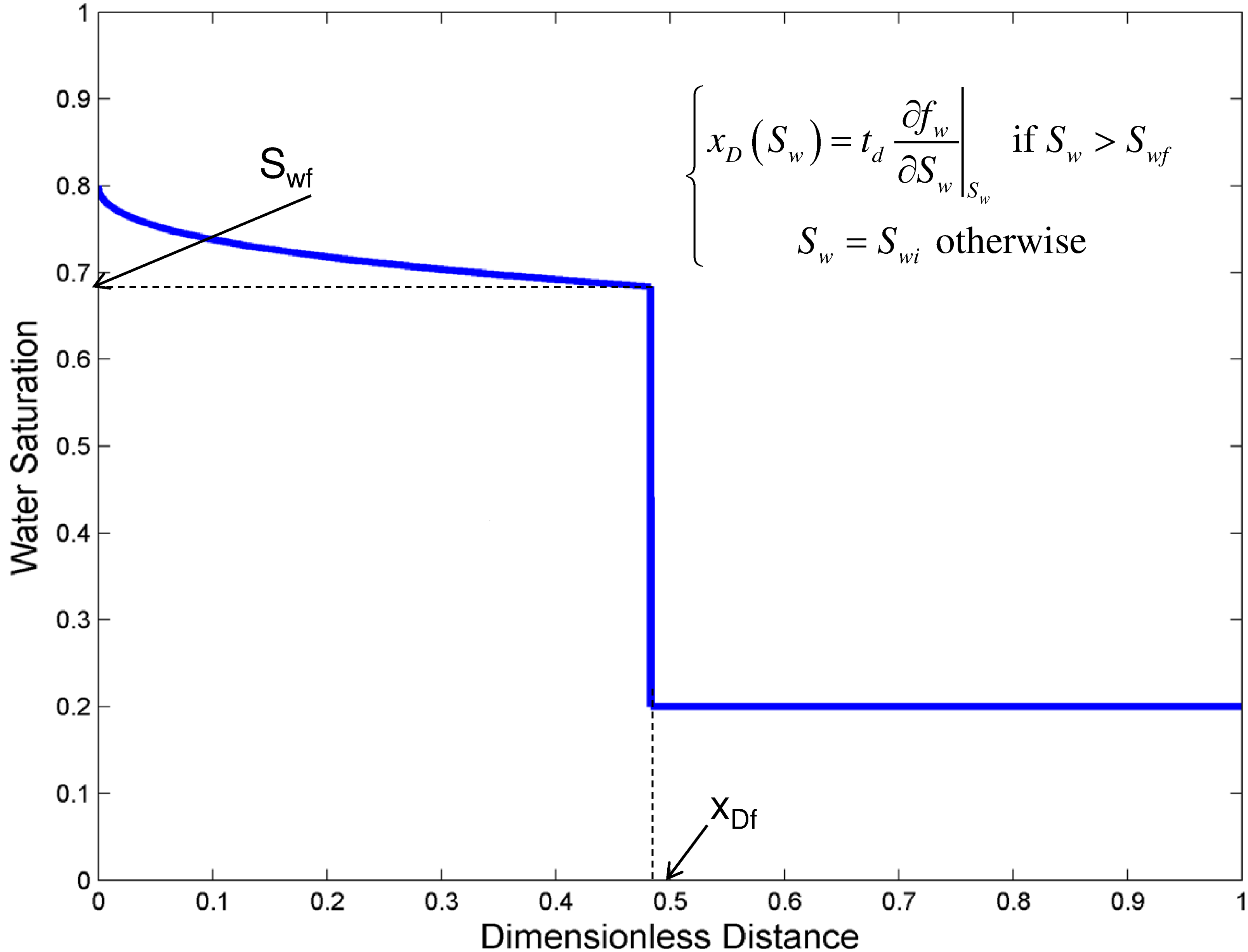
Water Saturation Profile from BL Theory



Steps in Reservoir Initialization (P_w, P_o, S_w)

1. Identify the water pressure at a reference point, e.g. the WOC

$$P_w = P_{w,WOC} \quad @ \quad z = WOC$$

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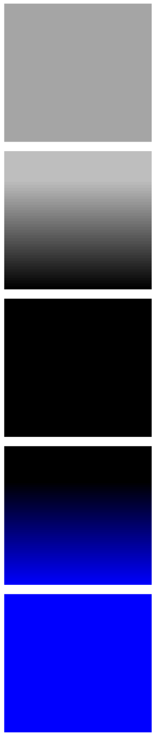
$$P_c^i = f\left(S_w^i \right)$$

Initial Conditions for Multiphase Flow



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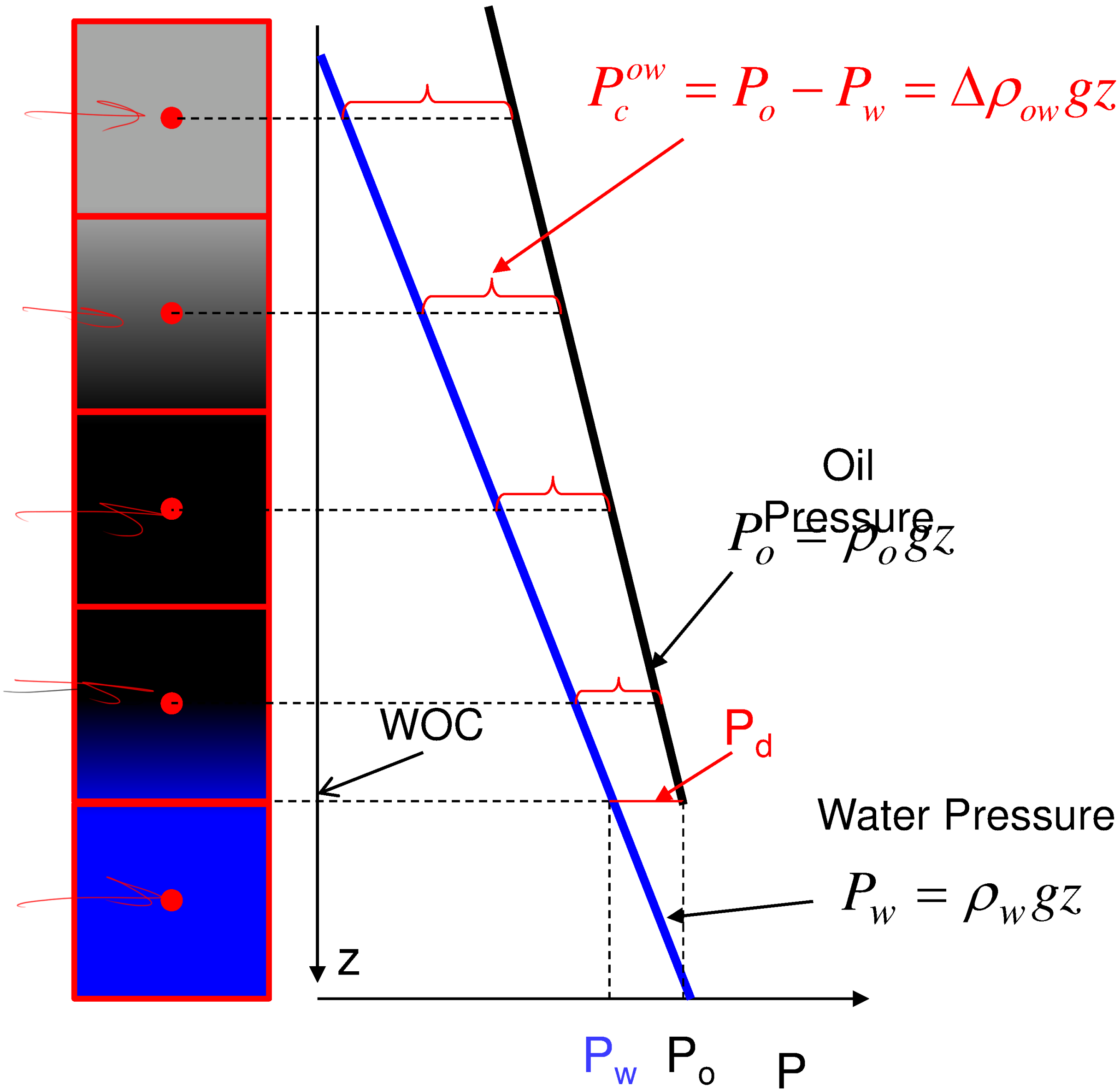
Pc Found Using Density Differences



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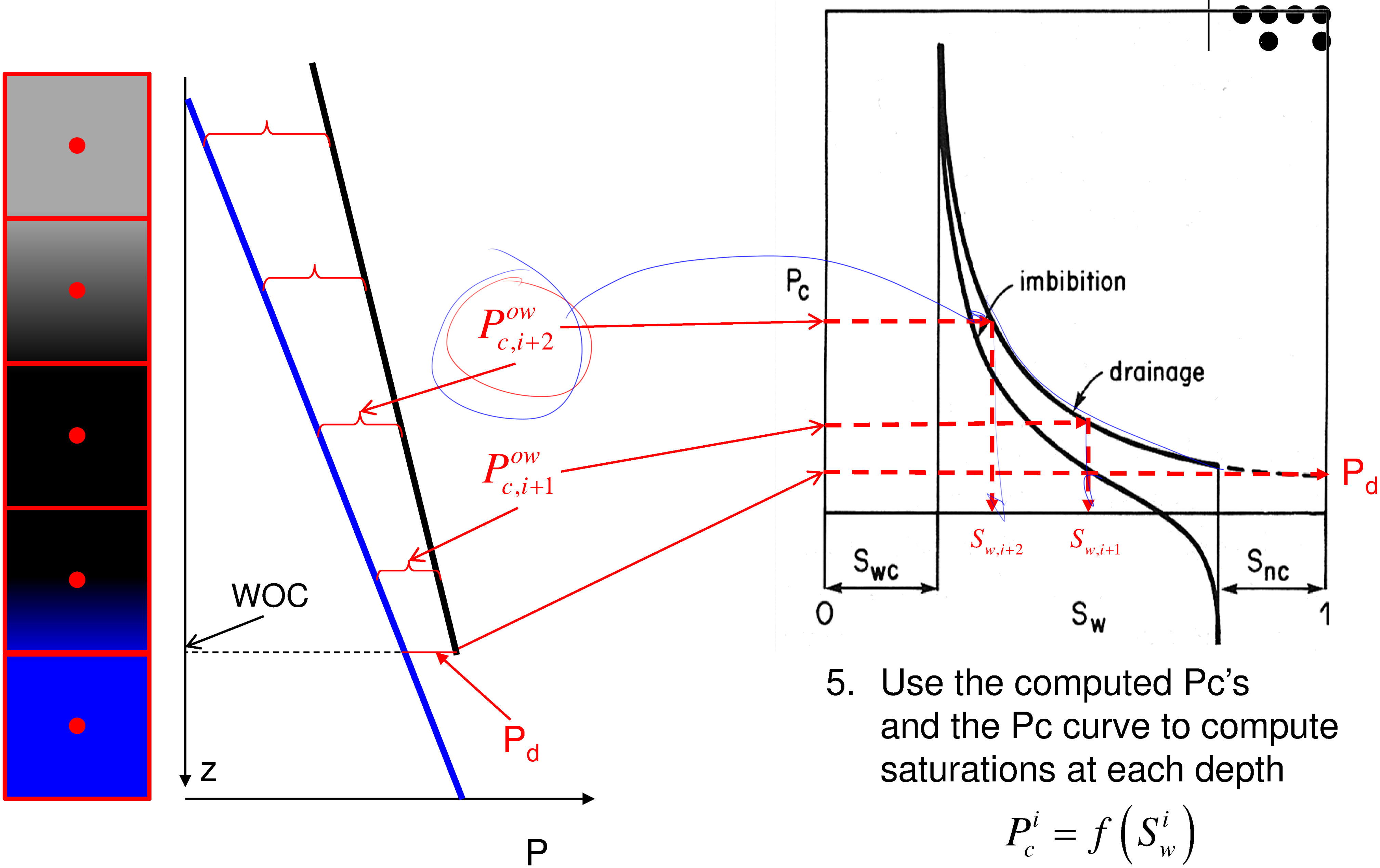

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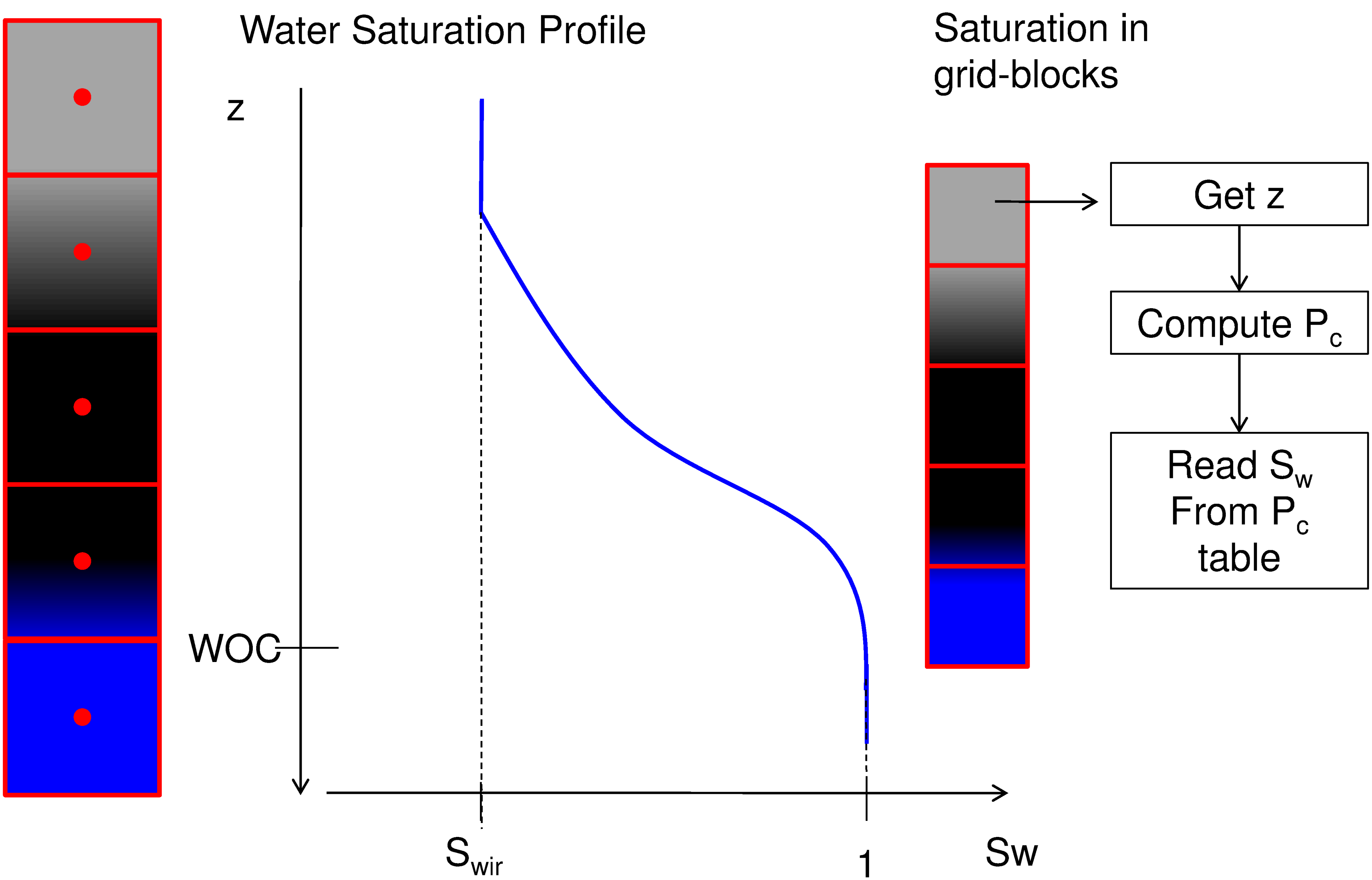
Saturation Profile with Depth



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Water

$$\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right) = \frac{\partial}{\partial x}\left(\frac{k\, k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right) + \tilde{q}^w$$

Oil

$$\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) = \frac{\partial}{\partial x}\left(\frac{k\, k_{ro}}{\mu_o B_o}\frac{\partial p}{\partial x}\right) + \tilde{q}^o$$

$\underbrace{\qquad\qquad}_{\text{product rule}}$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) = \frac{\phi}{B_o}\frac{\partial S_o}{\partial t} + \underbrace{\phi S_o \frac{\partial}{\partial t}\left(\frac{1}{B_o}\right) + \frac{S_o}{B_o}\frac{\partial}{\partial t}(\phi)}_{\text{chain rule}}$$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) = \frac{\phi}{B_o}\frac{\partial S_o}{\partial t} + \left[\phi S_o \frac{\partial}{\partial p}\left(\frac{1}{B_o}\right) + \frac{S_o}{B_o}\frac{\partial \phi}{\partial p}\right]\frac{\partial p}{\partial t}$$

Recall

$$c_o = B_o \frac{\partial}{\partial p}\left[\frac{1}{B_o}\right], \quad c_w = B_w \frac{\partial}{\partial p}\left[\frac{1}{B_w}\right] \quad c_R = \frac{1}{\phi}\frac{\partial \phi}{\partial p}$$

Oil

$$\frac{B_o}{B_w}\left\{\frac{\phi}{B_o}\frac{\partial S_o}{\partial t}+\left[\frac{\phi S_o}{B_o}c_o+\frac{\phi S_o}{B_o}c_R\right]\frac{\partial p}{\partial t}\right\}=\left\{\frac{\partial}{\partial x}\left[\frac{k k_{ro}}{\mu_o B_o}\frac{\partial p}{\partial x}\right]+\tilde{q}_o\right\}\frac{B_o}{B_w}$$

$$+\quad \frac{\phi}{B_w}\frac{\partial S_w}{\partial t}+\left[\frac{\phi S_w}{B_w}c_w+\frac{\phi S_w}{B_w}c_R\right]\frac{\partial p}{\partial t}=\frac{\partial}{\partial x}\left[\frac{k k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right]+\tilde{q}_w$$

$$c_t = S_o c_o + S_w c_w + c_R$$

$$\frac{\phi}{B_w}\underbrace{\left(\frac{\partial S_o}{\partial t}+\frac{\partial S_w}{\partial t}\right)}_{0}+\underbrace{\left[\frac{\phi S_o}{B_w}c_o+\frac{\phi S_w}{B_w}c_w+\frac{\phi S_o}{B_w}c_R+\frac{\phi S_w}{B_w}c_R\right]}_{\frac{c_t\phi}{B_w}}\frac{\partial p}{\partial t}=$$

$$\frac{B_o}{B_w}\frac{\partial}{\partial x}\left[\frac{k k_o}{\mu_o B_o}\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial x}\left(\frac{k k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right)+\frac{B_o}{B_w}\tilde{q}_o+\tilde{q}_w$$

$$\frac{\phi}{B_w}\frac{\partial}{\partial t}\underbrace{(S_o+S_w)}_{1}$$

Overall

$$\frac{\phi c_t}{B_w}\frac{\partial p}{\partial t} = \frac{B_o}{B_w}\frac{\partial}{\partial x}\left(\frac{k k_{ro}}{\mu_o B_o}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial x}\left(\frac{k k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right) + \left(\frac{B_o}{B_w}\right)\tilde{q}_o + \tilde{q}_w$$

Solve Implicit for P

Water

$$\frac{\phi}{B_w}\frac{\partial S_w}{\partial t} + \frac{\phi S_w}{B_w}(c_R + c_w)\frac{\partial p}{\partial t} = \frac{\partial}{\partial x}\left(\frac{k k_{rw}}{\mu_w B_w}\frac{\partial p}{\partial x}\right) + \tilde{q}_w$$

Solve Explicit for S

IMPES

Implicit Pressure, Explicit Saturation

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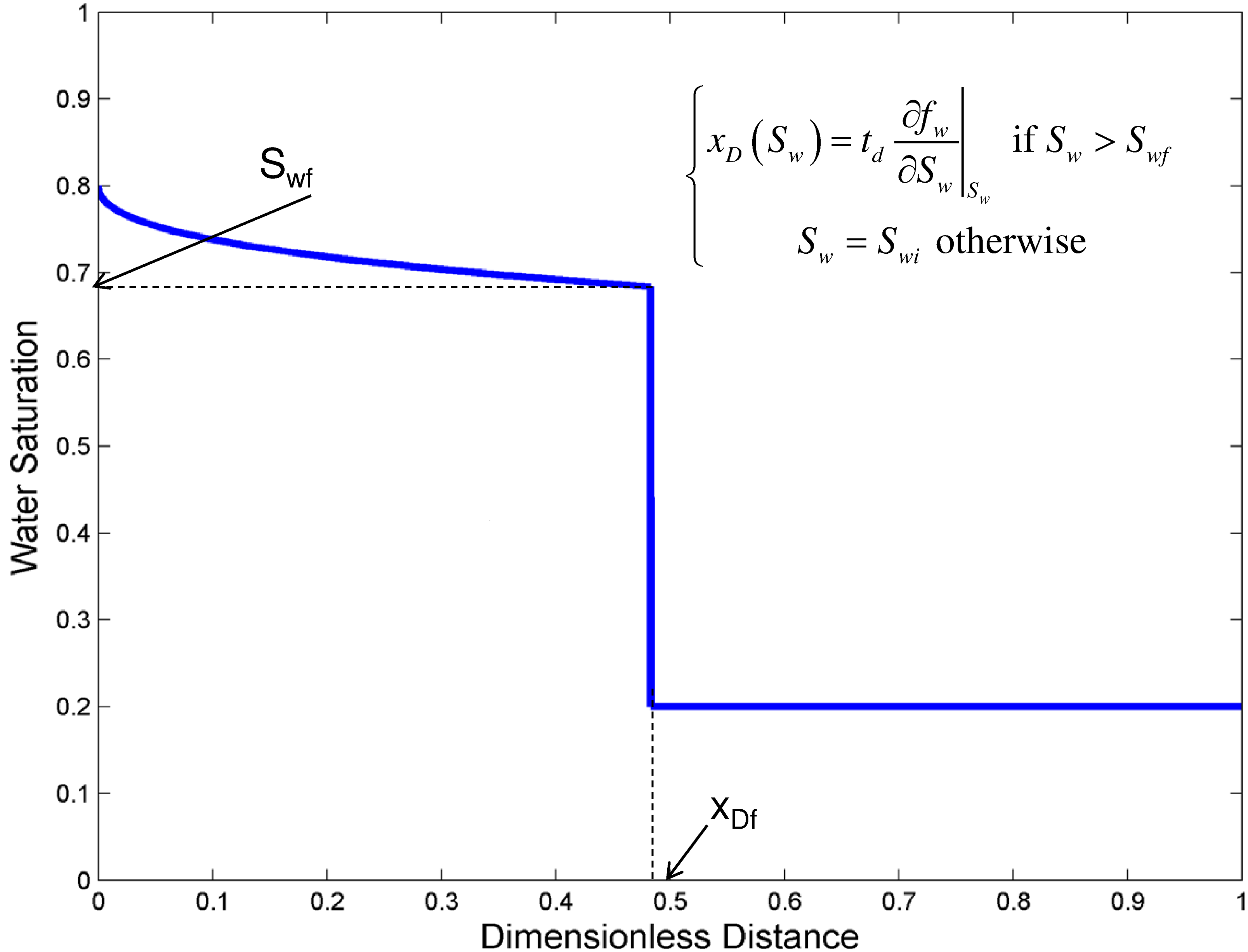

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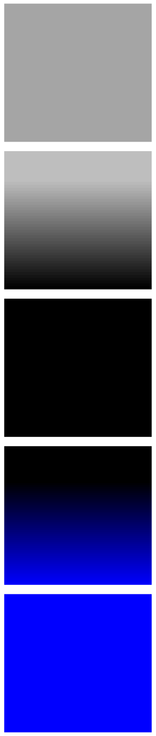

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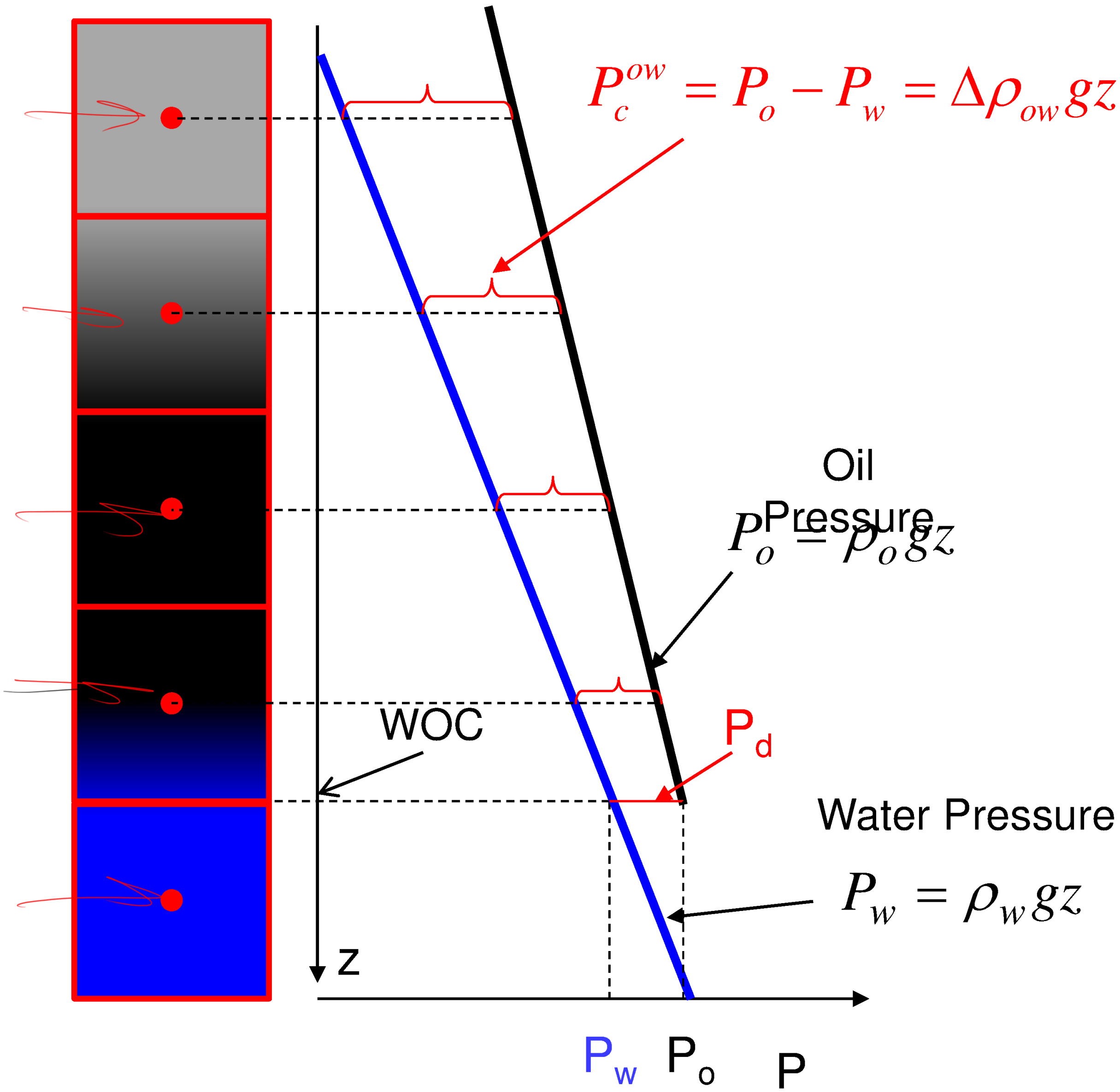

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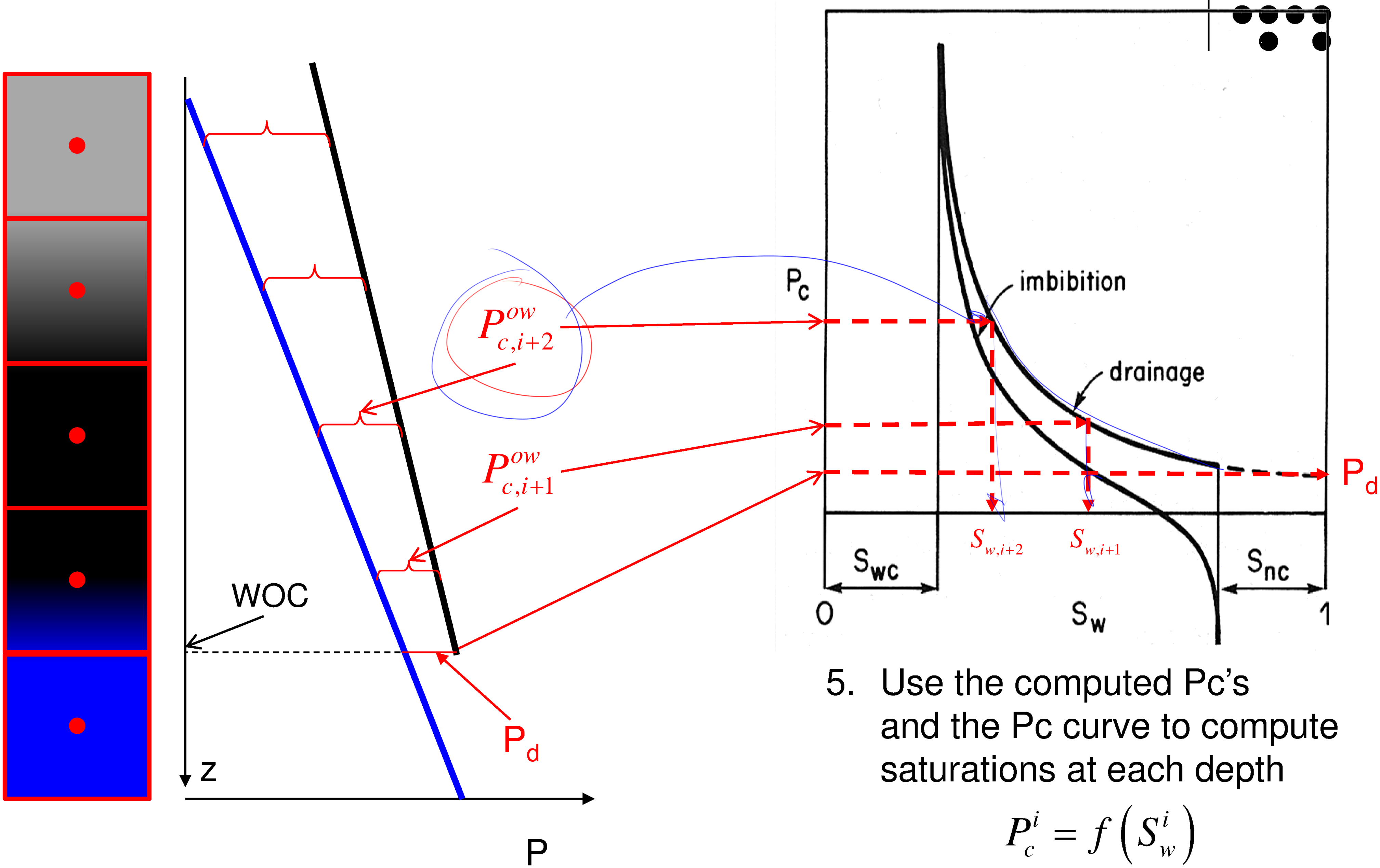

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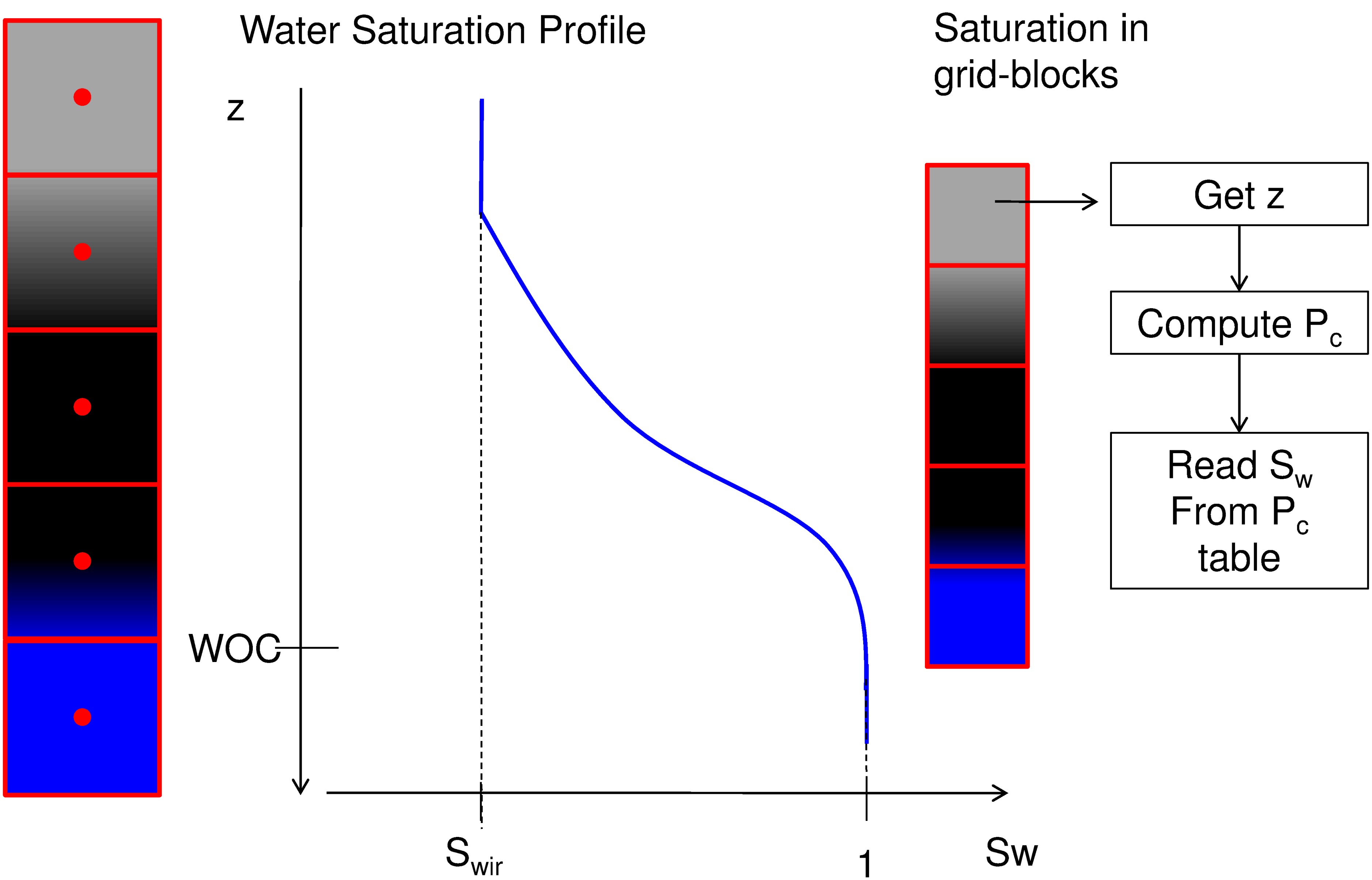

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