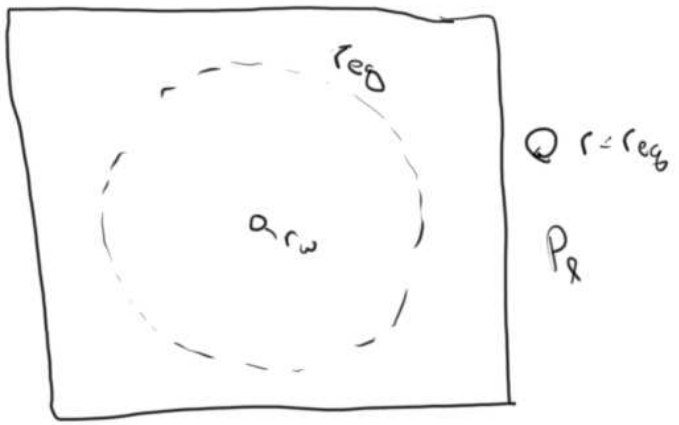
Wells

(1) Constant Rate → P_{wf} vs. time

(2) Constant BHP → Q^{x} vs time

$$q_w = -J_e^w (P_e - P_{wf})$$



Flow around a wellbore

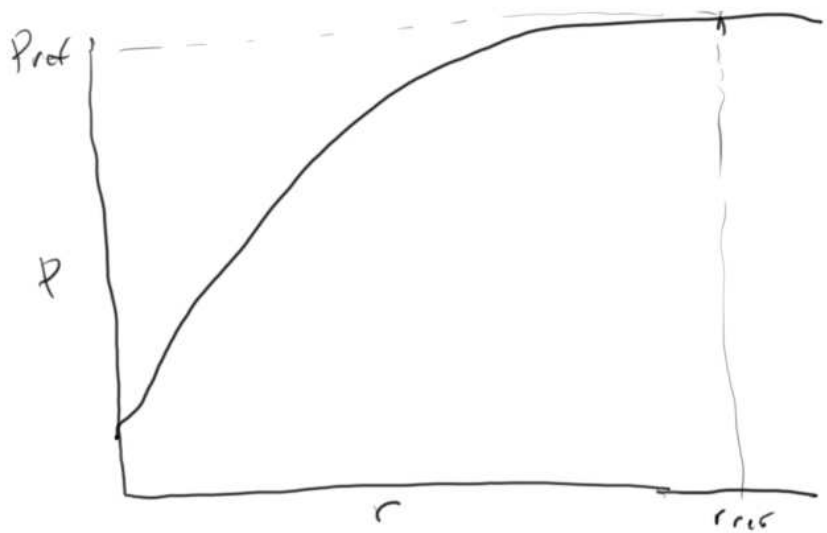
$\frac{\partial p}{\partial t} = 0$ S.S.

$$\frac{\phi \mu c_t}{k} \cancel{\frac{\partial p}{\partial t}} = \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial p}{\partial r}\right)$$ radial diffusivity equation

B.C.'s #1 $\lim_{r \to 0} \left(r \frac{\partial p}{\partial r}\right) = \frac{-q \mu B_w}{2\pi k h}$

#2 $P = P_{ref}$ @ $r = r_{ref}$

$$P(r) = P_{ref} - \frac{q_w \mu B_w}{2\pi k h} \ln\left(\frac{r}{r_{ref}}\right)$$



$$P(r) = P_{ref} - \frac{q_w \mu B_w}{2\pi kh} \ln\left(\frac{r}{r_{ref}}\right)$$

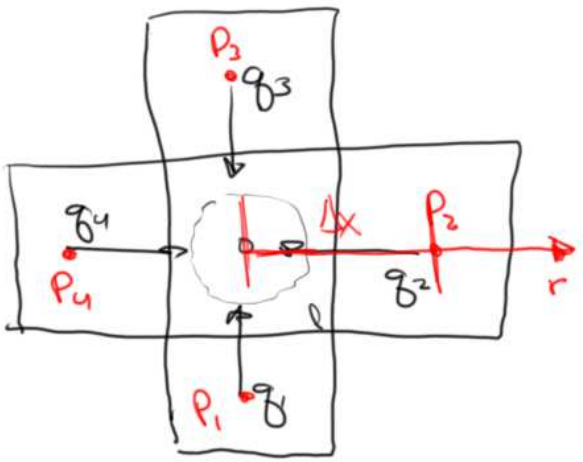
Let $P_{ref} = P_{wf}$ $\quad r_{ref} = r_w$

$$P(r) = P_{wf} - \frac{q_w \mu B_w}{2\pi kh} \ln\left(\frac{r}{r_w}\right)$$

$$P(r_{eq}) = P_{wf} - \frac{q_w \mu B_w}{2\pi kh} \ln\left(\frac{r_{eq}}{r_w}\right)$$

P_e

$$q_w = -J_e^w (P_e - P_{wf})$$



$$q_w + (q_1 + q_2 + q_3 + q_4) = 0$$

$$q_1 = \frac{kh\Delta x}{\mu B_w \Delta y}(P_1 - P_e)$$

$$q_2 = \frac{kh\Delta y}{\mu B_w \Delta x}(P_2 - P_e)$$

$$q_3 = \frac{kh\Delta x}{\mu B_w \Delta y}(P_3 - P_e)$$

$$q_4 = \frac{kh\Delta y}{\mu B_w \Delta x}(P_4 - P_e)$$

Blocks are square $\Delta x = \Delta y$

$$q_w = \frac{-kh}{\mu B_w}(P_1 + P_2 + P_3 + P_4 - 4P_e)$$

$$P_4 = P_3 = P_2 = P_1 = P_e - \frac{q_w \mu B_w}{2\pi kh} \ln\left(\frac{\Delta x}{r_{eq}}\right)$$

$$q_w = \frac{-kh}{\mu B_w}\left(P_1 + P_2 + P_3 + P_4 - 4P_e\right)$$

$$q_w = \frac{+kh}{\mu B_w}\left[4\left(P_e + \frac{q_w \mu B_w}{2\pi kh} \ln\left(\frac{\Delta x}{r_{eq}}\right)\right) - 4P_e\right]$$

$$\frac{\pi}{2} = \ln\left(\frac{\Delta x}{r_{eq}}\right) \Rightarrow r_{eq} = \Delta x\, e^{-\pi/2} \approx 0.20788\,\Delta x$$

"Peaceman correction"

$$P_e = P_{wf} - \frac{q_w \mu B_w}{2\pi kh} \ln\left(\frac{r_{eq}}{r_w}\right)$$

$$q_w = -J_e^w\left(P_e - P_{wf}\right) \qquad J^w = \frac{2\pi kh}{\mu B_w\left[\ln\left(\frac{r_{eq}}{r_w}\right) + s\right]}$$

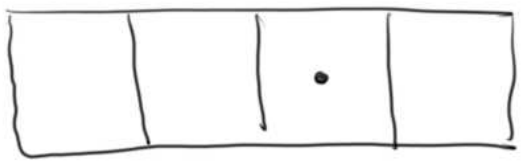
$r_{eq} \approx 0.2\,\Delta x$

$$T\left(P_{i-1}^{n+1} - P_i^{n+1}\right) + T\left(P_{i+1}^{n+1} - P_i^{n+1}\right) = \frac{1}{\Delta t} B_i \left(P_i^{n+1} - P_i^{n}\right) - Q_i$$

$$\| \quad J_i^{\omega}\left(P_i^{n+1} - P_{wf}\right)$$

$$-T P_{i-1}^{n+1} + \left(2T + J_i^{\omega} + \frac{1}{\Delta t} B_i\right) P_i^{n+1} - T P_{i+1}^{n+1} = \frac{1}{\Delta t} B_i P_i^{n} + J_i P_{wf}$$

$$\left(\vec{T}^{\omega} + \vec{J} + \frac{1}{\Delta t}\vec{B}\right)\vec{P}^{n+1} = \frac{1}{\Delta t}\vec{B}\,\vec{P}^{n} + \vec{Q}$$



$$\vec{J}^{\omega} = \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & J_3 & \\ & & & 0 \end{bmatrix} \Rightarrow \quad \vec{Q} = \begin{bmatrix} 0 \\ 0 \\ J_3 P_{wf} \\ 0 \end{bmatrix}$$