MA3006 sem 2 18/19

1(a) Consider 2 cases

① point 1 to 2 — let pt A be pt 4

② point 2 to 3 & 2 to 4 — let pt B be pt 3

Assuming frictionless flow ($V_2 \approx V_3 \approx V_4$)

1(a)(i) $\Sigma F_x = \Sigma \dot{m}(V_{out})_x - \dot{m}(V_{in})_x$

$0 = \dot{m}_4 V_{4x} - \dot{m}_3 V_{3x} - \dot{m}_2 V_{2x}$

$V_3 = V_{3x}$ & $V_4 = V_{4x}$

$V_{3y} = V_{4y} = 0$

$\Sigma F_y = \Sigma \dot{m}(V_{out})_y - \dot{m}(V_{in})_y$

$-25 = 0 - \dot{m}_2 V_{2y}$

1(a)(ii)

1→2

$\cancel{\frac{P_1}{\rho g}} + \frac{V_1^2}{2g} + \cancel{Z_1} = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + Z_2 \Rightarrow V_1^2 = V_2^2 + 4(9.81) \Rightarrow V_2 = \sqrt{10^2 - 4(9.81)} = 7.79 m/s$

$V_{2y} = 7.79 \sin 60°$ & $V_{2x} = 7.79 \cos 60°$)

$\dot{m}_2 = \frac{25}{7.79 \sin 60°} = 3.703 kg/s$

$\dot{m}_2 = \dot{m}_3 + \dot{m}_4 = 3.703$

$0 = \dot{m}_4(7.79) - \dot{m}_3(7.79) - 3.703(7.79 \cos 60°)$

form 2 eq^{n}s then use GC! Don't waste time solving

order of eq^n change

$-7.79 \dot{m}_3 + 7.79 \dot{m}_4 = 14.433$

$\dot{m}_3 = 0.925 kg/s \Rightarrow \dot{m}_B = 0.925 kg/s$

$\dot{m}_4 = 2.78 kg/s \Rightarrow \dot{m}_A = 2.78 kg/s$

$\dot{m} = \rho A V$

$A = \frac{3.703}{1000 \times 10} = 3.703 \times 10^{-4} m^2$ & $A = \frac{\pi}{4} d^2 \Rightarrow d = \sqrt{\frac{4A}{\pi}} = 0.0217 m$

$\star \dot{m} = \rho Q$

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1(b)(i) $V_{rel} = \frac{Q}{nA}$ $T = \dot{m}R(V_{rel}\cos\theta - R\omega)$ ↗ eqⁿ

$R = 0.18m$ (circular

$r = 0.004m$ (nozzle)

$V_{rel} = \frac{0.005}{4(\pi)(0.004)^2} = 24.868 m/s$ $\dot{m} = 0.005 \times 1000 = 5 kg/s$

when $T=0$,

$0 = 5(0.18)(24.868\cos 30° - 0.18\omega)$ OR just use $V_{rel}\cos\theta - R\omega = 0$ can alr

$\omega = \frac{24.868\cos 30°}{0.18} = 119.6 rad/s$

1(b)(ii) when $\omega = 10 rad/s$

$T = 5(0.18)(24.868\cos 30° - 0.18 \times 10) = 17.76 Nm$

1(b)(iii) at $\theta = 90°$, $\cos 90° = 0$

$T = 0$ (shown)

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2(a)

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2(a) $\beta = \frac{48}{80} = 0.6$ & $Re = \frac{1000 \times 1.99 \times 0.08}{1.002 \times 10^{-3}} = 1.588 \times 10^5$

$\dot{m} = \rho A V$

$V = \frac{10}{1000 \times \pi (0.04)^2}$

$= 1.99 m/s$

$C_0 = 0.612$

$Q_{actual} = AV = 0.01 m^3/s$

$Q_{actual} = C_0 A_2 \sqrt{\frac{2\Delta P}{1-\beta^4}}$ $\times$ ($\star$ $Q_{actual} = C_0 A_2 \sqrt{\frac{2\Delta P}{\rho(1-\beta^4)}}$)

$0.01 = 0.612 \times \pi (0.024)^2 \sqrt{\frac{2\Delta P}{1000(1-0.6^4)}} \Rightarrow \Delta P = 35.5 kPa$ ✓

2(b) $W = f(d, l, \omega, V, \rho, \mu)$

4 π terms!

$\pi_1 : W$, $\pi_2 : l$

$\pi_3 : V$, $\pi_4 : \mu$

$\pi_1 = W(\rho)^a (d)^b (\omega)^c$

$= \frac{kg\, m^2}{s^3} \left(\frac{kg}{m^3}\right)^a (m)^b \left(\frac{1}{s}\right)^c$

$\underbrace{\rho g Q H}$ find the SI units for each of them!

$W = \left(\frac{kg}{m^3}\right)\left(\frac{m}{s^2}\right)\left(\frac{m^3}{s}\right)(m)$

$= \frac{kg\, m^2}{s^3}$

kg: $1 + a = 0 \Rightarrow a = -1$

m: $2 - 3a + b = 0 \Rightarrow b = -5$

s: $-3 - c = 0 \Rightarrow c = -3$

$\mu = \frac{kg\, m}{s^2}\left(\frac{s}{m^2}\right) = \frac{kg}{ms}$

$\omega = \frac{1}{s}$

$\pi_1 = \frac{W}{\rho \omega^3 d^5}$

$\pi_2 = l(\rho)^a (d)^b (\omega)^c$ ← no need do this - l & d have the same parameter

$= m\left(\frac{kg}{m^3}\right)^a (m)^b \left(\frac{1}{s}\right)^c$ - just straightaway write $\pi_2 = \frac{l}{d}$

kg: $a = 0$

s: $c = 0$

m: $1 - 3a + b = 0 \Rightarrow b = -1$

$\pi_2 = \frac{l}{d}$

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same parameter as d

$$V = \frac{m}{s} = m\left(\frac{1}{s}\right)$$

same parameter as ω

straightaway write

$$\Pi_3 = \frac{V}{d\omega}$$

$$\Pi_3 = V(\rho)^a(d)^b(\omega)^c$$
$$= \frac{m}{s}\left(\frac{kg}{m^3}\right)^a(m)^b\left(\frac{1}{s}\right)^c$$

kg: $a = 0$

m: $1 - 3a + b = 0 \Rightarrow b = -1$

s: $-1 - c = 0 \Rightarrow c = -1$

$$\Pi_3 = \frac{V}{d\omega}$$

$$\Pi_4 = \mu(\rho)^a(d)^b(\omega)^c$$
$$= \frac{kg}{ms}\left(\frac{kg}{m^3}\right)^a(m)^b\left(\frac{1}{s}\right)^c$$

kg: $1 + a = 0 \Rightarrow a = -1$

m: $-1 - 3a + b = 0 \Rightarrow b = -2$

s: $-1 - c = 0 \Rightarrow c = -1$

$$\Pi_4 = \frac{\mu}{\rho d^2 \omega}$$

$$\frac{W}{\rho\omega^3 d^5} = f\left(\frac{V}{d\omega}, \frac{\lambda}{d}, \frac{\mu}{\rho d^2 \omega}\right)$$

$V_{avg} = \frac{1}{2} V_c$ (laminar flow)

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3(a) $V_c = 1 m/s \Rightarrow V_{avg} = 0.5 m/s$

$\cancel{*}$ $\downarrow \Rightarrow (+)$ & $\uparrow \Rightarrow (-)$ for travelling mtd

$Re = \frac{\rho V d}{\mu} = \frac{1200 \times 0.5 \times 0.1}{0.1} = 600$ (laminar)

Since laminar flow frm (left to right) → dir of flow impt!

$V = \frac{\Delta P D^2}{32 \mu l}$ & $\Delta P, l$ is unknown!

$P_1 + \rho_l g(0.2 + \cancel{x}) - \rho_f g(0.2) - \cancel{\rho_l g(x)} = P_2$ $\rho_l = 1200$ & $\rho_f = 1600$

$P_1 - P_2 = (\rho_f - \rho_l) 0.2g = 400(0.2)(9.81) = 784.8 Pa$

$l = \frac{\Delta P D^2}{32 \mu V} = \frac{784.8 \times 0.1^{\cancel{2}}}{32(\cancel{0.1})(0.5)} = 4.905 m$

$\uparrow$ in $\Delta P \Rightarrow \uparrow l$, if (flow is inviscid) there is no viscosity & thus no friction. flow is frictionless

$h \downarrow$ to 0 & $l \to \infty$ since there's [no pressure change]

3(b) $Q = AV$ & dir flow is frm $A \to B$

$V = \frac{0.02\cancel{\pi}}{\cancel{\pi}(0.1)^2} = 2 m/s$ & $Re = \frac{\rho V d}{\mu} = \frac{1000 \times 2 \times 0.2}{0.001} = 400000$ (turbulent)

$f = \frac{0.316}{(400000)^{0.25}} = \cancel{0.0223}\ 0.012565$

$\frac{\cancel{P_A}}{\cancel{\rho g}} + \frac{\cancel{V_A^2}}{\cancel{2g}} + \cancel{Z_A} + H_p - h_L = \frac{P_B}{\rho g} + \frac{\cancel{V_B^2}}{\cancel{2g}} + \cancel{Z_B} \Rightarrow H_p = h_L$

$h_L = \left(f_1 \frac{l_1}{d_1} + \Sigma K_{L1}\right)\frac{V^2}{2g} + \left(f_2 \frac{l_2}{d_2} + \Sigma K_{L2}\right)\frac{V^2}{2g}$

$= \left(\frac{0.0126(50)}{0.2} + 2 + \frac{0.0126(400)}{0.2} + 5\right)\frac{2^2}{19.62} = 7.19 m$

$7.19 = K - 2000(0.02\pi)^2 \Rightarrow K = 15.1$

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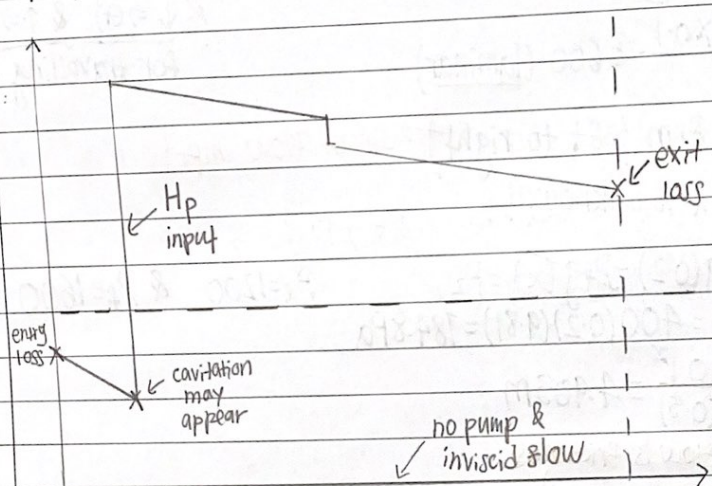

Cavitation most likely to occur at the pt b4 the pump ⇒ ~~EGL~~ pressure head @ lowest pt

w/o pump & inviscid flow ⇒ no flow is present frm A to B is there is equal pressures ~~@~~ & elevations at A&B ⇒ EGL will just be a straight line

$Z_2 - Z_1$ head loss

4(a)(i) System ~~chara~~ dd $= 60 + 500Q^2$
difference in elevation = 60m

4(a)(ii) $\omega = 2000 \times \frac{2\pi}{60} = 209.44$ rad/s

$H_p = E \Rightarrow 100 - 625Q^2 = 60 + 500Q^2 \Rightarrow Q = 0.1886 m^3/s$

$H_p = 100 - 625(0.1886)^2 = 77.78m$

$N_s = \frac{209.44\sqrt{0.1886}}{(9.81 \times 77.78)^{3/4}} = 0.626$ (radial flow)

4(a)(iii) $\eta = 600(0.1886) - 3750(0.1886)^3 = 88.0\%$

Flow (pump) $= \rho g Q H_p = 9810(0.1886)(77.78) = 143.9kW$

Input power $= \frac{143.9}{0.88} = 163.5kW$

4(a)(iv) 2 pumps in // $\Rightarrow$ Q becomes $\frac{Q}{2}$ & ~~Hp remains the same~~

$60 + 500Q^2 = 100 - 625(\frac{Q}{2})^2$

$656.25Q^2 = 40 \Rightarrow Q = 0.2469 m^3/s$

$H_p = 100 - \frac{625}{4}(0.2469)^2 = 90.48m$

~~Power~~ Pump flow $= 9810(0.2469)(90.48) = 219.14kW$

$\eta = 600(0.2469) - 3750(0.2469)^3 = 91.7\%$

Input power $= \frac{219.14}{0.917} = 239.0kW$

4(b) $NPSH_R = NPSH_A$ (minimum)

Pt 1 : @ atmosphere
Pt S : @ pt b4 the pump

① → Ⓢ

$$\frac{P_1 - P_v}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} - \overset{2m}{h_L} = \overset{5m}{\left(\frac{P_s - P_v}{\rho g} + \frac{V_s^2}{2g}\right)} + Z_s$$

$$V = \frac{0.1}{\pi (0.1)^2} = \overset{3.183}{\cancel{0.0318}} \text{ m/s}$$

$$\frac{10^5 - 2340}{9810} + \frac{3.183^2}{19.62} - 2 - 5 = Z_s$$

$$Z_s = 3.47 \text{ m}$$

∴ Max height of pump = 3.47m

$$5 = \frac{P_s - 2340}{9810} + \frac{3.183^2}{19.62} \Rightarrow P_s = 46.3 \text{ kPa}$$

∴ Min pressure at pump inlet = 46.3 kPa

new $Z_s = 1.6$m

$$\frac{P_{atm} - P_v}{\rho g} + \frac{V_1^2}{2g} - 2 - 1.6 = 5 \Rightarrow \frac{P_{atm} - 2340}{9810} = 8.0836$$

$$P_{atm} = 81640 \text{ Pa} = 81.6 \text{ kPa}$$

$$81.64 = 100\left[1 - \left(\frac{0.0065}{288}\right) Z\right]^{5.2586}$$

$$1 - \left(\frac{0.0065}{288}\right) Z = 0.962 \Rightarrow Z = \cancel{168m}\ 1680 \text{m}$$

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