Pointed Hopf actions on quantum generalized Weyl algebras

Jason Gaddis

Workshop on Noncommutative Geometry and Noncommutative Invariant Theory
Banff International Research Station, Banff, Canada

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(Joint work with Robert Won)

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is $\mathbb{Z}$-graded (set $\deg(x) = 1$ and $\deg(y) = -1$) but exhibits no finite-dimensional quantum symmetry (Cuadra-Etingof-Walton).

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Our interest is in actions on generalized Weyl algebras (GWAs) over a polynomial ring in one variable. These algebras are known to be twisted Calabi-Yau (Liu).

So, this is a natural extension of the problem of studying Hopf actions on connected $\mathbb{N}$-graded twisted Calabi-Yau algebras (i.e., Artin-Schelter regular algebras).

Quantum GWAs

Definition

Let $q \in \Bbbk^\times$ and let $h(t) \in \Bbbk[t]$ be non-constant. The corresponding *quantum generalized Weyl algebra* is

$$\Bbbk[t](u, v, q, h) = \Bbbk\langle u, v, t \mid ut - qtu, vt - q^{-1}tv, vu = h(t), uv = h(qt)\rangle.$$

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- Setting $h = t - 1$, we obtain the *quantum Weyl algebras:*

$$A_1^q(\mathbb{k}) = \mathbb{k}\langle u, v \mid uv - qvu - 1\rangle$$

(Generalized) Taft algebras

Definition

Let $m, n \in \mathbb{N}$ such that $m > 1$ and $m \mid n$, and let $\lambda \in \Bbbk$ be a primitive m^{th} root of unity. The *generalized Taft algebra* corresponding to this data is

$$T_n(\lambda, m) := \Bbbk\langle x, g \mid g^n - 1, x^m, gx - \lambda xg \rangle.$$

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The actions we consider here are generally distinct from those studied above.

Weakly $\mathbb{Z}$-graded actions

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The weakly $\mathbb{Z}$-graded setting captures group actions that preserve the $\mathbb{Z}$-grading of A up to the automorphism of $\mathbb{Z}$ which sends 1 to -1.

Quantum thickenings

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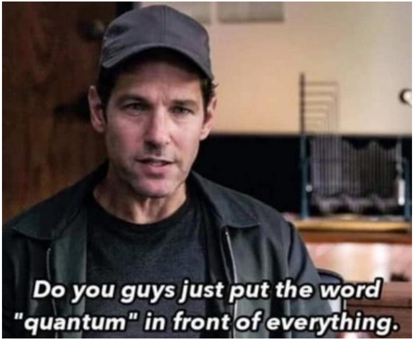

Automorphisms of quantum GWAs (Suárez-Alvarez and Vivas)

Let $A = \mathbb{k}[t](u, v, q, h)$. Write $h = \sum h_i t^i$ and let $\ell = \gcd\{i - j \mid h_i h_j \neq 0\}$. Set

$$C_\ell = \begin{cases} \mathbb{k}^\times & \text{if } h \text{ is a monomial} \\ \{\ell^{\text{th}} \text{ roots of unity}\} & \text{otherwise.} \end{cases}$$

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For $(\gamma, \mu) \in C_\ell \times \mathbb{k}^\times$, define $\eta_{\gamma,\mu} \in \mathrm{Aut}(A)$ by

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If $q = -1$, then there is an order 2 automorphism Ω defined by

$$\Omega(t) = -t, \quad \Omega(v) = u, \quad \Omega(u) = v.$$

In this case, every automorphism of A is either some $\eta_{\gamma,\mu}$ or else $\Omega \circ \eta_{\gamma,\mu}$.

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So, every automorphism of a quantum GWA is weakly $\mathbb{Z}$-graded. But, when $q \neq -1$, every automorphism is actually $\mathbb{Z}$-graded.

Actions on the polynomial ring in one variable

For our main result, it was necessary to first study actions of generalized Taft actions on the polynomial base ring $\mathbb{k}[t]$.

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Let $T = T_n(\lambda, m)$. Let $\gamma \in \Bbbk \setminus \{0, 1\}$ and $0 \neq \phi \in \Bbbk[t]$ with $\deg_t(\phi) = d$.

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I. If $\Bbbk[t]$ is a T-module algebra with $g(t) = \gamma t$ and $x(t) = \phi$, then

(1) *γ is a primitive m^{th} root of unity,*

(2) *$\lambda = \gamma^{d-1}$ and $\gcd(d-1, m) = 1$, and*

(3) $\mathrm{supp}(\phi) \subseteq \{d, d-m, d-2m, \ldots\}$.

Furthermore, the action is inner-faithful if and only if $m = n$.

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II. Conversely, if γ and ϕ satisfy the conditions (1)—(3), then there is a unique T-module algebra structure on $\Bbbk[t]$ such that $g(t) = \gamma t$ and $x(t) = \phi$.

Main results

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(A) There is an inner-faithful weakly $\mathbb{Z}$*-graded* T*-module algebra structure on* A *if and only if*

1. $\mathrm{supp}(h)$ *is contained in a single congruence class modulo* m*, and*
2. *there exists an integer* k *coprime to* m *such that* $\mathrm{lcm}(m, \mathrm{ord}(q^k)) = n$.

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(B) Assuming the conditions in (A) are satisfied, the inner-faithful weakly $\mathbb{Z}$-graded T-module algebra structures on A are parametrized by $\gamma, \mu \in \Bbbk^\times$ and $\phi(t) \in \Bbbk[t]$ of degree d such that

1. $\operatorname{ord}(\gamma) = m$ *and* $\lambda = \gamma^{d-1}$,
2. $\operatorname{lcm}(m, \operatorname{ord}(\mu)) = n$,
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These conditions guarantee an action even in the case of $q = -1$. However, when $q = -1$, there may be additional T-actions.

Main results

We can also frame our results in terms of quantum thickenings.

Theorem

Let $A = \Bbbk[t](u, v, q, h)$ with $q^2 \neq 1$. Let $G = \langle \eta_{\gamma,\mu} \rangle$ be a cyclic subgroup of $\mathrm{Aut}(A)$ *of order n. Let $m = \mathrm{ord}(\gamma)$ so $m \mid n$.*

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(A) The action of G is the restriction of the action to the group of group-likes of an inner-faithful weakly $\mathbb{Z}$-graded $T_n(\lambda, m)$-module algebra action if and only if there exists an integer k coprime to m such that μq^k is an m^{th} root of unity.

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(B) The actions of each $T_n(\lambda, m)$ *whose group-like elements restrict to the action of* G *are parameterized by nonzero polynomials* $\phi(t) \in \Bbbk[t]$ *of degree* d *such that*

1. $\gcd(d - 1, m) = 1$*, and*
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Invariants

For a Hopf algebra H and an H-module algebra A, the *fixed ring of A by H* is

$$A^H = \{a \in H \mid h(a) = \epsilon(h)a \text{ for all } h \in H\}.$$

Theorem

Let $A = \Bbbk[t](u, v, q, h)$ with q a root of unity, $q \neq 1$, and let $T = T_n(\lambda, m)$. Suppose that A is an inner-faithful weakly $\mathbb{Z}$-graded T-module algebra where g acts as $\eta_{\gamma,\mu} \in \mathrm{Aut}(A)$ with $\gamma \neq 1$.

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Thank You!