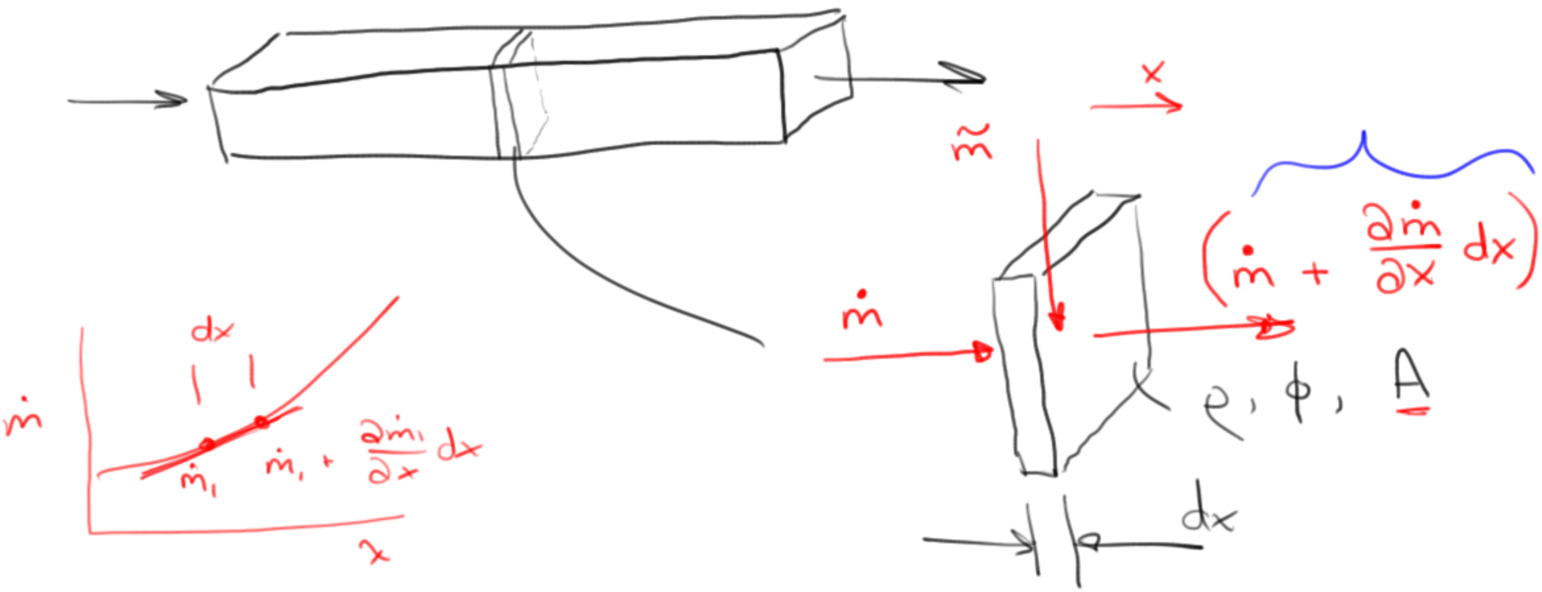

$$\frac{(\text{mass in})}{\Delta t} - \frac{(\text{mass out})}{\Delta t} + \frac{\text{generation/consumption}}{\Delta t} = \frac{\text{accumulation}}{\Delta t}$$

(rate of mass in) − (rate of mass out) + (rate gen/cons.) = rate of acc.

$$\dot{m} = \rho u \qquad \frac{M}{L^2}\,\frac{1}{T} = \frac{M}{L^{2}}\,\frac{\cancel{L}}{T}$$

$$\rho A\, u - \left(\rho A\, u + \frac{\partial(\rho u) A}{\partial x} dx\right) + \tilde{m} = \frac{\partial(\rho \phi \cancel{A\,dx})}{\partial t}$$

1D

$$\boxed{-\frac{\partial(\rho u)}{\partial x} = \frac{\partial(\rho\phi)}{\partial t} - \tilde{m}}$$

mass conservation

continuity eqn.

3D Cartesian coords.

$$-\left[\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} \right] = \frac{\partial(\rho\phi)}{\partial t} - \tilde{m}$$

$$-\nabla\cdot(\rho\vec{u}) = \frac{\partial(\rho\phi)}{\partial t} - \tilde{m}$$

$$a = [a_1 \quad a_2 \quad a_3]$$

$$a\cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right]$$

$$u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

$$\nabla\cdot u = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$$

1D

$$\left[-\frac{\partial(\rho u)}{\partial x} = \frac{\partial(\rho\phi)}{\partial t} - \tilde{m}\right]\frac{1}{\rho_{sc}}$$

Darcy's law

$$u = -\frac{k}{\mu}\frac{\partial p}{\partial x}$$

$$\vec{u} = -\frac{\bar{\bar{K}}}{\mu}\left(\nabla p + \rho g \vec{z}\right)$$

Formation volume factor

$$B_\ell = \frac{V_{\ell, RC}}{V_{\ell, SC}} = \frac{\rho_{\ell, SC}}{\rho_{\ell, RC}}$$

gas << 1

oil > 1

water ≈ 1

$$\phi = \phi(p)$$

$$\frac{\partial\phi}{\partial t} = \frac{\partial\phi}{\partial p}\frac{\partial p}{\partial t}$$

$$\frac{\partial}{\partial x}\left[\frac{k}{B_w\mu}\frac{\partial p}{\partial x}\right] = \underbrace{\frac{\partial}{\partial t}\left(\frac{\phi}{B_w}\right)} - \frac{\tilde{m}}{\rho_{sc}}$$

$$\frac{\partial}{\partial t}\left[\frac{\phi}{B_w}\right] = \frac{\partial}{\partial p}\left(\frac{\phi}{B_w}\right)\frac{\partial p}{\partial t} = \left[\frac{1}{B_w}\frac{\partial\phi}{\partial p} + \phi\frac{\partial}{\partial p}\left(\frac{1}{B_w}\right)\right]\frac{\partial p}{\partial t}$$

$$c_R = \frac{1}{\phi}\frac{\partial\phi}{\partial p} \qquad c_f = B_w\frac{\partial}{\partial p}\left(\frac{1}{B_w}\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B_w}\right) = \frac{\phi}{B_w}\Bigg[\underbrace{\frac{1}{\phi}\frac{\partial \phi}{\partial p}} + B_w\frac{\partial}{\partial p}\left(\frac{1}{B_w}\right)\Bigg]\frac{\partial p}{\partial t} = \frac{\phi}{B_w}\, c_T\, \frac{\partial p}{\partial t}$$

$$c_R + c_f = c_T$$

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B_w}\frac{\partial p}{\partial x}\right) = \frac{\phi}{B_w}\, c_T\, \frac{\partial p}{\partial t} - \frac{\tilde{m}}{\rho_{sc}}$$

$$\frac{k}{\mu}\left[\frac{1}{B_w}\frac{\partial^2 p}{\partial x^2} + \underbrace{\frac{\partial}{\partial x}\left(\frac{1}{B_w}\right)}\frac{\partial p}{\partial x}\right] = \frac{\phi}{B_w}\, c_T\, \frac{\partial p}{\partial t} - \frac{\tilde{m}}{\rho_{sc}}$$

$$\frac{k}{\mu}\left[\frac{1}{B_w}\frac{\partial^2 p}{\partial x^2} + \cancelto{0}{\underbrace{\frac{\partial}{\partial p}\left(\frac{1}{B_w}\right)}_{\frac{c_F}{B_w}\,\approx\, 0}\left(\frac{\partial p}{\partial x}\right)^2}\right] = \frac{\phi}{B_w}\, c_T\, \frac{\partial p}{\partial t} - \frac{\tilde{m}}{\rho_{sc}}$$

$$\boxed{\frac{k}{\mu B_w}\frac{\partial^2 p}{\partial x^2} = \frac{\phi\, c_T}{B_w}\frac{\partial p}{\partial t} - \frac{\tilde{m}}{\rho_{sc}}}$$

$$\alpha = \frac{k}{\mu \phi c_T}$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{\mu \phi c_T}{k}\frac{\partial p}{\partial t} - \cancelto{0}{\frac{\tilde{m}}{\rho_{sc}}}$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{\alpha}\frac{\partial p}{\partial t} \leftarrow$$ Diffisivity eqn. Heat eqn.