# 22/23 S1

✓ unsure:

Q2a

## Q1

a) 300 pulses per full rotation of power joint

800 rotation of power joint = 1 rotation of torque joint

$$\frac{15°}{360} \times 300 \times 800 = 10{,}000 \,\#$$

b) angular error at link joint 2: $\Delta\theta_2 = \frac{360°}{200 \times 500} = 0.0036° \,\#$

c) $x = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$

$$\Delta x = \Delta\theta_1 \cdot (-L_1\sin\theta_1) + (\Delta\theta_1 + \Delta\theta_2)(-L_2\sin(\theta_1+\theta_2))$$

$$\Delta\theta_1 = \frac{360°}{300 \times 800} = 0.0015°$$

$$\therefore \max|\Delta x| = \frac{\pi}{180}\left[0.0015 \times 35 + (0.0015 + 0.0036)(32)\right]$$

$$= 0.00371\,\text{cm} \,\#$$

TBC

## Q2

2. As shown in Figure 2(a), a disk (rigidly attached to the end of a rod) is rotating at constant angular velocity $\dot\theta$ about axis $Z$ at Point A. The rod is spinning at constant $\dot\beta$ about axis $X$. Note that Point O is the origin of fixed frame $XYZ$. Point $P$ is moving at constant velocity $\dot s$. Note that $\dot s$ is the tangential velocity perpendicular to the inner circle with radius $r$, as shown in Figure 2(b).

TBC: when? Is it $\dot s = r\dot\theta$

(a) Find the absolute velocity of Point P in terms of the velocities ($\dot\theta$, $\dot\beta$, and $\dot s$).

(12 marks)

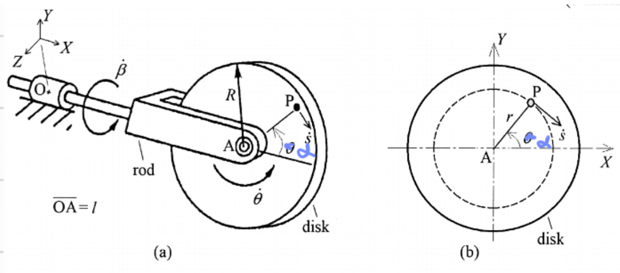



Figure 2

Q: $\dot\theta$ is arbitrarily; not related to $\dot\theta$ of local frame?

interpretation #1: $\dot s$ is always tangential

#2: $\dot s$ is NOT always tangential

a) let local coord sys 1 be attached to point O spinning at $\dot\beta$ while local coord sys 2 is attached to point A spinning at $\dot\theta$

then ${}^0P_p = {}^0P_{\overline{OA}} + {}^0P_{\overline{AP}} = R_{01}\,{}^1P_{\overline{OA}} + R_{02}\,{}^2P_{\overline{AP}}$

this shd be a vector in local frame, not a point

$${}^1P_{OA} = \begin{pmatrix} l \\ 0 \\ 0 \end{pmatrix} \text{ arbitrary; trivial}$$

$$R_{01} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{pmatrix} \sim \text{rot abt } X$$

$$\boxed{\begin{aligned} {}^2P_{\overline{AP}} &= \begin{pmatrix} r\cos\alpha \\ r\sin\alpha \\ 0 \end{pmatrix} \\ R_{12} &= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \text{rot. abt } Z \\ {}^0P_p &= R_{01}\,{}^1P_{\overline{OA}} + R_{02}\,{}^2P_{\overline{AP}} \end{aligned}}$$

motion in a circle at const $\dot s$

$x = r\cos\alpha, \quad y = r\sin\alpha$

$\dot x = -r\dot\theta\sin\alpha, \quad \dot y = r\dot\theta\cos\alpha$

$\sqrt{\dot x^2 + \dot y^2} = \sqrt{r^2\dot\alpha^2\sin^2\alpha + r^2\dot\alpha^2\cos^2\alpha}$

$= r\dot\alpha = \dot s$

$\dot\alpha = \dot s / r$

or ${}^0P_p = R_{01}\,{}^1P_{\overrightarrow{OA}} + R_{01}\,{}^1P_{\overrightarrow{AP}}$

where $x_1y_1z_1$ is a local frame at A NOT rotating w/ the disc:

$${}^1P_{\overrightarrow{AP}} = \begin{bmatrix} r\cos\theta + \dot s t\sin\theta \\ r\sin\theta + \dot s t\cos\theta \\ 0 \end{bmatrix}$$

?

$$\Rightarrow {}^0\dot P_p = \underbrace{\dot R_{01}\,{}^1P_{\overline{OA}}}_{=0} + \underbrace{R_{01}\,{}^1\dot P_{\overline{OA}}}_{=0} + \dot R_{01}R_{12}\,{}^2P_{\overline{AP}} + R_{01}\dot R_{12}\,{}^2P_{\overline{AP}} + R_{01}R_{12}\,{}^2\dot P_{\overline{AP}}$$

$$= \dot\beta \cancel{\begin{pmatrix}0&0&0\\0&-\sin\beta&-\cos\beta\\0&\cos\beta&-\sin\beta\end{pmatrix}\begin{pmatrix}L\\0\\0\end{pmatrix}}^{\,0} + \cancel{\begin{pmatrix}1&0&0\\0&\cos\beta&-\sin\beta\\0&\sin\beta&\cos\beta\end{pmatrix}\begin{pmatrix}0\\0\\0\end{pmatrix}}^{\,0} + \dot\beta\begin{pmatrix}0&0&0\\0&-\sin\beta&-\cos\beta\\0&\cos\beta&-\sin\beta\end{pmatrix}\begin{pmatrix}\cos\theta&-\sin\theta&0\\\sin\theta&\cos\theta&0\\0&0&1\end{pmatrix}\begin{pmatrix}r\cos\alpha\\r\sin\alpha\\0\end{pmatrix}$$

$$+\begin{pmatrix}1&0&0\\0&\cos\beta&-\sin\beta\\0&\sin\beta&\cos\beta\end{pmatrix}\begin{pmatrix}-\sin\theta&-\cos\theta&0\\\cos\theta&-\sin\theta&0\\0&0&0\end{pmatrix}\dot\theta\begin{pmatrix}r\cos\alpha\\r\sin\alpha\\0\end{pmatrix} + \begin{pmatrix}1&0&0\\0&\cos\beta&-\sin\beta\\0&\sin\beta&\cos\beta\end{pmatrix}\begin{pmatrix}\cos\theta&-\sin\theta&0\\\sin\theta&\cos\theta&0\\0&0&1\end{pmatrix}\begin{pmatrix}-r\sin\alpha\\r\cos\alpha\\0\end{pmatrix}\dot\alpha$$

$$\cancel{\Rightarrow \text{subst } \dot\alpha = \frac{\dot s}{r}}$$

$$\cancel{\therefore\ {}^0\dot P_p = \dot\beta\begin{bmatrix}0&0&0\\-\sin\beta\sin\theta&-\sin\beta\cos\theta&-\cos\beta\\\cos\beta\sin\theta&\cos\beta\cos\theta&-\sin\beta\end{bmatrix}\begin{pmatrix}r\cos \dot s/r\\r\sin \dot s/r\\0\end{pmatrix} + \dot\theta\begin{bmatrix}-\sin\theta&-\cos\theta&0\\\cos\beta\cos\theta&-\cos\beta\sin\theta&0\\\sin\beta\cos\theta&-\sin\beta\sin\theta&0\end{bmatrix}\begin{pmatrix}r\cos \dot s/r\\r\sin \dot s/r\\0\end{pmatrix}\ \#}$$

b) $${}^0\dot P_A = \dot R_{01}\,{}^1P_{\vec{OA}} + R_{01}\,{}^1\dot P_{\vec{OA}} = \dot\beta\cancel{\begin{pmatrix}0&0&0\\0&-\sin\beta&-\cos\beta\\0&\cos\beta&-\sin\beta\end{pmatrix}\begin{pmatrix}L\\0\\0\end{pmatrix}}^{\,0} + \cancel{\begin{pmatrix}1&0&0\\0&\cos\beta&-\sin\beta\\0&\sin\beta&\cos\beta\end{pmatrix}\begin{pmatrix}0\\0\\0\end{pmatrix}}^{\,0} = 0\ \#$$

c) . abs ang vel? about fixed origin ok, but abt rotating coord sys??

rotate abt X

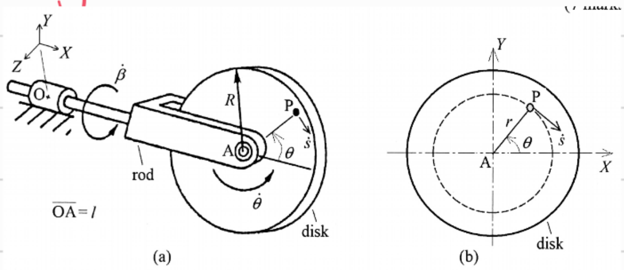


Figure 2

$${}^0\omega_A = \begin{pmatrix}\dot\beta\\0\\0\end{pmatrix} + \begin{pmatrix}1&0&0\\0&\cos\beta&-\sin\beta\\0&\sin\beta&\cos\beta\end{pmatrix}\begin{pmatrix}0\\0\\\dot\theta\end{pmatrix}$$

$$= \begin{pmatrix}\dot\beta\\-\dot\theta\sin\beta\\\dot\theta\cos\beta\end{pmatrix}$$

~ makes sense: $\dot\theta$ has Y and Z components as it rotates about global X.

$${}^2\omega_A = R_{20}\,{}^0\omega_A = \underbrace{\begin{pmatrix}\cos\theta&-\sin\theta&0\\\sin\theta&\cos\theta&0\\0&0&1\end{pmatrix}}_{R_{21}:\ \text{local } z}\underbrace{\begin{pmatrix}1&0&0\\0&\cos\beta&-\sin\beta\\0&\sin\beta&\cos\beta\end{pmatrix}}_{R_{10}:\ \text{local } x}\begin{pmatrix}\dot\beta\\-\dot\theta\sin\beta\\\dot\theta\cos\beta\end{pmatrix}$$

$$= \begin{pmatrix}\cos\theta&-\sin\theta&0\\\sin\theta&\cos\theta&0\\0&0&1\end{pmatrix}\begin{pmatrix}\dot\beta\\-2\dot\theta\sin\beta\cos\beta\\-\dot\theta\sin^2\beta+\dot\theta\cos^2\beta\end{pmatrix}\ \#$$

??

(d) Assuming the rod is massless, while the disk is uniform with mass $m$ and its major mass moment of inertia about Point A and along the axis normal to the disk be $I_z$. Note that two other inertia components are negligible. By making use of the results obtained above, express the steps to find the kinetic and potential energies of the system shown in Figure 2(a) if Point P is a lumped mass with mass $M$. (Note: Do Not expand the expressions.)

(7 marks)

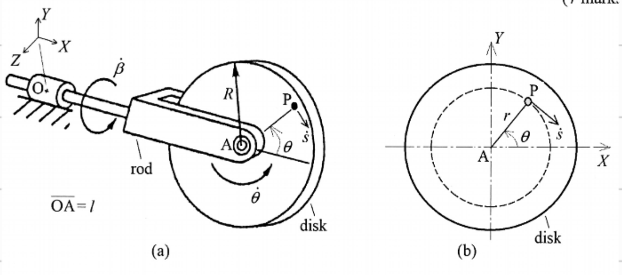



Figure 2

TBC: [how to make use of above results??]

d) Kinetic energy of rotating disc about $z_A$-axis (neglecting rotation about $x_A$-axis)

$$= \frac{1}{2} I_z \omega_z^2 = \frac{1}{2} I_z \dot{\theta}^2 \quad \text{—— (1)}$$

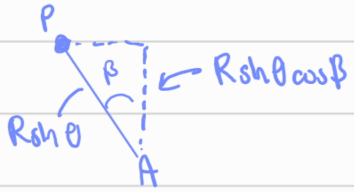
mass moment of inertia of point mass P about $z_A$-axis $= MR^2$

" about $x_A$-axis $= M(R\sin\theta)^2 = MR^2\sin^2\theta$

Kinetic energy of lumped mass $= \frac{1}{2}(MR^2)\left(\frac{\dot{s}}{r} - \dot{\theta}\right)^2 + \frac{1}{2}(MR^2\sin^2\theta)(\dot{\beta})^2$ —— (2)

$\therefore$ total $KE = \frac{1}{2} I_z \dot{\theta}^2 + \frac{1}{2}(MR^2)\left(\frac{\dot{s}}{r} - \dot{\theta}\right)^2 + \frac{1}{2}(MR^2\sin^2\theta)(\dot{\beta})^2$ #

total PE = PE of lumped mass (taking A as reference) $= MR\sin\theta\cos\beta$ #



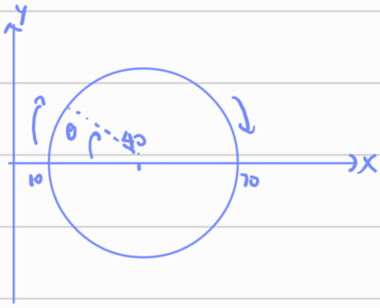

check: when $\theta = 90°$, $\beta = 180°$ : $PE = MR(1)(-1) = -MR$ ✓

$\theta = 180°$ : $PE = 0$ ✓

$\beta = 90°$ : $PE = 0$ ✓

$\boxed{Q3}$

a)




eqn of circle: $(x-a)^2+(y-b)^2=r^2$

$(x-40)^2+y^2=30^2$ #

b) parametric eqn of circle: $\begin{cases} X = 40 - R\cos\theta \\ Y = R\sin\theta \end{cases}$

to complete motion of $\theta = 180°$, $180° = \left[\frac{\dot\theta}{2}\times 2\right]\times 2 + \dot\theta \times 10$

$= 12\dot\theta \Rightarrow \dot\theta = 15° s^{-1}$

$\dot\theta_{cruise} = 15° s^{-1} = \frac{\pi}{12} \text{ rad } s^{-1}$

$\dot\theta(t=2) = \frac{\pi}{12}$ rad/s

from $t=0$ to $t=2$, $\ddot\theta = \frac{\pi}{24}$ rad $s^{-1}$

$\dot\theta = \frac{\pi}{24}t$

$\theta = \frac{\pi}{48}t^2$ #

for $t=0$ to $t=2$, $\dot\theta_{avg} = \frac{\pi}{24}t$ rad $s^{-1}$ ✗ wrong method?

$\theta = \frac{\pi}{48}t^2$ rad ✗ $\because \theta(t=0)=0$

$\therefore X = 40 - 30\cos\frac{\pi}{48}t^2$ # ✓

sub in $t=2$: $\theta = \frac{\pi}{12}$

check: ✓ $\frac{\pi - \frac{\pi}{12}\times 10}{2} = \frac{\pi}{12}$

for $t=2$ to $t=12$, $\dot\theta = \frac{\pi}{12}$ rad/s

$\theta = \frac{\pi}{12}(t-2) + \frac{\pi}{12}$ $\because \theta(t=2) = \frac{\pi}{12}$

$= \frac{\pi}{12}(t-1)$

(messy)

check: at $t=7$ (midpoint),

$\theta = \frac{\pi}{12}(7-1) = \pi/2$ ✓

$\therefore X = 40 - 30\cos\left[\left(\frac{\pi}{12}\right)(t-1)\right]$ # ✓

same angular displacement for slowdown & speed up

for $t=12$ to $t=14$, $\dot\theta = \frac{\pi}{12} - \frac{\pi}{24}(t-12) = \frac{7\pi}{12} - \frac{\pi}{24}t$

$\theta = \frac{7\pi}{12}t - \frac{\pi}{48}t^2 + C$, $C = 11\pi/12$

$\therefore X = 40 - 30\cos\left(\frac{7\pi}{12}t - \frac{\pi}{48}t^2 + \frac{11\pi}{12}\right)$ # ✗

$X = 40 - 30\cos\left[\frac{\pi}{12}(t-12) - \frac{\pi}{48}(t-12)^2 + \frac{11\pi}{12}\right]$ #

For $t=12$ to $14$,

$\dot\theta = \frac{\pi}{12}$

$\ddot\theta = -\frac{\pi}{24}$

$\dot\theta = -\frac{\pi}{24}t$

$\theta = -\frac{\pi}{48}t^2$

$\therefore X = 40 + 30\cos\left(-\frac{\pi}{48}(t-14)^2\right)$ #

(should be) ✓

$\boxed{\text{better.}}$ more intuition..

$\dot\theta = \frac{\pi}{12} - \frac{\pi}{24}t$

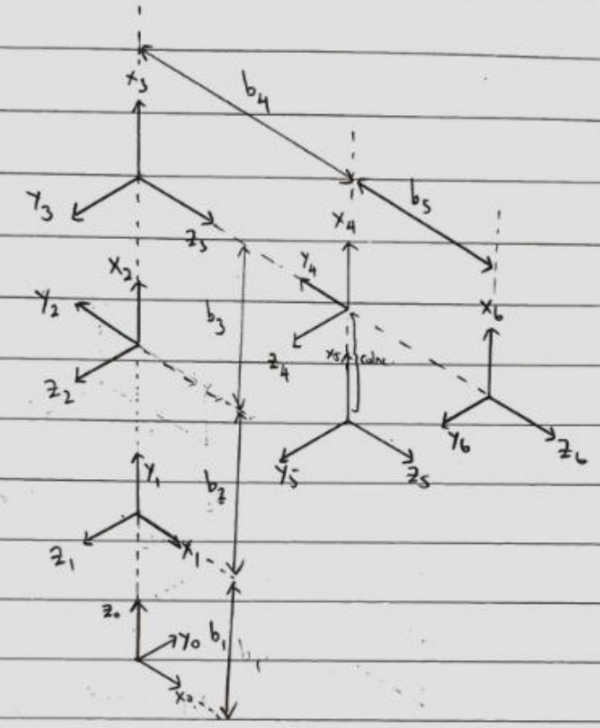
integrating, $\theta = \frac{\pi}{12}t - \frac{\pi}{48}t^2 + C$; $C = \frac{11\pi}{12}$

replace $t$ with $t-12$: $\theta = \frac{\pi}{12}(t-12) - \frac{\pi}{48}(t-12)^2 + \frac{11\pi}{12}$

22/23 S1

**Q4**

a)




b)

| | ← $X_{i-1} \to X_i$ along $z_{i-1}$ → | | ← $z_{i-1}$ to $z_i$ along $x_i$ → | | | |
|---|---|---|---|---|---|---|
| Joint | $\theta_i$ | $d_i$ | $a_i$ | $\alpha_i$ | $\cos\alpha_i$ | $\sin\alpha_i$ |
| 1 | 0 | $b_1$ | 0 | 90 | 0 | 1 |
| 2 | 90 | 0 | $b_2$ | 0 | 1 | 0 |
| 3 | 0 | 0 | $b_3$ | 90 | 0 | 1 |
| 4 | 0 | $b_4$ | 0 | -90 | 0 | -1 |
| 5 | 0 | 0 | 0 | 90 | 0 | 1 |
| 6 | 0 | $b_5$ | 0 | 0 | 1 | 0 |

c) $H_{03} = H_{01} H_{12} H_{23}$ given $H_{i-1,i} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\therefore H_{03} = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & b_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & b_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & 0 & \sin\theta_3 & b_3\cos\theta_3 \\ \sin\theta_3 & 0 & -\cos\theta_3 & b_3\sin\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#

Q4 (cont.)

(d) Figure 5 shows the base of a triangular block placing above a cube. Note that Plane ijkgh and Plane ehgf are perpendicular to each other. The right-handed coordinate frames $x_0y_0$ and $x_1y_1$ are located at points b and j, respectively. Note that axes $x_1$, $y_1$, and line jg are all lie on the same plane (Plane ijkgh). Given the dimensions shown in Figure 5, find the 4×4 homogenous matrix $[T_{01}]$, which is the matrix to transform the coordinates vector from frame 1 to frame 0.

(5 marks)

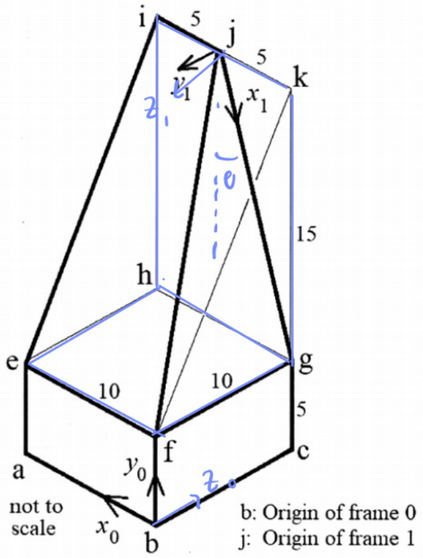
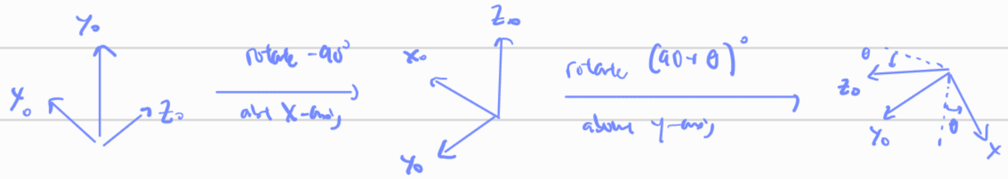



Figure 5

frame 1 to frame 0: ${}^0P_1 = \vec{BJ} = \begin{pmatrix}5\\20\\10\end{pmatrix}$

$\theta = \tan^{-1}\left(5/15\right) = 0.322\text{rad}$

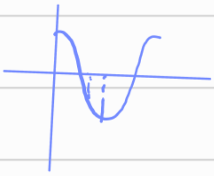



$$R_{01} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

(x-axis) (y-axis)

L7.2 S.29

to find $R_{01}$, go from 0 to 1 (not 1 to 0) then see $R_{ij}$ rotation matrix



(Sub $\alpha = -\frac{\pi}{2}$, $\phi = \frac{\pi}{2} + 0.322$)

$$R_{01} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \sin 0.322 & 0 & \cos 0.322 \\ 0 & 1 & 0 \\ -\cos 0.322 & 0 & \sin 0.322 \end{bmatrix}$$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cancel{\cos\frac{\pi}{2}\cos 0.322} - \sin\frac{\pi}{2}\sin 0.322$

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\sin\frac{\pi}{2}\cos 0.322 + \cancel{\sin 0.322\cos\frac{\pi}{2}}$

$$= \begin{bmatrix} \sin 0.322 & 0 & \cos 0.322 \\ -\cos 0.322 & 0 & \sin 0.322 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\therefore H_{01} = \begin{bmatrix} \sin 0.322 & 0 & \cos 0.322 & 5 \\ -\cos 0.322 & 0 & \sin 0.322 & 20 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \#$$

# NANYANG TECHNOLOGICAL UNIVERSITY

## SEMESTER 1 EXAMINATION 2022-2023

## MA4825 – ROBOTICS

November / December 2022 Time Allowed: 2 hours

## INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is an **OPEN-BOOK** examination.

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1. A planar robot arm with two links is shown in Figure 1. Link 1's length is 35.0 cm while link 2's length is 32.0 cm.

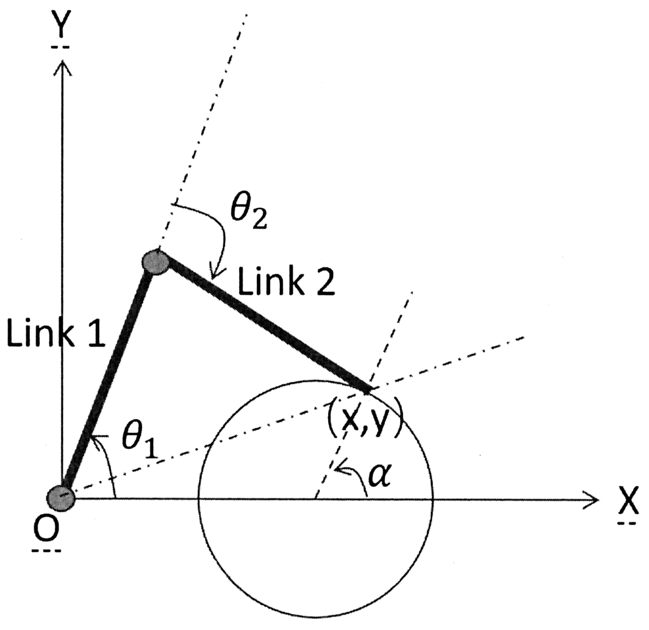



Figure 1

(a) Robot's joint 1 consists of link joint coupled with power joint (i.e. electric motor 1) through torque joint. The torque joint's reduction ratio is 800. The incremental encoder placed at the power joint will output 300 pulses per full rotation. Determine the new increment of the desired number of pulses (i.e., the motor shaft's new angular displacement in terms of pulses) to the position control loop of the power joint, if the link joint needs to make an angular displacement of 15.0°.

(10 marks)

Note: Question 1 continues on page 2.

(b) Robot's joint 2 also consists of link joint coupled with power joint (i.e. electric motor 2) through torque joint. The torque joint's reduction ratio is 500. The incremental encoder placed at the power joint will output 200 pulses per full rotation. Assume that the position control system itself will be able to achieve 100% accuracy. Determine the angular error which could be observed at the link joint of robot's joint 2?

(5 marks)

(c) Determine the maximum error of the tool's x coordinate if the robot's link 2 carries a tool at its distal end.

(10 marks)

2. As shown in Figure 2(a), a disk (rigidly attached to the end of a rod) is rotating at constant angular velocity $\dot{\theta}$ about axis $Z$ at Point A. The rod is spinning at constant $\dot{\beta}$ about axis $X$. Note that Point O is the origin of fixed frame $XYZ$. Point $P$ is moving at constant velocity $\dot{s}$. Note that $\dot{s}$ is the tangential velocity perpendicular to the inner circle with radius $r$, as shown in Figure 2(b).

TBC: When? but $\dot{s} = r\dot{\theta}$

(a) Find the absolute velocity of Point P in terms of the velocities ($\dot{\theta}$, $\dot{\beta}$, and $\dot{s}$).

(12 marks)

(b) Find the absolute velocity of Point A, in terms of the velocities ($\dot{\theta}$, and $\dot{\beta}$).

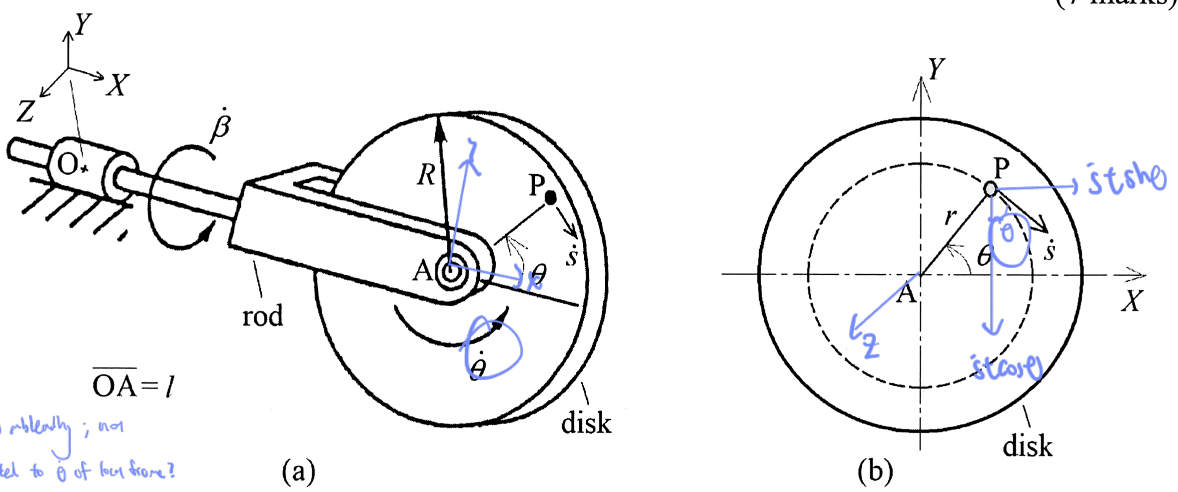
(2 marks)

(c) Find the absolute angular velocity of Point A expressed in the local coordinate system attached to the disk. TBC

(4 marks)

(d) Assuming the rod is massless, while the disk is uniform with mass $m$ and its major mass moment of inertia about Point A and along the axis normal to the disk be $I_z$. Note that two other inertia components are negligible. By making use of the results obtained above, express the steps to find the kinetic and potential energies of the system shown in Figure 2(a) if Point P is a lumped mass with mass $M$. (Note: Do Not expand the expressions.)

(7 marks)




Q: $\ddot{\theta}$ is absolutely; not related to $\dot{\theta}$ of local frame?

interpretation #1: $\dot{s}$ is always tangential

#2: $\dot{s}$ is NOT always tangential

Figure 2

3. A robotic arm carrying a gripper is shown in Figure 3. Initially, the gripper is at rest at point A. Now, the robotic arm will move the gripper to stop at point B by following a circular path/trajectory. Assume that the coordinates of point A are (10.0, 0.0) (cm) while the coordinates of point B are (70.0, 0.0) (cm). The orientation of gripper at point A is +90.0° while the orientation of gripper at point B is -90.0°. Both orientations are tangential to the circular path/trajectory.

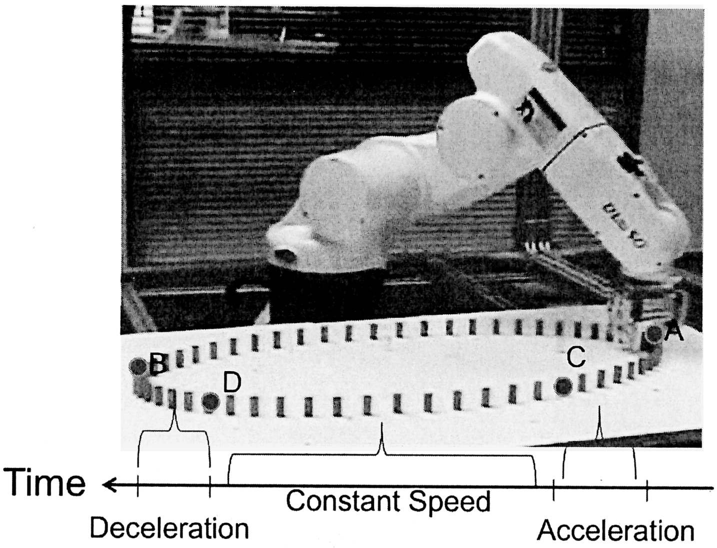



Figure 3

(a) Determine the equation of the path to be followed by the gripper's position.

(5 marks)

(b) The motion of the gripper's position has three phases: speed-up, cruising, and slow-down. The time intervals for these three phases are 2.0 s, 10.0 s, and 2.0 s, respectively. Determine the equations of trajectory to be followed by the gripper's x-coordinate.

(15 marks)

(c) Determine the value of acceleration and deceleration in the above equations of trajectory.

(5 marks)

4. A schematic diagram of an industrial robot (*ABB* IRB120 robot) used for the assembly of industrial products is shown in Figure 4(a). The arm has six joints: $\boldsymbol{q} = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6\}^T$. The views of the robot with dimensions are illustrated in Figure 4(b).

(a) Given that the origins of respective joint and the hand coordinates of the robot are as indicated in Figure 4(c), specify and draw the right-handed coordinate systems $i$ ($i = 1$ to 6). Also, indicate the corresponding values $b_j$ ($j = 1$ to 5) in the figure. (Note: Reproduce Figure 4(c) by connecting the origins in your answer booklet for your solution.)

(8 marks)

(b) Tabulate the Denavit-Hartenberg parameters for the position of the robot shown in Figure 4(c). Note that $\theta_i$ ($i = 1$ to 6) are the joint variables.

(7 marks)

(c) Generate the generic arm matrix $\boldsymbol{H}_{\text{base}}^{\text{elbow}}$ for the robot, where $\boldsymbol{H}_{\text{base}}^{\text{elbow}}$ is a 4×4 homogeneous matrix transforming a vector from the coordinates ($x_3y_3z_3$) of the elbow to the base coordinates ($x_0y_0z_0$). DO NOT multiply the individual matrices ([$H_{01}$], [$H_{12}$], and [$H_{23}$]).

(5 marks)

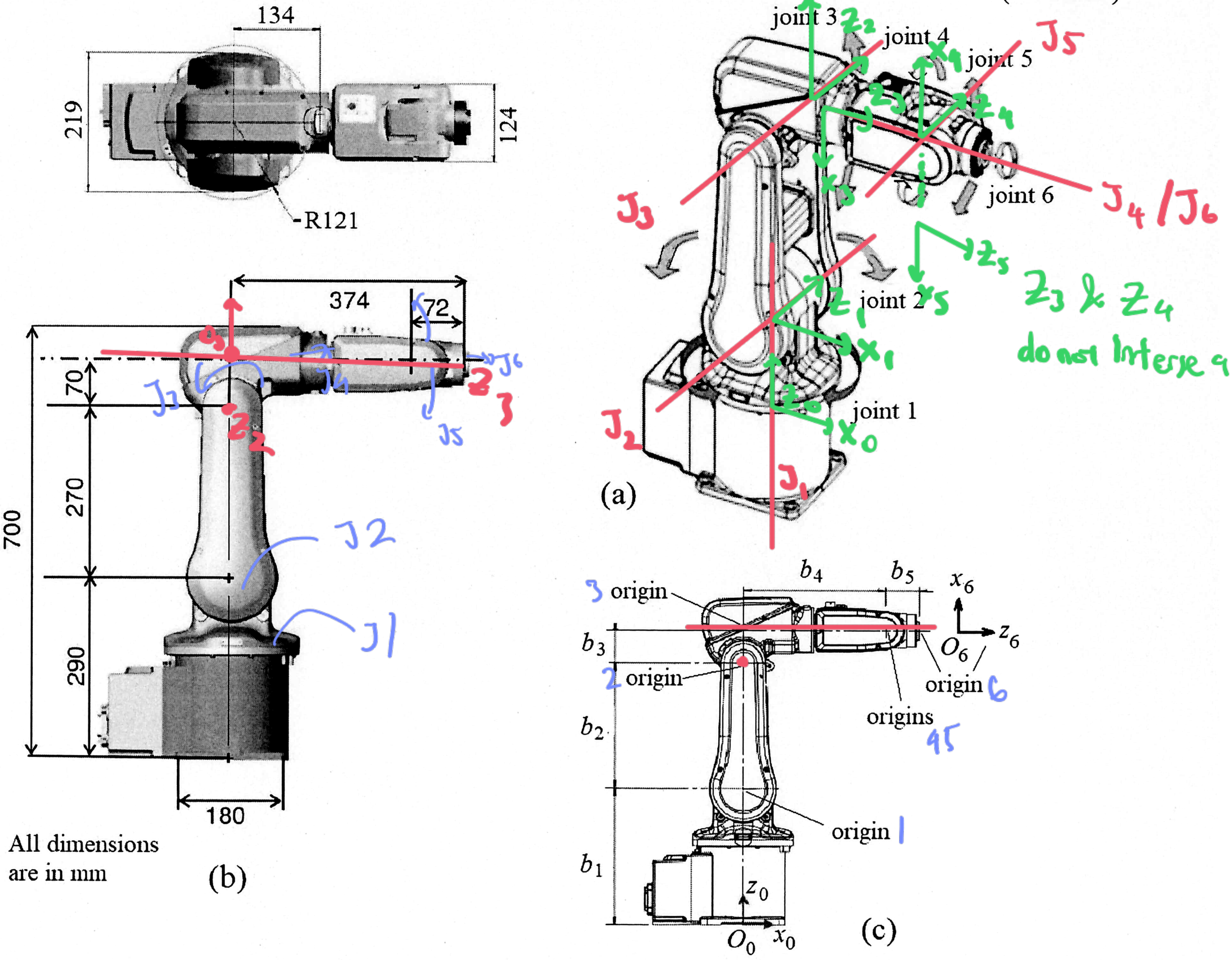



Figure 4

Note: Question 4 continues on page 5.

(d) Figure 5 shows the base of a triangular block placing above a cube. Note that Plane ijkgh and Plane ehgf are perpendicular to each other. The right-handed coordinate frames $x_0y_0$ and $x_1y_1$ are located at points b and j, respectively. Note that axes $x_1$, $y_1$, and line jg are all lie on the same plane (Plane ijkgh). Given the dimensions shown in Figure 5, find the 4×4 homogenous matrix $[T_{01}]$, which is the matrix to transform the coordinates vector from frame 1 to frame 0.

(5 marks)

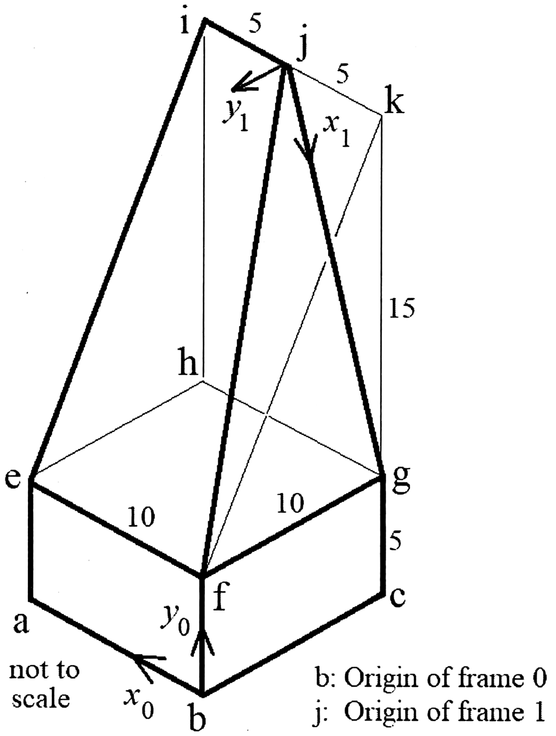



Figure 5

END OF PAPER