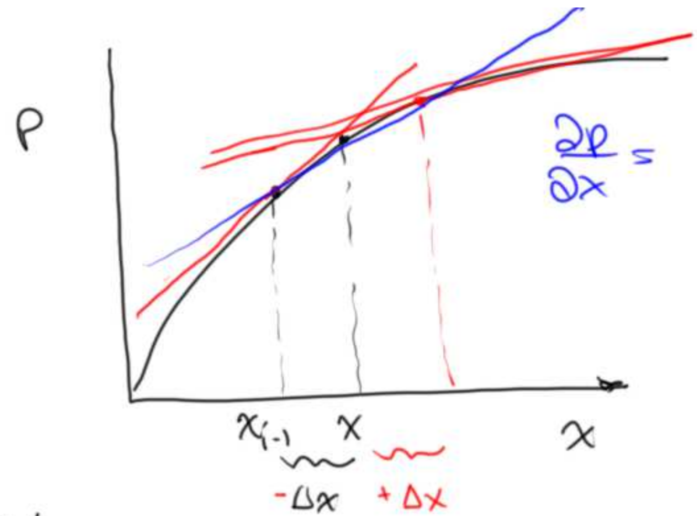


$$\frac{\partial P}{\partial x} = \frac{P(x+\Delta x) - P(x-\Delta x)}{2\Delta x}$$

$$P(x-\Delta x) = P(x) - \left.\frac{\partial P}{\partial x}\right|_x \Delta x + \quad \cdots$$

$$\left.\frac{\partial P}{\partial x}\right|_x = \frac{P(x) - P(x-\Delta x)}{\Delta x} + O(\|\Delta x\|)$$

Backward difference approx.

Central difference

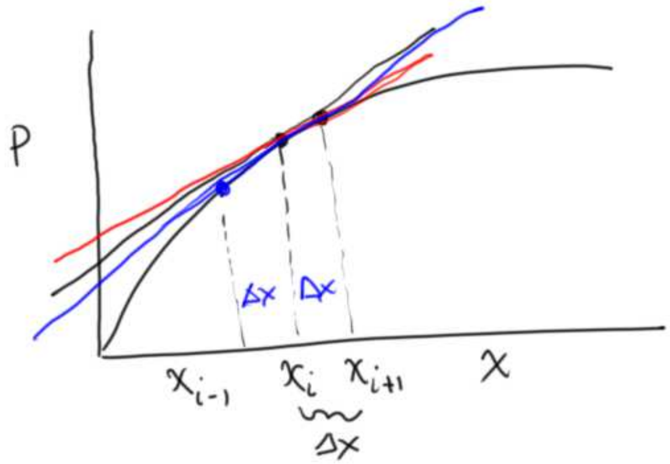
$$P(x+\Delta x) = P(x) + \left.\frac{\partial P}{\partial x}\right|_x \Delta x + \frac{1}{2!}\frac{\partial^2 P}{\partial x^2}\Delta x^2 + \frac{1}{3!}\frac{\partial^3 P}{\partial x^3}(\Delta x)^3 + K$$

$$-\left(P(x-\Delta x) = P(x) - \left.\frac{\partial P}{\partial x}\right|_x \Delta x + \frac{1}{2!}\frac{\partial^2 P}{\partial x^2}\Delta x^2 - \frac{1}{3!}\frac{\partial^3 P}{\partial x^3}(\Delta x)^3\right)$$

$$P(x+\Delta x) - P(x-\Delta x) = 2\left.\frac{\partial P}{\partial x}\right|_x + \frac{2}{3!}\frac{\partial^3 P}{\partial x^3}\Delta x^3 \Rightarrow \left.\frac{\partial P}{\partial x}\right|_x = \frac{P(x+\Delta x) - P(x-\Delta x)}{2\Delta x} + O(\|\Delta x^2\|)$$

$$\frac{\partial p}{\partial t} = \alpha \frac{\partial^2 p}{\partial x^2}$$

$$\frac{\partial p}{\partial x} \approx \frac{p(x_{i+1}) - p(x_i)}{x_{i+1} - x_i}$$



$$p'(x_i) = \left.\frac{\partial p}{\partial x}\right|_{x=x_i}$$

$$\frac{\partial p}{\partial x} \approx \frac{p(x_{i+1}) - p(x_i)}{\Delta x}$$

$$p'(x_i) = \frac{p(x_{i+1}) - p(x_{i-1})}{2\Delta x}$$

$$\approx \frac{p(x_i + \Delta x) - p(x_i)}{\Delta x}$$ Forward difference approx.

Taylor series

$$f(x) = f(x_i) + f'(x_i)(x - x_i) + \frac{f''(x_i)}{2!}(x - x_i)^2 + \frac{f'''(x_i)}{3!}(x - x_i)^3 + \ldots$$

$$p(x_{i+1}) = p(x_i) + p'(x_i)(x_{i+1} - x_i) + \frac{p''(x_i)}{2!}(x_{i+1} - x_i)^2 + \text{H.O.T.}$$

$$p'(x_i) \approx \left.\frac{\partial p}{\partial x}\right|_{x=x_i} = \frac{p(x_{i+1}) - p(x_i)}{\underbrace{(x_{i+1} - x_i)}_{\Delta x}} - \frac{p''(x_i)}{2!}(\underbrace{x_{i+1} - x_i}_{\Delta x}) + \ldots + \ldots$$

$$p'(x_i) = \frac{p(x_{i+1}) - p(x_i)}{\Delta x} + O(\|\Delta x\|)$$

FORWARD

$\Rightarrow$

$$p'(x_i) = \frac{p(x_i) - p(x_{i-1})}{\Delta x} + O(\|\Delta x\|)$$

BACKWARD

$$P(x+\Delta x) = P(x) + \left.\frac{\partial P}{\partial x}\right|_x \Delta x + \cancel{\frac{1}{2!}\left.\frac{\partial^2 P}{\partial x^2}\right|_x} \Delta x^2 + \frac{1}{3!}\left.\frac{\partial^3 P}{\partial x^3}\right|_x \Delta x^3 + \ldots\ldots$$

$$-\left(P(x-\Delta x) = P(x) - \left.\frac{\partial P}{\partial x}\right|_x \Delta x + \cancel{\frac{1}{2!}\left.\frac{\partial^2 P}{\partial x^2}\right|_x} \Delta x^2 - \frac{1}{3!}\left.\frac{\partial^3 P}{\partial x^3}\right|_x \Delta x^3 + \ldots\ldots \right)$$

$$\underbrace{\frac{P(x+\Delta x) - P(x-\Delta x)}{2\Delta x}}_{\text{Central difference}} = \cancel{2}\left.\frac{\partial P}{\partial x}\right|_x \cancel{\Delta x} + \underbrace{\frac{\cancel{2}}{3!\,\cancel{2}}\frac{\partial^3 P}{\partial x^3}\frac{\Delta x^{\cancel{3}\,2}}{\cancel{\Delta x}}}_{\mathcal{O}(\|\Delta x\|^2)} + \ldots\ldots$$

$$\left.\frac{\partial P}{\partial x}\right|_x = \frac{P(x+\Delta x) - P(x-\Delta x)}{2\Delta x} + \mathcal{O}(\|\Delta x\|^2)$$