2017/18 Sem 1

$\dot{m} = A v \rho$

1a)

(i) $\cancel{\frac{P_1}{\rho g}} + \frac{V_1^2}{2g} + \cancel{z_1} = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{z_2}$

$V_2 = -V_1$

$\sum F_x = \dot{m}(V_{2x} - V_{1x})$ (positive : right)

$RF = \rho A V (-12 - 12)$

$RF = 1000 \left(\frac{\pi}{4} \times 0.04^2\right)(12)(-24)$

$RF = -361.91 \text{ N}$

(ii) $361.91 = ma$

$a = 1.8096 \text{ m/s}^2$

$v = u + at$

$3 = 1.8096t$ $\quad t = 1.6579 \text{ s}$

$\frac{\rho}{2}$

NO: $E_{inlet} = E_{out}$ DATE:

1b) BE : 3 → nozzle (derivation of Q orifice eqn)

$$\frac{P_3}{\rho g} + \frac{V_3^2}{2g} + \cancel{z_3} = \frac{P_0}{\rho g} + \frac{V_0^2}{2g} + \cancel{z_0}$$

$$\frac{P_3 - P_0}{\rho g} = \frac{1}{2g}\left(V_0^2 - V_3^2\right)$$

$$\frac{P_3 - P_0}{\rho g} = \frac{1}{2g}\left(\frac{1}{A_0^2} - \frac{1}{A_3^2}\right) Q^2$$

$$\frac{P_3 - P_0}{\rho g} = \frac{1}{2g}\frac{1}{A_0^2}\left(1 - \frac{A_0^2}{A_3^2}\right) Q^2$$

$$\frac{P_3 - P_0}{\rho \cancel{g}} = \frac{1}{2\cancel{g}}\frac{1}{A_0^2}\left(1 - \left(\frac{D_0}{D_3}\right)^4\right) Q^2$$

$$Q_{ideal} = \sqrt{\frac{\frac{2}{\rho} A_0^2 (P_3 - P_0)}{1 - \left(\frac{D_0}{D_3}\right)^4}} \Longrightarrow 3.01646 \ m^3/s$$

$$Q_{actual} = C_n Q_{ideal} \Longrightarrow 1.8702 \ m^3/s$$

$$V_3 = \frac{Q}{A_3} = 3.72065 \ m/s$$

$$A_1 V_1 + A_2 V_2 = A_3 V_3$$

$$\frac{\pi}{4}(0.4)^2 \times 8 + \frac{\pi}{4}(0.3)^2 V_2 = \frac{\pi}{4}(0.8)^2 \times 3.72065$$

$$V_2 = 12.2357 \ m/s$$

$$\left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1}\right) + \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}\right) - h_{turb} - h_L = \left(\frac{P_3}{\rho g} + \frac{V_3^2}{2g} + \cancel{z_3}\right)$$

$$\frac{200000}{\rho g} + \frac{8^2}{2g} + \frac{300000}{\rho g} + \frac{12.2357^2}{2g} - h_{turb} - \frac{50000}{\rho g Q} = \frac{100000}{\rho g} + \frac{3.72065^2}{2g}$$

$$h_{turb} = 48.2364$$

$$W_{out,ideal} = \rho g Q h_{turb} = 884978 \ W$$

$$W_{out,actual} = \eta W_{out,ideal} = 707982 \ W$$

8 variables
3 units $\longrightarrow$ 5 π

kg $m^{-1}s^{-1}$ m ms^{-1}

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2a) $F_L = g(V, l, S, \alpha, c, \rho, \mu)$ repeating variables: μ, S, c

$$\pi_1 = F_L(S^a \rho^b \mu^c)$$
$$= (MLT^{-2})(L^a)(ML^{-3})^b(ML^{-1}T^{-1})^c$$
$$= M^{1+b+c} L^{1+a-3b-c} T^{-2-c}$$

$a = 0$
$b = 1$
$c = -2$

$$\pi_1 = F_L \frac{\rho}{\mu^2}$$

$$\pi_2 = l(S^a \rho^b \mu^c)$$

$$\pi_2 = \frac{l}{S}$$

$$\pi_3 = \alpha$$

$$\pi_4 = c(S^a \rho^b \mu^c)$$
$$= (LT^{-1})(L^a)(ML^{-3})^b(ML^{-1}T^{-1})^c$$
$$= M^{b+c} L^{1+a-3b-c} T^{-1-c}$$

$a = 1$
$b = 1$
$c = -1$

$$\pi_4 = \frac{cS\rho}{\mu}$$

$$\pi_5 = V(S^a \rho^b \mu^c)$$
$$\pi_5 = \frac{VS\rho}{\mu}$$

$$F_L \frac{\rho}{\mu^2} = \phi\left(\frac{VS\rho}{\mu}, \frac{l}{S}, \alpha, \frac{cS\rho}{\mu}\right)$$

$$\frac{\pi_5}{\pi_4} = \frac{V}{c} \text{ (mach no.)}$$

$$\frac{\rho}{\mu^2} \propto \frac{\mu}{VS\rho}$$

$$\frac{\pi_1}{\pi_5} = \frac{F_L}{V}\frac{1}{S}\frac{1}{\mu}$$

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fully submerged in fluid (air)

2b) aircraft, dynamic similarity $\Rightarrow$ Reynold's $\Rightarrow \frac{V S \rho}{\mu}$

$$Re_m = Re_p$$

$$\left(\frac{VS\rho}{\mu}\right)_m = \left(\frac{VS\rho}{\mu}\right)_p$$

$$\frac{V_m}{V_p}\,\frac{S_m}{S_p}\,\frac{\rho_m}{\rho_p}\,\frac{\mu_p}{\mu_m} = 1$$

$$\frac{V_m}{V_p}\left(\frac{1}{20}\right)\left(\frac{1.204}{1.514}\right)\left(\frac{1.57\cancel{\times 10^{-5}}}{1.82\cancel{\times 10^{-5}}}\right) = 1$$

$$\frac{V_p}{V_m} = 0.0343$$

$$\frac{\pi_5}{\pi_4} = \frac{V}{c} \text{ (mach no.)}$$

$$\frac{\rho}{\cancel{\mu^2}} \times \left(\frac{\cancel{\mu}}{VS\rho}\right)^2 = \frac{1}{V^2S^2\rho}$$

$$\frac{\pi_1}{(\pi_5)^2} = \frac{F_L}{V^2S^2\rho}$$ $\Leftarrow$ new dimensionless grp (lift force coefficient)

$$\left(\frac{F_L}{V^2S^2\rho}\right)_m = \left(\frac{F_L}{V^2S^2\rho}\right)_p$$ (drag/lift force coefficient will always the same) for model and prototype

$$\left(\frac{V_p}{V_m}\right)^2\left(\frac{S_p}{S_m}\right)^2\left(\frac{\rho_p}{\rho_m}\right) = \frac{(F_L)_p}{(F_L)_m}$$

$$(0.0343)^2(20^2)\left(\frac{1.514}{1.208}\right) = \frac{(F_L)_p}{1.25}$$

$$(F_L)_p = 0.739 \text{ kN}$$

2c) $u_r = V_c\left[1-\left(\frac{2r}{D}\right)^2\right]$

$= \left(\frac{\Delta p D^2}{16\mu l}\right)\left[1-\left(\frac{2r}{D}\right)^2\right]$

$Q = \int u_r \, dA$

$= \int u_r \, 2\pi r \, dr$

$= \int 2\pi V_c\left[1-\left(\frac{2r}{D}\right)^2\right] r \, dr$

$= 2\pi \int \left(\frac{\Delta p D^2}{16\mu l}\right)\left[1-\left(\frac{2r}{D}\right)^2\right] r \, dr$

$= \pi \cdot \frac{\Delta p D^2}{8\mu l} \int_0^R \left(1-\left(\frac{\cancel{2}r}{\cancel{2}R}\right)^2\right) r \, dr$

$= \pi \cdot \frac{\Delta p D^2}{8\mu l}\left[\frac{1}{2}r^2 - \frac{1}{R^2}\frac{1}{4}r^4\right]_0^R$

$= \pi \cdot \frac{\Delta p D^2}{8\mu l}\left[\frac{1}{2}R^2 - \frac{1}{\cancel{R^2}}\frac{1}{4}R^{\cancel{4}\,2}\right]$

$= \frac{\pi D^2 \Delta p}{8\mu l}\left(\frac{1}{4}R^2\right)$

$= \frac{\pi D^2 \Delta p}{8\mu l}\left(\frac{1}{4}\frac{1}{4}D^2\right)$

$= \frac{\pi D^4 \Delta p}{128\mu l}$ //

$Re = 800 = \frac{\rho V_{avg} D}{\mu}$

$800 = \frac{1050 V_{avg}(0.01)}{0.00025}$

$V_{avg} = 0.019048$

$V_c = 2V_{avg} = 0.3809 \text{ m/s}$

pressure gradient $= \frac{\Delta p}{l}$

$V_c = \frac{\Delta p D^2}{16\mu l}$ (given)

$0.3809 = \frac{\Delta p}{l}\left(\frac{0.01^2}{16\mu}\right)$

$\frac{\Delta p}{l} = 1.5238 \text{ Pa/m}$ //

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3a) $\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} - h_L = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}$

$$\frac{P_1 - P_2}{\rho g} = f\frac{l}{d}\frac{V^2}{2g}$$

$$\frac{\Delta P}{\rho \cancel{g}}\frac{d}{l}\frac{1}{f} = \frac{V^2}{2\cancel{g}}$$

$$V = \sqrt{\frac{2\Delta p}{f}\frac{d}{l}\frac{1}{\rho}} \quad \text{(QED)}$$

$$Re = \frac{\rho V D}{\mu} = \frac{1000V(0.01)}{0.001} = 10000\,V$$

$$f = \frac{64}{Re} = \frac{64}{10000\,V}$$

$$V = \sqrt{\frac{2\Delta p}{\frac{64}{10000V}}\frac{d}{l}\frac{1}{\rho}}$$

$$V = \sqrt{\frac{2\times 10000(500)}{64}\cdot\frac{0.01}{10}\cdot\frac{1}{1000}V}$$

$$\sqrt{V} = \frac{\sqrt{10}}{8}$$

$$V = 0.156 \text{ m/s}$$

b)

(i) $\frac{P_A}{\rho g} + \cancel{\frac{V^2}{2g}} + z_A - h_L = \cancel{\frac{P_B}{\rho g}} + \frac{V_B^2}{2g} + z_B$

$$\frac{120\,000}{\rho g} + 2 - 2.5\frac{V^2}{2g} = \frac{1}{2g}\left(\frac{Q}{A_B}\right)^2$$

$$\frac{120\,000}{\rho g} + 2 - 2.5\frac{1}{2g}\left(\frac{Q}{A_1}\right)^2 = \frac{1}{2g}\left(\frac{Q}{A_B}\right)^2$$

$$\frac{120\,000}{\rho g} + 2 = 2.5\frac{1}{2g}\left(\frac{Q}{\frac{\pi}{4}(0.1)^2}\right)^2 + \frac{1}{2g}\left(\frac{Q}{\frac{\pi}{4}(0.025)^2}\right)^2$$

$$\frac{120\,000}{\rho g} + 2 = \frac{Q^2}{2g}\left(\frac{2.5}{(\frac{\pi}{4}\times 0.1^2)^2} + \frac{1}{(\frac{\pi}{4}\times 0.025^2)^2}\right)$$

$$Q = 0.008163 \text{ m}^3/\text{s}$$

nozzle velocity head $= \frac{V^2}{2g} = \frac{1}{2g}\left(\frac{Q}{\pi/4\,(0.025)^2}\right)^2$

$= 14.09$ m

(ii) $\cancel{\frac{P_A}{\rho g}} + \cancel{\frac{V_A^2}{2g}} + z_A + H_p - h_L = \cancel{\frac{P_B}{\rho g}} + \frac{V_B^2}{2g} + z_B$

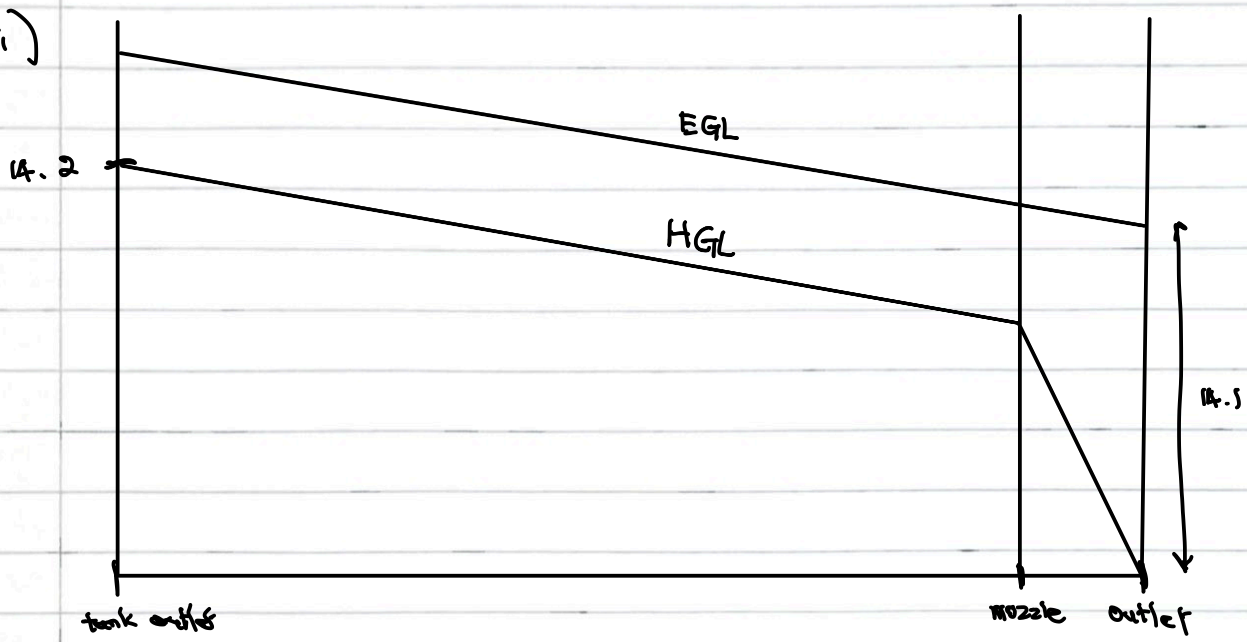
$$2 + H_p - 2.5\frac{V^2}{2g} = \frac{1}{2g}\left(\frac{Q}{\frac{\pi}{4}(0.025)^2}\right)^2$$

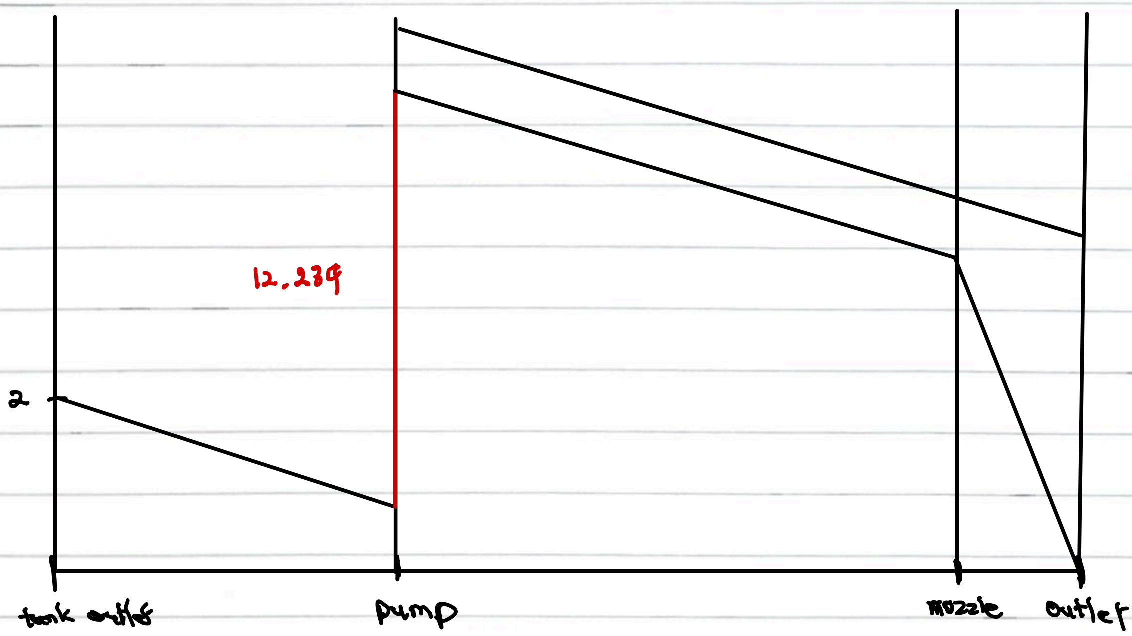
$$2 + H_p - \frac{2.5}{2g}\left(\frac{Q}{\frac{\pi}{4}(0.1)^2}\right)^2 = \frac{1}{2g}\left(\frac{Q}{\frac{\pi}{4}(0.025)^2}\right)^2$$

$$H_p = 12.2324 \text{ m}$$

$$W_p = \frac{\rho g Q H_p}{\eta} = 1399.37 \text{ W}$$

(iii)





$$EGL = \frac{P}{\rho g} + \frac{V^2}{2g}$$

$$HGL = \frac{P}{\rho g}$$

$$Q = AV$$

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4a) $$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + Z_1 + H_p - h_l = \cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + Z_2$$

$$H_p = \left(f_1 \frac{l_1}{d_1}\frac{V_1^2}{2g} + f_2 \frac{l_2}{d_2}\frac{V_2^2}{2g}\right) + Z_2 - Z_1$$

$$H_p = (Z_2 - Z_1) + \left(f_1 \frac{l_1}{d_1}\frac{1}{2g}\left(\frac{Q}{\frac{\pi}{4}d_1^2}\right)^2 + f_2 \frac{l_2}{d_2}\frac{1}{2g}\left(\frac{Q}{\frac{\pi}{4}d_2^2}\right)^2\right)$$

$$H_p = (Z_2 - Z_1) + \frac{1}{2g} \times \left(\frac{4}{\pi}\right)^2 \left(f_1 \frac{l_1}{d_1^5} Q^2 + f_2 \frac{l_2}{d_2^5} Q^2\right)$$

$$H_p = (Z_2 - Z_1) + 0.8262\left(f_1 \frac{l_1}{d_1^5} + f_2 \frac{l_2}{d_2^5}\right) Q^2 \quad \text{(QED)}$$

b)

(i) (undoable without graph)

- find Q by equating pump and system curve
- $W_p = \frac{\rho g Q H}{\eta}$ $\quad \eta(Q = \_\_) =$

(ii) Pump suction inlet

Here, the local pressure is usually lower than vapour pressure, right before the pump adds head to the flow.

(iii) $$NPSH_A = \frac{P_{atm} - P_v}{\rho g} - h_l + z_1 - z_2$$

$$= \frac{100\,000 - 2750}{1020\, g} - 0.8262\left(f_1 \frac{l_1}{d_1^5} Q^2\right) - 1 =$$

$NPSH_R$ (Q = $\quad$) =

$NPSH_A$ ____ $NPSH_R$ $\therefore$ cavitation (will / will not) occur

(iv) $$NPSH_A = \frac{100\,000 - 2750}{1020\, g} - 0.8262\left(f_1 \frac{l_1}{d_1^5} Q^2\right) - \Delta z$$

Cavitation onset : $NPSH_A = NPSH_R$

$\Delta z =$ ____