

mass in - mass out + injected/produced = accumulation

$$\rho^{RC} q_{i-1/2} \Delta t - \rho^{RC} q_{i+1/2} \Delta t + \rho^{RC} q_i^{RC} \Delta t = m_i^{n+1} - m_i^n$$

$$m_i = \rho^{RC} \phi V_i, \quad q_{i-1/2} = \frac{kA}{\mu B_w} \frac{P_i - P_{i-1}}{\Delta x} = T_{i-1/2} B_w (P_i - P_{i-1})$$

$$q_i^{RC} = B_w \underline{Q_i^{sc}}$$

$$\rho^{rc} B_w T_{i-1/2} (P_i - P_{i-1}) \Delta t + \rho^{rc} B_w T_{i+1/2} (P_i - P_{i+1}) \Delta t + \rho^{rc} B_w Q_i^{sc} \Delta t = V_i [(\rho^{rc} \phi)^{n+1} - (\rho^{rc} \phi)^n]_i$$

Divide by $\rho^{sc} \Delta t$

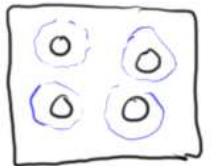
$$T_{i-1/2} (P_i - P_{i-1}) + T_{i+1/2} (P_i - P_{i+1}) + Q_i^{sc} = \frac{V_i}{\Delta t} \left[\left(\frac{\phi}{B_w} \right)^{n+1} - \left(\frac{\phi}{B_w} \right)^n \right]_i$$

$$\begin{aligned} \left(\frac{\phi}{B_w} \right)^{n+1} - \left(\frac{\phi}{B_w} \right)^n &= \left(\frac{\phi}{B_w} \right)^{n+1} - \left[\frac{\phi^{n+1}}{B_w^n} - \frac{\phi^{n+1}}{B_w^n} \right] - \left(\frac{\phi}{B_w} \right)^n \\ &= \phi^{n+1} \left(\frac{1}{B_w^{n+1}} - \frac{1}{B_w^n} \right) + \frac{1}{B_w^n} (\phi^{n+1} - \phi^n) \end{aligned}$$

Recall linear approx,

$$\frac{1}{B_w^n} \approx \frac{1}{B_w^0} [1 + c_f (P^n - P^0)]$$

$$\phi^n \approx \phi^0 [1 + c_r (P^n - P^0)]$$



$$\left(\frac{1}{B_\omega^{n+1}} - \frac{1}{B_\omega^n} \right) \approx \frac{1}{B_\omega^0} \cancel{\left(1 + C_f (p^{n+1} - p^0) \right)} - \frac{1}{B_\omega^0} \cancel{\left(1 + C_f (p^n - p^0) \right)} = \frac{C_f}{B_\omega^0} (p^{n+1} - p^n)$$

$$(\phi^{n+1} - \phi^n) \approx \phi^0 \cancel{\left(1 + C_r (p^{n+1} - p^0) \right)} - \phi^0 \cancel{\left(1 + C_r (p^n - p^0) \right)} = \phi^0 C_r (p^{n+1} - p^n)$$

$$\left(\frac{\phi}{B_\omega} \right)^{n+1} - \left(\frac{\phi}{B_\omega} \right)^n = \frac{C_f \phi^{n+1}}{B_\omega^0} (p^{n+1} - p^n) + \frac{\phi^0 C_r}{B_\omega^0} (p^{n+1} - p^n)$$

$$= \frac{\phi}{B_\omega} (C_f + C_r) (p^{n+1} - p^n) = \frac{\phi C_t}{B_\omega} (p^{n+1} - p^n)$$

$$T_{i-1/2} (p_{i-1} - p_i) + T_{i+1/2} (p_{i+1} - p_i) = \frac{V_i \phi C_t}{B_\omega \Delta t} (p_i^{n+1} - p_i^n) - Q_i^{sc}$$

$$T_{i-1/2} p_{i-1}^n - (T_{i-1/2} + T_{i+1/2}) p_i^n + T_{i+1/2} p_{i+1}^n = \text{RHS}$$