$$\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) = \frac{\partial}{\partial x}\left(\frac{k k_{ro}}{\mu_o B_o}\frac{\partial p}{\partial x}\right) + \tilde{q}_o \qquad \text{(Oil)}$$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_o}\right) = \frac{\partial}{\partial x}\left(\frac{k k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right) + \tilde{q}_w \qquad \text{(Water)}$$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) = \frac{\phi}{B_o}\frac{\partial}{\partial t}\left(S_o\right) + \phi S_o \left(\frac{1}{B_o}\right)\frac{\partial}{\partial t}(\phi)$$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) = \frac{\phi}{B_o}\frac{\partial S_o}{\partial t} + \left[\frac{B_o}{B_o}\phi S_o \underbrace{\frac{\partial}{\partial p}\left(\frac{1}{B_o}\right)}_{=c_o} + \frac{\phi}{\phi}\frac{S_o}{B_o}\underbrace{\frac{\partial \phi}{\partial p}}_{=c_R}\right]\frac{\partial p}{\partial t}$$

$$\phi = \phi(p(t))$$

$$\frac{1}{B_o} = \frac{1}{B_o}(p(t))$$

$$c_o = B_o \frac{\partial}{\partial p}\left(\frac{1}{B_o}\right) \qquad c_r = \frac{1}{\phi}\frac{\partial \phi}{\partial p}$$

$$\frac{B_o}{B_w}\frac{\phi}{B_o}\frac{\partial S_o}{\partial t} + \frac{B_o}{B_w}\left[\frac{\phi S_o}{B_o}c_o + \frac{\phi S_o}{B_o}c_R\right]\frac{\partial p}{\partial t} = \frac{B_o}{B_w}\frac{\partial}{\partial x}\left[\frac{k k_{ro}}{\mu B_o}\frac{\partial p}{\partial x}\right] + \frac{B_o}{B_w}\tilde{q}_o \qquad \text{(oil)}$$

$$c_w = B_w \frac{\partial}{\partial p}\left(\frac{1}{B_w}\right)$$

$$+ \frac{\phi}{B_w}\frac{\partial S_w}{\partial t} + \left[\frac{\phi S_w}{B_w}c_w + \frac{\phi S_w}{B_w}c_R\right]\frac{\partial p}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right] + \tilde{q}_w \qquad \text{(water)}$$

$$\frac{\phi}{B_w}\cancelto{0}{\underbrace{\left(\frac{\partial S_o}{\partial t} + \frac{\partial S_w}{\partial t}\right)}_{\frac{\partial}{\partial t}(\underbrace{S_o+S_w}_{=1})}} + \underbrace{\frac{\phi}{B_w}\left[S_o c_o + S_w c_w + c_R(\underbrace{S_o + S_w}_{1})\right]}_{\frac{\phi}{B_w}c_t}\frac{\partial p}{\partial t} =$$

$$\frac{B_o}{B_w}\frac{\partial}{\partial x}\left(\frac{k k_{ro}}{\mu_o B_o}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial x}\left(\frac{k k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right) + \left(\frac{B_o}{B_w}\right)\tilde{q}_o + \tilde{q}_w$$

$$c_t = S_o c_o + S_w c_w + c_R$$

$$S_o + S_w = 1$$

$$\frac{\phi c_t}{B_w}\frac{\partial p}{\partial t} = \frac{B_o}{B_w}\frac{\partial}{\partial x}\left(\frac{k k_{ro}}{\mu_o B_o}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial x}\left(\frac{k k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right) + \frac{B_o}{B_w}\tilde{q}_o + \tilde{q}_w$$

"pressure equation"

$$\frac{\phi}{B_w}\frac{\partial S_w}{\partial t} + \left[\frac{\phi S_w}{B_w}c_w + \frac{\phi S_w}{B_w}c_r\right]\frac{\partial p}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right] + \tilde{q}_w$$

"saturation equation"

$$\frac{B_o}{B_w}\frac{\partial}{\partial x}\left(\frac{k k_{ro}}{\mu_o B_o}\frac{\partial p}{\partial x}\right) = \frac{1}{\Delta x_i}\left(\frac{B_o}{B_w}\right)\left[\lambda_{o,i-1/2}\frac{\left(p_{i-1}^{n+1} - p_i^{n+1}\right)}{\Delta x_{i-1/2}} + \lambda_{o,i+1/2}\frac{\left(p_{i+1}^{n+1} - p_i^{n+1}\right)}{\Delta x_{i+1/2}}\right]$$

$$\frac{\partial}{\partial x}\left(\frac{k k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right) = \frac{1}{\Delta x_i}\left(\lambda_{o,i-1/2}\frac{\left(p_{i-1}^{n+1} - p_i^{n+1}\right)}{\Delta x_{i-1/2}} + \lambda_{w,i+1/2}\frac{\left(p_{i+1}^{n+1} - p_i^{n+1}\right)}{\Delta x_{i+1/2}}\right)$$

$$\frac{\phi c_t}{B_w}\frac{\partial p}{\partial t} = \frac{\phi c_t}{B_w}\frac{\left(p_i^{n+1} - p_i^{n}\right)}{\Delta t}$$

$$\lambda_{o,i+1/2} = \frac{k_{i+1/2}\, k_{ro,i+1/2}}{\mu_o B_o}$$

$$\frac{1}{\Delta x_i}\frac{B_o}{B_w}\left[\lambda_{o,i-1/2}\frac{\left(p_{i-1}^{n+1}-p_i^{n+1}\right)}{\Delta x_{i-1/2}}+\lambda_{o,i+1/2}\frac{\left(p_{i+1}^{n+1}-p_i^{n+1}\right)}{\Delta x_{i+1/2}}\right]$$

$$+\left[\qquad\qquad '' \qquad\qquad\right]_w = \frac{\phi c_t}{B_w}\frac{p_i^{n+1}-p_i^n}{\Delta t}-\left(\frac{B_o}{B_w}\right)\tilde{q}_{o,i}$$

$$-\tilde{q}_{w,i}$$

Multiply $V_i = \Delta x A_i$

$$\left(\frac{B_o}{B_w}T_{o,i-1/2}+T_{w,i-1/2}\right)\left(p_{i-1}^{n+1}-p_i^{n+1}\right)+\left(\frac{B_o}{B_w}T_{o,i+1/2}+T_{w,i+1/2}\right)\left(p_{i-1}^{n+1}-p_i^{n+1}\right)$$

$$=\frac{V_i\phi c_t}{B_w}\frac{p_i^{n+1}-p_i^n}{\Delta t}-\left(\frac{B_o}{B_w}\right)_i Q_{o,i}-Q_{w,i}$$

$$\left(\overline{\overline{T}}+\frac{\overline{\overline{B}}}{\Delta t}\right)\vec{p}^{\,n+1}=\frac{\overline{\overline{B}}}{\Delta t}\vec{p}^{\,n}+\vec{Q}$$

<u>ImPES</u>