Part 1a: Perfect Competition and Monopoly

Instructor: Oleg Baranov

Microeconomics 2 *(Module 1, 2023)*.

Suggested Readings for this part

Readings in Class Textbooks:

- **Varian, 8th edition:**
  - Ch 22: Firm Supply
  - Ch 23: Industry Supply
  - Ch 24: Monopoly

- **Carlton and Perloff:**
  - Ch 3: Competition
  - Ch 4: Monopolies, Monopsonies and Dominant Firms

Additional Readings on Smart LMS:

- Amazon and Hachette Dispute

Review: Cost Concepts

Cost Concepts - I

- Denote F fixed costs (independent of quantity q) and $VC(q)$ variable cost of producing quantity q
- The total cost of producing quantity q is given by

$$C(q) = F + VC(q)$$

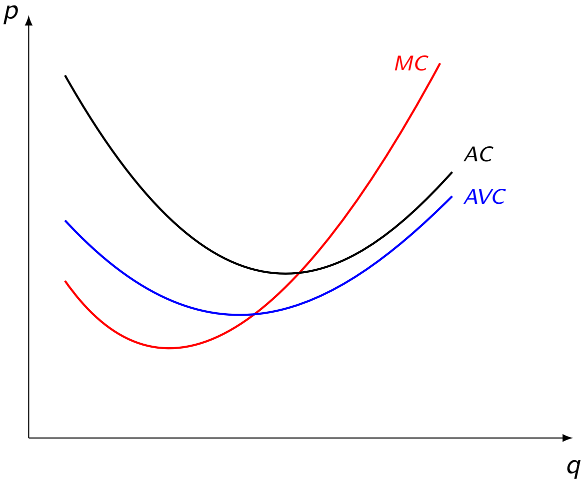
- Marginal *(incremental)* cost for quantity q is

$$MC(q) = C'(q) = VC'(q)$$

- Average costs (AC) and average variable costs (AVC) for quantity $q > 0$ are

$$AC(q) = \frac{C(q)}{q} \qquad \text{and} \qquad AVC(q) = \frac{VC(q)}{q}$$

Cost Concepts



Note that MC curve hits AC and AVC curves at their minimums (interior).

Cost Concepts - II

- Fixed costs F can be further decomposed into sunk costs and avoidable fixed costs.
- The total cost of producing quantity q is given by

$$C(q) = \begin{cases} S & q = 0 \\ F + VC(q) & q > 0 \end{cases}$$

where

- o S is the sunk cost part of fixed costs ($S \leq F$)
- o $F - S$ is the avoidable part of fixed costs

Perfect Competition

Competitive markets

Perfectly competitive market is an **ideal** (a benchmark) against which other models/markets are compared.

Main assumptions of perfect competition:

- Homogeneous good
- Perfectly divisible output
- Perfect information
- No transaction costs
- No externalities
- **Price-taking**

Perfectly competitive markets are rarely observed in real world (if ever).

Perfect competition

How much does a firm produce?

Firm believes that the market price is not affected by its output choice (i.e., **price-taking behavior**).

- Revenue $R(q) = p \cdot q$
- Profit $\pi(q) = R(q) - C(q)$
- Firm maximizes its profit $\pi(q)$ by choosing $q \geq 0$

Firm's decision process (a two-step process):

1. What positive output q maximizes the profit?
2. Does the firm make enough profit to cover its costs, or is it better off by shutting down ($q = 0$)?

Step 1: Maximize profit when $q > 0$

Firm's profit is given by

$$\max_{q>0} \quad p \cdot q \; - \; C(q)$$

FOC (interior optimum) — price equals marginal cost at $q^* > 0$:

$$\boxed{p \; = \; MC(q^*)}$$

Not done yet! We have to check the SOC:

$$-MC'(q^*) \; \leq \; 0 \qquad \Rightarrow \qquad MC'(q^*) \geq 0$$

Intuitively, marginal costs must be increasing at q^*!

Step 2: Check the shutdown condition

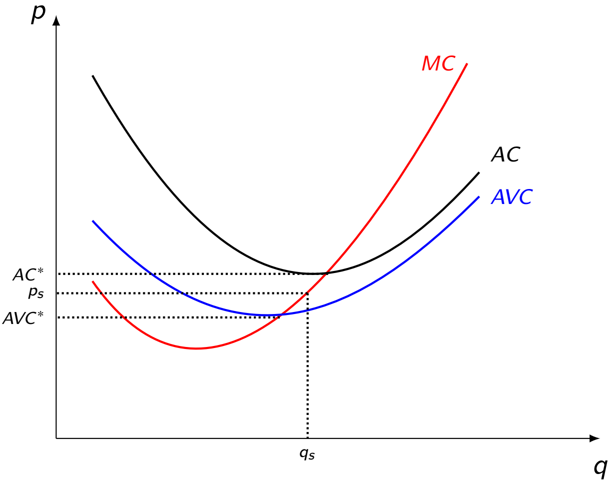
- We have to check that shutdown is suboptimal:

$$
\begin{aligned}
\pi(q^*) &\geq \pi(0) \\
p &\geq AVC(q^*) + \tfrac{F-S}{q^*} \\
&\text{or} \\
MC(q^*) &\geq AVC(q^*) + \tfrac{F-S}{q^*}
\end{aligned}
$$

- the price is high enough to cover all avoidable costs
  - o if $F = S$ *(all fixed costs are sunk)*, then $p \geq AVC(q^*)$
  - o if $S = 0$ *(no sunk costs)*, then $p \geq AC(q^*)$
- Solve for q_s and p_s using the following equations:

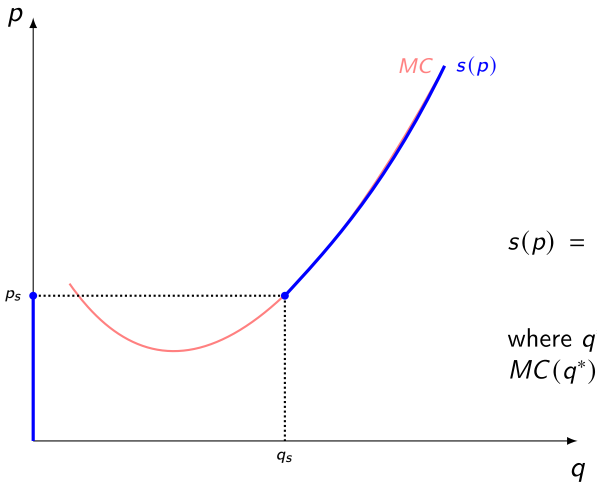
$$\boxed{MC(q_s) = AVC(q_s) + \frac{F-S}{q_s}} \qquad \boxed{p_s = MC(q_s)}$$

When to shutdown?



The shutdown price p_S can be as low as AVC^* or as high as AC^*.

Firm's Supply – optimal output level



$$s(p) = \begin{cases} 0 & p < p_s \\ 0 \text{ or } q_s & p = p_s \\ q^* & p > p_s \end{cases}$$

where q^* is determined by $MC(q^*) = p$.

Class Example - 1

- Suppose that the firm's total cost of producing q is given by

$$C(q) = \begin{cases} 12 & q = 0 \\ 16 + q^2 & q > 0 \end{cases}$$

- Construct firm's supply function $s(p)$

Competitive Equilibrium

- **Demand side:** market demand is given by $D(p)$
- **Supply side:** N firms with supply curves $s_1(p), \ldots, s_N(p)$, then the **market supply** is given by

$$S(p) = \sum_{i=1}^{N} s_i(p)$$

- **Market clearing price:** Price p^* that equalizes supply and demand

$$S(p^*) = D(p^*)$$

  - Note that such p might fail to exist
  - There can be firms with $s_i(p^*) = 0$, $\pi > 0$ or $\pi < 0$.
- This concept is known as the short-run equilibrium.

Class Example - 2

- Market demand is given by

$$D(p) = 20 - p$$

- Suppose that all $N \geq 1$ firms are identical.
- Suppose that a firm's total cost of producing q is given by

$$C(q) = \begin{cases} 12 & q = 0 \\ 16 + q^2 & q > 0 \end{cases}$$

- Find the short-run equilibrium for all N (if it exists)

Competitive equilibrium with free entry/exit

- Suppose that all firms are identical and entry/exit is free.
- This concept is known as the long-run equilibrium.
- **Main Idea:** # of firms adjusts in the long-run (entry and exit)
- **A long-run equilibrium** is a **market price** p^* and a **number of firms** N such that
  1. Market supply equals market demand, i.e.
  $$N \cdot s(p^*) \;=\; D(p^*)\;, \text{ and}$$
  2. Free entry/exit
  $$\pi\left(s(p^*)\right) \;=\; 0$$
  This condition is equivalent to $p^* \;=\; AC(q^*)$.

Inverse Demand Function

- Demand function $D(p)$ tells us a quantity the consumer wants to buy at a **per unit price** p.
- The **inverse demand function** is given by

$$p(q) \; = \; D^{-1}(q)$$

- Inverse demand function $p(q)$ tells us the maximal price a consumer can pay for the q-th unit of the good, so

$$\boxed{q \; = \; D(p(q))}$$

Social welfare

- If q units are bought at $p(q)$, then **consumer surplus (CS)**

$$CS := \underbrace{\int_0^q p(x)dx}_{\text{value}} - \underbrace{p(q) \cdot q}_{\text{total payment}}$$

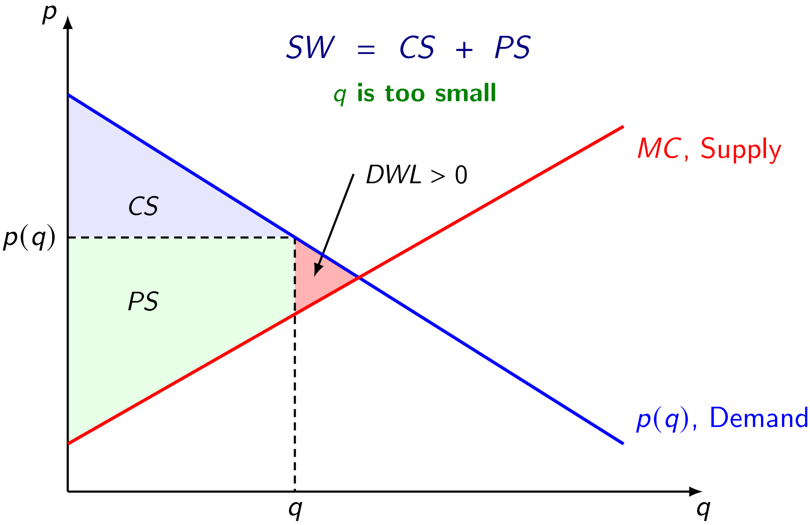
- Similarly, if q units are sold, the **producer surplus (PS)**

$$PS := \underbrace{p(q) \cdot q}_{\text{total payment}} - \underbrace{C(q)}_{\text{production costs}}$$

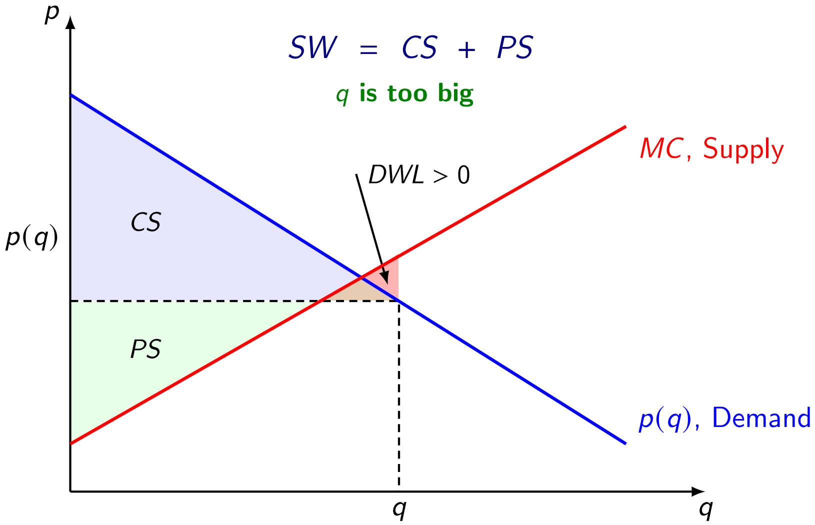
- The sum of CS and PS is called **social welfare (SW)**

$$SW := CS + PS$$

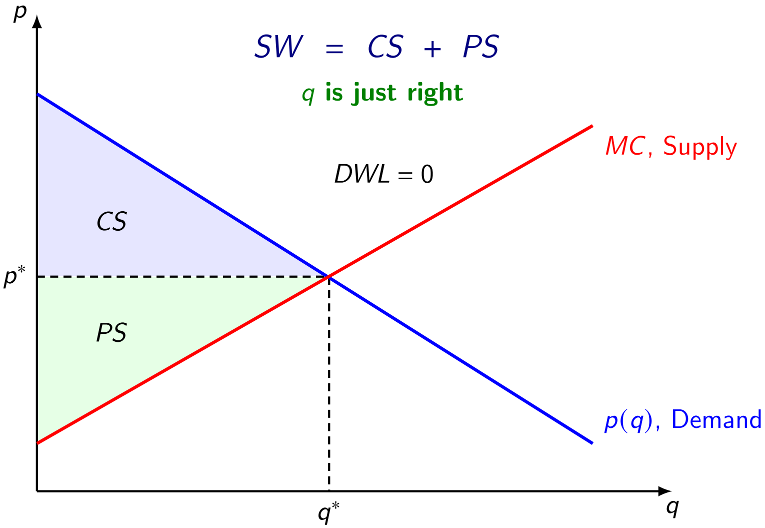
Maximizing Social Welfare



Maximizing Social Welfare



Maximizing Social Welfare



This is why perfect competition is an ideal. **It maximizes the social welfare**.

Class Example - 3

- Market demand is given by

$$D(p) = 30 - p$$

- Suppose that a firm's total cost of producing q is given by

$$C(q) = q^2 \qquad \forall q \geq 0$$

- Calculate CS, PS, DWL and SW in the perfect competition model with a single firm.

Monopoly

Monopoly and its instruments

A firm is a **monopoly** if it is the only seller of a product in a given market.

What can a monopolist do? Many things, including:

- Post any per unit price p.
- Charge some fixed fee upfront and post a per unit price – two part tariff *(e.g. rent a scooter)*
- Post different prices for different consumers – 3rd degree price discrimination *(e.g. museums)*
- Offer different varieties/quantities of the same good at different prices – 2nd degree price discrimination *(e.g., volume discounts, drinks in bottles/cans)*
- If several goods are available, then sell them as a package – bundling *(e.g. business lunch)*

What leads to a monopolized market?

There are two key drivers:

- **Technology** – the average cost curve and the minimum efficient scale (AC^*) in the long-run
- **The size of the market** — demand curve

If only 1-2 firms can potentially coexist in the market in the long run due to

- high AC^*, and
- small market (small relative to AC^*)

we should expect monopolies to appear.

Monopoly: Linear Pricing

Linear pricing

- Suppose that a monopolist charges every consumer the same per unit price p. This is called linear pricing.
- The total sales are then given by the market demand

$$q = D(p)$$

- The profit of the monopolist is then given by

$$\pi(p) = p \cdot D(p) - C(D(p))$$

- What price should the monopolist set?

$$\max_{p \geq 0} \pi(p)$$

Equivalent problem

- The inverse demand function is given by $p(q) = D^{-1}(q)$.
- The monopolist revenue

$$R(q) = p \cdot D(p) = p(q) \cdot q$$

- The monopolist chooses output level to maximize its profit

$$\max_{q \geq 0} \pi(q) = R(q) - C(q)$$

- Monopolist's decision process:
  - What positive output q maximizes the profit?
  - Does the firm make enough profit to cover its costs, or it is better off by shutting down ($q = 0$)?

Marginal revenue

- The monopolist revenue is given by

$$R(q) \; = \; p \cdot D(p) \; = \; p(q) \cdot q$$

- Then marginal revenue (abbreviated as MR) is as follows:

$$MR(q) \; = \; R'(q) \; = \; p(q) \; + \; p'(q) \cdot q$$

- Equivalently:

$$MR(q) \; = \; p(q) \cdot \left(1 \; + \; \frac{1}{\epsilon(q)}\right)$$

where $\epsilon(q)$ is a **demand elasticity**,i.e., $\epsilon(q) \; = \; \frac{dlnq}{dlnp} \; = \; \frac{p(q)}{p'(q) \cdot q}$

- Note that $MR(q) < 0$ for inelastic demands ($-1 < \epsilon(q) < 0$).

Monopoly profit maximization

FOC (interior optimum) — marginal revenue equals marginal cost:

$$\boxed{MR(q^m) \ = \ MC(q^m)}$$

We are not done yet!

1. Need to check SOC

$$MR'(q^m) \ - \ MC'(q^m) \leq 0$$

2. Have to check that shutdown is suboptimal

$$\begin{aligned} \pi(q^m) \ &\geq \ \pi(0) \\ p \ &\geq \ AVC(q^m) \ + \ \tfrac{F-S}{q^m} \\ MC(q^m) \ - \ p'(q^m)q^m \ &\geq \ AVC(q^m) \ + \ \tfrac{F-S}{q^m} \end{aligned}$$

This inequality is easier to satisfy in compar. with the PC scenario.

Class Example - 4

- Inverse demand function is given by

$$p(q) \; = \; K \; - \; q \qquad \text{where} \quad K > 0$$

- Suppose that the firm's total cost of producing q is given by

$$C(q) \; = \; \begin{cases} 12 & q \; = \; 0 \\ 16 \; + \; q^2 & q \; > \; 0 \end{cases}$$

- Find the optimal monopoly quantity-price pair for $K = 4$ and $K = 8$.

Lerner's Index of Market Power

We can rewrite FOC using demand elasticity:

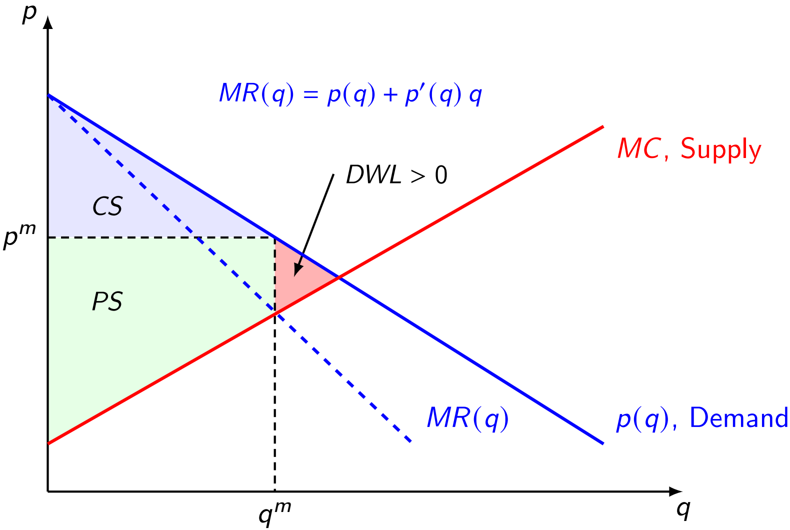
$$\frac{p(q) - MC(q)}{p(q)} = -\frac{1}{\epsilon(q)}$$

- LHS is a price-cost margin. It is often called *Lerner's index of market power*
  - the index takes values in $[0, 1]$ interval
  - higher index indicates larger market power, and index of a competitive firm is zero
- For a monopoly, the price-cost margin depends only on the elasticity of demand
  - Inelastic demand leads to very high margins. For example:

    if $\epsilon = -11$, then $p = 1.1MC$ and $LI = 0.01$.

    if $\epsilon = -1.1$, then $p = 11MC$ and $LI = 0.91$.
  - What happens when $-1 < \epsilon < 0$?

Graphical Analysis



Since $MR(q) < p(q)$, there is always undersupply $q^m < q^*$ and positive DWL.

Social costs

Why monopoly is bad?

- The higher prices, the lower the **consumer surplus** CS.
- In general, a monopoly is inefficient resulting in **deadweight loss** DWL.
- The total welfare loss can be even higher than DWL due to rent-seeking (profits can be used to gain and protect the market power) *e.g. lobbying, advertising*.

Government often regulates and limits market power:

- Price caps — oil and gas prices.
- Limitations on mergers (consolidation) — need to get an approval from the government.

Social benefits

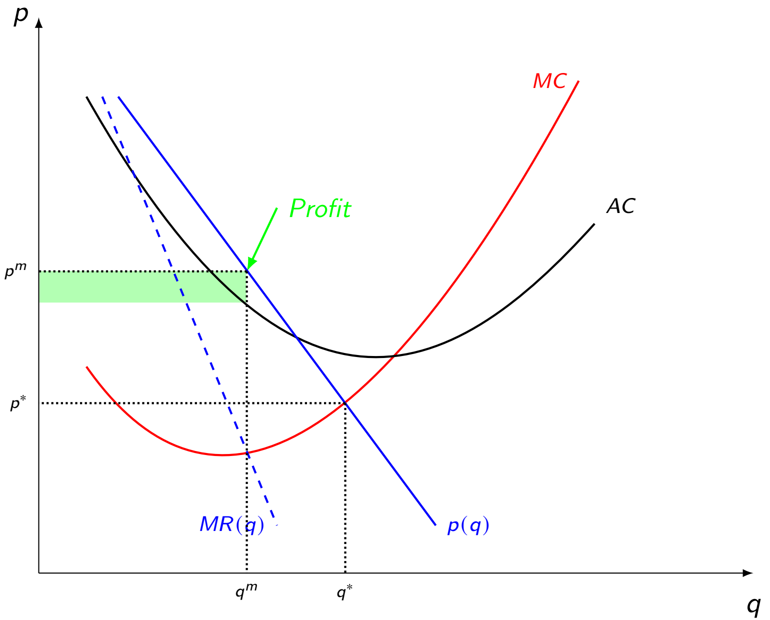
Can monopoly be good?

- Incentives to innovate *(new products, more efficient production)*, patents *(protects R&D)*.
- Sometimes a monopoly may be the most efficient way to produce: *postal service, subway*. This situation is called a natural monopoly.
- A natural monopoly may exist if the industry cost function is **subadditive**:

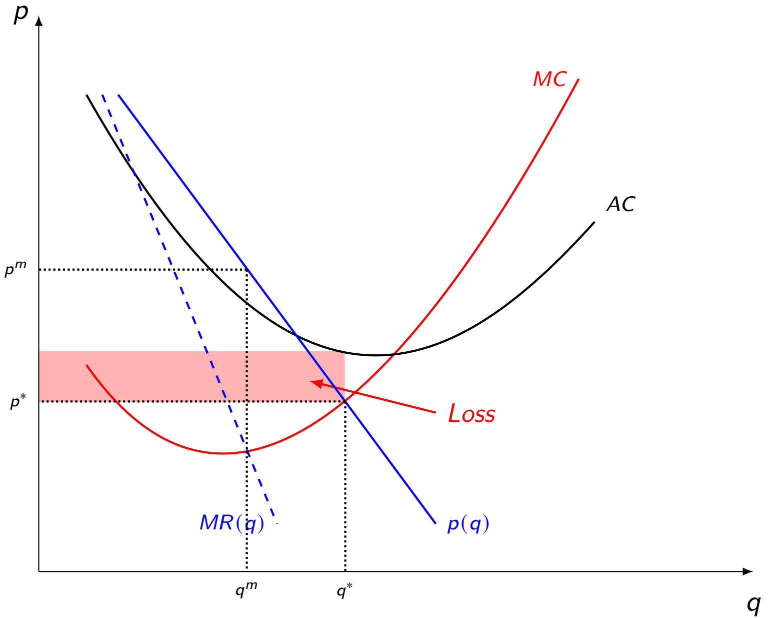
$$C(q_1 + q_2) \leq C(q_1) + C(q_2) \quad \forall q_1, q_2 \geq 0$$

- This is very common for industries with large fixed costs (e.g., public utilities)

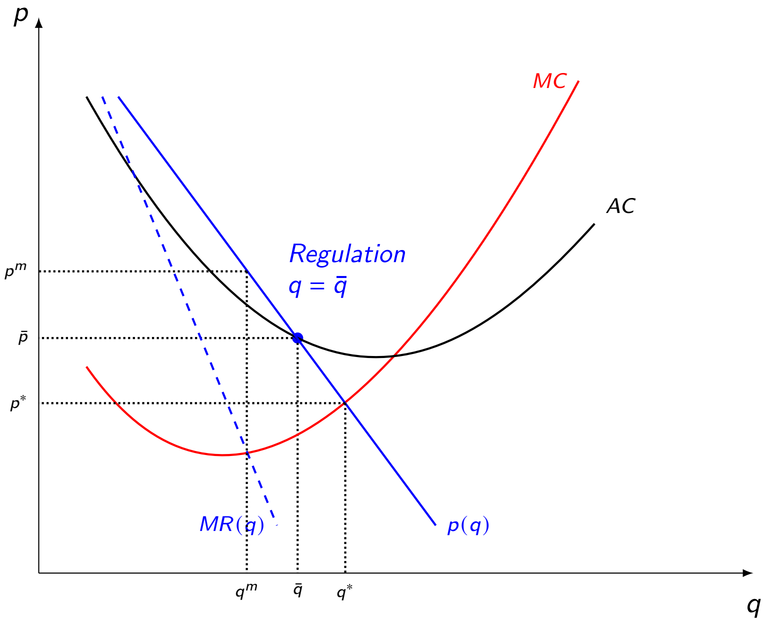
Natural Monopoly



Natural Monopoly



Natural Monopoly



How to regulate natural monopolies?

Main problem: if we require (p^*, q^*), the monopoly will shutdown

Three potential approaches:

1. No regulation — monopoly picks (p^m, q^m). Inefficiency.
2. Require a point like $(\bar{p}, \bar{q})$ such that the monopoly can operate. However, the social surplus is not maximized here (still inefficiency, but less).
3. Require (p^*, q^*) and subsidize the monopoly from other sources (taxes). Efficient, but might distort other markets.

Furthermore, one should also account for the cost of regulation itself as proper discovery of $(\bar{p}, \bar{q})$ or (p^*, q^*) requires a lot of information and work.

Monopsony

A firm is a **monopsony** if it is the only buyer of a product in a given market — "Nornikel" hiring labor in Norilsk.

Monopsony is a flipside of monopoly:

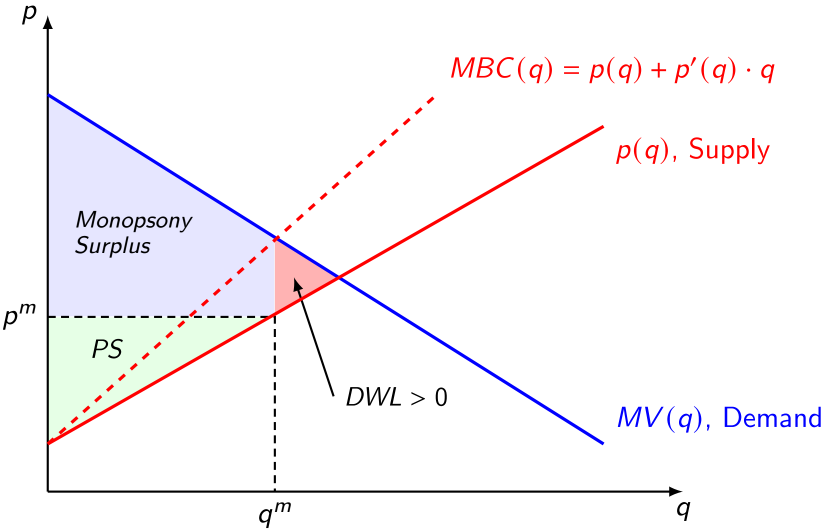
- Value of buying quantity q: $V(q)$ *(MV for marginal)*
- $p(q)$ is the **inverse supply**.
- Cost of buying q units: $BC(q) = p(q) \cdot q$
- Marginal cost of buying q units

$$MBC(q) = p(q) + p'(q) \cdot q$$

- Monopsony's profit: $\pi(q) = V(q) - BC(q)$
- Monopsony will chose q^m to satisfy

$$MV(q^m) = MBC(q^m)$$

Graphical Analysis



Since $MBC(q) > p(q)$, there is always undersupply $q^m < q^*$ and positive DWL.

Case Study: Amazon

Background

Amazon was founded in 1986 as an online book retailer.

Currently Amazon is the largest online retailer.

Issue: Dispute with Hachette over cut of supply prices for books

- Amazon wanted a larger share.
- Hachette refused.
- Amazon began disrupting sales of Hachette books:
  - Delay deliveries.
  - Steer away customers.
  - Raise prices.
  - Prevent pre-orders.

Market power

- Amazon is the largest seller, but it does not use its monopoly power — keeps final prices low for its customers. **Why?**
- Amazon is also a monopsony as it is the largest buyer of books from publishers.
- Amazon is abusing its monopsony power by driving supply prices down

"Amazon has too much power and it uses that power in ways to hurt America" - Paul Krugman

Pros & Cons

On the one hand:

- Hurts publishers and writers — low wages.
- squeeze out publishers — reduced quality of books.

On the other hand:

- Powerful tool for selling books — e-book platform.
- This tool is used efficiently in many ways:
  - Low transaction costs.
  - Low search costs.
  - Low inventory costs.

How to solve problems with multiple consumers?

Step 1: Demand Aggregation

- Demand function $D(p)$ tells us the quantity that a consumer wants to buy at **per unit price** p.
- Inverse demand function tells a maximal price $p(q)$ a consumer can pay for the q-th unit of the good
- If we have N consumers with demand functions $D_1, \ldots, D_N$, then a market demand is given by

$$D(p) \;=\; \sum_{i=1}^{N} D_i(p)$$

- The market demand function itself can be inverted, i.e., $p(q) \;=\; D^{-1}(q)$. This is inverse market demand.
- In general,

$$p(q) \;\neq\; \sum_{i=1}^{N} p_i(q)$$

Step 2: Finding monopoly outcome

- Suppose that there are N consumers with individual demands $D_i(p)$ and inverses $p_i(q)$.
- Market demand and inverse market demand are given by

$$D(p) = \sum_{i=1}^{N} D_i(p) \qquad \Rightarrow \qquad p(q) = D^{-1}(q)$$

- The optimal quantity q^m and per unit price p^m are given by

$$MR(q^m) = MC(q^m) \qquad \Rightarrow \qquad p^m = p(q^m)$$

- Individual quantities bought by consumers are given by

$$q_i = D_i(p^m)$$

Skill checklist for Part 1a

1. Find a market demand/inverse demand.
2. Find a market supply with N competitive firms.
3. Find a competitive equilibrium: price, each consumer's/firm's quantity.
4. Determine a number of competitive firms with free/costly entry.
5. Find a monopoly/monopsony uniform price and each consumer's quantity *(FOC, SOC, shutdown condition)*.
6. Compute CS, PS, SW, DWL in each case.

Part 1b: Price Discrimination

Instructor: Oleg Baranov

Microeconomics 2 *(Module 1, 2023)*.

Suggested readings for this part

Readings in Class Textbooks:

- **Varian, 8th edition:**
  - Ch 25: Monopoly Behavior
- **Carlton and Perloff:**
  - Ch 9: Price Discrimination

Overview

- Up to now, we considered situations in which the monopoly used one per unit price (i.e., linear pricing).
- Recall that

$$MR(q) \;=\; p(q) \;+\; q \cdot \underbrace{p'(q)}_{<0}$$

- To sell more, the firm has to reduce its price — **price discrimination** aims to eliminate this effect.

How a monopoly can increase sales without price reduction?

- Treat consumers differently — consumer specific prices
- Offer each unit of the good at a different price — quantity specific prices
- *Discrimination is feasible only if resale is banned or too costly.*

Tariff Functions

- Before we only considered non-discriminatory linear pricing
- Mathematically, any consumer i has to pay $T_i(q)$ when he or she buys q units of the good, i.e.,

$$T_i(q) = p \cdot q \qquad \forall i = 1, ..., N$$

- But, in principle, the tariff function (payment function) can be quite different.
- For example,

$$T_1(q) = 10 \qquad T_2(q) = 5 + q \qquad T_3(q) = q^2 + \sqrt{q}$$

- In fact, consumers can be a given a menu of tariffs to choose from

$$\{T_1(q),\ T_2(q),\ T_3(q), ...\}$$

Price Discrimination

We will cover three types of price discrimination:

1. Third-degree PD
2. Second-degree PD
3. First-degree PD

Third-degree Price Discrimination

- linear prices, but **consumer** specific
- Consumer i has to pay $T_i(q) = p_i \cdot q$ to buy a quantity q.
- Typically consumer i means a group of consumers with similar observables — location, age, etc.
- Most widespread — senior citizen/student discounts, iPhones (Russia vs. USA), Steam.

08 сентября 2022, 16:46 Технологии

Почему iPhone 14 в России продают по курсу 106 рублей за доллар

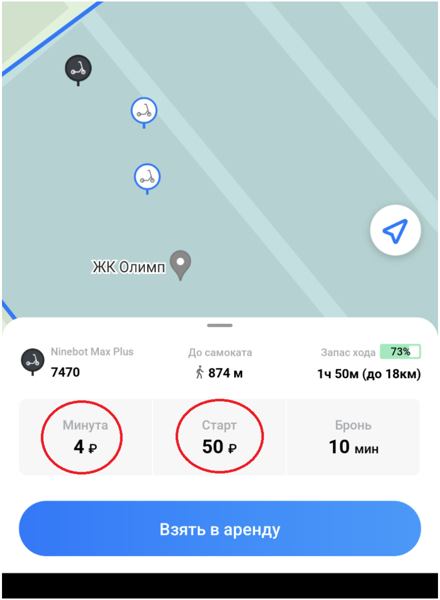
Муртазин из Mobile Research Group назвал предзаказ на iPhone 14 в России «продажей воздуха»

Валерий Романов

Линейку iPhone 14 в России предлагают купить фактически по курсу 106 рублей за доллар. Высокая цена на гаджеты связана с тем, что первые поставки будут идти из стран Европы, где Apple повысила стоимость своих устройств, в отличие от

Second-degree (non-linear pricing) Price Discrimination

- **quantity** specific prices only.
- Any tariff functions are allowed as long as they offered to all consumers (no consumer discrimination)
- Each consumer has to pay $T(q)$ to buy a quantity q.
- Very widespread — *soft drinks, mobile service, rentals*.



Tariff function in the Fall 2022:

$$T(q) = 50 + 4q$$

First-degree (Perfect) Price Discrimination

- **consumer & quantity** specific prices
- Consumer i has to pay $T_i(q)$ to buy a quantity q.
- Often infeasible — needs a lot of consumer data.
- Often illegal — medical insurance cannot be too discriminatory.
- Became more feasible in online markets — Amazon, Google, Yandex et al. store your purchasing data.

Price Discrimination

Unit Demand Setting

What is Unit Demand?

- Suppose that the good is indivisible
- A consumer has a **unit demand** whenever there exists $\omega \geq 0$ such that:

$$D(p) = \begin{cases} 1 & p \leq \omega \\ 0 & p > \omega \end{cases}$$

- This ω is often called **willingness to pay (wtp)** .

Setting and Socially Optimal Outcome

- There are N consumers with unit demands such that

$$\omega_1 > \omega_2 > \ldots > \omega_N$$

- The monopolist can produce q units at a cost of $C(q)$.
- The socially optimal level of production q^* is given by

$$\max_{q \in \{1,\ldots,N\}} \quad \sum_{i=1}^{q} \omega_i - C(q)$$

How to solve a monopoly problem?

- Suppose there are N consumers with unit demands such that

$$\omega_1 > \omega_2 > \ldots > \omega_N$$

- The monopolist can produce q units at a cost of $C(q)$.
- Monopoly's profit maximization can be broken down into two steps:

  1. Suppose that the monopoly wants to sell exactly k units of the good. **How should monopoly price these k units to maximize the revenue?**
  2. Pick the optimal number of units to sell k^* accounting for the production cost.

Linear (non-discriminatory) pricing

- What non-discriminatory price should a monopolist pick?
- To sell k units, the linear price p must be

$$\omega_{k+1} \leq p \leq \omega_k .$$

- The best linear price to sell k units is given by

$$p = \omega_k \qquad \text{and} \qquad Revenue = k\omega_k .$$

- The monopoly picks k^m to maximize its profit, i.e.:

$$\max_{k \in \{1,\ldots,N\}} k\omega_k - C(k) \qquad \Rightarrow \qquad p^m = \omega_{k^m}$$

- Generally, the monopolist produces less than socially optimal quantity, so

$$k^m < q^*$$

1st-degree PD

- To sell k units, the best **consumer specific prices** are

$$p_i = \begin{cases} \omega_i & i < k \\ \omega_k & i \geq k \end{cases} \qquad \text{and} \qquad Revenue = \sum_i^k \omega_i \, .$$

- The monopolist extracts the CS of **every served consumer**!
- Note that

$$\sum_{i=1}^k \omega_i > k\omega_k \qquad \forall k \geq 2$$

where $k\omega_k$ is the revenue without discrimination.

- The monopoly chooses k^m to maximize its profit, i.e.:

$$\max_{k \in \{1,\dots,N\}} \sum_{i=1}^k \omega_i - C(k)$$

- Note that k^m equals to socially optimal q^*.

Class Example - 1

- There are three unit demand consumers with the following wtps:

$$\omega_1 = 10 \qquad \omega_2 = 7 \qquad \omega_3 = 6$$

- The cost function of the monopoly is $C(q) = q^2$.

- **Question 1.** Determine the socially optimal level of q.

- **Question 2.** Find the optimal price for the monopoly when it is limited to linear prices (non-discriminatory).

- **Question 3.** Find the optimal prices for the monopoly when 1-st degree discrimination is allowed.

2nd-degree PD

- On the one hand, the quantity specific prices have no discriminatory power in unit demand settings *(they are only useful with multi-unit demands, e.g., volume discounts)*
- On the other hand, 2rd-degree PD does not allow consumer specific prices — any tariff(s) has to be offered to all customers with no exceptions.
- Thus, in the unit demand setting, **the 2nd-degree PD cannot generate any extra gains relative to linear pricing.**
- *(Later in Part 1c)* When customers only buy one unit of the good, a monopoly can differentiate by other attributes, e.g., quality — *airline pricing*.

3rd-degree PD

- The quantity specific prices have no discriminatory power in unit demand settings
- Then 1st-degree PD must fully rely on the discriminatory power of consumer specific prices.
- The 3rd-degree PD allows consumer specific per unit price, i.e.,

$$T_i(q) \;=\; p_i \cdot q \qquad \forall q \geq 0$$

- **Thus, in the unit demand setting, the 1st-degree PD and 3rd-degree PD are equally powerful and extract all surplus from consumers**.

Summary for the Unit Demand Setting

Assumptions:

- A monopoly producing **homogeneous** good.
- Multiple customers with **unit demand** for the good

Results:

- Quantity specific prices have no discrimination power.
  - 2nd-degree PD does not give any advantage to the monopoly
  - 2nd-degree is equivalent to nondiscriminatory linear pricing
- Consumer specific prices have full (perfect) discrimination power.
  - even 3rd-degree PD extracts all surplus from consumers
  - 3rd-degree PD is equivalent to 1st-degree PD

Price Discrimination

General Setting

1st-degree PD: Single Consumer

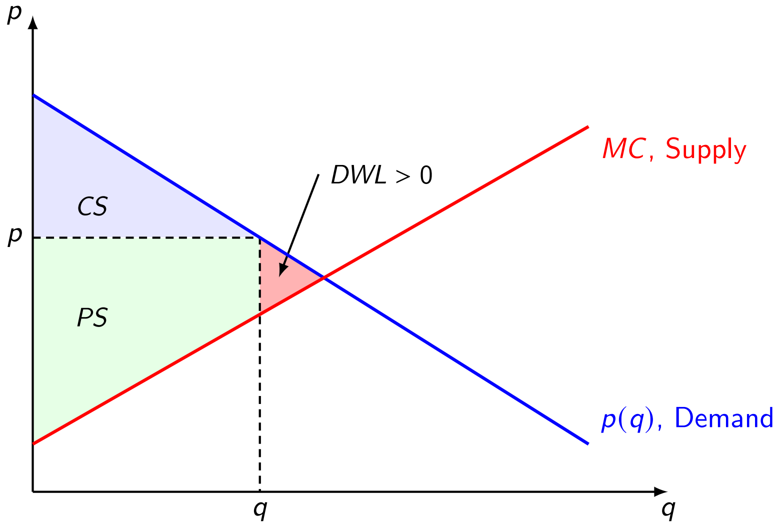
- Suppose that there is a single consumer with demand $D(p)$ and the inverse $p(q)$
- *Note that $D(p)$ does not depend on the budget/wealth of the consumer, i.e., - quasilinear utility.*
- There are two sources of extra profit for the monopoly: *consumer surplus (CS)* and *deadweight loss (DWL)*.
- It is easy to show that the monopoly can extract all social welfare in this market with a simple two-part tariff

$$T(q) \ = \ F \ + \ p \cdot q$$

  where F is an upfront fee ("entry fee") and p is per unit price.
- Let's verify this claim by using a graphical analysis

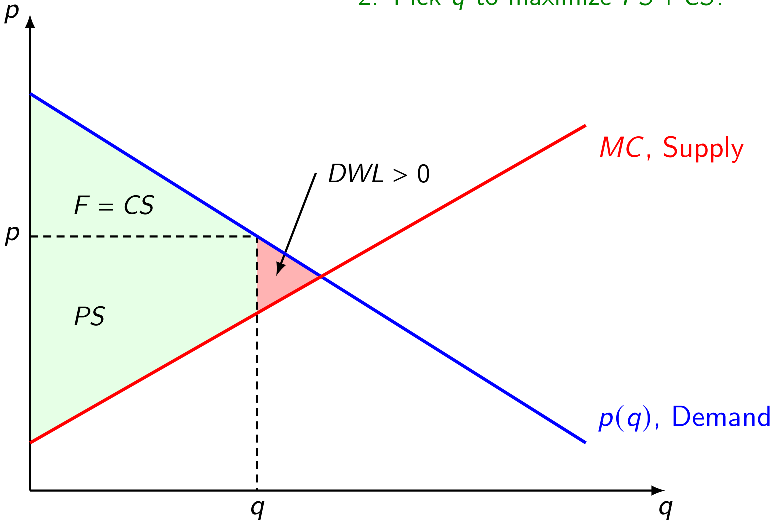
1st degree PD: Graphical Illustration

1. For any q, set flat fee $F = CS$.



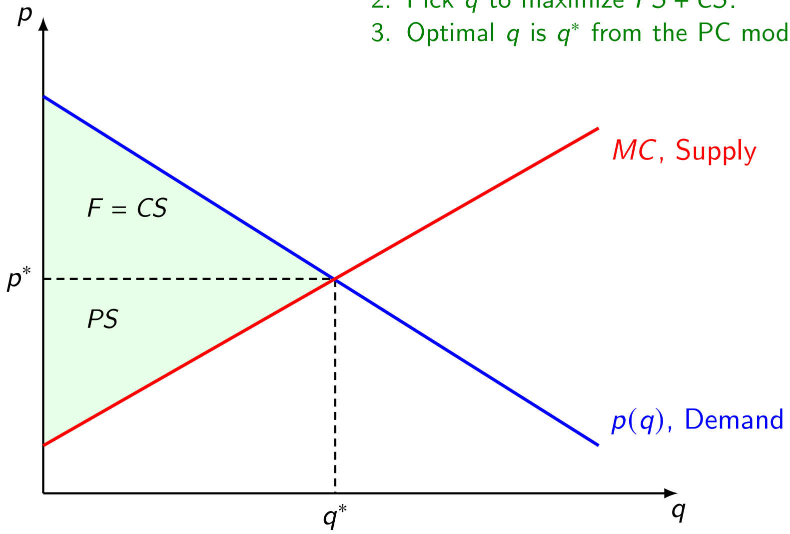
1st degree PD: Graphical Illustration

1. For any q, set flat fee $F = CS$.
2. Pick q to maximize $PS + CS$.



1st degree PD: Graphical Illustration

1. For any q, set flat fee $F = CS$.
2. Pick q to maximize $PS + CS$.
3. Optimal q is q^* from the PC model.



1st degree PD: Single Consumer

- Recall that for the perfectly competitive outcome, we have

$$q^* : MC(q^*) = p(q^*) \qquad \text{and} \qquad CS^* = \int_o^{q^*} p(x)dx - p^* \cdot q^*$$

- Monopoly can extract all SW^* by using the following two-part tariff

$$T(q) = CS(q^*) + p^* \cdot q$$

  In other words, charge an "entry fee" of $CS(q^*)$ and then charge per unit price of p^* for any quantity desired.

- Given $T(q)$, the consumer is indifferent between buying 0 and q^* (buying any other quantity is strictly worse).

- Note that the social welfare is maximized ($DWL = 0$), but $CS = 0$ under the 1st-degree PD.

1st degree PD: N Consumers

- Suppose that there are N consumers with individual demands $D_i(p)$ and inverses $p_i(q)$.
- Now the monopolist can extract all social welfare by using a consumer specific two-part tariffs
- For a perfectly competitive outcome, we have

$$q^* : \; MC(q^*) = p(q^*) = p^* \qquad \text{and} \qquad q_i^* = D_i(p^*)$$

- Monopoly extracts everything from consumer i by using a two-part tariff

$$T_i(q) = CS_i(q_i^*) + p^* \cdot q$$

- **Summary:** First-degree price discrimination is hard to implement (it needs a lot of data). Also, it is often **illegal** since it is so bad for consumers.

2nd-degree PD

- 1st-degree PD requires consumer specific prices (recall entry fees). This approach is infeasible under 2nd-degree PD.

- The monopolist can use any non-linear tariff(s) that do not depend on identifies of consumers. This is a very large set.

A single non-linear tariff:	$T(q) \;=\; F \;+\; p\,q \;+\; k\,q^2$
A single two-part tariff:	$T(q) \;=\; F \;+\; p\,q$
A menu of different tariffs:	$\{T_1(q), T_2(q), ..., T_K(q)\}$

- Solving for the optimal 2nd-degree PD tariff(s) is a **very challenging problem**. In general, when consumers are different, **the full surplus extraction is infeasible.**

- In practice, firms tend to use very simple tariff(s): *a couple of two-part tariffs, or a progressive discount on volume, etc ...*

2nd degree PD: Limited Setting

Assumptions:

- Suppose that there are two consumers $i = 1, 2$ with demands $D_i(p)$ and inverses $p_i(q)$
- Consumer 1 is **high-value** in a sense of

$$D_1(p) > D_2(p) \qquad \forall p \in [\,0, p_1(0)\,)$$

- Suppose that the monopolist has to offer the same two-part tariff to both consumers, i.e.,

$$T(q) = F + p \cdot q$$

What is the best two-part tariff? *(essentially, we just need to pick two numbers, F and p).*

2nd-degree PD: optimal two-part tariff

Consumer i will buy exactly $q_i = D_i(p)$ as long as F is smaller than consumer's value from the transaction, i.e.,

$$F \leq CS_i(q_i) \; .$$

There are only two possibilities here:

1. **Only serve a high-value consumer** — set $F = CS_1 > CS_2$
   - the monopolist does 1st-degree PD on consumer 1 only
   - full surplus extraction from consumer 1

2. **Serve both consumers** — set $F = CS_2 < CS_1$
   - the monopolist maximizes the following profit function

     $$\max_q \; p(q) \cdot q \; - \; C(q) \; + \; 2 \cdot CS_2(q)$$

   - full surplus extraction from consumer 2
   - partial extraction from consumer 1

2nd-degree PD: limited setting

Comments:

- It is better to serve both types when D_2 is relatively close to D_1. Otherwise, just serve D_1.
- Naturally, the monopolist **can do better by going beyond a single two-part tariff** here.
- For example, a monopolist can offer a pair of two-part tariffs:

$$\begin{aligned} T_1(q) &= F_1 + p_1 \cdot q \\ T_2(q) &= F_2 + p_2 \cdot q \end{aligned}$$

such that

$$F_1 > F_2 \qquad \text{and} \qquad p_1 < p_2$$

and tailor them such that T_1 is more appealing to Consumer 1 and T_2 is more appealing to Consumer 2.

3rd-degree PD: Setting

- Suppose that there are N consumers with individual demands $D_i(p)$ and inverses $p_i(q)$.
- Charge every consumer a different linear price
- Consumer i has to pay

$$T_i(q) \;=\; p_i \cdot q \qquad \forall q \geq 0$$

- To sell q_i units to Consumer i, the monopoly should set its price to

$$p_i \;=\; p_i(q_i)$$

3rd-degree PD: Solution

- **How much to produce?** Find $q_1, \ldots, q_N \geq 0$ which solve:

$$\max_{q_1,\ldots,q_N \geq 0} \quad \sum_{i=1}^{N} q_i \cdot p_i(q_i) \; - \; C\left(\sum_{i=1}^{N} q_i\right)$$

- At the optimal solution (**FOC**), we have:

$$\begin{aligned} MR_i(q_i^*) &= MC\left(\textstyle\sum_{j=1}^{N} q_j^*\right) && \forall i : \; q_i^* > 0 \\ MR_i(0) &\leq MC\left(\textstyle\sum_{j=1}^{N} q_j^*\right) && \forall i : \; q_i^* = 0 \end{aligned}$$

- For every served consumer, its marginal revenue equals marginal cost of the total production.

- **Hint for solving problems:** Note that marginal revenues $MR_i(q_i^*)$ are equal to each other for all $q_i^* > 0$.

Class Example - 2

- Suppose that there are two consumers with

$$D_1(p) = 10 - p \qquad \text{and} \qquad D_2(p) = 6 - p$$

- The cost function of the monopolist is given by

$$C(q) = 0 \qquad \forall q \geq 0$$

- **Question 1:** Solve for a perfectly competitive outcome.
- **Question 2:** Solve for a non-discriminatory monopoly outcome.
- **Question 3:** Solve for a monopoly outcome under 1st degree PD.
- **Question 4:** Solve for a monopoly outcome under 2nd-degree PD (limited to a single two-part tariff).
- **Question 5:** Solve for a monopoly outcome under 3rd-degree PD.

Summary on Price Discrimination

- A monopoly always prefers to discriminate (than not). It is impossible for the producer surplus to go down.
- Unless 1st degree, other welfare effects are ambiguous.
- "High" / "Low" demand consumers typically loose/gain, because discrimination $\uparrow$/$\downarrow$ their prices.
- The social welfare goes down when the discriminating firm produces less.
- Or the social welfare goes up **(and even consumers can gain)** when a discriminating firm produces more.

Some Hints for Solving Problems

Non-discriminatory pricing (same per unit price):

- **Easy case:** when all $p_i(0)$ are the same, the monopolist always serves all markets. Example of a setting with the same $p_i(0)$

$$D_i(p) = A_i(K - p) \qquad \Rightarrow \qquad p_i(0) = K$$

- **Harder case:** when $p_i(0)$ are different across consumers , then it can be optimal for a monopolist to not serve low-value markets.

3rd-degree discriminatory pricing (different per unit price):

- **Easy case:** A monopolist with a constant marginal costs $C(q) = cq$. Then the monopolist serves all consumers with $p_i(0) > c$. This leads to a very simple system of equations.

- **Harder case:** Non-constant marginal cost for a monopolist leads to a complex system of equations. Solving requires making "educated guesses" about which consumers are served.

Skill checklist for Part 1b

1. Demand and inverse demand aggregation
2. First-degree PD: consumer specific two-part tariffs (general demand) or individual prices (unit demand)
3. Second-degree PD: optimal tariff in a limited setting (general demand) or linear price (unit demand).
4. Third-degree PD: consumer specific linear prices (general case) or individual prices (unit demand)
5. Compute CS, PS, SW, DWL in each case.

Part 1c: More on 2nd-degree Price Discrimination

Instructor: Oleg Baranov

Microeconomics 2 *(Module 1, 2023)*.

Suggested readings for this part

Readings in Class Textbooks:

- **Varian, 8th edition:**
  - Ch 25: Monopoly Behavior
- **Carlton and Perloff:**
  - Ch 9: Price Discrimination
  - Ch 10: Advanced Topics in Pricing

Remainder: 2nd-degree PD

- Consider a **unit demand setting** with a homogeneous good
- We have concluded (see Part 1b) that the 2nd-degree PD is equivalent to the nondiscriminatory linear pricing
- **Interesting question:** Is there anything a monopoly can do to increase its profit when consumer specific prices are not feasible (illegal)?
- Actually, YES. It turns out that a monopoly firm can mess around with homogeneity by differentiating its product.

Vertical Differentiation

Example

- A monopolist has two units of the good (already produced) and values them at zero.
- Suppose that there are two unit demand customers with

$$\omega_1 = 10 \qquad \text{and} \qquad \omega_2 = 4$$

- Without consumer specific prices, **the best option is to sell one for 10**.
- What if the monopolist change "the quality" of items, effectively differentiating them into varieties which can be **priced individually**.
- This is called vertical differentiation.

Model

- There are two consumers ($N = 2$) with unit demands
- Good is offered in two varieties — high and low quality
- The monopolist has one unit of each variety and has no value from keeping any items.
- The wtps of consumers are as follows:

	High	*Low*
Consumer 1	U_H	U_L
Consumer 2	V_H	V_L

- Let's assume that

$$\begin{array}{rl} (i) & U_H > U_L \geq 0 \\ (ii) & V_H > V_L \geq 0 \\ (iii) & U_H - U_L > V_H - V_L \end{array}$$

Social Optimum

- To be concrete, let's assume the following values:

	High	*Low*
Consumer 1	$U_H = 10$	$U_L = 5$
Consumer 2	$V_H = 4$	$V_L = 3$

- The social optimum is given by
  - *High* good $\Rightarrow$ Consumer 1
  - *Low* good $\Rightarrow$ Consumer 2

  This is due to $U_H - U_L > V_H - V_L$.

- Social welfare

$$SW = U_H + V_L = 13$$

Uniform and Consumer-specific Pricing

- **Uniform price (same for both varieties)**
  - C1 receives H for $p = 10$.
  - $PS = 10, \; CS = 0, \; SW = 10, \; DWL = 3$.

- **1st or 3rd degree PD (known identities)**
  - C1 gets H for $p_H = 10$, and C2 gets L for $p_L = 3$.
  - $PS = 13, \; CS = 0, \; SW = 13, \; DWL = 0$.

- **Will $p_H = 10, \; p_L = 3$ work when identities unknown?**
  - No! Consumer 1 will buy *Low* instead (a surplus of 2 instead of 0).

Heterogeneous Pricing (2nd-degree PD)

What is the best way to assign H to 1 and L to 2?

Choose p_H and p_L to

$$\max_{p_H, p_L} \quad p_H + p_L$$

such that:

$$\begin{array}{lr}
\text{Consumer 1 wants to buy } H & 10 - p_H \geq 0 \quad (1) \\
\text{Consumer 2 wants to buy } L & 3 - p_L \geq 0 \quad (2) \\
\text{Consumer 1 wants to buy } H \text{ instead of } L & 10 - p_H \geq 5 - p_L \quad (3) \\
\text{Consumer 2 wants to buy } L \text{ instead of } H & 3 - p_L \geq 4 - p_H \quad (4)
\end{array}$$

Note that

$$10 - p_H \geq 5 - p_L \implies p_H - p_L \leq 5$$
$$3 - p_L \geq 4 - p_H \implies p_H - p_L \geq 1$$

Solution to 2nd-degree PD

- Let's simplify the system of constraints
- (1) is redundant because of (3) and (2)

$$10 - p_H \geq 5 - p_L > 3 - p_L \geq 0$$

- Then (2) must bind , i.e.,

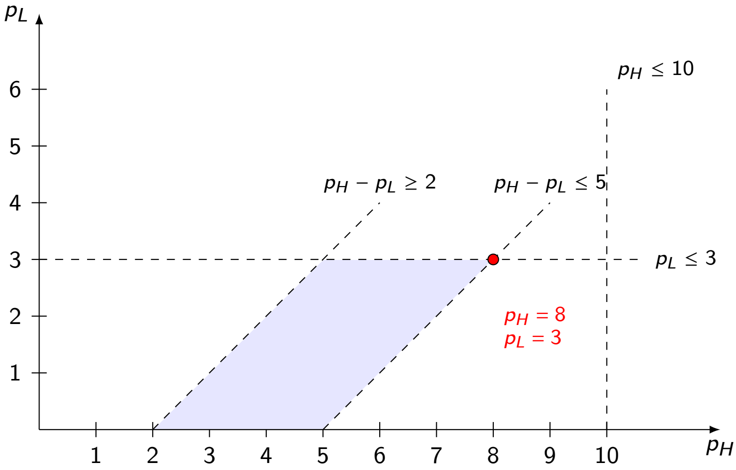
$$p_L = 3$$

- Finally, (3) must bind, i.e.,

$$10 - p_H = 5 - p_L \qquad \Rightarrow \qquad p_H = 8$$

- Then $PS = 11$, $CS = 2$, $SW = 13$, $DWL = 0$.

Alternative Graphical Solution



Summary – Discrimination Strategy

- Offer a menu of products and let consumers self-select.
- Separate consumers with different wtp's.
- Charge low-wtp consumers their wtp ($CS = 0$).
- Charge high-wtp consumers so that they are indifferent ($CS > 0$).
- The **monopoly profit** is higher than under the uniform pricing, but worse than under the 1st-degree PD.

Summary on Vertical Differentiation

- Typically consumers have differentiated tastes for varieties — some customers might really dislike the “low” quality items.
- Individual pricing of varieties allows the monopoly to extract surplus of variety-loving consumers without using consumer specific prices.
- Thus, infeasible consumer specific prices are effectively replaced with feasible variety specific prices.

Bundling (Tie-in Sales)

Setting

- Two goods, A and B. Let's assume zero cost of production for simplicity.

- Multiple consumers with **independent unit demand** for each product described by its willingness to pay (wtp):

$$\omega_A \text{ for good } A \qquad \text{and} \qquad \omega_B \text{ for good } B$$

- Three pricing regimes:

  - **Individual Pricing**: p_A and p_B.
  - **Pure Bundling:** p_{AB} (price of the bundle).
  - **Mixed Bundling:** p_A, p_B and p_{AB} such that

$$p_{AB} \leq p_A + p_B$$

Which one is better for the firm?

- Three pricing regimes:
  - **Individual Pricing**: p_A and p_B.
  - **Pure Bundling:** p_{AB} (price of a bundle).
  - **Mixed Bundling:** p_A, p_B and p_{AB} such that

$$p_{AB} \leq p_A + p_B$$

- What is the profit-maximizing pricing strategy here? Depends
  - a comparison between **indiv. pricing** and **pure bundling** is ambiguous (depends on preferences)
  - **mixed bundling** is less restricted, so it can never be worse than **indiv. pricing** or **pure bundling**.
  - **mixed bundling** can be strictly better sometimes (depends on preferences).

Example - I

	Steak	*Cake*	*Meal*
Romeo	80	20	100
Juliet	70	30	100

- Different relative tastes for components
- But similar value for the whole meal
- Here, **Pure Bundling** is better than **Individual Pricing**
- What about **Mixed Bundling**?

Example - II

	Steak	*Cake*	*Meal*
Romeo	80	30	110
Juliet	70	10	80

- Similar relative tastes for components
- But different value for the whole meal
- Here, **Individual Pricing** is better than **Pure Bundling**
- What about **Mixed Bundling**?

Example - III

	Steak	*Cake*	*Meal*
Romeo	80	10	90
Juliet	10	40	50

- Different value for the whole meal
- And very different relative tastes for components
- Here, **Individual Pricing** is better than **Pure Bundling**
- But we can do even better by using **Mixed Bundling** here

Quality Choice Model

Product Differentiation

A monopoly can increase its market power by differentiating products:

- Vertical Differentiation — different quality.
  - if prices are the same, people only buy the highest quality product.
  - **Examples:** *health insurance, first-class tickets*
- Horizontal Differentiation — different features, similar quality.
  - if prices are the same, people buy different products.
  - **Examples:** *taste, color*

So, quality by itself is a firm's choice!

Quality Choice for a Monopoly

- Before selling its products, a monopoly can design products with specific quality levels
- All consumers prefer higher quality, but have a different willingness to pay (wtp) for quality.
- When consumers' tastes are different, it is optimal for the monopoly to engage in vertical differentiation

Setting

- There are two types of travelers $\theta \in \{\theta_B, \theta_E\}$, Business and Economy, with unit demands. We assume that

$$\theta_B > \theta_E > 0 .$$

- There are 100 travelers, and λ of them are *Economy*.

- A traveler's monetary value depends on quality of travel $q \geq 0$ and the type of the traveler:

$$v(\theta, q) = \theta \cdot q$$

  Both types like quality, but *Business* types like quality more than *Economy*

- Cost of producing quality q for each type is independent and is given by

$$c(q) = \frac{q^2}{2} \qquad (c'(q) = q)$$

Quality Choice: 1st-degree PD

The airline designs a quality-price bundle for each type

$$(q_B,\ p_B) \qquad \text{and} \qquad (q_E,\ p_E)$$

subject to **participation** constraints:

$$\text{Business wants to buy B:} \qquad \theta_B\, q_B \ - \ p_B \ \geq \ 0 \tag{1}$$

$$\text{Economy wants to buy E:} \qquad \theta_E\, q_E \ - \ p_E \ \geq \ 0 \tag{2}$$

$$\max_{(q_B,p_B),(q_E,p_E)} \quad \lambda(p_E \ - \ c(q_E)) \ + \ (100 - \lambda)(p_B \ - \ c(q_B))$$

Solution for the 1st-degree PD (without derivation)

1. **What are the optimal prices?**

$$p_E \;=\; \theta_E \, q_E \qquad \text{and} \qquad p_B \;=\; \theta_B \, q_B$$

   Such that each type is indifferent between buying and not buying.

2. **Optimal choice of q_B and q_E is given by:**

$$q_E \;=\; \theta_E \qquad \text{and} \qquad q_B \;=\; \theta_B$$

* Note that the monopoly picks the socially optimal quality level *(marginal benefit equals marginal cost)* for each type
* Each consumer gets zero surplus.

Quality Choice: 2nd-degree PD

The airline designs a quality-price bundle for each type

$$(q_B,\ p_B) \qquad \text{and} \qquad (q_E,\ p_E)$$

subject to **participation** and **self-selection constraints**:

$$\begin{array}{lrcll}
\text{Business wants to buy B:} & \theta_B\, q_B - p_B & \geq & 0 & (1)\\
\text{Economy wants to buy E:} & \theta_E\, q_E - p_E & \geq & 0 & (2)\\
\text{Business prefers B to E:} & \theta_B\, q_B - p_B & \geq & \theta_B\, q_E - p_E & (3)\\
\text{Economy prefers E to B:} & \theta_E\, q_E - p_E & \geq & \theta_E\, q_B - p_B & (4)
\end{array}$$

$$\max_{(q_B, p_B), (q_E, p_E)} \quad \lambda\,[p_E - c(q_E)] + (100 - \lambda)\,[p_B - c(q_B)]$$

Solution to 2nd-degree PD (without derivation)

- If λ is small, the monopoly will offer

$$(q_B,\ p_B) \qquad \text{and} \qquad (0,\ 0)$$

where

$$p_B \ = \ \theta_B\, q_B \qquad \text{and} \qquad q_B \ = \ \theta_B$$

- In other words, the monopoly serves *Business* type only, and extracts all surplus from Business consumers (since $p_B = \theta_B\, q_B$).

Solution to 2nd-degree PD (without derivation)

- If λ is large, the monopoly will offer

$$(q_B,\ p_B) \qquad \text{and} \qquad (q_E,\ p_E)$$

where

$$\begin{aligned} p_E &= \theta_E\, q_E \\ p_B &= \theta_B\, q_B \ - \ (\theta_B - \theta_E)\, q_E \\ q_B &= \theta_B \\ q_E &= \theta_E \ - \ \left(\tfrac{100-\lambda}{\lambda}\right) (\theta_B \ - \ \theta_E) \end{aligned}$$

- Note that *Business* type gets positive surplus.
- Note that q_B is at socially optimal level but q_E is lower (than social. optimal)

Recipe for a real-world airline

- **How to set quality level?**
  - set quality in *Business* to social. optimum
  - set quality in *Economy* to low level (lower than it should be)
- **How to set airfare (price)?**
  - set p_B lower than the value of the *Business* customer
  - set p_E to extract all value from the *Economy* customer
- *In other words, make a passenger life miserable when traveling in the Economy class.*

Checklist for Part 1c

1. 2nd-degree PD with vertical differentiation and unit demands.
2. **Bundling:** Individual Pricing vs. Pure Bundling vs. Mixed Bundling (with unit demands).

Part 2a: Static Game Theory

Instructor: Oleg Baranov

Microeconomics 2 *(Module 1, 2022)*.

Suggested Readings for this part

Readings in Class Textbooks:

- **Varian, 8th edition:**
  - Ch 28: Game Theory
  - Ch 29: Game Applications
- **Carlton and Perloff:**
  - Ch 6: Oligopoly

Grade Game

Each student have to pick between two options: A or B.

I will randomly pair your form with one other form. Neither you nor your pair will ever know with whom you were paired. We will be using the **HSE 10-point grading system**. Here is how grades may be assigned for this class (hypothetically).

- If you put **A** and your pair puts **B**, then you will get **9**, and your pair will get **4** .
- If both of you put **A**, then you both will get grade **5**.
- If you put **B** and your pair puts **A**, then you will get grade **4**, and your pair will get grade **9**.
- If both of you put **B**, then you will both get grade **7**.

Make your choice

Guess 2/3 of the Average Game

Rules of the game:

Each of you have to choose an integer between 1 and 100 in order to guess *"2/3 of the average of the responses given by all students in the class"*. Each student who guesses the integer closest to the 2/3 of the average of all the responses, wins.

Example: (with only 3 students)

Choices made:	45, 10, 80
The average:	45
2/3 of the Average	30
The winner	Student who wrote 45

What is your guess?

Motivation

- Our focus has been on markets dominated by a single firm.
  - **Goal:** how to create and extract surplus from consumers.
  - **Choices:** pricing (linear and non-linear), consumer pricing, product differentiation, product design (vertical differentiation).
- But the actual monopoly markets are rare in practice.
- At the same time, many markets are dominated by a small number of large firms. This situation is called **oligopoly**.

Oligopoly

Many markets are dominated by a small number of large firms:

- Soft drinks — Coke and Pepsi.
- Smartphones — Apple, Sony, Samsung,....
- Each firm's action affects others, and prompts reactions.
- Firms should take these reactions into account when formulating strategies for pricing, output, new products, ...

Thus, strategic component (interaction among firms) is important here.

Oligopoly

- A competitive firm has no market power (price-taking)

$$\max_{q \geq 0} \; p \cdot q \; - \; C(q)$$

- A monopoly has all market power (sets any price/quantity)

$$\max_{q \geq 0} \; p(q) \cdot q \; - \; C(q)$$

- With oligopoly, there are several big firms. Each one maximizes its own profit by
  - setting prices/quantities
  - while accounting for choices of others since they also affect profit
- We have to use **game theory** – a tool to analyze strategic interactions and conflicts of interest.

Static Game Theory

(One-period games)

Example of a static game

Competition between **Aeroflot** and **S7** on the route between Moscow and Saint-Petersburg.

- **Possible actions**: time to fly — $\{$Morning, Evening$\}$.
- **Profits**:
  - Demand for morning $=$ 30
  - Demand for evening $=$ A ($A \geq 30$)
  - Each airline can serve 200 passengers at a time.
  - Constant marginal cost $c = 0$ and a price fixed at $p = 1$.
  - If they fly at the same time, they split the market.
  - If they fly at different times, each gets the whole market.

Preview: Large A

- Suppose that $A = 70$

		S7	
		Morning	Evening
Aeroflot	Morning	15, 15	30, 70
	Evening	70, 30	$\boxed{35, 35}$

- One prediction
- Morning is strictly dominated for each airline
- And Evening is strictly dominant for each airline (strictly dominates all other strategies)
- This game is known as the "Prisoner's Dilemma".

Preview: Small A

- Suppose that $A = 40$

		S7	
		Morning	Evening
Aeroflot	Morning	15, 15	$\boxed{30, 40}$
	Evening	$\boxed{40, 30}$	20, 20

- Two predictions
- Later we will say that the game has two Nash equilibria
- This game is known as "coordination game".

Preview: Intermediate A

- Suppose that $A = 60$

		S7	
		Morning	Evening
Aeroflot	Morning	$15, 15$	$\boxed{30, 60}$
	Evening	$\boxed{60, 30}$	$\boxed{30, 30}$

- Three predictions
- Morning is weakly dominated for each airline
- And Evening is weakly dominant for each airline (weakly dominates all other strategies)

Definition of Simultaneous Game

- Set of **players** N
- For each player from N, set of **actions/pure strategies** available to him/her:
  - S_i - set of all possible actions for player i
  - s_i - a particular play of player i
  - $s = (s_1, ..., s_N)$ a particular play of all players (strategy profile)
  - s_{-i} a particular play of all players except player i
- **Payoffs/utilities/profits**: A payoff of player i at strategy profile s (particular play of the game)

$$u_i(s_1, s_2, ..., s_N)$$

$$u_i(s)$$

$$u_i(s_i, s_{-i})$$

Example

Player 2

		L	R
Player 1	T	1, 5	4, 6
	M	3, 11	4, 0
	B	0, 0	0, 2

Players:

1,2

Strategy Sets:

$S_1 = \{T, M, B\}$

$S_2 = \{L, R\}$

Payoffs:

$u_1(T, L) = 1$

$u_2(T, L) = 5$

...

First Solution Concept – *Dominance*

Definition: Strict Dominance

For player i, strategy x **strictly dominates** some other strategy y if strategy x generates a **strictly higher** payoff for player i than strategy y **no matter what other players are playing**, i.e.,

$$u_i(x, s_{-i}) \ > \ u_i(y, s_{-i}) \qquad \text{for all} \qquad s_{-i}.$$

Extra language:

- If x **strictly dominates** y, then y is **strictly dominated** by x.
- If x strictly dominates ALL other strategies in S_i, then x is **strictly dominant** strategy for player i.

Class Example 1

		Player 2	
		L	R
	T	1, 5	4, 6
Player 1	M	3, 11	4, 0
	B	0, 0	0, 2

First Solution Concept – *Dominance*

Definition: Weak Dominance

For player i, strategy x **weakly dominates** some other strategy y if strategy x generates a **weakly higher** payoff for player i than strategy y no matter what other players are playing, i.e.,

$$u_i(x, s_{-i}) \geq u_i(y, s_{-i}) \qquad \text{for all} \quad s_{-i}$$

$$u_i(x, s_{-i}) > u_i(y, s_{-i}) \qquad \text{for some} \quad s_{-i} \quad \textit{(at least one } s_{-i}\textit{)}$$

Extra definitions:

- If x **weakly dominates** y, then y is **weakly dominated** by x
- If x weakly dominates ALL other strategies in S_i, then x is **weakly dominant** strategy for player i.

First Solution Concept – Dominance (strict and weak)

Is it a good solution concept? Do we believe in its prediction?

- Dominance makes sense

  There are some empirical evidence that people do not play dominated strategies

- But it is not powerful enough to solve majority of interesting games (no clear predictions).

  Example: A "coordination" game.

Guess 2/3 of the Average Game

Rules of the game:

Each of you have to choose an integer between 1 and 100 in order to guess *"2/3 of the average of the responses given by all students in the class"*. Each student who guesses the integer closest to the 2/3 of the average of all the responses, wins.

Example: (with only 3 students)

Choices made:	45, 10, 80
The average:	45
2/3 of the Average	30
The winner	Student who wrote 45

How did you play this game?

Guess 2/3 of the Average Game: Solving

Strategies:	**Why not to play?**	**Requirements**
68, 69,..., 100	weakly dominated by 67	my rationality (R)
45,..., 67	not weakly dominated in the original game but weakly dominated by 44 once we eliminated 68-100	my rationality (R) + knowledge that others are rational (KR)
30,..., 44	weak. dominated by 29 ...	R, KR, KKR
20,..., 29	weak. dominated by 19	...
...	...	...
2	weak. dominated by 1	...

After multiple rounds of elimination, only 1 survives.

Second Solution Concept - Iterative Elimination of Strictly Dominated Strategies (IESDS)

1st round of elimination:

- Start with a game with actions sets: A_i for $i = 1, \ldots, N$.
- For each player i, delete all actions which are strictly dominated.
- Obtain new game with smaller action sets: $B_i = [\text{undominated actions of player } i] \subseteq A_i$ for $i = 1, \ldots, N$.

2nd round of elimination:

- Start with a game with actions sets: B_i for $i = 1, \ldots, N$.
- For each player i, delete all actions a_i which are strictly dominated.
- Obtain new game with smaller action sets: $C_i = [\text{undominated actions of player } i] \subseteq B_i$ for $i = 1, \ldots, N$.

Repeat until no strictly dominated strategies left.

Class Example 2

		Player 2	
		Left	Right
Player 1	Top	1, 2	4, 1
	Middle	3, 2	2, 1
	Bottom	2, 1	1, 3

Properties of Iterated Elimination

Strict Version (IESDS):

- If each player is left with one action, then a unique prediction.
- Order of elimination does not matter — a final set of actions is always the same.
- Produces a unique equilibrium very rarely.

Weak Version (IEWDS):

- If each player is left with one action, then one of possible predictions.
- Order of elimination **does matter** — a final set of actions can change.
- Works well quite often, but might lead to unrealistic predictions.

Iterated Elimination of Dominated Strategies (IEDS)

Is it a good solution concept? Do we believe in its predictions?

1. Application of IEDS requires the knowledge of other players' payoffs (unlike Dominance).
2. Also requires the knowledge of other players' rationality (unlike Dominance).
3. Might require too many layers of rationality.
   Example: "2/3 of the average" game
4. IEDS still can be powerless in many games (no clear predictions).
   Example: A "coordination" game.

Nash Equilibrium

Third Solution Concept - Nash Equilibrium

- **Intuition:** given a conjecture about the others, each player chooses her best action (playing your best response)
- **Definition:** Player i's strategy x is a **best response** to the strategy profile s_{-i} of other players (denoted as $BR_i(s_{-i})$) if

$$u_i(x, s_{-i}) \;\geq\; u_i(y, s_{-i}) \qquad \text{for all} \qquad y \in S_i$$

- **In the equilibrium, all conjectures are correct and each player is playing its best response.**
- Formally, a pure strategy Nash equilibrium is a strategy profile

$$s^* \;=\; (s_1^*, \ldots, s_N^*) \qquad \text{such that} \qquad s_i^* \;\in\; BR_i(s_{-i}^*) \quad \forall i$$

Class Example 3

		Player 2		
		Left	Center	Right
	Top	0, 4	4, 2	5, 2
Player 1	Middle	4, 0	3, 4	4, 2
	Bottom	2, 4	6, 5	3, 5

- Find all Nash equilibria

Nash equilibrium

NE concept is justified as a reasonable prediction of the outcome:

- If agreed beforehand on this set of actions — nobody has an incentive to deviate.
- If one announces that she will play its part of NE, the others will also play the NE.
- Long-term interaction (play multiple times) where players learn what their opponents do.

Main weakness: How can a player form a right conjecture?

Properties

- (Pure strategy) Nash equilibrium might fail to exist.
- A game can have several Nash equilibria.
- All Nash equilibria **survive** the iterative elimination of **strictly** dominated strategies (IESDS).
- Some Nash equilibria can be lost during the iterative elimination of **weakly** dominated strategies (IEWDS).
- At least one Nash equilibrium always survives during IEWDS.

Additional Examples

Unique pure NE — unique outcome surviving IESDS

$$\begin{array}{ccc} & L & R \\ U & 2,2 & -1,3 \\ D & 3,-1 & \boxed{0,0} \end{array}$$

Two pure NE — IESDS & IEWDS are useless

$$\begin{array}{ccc} & L & R \\ U & \boxed{2,1} & 0,0 \\ D & 0,0 & \boxed{1,2} \end{array}$$

Three pure NE — IESDS is useless & IEWDS can pick any

$$\begin{array}{ccc} & L & R \\ U & 0,0 & \boxed{1,2} \\ D & \boxed{2,1} & \boxed{1,1} \end{array}$$

Mixed Strategies

“Rock, Paper, Scissors” Game

Player II

Player I		Rock	Paper	Scissors
	Rock	$0, \; 0$	$-1, \; 1$	$1, -1$
	Paper	$1, -1$	$0, \; 0$	$-1, \; 1$
	Scissors	$-1, \; 1$	$1, -1$	$0, \; 0$

"Rock, Paper, Scissors" Game

		Player II		
		Rock	Paper	Scissors
	Rock	$0, \; 0$	$-1, \; 1$	$1, -1$
Player I	Paper	$1, -1$	$0, \; 0$	$-1, \; 1$
	Scissors	$-1, \; 1$	$1, -1$	$0, \; 0$

- No dominated strategies.
- Nash equilibrium in "pure" strategies does not exist.

  Why? *If I play* **Rock**, *my opponent replies with* **Paper**, *but then my best response is to play* **Scissors**, *and so on ...*

  Mutual best response does not exist in this game!

Mixed strategies

A **mixed strategy** for player $i \in N$ is a probability distribution p_i over S_i, that is

$$p_i(s_i) \geq 0 \quad \forall s_i \in S_i \qquad \text{and} \qquad \sum_{s_i \in S_i} p_i(s_i) = 1$$

where $p_i(s_i)$ is probability that player i plays s_i.

Example

Suppose $S_i = \{Rock, Paper, Scissors\}$

- $p_i = (1/3, 1/3, 1/3)$ corresponds to player i choosing each action equally likely;
- $p_i = (0.5, 0.5, 0)$ corresponds to playing *Rock* and *Paper* with 50% probability each, and never playing *Scissors*

Expected Payoffs

- Suppose that there are only **two** players.

- Player 1's expected payoff from a **pure strategy** s_1 when another player is using a mixed strategy p_2:

$$EU_1(s_1, p_2) \;=\; \sum_{s_2 \in S_2} u_1(s_1, s_2) \cdot p_2(s_2)$$

- Player 1's expected payoff from a **mixed strategy** p_1 when another player is using a mixed strategy p_2:

$$EU_1(p_1, p_2) \;=\; \sum_{s_1 \in S_1} EU_1(s_1, p_2) \cdot p_1(s_1)$$

- We can define payoffs for Player 2 analogously.

Class Example 4

The class odds in Fall 2022 (139 responses):

Rock	Paper	Scissors
35%	40%	25%

Class Example 4

The class odds in Fall 2022 (139 responses):

Rock	Paper	Scissors
35%	40%	25%

Expected Payoff Calculation for $p_2 \;=\; (0.35, 0.40, 0.25)$

$$
\begin{aligned}
EU_1(Rock, p_2) &= 0.35\,[0] + 0.40\,[-1] + 0.25\,[1] = -0.15 \\
EU_1(Paper, p_2) &= 0.35\,[1] + 0.40\,[0] + 0.25\,[-1] = 0.10 \\
EU_1(Scissors, p_2) &= 0.35\,[-1] + 0.40\,[1] + 0.25\,[0] = 0.05
\end{aligned}
$$

Expected Payoff Calculation for $p_1 \;=\; (1/3, 1/3, 1/3)$

$$EU_1(p_1, p_2) \;=\; \frac{1}{3}[-0.15] \;+\; \frac{1}{3}[0.10] \;+\; \frac{1}{3}[0.05] \;=\; 0$$

Can I do better than p_1 above? Yes, I should play $p_1 = (0, 1, 0)$.

Rock, Paper, Scissors Game — HSE

Class	Rock	Paper	Scissors	Best Play
Fall 2022	35%	40%	25%	$P > S > R$

Rock, Paper, Scissors Game — University of Colorado

Class	Rock	Paper	Scissors	Best Play
Summer 2022	37%	39%	24%	$P > S > R$
Spring 2022	41%	33%	26%	$P > R > S$
Fall 2021	27%	44%	29%	$S > P > R$
Spring 2021	40%	43%	17%	$P > S > R$
Fall 2020	33%	41%	25%	$P = S > R$
Spring 2020	32.5%	45.0%	22.5%	$S > P > R$
Spring 2019	38.1%	47.6%	14.3%	$P > S > R$
Fall 2018	62.5%	31.5%	6%	$P > R > S$

Best Response in mixed strategies

- Fix p_2, and calculate the highest payoff that Player 1 can obtain from playing its pure strategies, i.e.,

$$EU_1^* \;=\; \max_{s_1 \in S_1} \; EU_1(s_1,\, p_2)$$

- Then the full best response of Player 1 (including mixed strategies) to Player 2 playing p_2 is

$$BR_1(p_2) \;=\; \{\, p_1 : \;\; EU_1(p_1,\, p_2) \;=\; EU_1^* \,\}$$

- The only possibility for $p_1 \in BR_1(p_2)$ is when
  - For any action s_1 with $p_1(s_1) > 0$, player i gets EU_1^*.
  - For any action s_1 with $p_1(s_1) = 0$, player i's payoff is $\leq EU_1^*$.

Nash equilibrium in mixed strategies

- A mixed strategy Nash equilibrium is a profile $p = (p_1, p_2)$ such that

$$p_1 \in BR_1(p_2) \qquad \text{and} \qquad p_2 \in BR_2(p_1)$$

- **Nash Theorem:** In any *finite game*, **at least one NE always exists** (maybe in mixed strategies).

- *finite game:* finite number of players, each with finite number of pure strategies.

Nash equilibrium of RPS game

Expected Payoff Calculation for $p_2 = (1/3, 1/3, 1/3)$

$$\begin{aligned}
EU_1(Rock, p_2) &= \tfrac{1}{3}[0] + \tfrac{1}{3}[-1] + \tfrac{1}{3}[1] = 0 \\
EU_1(Paper, p_2) &= \tfrac{1}{3}[1] + \tfrac{1}{3}[0] + \tfrac{1}{3}[-1] = 0 \\
EU_1(Scissors, p_2) &= \tfrac{1}{3}[-1] + \tfrac{1}{3}[1] + \tfrac{1}{3}[0] = 0
\end{aligned}$$

Expected Payoff Calculation for $p_1 = (1/3, 1/3, 1/3)$

$$EU_1(p_1, p_2) = \frac{1}{3}[0] + \frac{1}{3}[0] + \frac{1}{3}[0] = 0$$

Can I do better than p_1 above?

- No, player 1 cannot do better.
- And player 2 cannot do better against p_1.
- This is a mixed Nash equilibrium for RPS game. **So what is game theory actually predicts for this game?**

Class Example 5

		S7	
		Morning	Evening
Aeroflot	Morning	15, 15	30, 40
	Evening	40, 30	20, 20

- Find a mixed strategy NE (if exists).

Extending dominance to mixed strategies

- For player 1, a pure strategy s_1 is **strictly dominated** by a mixed strategy p_1 if

$$EU_1(p_1, s_2) > u_1(s_1, s_2) \qquad \text{for all } s_2 \in S_2$$

- For player 1, a pure strategy s_1 is **weakly dominated** by a mixed strategy p_1 if

$$\begin{aligned} EU_1(p_1, s_2) &\geq u_1(s_1, s_2) \qquad && \text{for all } s_2 \in S_2 \\ EU_1(p_1, s_2) &> u_1(s_1, s_2) \qquad && \text{for some } s_2 \in S_2 \end{aligned}$$

- We can also extend IESDS and IEWDS to allow for mixed strategies, i.e. at each stage, **eliminate pure strategies** that are dominated by some mixed strategies.

Class Example 6

		Player 2	
		L	R
Player 1	T	1, 5	5, 6
	M	2, 11	2, 0
	B	5, 0	1, 2

Skill checklist for Part 2a

1. Specify all components of a game
2. Find strictly and weakly dominated strategies.
3. Identify strategies that can be eliminated by IESDS and IEWDS.
4. Find Nash equilibria in pure strategies.
5. Find Nash equilibria in mixed strategies.

Part 2b: Oligopoly Models

Instructor: Oleg Baranov

Microeconomics 2 *(Module 1, 2022)*.

Suggested Readings for this part

Readings in Class Textbooks:

- **Varian, 8th edition:**
  - Ch 27: Oligopoly
- **Carlton and Perloff:**
  - Ch 6: Noncooperative Oligopoly
  - Ch 7: Product Differentiation

Oligopoly

Models of oligopoly

- Two classical models with homogeneous goods:
  - **Bertrand** — price-setting: firms choose prices, a market demand determines a quantity.
  - **Cournot** — quantity-setting: firms choose quantities, a market demand determines a price at which the market clears.
- The models lead to very different market predictions.
- This contrasts with a monopoly setting where choosing **prices** or **quantities** is the same

Cournot competition: Setting

- N firms indexed by $i = 1, \ldots, N$
- the cost function of each firm is $C_i(q_i)$
- Total quantity is

$$Q = q_1 + q_2 + ...q_N$$

- The market price is given by an inverse demand function $p(Q)$
- Firm $i's$ profit is the following function of $q = (q_1, \ldots, q_N)$:

$$\pi_i(q) = p\left(\sum_{j \neq i} q_j + q_i\right) q_i - C_i(q_i).$$

Cournot competition: Solution

- We look for a **pure strategy Nash equilibrium**. To be specific, we look for strategy profile $q^* = (q_1^*, q_2^*, ..., q_N^*)$ such that each firm i

$$\pi_i(q_i^*, q_{-i}^*) \;\geq\; \pi_i(q_i, q_{-i}^*) \qquad \forall q_i \geq 0.$$

- **How to solve for a Nash equilibrium?**

1. Fix arbitrary q_{-i} and solve for the firm i's best response, i.e.

$$\max_{q_i \geq 0} \; \pi_i(q_i,\, q_{-i}).$$

   Again: **FOC, SOC, shutdown** $\implies BR_i(q_{-i})$.

2. Then solve the system of equations in N variables $q^* = (q_1^*, \ldots, q_N^*)$:

$$q_i^* \;=\; BR_i(q_{-i}^*) \qquad i \;=\; 1, \ldots, N$$

Cournot competition: Characterization

- In the Nash equilibrium, for any firm i with $q_i^* > 0$, it must be

$$p(Q^*) \; + \; p'(Q^*)\, q_i^* \; = \; MC_i(q_i^*)$$

where

$$Q^* \; = \; q_1^* \; + \; q_2^* \; + \; \dots \; + \; q_N^*$$

- In other words, "marginal revenue equals marginal costs" but accounting for the share occupied by other firms
- In contrast, **for the monopoly**, the optimal production level Q^* is given by

$$MR(Q^*) \; = \; p(Q^*) \; + \; p'(Q^*)\, Q^* \; = \; MC(Q^*)$$

Cournot competition: Final Remarks

- Some firms with high costs might shutdown voluntarily in the equilibrium.
- Typically, without fixed costs, a NE is unique.
- With fixed costs, multiple equilibria possible including the ones with different number of active firms (with $q_i^* > 0$).
- Firms are better off compared to perfect competition, but worse off compared to the monopoly setting

Class Example 1

- Inverse market demand is given by

$$p(Q) = 8 - Q$$

- N firms, each with $C(q) = 2q$
- Firms compete in **quantities**

- **Question 1:** Solve for a market outcome when $N = 1$.
- **Question 2:** Solve for a market outcome when $N = 2$.
- **Question 3:** For $N = 2$, draw the best response diagram in the (q_1, q_2) coordinates.

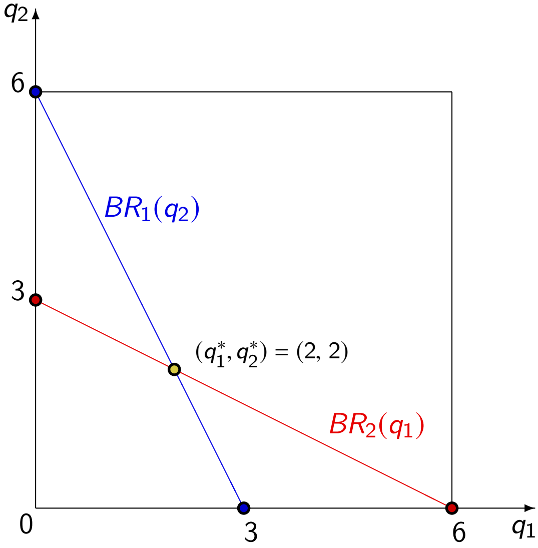
Best Response Diagram

- Best Response functions:

$$BR_1(q_2) = 3 - \frac{q_2}{2}$$
$$BR_2(q_1) = 3 - \frac{q_1}{2}$$

- The Nash equilibrium is

$$(q_1^*, q_2^*) = (2, 2)$$



Bertrand competition: Setting

- Two firms indexed by $i = 1, 2$, with cost function $C_i(q_i)$
- The market demand function is $D(p)$
- Firms simultaneously choose prices, p_1 and p_2
  - o Consumers purchase from the firm with the **lowest price**.
  - o When sellers charge the same price, they **split the market**.
- Firm i's profit as a function of p_i, p_j:

$$\pi_i(p_i, p_j) \ = \ \begin{cases} p_i \cdot D(p_i) \ - \ C_i(D(p_i)) & p_i < p_j \\ p_i \cdot D(p_i)/2 \ - \ C_i(D(p_i)/2) & p_i = p_j \\ -C_i(0) & p_i > p_j \end{cases}$$

Bertrand competition: Solution

- We look for a pure strategy Nash equilibrium $p^* = (p_1^*, p_2^*)$ such that for each firm $i = 1, 2$:

$$\pi_i(p_i^*, p_j^*) \;\geq\; \pi_i(p_i, p_j^*) \qquad \forall p_i \geq 0$$

- We can work with the best responses as before, that is

$$\max_{p_i \geq 0} \; \pi_i(p_i, p_j) \qquad \Longrightarrow \qquad BR_i(p_j).$$

- **But the profit function is non-continuous and best response is frequently undefined**
- In Bertrand models, it is easier to find NE by making educated observations
  - Firms with positive sales have to charge the same price;
  - Firms should not lose money when they sell $(\pi_i^* \geq -C_i(0))$
  - No incentives for undercutting *(charging a slightly lower price)*

Class Example 2

- Market demand is given by

$$D(p) = 8 - p$$

- Two firms, each with $C(q) = 2q$
- Firms compete in **prices**

- **Question 1:** Solve for a market outcome.

Remarks on Bertrand

- With constant marginal costs, the NE is either unique, or does not even exist
- NE is usually not unique when marginal costs are increasing (weaker incentives to engage in undercutting)
- All firms with high costs shutdown — costs advantage is crucial in Bertrand setting.
- **Bertrand paradox:** set of equilibria frequently includes (or even consists of) the one corresponding to perfect competition

Bertrand paradox

Bertrand paradox: with homogeneous good and constant marginal costs, Bertrand equilibrium implements a perfectly competitive outcome.

In practice, it does not play out because:

- Discrete prices
- Dynamic interaction & collusion
- Consumers' search *(can be hard to find the best price)*
- firms' capacity constraints *(can be hard to serve all customers)*
- **Product differentiation: horizontal or vertical**

Bertrand paradox

Bertrand paradox: with homogeneous good and constant marginal costs, Bertrand equilibrium implements a perfectly competitive outcome.

One possible resolution — product differentiation:

- Vertical — different quality.
  - if prices are the same, people only buy the highest quality product.
  - Example: *iPad 32GB vs. iPad 128 GB*
- Horizontal — similar quality, different features (location, color).
  - if prices are the same, people buy different products.
  - Example: *iPad Black vs. iPad White*

Hotelling Model ("Hotelling Beach")

- A beach town with a 1km broadwalk
- Two identical firms located at $0 \leq a_1 \leq a_2 \leq 1$
- Each firm can produce the good at constant marginal cost of c
- Consumers with unit demand (each with **wtp** of w)
- Consumers are evenly spread out along the broadwalk (uniform distribution)
- Consumers can buy from either firm, but they have to walk first. Walking costs $t \cdot d$ for any distance d (in monetary equivalent)
- Let's assume

$$t > 0 \qquad \text{and} \qquad w > c + t$$

What would a monopoly do here?

- It is easy to show that the monopoly wants to **serve all consumers** in this market
- Where should the monopoly set its shop?
- Fix location at $a \in [0, 1]$. The longest travel for a consumer is given by $\max\{a, 1 - a\}$. To serve everyone, the optimal monopoly price is given by

$$p = w - t \cdot \max\{a, 1 - a\}$$

- The best location when serving everyone is $a = 0.5$
- In the absence of competition, firms go to consumers to minimize their costs (and extract more surplus).

  This is the principle of minimum differentiation

Hotelling Model: Same Location

- Two firms located at the same location ($a_1 = a_2$)
- Then we have the same product, the same location and firms competing in prices
- **This is the Bertrand model**
- The unique Nash equilibrium:

$$p_1^* = p_2^* = c$$

and

$$\pi_1^* = \pi_2^* = 0$$

Back to Hotelling Model

- Two firms are located at the ends of the broadwalk ($a_1 = 0$, $a_2 = 1$)
- Let's assume that all consumers will buy the good (w is large enough, need to check later)
- Solving for an indifferent consumer *(same value of buying)*

$$\begin{aligned} w - p_1 - t\hat{x} &= w - p_2 - t(1-\hat{x}) \\ \hat{x} &= \tfrac{1}{2} + \tfrac{p_2 - p_1}{2t} \end{aligned}$$

- Then demands for each firm are given by

$$D_1(p_1, p_2) = \hat{x} = \tfrac{1}{2} + \tfrac{p_2 - p_1}{2t} \qquad D_2(p_1, p_2) = 1 - \hat{x} = \tfrac{1}{2} + \tfrac{p_1 - p_2}{2t}$$

- And profits of firms are given by

$$\pi_1(p_1, p_2) = (p_1 - c) \cdot D_1(p_1, p_2) \qquad \pi_2(p_1, p_2) = (p_2 - c) \cdot D_2(p_1, p_2)$$

Hotelling Model: Solution

- We look for a pure strategy Nash equilibrium $p^* = (p_1^*, p_2^*)$ such that for each firm $i = 1, 2$:

$$\pi_i(p_i^*, p_j^*) \ \geq \ \pi_i(p_i, p_j^*) \qquad \forall p_i \geq 0$$

- Fix p_j and solve for firm i's best response, i.e.,

$$\max_{p_i \geq 0} \ \pi_i(p_i, p_j) \qquad \Longrightarrow \qquad BR_i(p_j).$$

- Solve for (p_1^*, p_2^*) such that

$$p_1^* \ = \ BR_1(p_2^*) \qquad \text{and} \qquad p_2^* \ = \ BR_2(p_1^*)$$

Hotelling Model: Nash equilibrium

- Best response functions are given by

$$BR_1(p_2) = \frac{t + c + p_2}{2} \quad \text{and} \quad BR_2(p_1) = \frac{t + c + p_1}{2}$$

- Then Nash equilibrium (p_1^*, p_2^*)

$$p_1^* = p_2^* = t + c$$

- Equilibrium profits are

$$\pi_1^* = \pi_2^* = \frac{t}{2} > 0$$

- **Last check:** max cost in NE is $p^* + t/2$, so we need

$$w \geq c + \frac{3}{2}t$$

Discussion

- In the Hotelling model, prices and profits increase with
  - the distance between firms; and
  - the cost of traveling t.
- When facing competition, firms differentiate themselves as much as possible

  The principle of maximum differentiation
- We do not have to think about consumer location as literally *"physical location"*. Instead, we can think of $x \in [0,1]$ as a bundle of characteristics, i.e., color, taste, shape, convenience, inconvenience, ambience.

Alternative Model of Differentiated Products

- Two firms $i = 1, 2$, each with cost function $C_i(q_i)$
- Market demand for Firm i depends on p_i and p_j

$$D_i(p_i, p_j) \ = \ 1 \ - \ bp_i \ + \ ap_j$$

  where
  - $b > 0$ — negative impact of p_i on D_i (as usual)
  - $a > 0$ — goods are substitutes ($p_j \uparrow \Rightarrow \ D_i \uparrow$)
  - $a < 0$ — goods are complements ($p_j \uparrow \Rightarrow \ D_i \downarrow$)
  - $a = 0$ — unrelated goods

- Firm i's profit is given by

$$\pi_i(p_i, p_j) \ = \ p_i \cdot D_i(p_i, p_j) \ - \ C_i(D_i(p_i, p_j))$$

- You can find NE using the standard solution process (the one we used for solving Cournot and Hotelling models)

Skill checklist for Part 2b

1. Solving for NE in different Cournot models
2. Solving for NE in different Bertrand models
3. Solving for NE in different Hotelling models
4. Calculating various market characteristics for each model

Part 2c: Dynamic Games

Instructor: Oleg Baranov

Microeconomics 2 *(Module 1, 2022)*.

Suggested Readings for this part

Readings in Class Textbooks:

- **Varian, 8th edition:**
  - Ch 28: Game Theory
- **Carlton and Perloff:**
  - Ch 6: Noncooperative Oligopoly

Recap

Up to now we studied situations where firms act **simultaneously and once:**

- **Bertrand** — at the same time, each firm picks a price and consumers decide whom to buy from.
  - perfect substitutes/homogeneous goods
  - horizontally differentiated goods (*Hotelling*)
- **Cournot** — at the same time, each firm picks a quantity/order/capacity and prices determined to clear the market.
  - perfect substitutes/homogeneous goods
  - horizontally differentiated goods *(can be formulated)*

Question I — But what if firms act sequentially?

Question II — And what if firms act more than once?

Dynamic strategic interaction

- Often firms act
  - Sequentially — the official BMW dealer can place order earlier than other dealers
  - More than once — first decide on how much to order, then post a price
  - Repeateadly — change prices every month/quarter/year
- The timing of competition is important
- We need new game theory tools

Game Theory for Dynamic Games

Example - Date Game

		Romeo: Movie	Romeo: Park
Juliet	Movie	$1, 2$	$0, 0$
	Park	$0, 0$	$2, 1$

Example - Date Game

		Romeo: Movie	Romeo: Park
Juliet	Movie	$1, 2$	$0, 0$
	Park	$0, 0$	$2, 1$

Timing:

1. Juliet goes to the Movie theater or to the Park (and checks-in)
2. Romeo finds out Juliet's location
3. Romeo decides where to go

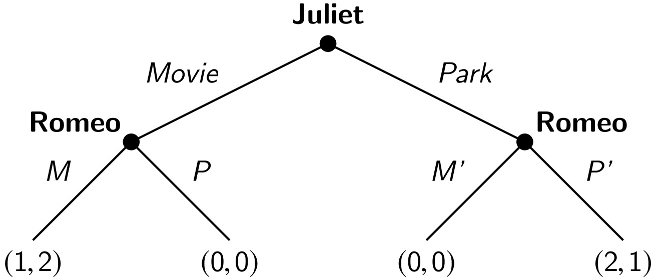
Extensive form of a game

Definition of a dynamic game:

- Players: $i = 1, ..., N$
- The order of players' moves and actions each player can take at each point — **one player at a time!**
- The information of players' when they choose their actions.
- Payoffs/utilities players receive at the end.

All this elements can by represented by a **game tree**

Extensive Form: Sequential Date Game



Game Tree (but growing down):

- **Leafs:** payoffs
- **Node:** a player who is making a choice
- **Branches:** available choices

Strategies and Information

Strategy:

- Player's strategy is a **COMPLETE PLAN** that tells a player what to do at **ALL** nodes where he or she can take an action

Information:

- Information is perfect when each player knows choices of all other players who moved before him or her
- Otherwise the information is imperfect.

Date Game — sequential with observing

- **Players:** Romeo and Juliet.
- **Order & actions:** Juliet picks first, then Romeo
- **Information:** perfect — Romeo observes Juliet's choice.
- **Strategies:**
  - Juliet: { *Movie, Park* }
  - Romeo: { *MM', MP', PM', PP'* }

Date Game — sequential without observing

- **Players:** Romeo and Juliet.
- **Order & actions:** Juliet picks first, then Romeo
- **Information:** imperfect — Romeo does not observe Juliet's choice.
- **Strategies:**
  - Juliet: { *Movie, Park* }
  - Romeo: { *Movie, Park* }
- It is **exactly the same strategic situation** when
  - Juliet moves first, but Romeo does not observe Juliet's choice
  - players move simultaneously

Extensive form vs. Normal form

Extensive form of a game:

- Players: $i = 1, \ldots, N$
- The order of moves, actions and information
- Payoffs/utilities

Normal form of a game:

- Players: $i = 1, \ldots, N$
- Set of strategies
- Payoffs/utilities

Date Game – sequential without observing

- It is **exactly the same strategic situation** when
  - o Juliet moves first, but Romeo does not observe Juliet's choice
  - o players move simultaneously
- Game matrix is as follows:

		Romeo	
		Movie	Park
Juliet	Movie	$\underline{1}$, $\underline{2}$	0, 0
	Park	0, 0	$\underline{2}$, $\underline{1}$

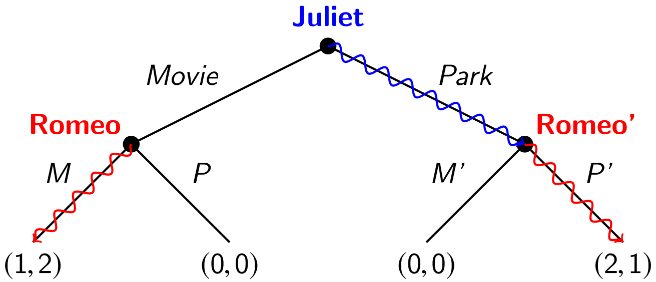
- **Two NE:** *(Movie, Movie)* and *(Park, Park)*
- Both Nash equilibria are reasonable!

Backward induction with perfect information

To solve a dynamic game, proceed backwards:

- Look at the last stage, and find the best action
- Assume that this best action will be played at the end, then find the best action at the stage preceding the last one
- Repeat until you reach the first stage

Extensive Form: Sequential Date Game



Backward Induction Outcome:

- **Romeo(1)** selects *Movie (M)*
- **Romeo(2)** selects *Park (P')*
- **Juliet** selects *Park*

Date Game – sequential with observing

- **Nash equilibrium of an extensive form game is a Nash equilibrium of its normal form**
- Then the game matrix is as follows:

		Romeo			
		(M, M’)	(M, P’)	(P, M’)	(P, P’)
Juliet	Movie	$\underline{1}$, $\underline{2}$	1, $\underline{2}$	$\underline{0}$, 0	0, 0
	Park	0, 0	$\underline{2}$, $\underline{1}$	$\underline{0}$, 0	$\underline{2}$, $\underline{1}$

- **Three NE:** *(Movie, M, M’)*, *(Park, M, P’)*, *(Park, P, P’)*
- **Two equilibrium outcomes:** *Movie* or *Park*

Date Game – sequential with observing

		Romeo			
		(M, M')	(M, P')	(P, M')	(P, P')
Juliet	Movie	$\underline{1}$, $\underline{2}$	1, $\underline{2}$	$\underline{0}$, 0	0, 0
	Park	0, 0	$\underline{2}$, $\underline{1}$	$\underline{0}$, 0	$\underline{2}$, $\underline{1}$

Analysis of different Nash equilibria:

- *(Park, M, P')* coincides with the Backward Induction outcome
- *(Park, P, P')* includes an irrational play by Romeo, but no harm
- *(Movie, M, M')* includes an **empty threat** by Romeo that affects the outcome

Only one NE is reasonable — (Park, M, P')

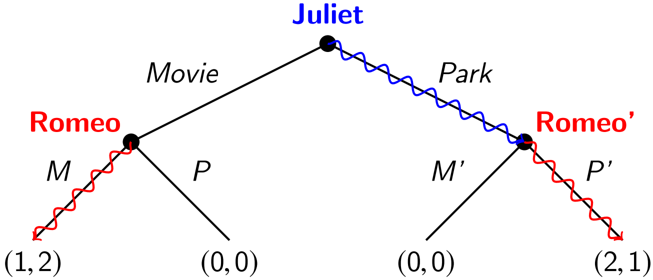
Inadequacy of Nash Equilibria

- *A Nash equilibrium assumes that each player commits to its equilibrium strategy and never revises it as the game unfolds*
- As a result, it can predict **irrational choices and silly outcomes**
- We need a stronger solution concept. Something that would eliminate noncredible threats. Something that would predict the same outcome as the one obtained by Backward Induction.
- This solution concept is known as **Subgame Perfect Nash Equilibrium**

Subgame Perfect Nash Equilibrium (SPNE)

- A **subgame** is what is left of the original game after some moves have been played.
- In a dynamic game, a strategy profile is said to form a subgame-perfect Nash equilibrium (SPNE) if the strategies constitute Nash equilibria in every subgame of the game.
- In other words, SPNE is a NE of the original game that satisfies additional rationality checks.
- **Intuition:** eliminates all irrational choices made by players in all nodes.

Extensive Form: Sequential Date Game



- **Three NE:** *(Movie, M, M')*, *(Park, M, P')*, *(Park, P, P')*
- **But only one SPNE:** *(Park, M, P')*

Multiple Subgame Perfect Nash Equilibria

- *Multiple SPNE are quite possible when players are indifferent between their best options*
- Example:
  - Player 1 chooses between A and B.
  - choosing B ends the game and gives $(5, 5)$.
  - is A is chosen, then Player 2 decides among Yes/No/Kill Everyone.
  - Yes gives $(10, 0)$, No gives $(0, 0)$ and Kill Everyone gives $(-100, -100)$.
- Two SPNE: (B, No) and (A, Yes)
- The game has another NE (B, Kill Everyone), but it is not SPNE

Stackelberg Model

Stackelberg Model

- The Stackelberg model is a Cournot model with sequential order of moves.
- Suppose that there are two firms $N = 2$, each with

$$C_i(q) = c \cdot q \qquad \text{where} \qquad c < 1$$

- The market inverse demand is given by

$$p(Q) = 1 - Q$$

- Firm 1 **(the leader)** chooses its quantity q_1 first.
- Firm 2 **(the follower)** observes q_1, and then chooses its own quantity q_2.
- We solve this game by **backward induction**

Stackelberg Model: Solution

- Firm 2's profit:

$$\pi_2(q_1, q_2) = (1 - q_1 - q_2)q_2 - cq_2$$

- Firm 2 chooses q_2 by:

$$\max_{q_2 \geq 0} \pi_2(q_1, q_2)$$

- Firm 2's best response is given by

$$BR_2(q_1) = \begin{cases} \frac{1-c-q_1}{2} & q_1 \leq 1 - c \\ 0 & q_1 > 1 - c \end{cases}$$

Stackelberg Model: Solution

- Firm 1 will never choose $q_1 > 1 - c$ since it implies $\pi_1 \leq 0$
- Then Firm 2 will select

$$q_2 = BR_2(q_1) = \frac{1 - c - q_1}{2}$$

- Firm 1's profit

$$\pi_1(q_1) = [1 - q_1 - BR_2(q_1)]q_1 - cq_1$$

- Optimal quantity for Firm 1

$$q_1^* = \frac{1 - c}{2}$$

- Then Firm 2 will pick

$$q_2^* = BR_2(q_1^*) = \frac{1 - c}{4}$$

Stackelberg Model: SPNE and Equilibrium Outcome

- **SPNE:** *(must be strategies at each node)*

$$q_1^* = \frac{1-c}{2} \qquad \text{and} \qquad BR_2(q_1) = \begin{cases} \frac{1-c-q_1}{2} & q_1 \leq 1-c \\ 0 & q_1 > 1-c \end{cases}$$

- **Equilibrium Outcome:** *(quantities actually selected)*

$$q_1^* = \frac{1-c}{2} \qquad \text{and} \qquad q_2^* = \frac{1-c}{4}$$

- **Note:** *Here Firm 1 produces its monopoly output. However, this is just a coincidence rather than a general result.*

Stackelberg model with $N \geq 2$

- N firms indexed by $i = 1, \ldots, N$, each with cost function $C_i(q_i)$
- The inverse market demand $p(Q)$ where

$$Q \;=\; q_1 \;+\; \ldots \;+\; q_N$$

- Firm i set its own q_i sequentially (firm $i+1$ acts after firm i)
- Firm $i's$ profit is the following function of $q \;=\; (q_1, \ldots, q_N)$:

$$\pi_i(q_1, \ldots, q_N) \;=\; p(Q)q_i \;-\; C_i(q_i)$$

- Solve using **the backward induction principle**

Solving

- Fix $q_1, \ldots, q_{N-1}$. Solve for $BR_N(q_1, ..., q_{N-1})$ (the best response of firm N)
- Fix $q_1, \ldots, q_{N-2}$. Plug BR_N for q_N. Solve for $BR_{N-1}(q_1, ..., q_{N-2})$ (the best response of firm $N-1$)

  ...

- Solve for q_1^*.
- **SPNE:** $\left\{q_1^*,\ BR_2(q_1),\ BR_3(q_1, q_2),\ ...,\ BR_N(q_1, ..., q_{N-1})\right\}$
- Calculate all other equilibrium quantities

  $$q_2^* = BR_2(q_1^*) \qquad q_3^* = BR_3(q_1^*, q_2^*) \quad \ldots \quad q_N^* = BR_N(q_1^*, \ldots, q_{N-1}^*)$$

- **Equilibrium Outcome:** $\left\{q_1^*,\ q_2^*,\ \ldots,\ q_N^*\right\}$

Comparing different models

- Homogeneous good, $N = 2$, and $C_i(q_i) = cq_i$
- Inverse demand $p(Q) = 1 - Q$ where $Q = q_1 + q_2$
- **Collusion** is a monopoly outcome split between two firms

	Competition	**Bertrand**	**Stakelberg**	**Cournot**	**Collusion**
q_1	$\frac{1-c}{2}$	$\frac{1-c}{2}$	$\frac{1-c}{2}$	$\frac{1-c}{3}$	$\frac{1-c}{4}$
q_2	$\frac{1-c}{2}$	$\frac{1-c}{2}$	$\frac{1-c}{4}$	$\frac{1-c}{3}$	$\frac{1-c}{4}$
π_1	0	0	$\frac{(1-c)^2}{8}$	$\frac{(1-c)^2}{9}$	$\frac{(1-c)^2}{8}$
π_2	0	0	$\frac{(1-c)^2}{16}$	$\frac{(1-c)^2}{9}$	$\frac{(1-c)^2}{8}$
Q	$1-c$	$1-c$	$\frac{3(1-c)}{4}$	$\frac{2(1-c)}{3}$	$\frac{1-c}{2}$
p	c	c	$\frac{1+3c}{4}$	$\frac{1+2c}{3}$	$\frac{1+c}{2}$
CS	$\frac{(1-c)^2}{2}$	$\frac{(1-c)^2}{2}$	$\frac{9(1-c)^2}{32}$	$\frac{2(1-c)^2}{9}$	$\frac{(1-c)^2}{8}$

Firms acting more than once

Often firms **compete/decide in multiple stages**. For example:

- **Hotelling Model** *(will be covered in a seminar)*
  1. First, firms simultaneously set their locations.
  2. Observe their locations and simultaneously set their prices.

- **Cournot & Bertrand/price competition with capacities**
  1. Firms simultaneously set their capacities/orders.
  2. Observe capacities and set simultaneously their prices.

- **Stackelberg Model & entry deterrence**
  1. Firm 1 sets its quantity.
  2. Firm 2 observes Firm 1's choice and decides to enter & pay entry costs/not.

Entry Deterrence

Stackelberg Model with Entry Costs

- Suppose that there are two firms $N = 2$, with the following cost functions

$$C_1(q) = 0 \qquad \text{and} \qquad C_2(q) = \begin{cases} 0 & q = 0 \\ F & q > 0 \end{cases}$$

  where $F > 0$.

- The market inverse demand is given by

$$p(Q) = 1 - Q$$

- Firm 1 chooses its quantity q_1 first.
- Firm 2 observes q_1, and then chooses whether to enter this market and how much to produce q_2.

Naive Solution

- In the standard Stackelberg model with $c = 0$, in the SPNE firms produce

$$q_1^* = \frac{1}{2} \qquad \text{and} \qquad q_2^* = \frac{1}{4}$$

- And their profits are given by

$$\pi_1^* = \frac{1}{8} \qquad \text{and} \qquad \pi_2^* = \frac{1}{16}$$

- Therefore, if $F \leq \frac{1}{16}$, the equilibrium must stay the same
- If $F > \frac{1}{16}$, then Firm 1 should act as monopolist because Firm 2 will not enter

Naive Solution

- In the standard Stackelberg model with $c = 0$, in the SPNE firms produce

$$q_1^* = \frac{1}{2} \qquad \text{and} \qquad q_2^* = \frac{1}{4}$$

- And their profits are given by

$$\pi_1^* = \frac{1}{8} \qquad \text{and} \qquad \pi_2^* = \frac{1}{16}$$

- Therefore, if $F \leq \frac{1}{16}$, the equilibrium must stay the same
- If $F > \frac{1}{16}$, then Firm 1 should act as monopolist because Firm 2 will not enter
- **THIS SOLUTION IS INCORRECT**

Correct Solution

- Now we solve this game by **backward induction**
- Firm 2's profit:

$$\pi_2(q_1, q_2) = (1 - q_1 - q_2)q_2 - F$$

- Firm 2's best response is given by

$$BR_2(q_1) = \begin{cases} \frac{1-q_1}{2} & q_1 < 1 - 2\sqrt{F} \\ 0 & q_1 \geq 1 - 2\sqrt{F} \end{cases}$$

Correct Solution

- Firm 1 has to make a choice
  - produce less then $1 - 2\sqrt{F}$ such that Firm 2 enters, or
  - produce more, and prevent Firm 2's entry
- If Firm 1 was a monopolist, it will optimally produce

$$q_1 = \frac{1}{2}$$

- Note that for

$$1 - 2\sqrt{F} \leq \frac{1}{2} \qquad \Rightarrow \qquad F \geq \frac{1}{16}$$

- Thus, when $F \geq 1/16$, Firm 1 can safely produce the monopoly output and Firm 2 will stay out

Correct Solution

- Now suppose that $F < 1/16$ (≈ 0.06)
- If Firm 1 decides to leave space for Firm 2, then it should produce like in standard Stackelberg

$$q_1 = \frac{1}{2} \qquad \pi_1 = \frac{1}{8}$$

- If Firm 1 decides to prevent entry, then it should produce more, i.e.,

$$q_1 = 1 - 2\sqrt{F} \qquad \pi_1 = 2\sqrt{F}(1 - 2\sqrt{F})$$

- Comparing profits

$$\begin{aligned} 2\sqrt{F}(1 - 2\sqrt{F}) &\geq \tfrac{1}{8} \\ F &\geq \left(\tfrac{2-\sqrt{2}}{8}\right)^2 = \hat{F} \approx 0.005 \end{aligned}$$

Stackelberg Model: SPNE and Equilibrium Outcome

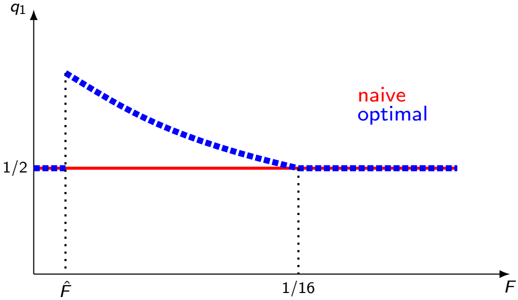
- **SPNE:** *(must be strategies at each node)*

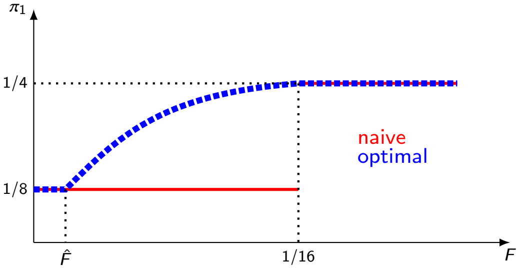
$$BR_2(q_1) = \begin{cases} \frac{1-q_1}{2} & q_1 < 1 - 2\sqrt{F} \\ 0 & q_1 \geq 1 - 2\sqrt{F} \end{cases}$$

$$q_1^* = \begin{cases} \frac{1}{2} & F < \hat{F} \\ 1 - 2\sqrt{F} & \hat{F} \leq F < 1/16 \quad \textbf{(entry deterrence)} \\ \frac{1}{2} & F \geq 1/16 \end{cases}$$

- **Equilibrium Outcome:** *(quantities actually selected)*
  - if $F < \hat{F}$, then $q_1^* = 1/2$ and $q_2^* = 1/4$
  - if $\hat{F} \leq F < 1/16$, then $q_1^* = 1 - 2\sqrt{F}$ and $q_2^* = 0$
  - if $F \geq 1/16$, then $q_1^* = 1/2$ and $q_2^* = 0$

Entry Deterrence ($\hat{F} \leq F < 1/16$)





Skill checklist for Part 2c

1. Draw a game tree of a dynamic game
2. Backward induction outcome and SPNE
3. Normal form of a dynamic game
4. Nash equilibria of dynamic games
5. Stackelberg model
   - $N = 2$
   - $N \geq 2$
   - entry deterrence

Part 2d: Collusion Games

Instructor: Oleg Baranov

Microeconomics 2 *(Module 1)*

Suggested Readings for this part

Readings in Class Textbooks:

- **Varian, 8th edition:**
  - Ch 28: Game Theory
- **Carlton and Perloff:**
  - Ch 6: Oligopoly

Collusion Game

		Player 2	
		Collude	Rip Off
Player 1	Collude	$2, 2$	$0, 4$
	Rip Off	$4, 0$	$1, 1$

Collusion Game

		Player 2	
		Collude	Rip Off
Player 1	Collude	$2, 2$	$0, 4$
	Rip Off	$4, 0$	$1, 1$

- If this game played once, we expect the *(Rip Off, Rip Off)* outcome
- What if this game is played **multiple** times?

Finitely-Repeated Games

Collusion Game with $T = 2$

Suppose that the game is played for two periods:

- First, both players simultaneously choose between *Collude* and *Rip Off*. Observe the outcome. Then play again.
- The game payoff is the **sum of payoffs** from period 1 and period 2.
- This is a **dynamic game**. We can solve it by backward induction.

Can players collude in the SPNE?

Collusion Game with $T = 2$

Suppose that the game is played for two periods:

- First, both players simultaneously choose between *Collude* and *Rip Off*. Observe the outcome. Then play again.
- The game payoff is the **sum of payoffs** from period 1 and period 2.
- This is a **dynamic game**. We can solve it by backward induction.

Can players collude in the SPNE?

- Players will **never collude** in the **second period**.
- But then there is no point in colluding in the first period.

Collusion Game with finite T

Suppose the game is played for T periods:

- First, both players simultaneously choose between *Collude* and *Rip Off*. Observe the outcome. Then play again, and so on.
- The game payoff is the **sum of payoffs** from all periods.

Can players collude in the SPNE?

- Players will never collude in **period** T.
- Then players will never collude in **period** $T - 1$.
- ...
- Players never collude in **period 1**.

If you trust **backward induction principle**, it predicts that players will **never collude**, even when $T = 100$ or $T = 1000000$.

Theory Results

- Take any static *(one-period)* game with the unique Nash equilibrium (in pure **and** mixed strategies).
- Suppose that players play this game finitely many times, observing the outcomes after each time.
- Players' payoffs is the sum *(or a discounted sum)* of payoffs from all periods.
- **Theorem:** The resulting dynamic game has the unique subgame perfect Nash equilibrium (SPNE) that consists of **all players playing the Nash equilibrium in every period**.

Infinitely-Repeated Games

Collusion Game with infinite T

Now suppose the game is played for ∞ periods:

- First, both players simultaneously choose between *Collude* and *Rip Off*. Observe the outcome. Then play again, and so on.
- We need a discount factor $\delta \in [0, 1)$. Low δ means *impatient*, and high δ means *patient*.
- The game payoff is the **discounted sum of payoffs**:

$$U_i = \sum_{t=1}^{\infty} \delta^{t-1} u_i^t$$

Can players collude in a SPNE?

Preview: It turns out that the answer is **YES** as long as δ is sufficiently close to 1 (*when players are sufficiently patient*).

Collusion Game with infinite T

Trigger Strategy

- Play *Collude* in Period 1.
- In period $t \geq 2$, play *Collude* if both players played collude in periods $1, ..., t-1$ (all previous periods)
- If somebody played *Rip Off* before (even when it was you), play *Rip Off* in t.

Intuitively, collude until everyone colludes. Once ripped off, trigger a punishment phase (playing Rip-Off) forever.

Collusion Game with infinite T

SPNE analysis:

- Playing *Rip Off* is obviously optimal in any subgame where somebody cheated before.
- How about subgames in which nobody cheated before?
- Players will continue to collude as long as

$$\sum_{t=1}^{\infty} \delta^{t-1} 2 \ \geq \ 4 \ + \ \sum_{t=2}^{\infty} \delta^{t-1} 1 \quad \Longleftrightarrow \quad \delta \ \geq \ \frac{2}{3}$$

- Thus, collusive SPNE exists when $\delta \geq 2/3$.

Infinitely-Repeated Games
Theory

Setting

- Take any static (one period) game with N players that have **at least one Nash equilibrium**.
- Pick any Nash equilibrium: strategy profile $(s_1, ..., s_N)$ and payoffs $(u_1, ..., u_N)$. We will use it as a **punishment**.
- Suppose that players play this game infinitely many times, observing the outcomes after each time. Players' payoffs is the discounted sum of payoffs from all periods.
- Pick ANY PATH through the game tree that you want players to follow. We will refer to it as collusive path.

Trigger Strategy and Folk Theorem

Trigger Strategy for player i

- if nobody deviated from the **collusive path** in periods $1, \ldots, t-1$, continue to follow the collusive path in period t
- if somebody deviated (including yourself) in periods $1, \ldots, t-1$, play s_i

Folk Theorem

The trigger strategy can be used to implement **any collusive path** as a subgame perfect Nash equilibrium provided that

- For each player i, its payoff from staying on the collusive path is strictly higher than u_i (its payoff in NE that is used for punishment); and
- the discount factor δ is close enough to 1

Class Example 1

		Player 2	
		Left	Right
Player 1	Top	$1, 2$	$5, 0$
	Bottom	$0, 6$	$4, 3$

Game Theory Paradox

- Take a finitely-repeated Collusion Game with $T = 20$ and a discount factor of $\delta = 2/3$. The game has a **unique SPNE** – *rip off* in every period.
- Now take an infinitely-repeated Collusion Game with a discount factor of $\delta = 2/3$. It has a SPNE in which players *collude* in every period.
- When looking from the perspective of the first period, 20 periods account for 99.97% of the total payoff in the infinitely-repeated game

$$\begin{aligned} \text{From 1st period:} \quad & \frac{2}{1-\delta} &=& \quad 6 \\ \text{From 21st period:} \quad & \frac{2\,\delta^{20}}{1-\delta} &=& \quad 6\left(\frac{2}{3}\right)^{20} \approx 0.002 \end{aligned}$$

- **Is there a meaningful difference between games with $T = 20$ and $T = \infty$ when you start playing them?**

Collusion in Oligopoly Games

Collusion in Bertrand Model

- Two firms, each with $C_i(q) = 0$. Market demand is given by

$$D(p) = 1 - p$$

- Firms can collude and set the monopoly price which solves:

$$\max_{p \geq 0} (1 - p)\, p \qquad \Longrightarrow \qquad p^m = \frac{1}{2}$$

- If both firms set $p_1 = p_2 = 1/2$, then they split the market equally:

$$q_1 = q_2 = \frac{1}{4} \qquad \text{and} \qquad \pi_1 = \pi_2 = \frac{1}{8}$$

- Obviously, it is not a NE: a slight reduction in price (an undercut) yields "almost" 1/4.

- **Can firms collude if they interact repeatedly?**

Bertrand Model with finite T

Suppose firms live for 2 **periods**:

- First, simultaneously choose $p^1 = (p_1^1, p_2^1)$.
- Observe p^1, earn $\pi_1(p^1)$ and $\pi_2(p^1)$.
- Second, simultaneously choose $p^2 = (p_1^2, p_2^2)$.
- Observe p^2, earn $\pi_1(p^2)$ and $\pi_2(p^2)$.
- Firm i's total profit is given by

$$\pi_i = \pi_i(p^1) + \pi_i(p^2)$$

Bertrand Model with finite T

Suppose firms live for 2 **periods**:

- First, simultaneously choose $p^1 = (p_1^1, p_2^1)$.
- Observe p^1, earn $\pi_1(p^1)$ and $\pi_2(p^1)$.
- Second, simultaneously choose $p^2 = (p_1^2, p_2^2)$.
- Observe p^2, earn $\pi_1(p^2)$ and $\pi_2(p^2)$.
- Firm i's total profit is given by

$$\pi_i = \pi_i(p^1) + \pi_i(p^2)$$

- **Can players collude in the SPNE?**

Bertrand Model with finite T

Can players collude in the SPNE?

- The static Bertrand game has a unique Nash equilibrium

$$p_1 = p_2 = 0 \qquad \Rightarrow \qquad \pi_1 = \pi_2 = 0$$

- By the theorem, the finitely-repeated Bertrand game has only one SPNE – play NE in every stage.
- Thus, firms will never collude when T is finite.

Infinitely-repeated Bertrand game

Suppose firms live for $T = \infty$ **periods**:

- At time t, observe $p^1, \ldots, p^{t-1}$ and simultaneously choose $p^t = (p_1^t, p_2^t)$
- Firm i's total profit is $\sum_{t=1}^{\infty} \delta^{t-1} \pi_i(p^t)$.

Can firms collude in SPNE?

Infinitely-repeated Bertrand game

Suppose firms live for $T = \infty$ periods:

- At time t, observe $p^1, \ldots, p^{t-1}$ and simultaneously choose $p^t = (p_1^t, p_2^t)$
- Firm i's total profit is $\sum_{t=1}^{\infty} \delta^{t-1} \pi_i(p^t)$.

Can firms collude in SPNE? Yes, we can use a trigger strategy to support collusion.

Infinitely-repeated Bertrand game

One possible collusive path:

$$\left(\frac{1}{2},\ \frac{1}{2}\right),\ \left(\frac{1}{2},\ \frac{1}{2}\right),\ \left(\frac{1}{2},\ \frac{1}{2}\right),\ \dots$$

Supporting Trigger Strategy for players $i = 1, 2$:

- Play $p_i = 1/2$ in Period 1.
- In period $t \geq 2$, play $p_i = 1/2$ if both players played $p = 1/2$ in periods $1, \dots, t-1$ (all previous periods)
- If somebody played $p \neq 1/2$ before, play $p_i = 0$ in t.

Infinitely-repeated Bertrand game

SPNE analysis:

- Playing $p_i = 0$ is obviously optimal in any subgame where somebody already deviated from colluding.
- How about subgames in which nobody deviated from collusion?
- Players will continue to collude as long as

$$\sum_{t=1}^{\infty} \delta^{t-1} \frac{1}{8} \;\geq\; \frac{1}{4} + \sum_{t=2}^{\infty} \delta^{t-1}\, 0 \qquad \Longleftrightarrow \qquad \delta \;\geq\; \frac{1}{2}$$

- Thus, collusive SPNE exists in this game for $\delta \geq 1/2$.

Collusion in Cournot Model

- Two firms, each with $C_i(q) = 0$. The inverse market demand is given by
$$P(Q) = 1 - Q$$
- Firms can collude and set the monopoly quantity which solves:
$$\max_{q \geq 0} (1 - q)\, q \qquad \Rightarrow \qquad q^m = \frac{1}{2}$$
- If firms set $q_1 = q_2 = 1/4$, they split the market equally:
$$q_1 = q_2 = \frac{1}{4} \qquad \text{and} \qquad \pi_1 = \pi_2 = \frac{1}{8}$$
- Obviously, it is not a NE. If $q_2 = 1/4$, the best deviation for Firm 1 is given by its best response
$$q_1^d = BR_1(q_2) = \frac{1 - q_2}{2} = \frac{3}{8} \qquad \Rightarrow \qquad \pi_1^d = \frac{9}{64} > \frac{1}{8}$$

Cournot Model with finite T

Can players collude in the SPNE?

- The static Cournot game has a unique Nash equilibrium

$$q_1 = q_2 = \frac{1}{3} \qquad \Rightarrow \qquad \pi_1 = \pi_2 = \frac{1}{9}$$

- By the theorem, the finitely-repeated Cournot game has only one SPNE – play NE in every stage.
- Thus, firms will never collude when T is finite.

Infinitely-repeated Cournot Model

Can firms collude in SPNE? Yes, just use a trigger strategy

One possible collusive path:

$$\left(\frac{1}{4},\ \frac{1}{4}\right),\ \left(\frac{1}{4},\ \frac{1}{4}\right),\ \left(\frac{1}{4},\ \frac{1}{4}\right),\ \ldots$$

Supporting Trigger Strategy for player $i = 1, 2$:

- Play $q_i = 1/4$ in period 1
- In period $t \geq 2$, play $q_i = 1/4$ if both players played $1/4$ in periods $1, \ldots, t-1$ (all previous periods)
- If somebody played $q \neq 1/4$ before (even when it was you), play $q_i = 1/3$ in t.

Infinitely-repeated Cournot game

SPNE analysis:

- Playing $q_i = 1/3$ is obviously optimal in any subgame where somebody deviated from colluding before.
- How about subgames in which nobody deviated from collusion?
- Players will continue to collude as long as

$$\sum_{t=1}^{\infty} \delta^{t-1} \frac{1}{8} \geq \frac{9}{64} + \sum_{t=2}^{\infty} \delta^{t-1} \frac{1}{9} \quad \Longleftrightarrow \quad \delta \geq \frac{9}{17} \approx 0.53$$

- Thus, collusive SPNE exists in this game when $\delta \geq 9/17$.

Skill checklist for Part 2d

1. Finitely-repeated games
2. Infinitely-repeated games
3. Deriving the lower bound for the discount factor