CHAPTER 7.
WELLS AND WELL MODELS

Basic Well Model in Simulation Equations

Well model can be written with a "productivity index" or "well index"

$$q_w = -J_l^w (P_l - P_{wf}) \qquad J_l^w = \frac{2\pi kh}{\mu B_w \left[\ln\left(\frac{r_{eq}}{r_w}\right) + s \right]}$$

Skin factor

Mass balance equation written implicitly in 1D:

$$T(P_{l-1}{}^{n+1} - P_l^{n+1}) + T(P_{l+1}{}^{n+1} - P_l^{n+1}) = \frac{1}{\Delta t} B_l (P_l^{n+1} - P_l^n) - Q_l$$

For a constant bottom hole pressure well:

$$T(P_{l-1}{}^{n+1} - P_l^{n+1}) + T(P_{l+1}{}^{n+1} - P_l^{n+1}) = \frac{1}{\Delta t} B_l (P_l^{n+1} - P_l^n) + \underbrace{J_l^w \left(P_l^{n+1} - P_{wf} \right)}$$

$T \equiv Transmissibility \left[\frac{L^4 T}{M}\right]$ | $P \equiv Pressure \left[\frac{M}{LT^2}\right]$ | $B \equiv Compressibility \left[\frac{L^4 T^2}{M}\right]$ | $Q \equiv Source \left[\frac{L^3}{T}\right]$ | $J^w \equiv Productivity\ Index \left[\frac{L^4 T}{M}\right]$

Forming the System of Equations

$$T(P_{l-1}^{n+1} - P_l^{n+1}) + T(P_{l+1}^{n+1} - P_l^{n+1}) = \frac{1}{\Delta t} B_l (P_l^{n+1} - P_l^n) - \boxed{J_l^w (P_{wf} - P_l^{n+1})}$$

Grouping like terms of Pressure and multiplying through by "-1"

$$-TP_{l-1}^{n+1} + (2T + \boxed{J_l^w} + \frac{1}{\Delta t} B_l) P_l^{n+1} - TP_{l+1}^{n+1} = \frac{1}{\Delta t} B_l P_l^n + \boxed{J_l^w P_{wf}}$$

We get a system of equations of the usual form, but with a new diagonal J matrix

$$\boxed{\left(\mathbf{T} + \mathbf{J} + \frac{1}{\Delta t}\mathbf{B}\right)\vec{P}^{n+1} = \frac{1}{\Delta t}\mathbf{B}\vec{P}^n + \vec{Q}}$$

Diagonal Matrix

Includes J^*P_{wf} terms + constant rate wells + BCs

$T \equiv Transmissibility \left[\frac{L^4T}{M}\right]$ | $P \equiv Pressure \left[\frac{M}{LT^2}\right]$ | $B \equiv Compressibility \left[\frac{L^4T^2}{M}\right]$ | $Q \equiv Source \left[\frac{L^3}{T}\right]$ | $J^w \equiv Productivity\ Index \left[\frac{L^4T}{M}\right]$

Matrix Equations for Constant BHP Well

$$\boxed{\left(\mathbf{T}+\mathbf{J}+\frac{1}{\Delta t}\mathbf{B}\right)\vec{P}^{n+1}=\frac{1}{\Delta t}\mathbf{B}\vec{P}^{n}+\vec{Q}}$$

Consider a 1D, 4-block homogeneous problem with no-flow BCs a constant BHP well in block #3

$$\mathbf{T}=\begin{bmatrix} T & -T & & \\ -T & 2T & -T & \\ & -T & 2T & -T \\ & & -T & T \end{bmatrix} \quad \mathbf{J}=\begin{bmatrix} 0 & & & \\ & 0 & & \\ & & J_3 & \\ & & & 0 \end{bmatrix} \quad \mathbf{B}=\begin{bmatrix} B_1 & & & \\ & B_2 & & \\ & & B_3 & \\ & & & B_4 \end{bmatrix} \quad Q=\begin{bmatrix} 0 \\ 0 \\ J_3 P_{wf} \\ 0 \end{bmatrix} + q_w$$

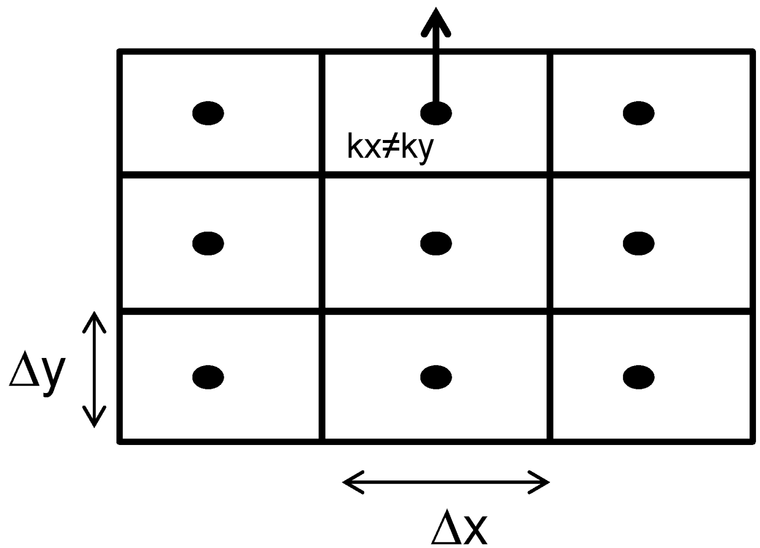
- **B** and **T** matrix are no different now that wells are present
- **J** is a diagonal matrix and diagonal elements are non-zero if a BHP well exists in that block. In which case J_i is always **positive**
- **Q** is source vector; it includes a **positive** ($J*P_{wf}$) regardless of whether we have an injector or producer BHP well. Note: This is different from constant rate wells where the sign depends on producer or injector

$T \equiv Transmissibility \left[\frac{L^4T}{M}\right]$	$P \equiv Pressure \left[\frac{M}{LT^2}\right]$	$B \equiv Compressibility \left[\frac{L^4T^2}{M}\right]$	$Q \equiv Source \left[\frac{L^3}{T}\right]$	$J^w \equiv Productivity\ Index \left[\frac{L^4T}{M}\right]$

Anisotropy and Rectangular Grids

If the porous medium is anisotropic and/or has rectangular grids:

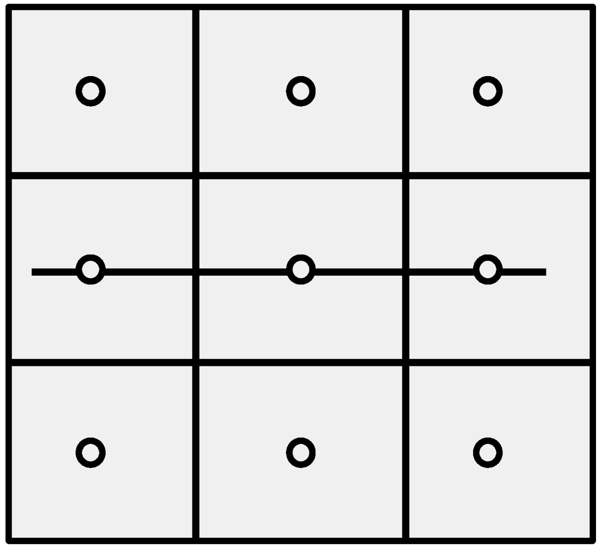
$$J_l^w = \frac{2\pi h\sqrt{k_x k_y}}{\mu B_w \left[\ln\left(\frac{r_{eq}}{r_w}\right) + s\right]}; \quad r_{eq} = 0.28\frac{\left[\left(k_y/k_x\right)^{1/2}\Delta x^2 + \left(k_x/k_y\right)^{1/2}\Delta y^2\right]^{1/2}}{\left(k_y/k_x\right)^{1/4} + \left(k_x/k_y\right)^{1/4}}$$



- Numerator of productivity index includes geometric mean of permeability
- Equivalent radius has ratios of permeability and includes rectangular grid sizes
- Note that the expressions reduce to the simpler case when medium is isotropic and grids are square

$P \equiv Pressure \left[\frac{M}{LT^2}\right]$ | $q \equiv Flow\ Rate \left[\frac{L^3}{T}\right]$ | $\mu \equiv Viscosity \left[\frac{M}{LT}\right]$ | $k \equiv Permeability \left[L^2\right]$ | $h \equiv Thickness\ [L]$ | $r \equiv Radius\ [L]$

Horizontal Wells



Horizontal wells have the following issues:

1. Much more likely anisotropic ($k_z \neq k_x$ or k_y)
2. Well traverses horizontally, so dimensions changed
3. Well passes through multiple grids in x-y plane, so J matrix has entries in each grid

$$J_l^w = \frac{2\pi \Delta x \sqrt{k_y k_z}}{\mu B_w \left[\ln\left(\frac{r_{eq}}{r_w} \right) + s \right]};$$

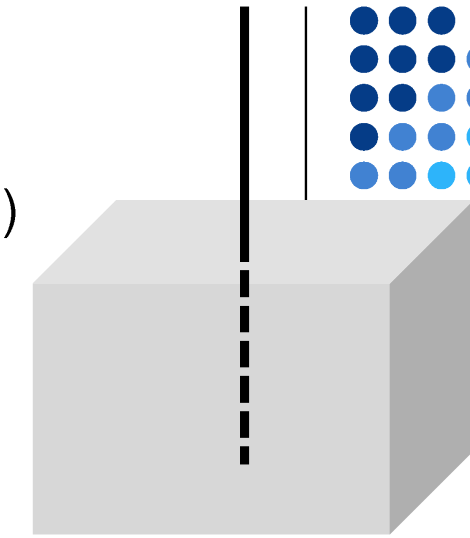
$$\mathrm{r}_{eq} = 0.28 \frac{\left[\left(k_z / k_y \right)^{1/2} \Delta y^2 + \left(k_y / k_z \right)^{1/2} \Delta z^2 \right]^{1/2}}{\left(k_z / k_y \right)^{1/4} + \left(k_y / k_z \right)^{1/4}}$$

- Assumes that well is in x-direction, not y-direction
- Δz = h, the reservoir thickness in 2D

$$\mathbf{J} = \begin{pmatrix} 0 & & & & & & & & \\ & 0 & & & & & & & \\ & & 0 & & & & & & \\ & & & J_4 & & & & & \\ & & & & J_5 & & & & \\ & & & & & J_6 & & & \\ & & & & & & 0 & & \\ & & & & & & & 0 & \\ & & & & & & & & 0 \end{pmatrix}$$

Well traverses through blocks 4,5, and 6

Treatment of Wells in Reservoir Simulation



- Wells are treated as sources (injectors) or sinks (producers) of mass

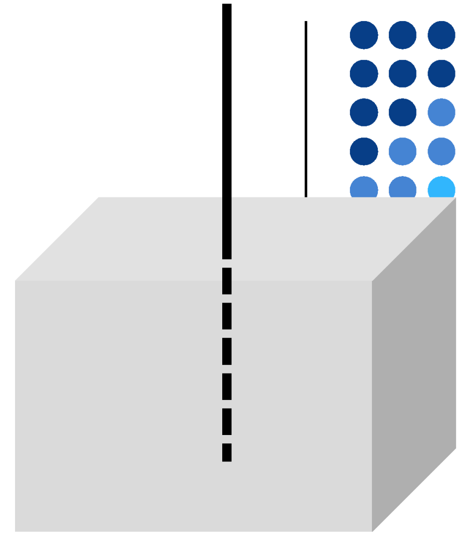
- Well is a line source or sink inside the grid block. Mass flux for the source/sink is distributed over the whole grid block

- Well pressure is <u>**DIFFERENT**</u> than block pressure, which is the average pressure in the block and we locate it at its center

- Well and block pressure are linked to each other through a **well model.** It can be constrained either by pressure (P_{wf} is given) or by rates (q_w is given).

Well constraints
- Rate $\longrightarrow$ Need to calculate P_{wf}
- Pressure $\longrightarrow$ Need to calculate rate

Radial Flow around the Well



We assume that flow occurs radially around a well. Radial Diffusivity equation around a well is given by the following equation; we assume flow is steady state

Steady state

$$\cancel{\frac{\phi\mu c_t}{k}\frac{\partial P}{\partial t}} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = 0$$

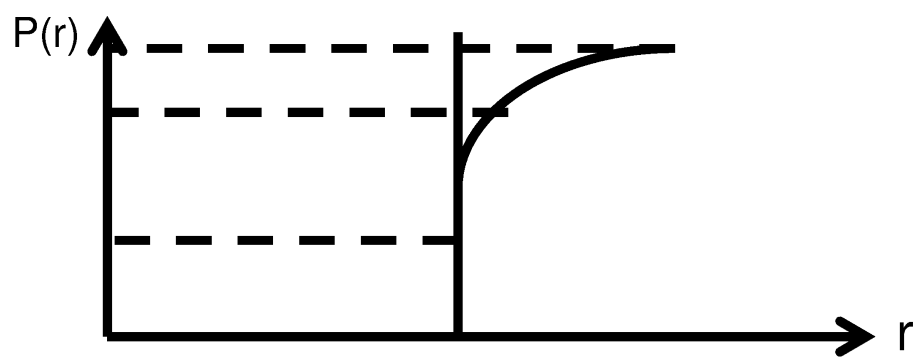
$$\text{BC \#1: } \lim_{r\to 0}\left(r\frac{\partial P}{\partial r}\right) = -\frac{q_w \mu B_w}{2\pi kh}$$

$$\text{BC\#2}: P = P_{ref} \quad @ \quad r = r_{ref}$$

Solution to ODE with BCs can be solved using separation of variables

$$\boxed{P(r) = P_{ref} - \frac{q_w \mu B_w}{2\pi kh}\ln\left(\frac{r}{r_{ref}}\right)}$$

Pressure is known (P_{ref}) at reference radius, r_{ref}



$P \equiv Pressure \left[\frac{M}{LT^2}\right]$	$q \equiv Flow\ Rate \left[\frac{L^3}{T}\right]$	$\mu \equiv Viscosity \left[\frac{M}{LT}\right]$	$k \equiv Permeability \left[L^2\right]$	$h \equiv Thickness\ [L]$	$r \equiv Radius\ [L]$

Radial Flow around Well

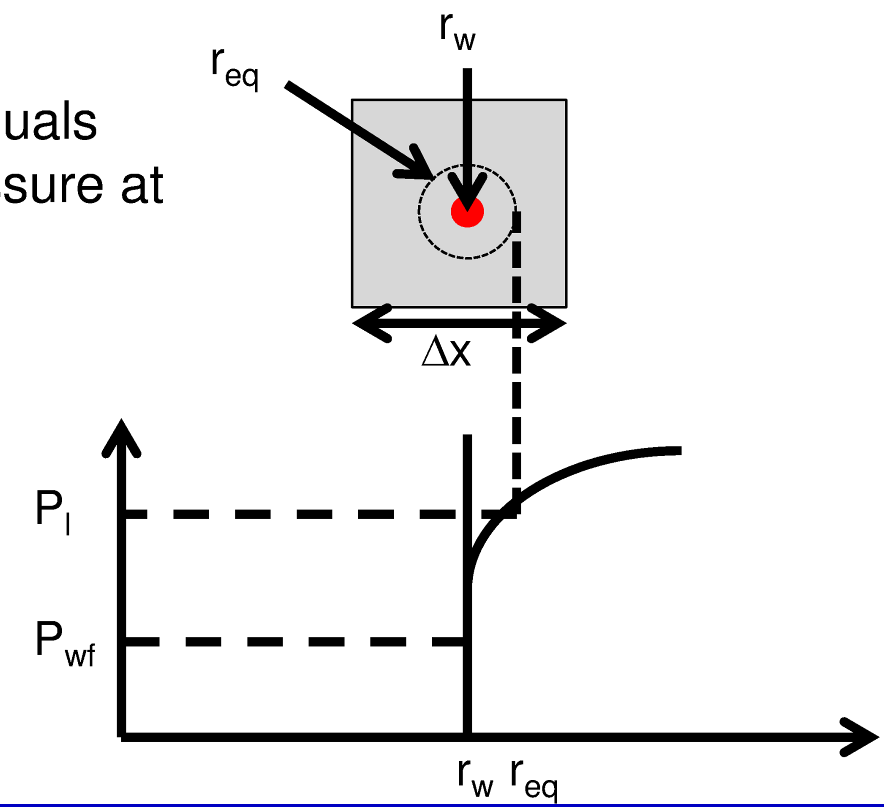
Consider the analytical solution for radial flow at steady state:

$$P(r) = P_{ref} - \frac{q_w \mu B_w}{2\pi k h} \ln\left(\frac{r}{r_{ref}}\right)$$

Now, let the reference pressure be the well pressure, P_{wf}. The reference radius is the well radius, r_w.

Define r_{eq} as radius where pressure, $P(r_{eq})$ equals average block pressure, P_l. Evaluate the pressure at the equivalent radius (r_{eq}) and we get:

$$\boxed{P_l = P_{wf} - \frac{q_w \mu B_w}{2\pi k h} \ln\left(\frac{r_{eq}}{r_w}\right)}$$



But how do we compute r_{eq}?

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Steady Mass Balance on Grids w/ Wells in 2D

Now consider steady state (no accumulation) flow in a grid, "l". Flow "in-out" from its 4 neighbors is equal to that injected/produced from a well

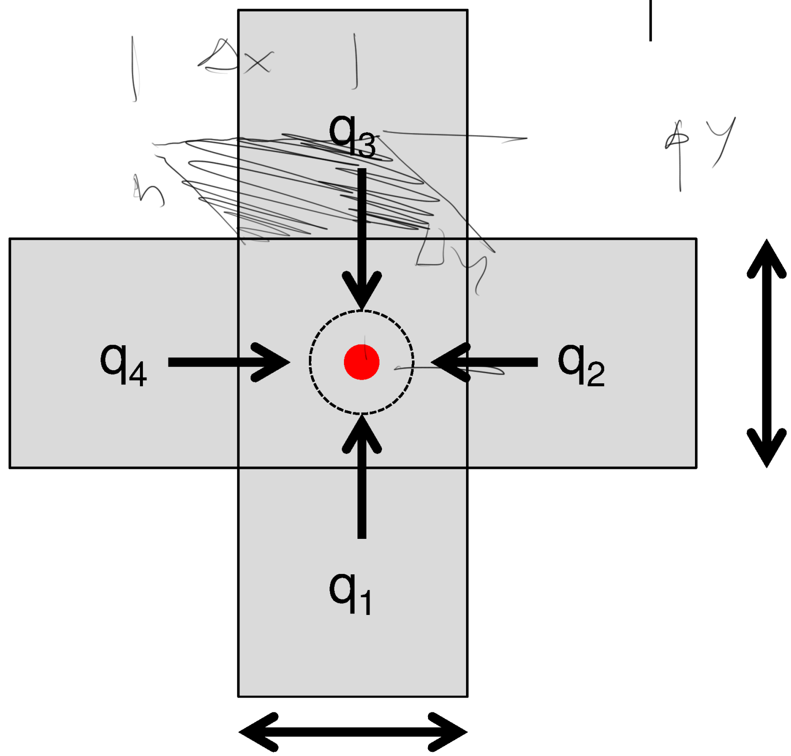
$$q_w + \left(q_1 + q_2 + q_3 + q_4\right) = 0$$

$$q_1 = \frac{kh\Delta x}{\mu B_w \Delta y}(P_1 - P_l)$$

$$q_2 = \frac{kh\Delta y}{\mu B_w \Delta x}(P_2 - P_l)$$

$$q_3 = \frac{kh\Delta x}{\mu B_w \Delta y}(P_3 - P_l)$$

$$q_4 = \frac{kh\Delta y}{\mu B_w \Delta x}(P_4 - P_l)$$



If the block is square, Δx=Δy, then the mass balance reduces to:

$$\boxed{q_w = -\frac{kh}{\mu B_w}(P_1 + P_2 + P_3 + P_4 - 4P_l)}$$

$q \equiv Flow\ Rate\left[\frac{L^3}{T}\right]$	$k \equiv Permeability\left[L^2\right]$	$h \equiv Thickness\ [L]$	$\mu \equiv Viscosity\left[\frac{M}{LT}\right]$	$P \equiv Pressure\left[\frac{M}{LT^2}\right]$

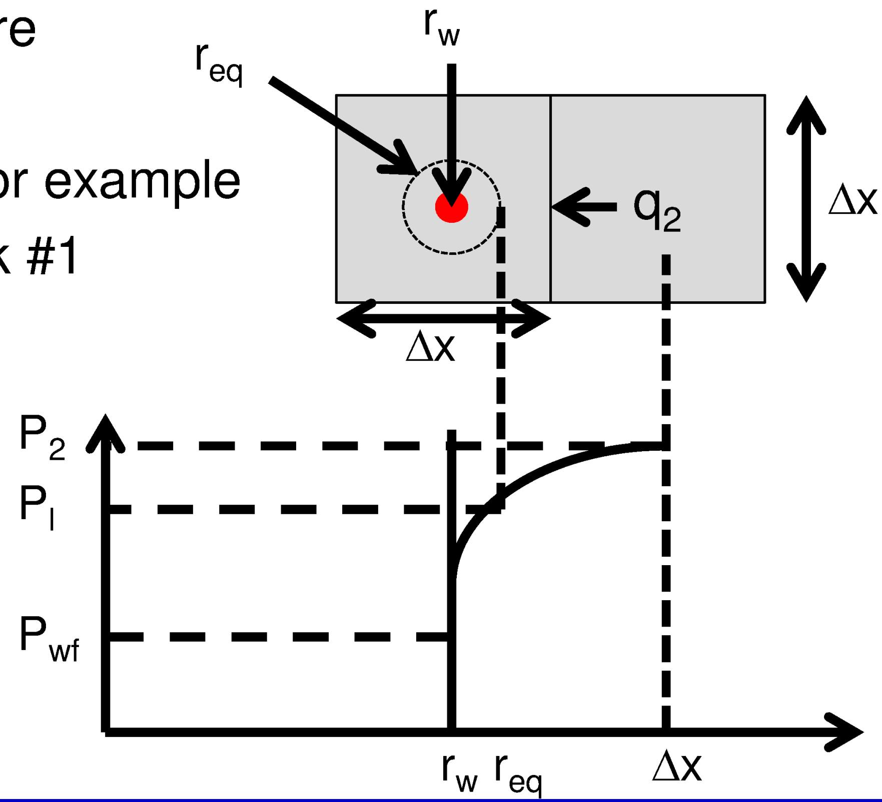
Radial Flow around Well to Adjacent Blocks

Recall our radial solution for pressure around a well

$$P(r) = P_{ref} - \frac{q_w \mu B_w}{2\pi k h} \ln\left(\frac{r}{r_{ref}}\right)$$

Now, let the reference pressure be the average block pressure of grid "l", P_l. The reference radius is the equivalent radius, r_{eq}, which again is the radius where the pressure is equal to the average block pressure

Evaluate the pressure in the adjacent block (for example Block #2), which is exactly Δx away from block #1



$$P_2 = P_l - \frac{q_w \mu B_w}{2\pi k h} \ln\left(\frac{\Delta x}{r_{eq}}\right)$$

Similar expressions can be made for 3 other neighboring blocks

$P \equiv Pressure \left[\frac{M}{LT^2}\right]$ | $q \equiv Flow\ Rate \left[\frac{L^3}{T}\right]$ | $\mu \equiv Viscosity \left[\frac{M}{LT}\right]$ | $k \equiv Permeability \left[L^2\right]$ | $h \equiv Thickness\ [L]$ | $r \equiv Radius\ [L]$

Basic Well Model

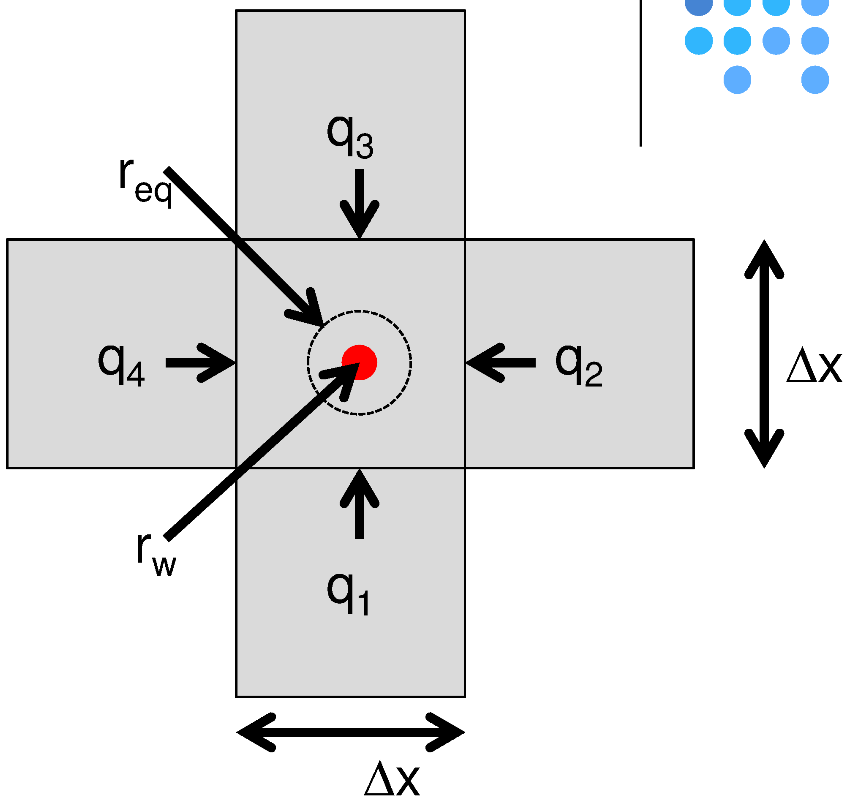
Recall the equation from the mas balance:

$$q_w = -\frac{kh}{\mu B_w}(P_1 + P_2 + P_3 + P_4 - 4P_l)$$

Derived equations for neighboring block pressures:

$$P_1 = P_l - \frac{q_w \mu B_w}{2\pi kh}\ln\left(\frac{\Delta x}{r_{eq}}\right) \qquad P_3 = P_l - \frac{q_w \mu B_w}{2\pi kh}\ln\left(\frac{\Delta x}{r_{eq}}\right)$$

$$P_2 = P_l - \frac{q_w \mu B_w}{2\pi kh}\ln\left(\frac{\Delta x}{r_{eq}}\right) \qquad P_4 = P_l - \frac{q_w \mu B_w}{2\pi kh}\ln\left(\frac{\Delta x}{r_{eq}}\right)$$



Substituting into the mass balance gives:

$$\boxed{q_w = -\frac{kh}{\mu B_w}\left(\sum_{j=1}^{4}\left[P_l - \frac{q_w \mu B_w}{2\pi kh}\ln\left(\frac{\Delta x}{r_{eq}}\right)\right] - 4P_l\right)}$$

$q \equiv Flow\ Rate\left[\frac{L^3}{T}\right]$ | $k \equiv Permeability\left[L^2\right]$ | $h \equiv Thickness\ \left[L\right]$ | $\mu \equiv Viscosity\left[\frac{M}{LT}\right]$ | $P \equiv Pressure\left[\frac{M}{LT^2}\right]$

Basic Well Model

Mass balance equation can be simplified:

$$q_w = -\frac{kh}{\mu B_w}\left(\sum_{j=1}^{4}\left[P_l - \frac{q_w \mu B_w}{2\pi kh}\ln\left(\frac{\Delta x}{r_{eq}}\right)\right] - 4P_l\right) = -\frac{kh}{\mu B_w}\left(\left[\cancel{4P_l} - \frac{4q_w \mu B_w}{2\pi kh}\ln\left(\frac{\Delta x}{r_{eq}}\right)\right] - \cancel{4P_l}\right)$$

And simplified further:

$$\cancel{q_w} = \frac{2\cancel{q_w}}{\pi}\ln\left(\frac{\Delta x}{r_{eq}}\right)$$

Solving for r_{eq} we get the famous "Peaceman Correction"

$$\frac{\pi}{2} = \ln\left(\frac{\Delta x}{r_{eq}}\right) \Rightarrow r_{eq} = \Delta x e^{-\frac{\pi}{2}} \Rightarrow \boxed{r_{eq} = 0.2078\Delta x \approx 0.2\Delta x}$$

*So radius at which the pressure is equal to the average grid block pressure is about 20% of the grid block size

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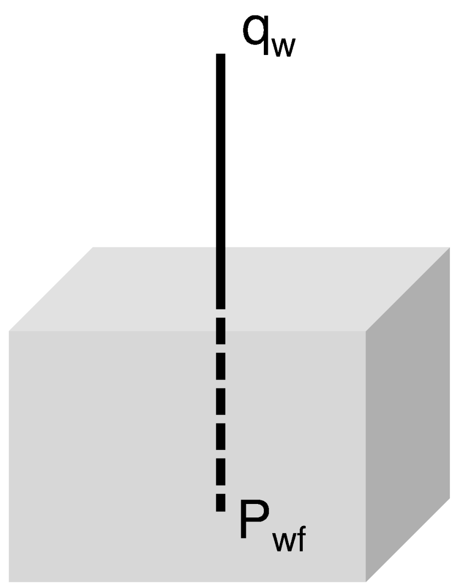
Wells can be constrained by rate or bottom hole pressure

Rate Constraint

We can solve for well pressure

$$P_{wf} = P_l + \frac{q_w \mu B_w}{2\pi kh} \ln\left(\frac{0.2078\Delta x}{r_w}\right) = P_l + \frac{q_w}{J_l^w}$$



Pressure Constraint

We can solve for well rate

$$q_w = -\frac{2\pi kh}{\mu B_w \ln\left(\dfrac{0.2078\Delta x}{r_w}\right)}(P_l - P_{wf}) = -J_l^w (P_l - P_{wf})$$

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