MA3006 Sem 1 19/20

1(a)(i) $V_{1x} = 12\cos30°$ $\quad V_{2x} = -\cancel{12\cos30°}\ 0$

$V_{1y} = -12\sin30°$ $\quad V_{2y} = -\cancel{12\sin30°}\ 0$

$\dot{m} = \rho AV = 1000\pi(0.004)^2(12) = 0.6032\,kg/s$

$\Sigma F_x = \Sigma\dot{m}(V_{out})_x - \Sigma\dot{m}(V_{in})_x$

$F_x = 0 - 0.6032(12\cos30°) = -6.268\,N$

$\Sigma F_y = \Sigma\dot{m}(V_{out})_y - \Sigma\dot{m}(V_{in})_y$

$F_y = 0 - 0.6032(-12\sin30°) = 3.619\,N$

$F = \sqrt{F_x^2 + F_y^2} = 7.24\,N$

1(a)(ii) $V_{2y} = 12$ & $V_{3y} = -12$ $\quad V_{2x} = V_{3x} = 0$

$F_x = -6.268\,N$

$\Sigma F_y = \dot{m}_2 V_{2y} + \dot{m}_3 V_{3y} - \dot{m}_1 V_{1y}$ $\qquad \dot{m}_1 = \dot{m}_2 + \dot{m}_3$

$F_y = 12\dot{m}_2 - 12\dot{m}_3 - 0.6032(-12\sin30°)$ $\qquad \dot{m}_3 = \dot{m}_1 - \dot{m}_2$

$0 = 12\dot{m}_2 - 12(\dot{m}_1 - \dot{m}_2) - 0.6032(-12\sin30°)$

$F_R = 6.27\,N$

$24\dot{m}_2 - 12(0.6032) + 0.6032(12\sin30°) = 0 \Rightarrow \dot{m}_2 = 0.1508\,kg/s$

$\dot{m}_3 = 0.452\,kg/s$

1(a)(iii) relative speed $= 12\cos30° + 3 = 13.39\,m/s$ ✓ → always find relative speed 1st

$\dot{m}_1 = \rho AV_1' = 1000 \times \pi \times (0.004)^2 \times 13.39 = 0.673\,kg/s$

~~P=FV~~ $P = F_x V$ (Power in x-dir) $\qquad$ $(F = ma = \dot{m}v)$ → get the idea by SI units

~~$= (0.673 \times 13.39\cos30°)(13.39)$~~ $\qquad = 0.673 \times 13.39 = 9.015\,N$

~~$= 105\,W$~~ $\qquad W = Fv$

$= 9.015 \times 3 = 27.05\,W$

☆ towards ⇒ (+) & away ⇒ (−)

1(b) $Q_1 = Q_2 = 2\times10^{-3}\ m^3/s$

$V_1 = \frac{Q_1}{A_1} = \frac{0.002}{\pi(0.015)^2} = 2.829\,m/s \quad V_2 = \frac{Q_2}{A_2} = \frac{0.002}{\pi(0.03)^2} = 0.7074\,m/s$

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2} + h_L \rightarrow 0.22$

$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} - 0.22 \Rightarrow P_1 - P_2 = \frac{1}{2}\rho(V_2^2 - V_1^2) - 9810(0.22)$

$P_1 - P_2 = 500(0.7074^2 - 2.829^2) - 9810(0.22) = -5910\,Pa$

$h = \frac{P_1 - P_2}{\rho g} = \frac{-5910}{9810} = -0.602\,m$

$\Delta P = \frac{ma}{area} = kg\left(\frac{m}{s^2}\right)\left(\frac{1}{m^2}\right) = \frac{kg}{m s^2}$

Qactual = AV NOT Qideal!

Refer to right soln at back!

2(a) $\beta = \frac{56}{80} = 0.7$ & $Re = \frac{\rho V d}{\mu} = \frac{1000 \times 1.8 \times 0.08}{1.002 \times 10^{-3}} = 1.437 \times 10^5$ $Q_{actual} = A_1 V_1$

Frm fig 3, $C_o =$ ~~0.675~~ 0.613

$Q_{actual} = C_o A_2 \sqrt{\frac{2\Delta P}{\rho(1-\beta^4)}}$ & $Q_{ideal} = \cancel{\frac{Q_{actual}}{C_o}} = A_1 V_1$

Q_{actual} ~~Q_{ideal}~~ $= \pi(0.04)^2 \times 1.8 = 9.0478 \times 10^{-3}\, m^3/s$

$Q_{ideal} = A_2 \sqrt{\frac{2\Delta P}{\rho(1-\beta^4)}} \Rightarrow 9.0478 \times 10^{-3} = \pi(0.028)^2 \sqrt{\frac{2\Delta P}{1000(1-0.7^4)}} \Rightarrow \Delta P = 5130\, Pa$

2(b) $\Delta P = f(\omega, D, Q, \rho, \mu)$ need to find for ΔP also (3 π terms!)

$\rho = \frac{kg}{m^3}$ $Q = \frac{m^3}{s}$

$\mu = \frac{Ns}{m^2} = \left(kg\frac{m}{s^2}\right)\left(\frac{s}{m^2}\right) = \frac{kg}{ms}$

π_2 ~~π_1~~ $= Q(\rho)^a(\omega)^b(D)^c$

$= \left(\frac{m^3}{s}\right)\left(\frac{kg}{m^3}\right)^a\left(\frac{1}{s}\right)^b(m)^c$

kg: $a = 0$

s: $-1-b=0 \Rightarrow b=-1$

m: $3-3a+c=0 \Rightarrow c=-3$

$\pi_2 = \frac{Q}{\omega D^3}$

π_3 ~~π_2~~ $= \mu(\rho)^a(\omega)^b(D)^c$

$= \left(\frac{kg}{ms}\right)\left(\frac{kg}{m^3}\right)^a\left(\frac{1}{s}\right)^b(m)^c$

kg: $1+a=0 \Rightarrow a=-1$

s: $-1-b=0 \Rightarrow b=-1$

m: $-1-3a+c=0 \Rightarrow -1+3+c=0 \Rightarrow c=-2$

$\pi_3 = \frac{\mu}{\rho\omega D^2}$

$\pi_1 = \Delta P(\rho)^a(\omega)^b(D)^c$

$= \left(\frac{kg}{ms^2}\right)\left(\frac{kg}{m^3}\right)^a\left(\frac{1}{s}\right)^b(m)^c$

kg: $1+a=0 \Rightarrow a=-1$

m: $-1-3a+c=0 \Rightarrow c=$ ~~4~~ -2

s: $-2-b=0 \Rightarrow b=-2$

$\pi_1 = \frac{\Delta P}{\rho\omega^2 D^2}$

got 6 terms

let ω, D, ρ be variables

~~$\Delta P = f\left(\frac{Q}{\omega D^3}, \frac{\mu}{\rho\omega D^2}\right)$~~ $\frac{\Delta P}{\rho\omega^2 D^2} = f\left(\frac{Q}{\omega D^3}, \frac{\mu}{\rho\omega D^2}\right)$

Power $= \rho g Q \Delta h \Rightarrow \Delta P Q$

power coefficient (C_P): $C_P = \frac{P}{\rho\omega^3 D^5}$

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$Q = VA \Rightarrow$ model & prototype have diff areas!

$\nearrow$ Q is not ~~dimens~~ the same for model & prototype!

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2(c) $\left(\frac{Q}{\omega D^3}\right)_m = \left(\frac{Q}{\omega D^3}\right)_p$ need use power!

$\frac{\cancel{15}}{\omega(0.64)^3} = \frac{\cancel{15}}{500(0.8)^3} \Rightarrow \omega = \cancel{\frac{500(0.8)^3}{(0.64)^3}} = 977\,rpm$

$\left(\frac{\mu}{\rho\omega D^2}\right)_p = \left(\frac{\mu}{\rho\omega D^2}\right)_m$

$\omega_m D_m^2 = \omega_p D_p^2$

$\omega_m = \omega_p \left(\frac{D_p}{D_m}\right)^2 = 500\left(\frac{0.8}{0.64}\right)^2 = 781.25\,rpm$

2(a) $Q_{actual} = A_1V_1 = \pi(0.04)^2 \times 1.8 = 9.0478 \times 10^{-3}\, m^3/s$

$Q_{actual} = C_0 A_2 \sqrt{\frac{2\Delta P}{\rho(1-\beta^4)}} = 9.0478 \times 10^{-3}\, m/s$

$0.613 \times \pi(0.028)^2 \sqrt{\frac{2\Delta P}{1000(1-0.7^4)}} = 9.0478 \times 10^{-3} \Rightarrow \Delta P = 13644\,Pa$

☆ $Q_{actual} = C_0 A_2 \sqrt{\frac{2\Delta P}{\rho(1-\beta^4)}} = A_1 V_1$

$Q_{ideal} = A_2 \sqrt{\frac{2\Delta P}{\rho(1-\beta^4)}}$

A_1: pipe's area
A_2: orifice area
V_1: velocity flow @ pipe

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3(a) Assume pipe 1 to be laminar & pipe 2 to be turbulent

$d_1 = d_2 = 0.1m$ $\quad Q_1 = Q_2 = 0.01 m^3/s$

$V_1 = \frac{Q_1}{A_1} = \frac{0.01}{\pi(0.05)^2} = 1.273 m/s \quad V_2 = 1.273 m/s$ } V_{avg} for both pipe 1 & 2

Pipe 1

$Re = \frac{\rho V d}{\mu} = \frac{800(1.273)\cancel{(0.1)}}{\cancel{0.1}} = 1020$ (laminar)

laminar
$V_{avg} = \frac{1}{2} V_c$

V_c @ pipe 1 = $2V_{avg}$ @ pipe 1 = $2(1.273) = 2.546 m/s$

Pipe 2

$Re = \frac{\rho V d}{\mu} = \frac{7(1.273)(0.1)}{0.00001} = 89110$ (turbulent)

turbulent
$V_{avg} = 2V_c \frac{n^2}{(n+1)(2n+1)}$

Frm fig 5, n=7

$0.01 = 2\pi(0.05)^2 V_c @ \text{pipe } 2 \ \frac{7^2}{8 \times 15}$

V_c @ pipe 2 = $1.559 m/s$

$\frac{V_c @ \text{pipe } 2}{V_c @ \text{pipe } 1} = \cancel{\frac{2.546}{1.559}} = \frac{1.559}{2.546} = 0.612$

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3(b) Consider ONLY 2 cases : ① A→C & ② B→C   ~~A→B / B→A~~

V @ reservoir $= V = 0 \Rightarrow V_A = V_B = V_C = 0$

$Q_1 = Q_2 = 0.2\,m^3/s \quad Q_3 = 0.4\,m^3/s$

A→C

$$\cancel{\frac{P_A}{\rho g}} + \cancel{\frac{V_A^2}{2g}} + \overset{40}{\cancel{Z_A}} - h_L = \cancel{\frac{P_C}{\rho g}} + \cancel{\frac{V_C^2}{2g}} + \cancel{Z_C}$$

$$V_3 = \frac{Q_3}{A_3} = \overset{0.4}{Q_3}\left(\frac{4}{\pi d_3^2}\right) = 0.5093\left(\frac{1}{d_3^2}\right)$$

$$40 = f_1 \frac{l_1}{d_1}\frac{V_1^2}{2g} + f_3 \frac{l_3}{d_3}\frac{V_3^2}{2g}$$

$$40 = \frac{0.02(1000)}{0.5} \times \frac{1.019^2}{19.62} + \frac{0.02(500)}{d_3} \times \frac{1}{19.62} \times \left[0.5093\left(\frac{1}{d_3^2}\right)\right]^2 \qquad V_1 = \frac{0.2}{\pi(0.25)^2} = 1.019\,m/s$$

$$37.883 = 0.1322\,\frac{1}{d_3^5} \Rightarrow d_3 = 0.322\,m$$

B→C

$$V_2 = \frac{0.2}{\pi(0.2)^2} = 1.592\,m/s$$

$$45 = f_2 \frac{l_2}{d_2}\frac{V_2^2}{2g} + f_3 \frac{l_3}{d_3}\frac{V_3^2}{2g}$$

$$45 = \frac{0.02\,l_2}{0.4} \times \frac{1.592^2}{19.62} + \frac{0.02(500)}{0.3225} \times \frac{1}{19.62} \times \left[0.5093\left(\frac{1}{0.3225^2}\right)\right]^2$$

$$7.103 = 0.00646\,l_2 \Rightarrow l_2 = 1100\,m$$

Pipe 1: $f_1 \frac{1}{d_1}\frac{V_1^2}{2g} = \frac{0.02}{0.5} \times \frac{1.019^2}{19.62} = 0.00212$

Pipe 2: $f_2 \frac{1}{d_2}\frac{V_2^2}{2g} = \frac{0.02}{0.4} \times \frac{1.592^2}{19.62} = 0.00646$

Pipe 3: $f_3 \frac{1}{d_3}\frac{V_3^2}{2g} = \frac{0.02}{0.3225} \times \frac{1}{19.62} \times 0.5093^2 \times \frac{1}{0.3225^4} = 0.0758$

∴ Pipe 3 has the largest frictional head loss per unit length

$Q = AV$ $\quad A^2 = \pi^2(0.125)^4$ $\quad A = \pi(0.125)^2$

Always start $\bar{w}$ B·E if find Q!

4(i) $P_A = 200 \times 10^3$

$$\frac{P_A}{\rho g} + \cancel{\frac{V_A^2}{2g}} + \cancel{Z_A} - h_L + H_p = \cancel{\frac{P_C}{\rho g}} + \cancel{\frac{V_C^2}{2g}} + Z_C^{50} \quad \& \; h_L = \left(f\frac{l}{d} + \Sigma K\right)\frac{V^2}{2g}$$

$$\text{Total } h_L = \left[f^{0.02}\left(\frac{l_1+l_2+l_3}{0.25}\right)^{450} + (0+4+2)\right]\frac{Q^2}{2gA^2} = \cancel{43.6094Q^2}\; 888.404Q^2$$

$$\frac{200\times10^3}{9810} - 50 = \cancel{43.6094Q^2}^{\,888.404Q^2} - 100 + 400Q^2 \Rightarrow \cancel{443.609}^{\,1288.403}Q^2 = 70.387$$

$Q = 0.234\, m^3/s$

4(ii) $H_p = 100 - 400(0.0546) = 78.14m$ $\quad \omega = 2000 \times \frac{2\pi}{60} = 209.44\, rad/s$

$$N_s = \frac{\omega\sqrt{Q}}{(gH_p)^{0.75}} = \frac{209.44\sqrt{0.2337}}{(9.81 \times 78.14)^{0.75}} = 0.694 \text{ (centrifugal pump)}$$

4(iii) $V = \frac{Q}{A} = \frac{0.234}{\pi(0.125)^2} = 4.761 m/s$ $\quad \& \; Z_B = 70m$

$$\frac{P_A}{\rho g} + \cancel{\frac{V_A^2}{2g}} + \cancel{Z_A} - h_L + H_p = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B^{70} \quad \& \; h_L = \left[f\left(\frac{l_1+l_2}{d}\right)^{20} + \Sigma K_1^{0} + \Sigma K_2^{4}\right]\frac{4.761^2}{2(19.62)}$$

$$\frac{P_B}{\rho g} = \frac{200\times10^3}{9810} - 24 \times \frac{4.761^2}{19.62} - \frac{4.761^2}{19.62} + 78.14 - 70 = -0.3553 \Rightarrow P_B = -3485\, Pa$$

$P_B > P_v \Rightarrow$ no cavitation (focus on magnitude only)

4(iv)
$$\frac{P_A - P_v}{\rho g} + \frac{V_A^2}{2g} + Z_A - h_L = \frac{P_S - P_v}{\rho g} + \frac{V_S^2}{2g} + Z_S^{-2}$$

$$\frac{P_A - P_v}{\rho g} - \frac{1}{2g}\frac{Q^2}{\pi^2 R^4}\left(f_1\frac{l_1}{d_1} + \Sigma K_{L1}\right) = \underbrace{\left[\frac{P_S - P_v}{\rho g} + \frac{V_S^2}{2g}\right]}_{NPSH_A} + Z_S$$

$(-80+100) \times 10^3$

$$\frac{20\times10^3 - 2340}{9810} - \frac{0.18^2}{19.62}\left(\frac{1}{\pi^2(0.125)^4}\right)(0.8+2) + 2 = NPSH_A$$

$NPSH_A = 3.25m$

$NPSH_R = 3 + 100(0.18)^2 = 6.24$

$NPSH_A < NPSH_R \Rightarrow$ cavitation occurs!

Must rmber this! $A < R$ (cavitation)

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