Problem Statement: Intermediate (Requires some heat transfer knowledge, but needed equations are provided in the problem statement)

Our "super cool" coolant, "Boston cold" manufactured in Natick, MA is kept at a lower temperature than the ambient temperature. It flows through thin-walled, cylindrical pipes. We want to add insulation to the pipe. We assume that there is only one-dimensional, steady-state heat transfer in the radial direction. The insulation has constant thermal properties and the tube itself has negligible thermal resistance. The radiation heat exchange between the insulation surface and the surroundings is negligible.

Design an app using MATLAB App Designer to calculate the critical insulation thickness for this pipe. This app should have the following functionality:

- The temperature at the inner wall of the pipe is T_i and the ambient temperature is T_A and these values are entered by the user. If T_i is entered as higher than the ambient temperature, the user gets a warning to enter a lower value. The user can select the unit system, but if the temperature is not selected as Kelvin, it should be converted to Kelvin.
- The thermal conductivity, k of the insulating material is specified by the user. If it is not in $W/m \cdot K$, the units should be converted and displayed in $W/m \cdot K$
- Heat transfer coefficient of the ambient air, h is specified by the user. If the selected units are not in $W/m^2 \cdot K$, it should be converted.
- The app should calculate the critical insulation radius. Beyond the critical radius, increasing the insulation thickness increases the surface area of the pipe and leading to a greater heat loss by convection.
- For the inner pipe radius entered by the user, it should calculate the heat transfer rate per length of the pipe, outer surface temperature of the pipe.
- The app should plot the temperature profile of the insulation.
- The app should plot total thermal resistance vs. critical insulation radius
- The app should display the temperature profile equation.

Governing Equations:

$$R'_{tot} = \frac{ln(\frac{r_o}{r_i})}{2\pi k} + \frac{1}{2\pi r h}$$

$$R'_{tot} = total\ thermal\ resistance$$
$$k = thermal\ conductivity\ of\ the\ pipe$$
$$r_i = inner\ pipe\ radius$$
$$r_o = outer\ radius\ of\ the\ pipe$$

$$h = convection\ coefficient\ of\ the\ ambient\ air$$

$$q' = (T_A - T_i)/R'_{tot}$$

$$T_A = ambient\ air\ temperature$$

$$T_i = inner\ surface\ temperature\ of\ the\ pipe$$

$$q' = heat\ rate\ per\ pipe\ length$$

The heat rate per pipe length = conduction heat rate per pipe length through the insulation = convection heat rate per pipe length due to convection through the outer pipe surface

To obtain the temperature profile, integrate the heat equation with the boundary conditions, at the inner radius of the pipe, temperature should be T_i and at the outer surface, it should be T_o.

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0$$

The actual value of T_o can be obtained by setting heat rate per length equal to the convection rate per length:

$$q' = 2\,\pi\,r_o\,(T_A - T_o)$$

Further reading: [Fundamentals of Heat and Mass Transfer] by Bergman et. Al. or [Wikipedia – Thermal Insulation]