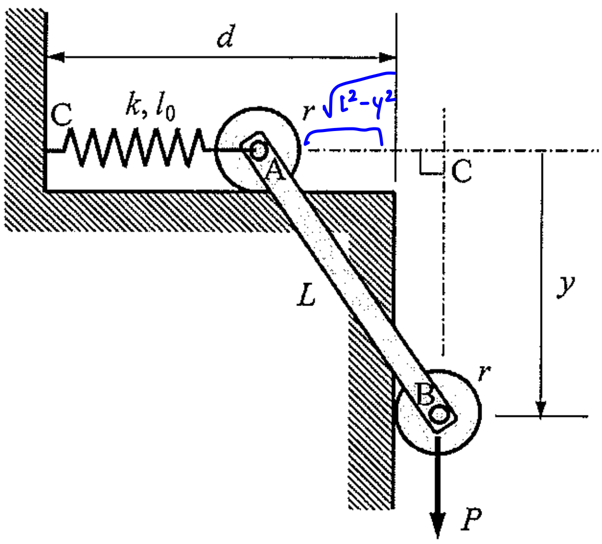
INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN-BOOK** examination. One double-sided A4 size reference sheet is allowed.

1(a) Figure 1 shows a rigid uniform bar AB of length L supported on rigid rollers pin-joined at the bar ends A and B. A spring of stiffness k and free length l_0 connects the end A of the bar to a rigid wall at C. A load P acts at the end B of the bar as shown. Ignore friction at all the joints and the weights of all the parts.

Using the *principle of virtual work*, derive an expression for the force P in terms of y and other parameters.

(10 marks)



$P\delta y = ke\,\delta e$

$e = d - \sqrt{L^2-y^2} - r - l_o$

$\delta e = \frac{de}{dy}\delta y$

$= \frac{2y}{\sqrt{L^2-y^2}}\delta y$

$P\cancel{\delta y} = k(d - \sqrt{L^2-y^2} - l_o - r)\left(\frac{2}{\sqrt{L^2-y^2}}\cancel{\delta y}\right)$

$P = 2k\left(\frac{d-l_o-r}{\sqrt{L^2-y^2}} - 1\right)$

Figure 1

Note: Question 1 continues on page 2.

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(b) Figure 2 shows a "Z" shaped beam ABCD of with segments AB and CD having the same length L, and segment BC having a length $\sqrt{2}L$. All the three segments have the same flexural rigidity EI. The beam is fixed to a rigid wall at A and carries a vertical load P at D. Consider only bending effects. Assume linear elastic behavior of the structure.

(i) Using the *unit load method*, derive an expression for the vertical deflection at D in terms of P, L and EI.

(10 marks)

Real Load:

$M_{DC} = Px$

$M_{CB} = PL - \frac{P}{\sqrt{2}}x$

$M_{BA} = Px$

Virtual:

$m_{DC} = x$

$m_{CB} = L - \frac{x}{\sqrt{2}}$

$m_{BA} = x$

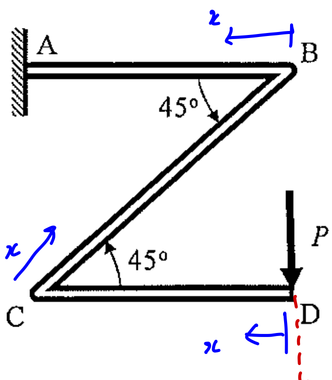


Figure 2

$$1 \cdot \Delta_D = \frac{1}{EI}\left\{ \int_0^L Px^2 dx + \int_0^{\sqrt{2}L} PL^2 - \frac{PxL}{\sqrt{2}} - \frac{PxL}{\sqrt{2}} + \frac{Px^2}{2} dx + \int_0^L Px^2 dx \right\}$$

$$= \frac{1}{EI}\left\{ 2\left[\frac{Px^3}{3}\right]_0^L + \left[PL^2x - \frac{\sqrt{2}Px^2L}{2} + \frac{Px^3}{6}\right]_0^{\sqrt{2}L} \right\}$$

$$= \frac{1}{EI}\left\{ \frac{2}{3}PL^3 + \sqrt{2}PL^3 - \sqrt{2}PL^3 + \frac{\sqrt{2}PL^3}{3} \right\} = \frac{PL^3}{3EI}(2+\sqrt{2})$$

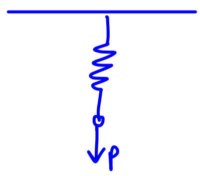
(ii) Now, instead of the static load P, consider an impact load due to a mass 10 kg falling from a height of 0.3 m directly above point D. Calculate the maximum vertical deflection at D caused by the impact load. List the usual simplifying assumptions made in such an analysis. Take $L = 1$ m and $EI = 10^4$ Nm^2 for the calculation.

(5 marks)

$F = kx$

$$k_{eff} = \frac{F}{x} = \frac{P}{\frac{PL^3}{3EI}(2+\sqrt{2})} = \frac{3EI}{L^3(2+\sqrt{2})}$$

Simplify into:



spring with constant k whereby $k = k_{eff} = \frac{F}{\Delta_s}$

$$\Delta_{static} = \frac{F}{k} = \frac{9.81 \times 10}{\frac{3\times10^4}{2+\sqrt{2}}}$$

$=$

$$\max \Delta = \Delta_{static}\left(1 + \sqrt{1 + \frac{2h}{\Delta_s}}\right)$$

$=$

2(a) A beam of a rectangular cross section with width $W = 40$ mm and thickness $t = 15$ mm is subjected to a steady bending moment $M = 700$ Nm as shown in Figure 3(a). The top surface of the beam has a through edge crack of length $a = 10$ mm. The beam is made of a material with fracture toughness $K_c = 42$ MNm$^{-3/2}$ and yield stress $\sigma_Y = 300$ MPa.

The tensile stress at the top surface is given by

$$\sigma = \frac{6M}{W^2 t} = \frac{6 \times 700}{0.04^2 \times 0.015} = 175 \text{ MPa}$$

The stress intensity factor at the crack tip is given by

$Y = 1.440736191$

$$K = \sigma\sqrt{\pi a}\,\sec\beta\left(\frac{\tan\beta}{\beta}\right)^2\left[0.923 + 0.199(1 - \sin\beta)\right]^4 \quad \text{where } \beta = \frac{\pi a}{2W} \text{ and } \sigma = \frac{6M}{W^2 t}$$

$K = 44.7$ MNm$^{-3/2}$ radian mode. $= 0.03927$

The beam can fail by yielding ($\sigma > \sigma_Y$) or by fracture ($K > K_c$). Ignore plastic zone correction for fracture calculations.

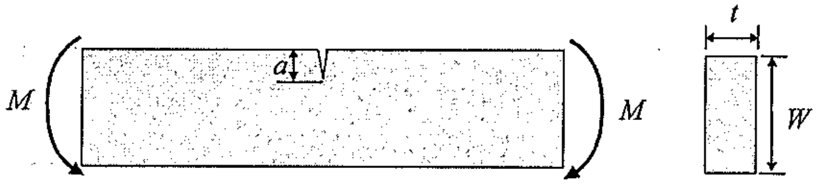


Figure 3(a)

(i) Determine safety factors against yield failure and fracture, and comment if the beam is safe under the given loading. Yield: $SF = \frac{300}{175} = 1.7143$

fracture: $SF = \frac{42}{44.7} < 1$ (5 marks)

∴ not safe

(ii) It is intended to remove a 6mm-thick layer of the material by grinding from the top surface of the beam as shown in Figure 3(b) so that the crack length reduces to 4 mm. Determine again the safety factors against yield failure and fracture, and comment if material removal helps in improving the safety factors.

new $W = 34$ mm

β and σ change.

K also change.

α also change

Lazy do.

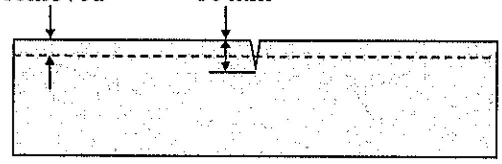


Figure 3(b)

(5 marks)

Note: Question 2 continues on page 4.

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(b) Figure 3(c) shows a large plate in a ship hull with a central hole subjected to a fully reversed uniaxial sinusoidal loading with the stress S fluctuating between +200 MPa and −200 MPa. The frequency of loading is 3 cycles per minute. In a routine inspection of the ship hull, it is discovered that the hole has two edge cracks of same length $a_0 = 0.005$ m symmetrically located as shown. $K_c = Y\sigma_f\sqrt{\pi a_f}$

$a_f = \left(\frac{K_c}{Y\sigma_f}\right)^2 \frac{1}{\pi} = \left(\frac{90}{1.24\times 200}\right)^2 \frac{1}{\pi} = 41.92$ mm

Determine the remaining fatigue life (in hours) of the plate. For this calculation, assume that the cracks grow symmetrically conforming to Paris' law. Because of symmetry, it is sufficient to consider only one of the two cracks. Take the following data for your calculations: $K_c = 90$ MNm$^{-3/2}$; radius of hole $r = 0.08$ m; Paris' law constants $m = 3$ and $C = 1.2\times10^{-11}$ (with $\frac{da}{dN}$ in m/cycle and ΔK in MNm$^{-3/2}$. Geometry factor $Y = 1.24$ (assumed to remain constant throughout the crack growth).

$S_R = 400$

after you find a_f can find N_f easily with Paris law.

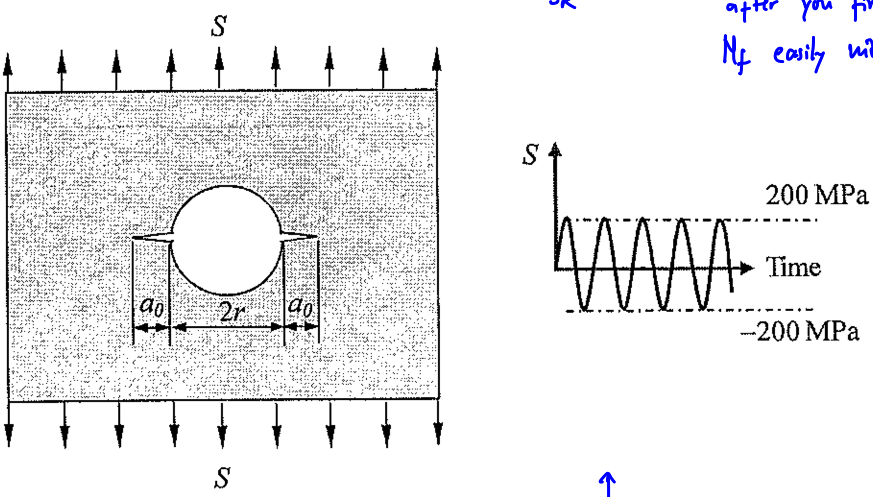
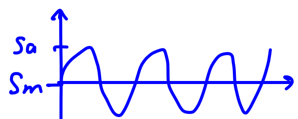


Figure 3(c)



(10 marks)

(c) Specimens made of a certain grade of steel material were subjected to a fully reversed uniaxial sinusoidal loading in an axial load fatigue testing machine. It was found that the material had a fatigue life of 755,243 cycles at a stress amplitude 240 MPa and a fatigue life of 214,286 cycles at a stress amplitude 300 MPa. It is intended to curve fit this fatigue test data so as to obtain an SN curve in the form $S = A + B/N$ where S is the stress amplitude, N is the life in number of cycles, and A and B are appropriate constants.

(i) Determine the values of constants A and B. simultaneous eq.

(3 marks)

(ii) Determine the endurance limit for the material.

(2 marks)

Goodman: $\frac{S_a}{S_e} + \frac{\cancel{S_m}^{0}}{S_u} \le 1$ Gerber: $\frac{S_a}{S_e} + \left(\frac{\cancel{S_m}^{0}}{S_u}\right)^2 \le 1$ Soderberg: $\frac{S_a}{S_e} + \frac{\cancel{S_m}^{0}}{S_Y} \le 1$ for no fatigue failure.

$\therefore$ All 3 reduces to $\frac{S_a}{S_e} \le 1$

$\therefore S_a = S_e = ?$

OR can assume $N = 50\times10^6$ cycles as long life.

$S = A + \frac{B}{50\times10^6}$

3(a) Figure 4 shows a pendulum, which consists of a rod of a mass m and a length L, pivoted at the point O of the celling. A spring of axial stiffness k and negligible mass is fixed to the rod next to the free end A.

(i) Find the natural frequency of this pendulum system, as shown in Figure 4(i).

$J\ddot{\theta} + k_\theta \theta = 0 \quad \frac{1}{3}mL^2\ddot{\theta} + \frac{mgL}{2}\theta = 0 \quad \therefore W_n = \sqrt{\frac{k_\theta}{J}} = \sqrt{\frac{\cancel{m}gL/2}{\frac{1}{3}\cancel{m}L^{\cancel{2}}}} = \sqrt{\frac{3g}{2L}}$ (5 marks)

(ii) Find the maximum angular velocity that the pendulum attains during its free swing after the release from an initial 30° angular position and a zero initial speed as shown in Figure 4(ii). $GPE \Rightarrow KE$.

$mg\Delta h = \frac{1}{2}J_0 w^2 \Rightarrow mg\left(\frac{L}{2} - \frac{L}{2}\cos 30^\circ\right) = \frac{1}{6}mL^2w^2 \Rightarrow w^2 = \frac{3g}{L}(1-\cos 30)$ (3 marks)

$\frac{mg}{2}L(1-\cos 30^\circ)$ $\quad w = \sqrt{\frac{3g}{L}(1-\cos 30)}$

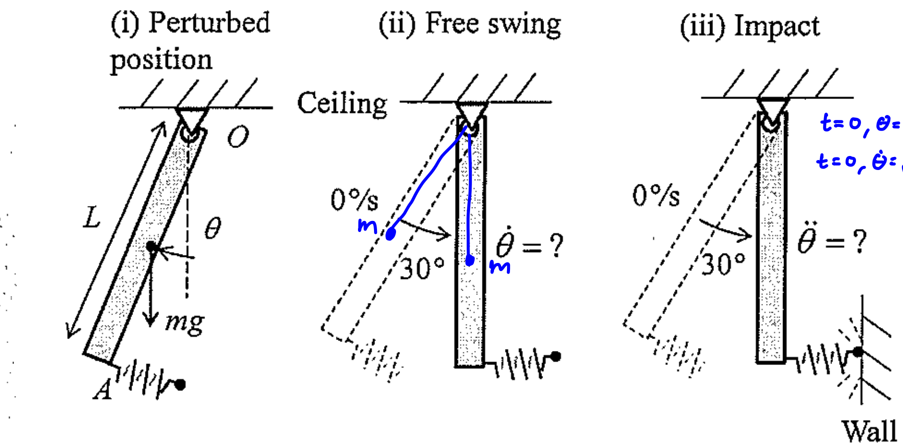
(iii) Derive the expression for the pendulum's angular acceleration upon its contact at 0° onto an adjacent wall after its release from an initial 30° angular position and a zero initial speed as shown in Figure 4(iii).

(7 marks)

EOM:

$\frac{1}{3}mL^2\ddot{\theta} + \left(\frac{mgL}{2} + kL^2\right)\theta = 0$

$\therefore W_n = \sqrt{\frac{\frac{mgL}{2} + kL^2}{\frac{1}{3}mL^2}} = \sqrt{\frac{3}{2}\frac{g}{L} + 3\frac{k}{m}}$



$t=0, \theta=0$

$t=0, \dot{\theta} = \sqrt{\frac{3g}{L}(1-\cos 30)}$

no damping → harmonic

$\theta = \Theta \sin(w_n t + \phi)$

$0 = \Theta \sin(W_n \times 0 + \phi)$

$\therefore \phi = 0$

$\dot{\theta} = \sqrt{\frac{3g}{L}(1-\cos 30)} = w_n \Theta \cos(W_n \times 0)$

$\therefore \Theta = \frac{\sqrt{\frac{3g}{L}(1-\cos 30)}}{\sqrt{\frac{3}{2}\frac{g}{L} + 3\frac{k}{m}}}$

Figure 4

$\therefore \ddot{\theta} = -w_n^2\theta$

$= -\sqrt{\frac{3g}{L}(1-\cos 30)\left(\frac{3}{2}\frac{g}{L} + 3\frac{k}{m}\right)} \sin\left(\sqrt{\frac{3}{2}\frac{g}{L} + 3\frac{k}{m}}\, t\right)$

(b) A 500 kg engine is mounted on rubber pads of negligible mass. The static loading by the engine's weight causes a static displacement of 5mm to depress the rubber pads. Its free vibration is observed to have the amplitudes decay in a ratio of 3 to 1 in each consecutive cycle. Find the amplitude of resonant vibration and the force transmission to the ground when this mounted engine is excited into resonant vibration by a harmonic vertical force with 1000N amplitude.

(10 marks)

$k_{eff} = \frac{F}{\Delta} = \frac{500 \times 9.81}{0.005} = 981000$

$\delta = \ln \frac{1}{3}$

$\zeta = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}} = 0.17223 = \frac{c}{c_c}$

$C_c = 2\sqrt{km}$
$= 44294.46$

$C = 7628.83$

$W_n = \sqrt{\frac{k}{m}} = 44.3$

$X = \frac{F_0}{\sqrt{(k - mw^2)^2 + (cw)^2}}$

$X = \frac{F_0}{cW} = 2.959\,mm$

$X_{res} = \frac{F_0}{k2\zeta} = 2.959\,mm$

$F_T = kX + mg$

$= 981000(X_{res} \sin W_n t) + 500 \times 9.81$

$= 2902 \sin 44.3t + 4905$

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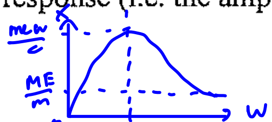
4(a) A table fan of mass M is spinning at a speed ω, and thus subjected to a centrifugal force due to a rotating unbalance mass m at a radius of eccentricity e. A mass-spring-damper system can model the vibration response of this fan on a table with a horizontal spring constant k and a viscous damping constant c.

$X = 0$ $\qquad X = \frac{me\omega^2}{c\omega} = \frac{me\omega}{c}$

Derive the expressions of the vibration amplitudes at zero spin speed, resonant spin speed, and a very high spin speed.

$X = \frac{me}{M}$ $\qquad X = \frac{me\omega^2}{\sqrt{(k-M\omega^2)^2+(c\omega)^2}}$

Plot and mark the frequency response (i.e. the amplitudes versus the spin speeds).



(10 marks)

(b) Figure 5 shows a pick-up truck of a total mass m_1 transporting a small cart of a mass m_2. The small cart is hitched through two springs of axial stiffness k each to the truck body. Absolute displacement of the truck is x_1 while that of the cart is x_2.

(i) Find the relative motion (x_2–x_1) of the cart when the truck is subjected to a harmonic force excitation $F_1\sin\omega t$.

(7 marks)

(ii) Find the natural frequencies and mode shapes of this two-degree-of-freedom system during the free vibrations. Sketch also the mode shapes.

(8 marks)

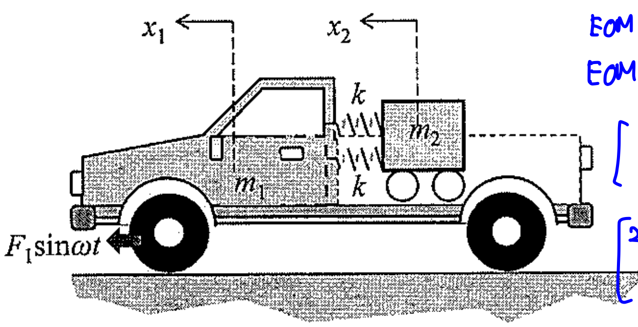


Figure 5

EOM 1: $m_1\ddot{x}_1 + 2k(x_1-x_2) = F\sin\omega t$

EOM 2: $m_2\ddot{x}_2 + 2k(x_2-x_1) = 0$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1\sin\omega t \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 2k-m_1\omega^2 & -2k \\ -2k & 2k-m_2\omega^2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 0 \end{Bmatrix}$$

Cramer's Rule: $X_1 = \frac{(2k-m\omega^2)F_1}{(2k-m_1\omega^2)(2k-m_2\omega^2)-4k^2}$

$X_2 = \frac{2F_1k}{(2k-m_1\omega^2)(2k-m_2\omega^2)-4k^2}$

End of Paper

$(x_2-x_1) = X_2\sin\omega t - X_1\sin\omega t$

$= \frac{F_1(\cancel{2k}-\cancel{2k}+m\omega^2)}{(2k-m_1\omega^2)(2k-m_2\omega^2)-4k^2}\sin\omega t$

find ω: $(2k-m_1\omega^2)(2k-m_2\omega^2)-4k=0$

$4k^2-2m_1k\omega^2-2m_2k\omega^2+m_1m_2\omega^4-4k=0$

$\omega^2(m_1m_2\omega^2-2(m_1+m_2)k)=0$

$\omega^2=0$ or $\omega^2=\frac{2(m_1+m_2)k}{m_1m_2}$

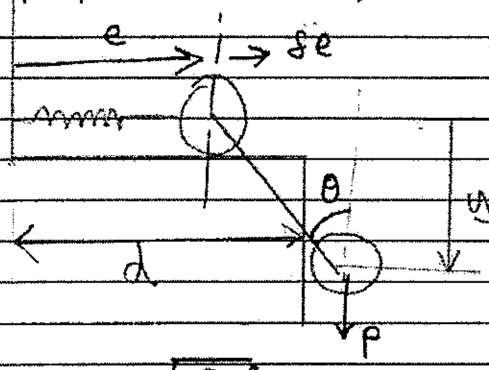
$\downarrow \qquad \downarrow$

$\frac{X_1}{X_2}=1 \qquad \frac{X_1}{X_2}=\frac{2k}{2k-m_1\omega^2}=\frac{2k}{2k-\frac{2(m_1+m_2)k}{m_2}}$

$=\frac{1}{1-\frac{m_1+m_2}{m_2}}$ (−ve), small value for $m_1 \gg m_2$.

$\overleftarrow{X_1} \quad \overleftarrow{X_2} \qquad \overleftarrow{X_1} \quad \overrightarrow{X_2}$

By principle of virtual work, $\delta U = \delta V$



$$P\,\delta y = Ke\,\delta e$$

From geometry,

$$e = d + r - L\sin\theta - l_0$$

$$\frac{de}{d\theta} = -L\cos\theta$$

$$y = L\cos\theta$$

$$\frac{dy}{d\theta} = -L\sin\theta$$

$$\sin\theta = \frac{\sqrt{L^2 - y^2}}{L}$$

$$\cos\theta = \frac{y}{L}$$

$$P(-L\sin\theta)\,\delta\theta = K(d + r - L\sin\theta - l_0).(-L\cos\theta)\,\delta\theta$$

$$e = d + r - L\sin\theta - l_0$$

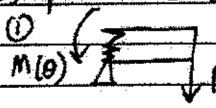
$$P = \frac{-LK(d + r - L\sin\theta - l_0)\cos\theta}{-L\sin\theta}$$

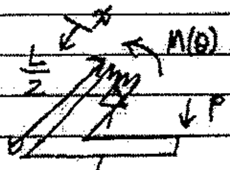
$$= \frac{K(d + r - l_0 - \sqrt{L^2 - y^2})\frac{y}{L}}{\frac{\sqrt{L^2 - y^2}}{L}}$$

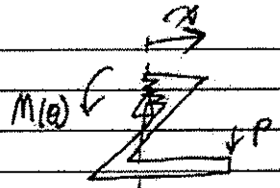
$$= \frac{Ky(d + r - l_0 - \sqrt{L^2 - y^2})}{\sqrt{L^2 - y^2}}$$

\# P(1)

b) Split into 3 segments.

① 

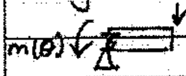


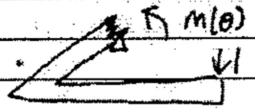


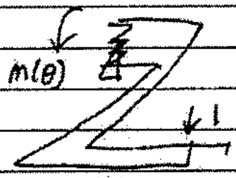
$\circlearrowleft^{+} M_{CO}: -M(\theta) + Px = 0$ $\quad \circlearrowleft^{+} M_{BD}: -M(\theta) + P\left(L - \frac{x}{\sqrt{2}}\right) = 0$ $\quad \circlearrowleft^{+} M_{AD}: -M(\theta) + Px = 0$

$M(\theta) = Px$ $\qquad\qquad M(\theta) = Px$

Using Same FBD for virtual load analysis







$m(\theta) = x$

$m(\theta) = 1\left(L - \frac{x}{\sqrt{2}}\right)$

$m(\theta) = x$

$$\int_0^L \frac{Px^2}{EI}dx + \int_0^L P\left(L - \frac{x}{\sqrt{2}}\right)\cdot\left(L - \frac{x}{\sqrt{2}}\right)\left(\frac{1}{EI}\right)dx + \int_0^L \frac{Px^2}{EI}dx$$

$$= \left[\frac{Px^3}{3EI}\right]_0^L + \frac{P}{EI}\int_0^{\sqrt{2}L}\left(L - \frac{x}{\sqrt{2}}\right)^2 + \left[\frac{Px^3}{EI}\right]_0^L$$

$$= \frac{2PL^3}{3EI} + \frac{P}{EI}\left[L^2x - \frac{2Lx^2}{2\sqrt{2}} + \frac{x^3}{6}\right]_0^{\sqrt{2}L}$$

$$= \frac{2}{3}\frac{PL^3}{EI} + \frac{P}{EI}\left(\sqrt{2}L^3 - \frac{2L^3}{\sqrt{2}} + \frac{(\sqrt{2})^3L}{6}\right)$$

$$= \frac{PL^3}{EI}\left(\frac{2}{3} + \sqrt{2} - \frac{2}{\sqrt{2}} + \frac{2\sqrt{2}}{6}\right)$$

$$= 1.1381\frac{PL^3}{EI}$$

ii) $\Delta_{max} = \Delta_s\left(1 + \sqrt{1 + \frac{2h}{\Delta_s}}\right)$

$= 0.0930\,m$ (sub in values only)

Material behavior is linear elastic

Mass of the beam is negligible. hence there is a small change in GPE of beam due to deformation is neglected

No energy loss

Solid block sticks to beam throughout impact & oscillation.

2a. Failure by yielding $\sigma > \sigma_Y$

$$\sigma = \frac{6(700)}{\left[(42\times10^6)^{-3/2}\right]^2 \times 15\times10^{-3}} = 175\times10^6 \text{ Pa}$$

$$\beta = \frac{\pi(10\times10^{-3})}{2(40\times10^{-3})} = 0.393$$

← Failure by fracture ($K > K_c$)
$K_c = 42 \text{ MN m}^{-3/2}$

$$K = \sigma\sqrt{\pi(10\times10^{-3})}\ \sec\beta\left(\frac{\tan\beta}{\beta}\right)^2\left[0.923 + 0.199(1-\sin\beta)\right]^4$$
$$= 44.69\times10^6 \text{ Nm}^{-3/2}$$

Since $K > K_c$, beam is not safe

Safety factor against yield stress: $\frac{300}{175} = 1.714$

against fracture: NIL

ii) new a = 10mm − 6mm = 4mm

new W = 40mm − 6mm = 34mm

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is:
http://bit.ly/2CAIRGf

A P(2)

b) Fully reversed uniaxial sinusoidal loading: $S_R = S_{max} = 200MPa$

$$K_c = Y\sigma \, (\pi a_f)^{1/2}$$

$$a_f = \left(\frac{K_c}{Y\sigma}\right)^2 \frac{1}{\pi} = 0.0419$$

$$N_f = \frac{2}{2(YS_R)^m \pi^{m/2}(2-m)} \left(a_f^{\,1-m/2} - a_0^{\,1-m/2}\right)$$

$$= 18167.2$$

$$\approx 18168$$

$$\frac{N_f}{3} = 6056_{min} \approx 101hr$$

c) $300 = A + \frac{B}{214286}$ —(1)

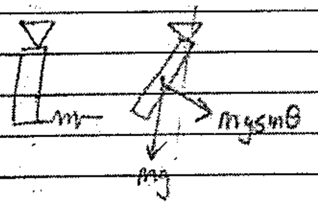
$240 = A + \frac{B}{755293}$ —(2)

$A = 216.233, \quad B = 17950188.44$

$S_e = 216.233 + \frac{17950188.44}{50\times10^6}$

$= 217MPa$

3a.



$J_0 = \frac{1}{3}mL^2$

$J_0\ddot{\theta} = -mg\sin\theta \cdot \frac{L}{2}$

$\frac{1}{3}mL^2\ddot{\theta} + mg\sin\theta \cdot \frac{L}{2} = 0.$

By small angle approx, $\frac{1}{3}mL^2\ddot{\theta} + \frac{mgL}{2}\theta = 0$

By $\theta(t) = A\sin(wt + \phi)$

$\ddot{\theta}(t) = -w^2 A\sin(wt + \phi)$

$\frac{1}{3}mL^2(-w^2)A + \frac{mgL}{2}A = 0$

$\frac{1}{3}mL^2 w^2 = \frac{mgL}{2}$

$w^2 = \frac{3g}{2L}$

$W_n = \sqrt{\frac{3g}{2L}}$

ii) $mgh = \frac{1}{2} J_0 \dot{\theta}^2$

$h = \frac{L}{2} - \frac{L}{2}\cos 30^\circ$

$= 0.06699L$

$mg(0.06699L) = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\dot{\theta}^2$

$\dot{\theta} = \sqrt{\frac{3.943}{L}}$

iii) $J_0 \ddot{\theta} + mg\frac{L}{2}\theta + kL^2\theta = 0$

$\frac{1}{3}mg^2\ddot{\theta} + \left(\frac{mgL}{2} + kL^2\right)\theta = 0$

$A\sin(\omega t - \phi) = \theta$

① When $t=0$, $\theta = 0$, $0 = A\sin(-\phi)$

$\phi = 0$

② $\dot{\theta} = \omega A\cos(\omega t)$

When $t=0$, $\dot{\theta} = \sqrt{\frac{3.943}{L}} = \omega_n A$

$\therefore \theta = \omega_n \frac{1}{\omega_n}\sqrt{\frac{3.943}{L}}\cos \omega_n t$

$= \sqrt{\frac{3.943}{L}}\cos\left(\sqrt{\frac{1.5mg + 3kL}{mL}}\, t\right)$

b) $K = \frac{500 \times 9.81}{0.005} = 981\ 000\ N/m$

$\zeta = \sqrt{\frac{\ln^2 \zeta}{4\pi^2 + \ln^2 \zeta}} = 0.1722$

Amplitude of resonant vibration $= \frac{F_0}{K2\zeta} = \frac{1000}{981000(2)(0.1722)} = 0.00296m$

EOM : $500\ddot{x} + kx + c\dot{x} = 1000\sin(\omega t)$

Force transmission $= \left|\frac{F_T}{F_0}\right| = \frac{1}{2\zeta} = 2.903$

$\ddagger$ P(3)

ii) $mgh = \frac{1}{2} J_o \dot{\theta}^2$

$h = \frac{L}{2} - \frac{L}{2}\cos 30°$

$= 0.06699L$

$mg(0.06699L) = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\dot{\theta}^2$

$\dot{\theta} = \sqrt{\frac{3.943}{L}}$

iii) $J_o\ddot{\theta} + mg\frac{L}{2}\theta + KL^2\theta = 0$

$\frac{1}{3}mg^2\ddot{\theta} + \left(\frac{mgL}{2} + KL^2\right)\theta = 0$

$A\sin(\omega t - \phi) = \theta$

① When $t=0$, $\theta = 0$, $0 = A\sin(-\phi)$

$\phi = 0$

② $\dot{\theta} = \omega A\cos(\omega t)$

When $t=0$, $\dot{\theta} = \sqrt{\frac{3.943}{L}} = \omega_n A$

$\therefore \theta = \cancel{\omega_n} \frac{1}{\omega_n}\sqrt{\frac{3.943}{L}} \cancel{\cos \omega_n t}$

$= \sqrt{\frac{3.943}{L}}\cos\left(\sqrt{\frac{1.5mg + 3KL}{mL}}\, t\right)$

b) $K = \frac{500 \times 9.81}{0.005} = 981\,000\ N/m$

$\xi = \sqrt{\frac{\ln^2 \xi}{4\pi^2 + \ln^2\xi}} = 0.1722$

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EOM: $500\ddot{x} + Kx + c\dot{x} = 1000\sin(\omega t)$

Force transmission $= \left|\frac{F_T}{F_o}\right| = \frac{1}{2\xi} = 2.903$

$$X_2 - X_1 = \frac{F_1 \omega^2 m_2}{(2k-\omega^2 m_1)(2k-\omega^2 m_2) - 4k^2}$$

ii) $\det \begin{vmatrix} 2k-\omega^2 m_2 & -2k \\ -2k & 2k-\omega^2 m_1 \end{vmatrix} = 0$

$$4k^2 - 2k\omega^2 m_1 - 2k\omega^2 m_2 + \omega^4 m_1 m_2 - 4k^2 = 0$$

$\omega_{n1} = 0$, $\quad -2km_1 - 2km_2 + \omega^2 m_1 m_2 = 0$

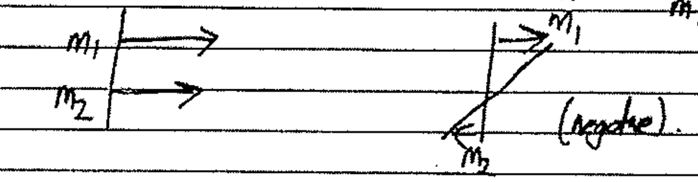
$$\omega_{n2} = \sqrt{\frac{2k(m_1+m_2)}{m_1 m_2}}$$

Mode shape: $2kX_1 = (2k-\omega^2 m_2)X_2$

$$\frac{X_2}{X_1} = \frac{2k}{2k-\omega^2 m_2}$$

When $\omega_n = 0$, $\left|\frac{X_2}{X_1}\right| = 1$

When $\omega_n = \sqrt{\frac{2k(m_1+m_2)}{m_1 m_2}}$, $\left|\frac{X_2}{X_1}\right| = \frac{2k}{2k - \frac{2k(m_1+m_2)}{m_1}} = \frac{1}{1-\frac{m_1+m_2}{m_1}}$



NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

MA3002 – SOLID MECHANICS AND VIBRATION

April/May 2018 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SEVEN (7)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN BOOK** examination. One double-sided A4 size reference sheet is allowed.

1(a) Figure 1 shows the equilibrium configuration of a system where a cylinder of mass M and radius r is carried by a pin-jointed structure consisting of two thin rigid weightless bars AB and BC, and a spring CD of stiffness k. The bars AB and BC are of lengths L_1 and L_2, respectively. In the configuration shown, the spring is compressed by the combined action of the weight of the cylinder and the force F applied at B. Assume the spring does not buckle. The free length of the spring (i.e, the initial length of the spring before it is compressed) is l_0. Assume that all the members in the structure except the spring are rigid. Ignore the effect of friction at joints and contacting surfaces.

Derive an expression for the force F in terms of θ and other system parameters using the *principle of virtual work*. Show all the coordinates and virtual displacements used for the purpose on a neat sketch of the system.

(13 marks)

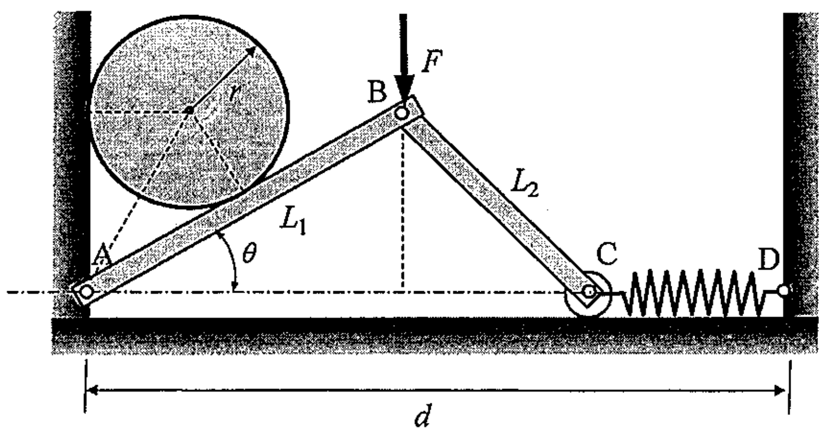


Figure 1

Note: Question 1 continues on page 2.

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(b) Figure 2 shows a statically indeterminate weightless beam structure ABC of flexural rigidity EI. The lengths of portion BC and AB are L and $L\cos 30^{\circ}$, respectively. End A is rigidly fixed to a wall and end C is guided to move vertically. A vertical load W is applied at point B as shown. Consider only bending effects and ignore friction at the vertical guide.

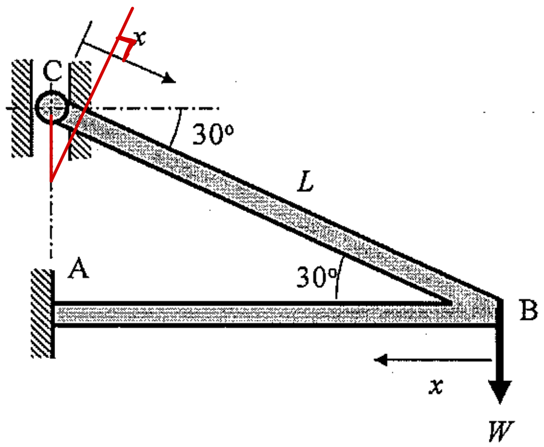


Figure 2

(i) Mark all unknown support reactions on a neat sketch and show that the degree of indeterminacy is 1.

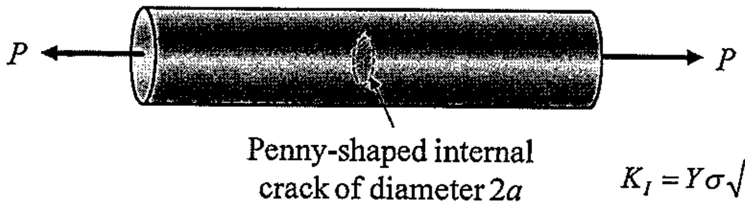
(2 marks)

(ii) Determine the horizontal reaction force at C by *unit load method*.

(10 marks)

2(a) A steel tension member in a crane has a circular cross section with 20 mm diameter. The supply of steel tension members from a vendor has yield stress σ_Y = 800 MPa and fracture toughness K_{Ic} = 30 $\text{MNm}^{-3/2}$, and is believed to have central penny-shaped internal cracks in the cross section. The non-destructive test facility available in the laboratory is not capable of detecting cracks smaller than 3 mm diameter.

Assume the cracks are well separated and interaction between cracks is negligible, and hence, a single isolated crack as shown in Figure 3 can be considered for the analysis. For penny-shaped cracks, the geometry correction factor $Y = 2/\pi$. Ignore plastic zone correction. Assume linear elastic fracture mechanics holds.



$$K_I = Y\sigma\sqrt{\pi a}$$

Figure 3

Note that the rod can fail by yielding ($\sigma > \sigma_Y$) or by fracture ($K_I > K_{Ic}$).

(i) Determine the maximum tensile load P that can be carried by the member purely based on yield failure (assuming no cracks). Also determine the maximum tensile load P that can be carried purely based on fracture consideration. Comment on how the presence of cracks affects the load carrying capacity of the rod.

(7 marks)

(ii) Determine the maximum crack size ($2a_c$) that can be permitted so that the maximum load carrying capacity with cracks is the same as that without cracks.

(4 marks)

Note: Question 2 continues on page 4.

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(b) A beam of width $W = 50$ mm and thickness $t = 50$ mm is subjected to 4-point bending as shown in Figure 4 where $b = 100$ mm. At the mid-section of the beam, there is a through edge crack of depth $a = 5$ mm on the bottom surface. The fracture toughness of the material is $K_{Ic} = 50$ MNm$^{-3/2}$. Assume linear elastic fracture mechanics holds.

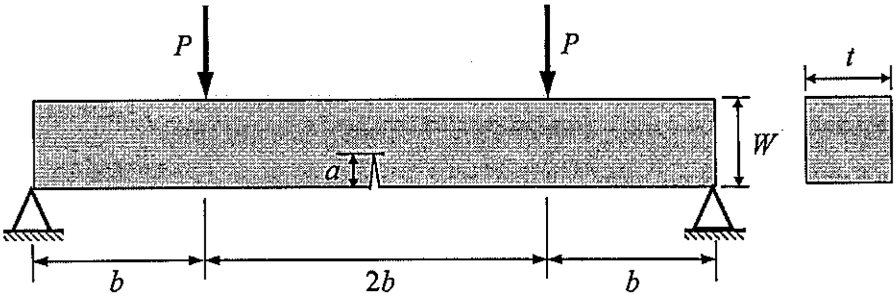

Figure 4

The formula for the stress intensity factor for the crack is as follows:

$$K_I = Y\sigma\sqrt{\pi a}$$

where $Y = 1.122 - 1.4\beta + 7.33\beta^2 - 13.08\beta^3 + 14.0\beta^4$ is the geometry correction factor, $\beta = a/W$, $\sigma = 6M/(tW^2)$ is the nominal bending stress and $M = Pb$ is the bending moment at the mid-section of the beam.

(i) Determine the maximum value of load P that the beam can carry before fracture.

(7 marks)

(ii) If the load P in Figure 4 fluctuates sinusoidally between 0 and 60 kN once every day, determine how many years it will take for the crack to grow in length by 20%. Assume the geometry correction factor (Y) to remain constant at the value you calculated in part (i). Use Paris law $da/dN = C(\Delta K)^m$ for this calculation where da/dN is in m/cycle and ΔK is in MNm$^{-3/2}$, $m = 3.5$ and $C = 0.25 \times 10^{-11}$.

(7 marks)

3(a) Figure 5 shows a durian fruit of mass 2.5 kg hanging from a tree branch. Free oscillation of the branch with the durian on is different from that of the branch alone (after durian falling). It is observed that the period of free oscillation for the branch with the durian on is 1.5 s. After the durian fell, the branch alone oscillates freely faster at a period of 1.0 s.

(i) By neglecting the damping, estimate the effective mass and effective stiffness of the branch. Estimate also the static displacement Δ of the branch upon unloading (after durian falling).

(6 marks)

(ii) Explain why the effective lumped mass calculated above is smaller than the actual mass of the branch.

(3 marks)

(iii) Calculate the damping constant based on the damped free vibration of the branch with durian whose amplitude decays in a ratio of 2 to 1 in a consecutive cycle.

(2 marks)

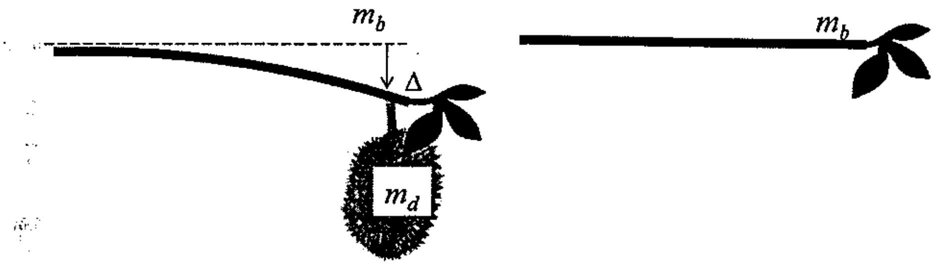

Figure 5

(iv) Estimate the damping ratio for the branch alone that undergoes damped free oscillation. (Use the solution obtained above in the Question 3a(i)-(iii).)

(3 marks)

(v) Identify and describe the possible damping mechanisms that dissipate the energy from this freely oscillating branch with the durian on.

(4 marks)

Note: Question 3 continues on page 6.

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(b) Figure 6 shows a boy sitting on a swing and being pumped (pushed) by a harmonic force $F_o \sin\omega t$ where F_o is the force amplitude and ω is the angular frequency of pumping in the horizontal direction. This forced oscillation can be modelled by a simple pendulum which consists of a lumped mass m swinging at a distance L from the pivot.

(i) Write the equation of motion for this pendulum under forced vibration.

(ii) Find the amplitude of angular swing based on the assumption of a small angle.

(7 marks)

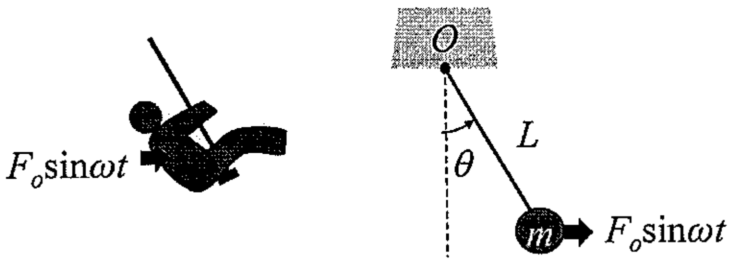

Figure 6

4(a) Figure 7 shows a ceramic vase of mass m being packed in a box with packaging foams of a total stiffness k. During the shipping by post, this box of vase is subjected to various handling conditions which include a base excitation (of conveyor belt) and a drop test.

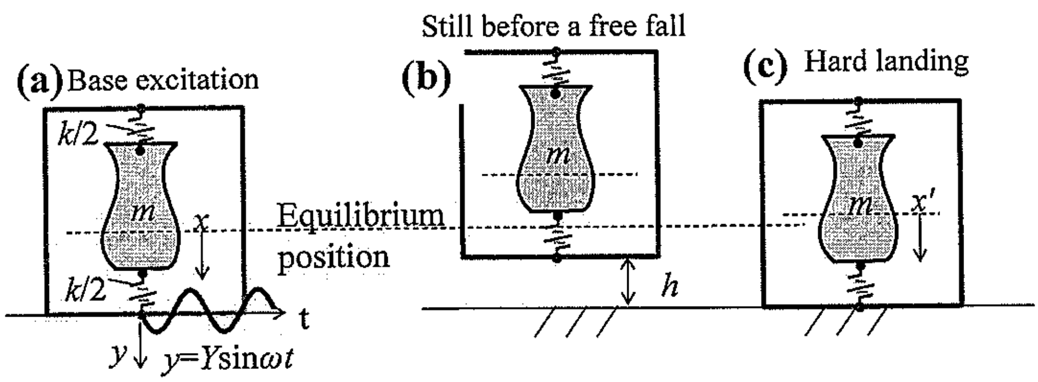

Figure 7

(i) Derive the expression of acceleration at which the vase undergoes due to base excitation $y = Y\sin\omega t$ that bounces the box where Y and ω are the amplitude and angular frequency respectively of the base motion.

(4 marks)

Note: Question 4 continues on page 7.

(ii) Derive the expression of the acceleration amplitude imparted to the vase during a drop test. The drop test has the box released from rest and falling through a height h before landing hard on the rigid ground.

(6 marks)

(b) Figure 8 shows the box of vase (as described in Question 4(a)) now being placed on a leverage plate with spring support. This plate of a moment of inertia J_o is pivoted at point O and supported by a spring of stiffness K at a distance b from the pivot O. The box of vase is located at a distance a from the pivot O.

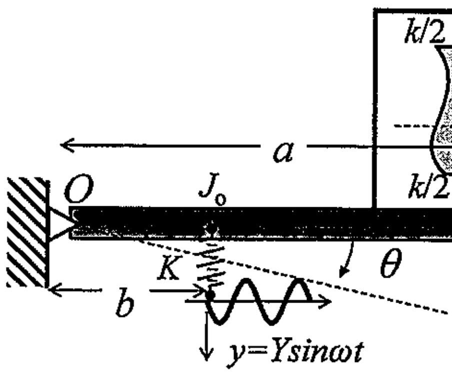

Figure 8

(i) Determine the amplitude of harmonic motion x of the vase due to the base excitation y acting at the lower spring support of the plate.

(7 marks)

(ii) Find the lowest natural frequency and the corresponding mode shape for this two-degree-of-freedom system undergoing free vibrations (upon removal of base excitation). Sketch the mode shape (i.e. the lever's angular displacement amplitude per unit vase's displacement amplitude). Given the leverage design having $J_o = ma^2$ and the base spring having the same stiffness as the foam, $K = k$.

(8 marks)

END OF PAPER

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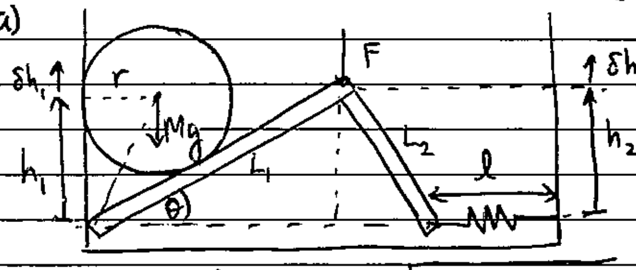
MA3002 2017/18 Sem 2 (April / May 2018)

① a)



$\cancel{\frac{r}{h_1} = \tan\left(\frac{1}{2}(90^\circ - \theta)\right)}$

$h_1 = r\cot(45^\circ - \frac{1}{2}\theta)$

$\delta h_1 = \frac{1}{2} r \csc^2(45^\circ - \frac{1}{2}\theta)\,\delta\theta$

$h_2 = L_1 \sin\theta$

$\delta h_2 = L_1 \cos\theta\,\delta\theta$

$l = d - L_1\cos\theta - \sqrt{L_2^2 - L_1^2\sin^2\theta}$

$e = l_0 - l$ (compressed)

$e = l_0 - d + L_1\cos\theta + \sqrt{L_2^2 - L_1^2\sin^2\theta}$

$\delta e = \left(-L_1\sin\theta + \frac{1}{2}(L_2^2 - L_1^2\sin^2\theta)^{-\frac{1}{2}}(-2L_1^2\sin\theta\cos\theta)\right)\delta\theta$

$= \left(-L_1\sin\theta - L_1^2\sin\theta\cos\theta\,(L_2^2 - L_1^2\sin^2\theta)^{-\frac{1}{2}}\right)\delta\theta$

Principle of virtual work

$\delta W = \delta U$

$-Mg\,\delta h_1 - F\delta h_2 = ke\,\delta e$

$-Mg\left(\frac{1}{2} r\csc^2(45^\circ - \frac{1}{2}\theta)\right)\delta\theta - F L_1\cos\theta\,\delta\theta =$

$k\left(l_0 - d + L_1\cos\theta + \sqrt{L_2^2 - L_1^2\sin^2\theta}\right)\left(-L_1\sin\theta - L_1^2\sin\theta\cos\theta\,(L_2^2 - L_1^2\sin^2\theta)^{-\frac{1}{2}}\right)\delta\theta$

Since $\delta\theta$ is arbitrary

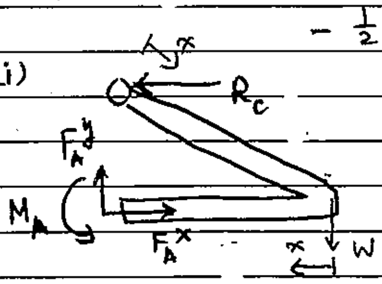
$\frac{1}{2}Mgr\csc^2(45^\circ - \frac{1}{2}\theta) + F L_1\cos\theta$

$= kL_1\sin\theta\left(l_0 - d + L_1\cos\theta + \sqrt{L_2^2 - L_1^2\sin^2\theta}\right)\left(1 + L_1\cos\theta\,(L_2^2 - L_1^2\sin^2\theta)^{-\frac{1}{2}}\right)\delta\theta$

$\cancel{F = \frac{1}{L_1\cos\theta}\left[kL_1\sin\theta\left(l_0 - d + L_1\cos\theta + \sqrt{L_2^2 - L_1^2\sin^2\theta}\right)\left(1 + \frac{L_1\cos\theta}{\sqrt{L_2^2 - L_1^2\sin^2\theta}}\right)\right.}$

$\left. - \frac{1}{2}Mgr\csc^2(45^\circ - \frac{1}{2}\theta)\right]$

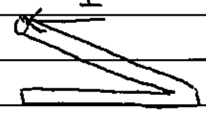
b) (i)



Unknown reactions: R_c, F_A^x, F_A^y, M_A → 4

Useful equations: 3 ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$)

· Degree of indeterminacy = 4 − 3 = $\underline{\underline{1}}$



(ii) Real load analysis

BC: $\circlearrowleft^{+}$ $M = R_c \sin 30^\circ x$

$= \frac{1}{2}R_c x$

AB: $\circlearrowleft^{+}$ $M = R_c \sin 30^\circ L - Wx$

$= \frac{1}{2}R_c L - Wx$

Virtual load analysis

BC: $\circlearrowleft^{+}$ $m = 1 \cdot \sin 30^\circ x$

$= \frac{1}{2}x$

AB: $\circlearrowleft^{+}$ $m = 1 \sin 30^\circ L$

$= \frac{1}{2}L$

1 b) (ii) Continued

$$\frac{1}{2}\Delta_c = \int_0^{L} \frac{(\frac{1}{2}R_c x)(\frac{1}{2}x)}{EI}\,dx + \int_0^{L\cos 30^\circ} \frac{(\frac{1}{2}R_c L - Wx)(\frac{1}{2}L)}{EI}\,dx$$

$\Delta_c = 0$

$$\rightarrow 0 = \int_0^{L} \frac{1}{4}R_c x^2\,dx + \int_0^{L\cos 30^\circ} \left(\frac{1}{4}R_c L^2 - \frac{1}{2}WLx\right)dx$$

$$0 = \left[\frac{1}{12}R_c x^3\right]_0^L + \left[\frac{1}{4}R_c L^2 x - \frac{1}{4}WLx^2\right]_0^{\frac{1}{2}\sqrt{3}L}$$

$$0 = \frac{1}{12}R_c L^3 + \frac{1}{8}\sqrt{3}R_c L^3 - \frac{3}{16}WL^3$$

$$R_c = 0.625W$$

2 a) (i) Yield: $\sigma_Y = \dfrac{P}{\frac{1}{4}\pi D^2}$

$$P = (800\times10^6)(\tfrac{1}{4}\pi\times0.02^2) = 251327.4\ N = 251.32\ kN$$

Fracture: $K_{IC} = Y\sigma(\pi a)^{\frac{1}{2}}$

$$30\times10^6 = \frac{2}{\pi}\left(\frac{P}{\frac{1}{4}\pi\times0.02^2}\right)\left(\pi\times\frac{0.003}{2}\right)^{\frac{1}{2}}$$

$$P = 215660.4\ N = 215.66\ kN$$

Presence of cracks reduces the maximum load that can be carried because under load, the crack will grow until the part fails.

(ii) $P = 251327.4\ N$

$$K_{IC} = Y\sigma(\pi a_c)^{\frac{1}{2}}$$

$$30\times10^6 = \frac{2}{\pi}\left(\frac{251327.4}{\frac{1}{4}\pi\times0.02^2}\right)(\pi a_c)^{\frac{1}{2}}$$

$$a_c = 1.1\times10^{-3}\ m$$

$$\rightarrow 2a_c = 2.2\ mm$$

b) (i) $\beta = \dfrac{a}{W} = \dfrac{5}{50} = 0.1$

$$Y = 1.122 - 1.4(0.1) + 7.33(0.1)^2 - 13.08(0.1)^3 + 14(0.1)^4$$

$$= 1.09362$$

$$K_{IC} = Y\left(\frac{6M}{tW^2}\right)\sqrt{\pi a}$$

$$50\times10^6 = 1.09362\left(\frac{6\times P\times0.1}{0.05\times0.05^2}\right)\sqrt{\pi\times0.005}$$

$$P = 79639\ N = 79.63\ kN$$

(ii) $F_R = 60 - 0 = 60\ kN$

$a_f = 1.2a_0 = 1.2\times5 = 6\ mm$

$$f_R = \frac{6(F_R)b}{tW^2} = \frac{6\times60000\times0.1}{0.05\times0.05^2} = 288\ MN/m^2$$

(2) b) (ii) Continued

$$N_f = \frac{2}{C(Y S_R)^m \pi^{m/2}(2-m)}\left(a_f^{1-\frac{m}{2}} - a_o^{1-\frac{m}{2}}\right)$$

$$= \frac{2}{0.25\times10^{-11}(1.09362\times 288)^{3.5}\pi^{\frac{3.5}{2}}(2-3.5)}\left(0.006^{1-\frac{3.5}{2}} - 0.005^{1-\frac{3.5}{2}}\right)$$

$= 1038.79$ days

$= 2.84$ years

(3) a) (i)

With durian

$(m_b + m_d)\ddot{x} + kx = 0$

$$\omega_1 = \sqrt{\frac{k}{m_b + m_d}} = \frac{2\pi}{T_1}$$

$$\frac{k}{m_b + 2.5} = \left(\frac{2\pi}{1.5}\right)^2$$

$$k = \frac{16\pi^2}{9}(2.5 + m_b)$$

Without durian

$m_b\ddot{x} + kx = 0$

$$\omega_2 = \sqrt{\frac{k}{m_b}} = \frac{2\pi}{T_2}$$

$$\frac{k}{m_b} = \left(\frac{2\pi}{1}\right)^2$$

$$k = 4\pi^2 m_b$$

$$4\pi^2 m_b = \frac{16\pi^2}{9}(2.5 + m_b)$$

$m_b = 2$ kg $\rightarrow k = 78.95$ N/m

$$\Delta = \frac{m_b g}{k} = \frac{2\times 9.8}{78.95} = 0.248 \text{ m}$$

(ii) Part of branch nearer to base oscillates at lower speed and displacement, hence the total energy of the branch would be less than if the whole branch oscillates with uniform speed & displacement $\rightarrow$ lumped mass is smaller

(iii) $\delta = \ln\frac{2}{1} = 0.693$

$$\zeta = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}} = \sqrt{\frac{0.693^2}{4\pi^2 + 0.693^2}} = 0.10965 = \frac{c}{2\sqrt{k(m_b + m_d)}}$$

$$\rightarrow c = 2\times 0.10965\sqrt{78.95(2 + 2.5)}$$

$= 4.13$ Ns/m

(iv) $C_r = 2\sqrt{k m_b} = 2\sqrt{78.95\times 2} = 25.13$ Ns/m

$$\zeta = \frac{c}{C_r} = \frac{4.13}{25.13} = 0.164$$

(v) $\rightarrow$ Air friction

$\rightarrow$ Internal friction within the branch

b) (i) $\sum M_o = mL^2\ddot{\theta}$

$(F_o \sin\omega t)(L\cos\theta) - mgL\sin\theta = mL^2\ddot{\theta}$; $\theta \ll \rightarrow \sin\theta \approx \theta, \cos\theta \approx 1$

$F_o L \sin\omega t - mgL\theta = mL^2\ddot{\theta}$

$(=)\quad mL\ddot{\theta} + mg\theta = F_o\sin\omega t$

P(2)

(3) b) (ii) Let $\theta = \theta_m \sin(\omega t - \phi) \rightarrow \ddot{\theta} = -\omega^2 \theta_m \sin(\omega t - \phi)$

Substituting : $(-mL\theta_m \omega^2 + mg\theta_m)\sin(\omega t - \phi) = F_0 \sin \omega t$

Comparing amplitude : $\theta_m = \dfrac{F_0}{mg - mL\omega^2}$

(4) a)(i) $\Sigma F = m\ddot{x}$

$k(y - x) = m\ddot{x}$

$kY \sin \omega t = m\ddot{x} + kx$

$x = X \sin(\omega t - \phi)$

$\ddot{x} = -\omega^2 X \sin(\omega t - \phi)$

$\rightarrow kY \sin \omega t = (-m\omega^2 X + kX)\sin(\omega t - \phi)$

Comparing amplitude : $X = \dfrac{kY}{k - m\omega^2}$; Comparing phase : $\omega t = \omega t - \phi$

$\phi = 0$

$\therefore \ddot{x} = \dfrac{-kY\omega^2}{k - m\omega^2} \sin \omega t$

(ii) Just before landing : $mgh = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gh}$

Initial condition : $\dot{x}'(0) = \sqrt{2gh}$

$x'(0) = 0$

$\Sigma F = m\ddot{x}'$

$mg - kx' = m\ddot{x}'$

$m\ddot{x}' + kx' = mg$

Solution is in form

$x' = A \sin \omega t + B \cos \omega t + \frac{mg}{k}$, where $\omega = \sqrt{\frac{k}{m}}$

$\rightarrow \dot{x}' = \omega A \cos \omega t - B\omega \sin \omega t$ (from EOM)

$\rightarrow$ Substituting initial condition

$x'(0) = 0 = A \sin 0 + B \cos 0 + \frac{mg}{k} \rightarrow B = -\frac{mg}{k}$

$\dot{x}'(0) = \sqrt{2gh} = \omega A \cos 0 - \omega B \sin 0 \rightarrow A = \dfrac{\sqrt{2gh}}{\omega}$

$= \sqrt{\dfrac{2mgh}{k}}$

$\therefore x' = \sqrt{\dfrac{2mgh}{k}} \sin \omega t - \frac{mg}{k} \cos \omega t + \frac{mg}{k}$

$\ddot{x}' = -\omega^2 \sqrt{\dfrac{2mgh}{k}} \sin \omega t + \omega^2 \frac{mg}{k} \cos \omega t$

$= -\sqrt{2gh\frac{k}{m}} \sin \omega t + g \cos \omega t$

Amplitude of $\ddot{x}' = \sqrt{2gh\frac{k}{m} + g^2}$

b) (i) Vase

$m\ddot{x} = -k(x - a\theta)$

$\Leftrightarrow m\ddot{x} + kx - ka\theta = 0$

Plate $\Sigma M_0 = J_0 \ddot{\theta}$

$-k(a\theta - x)a - K(b\theta - y)b = J_0 \ddot{\theta}$

$\Leftrightarrow J_0 \ddot{\theta} + (ka^2 + Kb^2)\theta - kax = Kby$

Let $x = X \sin \omega t$

$\theta = \theta_m \sin \omega t$

(4) b) (i) Continued

$$\begin{bmatrix} k-m\omega^2 & -ka \\ -ka & ka^2+Kb^2-J_0\omega^2 \end{bmatrix}\begin{bmatrix} X \\ \theta_m \end{bmatrix}\cancel{\sin\omega t} = \begin{bmatrix} 0 \\ KbY \end{bmatrix}\cancel{\sin\omega t}$$

$$X = \frac{\begin{vmatrix} 0 & -ka \\ KbY & ka^2+Kb^2-\omega^2 J_0 \end{vmatrix}}{\begin{vmatrix} k-m\omega^2 & -ka \\ -ka & ka^2+Kb^2-J_0\omega^2 \end{vmatrix}} = \frac{KkabY}{(k-m\omega^2)(ka^2+Kb^2-\omega^2J_0)-k^2a^2}$$

(ii) $y = 0$

$$\rightarrow \begin{bmatrix} k-m\omega^2 & -ka \\ -ka & ka^2+Kb^2-J_0\omega^2 \end{bmatrix}\begin{bmatrix} X \\ \theta_m \end{bmatrix}\cancel{\sin\omega t} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\cancel{\sin\omega t}$$

$K = k$, $J_0 = ma^2$

$$\begin{bmatrix} k-m\omega^2 & -ka \\ -ka & ka^2+kb^2-ma^2\omega^2 \end{bmatrix}\begin{bmatrix} X \\ \theta_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} k-m\omega^2 & -ka \\ -ka & ka^2+kb^2-ma^2\omega^2 \end{vmatrix} = 0$$

$$(k-m\omega^2)(ka^2+kb^2-ma^2\omega^2) - k^2a^2 = 0$$

$$\cancel{k^2a^2} + k^2b^2 - kma^2\omega^2 - kma^2\omega^2 - kmb^2\omega^2 + m^2a^2\omega^4 - \cancel{k^2a^2} = 0$$

$$m^2a^2\omega^4 - (2kma^2 + kmb^2)\omega^2 + k^2b^2 = 0$$

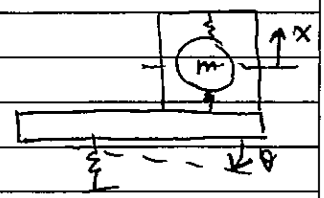
$$\omega^2 = \frac{(2kma^2+kmb^2) \pm \sqrt{(2kma^2+kmb^2)^2 - 4m^2a^2k^2b^2}}{2m^2a^2}$$

$$= \frac{k}{m}\left[1 + \frac{b^2}{2a^2} \pm \sqrt{1+\frac{b^4}{4a^4}}\right]$$

take (+) for ω_1, (−) for ω_2

From eq. 1:

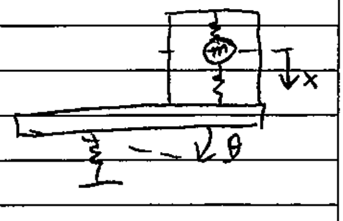
$$\frac{\theta_m}{X} = \frac{k-m\omega^2}{ka}$$



At ω_1:

$$\frac{\theta_m}{X} = \frac{k - k\left[1+\frac{b^2}{2a^2}+\sqrt{1+\frac{b^4}{4a^4}}\right]}{ka}$$

$$= -\frac{1}{a}\left[\frac{b^2}{2a^2} + \sqrt{1+\frac{b^4}{4a^4}}\right]$$



At ω_2:

$$\frac{\theta_m}{X} = \frac{k - k\left[1+\frac{b^2}{2a^2}-\sqrt{1+b^4/4a^4}\right]}{ka}$$

$$= \frac{1}{a}\left[\sqrt{1+\frac{b^4}{4a^4}} - \frac{b^2}{2a^2}\right]$$

P(3)

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2018-2019

MA3002– SOLID MECHANICS AND VIBRATION

April/May 2019 | Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SEVEN (7)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN-BOOK** examination. One double-sided A4 size reference sheet of paper is allowed.

1(a) A spring loaded lever mechanism has a horizontal force, F, applied at B, and is in equilibrium with the spring attached as shown in Figure 1. The rigid bar, ACB, has an angle, θ, relative to the horizontal plane at A. The spring CD, is connected to the rigid bar at C from a roller support at D. The linear spring has a stiffness of k. When the load, F, is removed, the spring will return the mechanism back to its initial position on the vertical plane where $\theta = 90°$.

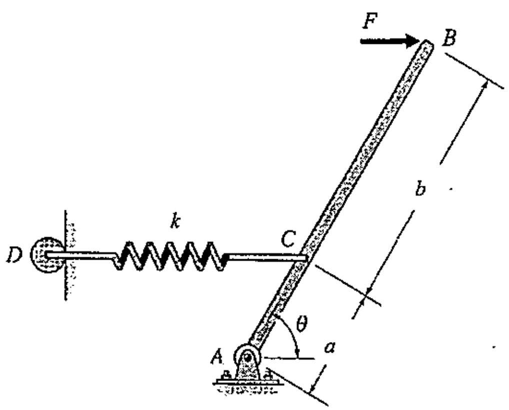


Figure 1

Note: Question 1 continues on page 2.

(i) Determine the relationship between the applied force (F) with respect to the spring stiffness (k), angle (θ), the bar dimension, a, and b. Use the Principle of Virtual Work (PVW) method. Draw and specify your datum. Neglect friction and self-weight of the bars.

(4 marks)

(ii) Calculate the applied force F in Newton (N), if the angle, $\theta = 45°$, the spring stiffness, $k = 50$ N/m. The bar dimension, $a = 1.0$ m, and $b = 2.0$ m respectively.

(2 marks)

(b) A welded beam ABC, has a horizontal segment of length, L, connected to a curved segment with radius R. The beam is fully fixed at point C, and has a vertical load P, applied at point A as shown in Figure 2. The deflection and rotation at any point of the beam can be calculated using the Unit Load and Unit Moment method.

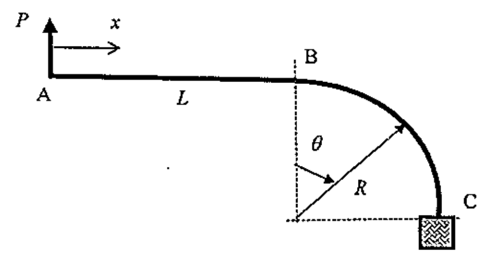


Figure 2

(i) Use the Unit Load method to determine the Vertical Deflection at A. Draw the Real Load diagram and Virtual Load diagram and write down the bending moment expressions for AB and BC segments respectively. Show your datum and moment sign convention.

(6 marks)

(ii) Determine the Vertical Deflection expression at point A using the Unit Load method. Consider only bending effects where the flexural rigidity of the beam is given by EI.

(6 marks)

(iii) Calculate the Vertical Deflection at A, given, $P = 100$ N; $L = 0.75$ m; $R = 0.5$ m, and $EI = 21{,}000$ Nm^2.

(2 marks)

(iv) Determine the Rotation expression at point B, using the Unit Moment method. Consider only bending effects where the flexural rigidity of the beam is EI. Calculate the angle of rotation at B, given, $P = 100$ N; $L = 0.75$ m; $R = 0.5$ m, and $EI = 21{,}000$ Nm^2.

(5 marks)

2(a) A Centre-Crack in an infinite width plate has a tensile stress normal to the crack indicated by σ, as shown in Figure 3. The crack length is $2a$, but the half crack length, a, will be used for Fracture and Fatigue calculations.

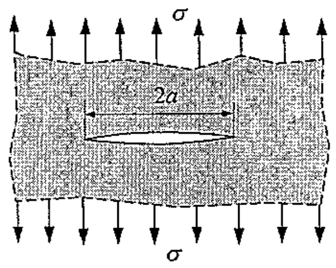


Figure 3

(i) Calculate the Stress Intensity Factor, K_I, if the half crack length, a = 0.01m, and the applied tensile stress, σ = 150MN/m^2.

(2 marks)

(ii) Calculate the critical half crack length, a_c, due to brittle fracture, if the plane strain fracture toughness, K_{IC} = 51 MN/m$^{3/2}$.

(2 marks)

(iii) Calculate the crack propagation life, N_f, if the half crack length propagates from an initial half crack length of a_o= 0.01m to a final half crack length (a_f) due to brittle fracture. The fatigue stress level is from σ_{min} = 0 MN/m^2 to σ_{max} = 150 MN/m^2 with a fatigue stress range of S_R = 150 MN/m^2.

The crack propagation life, N_f, expression is given in the equation below, where C = 2.4 x 10^{-12} (m/cycle); m = 3.0; a_o is an initial half crack length; and a_f is the final half crack length at brittle fracture.

$$N_f = \frac{2}{C(YS_R)^m \pi^{m/2}(2-m)}\left(a_f^{1-m/2} - a_0^{1-m/2}\right)$$

(5 marks)

(iv) Calculate a new initial half crack length, a_o, if the crack propagation life, N_f= 500,000 cycles. The fatigue stress range, S_R = 150 MN/m^2, remains the same.

(5 marks)

Note: Question no. 2 continues on page 4.

(v) Calculate the allowable fatigue stress range, S_R, to give a fatigue design life of 1 million cycles (1,000,000 cycles), for the initial half crack length of a_o = 0.01 m to a final half crack length (a_f) due to brittle fracture.

(5 marks)

(b) A structure is subjected to three blocks of repeated variable amplitude fatigue stress loading. The structure is operated for 8 hours per day. Table 1 provides the stress range (S_R), number of stress range cycles per day (n_i) and the S-N curve fatigue life cycles (N_i).

Table 1

S_R (Stress Range)	n_i (Load cycles)	N_i (Fatigue life)
400 MPa	2	50,000
300 MPa	8	400,000
200 MPa	20	2,000,000

(i) Calculate the cumulative damage summation ratio, D, using Miner's Law, with the data given in Table 1.

(2 marks)

(ii) Calculate the fatigue life of the structure as a number of years of service, before failure occurs when the cumulative damage ratio, D = 1.0.

(2 marks)

(iii) If the structure has to operate longer for 12 hours per day, what will be the expected fatigue life as a number of years of service? State your assumptions.

(2 marks)

3(a) Figure 4(a) shows a springboard in a swimming pool, which has a mass of 12 kg but remains straight with negligible deflection due to its self-weight. When the tip of the spring board is deflected downward by 0.1m and released, the spring board executes damped oscillations as shown in Figure 4(b).

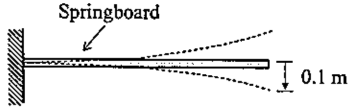


Figure 4(a)

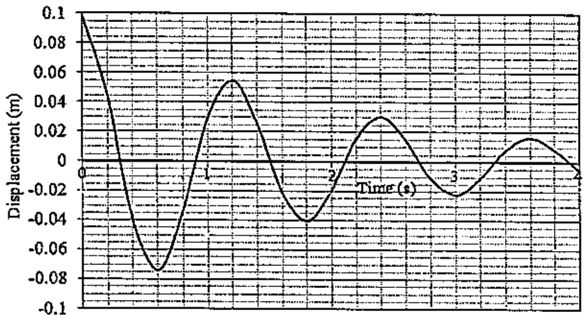


Figure 4(b)

(i) Determine the damping ratio (ζ) and natural frequency (ω_n) extracting necessary data for this calculation from the graph in Figure 4(b).

(4 marks)

(ii) The springboard is to be modelled as a spring-mass-damper system by lumping the effective mass of the springboard at its tip. Take the effective mass of the springboard as 1/4[th] of its total mass. Using your answers in part (i), determine the spring stiffness and damping constant for the spring-mass-damper system, and thereby write the equation of motion for the free vibration of the system.

(3 marks)

Note: Question 3 continues on page 6.

(iii) A diver of mass 70 kg drops from a height of 0.4 m above the tip of the springboard. As soon as he hits the tip of the springboard, he clings on to the tip of the springboard as shown in Figure 4(c). This results in damped oscillations of the springboard with the diver clinging on at the tip. Modelling the problem as a spring-mass-damper system, derive the equation of motion and identify the initial conditions. (You are not required to obtain the solution to the equation of motion.) Also determine the period of damped oscillations of the springboard with the diver clinging on at the tip.

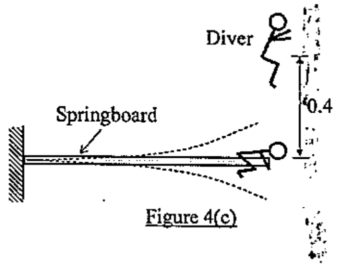


Figure 4(c)

(8 marks)

(b) Figure 5 shows a T-shaped frame hinged at A and connected to two springs of equal stiffness k and a damper of damping coefficient c as illustrated. The T-shaped frame is made of two uniform rigid bars of equal length L and equal mass M. In the position shown, the springs are free (unstrained).

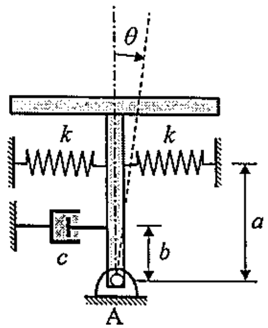


Figure 5

Derive the equation of motion for the system in terms of the coordinate θ shown in Figure 5, and thereby determine the natural frequency of the system (ω_n) in terms of a, k, M and L. Show your derivations clearly by drawing appropriate free body diagrams.

(10 marks)

36

4. Figure 6 shows a vibration absorber mounted on a reciprocating engine. The vibration absorber consists of a mass m_2 and two springs of same stiffness k_2. The casing of the vibration absorber is rigidly fixed to the engine as shown. The reciprocating engine is modelled as spring-mass system with a mass of M supported on two springs of total stiffness of k_1 as shown. The engine has an unbalanced mass of m_1. The total mass M includes m_1. The rotational speed of the engine is ω rad/s. The displacements x_1 and x_2 shown are measured from the respective static equilibrium positions of masses.

Note that the casing of the vibration absorber is rigidly fixed to the engine. Hence, at any instant of time, if engine mass M moves by a distance x_1, the casing of the vibration absorber also moves by the same distance x_1.

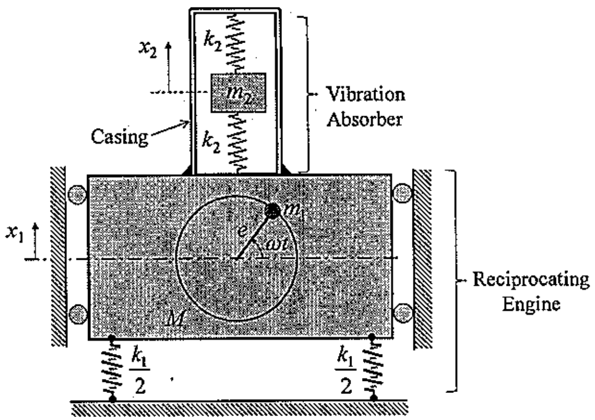


Figure 6

(a) Derive the equations of motion for this 2-DOF system by drawing neat free body diagrams of mass M and m_2, and applying Newton's 2nd law.

(10 marks)

(b) Taking $M = 200$ kg, $k_1 = 10$ kN/m, $m_1 = 1$ kg, $e = 0.1$ m, $m_2 = 2$ kg and $k_2 = 100$ N/m, determine the natural frequencies (ω_1, ω_2) of the system. You may not need all the numerical values given for your calculation.

(6 marks)

(c) Using the same numerical values given in part (b), determine the vibration amplitudes of engine mass (M) and absorber mass (m_1) at the excitation frequency $\omega = 7.5$ rad/s.

(6 marks)

(d) Using the same numerical values given in part (b), determine the values of excitation frequency (ω) at which the vibration amplitude of the engine mass (M) becomes zero.

(3 marks)

End of Paper

MA3002 SOLID MECHANICS & VIBRATION

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

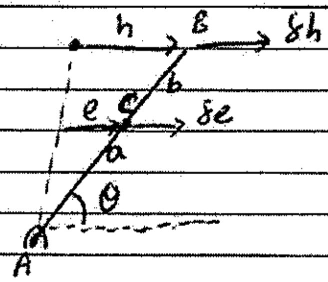
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.

Q1 a i) PVW, $\delta W = \delta U$ (spring)

$F\delta h = (k)(e)(\delta e)$ —— ①



$h = (a+b)\cos\theta$

$\delta h = -(a+b)\sin\theta\ \delta\theta$

$e = a\cos\theta$

$\delta e = -a\sin\theta\ \delta\theta$

Sub δe, e, δh into ①, $-(a+b)\sin\theta\,(F) = (k)(a\cos\theta)(-a\sin\theta)$

$\delta\theta$ is abitary, ignore.

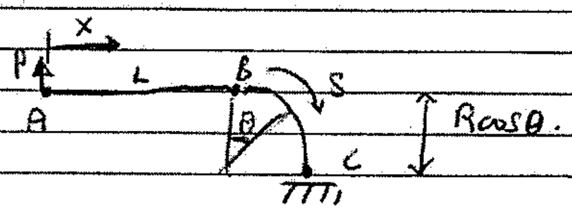
$$* \quad F = \frac{(k)(a\cos\theta)(a)}{a+b} \quad \#.$$

ii) Sub in values provided, $F = 11.79\,N$ #.

b i) Real Load:

$M_{AB} = Px$

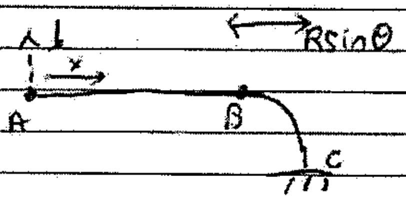
$M_{BC} = P(L + R\sin\theta)$



vir Load:

$m_{AB} = (1)x$

$m_{BC} = (1)(L + R\sin\theta)$



b ii) $ds = Rd\theta$, $\displaystyle\int_{AB} \frac{Mm}{EI}\,dx + \int_{BC}\frac{Mm}{EI}\,dS = \cancel{0}\ 1\cdot(\Delta A)_{vertical}$

$$\int_0^L \frac{(Px)(1x)}{EI}\,dx + \int_0^{\pi/2} \frac{P(L+R\sin\theta)(1)(L+R\sin\theta)R\,d\theta}{EI} = \cancel{0}\ 1\cdot(\Delta A)_{vert}$$

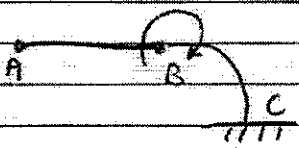
con't

1b ii) $\frac{P}{EI}\left[\frac{L^3}{3} + RL^2(\pi/2) + 2LR^2 + \pi/4(R^3)\right]$ #.

, $\int_0^{\pi/2}\sin\theta\,d\theta = 1$, $\int_0^{\pi/2}\sin^2\theta = \pi/4$

1b iii) Sub in values , vertical defl = 5.027 mm #.

1b iv) unit moment at B.



$M_{AB} = 0$
$M_{BC} = 1$

$1.\theta_B = \int_{AB}\frac{Mm}{EI}dx + \int_{BC}\frac{Mm}{EI}dS.$

$= 0 + \int_0^{\pi/2}\frac{P(L+R\sin\theta)(1)R\,d\theta}{EI}.$

$= (P/EI)(R)(\pi/2\,L + R)$ #.

Sub in values , $1.\theta_B = 3.995\times10^{-3}$ ° #.

Q2a i) $K = Y\sigma(\pi a)^{1/2}$
$= (1)(150\times10^6)(\pi)^{1/2}(0.01)^{1/2} = 26.59\times10^6\ Pa\sqrt{m}$ #.

ii) $a_c = \left(K/(Y\times\sigma\times\pi^{1/2})\right)^2$
$= \left[(51\times10^6)/(1)(150\times10^6)(\pi^{1/2})\right]^2 = 0.0368\ m$ #.

iii) $a_f = (K/Y\sigma)^2/\pi =$ ____ , $a_c = \underline{0.0368} = a_f$. #.
Sub in given values.
$N_f =$ ____

iv) Using a_f from (iii), sub into given formulae: N_f,
rearrange, $\left[\left[\frac{-N_f C(Y S_R)^m \pi^{m/2}(2-m)}{2}\right] + a_f^{1-m/2}\right]^{1/(1-m/2)} = a_0$. #.

av) Given $N_f = 1\times10^6$

Rearrange formula : $S_R = \left[\dfrac{2}{C Y^m N_f \pi^{m/2}(2-m)}\left(a_f^{1-m/2} - a_0^{1-m/2}\right)\right]^{1/m}$ #.

$=$ ______

2bi) Miner rule : $n_1/N_1 + n_2/N_2 + n_3/N_3 = 1$

$D = 2/50000 + 8/400\,000 + 20/2\times10^6$

$= 7\times10^{-5}$ per day.

2bii) $1/7\times10^{-5} = 14285.71$ days.

$= 14285.71/365$

$= 39.14$ years. $\approx$ 39 years.

2biii) 12hrs > 8hrs, the years of service will reduce.

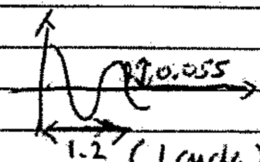
$7\times10^{-5} \times 12/8 = 1.05\times10^{-4}$

$1/1.05\times10^{-4} = 9523.81$ days. = 26 years.

Assumption : ① Same Environment, ~~Pre~~ acting constant.

3 a) i) $f = \frac{1}{\tau}$, $\omega_n = (2\pi)f = (2\pi)(\frac{1}{\tau})$

$= (2\pi)(1/1.2) = 5.24$



(1 cycle).

$\delta = \ln(x_0/x_1)$, $\dfrac{x_0}{x_n} = \left(\dfrac{x_0}{x_1}\right)^n$

$= \ln(0.1/0.055) = 0.598$

$\zeta = \dfrac{\delta}{\sqrt{4\pi^2+\delta^2}} = 0.0947$ #.

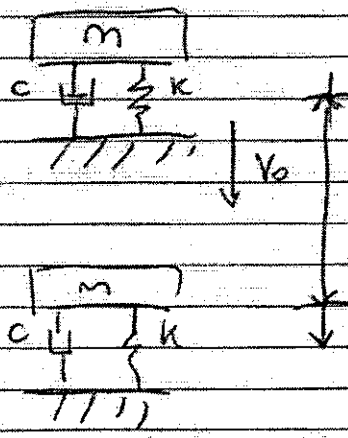
ii) $k = \omega_n^2 (m)$, $C_c = 2m\omega_n$

$C = 4(C_c) = 4(2m\omega_n)$

Let $m = 12$,

EOM => $(\frac{1}{4}m)\ddot{x} + c\dot{x} + kx = 0$ #.

3aiii) This can be modelled as drop test with Damping.



EOM: $m\ddot{x} + c\dot{x} + kx = mg \quad t \geq 0$

Similar to tutorial 11, Q5

Static Analysis before impact, $k\Delta = mg$

$x(t)$ EOM After impact:

initial condition ①: $x(0) = 0$

② : $\dot{x}(0) = v_0 = \sqrt{2gh} = \sqrt{2(9.81)(0.4)} = 2.801$ m/s.

$\zeta = 1 \quad x(t) = (C_1 + C_2 t)e^{\wedge}(-\omega_n t) + mg/k$

use I.C to determine C_1 & C_2.

Period $T_d = 2\pi/\omega_d$

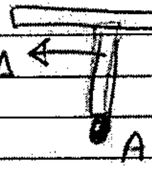
Find ω_d, $\omega_d = \sqrt{k/m}$

$T = 2\pi/(\sqrt{k/m})$

4/8

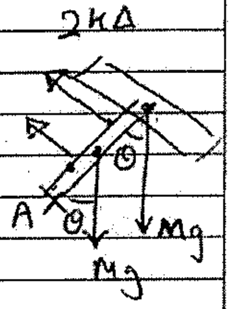
3b) Static Analysis: $\Sigma M_A = 0$ ⟲

$0 = 0$ (redundant.) $2k\Delta$ (unrestrain)



Dynamic Analysis: ⟲ $J_A \ddot{\theta} = \Sigma M_A$ ⟲

⟲ $J_A = \frac{ML^2}{3} + \left[\frac{ML^2}{12} + ML^2\right] = \frac{17}{12} ML^2$

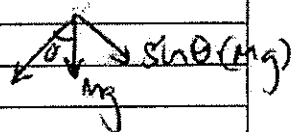


⟲ $M_A = Mg\frac{L}{2}\sin\theta + MgL\sin\theta$ (weight of Frame)

$+ (-)\ 2K\, a\sin\theta\ a\cos\theta$ (2 Spring)

$+ (-)\ c\ (b\cos\theta)\dot{\theta}\ (b\cos\theta)$

$c\frac{d}{dt}(b\sin\theta)$
$= c \cdot b\cos\theta \cdot \dot{\theta}$



$\sin\theta \approx \theta$, $\cos\theta \approx 1$ (small approx)

EOM:

$$\left(\frac{17}{12} ML^2\right)\ddot{\theta} + \left(cb^2\right)\dot{\theta} + \left(2ka^2 - \frac{3MgL}{2}\right)\theta = 0.$$

#.

$\omega_n = \sqrt{\frac{K_\theta}{J_o}} = \sqrt{\frac{2ka^2 - 3/2\ MgL}{17/12\ ML^2}}$ #.

DISCLAIMER: *The solutions are done by students who scored A or above in this subject. MAE Club and Campus supplies are not liable or responsible for any errors in the contents of these solutions. Students are advised to take the solutions as a guide rather than absolute answers to exam paper.*

Should there be any mistake identified, please proceed to the Facebook link encoded in the QR code to feedback or submit correct answers. The link is: http://bit.ly/2IW2C32

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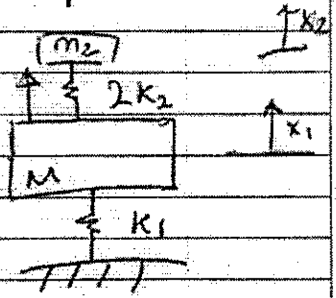
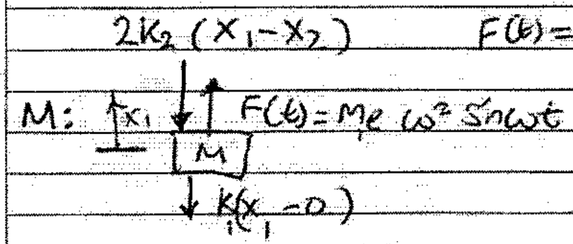
4 a) Assume $x_1 > x_2$ for M Rearrange Fig 6. equivalent.

$x_2 > x_1$ for m_2



$F(t) = m_1 e\omega^2 \sin\omega t$

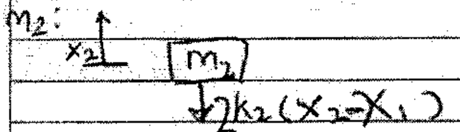


FBD ① ⟨Forced vibration of 2DOF sys⟩

$$M\ddot{x}_1 = \Sigma F_x$$

$$M\ddot{x}_1 = -k_1(x_1 - 0) + m_1 e\omega^2 \sin\omega t - 2k_2(x_1 - x_2)$$

$$M\ddot{x}_1 + k_1 x_1 + 2k_2(x_1 - x_2) - m_1 e\omega^2 \sin\omega t = 0 \quad (\text{EOM } ①)$$



$$m_2\ddot{x}_2 + 2k_2(x_2 - x_1) = 0 \quad (\text{EOM } ②)$$

b) EOM in matrix form

$$\begin{bmatrix} M & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + 2k_2 & -2k_2 \\ -2k_2 & 2k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} m_1 e\omega^2 \sin\omega t \\ 0 \end{Bmatrix}$$

$\ddot{x}_1 = -\omega^2 X_1 \sin\omega t$, $x_1 = X_1 \sin\omega t$

$\ddot{x}_2 = -\omega^2 X_2 \sin\omega t$, $x_2 = X_2 \sin\omega t$.

Sub into matrix & cancel out $\sin\omega t$ (common factor)

$$-\omega^2 \begin{bmatrix} M & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{bmatrix} k_1 + 2k_2 & -2k_2 \\ -2k_2 & 2k_2 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} m e\omega^2 \\ 0 \end{Bmatrix}$$

4b) con't

$$\begin{bmatrix} K_1+2K_2-M\omega^2 & -2K_2 \\ -2K_2 & 2K_2-M_2\omega^2 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} M_1 e \omega^2 \\ 0 \end{Bmatrix}$$

<~~Cramer's rule~~> <To find ω_{n1} & 2, take as free vibration analysis>

$$\begin{array}{cc} a & b \\ \end{array}$$

$$\det \begin{bmatrix} K_1+2K_2-M\omega^2 & -2K_2 \\ -2K_2 & 2K_2-M_2\omega^2 \end{bmatrix} = 0.$$

$$\begin{array}{cc} c & d. \end{array}$$

$(ad-bc) \Rightarrow$ ....

Sub in given values provided to obtain answer!

4c) <Forced vibration>

using cramer's rule, to calculate X_1 for M vibration amplitude. X_2 for M_2

Sub $\omega = 7.5$ rad/s. and given values in (4b)

$$X_1 = \frac{\det \begin{bmatrix} M_1 e\omega^2 & -2K_2 \\ 0 & 2K_2-M_2\omega^2 \end{bmatrix}}{\det \begin{bmatrix} 2K_2+K_1-M\omega^2 & -2K_2 \\ -2K_2 & 2K_2-M_2\omega^2 \end{bmatrix}}$$

$$X_2 = \frac{\det \begin{bmatrix} K_1+2K_2-M\omega^2 & M_1 e\omega^2 \\ -2K_2 & 0 \end{bmatrix}}{\det \begin{bmatrix} 2K_2+K_1-M\omega^2 & -2K_2 \\ -2K_2 & 2K_2-M_2\omega^2 \end{bmatrix}}$$

Refer notes on chap 8, Ex 8.2.

4d) Sub $X_1 = 0$ and given values on (4b)

$$0 = \frac{\det\begin{bmatrix} M_1 e\omega^2 & -2k_2 \\ 0 & 2k_2 - m_2\omega^2 \end{bmatrix}}{\det\begin{bmatrix} 2k_2 + k_1 - M\omega^2 & -2k_2 \\ -2k_2 & 2k_2 - m_2\omega^2 \end{bmatrix}}$$

Find ω as given answer

∴ Tips to score well!

1. Practice more PYP & tutorial Question, somewhat similar Question can be seen.
2. Draw FBD and equations to get as many points as possible.
3. Do Ask if you have Questions.
4. Simplify all complicated Figures to have a better understanding.

Good luck, All the Best :)

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

MA3002– SOLID MECHANICS AND VIBRATION

November/December 2019 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR** (4) questions and comprises **EIGHT** (8) pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED-OPEN BOOK** examination. One double-sided A4 size reference sheet of paper is allowed.

1(a) A spring loaded mechanism is made up of rigid bars AB, BC and CD pin jointed and subject to a vertical force, P, applied at point C. The mechanism is in equilibrium with the two springs shown in Figure 1. The rigid bars have uniform length of L. The mechanism has an angle, θ, relative to the horizontal surface. The two linear spring stiffness are indicated as k_1 and k_2. When the load, P, is removed, the springs will return the mechanism back to its initial position on the vertical plane when $\theta = 90°$.

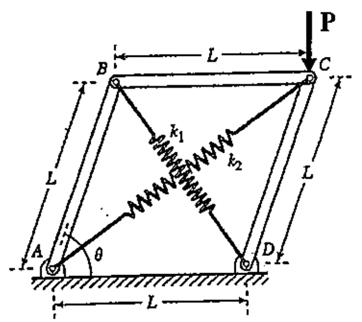


Figure 1

(i) Determine the relationship between the applied force (P) with respect to the spring stiffness k_1 and k_2, angle (θ), and the rigid bar length L. Use the Principle of Virtual Work (PVW) method. Draw and specify your datum. Neglect friction and self-weight of the bars.

(4 marks)

Note: Question 1 continues on page 2.

(ii) Calculate the applied force P in newton (N), if the bar dimension L = 1.0 m, the angle, $\theta = 60°$, the spring stiffness, k_1 = 1000 N/m and k_2 = 1000 N/m respectively.

(2 marks)

(b) A cantilevered curved beam (A-B-C-D) is subjected to a load, W, at the free end (D) as shown in Figure 2. Using the Unit Load method (consider only bending effects and state clearly your datum), determine the following:

(i) Vertical Displacement expression of the free end at D.
(ii) Horizontal Displacement expression of the free end at D.

Draw your REAL and VIRTUAL load diagrams and specify the relevant Bending Moment equations to be used. Calculate the Vertical and Horizontal Displacements in millimeters (mm) if the Point Load P = 100N, R = 1.0 m, and EI = 21,000 N/m^2.

(6 marks)

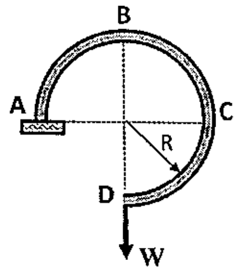


Figure 2

(c) A statically indeterminate structure is in a form of a square steel member is subjected to symmetrical Point Loads, P, as shown in Figure 3. A quarter symmetry segment is indicated by the position of A-B-C. The length of the square edge is given as L.

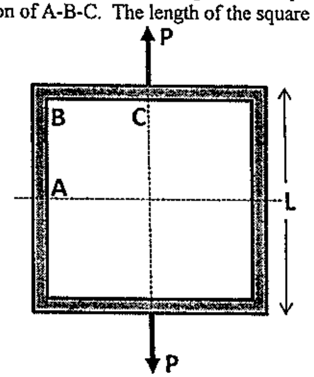


Figure 3

Note: Question 1 continues on page 3.

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(i) Draw the free body diagram for a Quarter Symmetry segment A-B-C, and indicate the equilibrium forces and moments in a free body diagram.
(4 marks)

(ii) Show that this problem is Statically Indeterminate to what degree of Indeterminacy.
(2 marks)

(iii) Determine the Vertical Displacement expression at the Load Point, C, for the Quarter Symmetry segment A-B-C using the Unit Load and Unit Moment methods. Consider bending effects only. Neglect friction and self-weight of the bars.
(5 marks)

(iv) Calculate the Vertical Displacement at the Load Point, C, for the Quarter Symmetry segment ABC in millimeters (mm) if the Point Load P = 1000N, L = 1.0 m, and EI = 26,000 N/m^2
(2 marks)

2(a) A Plane Strain fracture toughness test was conducted using a Compact Tension (CT) Specimen (with a thickness of B = 50mm) and the test data is recorded in Table 1.

Table 1: Compact Tension (CT) Specimen Test Data

P_{max}	= 170 kN	$K_Q = \frac{P_Q}{BW^{1/2}} f_2\left(\frac{a}{W}\right)$
P_Q	= 160 kN	
B	= 50mm (Thickness)	
a	= 50mm (Crack Length)	$f_2\left(\frac{a}{W}\right) = 9.6$
W	= 100mm (Width)	

(i) Calculate the Qualifying Stress Intensity Factor, K_Q value from the test data given in Table 1.
(4 marks)

(ii) Determine if the CT specimen test result meet the Plane Strain Fracture Toughness three-pronged checking criteria. Conclude if, K_Q is a valid K_{IC} (plane strain fracture toughness) test result?
(4 marks)

Note: Question 2 continues on page 4.

(b) An edge crack in an infinite width plate has a flaw in the form of an edge crack of length, a, as shown in Figure 4. The geometric factor, Y = 1.12, can be used in calculating the Stress Intensity Factor, solution.

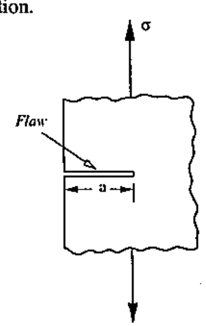

Figure 4

(i) Calculate the applied stress intensity factor, K_I, if the crack length, a = 1.0 mm (or 0.001m), and the applies stress, σ = 250 MN/m^2.
(2 marks)

(ii) Calculate the flaw size or critical crack length, a_{cr}, due to brittle fracture, if the plane strain fracture toughness of the material, K_{IC} = 56 MN/m$^{3/2}$.
(2 marks)

(iii) Calculate the crack propagation life, N_f, for a cyclic stress varying from zero to 250 MN/m^2 giving a stress range, S_R = 250 MN/m^2. The initial crack length, a_o = 1.0 mm (or 0.001m) and the final critical crack length a_f can be obtained from part (ii). The fatigue crack propagation life expression and the Paris Law constants are given below.

$$N_f = \frac{2}{C(YS_R)^m \pi^{m/2}(2-m)}\left(a_f^{1-m/2} - a_0^{1-m/2}\right)$$

where, $C = 4 \times 10^{-12}$ (m/cycle); $m = 3.3$
(6 marks)

Note: Question 2 continues on page 5.

(c) A steel specimen was tested under fatigue stress range conditions and the fatigue S-N curve test data is given below in Table 2.

Table 2: Fatigue S-N Curve Test Data

Test Specimen No.1
Stress Range : $S_R = 200$ MN/m^2.
Fatigue Life : $N_f = 900{,}000$ cycles

Test Specimen No.2
Stress Range : $S_R = 310$ MN/m^2.
Fatigue Life : $N_f = 12{,}000$ cycles

(i) The steel structure is subjected to two stress range level exposures every day and strain gage measurement show that there are 30 cycles of stress range of $S_R = 200$ MN/m^2 and 3 cycles of stress range of $S_R = 310$ MN/m^2, in one day of loading operation.

Using Miners Rule, calculate the number of years it will take for the structure to fail by fatigue when the Cumulative Damage Summation index is equal to D = 1.0. Use the Fatigue test data provided in Table 2.

(4 marks)

(ii) The S-N Curve equation can be curve-fitted to the fatigue test data when analyzed on a Log Stress versus Log Cycles plot.

Determine the S-N curve equation constants A and b using the fatigue test data from Table 2.

Note: A typical S-N Curve equation has two constants A and –b.

$$S_R = A \cdot (N_f)^{-b}$$

(3 marks)

3(a) It is intended to analyse the vertical oscillations of a small bridge by modelling it as a spring-mass system as shown in Figure 5(a) where k_{eff} is the effective stiffness of the bridge and m_{eff} is the corresponding effective mass. When the bridge is deflected down by imposing a certain vertical displacement at O (the mid-section of the beam) and allowed to oscillate freely, the bridge executes natural oscillations in the vertical direction as indicated. The natural period of vertical oscillations is found to be 0.4 s. When a mass 8000 kg is placed at the mid-section of the bridge at O, the natural period of the oscillation increases to 0.42 s. Determine k_{eff} and m_{eff} of the spring-mass system. Ignore damping effects.

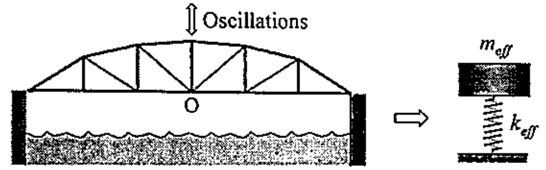

Figure 5(a)

(5 marks)

(b) When a chicken egg of mass $m = 50$ g is placed gently on a block of sponge material, it produces a static deflection of 5 mm as shown in Figure 5(b).

(i) Assuming that the sponge material behaves like a spring, determine the stiffness (k) of the sponge material.

(2 marks)

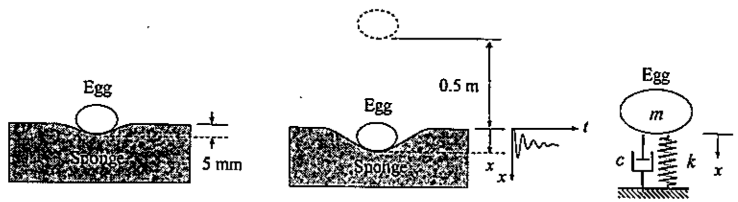

Figure 5(b) Figure 5(c) Spring 5(d)

Note: Question 3 continues on page 7.

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(ii) When the egg is dropped from a height of 0.5 m above the surface of sponge block (as shown in Figure 5(c)), the motion of egg after landing on sponge block is under-damped with successive vibration amplitudes reducing to 0.1 times that of the previous. Assume the egg does not rebound or break during the drop. Ignore the mass of sponge material. Determine the damping ratio, damping coefficient, undamped natural frequency (rad/s) and damped natural frequency (rad/s).
(5 marks)

(iii) As the egg is falling from the height of 0.5 m, the time t is taken as 0 when the egg just touches the sponge material. Using an appropriate free body diagram, derive the equation of motion of egg for time $t > 0$ by modelling the sponge-egg combination as a spring-mass-damper system as shown in Figure 5(d), and determine the initial conditions $x(0)$ and $\dot{x}(0)$.
(4 marks)

(iv) Determine the solution to the equation of motion derived in part (iii).
(7 marks)

(v) What should be the value of damping coefficient of sponge material for which the motion of egg is critically damped? Assume that sponge material has the same stiffness as calculated in part (i).
(2 marks)

4. Figure 6 shows a 2-DOF vibrating system in its static equilibrium configuration. The system consists of a rigid beam ABC of length l and mass m_1, pin-jointed at A and supported by two springs of same stiffness k_1 at the mid-section of the beam at B. The beam carries a spring-mass system of mass m_2 and spring stiffness k_2 at C. Mass m_2 is subjected to a harmonic force excitation of amplitude F_2 at excitation frequency ω.

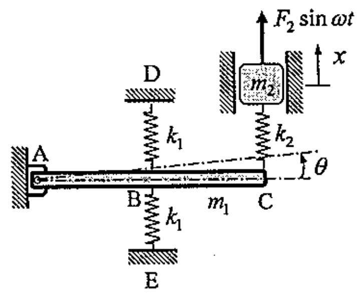

Figure 6

(a) Applying Newton's 2nd law, derive the equations of motion for this 2-DOF system in terms of the coordinates x and θ, and represent them in matrix form. Draw neat free body diagrams to aid your derivation. Static forces need not be shown on the free body diagrams as they are balanced.
(8 marks)

(b) Taking $l = 1$ m, $m_1 = 1$ kg, $k_1 = 1000$ N/m, $m_2 = 1$ kg, $k_2 = 1000$ N/m, determine the natural frequencies (in Hz) and the corresponding amplitude ratios of the system.
(10 marks)

(c) Taking $F_2 = 20$ N, determine the vibration amplitude of the beam (Θ) at the excitation frequency $\omega = 15$ rad/s.
(5 marks)

(d) What is the amplitude of force transmitted to each of the supports at E and D.
(2 marks)

End of Paper

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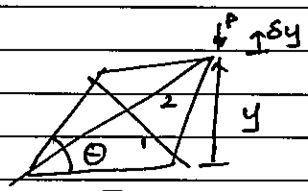
1(a) $\Sigma P \delta \Delta = \Sigma k e \delta e$



$-P\delta y = \Sigma k e \delta e$

Normal length of spring $= \sqrt{2}L$

~~Long~~ $y = L\sin\theta$

~~Length of~~ spring 1 $= -\sqrt{L^2\sin^2\theta + (L - L\cos\theta)^2} + \sqrt{2}L$

length of spring 2 $= \sqrt{L^2\sin^2\theta + (L + L\cos\theta)^2} - \sqrt{2}L$

$\frac{\delta y}{\delta\theta} = L\cos\theta \qquad \delta y = L\cos\theta\,\delta\theta$

$\cancel{\frac{\delta e_1}{\delta\theta} = \left(\frac{1}{2}\right)\frac{1}{\sqrt{L^2\sin^2\theta + (L - L\cos\theta)^2}}(-2L^2\sin}$

$\ell_1 = \sqrt{2}L - \sqrt{L^2\sin^2\theta + L^2 + L^2\cos^2\theta - 2L^2\cos\theta}$

$= \sqrt{2}L - \sqrt{2L^2(1-\cos\theta)}$

$\ell_2 = \sqrt{2L^2(1+\cos\theta)} - \sqrt{2}L$

$\frac{\delta e_1}{\delta\theta} = -\left(\frac{1}{2}\right)\frac{1}{\sqrt{2L^2(1-\cos\theta)}}(2L^2\sin\theta)$

$\frac{\delta e_2}{\delta\theta} = \left(\frac{1}{2}\right)\frac{1}{\sqrt{2L^2(1+\cos\theta)}}(2L^2)(-\sin\theta)$

$\cancel{-}P(L\cos\theta)\cancel{\delta\theta} = k_1 e_1 \delta e_1 + k_2 e_2 \delta e_2$

$= k_1\left(\sqrt{2}L - \sqrt{2L^2(1-\cos\theta)}\right)\left(\cancel{-}\frac{1}{2}\right)\frac{2L^2\sin\theta}{\sqrt{2L^2(1-\cos\theta)}}\cancel{\delta\theta}$

$+ k_2\left(\sqrt{2L^2(1+\cos\theta)} - \sqrt{2}L\right)\left(\frac{1}{2}\right)\frac{1}{\sqrt{2L^2(1+\cos\theta)}}(\cancel{-}\sin\theta\, 2L^2)\cancel{\delta\theta}$

Remove negative sign from every term, remove $\delta\theta$

$P = k_1\left(\sqrt{2}L - \sqrt{2L^2(1-\cos\theta)}\right)\left(\frac{1}{\cancel{2}}\right)\frac{\cancel{2}L^{\cancel{2}}\sin\theta}{\sqrt{2L^2(1-\cos\theta)}} + k_2\left(\sqrt{2L^2(1+\cos\theta)} - \sqrt{2}L\right)\left(\frac{1}{\cancel{2}}\right)\left(\frac{\cancel{2}L^{\cancel{2}}\sin\theta}{\sqrt{2L^2(1+\cos\theta)}}\right)$

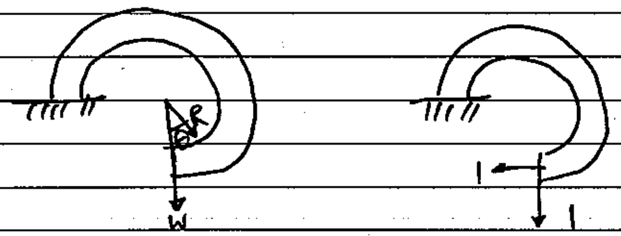
$\cancel{L}\cos\theta$

$= \frac{k_1\left(\sqrt{2}L - \sqrt{2L^2(1-\cos\theta)}\right)L\tan\theta}{\sqrt{2L^2(1-\cos\theta)}} + \frac{k_2\left(\sqrt{2L^2(1+\cos\theta)} - \sqrt{2}L\right)(L\tan\theta)}{\sqrt{2L^2(1+\cos\theta)}}$

b(ii) When $L = 1$ $\theta = 60°$ $k_1 = k_2 = 1000$ sub back into

$P = 1035\text{N}$ (downwards)

1b1)



Real load = $W(R\sin\theta)$

Virtual load (Vertical) = $R\sin\theta$

Virtual load (Hori) = $R - (R - R\cos\theta) = R\cos\theta - R$

Vertical Displacement = $\int_0^{l} \frac{Mm}{EI}\, ds = \int_0^{\frac{3\pi}{2}} \frac{Mm}{EI} R\, d\theta$

$= \int_0^{\frac{3\pi}{2}} \frac{WR\sin\theta(R\sin\theta)R}{EI}\, d\theta$

$= \frac{WR^3}{EI} \int_0^{\frac{3\pi}{2}} \sin^2\theta\, d\theta$ ← use GC dont waste time

$= \frac{WR3}{EI}(2.35619)$ downwards

* Hori displacement = $\int_0^{\frac{3\pi}{2}} \frac{Mm}{EI} R\, d\theta$

$= \frac{WR3}{EI} \int_0^{\frac{3\pi}{2}} \sin\theta(\cos\theta - 1)\, d\theta$

$= \frac{WR3}{EI}(-0.5)$ means rightwards

3b11) when ~~P=1000~~ W=100 R=1 EI= 21000

Vertical = $\frac{100(2.35619)}{21000}$

= 0.011 m (downward)

Hori = 0.00238 (rightward)

let Vertical Displacement

Virtual load (BC) = x

virtual load (AB) = $\frac{L}{2}$

Vertical Disp = $\int_0^{\frac{L}{2}} \frac{\left(\frac{P}{2}(x) - \frac{3PL}{16}\right)x}{EI} + \int_0^{\frac{L}{2}} \frac{\left(\frac{PL}{4} - \frac{3PL}{16}\right)\frac{L}{2}}{EI}$

$= \left[\frac{\frac{P}{2}\left(\frac{x^3}{3}\right) - \frac{3PL(x^2)}{32}}{EI} + \frac{\frac{3}{32}PL(x)}{EI} \right]_0^{\frac{L}{2}}$

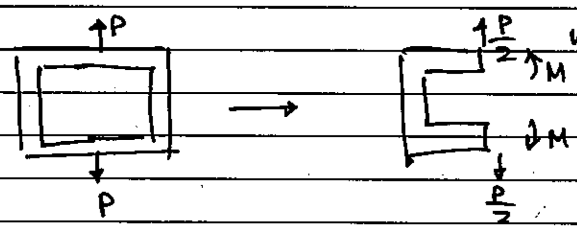
$= \frac{1}{EI}\left(\frac{PL^3}{3\times2\times8} - \frac{3PL^3}{32\times4} + \frac{3}{32}PL^2\left(\frac{1}{2}\right)\right)$

I(iv) when P = 1000 L = 1 EI = 26000

Vertical Disp = 0.0035 m

= 3.5 mm

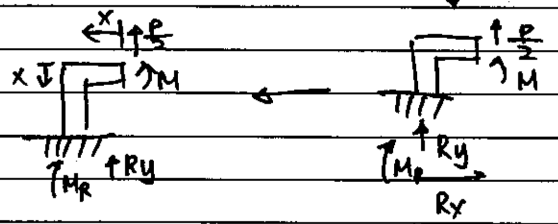
1c)



When no shear, max bending moment so there is bending moment internally

There is

1(i)



Since u cut at point away from P, there is definitely a bending moment M_R.

Rx is intact 0.

1(ii)

There is 3 unknown, M, Ry and MR but only 2 equilibrium equation, ΣF_y and ΣM that can be used. 1 degree of indeterminacy

P is known as it is the force you apply.

1(iii) At shear = 0 max bending moment, slope = 0

Real load (BC) $= \frac{P}{2}(x) + M$

Real load (AB) $= \frac{P}{2}\left(\frac{L}{2}\right) + M$

Virtual moment (BC) = 1

Virtual moment (AB) = 1

$$\Sigma \int_0^{\frac{L}{2}} \frac{Mm}{EI}\,dx = \int_0^{\frac{L}{2}} \frac{\frac{P(x)}{2}+M}{EI}\,dx + \int_0^{\frac{L}{2}} \frac{\frac{PL}{4}+M}{EI}\,dx$$

$$= \frac{1}{EI}\left[\frac{P}{2}\left(\frac{x^2}{2}\right) + Mx + \frac{PL}{4}x + Mx\right]_0^{\frac{L}{2}}$$

$$= \frac{P(L^{\cancel{2}})}{2\times2\times2\times\cancel{2}} + M\left(\frac{L}{2}\right) + \frac{PL}{4}\left(\frac{L}{2}\right) + M\left(\frac{L}{2}\right) = 0 \quad \text{since slope} = 0$$

$$= \frac{PL}{8} + \frac{PL}{4} + 2M = 0$$

$$\frac{3PL}{8} = -2M \qquad M = -\frac{3PL}{16}$$

2a) $K_Q = \dfrac{160 \times 1000}{50\times10^{-3} \times (100\times10^{-3})^{0.5}}$ 9.6

$= 6071573\text{l} \ Pam^{0.5}$

2aii) $\dfrac{a}{W} = \dfrac{50}{100} = 0.5$ $0.45 < \dfrac{a}{W} < 0.55$

$B, a, (W-a) \geq 2.5\left(\dfrac{K_Q}{\sigma_Y}\right)^2$ Sorry they only gave σY in exam so it's not on the paper.

2bi) $K = \sigma(\pi a)^{0.5} Y$

$= 250\times10^6 \times (\pi \times 0.001)^{0.5} \times 1.12$

$= 15693975 \ Pam^{0.5}$

2bii) $56\times10^6 = 1.12(250\times10^6)(\pi a)^{0.5}$

$a = \left(\dfrac{56\times10^6}{1.12\times250\times10^6}\right)^2 \dfrac{1}{\pi}$

$= 0.01273m$

2biii) $N_f = \dfrac{2}{4\times10^{-12}(1.12\times250\times10^6)^{3.3}\pi^{\frac{3.3}{2}}(2-3.3)}\left(0.01273^{1-\frac{3.3}{2}} - 0.001^{1-\frac{3.3}{2}}\right)$

$= 5.5829\times10^{-16}$

2ci) Damage fraction $= \dfrac{30}{900000} + \dfrac{3}{12000} = 2.833\times10^{-4}$

$1/2.833\times10^{-4} = 3529$ days $= 9.914$ years

2cii) $200 = A(900000)^{-b}$

$310 = A(12000)^{-b}$

$\dfrac{200}{310} = \left(\dfrac{900000}{12000}\right)^{-b}$

$\ln\left(\dfrac{200}{310}\right) = -b\ln\left(\dfrac{900000}{12000}\right)$

$-b = -0.1015$

$b = 0.1015$

$A = \dfrac{200}{900000^{-0.1015}} = 804.23$

3a) $\omega_{n_1} = \sqrt{\frac{k_{eff}}{m_{eff}}} = \frac{2\pi}{0.4}$

$\omega_{n_2} = \sqrt{\frac{k_{eff}}{m_{eff}+8000}} = \frac{2\pi}{0.42}$

$$\frac{\omega_{n_1}}{\omega_{n_2}} = \frac{\sqrt{\frac{k_{eff}}{m_{eff}}}}{\sqrt{\frac{k_{eff}}{m_{eff}+8000}}} = \frac{\frac{2\pi}{0.4}}{\frac{2\pi}{0.42}}$$

$$= \sqrt{\frac{m_{eff}+8000}{m_{eff}}} = \frac{0.42}{0.4}$$

$$m_{eff}+8000 = m_{eff}\left(\frac{0.42}{0.4}\right)^2$$

$$m_{eff} = \frac{8000}{\left(\frac{0.42}{0.4}\right)^2 - 1}$$

$$= 78048\ kg$$

$$k_{eff} = \left(\frac{2\pi}{0.4}\right)^2 \times 78048$$

$$= 19257572\ N/m$$

3bi) $F = kx$

$$k = \frac{F}{x} = \frac{0.05\times 9.81}{5\times 10^{-3}} = 98.1\ N/m$$

3bii) $\cancel{\tfrac{1}{2}v^2 = gh}$

$\cancel{v = \sqrt{2gh}}$

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln\left(\frac{x_1}{0.1x_1}\right) = \ln 10 = 2.3025$$

$$\zeta = \sqrt{\frac{2.3025^2}{4\pi^2 + 2.3025^2}} = 0.34407$$

$$c = c_c\zeta = 2\sqrt{km}\,(0.34407)$$

$$= 2\sqrt{98.1\times 0.05}\,(0.34407)$$

$$= 1.524$$

$$\omega_n = \sqrt{\frac{k_{eff}}{m_{eff}}} = \sqrt{\frac{98.1}{0.05}} = 44.294\ rad/s$$

$w_d = w_n \sqrt{1-\zeta^2}$

$= 44.294\sqrt{1-0.34407^2}$

$= 41.589 \text{ rads}^{-1}$

3b(iv)

~~3b(iii)~~) $x = Xe^{-\zeta w_n t} \sin(w_d t + \phi)$

Initial $v = \sqrt{2gh}$

$= \sqrt{2(9.81)(0.5)}$

$= 3.132 \text{ ms}^{-1}$

when $t = 0$ $x = 0$ $\therefore \phi = 0$

$\dot{x} = (-\zeta w_n e^{-\zeta w_n t} \sin(w_d t + \phi)$

$+ w_d \cos(w_d t) e^{-\zeta w_n t}) X$

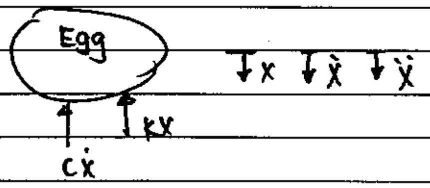
when $t = 0$ $\dot{x} = 3.132$

$\dot{x} = -\zeta w_n (1)(0) + 41.589(1)(1)(X)$

$X = \dfrac{3.132}{41.589} = 0.0753$

$x = 0.0753 e^{-15.457t} \sin(41.589t)$

3b(iii)

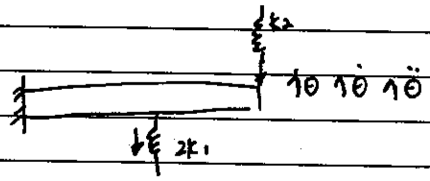


$m\ddot{x} = -kx - c\dot{x}$

$m\ddot{x} + kx + c\dot{x} = 0$

3b(v) $c_c = 2\sqrt{km} = 2\sqrt{98.1 \times 0.05} = 4.429$

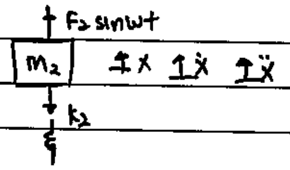
4a)



$2k_1$ spring equivalent to one spring of stiffness $2k_1$

$J_0 = \frac{m_1 l^2}{3}$ $\quad J_0\ddot{\theta} = -k_2(l\sin\theta - x)l - 2k_1\left(\frac{l}{2}\sin\theta\right)\frac{l}{2}$

$\frac{m_1 l^2}{3} + 2k_1\frac{l^2}{4}\theta + k_2(l\theta - x)l = 0$



$m_2\ddot{x} = F_2\sin\omega t - k_2(x - l\sin\theta)$

$m_2\ddot{x} + k_2(x - l\theta) = F_2\sin\omega t$

$$\begin{bmatrix} m_2 & 0 \\ 0 & \frac{m_1 l^2}{3} \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} k_2 & -k_2 l \\ -k_2 l & \frac{2k_1 l^2}{4} + k_2 l^2 \end{bmatrix}\begin{Bmatrix} x \\ \theta \end{Bmatrix} = F_2\sin\omega t$$

$x = X\sin\omega t \quad \theta = \Theta\sin\omega t \quad \ddot{x} = -\omega^2 X\sin\omega t \quad \ddot{\theta} = -\omega^2\Theta\sin\omega t$

$$\begin{bmatrix} k_2 - m_2\omega^2 & -k_2 l \\ -k_2 l & \frac{2k_1 l^2}{4} + k_2 l + \frac{m_1 l^2}{3}(-\omega^2) \end{bmatrix}\begin{Bmatrix} X \\ \Theta \end{Bmatrix} = \begin{bmatrix} F_2 \\ 0 \end{bmatrix}$$

4b) Sub in $k_1 = k_2 = 1000 \quad m_1 = m_2 = l = 1$

$$\begin{bmatrix} 1000 - \omega^2 & -1000 \\ -1000 & 1500 - \frac{\omega^2}{3} \end{bmatrix}\begin{Bmatrix} X \\ \Theta \end{Bmatrix} = \begin{bmatrix} F_2 \\ 0 \end{bmatrix}$$

To find ω_n det of Matrix $= 0$

$$\left(1500 - \frac{\omega^2}{3}\right)(1000 - \omega^2) - 1000000 = 0$$

$$1500000 - 1500\omega^2 - \frac{1000\omega^2}{3} + \frac{\omega^4}{3} - 1000000 = 0$$

Let $x = \omega^2$ and solve as quadratic

$$1500000 - 1500x - \frac{1000x}{3} + \frac{x^2}{3} - 1000000 = 0$$

$x = 287.785$ or 5212.214

$\omega_n = \sqrt{287.785}$ or $\sqrt{5212.214}$ (reject -ve values)

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To find amplitude ratios, make force vector as $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Sub in * w_n into equation and make X/Θ ~~as~~ or Θ/X as subject

$$\begin{bmatrix} 1000-w^2 & -1000 \\ -1000 & 1500-\frac{w^2}{3} \end{bmatrix} \begin{Bmatrix} X \\ \Theta \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(1000-287.785)X - 1000\Theta = 0$$

$$\frac{\Theta}{X} = \frac{1000-287.785}{1000} = 0.712$$

or

$$\frac{\Theta}{X} = \frac{1000-5212.214}{1000} = -4.212$$

4c) $\Theta = \dfrac{\det \begin{bmatrix} \cancel{20}\ 775 & \cancel{-1000}\ 20 \\ 0\ -1000 & \cancel{1500-\frac{15^2}{3}}\ 0 \end{bmatrix}}{\det \begin{bmatrix} 1000-15^2 & -1000 \\ -1000 & 1500-\frac{15^2}{3} \end{bmatrix}}$ Use GC

$= 0.\cancel{233}\ 1916$ rad

4d) $F = \frac{kl}{2} \sin\Theta$

$= 1000\left(\frac{1}{2}\right) \sin 0.1916$

$= 95.21 N$

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NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2020-2021

MA3002 – SOLID MECHANICS AND VIBRATION

November/December 2020 Time Allowed: 1 hour

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.

2. Answer **ALL** questions.

3. All questions carry equal marks.

4. This is an **OPEN BOOK E-EXAMINATION**. You are allowed to refer to hard copies of lecture and tutorial materials.

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1. The frame shown in Figure 1 is made of two segments: segment AB of length 5 m and BC of length 4 m. The flexural stiffness $EI = 10^5$ Nm^2 is constant for both segments. If the frame is subjected to point load $F_1 = 100$ N and $F_2 = 100$ N as shown, determine the rotation at point C (in rad). Use unit load method. Consider only bending effects.

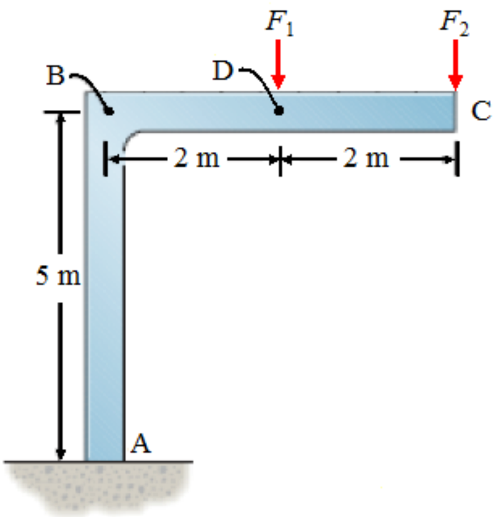


Figure 1

(25 marks)

2. The wall of the pressure vessel shown in Figure 2 is subjected to cyclic pressure loading. The stress engineer designs the wall to have a maximum stress of 1/4 of the yield strength of the material. Material data for the steel alloy is given in Table 1. Assume Y=1.0.

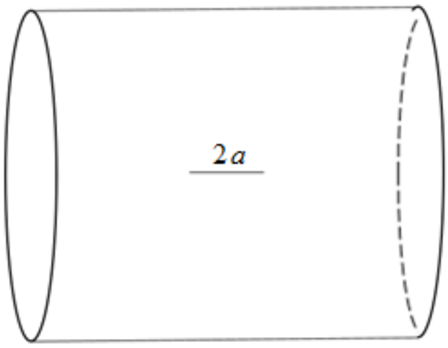


Figure 2

Table 1

Material	K_{IC} (MPa$\sqrt{m}$)	σ_{yield} (MPa)	$\sigma_{ultimate}$ (MPa)	Elongation limit (%)	C	m
Maraging Steel	82	1400	1600	12	1.5E-10	4.1

(a) Using Linear Elastic Fracture Mechanics, calculate the maximum defect size for the pressured vessel with the steel alloy.

(10 marks)

(b) The vessel is maintained with regular Nondestructive Inspection (NDI) using Fluorescent Penetrant Inspection (FPI). The FPI detection capability is 5 mm. Assuming the worst case that there is an initial crack size of 5 mm after inspection, determine the number of fatigue cycles to cause fracture for the steel.

(15 marks)

3. An electric motor of mass M = 30 kg is centrally mounted on a beam of negligible mass as shown in Figure 3. The static deflection (Δ_{st}) caused by the weight of the motor at the mid-section of the beam is 4 mm. If the motor is pushed downwards by 8 mm from the static equilibrium position, the amplitude of vertical oscillation of the motor is observed to dampen to 1 mm in 2 cycles.

(a) Determine the undamped and damped natural frequencies of the system in Hz.

(15 marks)

(b) Assuming that the motor has an unbalanced mass m with an eccentricity e, determine the magnification ratio MX/me at a rotational speed of 600 rpm.

(10 marks)

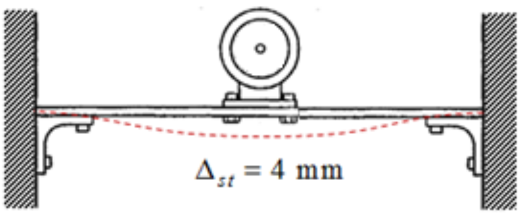


Figure 3

4. Figure 4 shows two rigid rods of equal length l and mass m hinged at points A and C and supported on two springs of equal stiffness k. In the configuration shown, the beams are in the horizontal position with the springs in the compressed state and the whole system is in static equilibrium. For free vibration analysis, the angular displacements θ_1 and θ_2 are measured with respect to the horizontal position (i.e., static equilibrium position) of the rods.

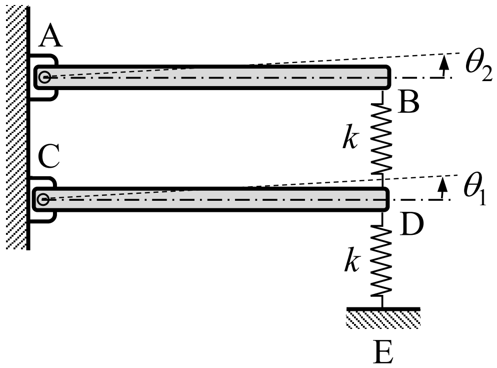


Figure 4

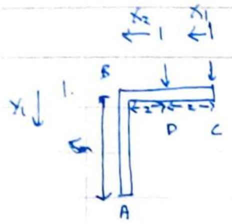
(a) Draw neat free body diagrams of rods AB and CD marking all the forces clearly. The static forces need not be included in the free body diagram for this problem. Explain the reason.

(10 marks)

(b) Applying Newton's 2nd law, derive the equations of motion for this 2-DOF system in terms of the coordinates θ_1 and θ_2 and write them in matrix form. You need not solve these equations.

(15 marks)

End of Paper

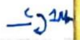


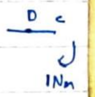
1. $EI = 10^5\ Nm^2$

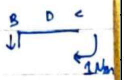
$F_1 = 100N$

$F_2 = 100N$

	Real	Virtual (moments)
C to D	$M = F_2(x_1)$	$m_{CD} = 1\,Nm$
~~B to D~~ D to B	$M = F_1 x_2 + F_2(2+x_2)$	$m_{DB} = 1\,Nm$
B to A	$M = F_1 \times 2 + F_2 \times 4 = 2F_1 + 4F_2$	$m_{BA} = 1\,Nm$







Using unit load method,

$$1 \cdot \theta = \delta U^*$$

$$= \underbrace{\int_0^2 \frac{Mm}{EI}\,dx_1}_{C\text{ to }D} + \underbrace{\int_0^2 \frac{Mm}{EI}\,dx_2}_{D\text{ to }B} + \underbrace{\int \frac{Mm}{EI}\,dy_1}_{B\text{ to }A}$$

$$= \int_0^2 \frac{F_2(x_1)(1)}{EI}\,dx_1 + \int_0^2 \frac{[F_1 x_2 + F_2(2+x_2)](1)}{EI}\,dx_2 + \int_0^5 \frac{(2F_1 + 4F_2)(1)}{EI}\,dy_1$$

$$= \frac{100}{10^5}\int_0^2 x_1\,dx_1 + \frac{1}{10^5}\int_0^2 100x_2 + 200 + 100x_2\,dx_2 + \frac{600}{10^5}\int_0^5 1\,dy_1$$

$$= \frac{100}{10^5}\left[\tfrac{1}{2}x_1^2\right]_0^2 + \frac{1}{10^5}\left[\frac{100}{2}x_2^2 + 200x + \frac{100}{2}x^2\right]_0^2 + \frac{600}{10^5}\left[y_1\right]_0^5$$

$$= \frac{100}{10^5}\left(\frac{2^2}{2}\right) + \frac{1}{10^5}\left(\frac{100}{2}(2)^2 + 200(2) + \frac{100}{2}(2)^2\right) + \frac{600}{10^5}(5)$$

$$= 0.04 \text{ rad} \quad (+\text{ve means CW})$$ //

2. $K_{Ic} = Y\sigma_f\sqrt{\pi a_c}$

maximum stress $= \frac{1}{4}$ yield strength

$= \frac{1400}{4} = 350\,MPa$

$K_{Ic} = 82\times10^6 = 350\times10^6(1)\sqrt{\pi(a_c)}$

$\Rightarrow a_c = 0.0174719$

$2a_c = 0.034939 \simeq 0.0349m$ //

b) $a_0 = \frac{5}{2}\times10^{-3}$

$$N_f = \frac{2}{C(Y S_R)^m \pi^{\frac{m}{2}}(2-m)}\left(a_f^{1-\frac{m}{2}} - a_0^{1-\frac{m}{2}}\right)$$

$Y=1$, $S_R = S_{max} = 350\,MPa$, $m = 4.1$, $a_f = 0.017471$

$a_0 = \frac{5}{2}\times10^{-3}$

$\Rightarrow N_f = 10.5839$ cycles.

3) i $m = 30\,kg$ $\qquad k = \frac{F}{x}$

$\Delta st = 4mm$ $\qquad = \frac{mg}{x} = \frac{mg}{0.004}$

$\Delta_{OY} = 8mm$ $\qquad = 73575\ N/m$

$\frac{x_1}{x_3} = \frac{8}{1}$ $\qquad w_n^2 = \frac{k}{m}$

$= \left(\frac{x_1}{x_2}\right)\left(\frac{x_2}{x_3}\right)$ $\qquad w_n = 49.5227\ rads^{-1}$

$= \left(\frac{x_1}{x_2}\right)^2$ $\qquad f_n = \frac{w_n}{2\pi}$

$\qquad = 7.8817 Hz$

$\frac{x_1}{x_2} = 2.828427$ $\qquad \approx 7.88 Hz$

$\zeta = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}} = 0.41048$ $\qquad w_d = w_n\sqrt{1-\zeta^2}$

$= 45.15826$

$f_d = \frac{w_d}{2\pi} = 7.18716$

$\approx 7.19 Hz$

b) spring mass w. rotating imbalance

$M\ddot{x} + kx = me\omega^2 \sin \omega t$

$\frac{MX}{me} = \frac{(w/w_n)^2}{|1-(w/w_n)^2|}$ $\qquad w = \frac{600 \times 2\pi}{60} = 62.83185$

$w = 62.83185\ rads^{-1}$

$w_n = 49.5227\ rads^{-1}$

$\frac{Mx}{me} = |-2.64| = 2.64$ //

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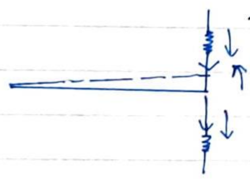
4)

AB

extension of k in AB = $l \sin\theta_2 - l \sin\theta_1$ (assuming $\theta_2 > \theta_1$)

$\approx l\theta_2 - l\theta_1$

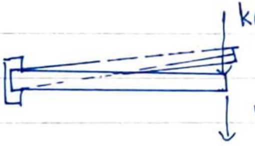
CD



$\rightarrow e = l\sin\theta_1 - l\sin\theta_2$ (assuming $\theta_1 > \theta_2$)

$\approx l\theta_1 - l\theta_2$

$\rightarrow e = l\sin\theta_1 \approx l\theta_1$



AB: $J_o \ddot{\theta}_2 = \Sigma M$

$\frac{1}{3} mL^2 \ddot{\theta}_2 = -k(l)(\theta_2 - \theta_1) \times l$

$= -kl^2(\theta_2 - \theta_1)$

$\frac{1}{3} mL^2 \ddot{\theta}_2 + kl^2(\theta_2 - \theta_1) = 0$

CD: $J_o \ddot{\theta}_1 = \Sigma M$

$= -k(l)(\theta_1 - \theta_2)(l) - k(l)^2\theta_1$

$= -kl^2(\theta_1 - \theta_2) - kl^2\theta_1$

$\Rightarrow \frac{1}{3} ml^2 \ddot{\theta}_1 + 2kl^2\theta_1 - kl^2\theta_2 = 0$

$\theta = \Theta \sin\omega t$

$\ddot{\theta} = \Theta \sin\omega t(-\omega^2)$

$\frac{1}{3} ml^2 \ddot{\theta}_2 + kl^2\theta_2 - kl^2\theta_1 = 0$

$\frac{1}{3} ml^2 \ddot{\theta}_1 + 2kl^2\theta_1 - kl^2\theta_2 = 0$

$$\Rightarrow \begin{bmatrix} \frac{1}{3}ml^2 & 0 \\ 0 & \frac{1}{3}ml^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_1 \end{bmatrix} + \begin{bmatrix} kl^2 & -kl^2 \\ -kl^2 & +2kl^2 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_1 \end{bmatrix} = 0$$

$$\Rightarrow -\omega^2 \begin{bmatrix} \frac{1}{3}ml^2 & 0 \\ 0 & \frac{1}{3}ml^2 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_1 \end{bmatrix} + \begin{bmatrix} kl^2 & -kl^2 \\ -kl^2 & +2kl^2 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_1 \end{bmatrix} = 0$$

$$= \begin{bmatrix} kl^2 - \frac{1}{3}\omega^2 ml^2 & -kl^2 \\ -kl^2 & 2kl^2 - \frac{ml^2}{3}\omega^2 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_1 \end{bmatrix} = 0$$

//.