CFRM 541 Homework 6

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Problem #1

Verify that solution of the estimating equations (8.6) and (8.7) gives the LS estimators (8.8) and (8.9).

Equations (8.6) and (8.7) are:

(8.6) $\Sigma_{t=1}^{T}(r_t - \alpha - \beta r_{f,t}) = 0$

(8.7) $\Sigma_{t=1}^{T} r_{f,t}(r_t - \alpha - \beta r_{f,t}) = 0$.

(8.8) and (8.9) are expressions for α and β deducible from (8.6) and (8.7):

(8.8) $\beta = \frac{\Sigma_{t=1}^{T}(r_t - \bar{r})(r_{f,t} - \bar{r}_f)}{\Sigma_{t=1}^{T}(r_{f,t} - \bar{r}_f)^2}$.

(8.9) $\alpha = \bar{r} - \beta \bar{r}_f$.

Since (8.9) expresses α in terms of β, let us begin by solving (8.6) for α:

$$
\begin{aligned}
\Sigma_{t=1}^{T}(r_t - \alpha - \beta r_{f,t}) &= 0 && (1)\\
\Sigma_{t=1}^{T}(r_t - \beta r_{f,t}) - T\alpha &= 0 && (2)\\
T\alpha &= \Sigma_{t=1}^{T}(r_t - \beta r_{f,t}) && (3)\\
\alpha &= \frac{\Sigma_{t=1}^{T}(r_t - \beta r_{f,t})}{T} && (4)\\
&= \frac{\Sigma_{t=1}^{T} r_t}{T} - \frac{\Sigma_{t=1}^{T} \beta r_{f,t}}{T} && (5)\\
&= \bar{r} - \beta \bar{r}_f. && (6)
\end{aligned}
$$

To find an expression for β, we first multiply (8.6) by $\bar{r}_f$, and then subtract the result from (8.7):

$$
\begin{aligned}
\bar{r}_f \Sigma_{t=1}^{T}(r_t - \alpha - \beta r_{f,t}) &= 0 && (7)\\
\Sigma_{t=1}^{T} \bar{r}_f (r_t - \alpha - \beta r_{f,t}) &= 0 && (8)
\end{aligned}
$$

Now:

$$
\begin{aligned}
\Sigma_{t=1}^{T} r_{f,t}(r_t - \alpha - \beta r_{f,t}) - \Sigma_{t=1}^{T} \bar{r}_f (r_t - \alpha - \beta r_{f,t}) &= 0 - 0 = 0 && (9)\\
\Sigma_{t=1}^{T}(r_{f,t} - \bar{r}_f)(r_t - \alpha - \beta r_{f,t}) &= 0. && (10)
\end{aligned}
$$

Now we plug in our result for α:

$$
\begin{aligned}
\Sigma_{t=1}^T (r_{f,t} - \bar{r}_f)(r_t - (\bar{r} - \beta\bar{r}_f) - \beta r_{f,t}) = 0 && (11)\\
\Sigma_{t=1}^T (r_{f,t} - \bar{r}_f)(r_t - \bar{r}) &= \beta \cdot \Sigma_{t=1}^T (r_{f,t} - \bar{r}_f)(r_{f,t} - \bar{r}_f) && (12)\\
\beta &= \frac{\Sigma_{t=1}^T (r_{f,t} - \bar{r}_f)(r_t - \bar{r})}{\Sigma_{t=1}^T (r_{f,t} - \bar{r}_f)^2} && (13)\\
&= \frac{\Sigma_{t=1}^T (r_t - \bar{r})(r_{f,t} - \bar{r}_f)}{\Sigma_{t=1}^T (r_{f,t} - \bar{r}_f)^2} && (14)
\end{aligned}
$$

Problem #2

Write down the mathematical form of an up and down market factor model with error term.

A single-factor up model would look like:

$r_{i,t} - r_{f,t} = \alpha_{up} + \beta_{up}(r_{M,t} - r_{f,t}) + \epsilon_{up}$.

(And a single-factor down model would look similar.) What we want is to have a model that makes use of α_{up} and β_{up} when the market is up, but that makes use of α_{dn} and β_{dn} when the market is down.

It won't do to write: $r_{i,t} - r_{f,t} = \alpha_{up} + \beta_{up}(r_{M,t} - r_{f,t}) + \epsilon_{up} + \alpha_{dn} + \beta_{dn}(r_{M,t} - r_{f,t}) + \epsilon_{dn}$, since this would improperly use all four parameters regardless of the state of the market. However, we can "silence" the relevant addend by making use of the indicator function $I(x)$, where

$$
I(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}
$$

We write:

$r_{i,t} - r_{f,t} = I(r_{M,t} - r_{f,t})(\alpha_{up} + \beta_{up}(r_{M,t} - r_{f,t})) + I(r_{f,t} - r_{M,t})(\alpha_{dn} + \beta_{dn}(r_{M,t} - r_{f,t})) + \epsilon_t$.

Problem #3

Use the 'plot' function with the optional argument 'SFM.line=T' on the results of using 'fitTsfm' to fit up and down market fits for the hedge fund managers HAM1, HAM2, HAM3, HAM4, HAM5, HAM6. Report your results for the manager whose up and down market betas in your judgment differ the most from the usual LS straight line fit.

```r
library(mpo)
```

```
## Loading required package: boot

## Loading required package: xts

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: lattice

##
## Attaching package: 'lattice'
```

```
## The following object is masked from 'package:boot':
##
##     melanoma

## Loading required package: mvtnorm

## Loading required package: rrcov

## Loading required package: robustbase

##
## Attaching package: 'robustbase'

## The following object is masked from 'package:boot':
##
##     salinity

## Scalable Robust Estimators with High Breakdown Point (version 1.4-3)

## Loading required package: robust

## Loading required package: fit.models

## Loading required package: MASS

## Loading required package: PortfolioAnalytics

## Loading required package: foreach

## Loading required package: PerformanceAnalytics

##
## Attaching package: 'PerformanceAnalytics'

## The following object is masked from 'package:graphics':
##
##     legend

## Loading required package: corpcor

## Loading required package: shiny

##
## Attaching package: 'mpo'

## The following object is masked from 'package:shiny':
##
##     runExample

## The following objects are masked from 'package:PerformanceAnalytics':
##
##     SFM.beta, StdDev.annualized
```

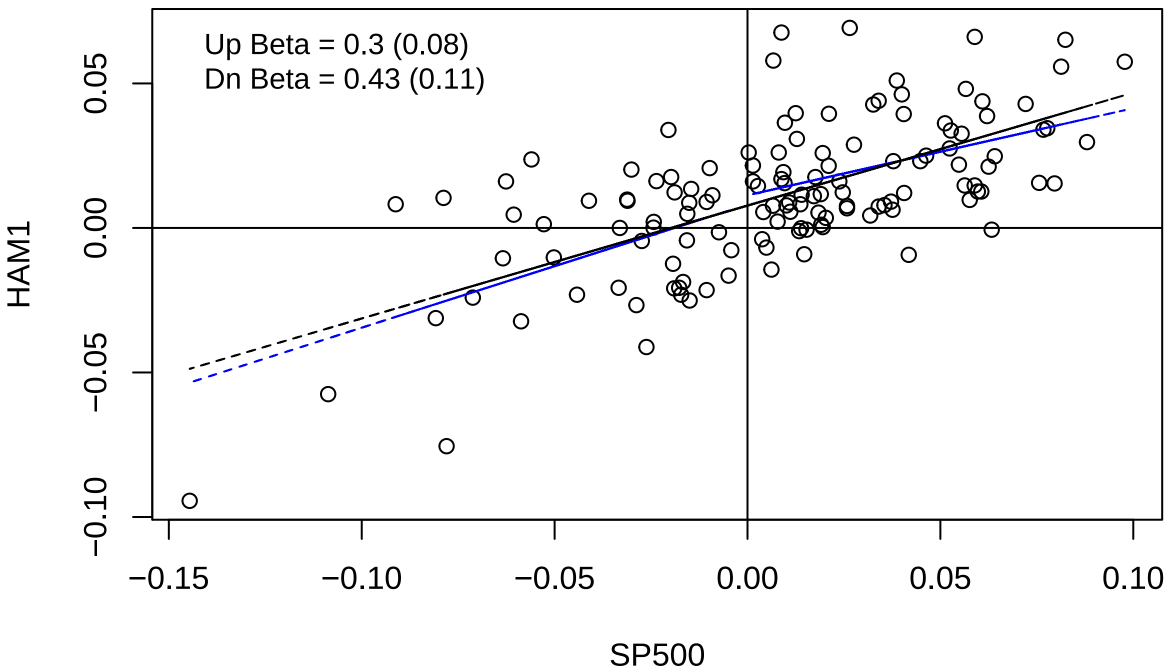
```r
library(robust)
library(factorAnalytics)

# Returns of the following: 6 hedge fund managers returns,
# a long-short equity hedge fund style index, sp500, long-term
# U.S. treasury 10 year bond, 3 month U.S treasury bill
data(managers)
names(managers)
```

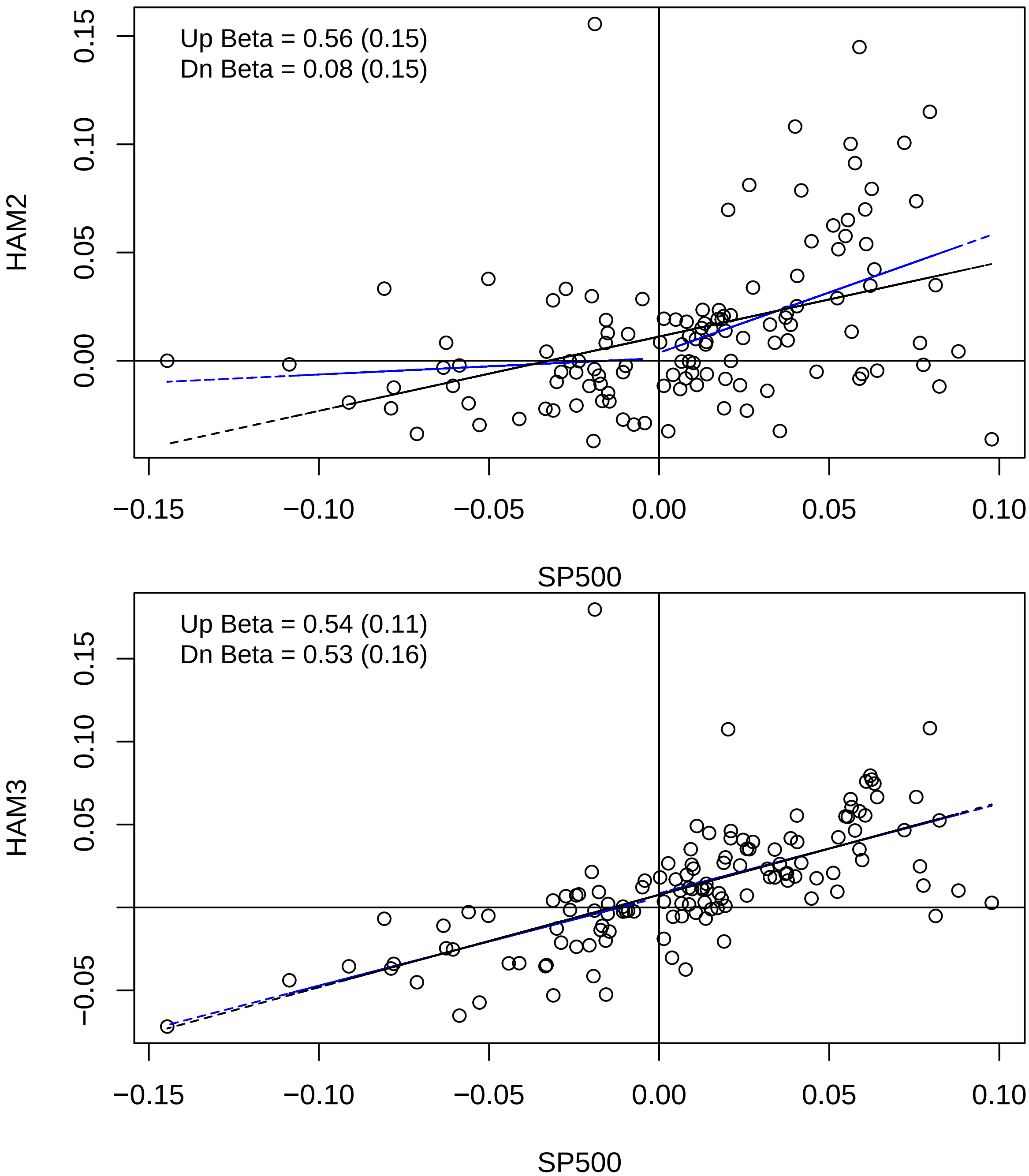
```
##  [1] "HAM1"        "HAM2"        "HAM3"        "HAM4"        "HAM5"
##  [6] "HAM6"        "EDHEC.LS.EQ" "SP500.TR"    "US.10Y.TR"   "US.3m.TR"
```

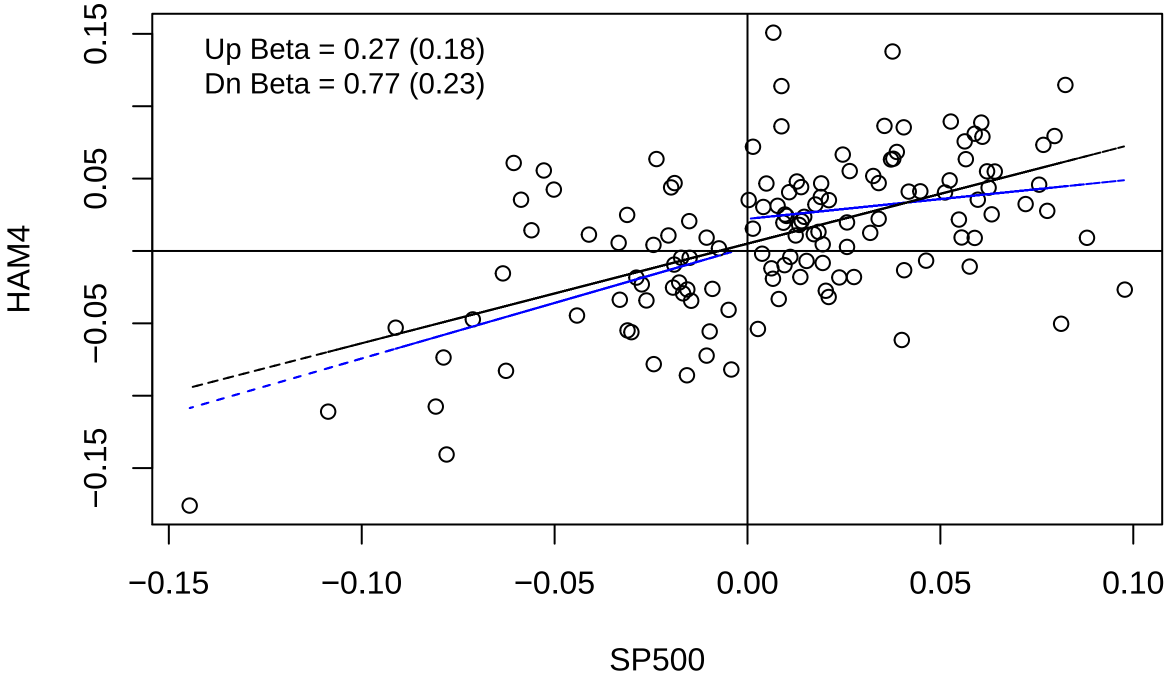
```r
# Renaming last four with somewhat shorter names
names(managers)[7:10] = c("LS.EQ.HF","SP500","US.10Y","UW.3M")
names(managers)
```

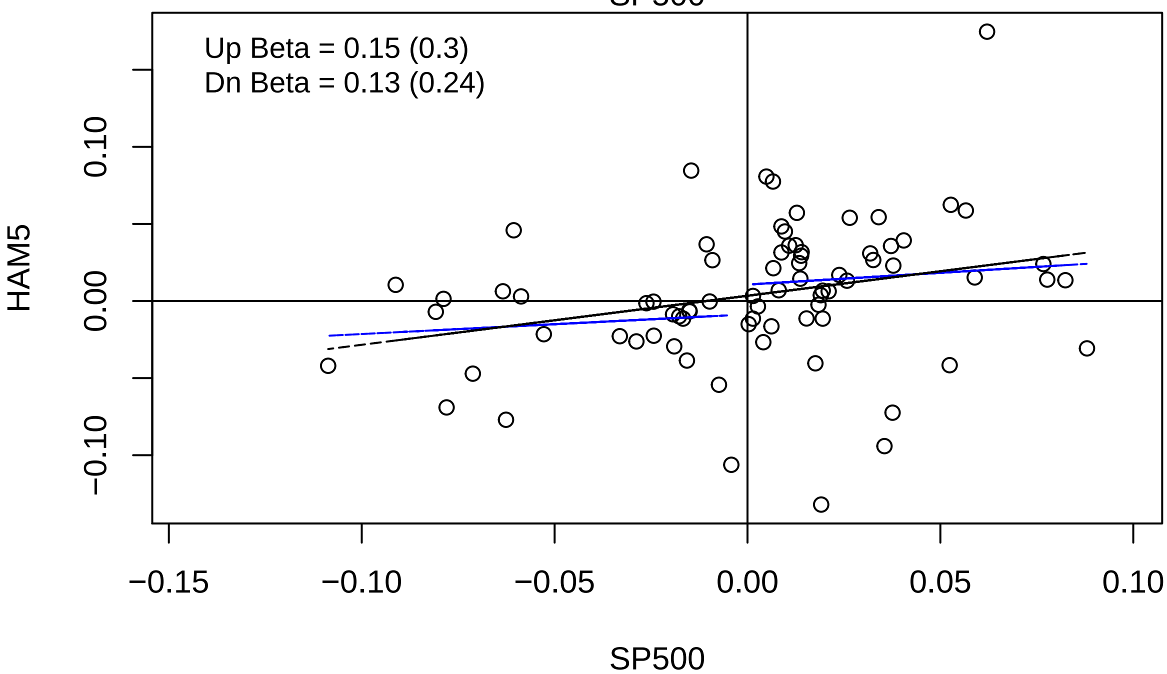
```
## [1] "HAM1"     "HAM2"     "HAM3"     "HAM4"     "HAM5"     "HAM6"
## [7] "LS.EQ.HF" "SP500"    "US.10Y"   "UW.3M"
```

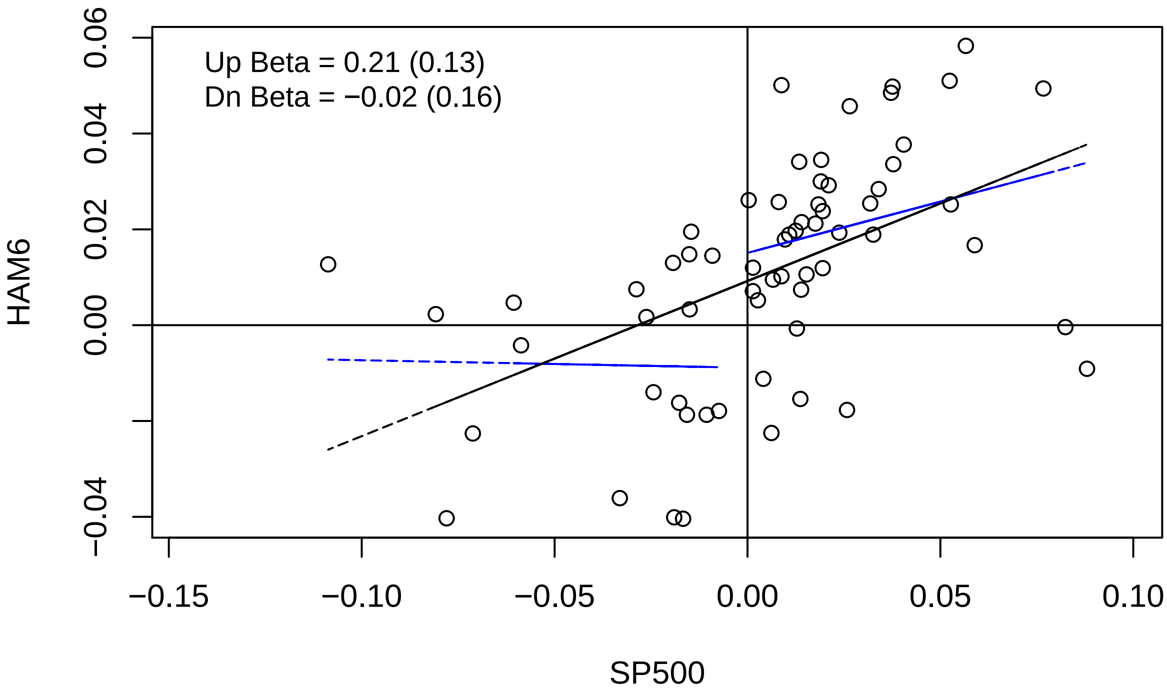
```r
# Select asset returns and fit up and down betas for each of HAM 1-6
for (j in 1:6){
asset = colnames(managers[,j])
fitUpDn <- fitTsfmUpDn(asset,mkt.name="SP500",data=managers)
plot(fitUpDn, SFM.line = T)
summary(fitUpDn)
}
```




Up Beta = 0.56 (0.15)
Dn Beta = 0.08 (0.15)
HAM2
0.15
0.10
0.05
0.00
−0.15
−0.10
−0.05
0.00
0.05
0.10
SP500
Up Beta = 0.54 (0.11)
Dn Beta = 0.53 (0.16)
HAM3
0.15
0.10
0.05
−0.05
−0.15
−0.10
−0.05
0.00
0.05
0.10
SP500


Up Beta = 0.27 (0.18)
Dn Beta = 0.77 (0.23)
HAM4
SP500


Up Beta = 0.15 (0.3)
Dn Beta = 0.13 (0.24)
HAM5
SP500



The sixth HAM seems to have up- and down-betas differing the most from the LS line.

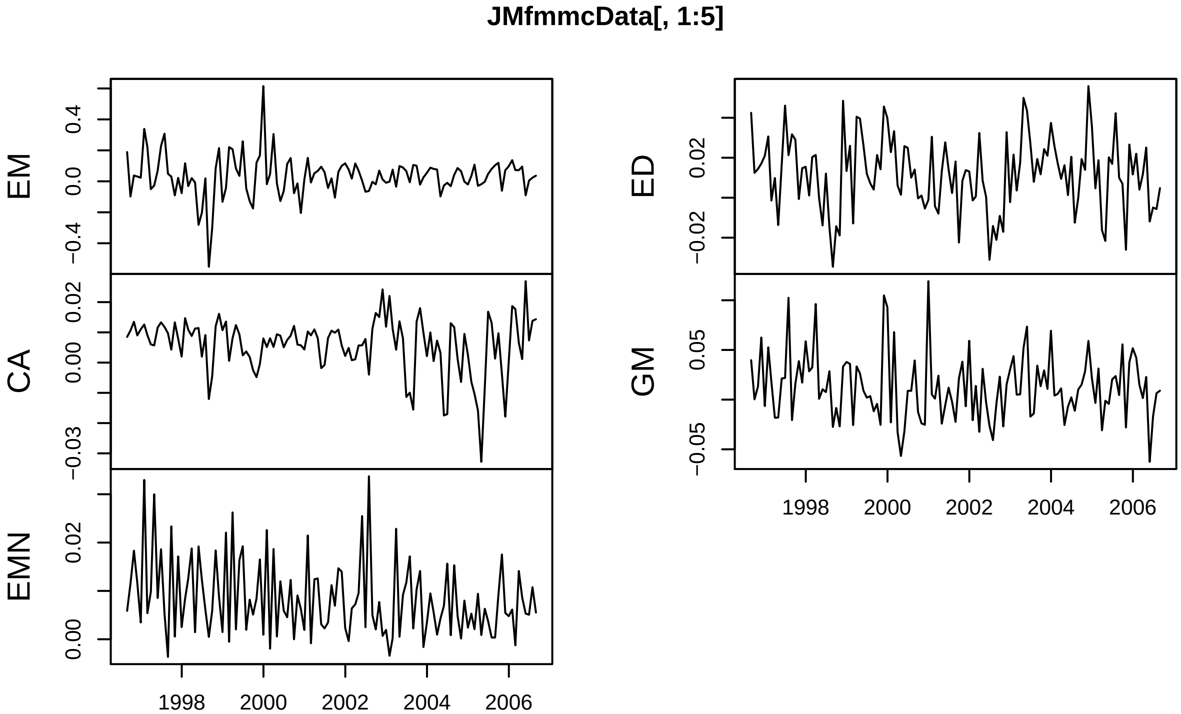
Problem #4

It is natural to conjecture that if we searched over all four single factor models with each of the hedge fund style indices in turn, we would probably have ended up with CST.EM being the best choice of a single factor. Confirm whether or not this is true by fitting single factor models using each of the hedge fund style indexes separately.

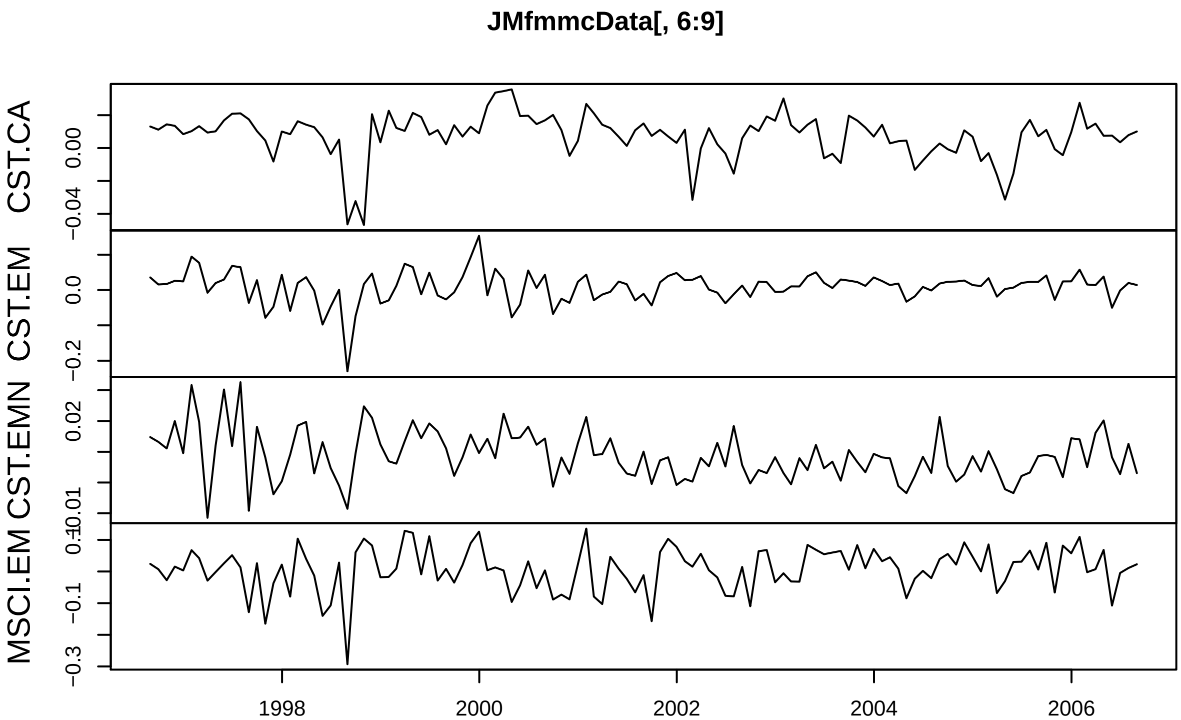
```
names(JMfmmcData)
```

```
## [1] "EM"       "CA"       "EMN"      "ED"       "GM"       "CST.CA"   "CST.EM"
## [8] "CST.EMN" "MSCI.EM"
```

```
plot.zoo(JMfmmcData[,1:5],xlab="",ylab = names(JMfmmcData[,1:5]))
```




```
plot.zoo(JMfmmcData[,6:9],xlab="",ylab = names(JMfmmcData[,6:9]))
```




```
# Fit EM hedge fund returns returns to each index (columns 6-9)
retHF = JMfmmcData
for (j in 6:9){
fac.names = names(retHF)[j]
fit.1fac = fitTsfm("EM",fac.names,data = retHF)
fitTsfmStats(fit.1fac,rsq = T)
```

```
}
```

```
##              Estimate  S.E. t-Stat p-Value
## (Intercept)    0.008 0.013  0.602   0.548
## CST.CA         3.388 0.810  4.184   0.000 ***
##
## R-Squared = 0.128
##              Estimate  S.E. t-Stat p-Value
## (Intercept)    0.012 0.007  1.692   0.093 .
## CST.EM         2.433 0.157 15.473   0.000 ***
##
## R-Squared = 0.668
##              Estimate  S.E. t-Stat p-Value
## (Intercept)   -0.020 0.016 -1.234    0.22
## CST.EMN        6.153 1.386  4.439    0.00 ***
##
## R-Squared = 0.142
##              Estimate  S.E. t-Stat p-Value
## (Intercept)    0.026 0.009  2.748   0.007 **
## MSCI.EM        1.154 0.135  8.562   0.000 ***
##
## R-Squared = 0.381
```

Because the fit to the CST.EM model has the highest R^2-term of the four models, this does indeed seem to be the best choice of a single factor.

Problem #5

Make LS fits of the EM hedge fund returns using the four hedge fund style indices for the entire EM returns series. Then do likewise for each of the following two time periods representing visually distinct regimes: 1996 through 2001, and 2002 through 2006. What do you conclude about the differences, if any, between your four factor model fits for each of those two regimes, and the differences between the results for each of the two regimes relative to the results for the entire returns history?

```
# Time periods
RETHFearly = window(retHF, start = "1996-08-31", end = "2001-12-31")
RETHFlate = window(retHF, start = "2002-01-31", end = "2006-08-31")

# LS fit of EM hedge fund returns to all 4 style indexes
fac.names = names(retHF)[6:9]
fit.4fac = fitTsfm("EM",fac.names,data = retHF)
fitTsfmStats(fit.4fac,rsq = T)
```

```
##              Estimate  S.E. t-Stat p-Value
## (Intercept)   -0.003 0.010 -0.288   0.774
## CST.CA         0.766 0.541  1.416   0.160
## CST.EM         2.606 0.286  9.119   0.000 ***
## CST.EMN        1.040 0.945  1.101   0.273
## MSCI.EM       -0.249 0.173 -1.440   0.152
##
## R-Squared = 0.687
```

```
# LS fit of EM hedge fund early period returns to all 4 style indexes
fac.names = names(retHF)[6:9]
```

```r
fit.4fac = fitTsfm("EM",fac.names,data = RETHFearly)
fitTsfmStats(fit.4fac,rsq = T)
```

```
##             Estimate  S.E. t-Stat p-Value
## (Intercept)   -0.007 0.022 -0.301   0.765
## CST.CA         0.908 0.990  0.917   0.363
## CST.EM         2.596 0.434  5.980   0.000 ***
## CST.EMN        1.406 1.615  0.871   0.387
## MSCI.EM       -0.278 0.293 -0.946   0.348
##
## R-Squared = 0.686
```

```r
# LS fit of EM hedge fund late period returns to all 4 style indexes
fac.names = names(retHF)[6:9]
fit.4fac = fitTsfm("EM",fac.names,data = RETHFlate)
fitTsfmStats(fit.4fac,rsq = T)
```

```
##             Estimate  S.E. t-Stat p-Value
## (Intercept)    0.002 0.007  0.294   0.770
## CST.CA         0.482 0.370  1.302   0.199
## CST.EM         2.540 0.435  5.837   0.000 ***
## CST.EMN       -0.043 0.817 -0.052   0.958
## MSCI.EM       -0.151 0.174 -0.873   0.387
##
## R-Squared = 0.716
```

The late-period return yields the higher R^2 value, so this seems to be the better fit.

The R^2 value for the whole period is slightly larger than the R^2 value for the early period (since the fit for the later period is significantly better). In all three tests it is only the CST.EM model that has a statistically (very) significant value, which is hardly surprising.