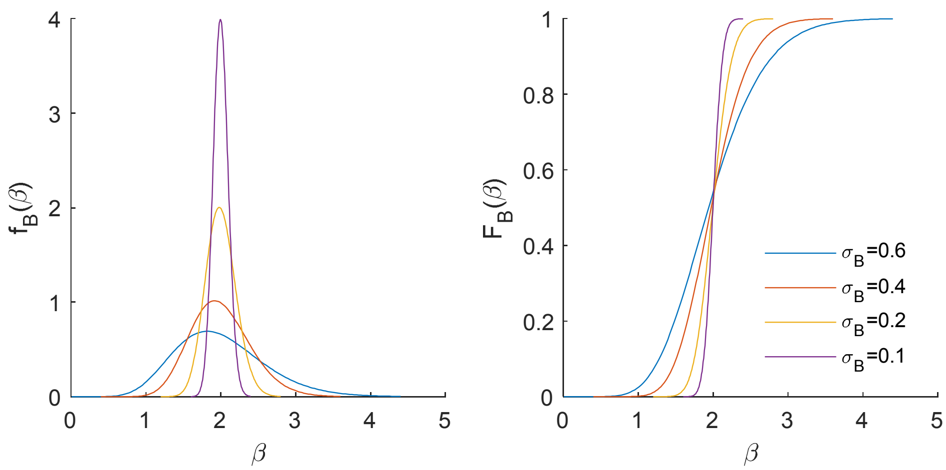
A single point source generates earthquakes of magnitude with a truncated exponential distribution (NM_{min}=2, $M_{min} = 4$, $M_{max} = 8$, $\bar{\beta} = 2$, $\sigma_\beta = \{0.1,\ 0.2,\ 0.4,\ 0.6\}$). Use the Sadigh et al. 1997 GMM (strike-slip, rock) and set $\sigma_{lnPGA} = 0$ to compute the seismic hazard curve for PGA for site located 10 km from the hypocenter.

To account for the uncertainty in β, use a Gamma probability density function (Keller, 2014) with mean $\bar{\beta} = 2$ and standard deviation σ_β. The functional for the Gamma pdf and cdf are, respectively

$$f_B(\beta) = \frac{\beta^{k-1} e^{-\beta/\theta}}{\Gamma(k\theta)\theta^k} \qquad \text{and} \qquad F_B(\beta) = \frac{1}{\Gamma(k)}\gamma(k, \beta/\theta)$$

where $k = \left(\frac{\bar{\beta}}{\sigma_\beta}\right)^2$ is the shape parameter, $\theta = \frac{\sigma_\beta^2}{\beta}$ is the scale parameter, and $\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$ is the incomplete lower Gamma function. For the different σ_β values considered, the pdf and cdf of β are shown herein.



Evaluating Sadigh et al. 1997 (strike-slip, rock) for PGA and $r = 10\ km$ leads to

$$\ln PGA = \begin{cases} -0.624 + 1.1m - 2.100\ln(10 + \exp(\ 1.29649\ + 0.250 \cdot m)) & if\ m \leq 6.5 \\ -1.274 + 1.1m - 2.100\ln(10 + \exp(-0.48451 + 0.524 \cdot m)) & if\ m > 6.5 \end{cases}$$

Then, the probability that PGA exceeds a test value y is simply $P(Sa > y|m) = H(PGA - y)$, where H is the Heaviside function.

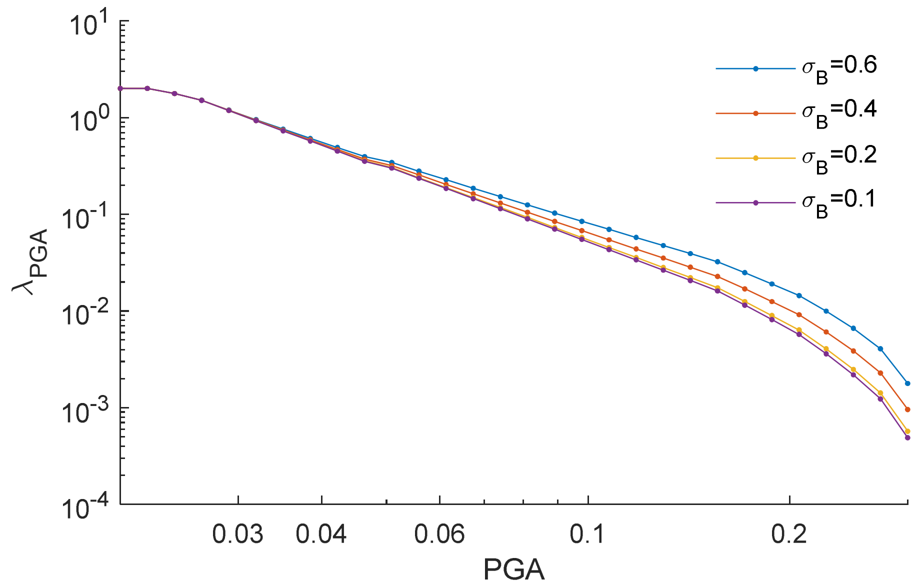
The probability density function for a Truncated Exponential magnitude is

$$f_M(m|\beta) = \frac{\beta\, e^{-\beta(m-M_{min})}}{1-e^{-\beta(M_{max}-M_{min})}}$$

Finally, based on total probabilities, the annual rate of exceedance λ_{PGA} is

$$\lambda_{PGA} = NM_{min}\int_{M_{min}}^{M_{max}}\int_0^{\infty} P(Sa > y|m) f_M(m|\beta) f_B(\beta) d\beta dm$$

$$\lambda_{PGA}(y) = NM_{min}\int_{M_{min}}^{M_{max}}\int_0^{\infty} H(PGA-y)\frac{\beta\, e^{-\beta(m-M_{min})}}{1-e^{-\beta(M_{max}-M_{min})}}\frac{\beta^{k-1}e^{-\beta/\theta}}{\Gamma(k\theta)\theta^k} d\beta dm$$



Conclusion: Variability of β values has very little effect on the seismic hazard curve. Recall that realistic values of σ_β are in the order of 0.06 to 0.15 (e.g., see Poulos et al 2019)

Independent MATLAB calculation

```
NMmin  = 2;
Mmin   = 4;
Mmax   = 8;
Beta   = 2;
sigmaB = [0.6 0.4 0.2 0.1];
rrup   = 10;

M      = linspace(Mmin,Mmax,100);
y      = logsp(0.02,0.3,30);

hold on
for ii=1:length(sigmaB)
    bint = [max(Beta-4*sigmaB(ii),0.01),Beta+4*sigmaB(ii)];
    b    = linspace(bint(1),bint(2),100);
    pd   = makedist('gamma',Beta^2/sigmaB(ii)^2,sigmaB(ii)^2/Beta);
    pd   = truncate(pd,bint(1),bint(2));

    [m,beta]= meshgrid(M,b);
    rrup    = ones(size(m))*20;
    lnpga   = Sadigh1997(0,m,rrup,'strike-slip','rock');
    PGA     = exp(lnpga);
    lambda = zeros(size(y));
    for i=1:length(y)
        P         = heaviside(PGA-y(i));
        fm        = beta.*exp(-beta.*(m-Mmin))./(1-exp(-beta*(Mmax-Mmin)));
        fB        = pdf(pd,beta);
        f         = P.*fm.*fB;
        lambda(i) = NMmin*trapz(M,trapz(b,f));
    end
    plot(y,lambda,'.-','displayname',['\sigma_B=',sprintf('%g',sigmaB(ii))]),
end
xlabel('PGA')
ylabel('\lambda_{PGA}')
set(gca,'yscale','log','xscale','log','xtick',[0.03 0.04 0.06 0.1 0.2 0.3])
L=legend;
L.Box='off';
```

Independent MATLAB implementation

```
NMmin  = 2;
Mmin   = 5;
Mmax   = 6.5;
b      = 1;
beta   = b*log(10);
rrup   = 20;
M      = linspace(Mmin,Mmax,100000);
C      = [-0.624 1.0 0.000 -2.100 1.29649 0.250 0.0];
lny    = C(1)+C(2)*M+C(4)*log(rrup+exp(C(5)+C(6)*M));

C      = [1.39 0.14 0.38 7.21];
sigma  = C(1)-C(2)*M;
y      = logsp(0.01,1,30);
lambda = zeros(size(y));
for i=1:length(y)
    xhat = (log(y(i))-lny)./sigma;
    P    = 1-normcdf(xhat);
    fm   = beta*exp(-beta*(M-Mmin))./(1-exp(-beta*(Mmax-Mmin)));
    lambda(i) = NMmin*trapz(M,P.*fm);
end
loglog(y,lambda,'.-')
```

References

- Keller, M., A. Pasanisi, M. Marcilhac, T. Yalamas, R. Secanell, and G. Senfaute (2014). A Bayesian methodology applied to the estimation of earthquake recurrence parameters for seismic hazard assessment, Qual. Reliab. Eng. Int. 30, no. 7, 921–933

- Poulos, A., Monsalve, M., Zamora, N., & de la Llera, J. C. (2018). An Updated Recurrence Model for Chilean Subduction Seismicity and Statistical Validation of Its Poisson Nature. Bulletin of the Seismological Society of America, 109(1), 66-74.