

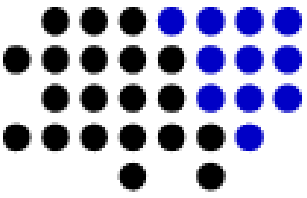


CHAPTER 1.

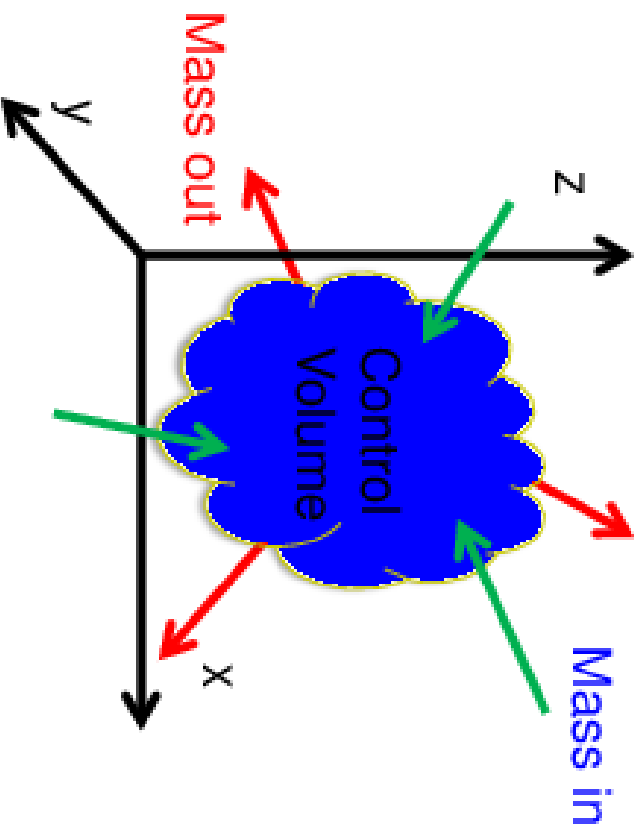
SINGLE PHASE FLOW

EQUATIONS

Single Phase Flow Equations



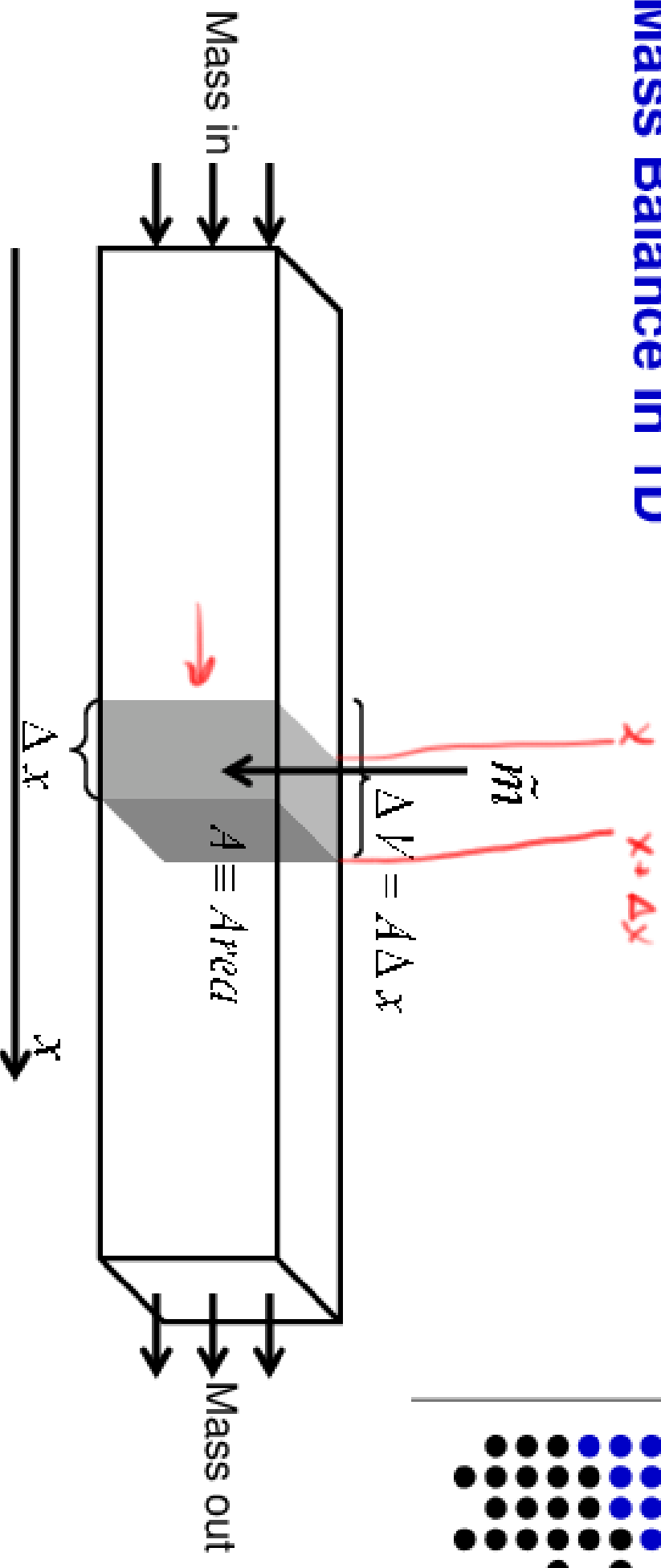
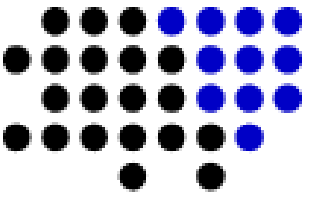
Mass In – Mass Out +/- Generation/Consumption = Accumulation



→ Control volume can be any shape

→ Mass enters and leaves the control volume in any direction

Mass Balance in 1D

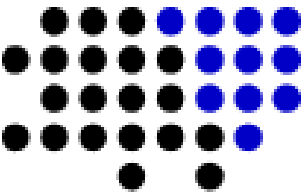


$$\underbrace{\dot{m}_x \bigg|_x}_{\text{mass in}} A \Delta t - \underbrace{\dot{m}_x \bigg|_{x+\Delta x}}_{\text{mass out}} A \Delta t + \underbrace{\tilde{m} \Delta V}_{\text{generation}} \Delta t = \underbrace{\cancel{\rho \phi} \Delta V}_{\text{accumulation}} \bigg|_{t+\Delta t} - \underbrace{\cancel{\rho \phi} \Delta V}_{\text{accumulation}} \bigg|_t$$

$\tilde{m} \equiv \text{mass of box}$	$\left[\frac{M}{L^3} \right]$	$A \equiv \text{Area}$	$\left[L^2 \right]$	$\tilde{m} \equiv \text{Sum of source rate}$	$\left[\frac{M}{L^3} \right]$	$\Delta V \equiv \text{Volume}$	$\left[L^3 \right]$	$\rho \equiv \text{Density}$	$\left[\frac{M}{L^3} \right]$	$\phi \equiv \text{Property}$	$\left[\frac{L}{L} \right]$
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Continuity Equation in 1D

$$\underbrace{\dot{m}_x|_x \Delta \Delta t}_{\text{mass in}} - \underbrace{\dot{m}_x|_{x+\Delta x} \Delta \Delta t}_{\text{mass out}} + \underbrace{\tilde{m} \Delta V \Delta t}_{\text{generation}} = \underbrace{\rho \phi \Delta V|_{t+\Delta t} - \rho \phi \Delta V|_t}_{\text{accumulation}}$$



Divide the mass balance by $\Delta x \Delta t$

$$\lim_{\Delta x \rightarrow 0} \lim_{\Delta t \rightarrow 0} \frac{\dot{m}_x|_x - \dot{m}_x|_{x+\Delta x}}{\Delta x} + \tilde{m} = \frac{\rho \phi|_{t+\Delta t} - \rho \phi|_t}{\Delta t}$$

Making Δx and Δt very small, they become dx and dt

$$\lim_{\Delta x \rightarrow 0} \frac{\dot{m}_x|_x - \dot{m}_x|_{x+\Delta x}}{\Delta x} = - \frac{\partial \dot{m}_x}{\partial x} \quad \text{and} \quad \lim_{\Delta t \rightarrow 0} \frac{\rho \phi|_{t+\Delta t} - \rho \phi|_t}{\Delta t} = \frac{\partial(\rho \phi)}{\partial t}$$

$\dot{m} = \rho u$

Expressing mass flux in terms of velocity ($\dot{m}_x = \rho u_x$), and subtracting \tilde{m} we get the continuity equation to be


$$\boxed{- \frac{\partial(\rho u_x)}{\partial x} = \frac{\partial(\rho \phi)}{\partial t} - \tilde{m}}$$

Continuity Equation

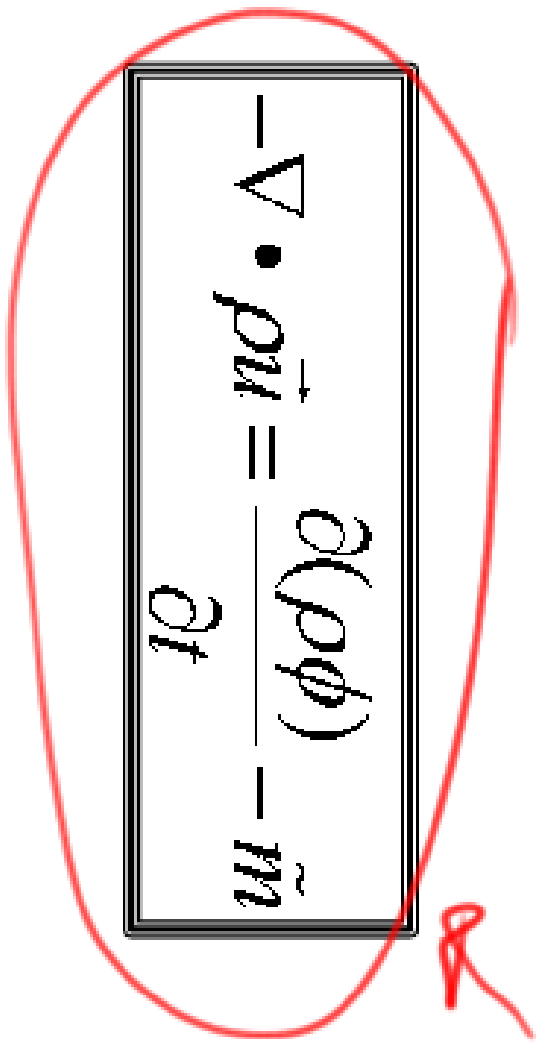
$\dot{m} \equiv \text{mass flux}$	$\left[\frac{\text{kg}}{\text{m}^2 \cdot \text{s}} \right]$	$\tilde{m} \equiv \text{Sink or source rate}$	$\left[\frac{\text{kg}}{\text{m}^3 \cdot \text{s}} \right]$	$\rho \equiv \text{Density}$	$\left[\frac{\text{kg}}{\text{m}^3} \right]$	$\phi = \text{fractional volume}$	$\left[\frac{\text{m}^3}{\text{m}^3} \right]$	$u = \text{velocity}$	$\left[\frac{\text{m}}{\text{s}} \right]$
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Continuity Equation in Multi-Dimensions

In 3D Cartesian coordinates the equation becomes

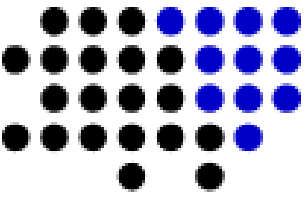

$$-\left[\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} \right] = \frac{\partial(\rho\phi)}{\partial t} - \tilde{m}$$

More general form of the Continuity Equation

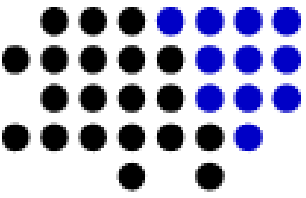

$$-\nabla \cdot \rho \vec{u} = \frac{\partial(\rho\phi)}{\partial t} - \tilde{m}$$

Other names:

- Mass Conservation Equation
- Material Balance Equation



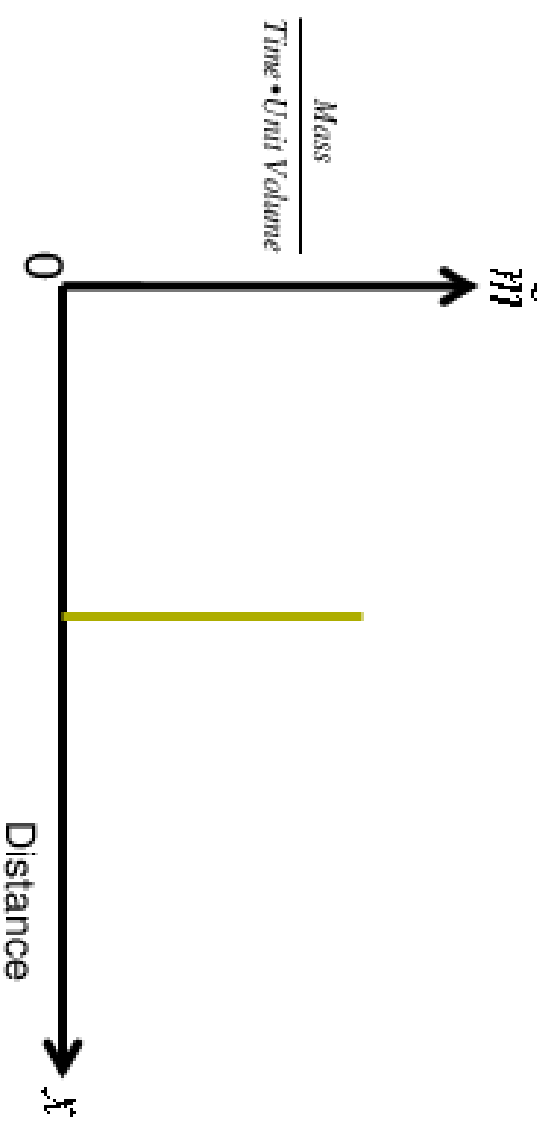
Continuity Equation: What is \tilde{m} ?



- \tilde{m} accounts for mass entering or leaving a control volume from a source or sink (e.g. well)
- We model injection/production wells as discrete sources/sinks so they occupy no volume
- Taking limits without using this variable creates difficulty

$$\tilde{m} = \frac{\text{mass}}{\text{time-volume}}$$

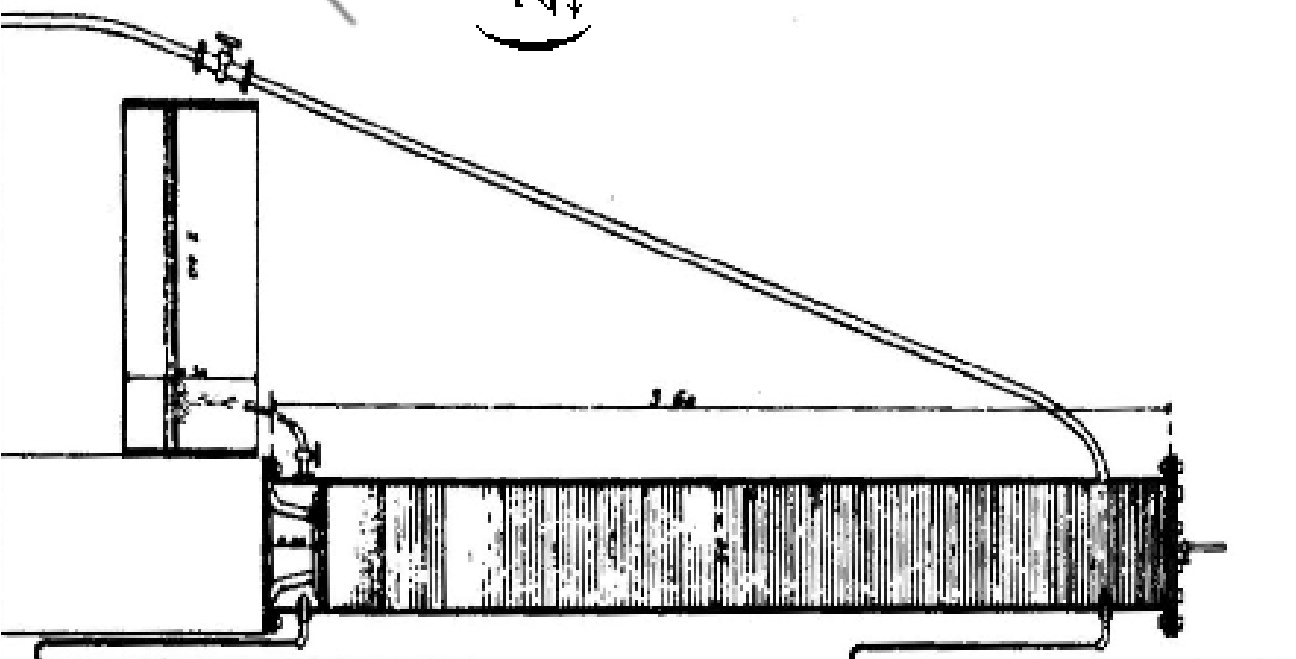
Units of mass/time per volume
of the control volume



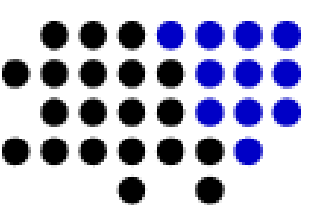
Darcy's Law

$$Q = - \frac{k_A \Delta \Phi}{\mu L}$$

$$\vec{n} = -\frac{\vec{k}}{\mu} \left(\nabla p + \rho \vec{g} \right)$$

$$1 \text{ darcy} = 9.869 \times 10^{-9} \text{ cm}^2$$
$$1 \text{ darcy} = 1.062 \times 10^{-11} \text{ ft}^2$$


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HERNAN LARREA
INVESTIGADOR EN EL INSTITUTO VENEZOLANO DE INVESTIGACIONES Y MEDICINA AMBIENTAL

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For example, the *Journal of the American Medical Association* (JAMA) has a policy that the copyright in the text of all its articles belongs to the publisher, not the author. The policy is that the author retains the right to use the text in other works, but the publisher owns the copyright. This policy is not unique to JAMA; many other journals have similar policies. The policy is based on the fact that the publisher is the one who bears the cost of publishing the article, and the publisher is the one who is responsible for the quality of the publication. The policy is also based on the fact that the publisher is the one who is responsible for the distribution of the publication. The policy is not based on the fact that the publisher is the one who is responsible for the content of the publication.

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VICTOR DANILIN[†], PHOTON

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Darcy's Experiment

"The Public Fountains of the Town of Dijon", Dalmont, Paris (1856)

$$Q \equiv \frac{1}{2} \ln \frac{I_{\text{max}}}{I_{\text{min}}}$$

$$k \equiv L^{2.0710(4)51(1)} \left[\frac{\text{cm}^3}{\text{g}} \right]$$

$$\Phi \equiv \frac{F_1}{F_2} \text{ or } \frac{F_1}{F_2} \Phi$$

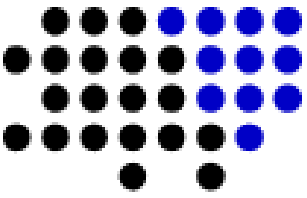
$$A \equiv \begin{bmatrix} 2.7 & 1.5 \\ 1.5 & 1.1 \end{bmatrix}$$

$$p \equiv p_{\text{cosmic}}$$

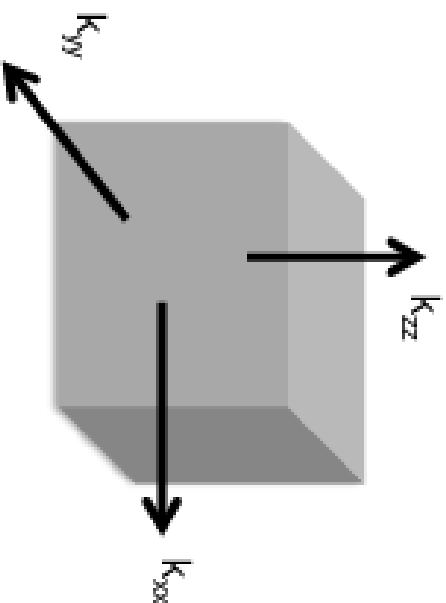
$\mathbb{R} \equiv \text{Growth}$

Permeability (k)

Medium is **homogeneous** if permeability does not vary with space, but **heterogeneous** if non-uniform, $k=k(x,y,z)$



Medium is **isotropic** if permeability is the same in all directions, but **anisotropic** if $k_x \neq k_y \neq k_z$. Oftentimes $k_x \approx k_y$ but $k_x/k_z \approx 1-10$



$$\underline{k} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \approx \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}$$

Off-diagonal terms are only zero if we choose a coordinate system that makes this true, but generally assume this is the case

Medium could be homogenous and anisotropic or heterogeneous and isotropic

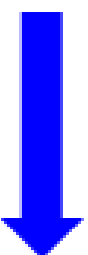
Darcy's Law into Continuity Equation

In 1D and horizontal flow (gravity neglected)

$$u_x = -\frac{k}{\mu} \nabla p = -\frac{k}{\mu} \frac{\partial p}{\partial x}$$

Replacing fluid velocity in the continuity equation

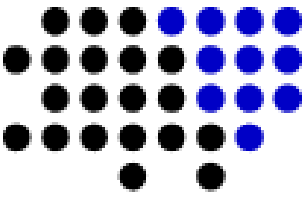
$$-\frac{\partial(\rho u_x)}{\partial x} = \frac{\partial(\rho \phi)}{\partial t} - \tilde{m}$$



$$\frac{\partial}{\partial x} \left(\rho \frac{k}{\mu} \frac{\partial p}{\partial x} \right) = \frac{\partial(\rho \phi)}{\partial t} - \tilde{m}$$

Assuming homogenous and isotropic permeability and viscosity (NOT always the case!)

$$\frac{k}{\mu} \frac{\partial}{\partial x} \left(\rho \frac{\partial p}{\partial x} \right) = \frac{\partial(\rho \phi)}{\partial t} - \tilde{m}$$



$k \equiv \text{Permeability} \left[\frac{L^2}{L^2} \right]$

$\mu \equiv \text{Viscosity} \left[\frac{L^2}{L^2} \right]$

$\rho \equiv \text{Porosity} \left[\frac{L^3}{L^3} \right]$

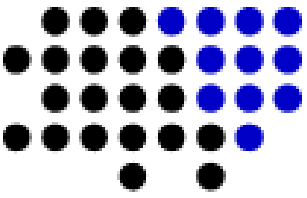
$\rho \equiv \text{Density} \left[\frac{M}{L^3} \right]$

$\phi = \text{Porosity} \left[\frac{L^3}{L^3} \right]$

$\tilde{m} \equiv \text{Sink or source rate} \left[\frac{L^3}{L^3} \right]$

Formation Volume Factor (B_w)

We measure volume at surface but do the mass balance at the reservoir



Surface/Standard Conditions

$$P = 14.7 \text{ psi}; T = 60^\circ\text{F}; V = V_{sc}$$

depth



Reservoir Conditions

$$P = P_R; T = T_R; V = V_R$$

$$\rho_{RC} = \frac{\rho_{SC}}{B_w}$$



$$B_o = \frac{V_{RC}}{V_{SC}} = \frac{\rho_{SC}}{\rho_{RC}}$$

Formation volume factor and ρ depend on both pressure and temperature

- B_o = formation volume factor for oil (> 1.0)
- B_g = formation volume factor for gas ($<< 1.0$)
- B_w = formation volume factor for water (~ 1.0)

$$\Delta \vec{r} = \text{Velocity} \left[\frac{m}{s} \right]$$

$$P \equiv \text{Pressure}$$

$$\left[\frac{N}{m^2} \right]$$

$$T = \text{Temperature} [K]$$

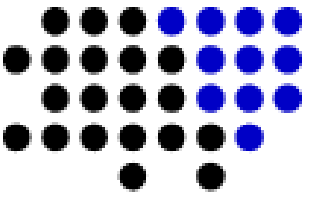
$$B_o \equiv \text{Volume Ratio (True)}$$

$$\left[\frac{V}{V} \right]$$

$$\rho \equiv \text{Density}$$

$$\left[\frac{M}{L^3} \right]$$

Formation Volume Factor—Continuity Equation



At reservoir conditions, density is:

$$\rho = \rho_{RC} = \frac{\rho_{sc}}{B_w}$$

Replacing density in continuity equation and divide through by ρ_{sc} (a constant)

$$\frac{k}{\mu} \frac{\partial}{\partial x} \left(\frac{\rho_{sc}}{B_w} \frac{\partial p}{\partial x} \right) = \frac{\partial}{\partial t} \left(\frac{\rho_{sc}}{B_w} \phi \right) - \tilde{m} \quad \rightarrow \quad \frac{k}{\mu} \frac{\partial}{\partial x} \left(\frac{1}{B_w} \frac{\partial p}{\partial x} \right) = \frac{\partial}{\partial t} \left(\frac{\phi}{B_w} \right) - \frac{\tilde{m}}{\rho_{sc}}$$

Using the product rule on the left-hand side of the equation

$$\frac{k}{\mu} \left[\frac{1}{B_w} \frac{\partial^2 p}{\partial x^2} + \frac{\partial}{\partial x} \left(\frac{1}{B_w} \right) \frac{\partial p}{\partial x} \right] = \frac{\partial}{\partial t} \left(\frac{\phi}{B_w} \right) - \frac{\tilde{m}}{\rho_{sc}}$$

...And chain rule on the left hand side,

$$\frac{k}{\mu} \left[\frac{1}{B_w} \frac{\partial^2 p}{\partial x^2} + \frac{\partial}{\partial x} \left(\frac{1}{B_w} \right) \left(\frac{\partial p}{\partial x} \right) \right] = \frac{\partial}{\partial t} \left(\frac{\phi}{B_w} \right) - \frac{\tilde{m}}{\rho_{sc}}$$

$\rho \equiv \text{Density}$	$\left[\frac{M}{L^3} \right]$	$V_f \equiv \text{Formation Volume}$	$\left[\frac{L^3}{L^3} \right]$	$k \equiv \text{Permeability}$	$\left[\frac{L^2}{L^2} \right]$	$\mu \equiv \text{Viscosity}$	$\left[\frac{M}{L T} \right]$	$\rho \equiv \text{Density}$	$\left[\frac{M}{L^3} \right]$	$\tilde{m} \equiv \text{Skin or wellbore effect}$	$\left[\frac{M}{L^3 T} \right]$
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