function Thalf(k, j)

returns T_{k-j}

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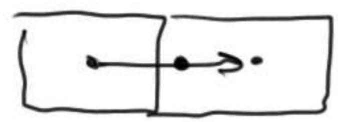
1. Define variables
2. Assemble T, B, Q

```
for i = 1:N
    T(i, i-1) = -Thalf(i, i-1)
    T(i, i+1) = -Thalf(i, i+1)
    T(i, i) = -sum(i)
    B(i, i)
    Q(i)
end
```



3. Update $\vec{p}^{\,n+1}$ w/ time

$$\frac{\partial p}{\partial t} = \frac{k}{\mu \phi c_t B_w} \frac{\partial^2 p}{\partial x^2}$$ 1D homogeneous

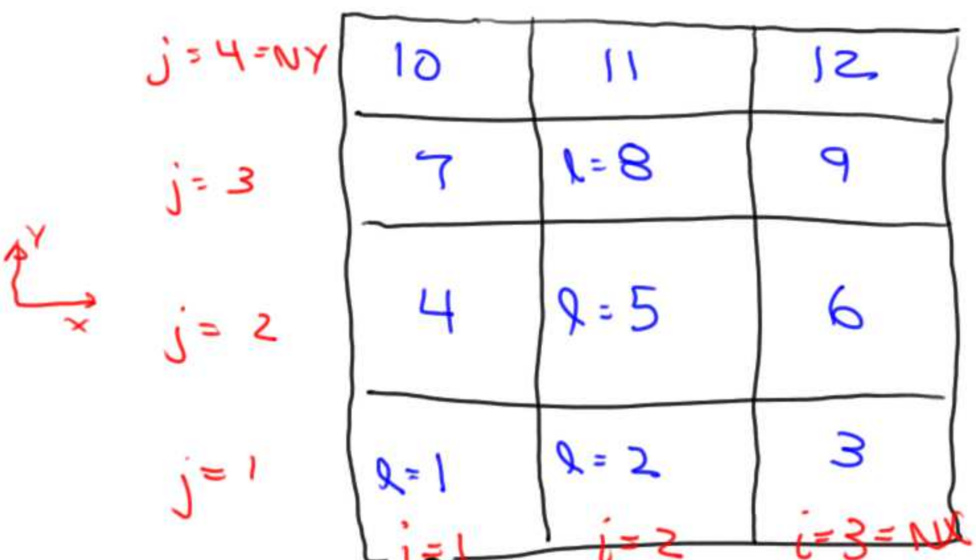
$$\phi c_t \frac{\partial p}{\partial x} = \frac{\partial}{\partial x}\left(\frac{k}{\mu B_w} \frac{\partial p}{\partial x} \right)$$ 1D heterogeneous

$$\phi c_t \frac{\partial p}{\partial t} = \frac{\partial}{\partial x}\underbrace{\left(\frac{\overset{k_x}{k_x}}{\mu B_w} \frac{\partial p}{\partial x} \right)} + \frac{\partial}{\partial y}\left(\frac{\overset{k_y}{k_y}}{\mu B_w} \frac{\partial p}{\partial y} \right) + \tilde{q}^{sc}$$ 2D heterogeneous, anisotropic w/ wells

To solve:

1.) Finite Diff.

2.) Control Volume

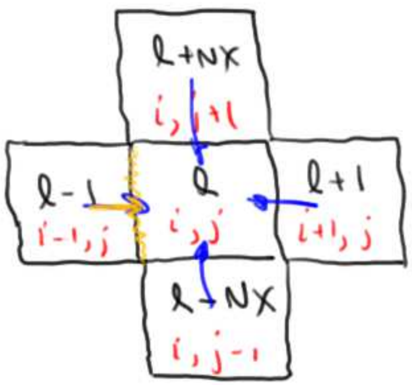


$l = 8 \rightarrow \begin{cases} i = 2 \\ j = 3 \end{cases}$

$$l = (j-1)\,NX + i$$

Mass balance on block "ℓ"

in - out + generation/. = accumulation



$$q^{sc}_{i-1/2,j} = TX_{i-1/2,j}\left(P_{i-1,j} - P_{i,j}\right)$$

$$= TX_{i-1/2,j}\left(P_{\ell-1} - P_{\ell}\right)$$

$$q^{sc}_{i+1/2,j} = TX_{i+1/2,j}\left(P_{i+1,j} - P_{i,j}\right)$$

$$= TX_{i+1/2,j}\left(P_{\ell+1} - P_{\ell}\right)$$

$$q^{sc}_{i,j-1} = TY_{i,j-1/2}\left(P_{i,j-1} - P_{i,j}\right)$$

$$= TY_{i,j-1/2}\left(P_{\ell-NX} - P_{\ell}\right)$$

$$q^{sc}_{i,j+1} = TY_{i,j+1/2}\left(P_{\ell+NX} - P_{\ell}\right)$$

Accumulation

$$B = \frac{V_{i,j}\, c_t\, \phi_{i,j}}{B_w \Delta t}\left(P_{i,j}^{n+1} - P_{i,j}^{n}\right) = \frac{V_\ell\, c_t\, \phi_\ell}{B_w \Delta t}\left(P_\ell^{n+1} - P_\ell^{n}\right)$$

Sources/Sinks

$$Q_\ell^{sc}$$

$$TX_{i-1/2,j}\left(P_{\ell-1} - P_\ell\right) + TX_{i+1/2,j}\left(P_{\ell+1} - P_\ell\right) + TY_{i,j-1}\left(P_{\ell-Nx} - P_\ell\right) + TY_{i,j}\left(P_{\ell+Nx} - P_\ell\right)$$

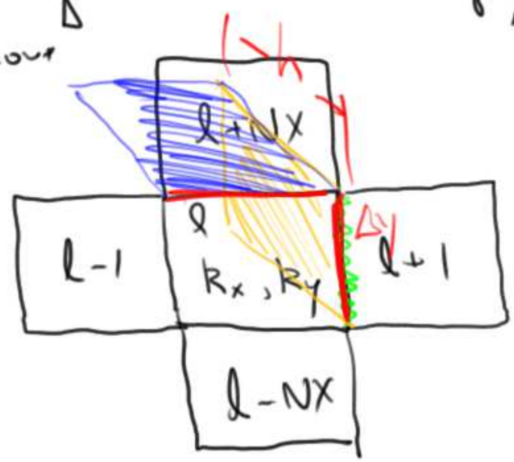
$$= \frac{V_\ell\, c_t\, \phi_\ell}{B_w \Delta t}\left(P_\ell^{n+1} - P_\ell^{n}\right) - Q_\ell^{sc}$$

Implicit

$$\left(\vec{\vec{T}} + \frac{\vec{B}}{\Delta t}\right)\vec{P}^{n+1} = \frac{B}{\Delta t}\vec{P}^{n} + \vec{Q}^{sc}$$

Heterogeneity + anisotropy

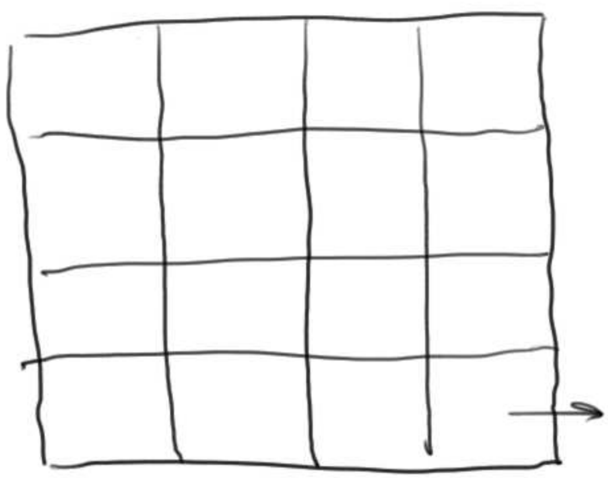
Heterogeneous



$$k_{x\,i+1/2,j} = \frac{\Delta x_{i,j} + \Delta x_{i+1,j}}{\dfrac{\Delta x_{i,j}}{k_{x\,i,j}} + \dfrac{\Delta x_{i+1,j}}{k_{x\,i+1,j}}}, \quad TX_{i+1/2,j} = \frac{k_{x\,i+1/2,j}\left(\Delta y_{i,j}\, h\right)}{\mu B_w \Delta x_{i+1/2,j}}$$

$$k_{y\,i,j+1/2} = \frac{\Delta y_{i,j} + \Delta y_{i,j+1}}{\dfrac{\Delta y_{i,j}}{k_{y\,i,j}} + \dfrac{\Delta y_{i,j+1}}{k_{y\,i,j+1}}}, \quad TY_{i,j+1/2} = \frac{k_{y\,i,j+1/2}\left(\Delta x_{i,j}\, h\right)}{\mu B_w \Delta y_{i,j+1/2}}$$

Boundary Conditions



$$q^{\ell}_{i-1/2,j} = \frac{k_x A}{\mu B_w \Delta x} \Delta P$$

$$= \frac{k_{x,\ell} (h \Delta y)}{\mu B_w (\Delta x / 2)} (P_B - P_\ell)$$

$$= 2 \underbrace{\frac{k_{x\ell} (h \Delta y)}{\mu B_w (\Delta x)}}_{TX_\ell} (P_B - P_\ell)$$

$$= 2\, TX_\ell (P_B - P_\ell)$$

Neumann (No flow, flow)

$$q_{i-1/2,j} = 0$$

$= Q^{sc}_\ell$ if flow BC (lump in w/ well vector)

Dirchelet

$$q^\ell = 2TX_\ell (P_B - P_\ell)$$