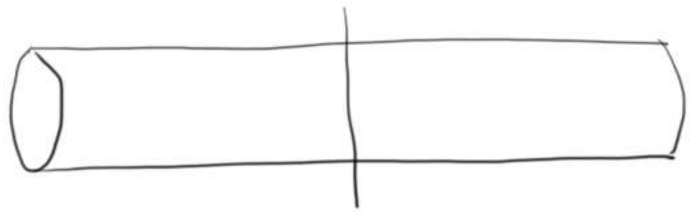


X_{Df}

$$\rho q_w\Big|_{x_{Df}-}\Delta t - \rho q_w\Big|_{x_{Df}+}\Delta t = \rho_w A\phi\,\Delta x\, S_w\Big|_{t+\Delta t} - \rho_w A\phi\,\Delta x\, S_w\Big|_{t}$$

$$q(f_{wf} - f_{wi})\Delta t = A\phi(S_{wf} - S_{wi})\Delta x$$

$$\frac{\Delta x}{\Delta t} = \frac{u(f_{wf} - f_{wi})}{\phi(S_{wf} - S_{wi})} \Rightarrow \frac{\Delta x_D}{\Delta t_D} = \frac{f_{wf}}{(S_{wf} - S_{wi})}$$

$$\frac{\partial x_D}{\partial t_D}\Big|_{S_{wf}} = \frac{\partial f}{\partial S_w}\Big|_{S_{wf}}$$

$$f_{wf} = \frac{\partial f_w}{\partial S_w}\Big|_{S_{wf}}(S_{wf} - S_{wi})$$

$$u_w = \frac{k k_{rw}}{\mu_w} \frac{\partial P_w}{\partial x}, \quad u_g = \frac{k k_{rg}}{\mu_g} \frac{\partial P_g}{\partial x}, \quad u_o = \frac{k k_{ro}}{\mu_o} \frac{\partial P_o}{\partial x}$$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right) = \frac{\partial}{\partial x}\left(\frac{1}{B_w} \frac{k k_{rw}}{\mu_w} \frac{\partial P_w}{\partial x}\right) + \tilde{q}_w$$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) = \frac{\partial}{\partial x}\left(\frac{1}{B_o} \frac{k k_{ro}}{\mu_o} \frac{\partial P_o}{\partial x}\right) + \tilde{q}_o$$

$$\frac{\partial}{\partial t}\left[\phi\left(\frac{1}{B_g} S_g + \frac{R_s}{B_o} S_o\right)\right] = \frac{\partial}{\partial x}\left(\frac{1}{B_g} \frac{k k_{rg}}{\mu_g} \frac{\partial P_g}{\partial x} + R_s \frac{1}{B_o} \frac{k k_{ro}}{\mu_o} \frac{\partial P_o}{\partial x}\right) + \tilde{q}_g + \tilde{R}_s \tilde{q}_o$$

$$S_w + S_o + S_g = 1$$

$$P_{c,ow}(S_w) = P_o - P_w$$

$$P_{c,og}(S_o) = P_g - P_o$$

$$k_{rw} = f(S_w)$$

$$k_{ro} = f(S_w, S_g)$$

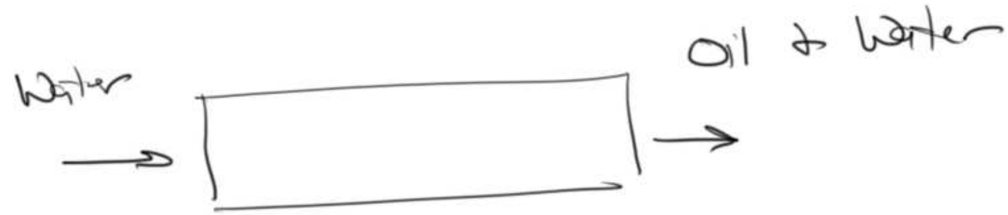
$$k_{rg} = f(S_g)$$

Buckley-Leverett Theory

- Semi-analytical
- Useful for verification of numerical model

Assumptions

- 2-phase flow (oil & water)
- 1D
- Incompressible fluids
- No capillary pressure / gravity
- No sources or sinks



I.C.'s & B.C.'s

- Core initially saturated w/ water @ residual water sat. ($S_{wi} = S_{wr}$)
- Constant injection rate (q) at $x = 0$
- Constant production rate (q) at $x = L$

$$\frac{\partial}{\partial t}\left(\phi \frac{S_w}{B_w}\right) = -\frac{\partial}{\partial x}\left(\frac{u_w}{B_w}\right)$$

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial u_w}{\partial x} \qquad u_w = \frac{q_w}{A} = \frac{q f_w}{A}$$

$$\phi \frac{\partial S_w}{\partial t} = -\frac{q}{A}\frac{\partial f_w}{\partial x} \qquad f_w(S_w)$$

$$\frac{\dfrac{\partial S_w}{\partial t}}{\dfrac{\partial S_w}{\partial t}} = -\frac{q}{A\phi}\frac{\partial f_w}{\partial S_w}\frac{\cancel{\dfrac{\partial S_w}{\partial x}}}{\cancel{\dfrac{\partial S_w}{\partial x}}}$$

$$S_w = S_w(x,t)$$

$$d\cancelto{0}{S_w} = \frac{\partial S_w}{\partial x}dx + \frac{\partial S_w}{\partial t}dt$$

$$u_w = \left.\frac{dx}{dt}\right|_{S_w} = -\frac{\left.\left(\dfrac{\partial S_w}{\partial t}\right)\right|_{S_w}}{\left.\left(\dfrac{\partial S_w}{\partial x}\right)\right|_{S_w}}$$

$$\left.\frac{\partial x}{\partial t}\right|_{S_w} = \frac{q}{A\phi}\frac{\partial f_w}{\partial S_w}$$

$$x_D = \frac{x}{L}$$

$$t_D = \frac{qt}{\phi A L} \equiv \frac{\text{pore volume injected}}{\text{pore volume core}}$$

$$\left.\frac{\partial x}{\partial t}\right|_{S_w} = \frac{q}{A\phi}\frac{\partial f_w}{\partial S_w} \Rightarrow \left.\frac{\partial x_D}{\partial t_D}\right|_{S_w} = \frac{\partial f_w}{\partial S_w}$$

$x_D = 0$ at $t_D = 0$

$$x_D(S_w) = t_D \left.\frac{\partial f_w}{\partial S_w}\right|_{S_w}$$