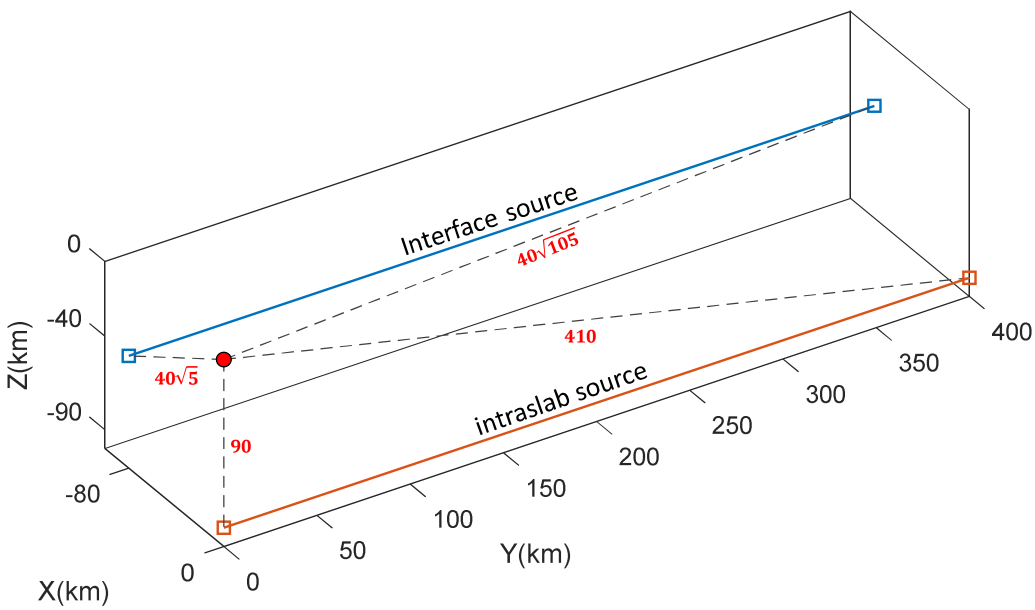
An interface line source 400 km long generates earthquakes of magnitude M=7 at a rate of $NM_{min} = 4$ events per year. A second intraslab line source also 400 km long generates earthquakes of magnitude M=7 at a rate of $NM_{min} = 6.5$ events per year. The interface source ends are located at XYZ(-80,0,-40) and XYZ(-80,400,-40), and the intraslab source ends are located at XYZ(0,0,-90km) and XYZ(0, 400, -90 km)

Use the Youngs et al. 1997 GMM to compute the seismic hazard curve for Sa(T=0.5s) at a rock site located at coordinates XYZ(10 km, 0, 0)



Hazard curve for the interface source (λ_y^1)

$$\ln Sa(0.5) = 0.2418 + 1.414M - 0.4 - 0.0048(10 - M)^3 - 2.36\ln(r + 1.7818e^{0.554M}) + 0.00607h$$

$$\ln Sa(0.5) = 9.853 - 2.36\ln(r + 86.1099) \ \text{ and } \quad \sigma \ = 1.45 - 0.1M = 0.75$$

The probability term $P(Sa > y|m = 7, r)$ is

$$P(Sa > y|m = 7, r) = 1 - \Phi\left(\frac{\log(y) - [9.853 - 2.36\ln(r + 86.1099)]}{0.75}\right)$$

With $f_M(m) = \delta(m - 7)$ and $f_R(r) = \frac{r}{L_f\sqrt{r^2 - r_{min}^2}}$, the hazard curve is

$$\lambda_y^1 = NM_{min}\int P(Sa > y|m, r)f_M(m)f_R(r)dmdr = NM_{min}\int_{r_{min}}^{r_{max}} P(Sa > y|m = 7, r)\frac{r}{L_f\sqrt{r^2 - r_{min}^2}}dr$$

$$\lambda_y^1 = 4\int_{40\sqrt{5}}^{40\sqrt{105}} \left\{1 - \Phi\left(\frac{\log(y) - [9.853 - 2.36\ln(r + 86.1099)]}{0.75}\right)\right\}\frac{r}{400\sqrt{r^2 - \left(40\sqrt{5}\right)^2}}dr$$

Hazard curve for the intraslab source (λ_y^2)

Analogously, the Youngs et al. 1997 model for the intraslab source gives

$$\ln Sa(0.5) = 0.2418 + 1.414M - 0.4 - 0.0048(10 - M)^3 - 2.36\ln(r + 1.7818e^{0.554M}) + 0.00607h + 0.3846$$

$$\ln Sa(0.5) = 10.5411 - 2.36\ln(r + 86.1099) \ \text{ and } \quad \sigma \ = 1.45 - 0.1M = 0.75$$

The probability term $P(Sa > y|m = 7, r)$ is

$$P(Sa > y|m = 7, r) = 1 - \Phi\left(\frac{\log(y) - [10.5411 - 2.36\ln(r + 86.1099)]}{0.75}\right)$$

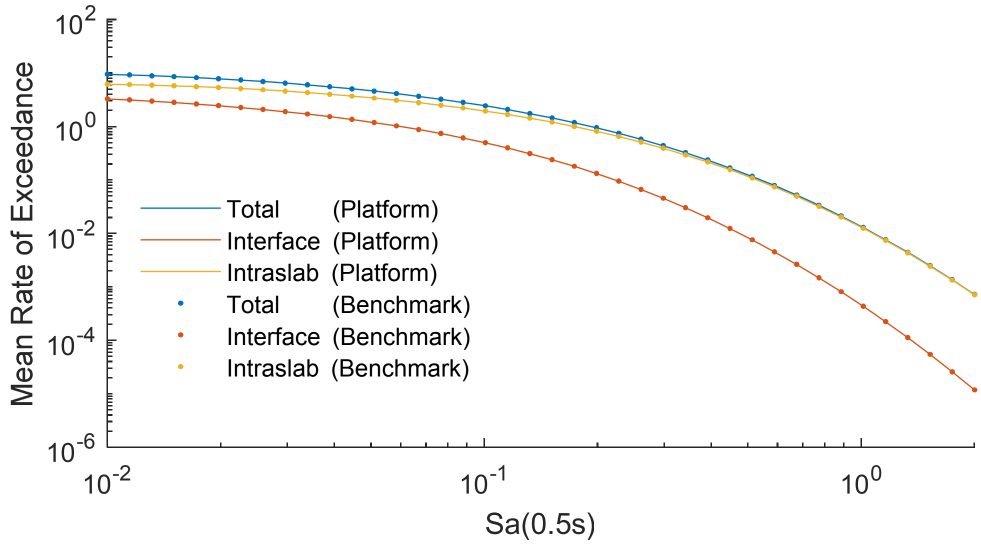
With $f_M(m) = \delta(m - 7)$ and $f_R(r) = \frac{r}{L_f\sqrt{r^2 - r_{min}^2}}$, the hazard curve is

$$\lambda_y^2 = NM_{min}\int P(Sa > y|m, r)f_M(m)f_R(r)dmdr = NM_{min}\int_{r_{min}}^{r_{max}} P(Sa > y|m = 6{,}5, r)\frac{r}{L_f\sqrt{r^2 - r_{min}^2}}dr$$

$$\lambda_y^2 = 6.5\int_{90}^{410} \left\{1 - \Phi\left(\frac{\log(y) - [10.5411 - 2.36\ln(r + 86.1099)]}{0.75}\right)\right\}\frac{r}{400\sqrt{r^2 - 90^2}}dr$$

Finally, the total hazard is computed by adding the contribution from both sources

$$\lambda_y = \lambda_y^1 + \lambda_y^2$$



Independent calculation in MATLAB:

```
% interface source
NMmin  = 4;
M      = 7; h  = 40;  ZT  = 0;
x1     = [-80,0  ,-40]; x2 = [-80,400,-40]; x0 = [0 0 0];
Lf     = norm(x1-x2);
rmin   = norm(x0-x1);
rmax   = norm(x0-x2);
r      = logsp(rmin+1e-6,rmax,1000000);
C      = [-0.400 -0.0048 -2.360 1.45 -0.1];
mu     = 0.2418+1.414*M+C(1)+C(2)*(10-
M).^3+C(3)*log(r+1.7818*exp(0.554*M))+0.00607*h+0.3846*ZT;
sigma  = C(4)+C(5)*min(M,8);

y       = logsp(0.01,2,40);
lambda1 = zeros(1,40);
fR = r./(Lf*sqrt(r.^2-rmin^2));
for i=1:length(y)
    P  = (1-normcdf((log(y(i)) - mu)/sigma));
    lambda1(i) = NMmin*trapz(r,P.*fR);
end

% intraslab source
NMmin  = 6.5;
M      = 7; h  = 90; ZT   = 1;
x1     = [0,0  ,-90]; x2  = [0,400,-90]; x0 = [0 0 0];
Lf     = norm(x1-x2);
rmin   = norm(x0-x1);
rmax   = norm(x0-x2);
r      = logsp(rmin+1e-6,rmax,1000000);
C      = [-0.400 -0.0048 -2.360 1.45 -0.1];
mu     = 0.2418+1.414*M+C(1)+C(2)*(10-
M).^3+C(3)*log(r+1.7818*exp(0.554*M))+0.00607*h+0.3846*ZT;
sigma  = C(4)+C(5)*min(M,8);
y      = logsp(0.01,2,40);
lambda2 = zeros(1,40);
fR = r./(Lf*sqrt(r.^2-rmin^2));
for i=1:length(y)
    P  = (1-normcdf((log(y(i)) - mu)/sigma));
    lambda2(i) = NMmin*trapz(r,P.*fR);
end

plot(y,[lambda1+lambda2;lambda1;lambda2],'.')
```

Computation of seismically induced slope displacements according to the Bray, Macedo & Travasarou 2017

$$P_0 = 1 - \Phi\left(-2.640 - 3.2\log k_y - 0.170\left(\log k_y\right)^2 - 0.490T_S\log k_y + 2.094T_s + 2.908\log Sa\right)$$

$$\ln D = -6.896 - 3.353\log k_y - 0.390\left(\log k_y\right)^2 + 0.538\log k_y \log Sa + 3.06\log Sa - 0.225\,(\log Sa)^2 + 3.801T_s + 0.55M - 0.803T_s^2$$

$$\sigma_{lnD} = 0.73$$

where $Sa = Sa(1.5T_s)$. For a slope with $k_y = 0.2$, $T_s = 0.333\ s$, and $M = 7$ these equations reduce to

$$P_0 = 1 - \Phi(2.658\log Sa + 3.218)$$

$$\overline{\ln D} = 2.518 + 2.194\log Sa - 0.225(\log Sa)^2$$

The probability of not exceeding a displacement value d conditioned on Sa is

$$P\left(D < d \middle| y^i\right) = P_0 + (1 - P_0)\cdot\Phi\left(\frac{\overline{\ln D} - \ln d}{\sigma_{lnD}}\right)$$

Therefore, the probability of exceedance is

$$P\left(D > d \middle| y^i\right) = 1 - P\left(D < d \middle| y^i\right)$$

The displacement hazard curve is computed by adding the hazard contribution from the two seismic sources

$$\lambda_D(d) = -\sum_{i=1}^{2}\int_0^\infty P\left(D > d \middle| y^i\right)\left(\frac{d\lambda_y^i}{dy^i}\right)\,dy^i$$

$$\lambda_D(d) = -\sum_{i=1}^{2}\int_0^\infty P\left(D > d \middle| y^i\right)d\lambda_y^i$$

This integral requires a numerical integration. In Matlab the following integration scheme was adopted

$$\lambda_D(d) = -\sum_{i=1}^{2} trapz(\lambda_y^i, P\left(D > d \middle| y^i\right))$$

Independent Matlab calculation

```
ky        = 0.2;
Ts        = 1/3;
M         = 7;
d         = logsp(1,200,40);
im        = logsp(0.001,3,200);
Nd        = length(d);
Pzero     = 1-normcdf(-2.640-3.200*log(ky)-0.170*log(ky)^2-
            0.490*Ts*log(ky)+2.094*Ts+2.908*log(im),0,1);
lnD       = -6.896-3.353*log(ky)-0.390*(log(ky))^2+ 0.538*log(ky)*log(im)+
            3.060*log(im)-0.225*(log(im)).^2+3.801*Ts+0.55*M-0.803*Ts^2;
sig       = 0.73;
lambdaD1 = zeros(1,Nd);
lambdaD2 = zeros(1,Nd);

for i=1:Nd
    CDF = Pzero + (1-Pzero).*(logncdf(d(i),lnD,sig));
    lambdaD1(i)=-trapz(lambda1,1-CDF);
    lambdaD2(i)=-trapz(lambda2,1-CDF);
end
```