$$-\eta P_{i-1}^{n+1} + (1+2\eta) P_i^{n+1} - \eta P_{i+1}^{n+1} = P_i^n \qquad [\equiv psi]$$

Transmissibility Form

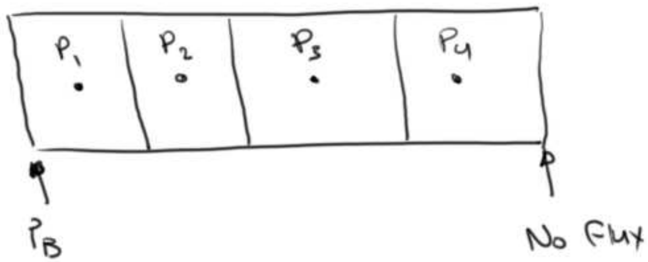
$$\eta = \frac{\alpha \Delta t}{(\Delta t)^2} = \frac{k}{\mu \phi c_t} \frac{\Delta t}{(\Delta x)^2}$$

Multiply both sides by

$$\frac{A \Delta x \phi c_t}{B_w \Delta t} = \frac{V_i \phi c_t}{B_w \Delta t}$$

$$-\underbrace{\frac{kA}{\mu B_w \Delta x}}_{T} P_{i-1}^{n+1} + \left(\underbrace{\frac{V_i \phi c_t}{B_w}}_{B} \frac{1}{\Delta t} + 2 \underbrace{\frac{kA}{\mu B_w \Delta x}}_{T} \right) P_i^{n+1} - \underbrace{\frac{kA}{\mu B_w \Delta x}}_{T} P_{i+1}^{n+1} = \underbrace{\frac{V_i \phi c_t}{B_w}}_{B} \frac{1}{\Delta t} P_i^n \qquad \left[\equiv \frac{ft^3}{day} \right]$$

$$-T P_{i-1}^{n+1} + \left(\frac{B_i}{\Delta t} + 2T \right) P_i^{n+1} - T P_{i+1}^{n+1} = \frac{B_i}{\Delta t} P_i^n$$



block 1: $\left(\frac{B_1}{\Delta t} + 3T\right) p_1^{n+1} - T p_2^{n+1} = \frac{B_1}{\Delta t} p_1^n + 2T p_B$

block 2: $-T p_1^{n+1} + \left(\frac{B_2}{\Delta t} + 2T\right) p_2^{n+1} - T p_3^{n+1} = \frac{B_2}{\Delta t} p_2^n$

block 3: $-T p_2^{n+1} + \left(\frac{B_3}{\Delta t} + 2T\right) p_3^{n+1} - T p_4^{n+1} = \frac{B_3}{\Delta t} p_3^n$

block 4: $-T p_3^{n+1} + \left(\frac{B_4}{\Delta t} + T\right) p_4^{n+1} = \frac{B_4}{\Delta t} p_4^n$

$$\left(\frac{1}{\Delta t}\underbrace{\begin{bmatrix} B_1 & 0 & 0 & 0 \\ 0 & B_2 & 0 & 0 \\ 0 & 0 & B_3 & 0 \\ 0 & 0 & 0 & B_4 \end{bmatrix}}_{\bar{\bar{B}}} + \underbrace{\begin{bmatrix} 3T & -T & 0 & 0 \\ -T & 2T & -T & 0 \\ 0 & -T & 2T & -T \\ 0 & 0 & -T & T \end{bmatrix}}_{\bar{\bar{T}}}\right) \underbrace{\begin{Bmatrix} p_1^{n+1} \\ p_2^{n+1} \\ p_3^{n+1} \\ p_4^{n+1} \end{Bmatrix}}_{\vec{p}^{n+1}} = \frac{1}{\Delta t}\underbrace{\begin{bmatrix} B_1 & 0 & 0 & 0 \\ 0 & B_2 & 0 & 0 \\ 0 & 0 & B_3 & 0 \\ 0 & 0 & 0 & B_4 \end{bmatrix}}_{\bar{\bar{B}}} \underbrace{\begin{Bmatrix} p_1^n \\ p_2^n \\ p_3^n \\ p_4^n \end{Bmatrix}}_{\vec{p}^n} + \underbrace{\begin{Bmatrix} 2Tp_B \\ 0 \\ 0 \\ 0 \end{Bmatrix}}_{\vec{Q}}$$

$$\left(\bar{\bar{T}} + \frac{1}{\Delta t}\bar{\bar{B}}\right)\vec{p}^{n+1} = \frac{1}{\Delta t}\bar{\bar{B}}\vec{p}^n + \vec{Q} \Rightarrow \vec{p}^{n+1} = \left(\bar{\bar{T}} + \frac{1}{\Delta t}\bar{\bar{B}}\right)^{-1}\left(\frac{1}{\Delta t}\bar{\bar{B}}\vec{p}^n + \vec{Q}\right)$$

$$\frac{B_i}{\Delta t} P_i^{n+1} = T P_{i-1}^n + \left(\frac{B_i}{\Delta t} - 2T \right) P_i^n + T P_{i+1}^n$$

Matrix Form

$$\frac{1}{\Delta t} \bar{\bar{B}} \vec{P}^{n+1} = \left(-\bar{\bar{T}} + \frac{1}{\Delta t} \bar{\bar{B}} \right) \vec{P}^n + \vec{Q}$$

$$\vec{P}^{n+1} = \vec{P}^n + \Delta t \, \bar{\bar{B}}^{-1} \left(\vec{Q} - T \vec{P}^n \right)$$

$$\bar{\bar{B}} = \begin{bmatrix} B_1 & & \\ & B_2 & \\ & & B_3 \end{bmatrix}$$

$$\bar{\bar{B}}^{-1} = \begin{bmatrix} \frac{1}{B_1} & & \\ & \frac{1}{B_2} & \\ & & \frac{1}{B_3} \end{bmatrix}$$

$$B B^{-1} = I$$

Mixed Methods

$$\Theta \left[\overset{=}{T} \vec{p}^{\,n} + \frac{1}{\Delta t} \overset{=}{B} \vec{p}^{\,n+1} = \frac{1}{\Delta t} \overset{=}{B} \vec{p}^{\,n} + \vec{Q} \right] +$$

Explicit

$$(1-\Theta) \left[\overset{=}{T} \vec{p}^{\,n+1} + \frac{1}{\Delta t} \overset{=}{B} \vec{p}^{\,n+1} = \frac{1}{\Delta t} \overset{=}{B} \vec{p}^{\,n} + \vec{Q} \right]$$

Implicit

$$\Theta \overset{=}{T} \vec{p}^{\,n} + (1-\Theta) \overset{=}{T} p^{n+1} + \frac{1}{\Delta t} \overset{=}{B} \vec{p}^{\,n+1} = \frac{1}{\Delta t} \vec{p}^{\,n} + \vec{Q}$$

for $\Theta = \frac{1}{2}$

"Crank-Nicholson" Method

$$\left((1-\Theta) \overset{=}{T} + \frac{1}{\Delta t} \overset{=}{B} \right) \vec{p}^{\,n+1} = \left(\frac{1}{\Delta t} \overset{=}{B} - \Theta \overset{=}{T} \right) \vec{p}^{\,n} + \vec{Q}$$

$$\vec{p}^{\,n+1} = \left((1-\Theta) \overset{=}{T} + \frac{1}{\Delta t} \overset{=}{B} \right)^{-1} \left\{ \left(\frac{1}{\Delta t} \overset{=}{B} - \Theta \overset{=}{T} \right) \vec{p}^{\,n} - \vec{Q} \right\}$$