Condition testing

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Contingency table:

		Condition (as determined by "Gold standard")			
	Total population	Condition positive	Condition negative	Prevalence = $\frac{\Sigma \text{ Condition positive}}{\Sigma \text{ Total population}}$	
Test outcome	Test outcome positive	**True positive**	**False positive** (Type I error)	Positive predictive value (PPV, Precision) = $\frac{\Sigma \text{ True positive}}{\Sigma \text{ Test outcome positive}}$	False discovery rate (FDR) = $\frac{\Sigma \text{ False positive}}{\Sigma \text{ Test outcome positive}}$
	Test outcome negative	**False negative** (Type II error)	**True negative**	False omission rate (FOR) = $\frac{\Sigma \text{ False negative}}{\Sigma \text{ Test outcome negative}}$	Negative predictive value (NPV) = $\frac{\Sigma \text{ True negative}}{\Sigma \text{ Test outcome negative}}$
	Positive likelihood ratio (**LR+**) = TPR/FPR	True positive rate (TPR, Sensitivity, Recall) = $\frac{\Sigma \text{ True positive}}{\Sigma \text{ Condition positive}}$	False positive rate (FPR, Fall-out) = $\frac{\Sigma \text{ False positive}}{\Sigma \text{ Condition negative}}$	Accuracy (ACC) = $\frac{\Sigma \text{ True positive} + \Sigma \text{ True negative}}{\Sigma \text{ Total population}}$	
	Negative likelihood ratio (**LR–**) = FNR/TNR	False negative rate (FNR) = $\frac{\Sigma \text{ False negative}}{\Sigma \text{ Condition positive}}$	True negative rate (TNR, Specificity, SPC) = $\frac{\Sigma \text{ True negative}}{\Sigma \text{ Condition negative}}$		
	Diagnostic odds ratio (**DOR**) = LR+/LR–				

In terms of probabilities:

Event	Event		Total
	B_1	B_2	
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

Joint Probability

Marginal (Simple) Probability

Marginal probabilities:

Relative size	Cond B	Cond $\bar{B}$	Total
Case A	w	x	w+x
Case $\bar{A}$	y	z	y+z
Total	w+y	x+z	w+x+y+z = 1

$$P(B) = \frac{w+y}{w+y+x+z} = w+y$$

$$P(A) = \frac{w+x}{w+y+x+z} = w+x$$

	Condition (determined by gold standard dataset)	
	Disease (D)	No Disease (D^c)
Test (+)	TP	FP
Test (-)	FN	TN
	P	N

(being the condition for example a Disease)

Probability of condition positive $\rightarrow p(D) =$ TP+FN $\Rightarrow$ **PREVALENCE**

Probability of condition negative $\rightarrow p(D^c) =$ FP+TN

Probability of test outcome positive $\rightarrow p(+) =$ TP+FP

Probability of test outcome negative $\rightarrow p(-) =$ FN+TN

Where:

$p(\text{total}) =$ TP+FP+FN+TN $= 1$

$p(D^c) = 1 - p(D) \equiv$ FP+TN $= 1 -$ (TP+FN)

$p(-) = 1 - p(+) \equiv$ FN+TN $= 1 -$ (TP+FP)

Joint probabilities (intersections):

Relative size	Cond B	Cond $\bar{B}$	Total
Case A	w	x	w+x
Case $\bar{A}$	y	z	y+z
Total	w+y	x+z	w+x+y+z = 1

$$P(AB) = \frac{w}{w+x+y+z} = w$$

	Condition (determined by gold standard dataset)	
	Disease (D)	No Disease (D^c)
Test (+)	TP	FP
Test (-)	FN	TN
	P	N

Probability of $+$ and $D \rightarrow p(+ \cap D) =$ TP

Probability of $-$ and $D \rightarrow p(- \cap D) =$ FN

Probability of $+$ and $D^c \rightarrow p(+ \cap D^c) =$ FP

Probability of $-$ and $D^c \rightarrow p(- \cap D^c) =$ TN

Conditional probabilities:

Relative size	Cond B	Cond $\bar{B}$	Total
Case A	w	x	$w+x$
Case $\bar{A}$	y	z	$y+z$
Total	$w+y$	$x+z$	$w+x+y+z$ = 1

$$P(A|B) = \frac{w}{w+y} = \frac{P(AB)}{P(B)}$$

$$P(B|A) = \frac{w}{w+x} = \frac{P(AB)}{P(A)}$$

	Condition (determined by gold standard dataset)	
	Disease (D)	No Disease (D^c)
Test (+)	TP	FP
Test (-)	FN	TN
	P	N

- ***Test outcome* conditioned on *condition***

(the probability of having a positive/negative test result for a patient that has/hasn't the disease)

Probability of $+$ given $D \rightarrow p(+|D) = \frac{p(+\cap D)}{p(D)} = \frac{\text{TP}}{\text{TP+FN}} \Rightarrow$ **SENSITIVITY**

Probability of $+$ given $D^c \rightarrow p(+|D^c) = \frac{p(+\cap D^c)}{p(D^c)} = \frac{\text{FP}}{\text{FP+TN}} \Rightarrow$ **FALSE POSITIVE RATE**

Probability of $-$ given $D^c \rightarrow p(-|D^c) = \frac{p(-\cap D^c)}{p(D^c)} = \frac{\text{TN}}{\text{FP+TN}} \Rightarrow$ **SPECIFICITY**

Probability of $-$ given $D \rightarrow p(-|D) = \frac{p(-\cap D)}{p(D)} = \frac{\text{FN}}{\text{TP+FN}} \Rightarrow$ **FALSE NEGATIVE RATE**

Where:

$p(+|D^c) = 1 - p(-|D^c) \equiv \frac{\text{FP}}{\text{FP+TN}} = 1 - \frac{\text{TN}}{\text{FP+TN}} \rightarrow \text{FPR} = 1 - \text{SPECIFICITY}$

$p(-|D) = 1 - p(+|D) \equiv \frac{\text{FN}}{\text{TP+FN}} = 1 - \frac{\text{TP}}{\text{TP+FN}} \rightarrow \text{FNR} = 1 - \text{SENSITIVITY}$

- ***Condition* conditioned on *test outcome***

(the probability of having/not having the disease if the test result is positive/negative)

Probability of D given $+ \rightarrow p(D|+) = \frac{p(+\cap D)}{p(+)} = \frac{\text{TP}}{\text{TP+FP}} \Rightarrow$ **POSITIVE PREDICTIVE VALUE (PPV, PRECISION)**

Probability of D given $- \rightarrow p(D|-) = \frac{p(+\cap D)}{p(-)} = \frac{\text{FN}}{\text{TP+FP}} \Rightarrow$ FALSE OMISSION RATE (FOR)

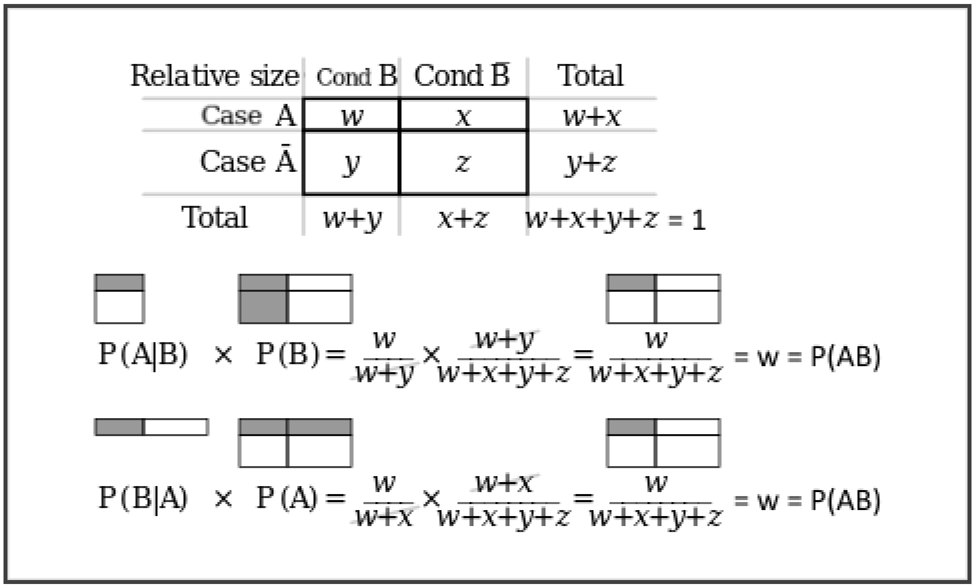
Probability of D^c given $+ \rightarrow p(D^c|+) = \frac{p(+\cap D^c)}{p(+)} = \frac{\text{FP}}{\text{TP+FP}} \Rightarrow$ FALSE DISCOVERY RATE (FDR)

Probability of D^c given $- \rightarrow p(D^c|-) = \frac{p(-\cap D^c)}{p(-)} = \frac{\text{TN}}{\text{FP+TN}} \Rightarrow$ **NEGATIVE PREDICTIVE VALUE (NPV)**

Bayes' theorem:

$p(A|B) = \frac{p(B|A)p(A)}{p(B)} \rightarrow p(A|B)p(B) = p(B|A)p(A)$

Demonstration:



SENSITIVITY : $p(+|D) = \frac{p(+\cap D)}{p(D)}$, PPV : $p(D|+) = \frac{p(+\cap D)}{p(+)}$

$\rightarrow p(+\cap D) = p(+|D)p(D) = p(D|+)p(+) \;\equiv\;$ TP = SENSITIVITY $*$ PREVALENCE = PPV $*$ (TP+FP)

Bayes' rule:

$p(B|A) = \frac{p(A|B)p(B)}{p(A|B)p(B)+p(A|B^c)p(B^c)}$

Bayes' theorem:

$p(B|A) = \frac{p(A|B)p(B)}{p(A)}$

Then:

$\rightarrow p(A) = p(A|B)p(B) + p(A|B^c)p(B^c)$

Demonstration:

$p(+) = p(+|D)p(D) + p(+|D^c)p(D^c) = p(+\cap D) + p(+\cap D^c) \rightarrow$ TP+FP = TP+FP

$p(D|+) = \frac{p(+|D)p(D)}{p(+|D)p(D)+p(+|D^c)p(D^c)} \;\equiv\;$ PPV $= \frac{\text{SENSITIVITY*PREVALENCE}}{\text{SENSITIVITY*PREVALENCE}+(1-\text{SPECIFICITY})*(1-\text{PREVALENCE})}$

Likelihood ratios:

Bayes' rule:

$\text{PPV} = p(D|+) = \frac{p(+|D)p(D)}{p(+|D)p(D)+p(+|D^c)p(D^c)}$, $\text{FDR} = p(D^c|+) = \frac{p(+|D^c)p(D^c)}{p(+|D)p(D)+p(+|D^c)p(D^c)}$

$\rightarrow \frac{p(D|+)}{p(D^c|+)} = \frac{p(+|D)}{p(+|D^c)} * \frac{P(D)}{P(D^c)} \equiv$ POST TEST ODDS = $\text{DLR}_+ *$ PRE TEST ODDS $\Rightarrow$

$\Rightarrow \text{DLR}_+ = \frac{\text{POST TEST ODDS}}{\text{PRE TEST ODDS}}$

DLR_+ relates the increase in the odds of the disease after a positive test result to the odds of disease prior to the test.

Similarly:

$\rightarrow \frac{p(D|-)}{p(D^c|-)} = \frac{p(-|D)}{p(-|D^c)} * \frac{P(D)}{P(D^c)} \equiv$ POST TEST ODDS = $\text{DLR}_- *$ PRE TEST ODDS $\Rightarrow$

$\Rightarrow \text{DLR}_- = \frac{\text{POST TEST ODDS}}{\text{PRE TEST ODDS}}$

DLR_- relates the decrease in the odds of the disease after a negative test result to the odds of disease prior to the test.

Type I and II errors

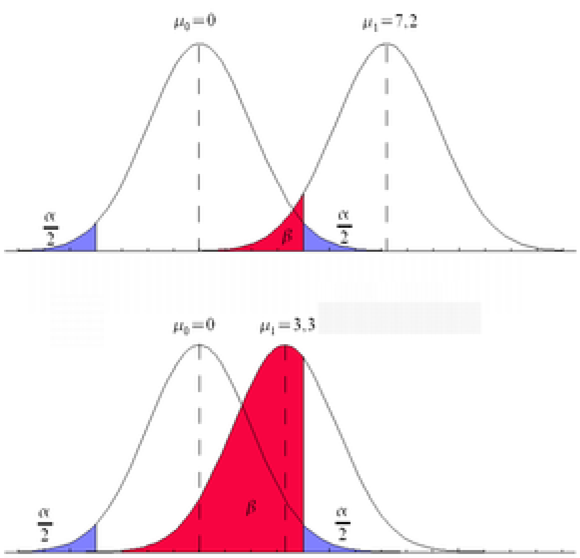
The **FPR** is a **Type I error** (α type error):

$$p(+|D^c) \equiv p(\text{rejecting H0} \mid \text{H0 is true})$$

We are rejecting the null hypothesis "there are no significant differences with the '*normal state*' population" when we souldn't, that is, we are considering that the pacient doesn't belong to the healthy population because he falls into the rejection region determined by the α significance level.

This happens because the two populations ('*normal state*' = condition negative = no disease, and '*altered state*' = condition positive = disease) overlap to some extent, so:

- the bigger the significance level, the more we are assuring the 'integrity' of the '*normal state*' population: we maximize true positives at the expense of maximizing false positives (bigger $\alpha \to$ more Type I error)
- the smaller the significance level, the more we are assuring the integrity of the '*altered state*' population: we minimize false positives at the expense of minimizing true positives (smaller $\alpha \to$ less Type I error)



The **FNR** is a **Type II error** (β type error):

$$p(-|D) \equiv p(\text{not rejecting H0} \mid \text{H0 is not true})$$

We aren't rejecting the null hypothesis when we should, that is, we are considering that the pacient is no significantly different than the '*normal state*' population because he doesn't fall into the α region.

Again, this happens because the two populations are overlapping: the more they do (the closer their means are to each other), the less we are able to discern between the two populations $\to$ the less we reject the null hypothesis (more false negatives). In this case we can't control the extension of the β region since it is determined by the mean of the alternative population.