$$\frac{\partial P}{\partial t} = \alpha \frac{\partial^2 P}{\partial x^2} \Rightarrow P_i^{n+1} = P_i^n + \underbrace{\frac{\alpha \Delta t}{(\Delta x)^2}}_{\equiv \eta}\left(P_{i-1}^n - 2P_i^n + P_{i+1}^n \right)$$

Explicit Method

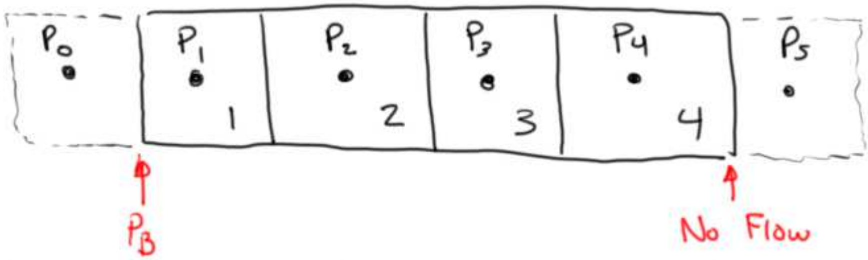
$$P_i^1 = P_i^0 + \eta\left(P_{i-1}^0 - 2P_i^0 + P_{i+1}^0 \right)$$

$$P_i^2 = P_i^1 + \eta\left(P_{i-1}^1 - 2P_i^1 + P_{i+1}^1 \right)$$

$n = 100$ max iterations

$$\vdots$$

$$P_i^{100} = P_i^{99} + \eta\left(P_{i-1}^{99} - 2P_i^{99} + P_{i+1}^{99} \right)$$



$$P_1^{n+1} = P_1^n + \eta\left(2P_B - P_1^n - 2P_1^n + P_2^n\right)$$

$$P_2^{n+1} = P_2^n + \eta\left(P_1^n - 2P_2^n + P_3^n\right)$$

$$P_3^{n+1} = P_3^n + \eta\left(P_2^n - 2P_3^n + P_4^n\right)$$

$$P_4^{n+1} = P_4^n + \eta\left(P_3^n - 2P_4^n + P_4^n\right)$$

$\eta \leq 0.5$ for stability

"Dirichlet" → Constan Pressure

"Neumann" → Constant/No Flux

Constant Pressure

$$P_B = \frac{P_0 + P_1}{2} \Rightarrow P_0 = 2P_B - P_1$$

No Flux

$$\cancel{q_x}^{0} = \frac{k}{\mu}\frac{\partial P}{\partial x} \Rightarrow \frac{P_5 - P_4}{\Delta x} = 0$$

$$\therefore P_4 = P_5$$

"refection"

$$q_{in} = \frac{k}{\mu}\frac{\partial P}{\partial x}$$

$$\frac{\mu q_{in}}{k} = \frac{P_5 - P_4}{\Delta x}$$

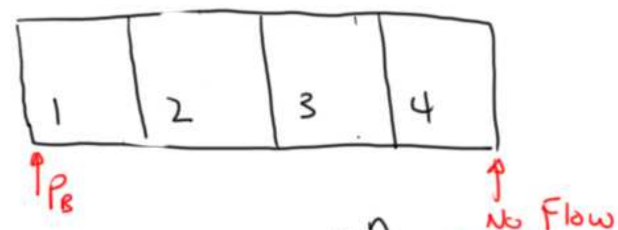
$$P_5 = \frac{\mu q_{in} \Delta x}{k} + P_4$$

Implicit method

$$\frac{\partial P}{\partial t} = \alpha \frac{\partial^2 P}{\partial x^2} \Rightarrow \frac{P_i^{n+1} - P_i^n}{\Delta t} = \alpha \frac{P_{i-1}^{n+1} - 2P_i^{n+1} + P_{i+1}^{n+1}}{(\Delta x)^2}$$

$$-\frac{\alpha \Delta t}{(\Delta x)^2} P_{i-1}^{n+1} + \left(1 + 2\frac{\alpha \Delta t}{(\Delta x)^2}\right) P_i^{n+1} - \frac{\alpha \Delta t}{(\Delta x)^2} P_{i+1}^{n+1} = P_i^n$$

$$-\eta P_{i-1}^{n+1} + (1+2\eta) P_i^{n+1} - \eta P_{i+1}^{n+1} = P_i^n$$



$P_0 = 2P_B - P_1$

$$\cancel{-\eta P_0^{n+1}} + (1+3\eta) P_1^{n+1} - \eta P_2^{n+1} = P_1^n + 2\eta P_B$$

$$-\eta P_1^{n+1} + (1+2\eta) P_2^{n+1} - \eta P_3^{n+1} = P_2^n$$

$$-\eta P_2^{n+1} + (1+2\eta) P_3^{n+1} - \eta P_4^{n+1} = P_3^n$$

$P_5 = P_4$

$$-\eta P_3^{n+1} + (1+\cancel{2}\eta) P_4^{n+1} \cancel{-\eta P_5^{n+1}} = P_4^n$$

$$\underbrace{\begin{bmatrix} 1+3\eta & -\eta & 0 & 0 \\ -\eta & 1+2\mu & -\eta & 0 \\ 0 & -\eta & 1+2\eta & -\eta \\ 0 & 0 & -\eta & 1+\eta \end{bmatrix}}_{A} \underbrace{\begin{Bmatrix} p_1^{n+1} \\ p_2^{n+1} \\ p_3^{n+1} \\ p_4^{n+1} \end{Bmatrix}} = \begin{Bmatrix} p_1^{n} \\ p_2^{n} \\ p_3^{n} \\ p_4^{n} \end{Bmatrix} + \begin{Bmatrix} 2\eta p_B \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\vec{p}^{\,n+1} \qquad = \qquad \vec{b} = \vec{p}^{\,n} + \begin{Bmatrix} 2\eta p_B \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$A \vec{p}^{\,n+1} = \vec{b}^{\,n}$$

$$\underbrace{A^{-1} A}\, \vec{p}^{\,n+1} = A^{-1} \vec{b}^{\,n}$$

$$I\, \vec{p}^{\,n+1} = A^{-1} \vec{b}^{\,n}$$

$$\vec{p}^{\,n+1} = A^{-1} \vec{b}^{\,n}$$

$$\vec{p}^{\,1} = A^{-1} b^{0}$$

$$\vec{p}^{\,2} = A^{-1} b^{1}$$

$$\vdots$$

$$\vec{p}^{\,100} = A^{-1} b^{99}$$