Math 252 Sec. A

Reading Assignment #1 (Sections 12.2-12.5)
Due Friday, Feb 1 at the beginning of class

This assignment should be completed on separate paper. Write neatly and organize your work using looseleaf paper and nor pages torn from a notebook. Staple multiple pages together. Write your name and assignment in the top right-hand corner of the page. Answers should appear in the order below.

1. Use the definition of the dot product from page 807 to compute the dot product of the vectors $\langle 3, 1, -4\rangle$ and $\langle 0, 2, -3\rangle$.
2. On page 808, the author works out the proof of properties 1 and 3 for dot products. Using these as a guide, work out property 4 on your own.
3. In Linear Algebra, one learns that a matrix is an array of numbers. Consider the two matrices of size $n \times n$,
$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}.$$
The product of these two matrices is again an $n \times n$ matrix. The entry c_{ij} in row i and column j of the product AB is given by the formula
$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}.$$
Restate this formula in terms of dot products.
4. Explain the geometric meaning of Theorem 3 on page 808 in your own words.
5. Explain in your own words the formulas for scalar and vector projections from page 811.
6. Using either the definition on page 815, or the method of determinants, compute the cross product of the vectors $\langle 3, 1, -4\rangle$ and $\langle 0, 2, -3\rangle$.
7. Using the proof of Theorem 8 (page 816) as a guide, verify that $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{b}$.
8. Give a justification for each step of the proof of Theorem 9 (page 817).
9. Explain how the three different equations of a line in three-dimensional space are related. Choose any two distinct points in three-dimensional space and write the equation of the line through those points in each of the three forms.
10. Explain in your own words the definition of a normal vector. Using Example 8 on page 827 to find the equation of the plane through the point $(1, 1, 2)$ with normal vector $\mathbf{n} = \langle 2, -1, 5\rangle$.

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Reading Assignment #2 (Sections 13.1-13.2)
Due Friday, Feb 8 at the beginning of class

This assignment should be completed on separate paper. Write neatly and organize your work using looseleaf paper and nor pages torn from a notebook. Staple multiple pages together. Write your name and assignment in the top right-hand corner of the page. Answers should appear in the order below.

1. In your own words, define a space curve.
2. Identify the component functions and domain of $\mathbf{r}(t) = \langle \ln(2+t), t/4, t^2 \rangle$.
3. Using Formula 1 on page 848, find the limit of $\mathbf{r}(t)$ as $t \to 0$, where
$$\mathbf{r}(t) = (e^{-t})i + (3 - t^2)j + (t\ln(t))k.$$
4. Find a vector equation and parametric equations for the line segment that joins the point $P(4, 1, -3)$ to the point $Q(2, 2, -1)$.
5. What is the significance of the tangent vector? That is, what does a the tangent vector of $\mathbf{r}(t)$ at a point $t = a$ tell us about the space curve defined by $\mathbf{r}(t)$?
6. Use Theorem 2 on page 856 to find the derivative of $\mathbf{r}(t) = (t^2e^t)i + \cos(t)j + (2+t)k$. Use Example 1 on page 857 as a guide.
7. The author works out the proof of property 4 of Theorem 3 (page 858). Using it as a guide, work out property 2 on your own.
8. The definite integral of a vector function is defined on page 859. What information does this tell us about the space curve? It might help to think about what the integral tells us for a curve in $\mathbb{R}^2$.

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Reading Assignment #2 (Sections 7.2, 7.3)

Due Monday, February 5 at the beginning of class

This assignment should be completed on separate paper. Write neatly and organize your work using looseleaf paper and nor pages torn from a notebook. Staple multiple pages together. Write your name and assignment in the top right-hand corner of the page. Answers should appear in the order below.

1. State the Pythagorean identities for $\sin x, \cos x$ and for $\tan x, \sec x$.
2. Follow Example 1 in Section 7.2 to evaluate $\int \sin^3 x \; dx$.
3. In your own words, explain the strategy for evaluating $\int \sin^m x \; \cos^n x \; dx$.
4. In your own words, explain the strategy for evaluating $\int \tan^m x \; \sec^n x \; dx$.
5. Explain, generally, Trig(onometric) Substitution in your own words. Copy down the rules from the Table in Section 7.3.
6. Explain why we cannot evaluate Example 3 in Section 7.3 using other techniques (such as substitution).
7. Follow Example 3 to evaluate
$$\int \frac{1}{x^2\sqrt{x^2+9}} \; dx.$$
8. Work Example 4 using trig substitution.

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Reading Assignment #3 (Sections 13.3-13.4)
Due Friday, Feb 15 at the beginning of class

This assignment should be completed on separate paper. Write neatly and organize your work using looseleaf paper and nor pages torn from a notebook. Staple multiple pages together. Write your name and assignment in the top right-hand corner of the page. Answers should appear in the order below.

1. Consider the helix with vector equation $\mathbf{r}(t) = a\cos(t)\mathbf{i} + a\sin(t)\mathbf{j} + bt\mathbf{k}$. Find the length of the arc from $(1, 0, 0)$ to $(1, 0, 2\pi)$. Here, a and b represent arbitrary nonzero real numbers. See Example 1 on page 862.

2. Reparameterize the helix from Question 1 with respect to arc length measured from $(1, 0, 0)$ in the direction of increasing t. See Example 2 on page 863.

3. In your own words, define curvature (see page 863).

4. Use Theorem 10 on page 864 to find the curvature of

$$\mathbf{r}(t) = \left\langle (1/\sqrt{2})\cos(t), (1/\sqrt{2})\cos(t), \sin(t) \right\rangle$$

   at $(0, 0, 1)$. See Example 4 in section 13.3.

5. In your own words, define/explain the normal plane, osculating plane, and osculating circle. See page 867.

6. Using the definitions in the text (see page 867 for a summary), show that

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}.$$

   It may help to look Formula 7 on page 863 and the argument for Formula 9 on page 864.

7. The position vector of an object moving in space is given by $\mathbf{r}(t) = (4t^2)\mathbf{i} + (t^3+1)\mathbf{j} + (te^t)\mathbf{k}$. Find the velocity, acceleration, and speed of the particle when $t = 2$. See Examples 1 and 2 on page 871.

8. A moving particle has initial position $\mathbf{r}(0) = \langle 1, 0, 0 \rangle$ and initial velocity $\mathbf{v}(0) = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$. Its acceleration is $\mathbf{a}(t) = (3t^2)\mathbf{i} + \mathbf{j} + 6t\mathbf{k}$. Find its velocity at time t. See Example 3 on page 871.

9. Read over the section on Kepler's Laws of Planetary Motion on page 875. Write down one step in the computations that confused you and why.

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Reading Assignment #4 (Sections 14.1-14.3)
Due Friday, March 1 at the beginning of class

This assignment should be completed on separate paper. Write neatly and organize your work using looseleaf paper and nor pages torn from a notebook. Staple multiple pages together. Write your name and assignment in the top right-hand corner of the page. Answers should appear in the order below.

1. Create your own function of two variables. Look up data to create a chart like Table 1 in Section 14.1 (page 889). Be sure to cite your source(s). Evaluate your function at three different points using your chart.
2. Sketch the domain of $f(x,y) = \frac{\sqrt{y-x^2}}{1-x^2}$. See Example 1 in Section 14.1 (page 888).
3. Explain in your own words the concept of a limit of a function of a two variables.
4. Show that
$$\lim_{(x,y)\to(0,0)} \frac{3x^2\cos^2 y}{x^2+y^2}$$
does not exist. See Example 1 in Section 14.2 (page 905).
5. Create your own function of two variables that is not continuous at $(x,y) = (1,1)$. Then following Example 8 in Section 14.2 (page 909), create a piecewise function that is continuous.
6. If $f(x,y) = 6x^2 + 2xy + 5y^3$, find $f_x(1,3)$ and $f_y(1,3)$. See Example 1 in Section 14.3 (page 914).
7. Find the four second partial derivatives of $f(x,y) = e^x y^2 + x^3 y + 3$. See Example 7 in Section 14.3 (page 918).
8. Show that the function $f(x,y) = \ln(\sqrt{x^2+y^2})$ satisfies the Laplace equation on page 920. See Example 9 in Section 14.3.

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Reading Assignment #5 (Sections 14.4-14.6)
Due Friday, March 8 at the beginning of class

This assignment should be completed on separate paper. Write neatly and organize your work using looseleaf paper and nor pages torn from a notebook. Staple multiple pages together. Write your name and assignment in the top right-hand corner of the page. Answers should appear in the order below.

1. State the definition of the tangent plane in your own words. See page 926 in Section 14.4.

2. Use Theorem 8 in Section 14.4 (page 931) to show that $f(x,y) = x^2y+4y+x$ is differentiable at $(1,0)$. Find the linearization of $f(x,y)$ and use it to approximate $f(0.09, 0.1)$. See Example 2 on page 931.

3. If $z = 4xy^2 + x^3y$, where $x = \ln(t)$ and $y = t^2$, find $\frac{\partial z}{\partial t}$ when $t = 1$. Use Example 1 in Section 14.5 as a reference (page 938).

4. Read the proof of The Chain Rule (Case 1) in Section 14.5 (page 938). Use this to prove the three-variable version of the problem stated below.

   Suppose that $w = f(x,y,z)$ is a differentiable function of x and y, where $x = g(t)$, $y = h(t)$, and $z = k(t)$ are all differentiable functions of t. Then w is a differentiable function of t and
$$\frac{dw}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}.$$

5. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $4x^3 + 2y^3 + z^3 + 3xyz = 5$ using Example 9 in Section 14.5 (page 943).

6. Explain the notion of a directional derivative from Section 14.6 in your own words.

7. If $f(x,y,z) = y^2z + x^3z^2$, find the gradient of f and find the directional derivative of f at $(0,2,1)$ in the direction of $v = 2i - k + j$. Refer to Example 5 in Section 14.6 (page 951).

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Reading Assignment #6 (Sections 14.7-15.1)
Due Friday, March 15 at the beginning of class

This assignment should be completed on separate paper. Write neatly and organize your work using looseleaf paper and nor pages torn from a notebook. Staple multiple pages together. Write your name and assignment in the top right-hand corner of the page. Answers should appear in the order below.

1. State the definitions of local maximum and absolute maximum in your own words (see Section 14.7). How are they different?
2. Find the local maximum and minimum values and saddle points of $f(x, y) = 4+x^3+y^3-3xy$. See Example 3 in Section 14.7 (page 961).
3. Using the proof of Theorem 3 part (a) in Section 14.7 (page 967), prove part (b).
4. Find the extreme values of $f(x, y) = 4x^2 + 10y^2$ on the disk $x^2 + y^2 = 4$ using the method of Lagrange Multipliers. See Example 2 in Section 14.8 (page 974).
5. Explain the geometric meaning of a double integral in Section 15.1 using your knowledge of definite integrals of a single variable.
6. Fubini's Theorem in Section 15.1 allows you to choose an order of integration for double integrals. Decide on an order and evaluate $\iint_R xe^{xy} dA$, where $R = [-1, 2] \times [0, 1]$. Explain why you chose that particular order.

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Reading Assignment #7 (Sections 15.2-15.4)
Due Monday, April 1 at the beginning of class

This assignment should be completed on separate paper. Write neatly and organize your work using looseleaf paper and nor pages torn from a notebook. Staple multiple pages together. Write your name and assignment in the top right-hand corner of the page. Answers should appear in the order below.

1. Describe in your own words the process of integrating over general regions compared to integrating over rectangles in Section 15.2 (page 1001).

2. Evaluate $\iint_D 4xy - y^3 \; dA$, where D is the region bounded by $y = \sqrt{x}$ and $y = x^3$. See Example 1 in Section 15.2 (page 1003).

3. Suppose c is a constant and f a continuous function of one variable x. In single variable calculus, one can use the fact that constants commute with sums and limits to show that

$$\int_a^b cf(x) \; dx = \lim_{n\to\infty} \sum_{i=1}^{n} cf(x_i^*) \; \Delta x = c \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*) \; \Delta x = c \int_a^b f(x) \; dx.$$

   Use a similar idea to prove that if c is a constant and f a continuous function of two variables, then

$$\iint_D cf(x,y) \; dA = c \iint_D f(x,y) \; dA.$$

   This is Property 7 in Section 15.2 (page 1006).

4. When making the change to polar coordinates in a double integral, one must take care to include the extra factor of r along with the function $f(x,y)$ converted to polar coordinates. See Formula 2 in Section 15.3 (page 1012). Read over the argument at the beginning of Section 15.3 and summarize *briefly* and *in your own words* where this factor comes from (or why we must include it).

5. Use polar coordinates to evaluate $\iint_R 4xy \; dA$, where R is the region in the first quadrant bounded by the circles $x^2 + y^2 = 2$ and $x^2 + y^2 = 4$. Hint: Use a double angle formula to make integration easier. See Example 1 in Section 15.3 (page 1012).

6. Find the mass and center of mass of a triangular lamina with vertices $(0, 1)$, $(2, 1)$, and $(2, 3)$ if the density function is $p(x, y) = 2y + 3x^2$. See Example 2 in Section 15.4 (page 1018).

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Reading Assignment #8 (Sections 15.7-15.9)

Due Friday, April 12 at the beginning of class

1. Suppose a point P has rectangular coordinates $(-\sqrt{2}, \sqrt{6}, 2\sqrt{2})$. Find the cylindrical and spherical coordinates of P. See Example 1 in Section 15.7 (page 1041) and Example 2 in Section 15.8 (page 1046).

2. Suppose E is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the xy-plane (the upper hemisphere). Make a change to cylindrical coordinates to evaluate the integral
$$\iiint_E (x^2 + y^2)\ \mathrm{dV}.$$
See Example 3 in Section 15.7 (page 1042). (You should be able to do this problem without cylindrical coordinates so you can check your answer).

3. Evaluate the same integral as in the previous problem but using spherical coordinates. See Example 4 in Section 15.8 (page 1048). (You may have to remind yourself of some trig integration from Calc II.)

4. Read over the argument in Section 15.9 and explain the main idea of the Jacobian in your own words. (Be sure to describe how it is obtained, not just what it is.)

5. Compute the Jacobian to derive Formula 4 in Section 15.7. That is, find the Jacobian corresponding to the change to cylindrical coordinates. See Example 4 in Section 15.9 (page 1059).

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Reading Assignment #9 (Sections 16.1-16.3)

Due Friday, April 19 at the beginning of class

1. Define a line integral in your own words. See Section 16.2.
2. Evaluate $\int_C (x^2 - y)\, ds$ where C is the line segment from $(1,3)$ to $(5,-2)$ using Example 2 in Section 16.2 as a reference.
3. Prove Theorem 2 in Section 16.3 for piecewise-smooth curves. Hint: Use the proof of Theorem 2 for smooth curves as a reference.
4. Using Theorem 6 in Section 16.3, determine whether or not the vector field $F(x,y) = 10xi + (5x^2 + 20yz)j$ is conservative.

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Reading Assignment #10 (Sections 16.4-16.6)

Due Friday, April 26 at the beginning of class

1. Read the proof of Green's Theorem for equation 2 in Section 16.4. Prove equation 3 in a similar way by expressing D as a type II region to obtain Green's Theorem.

2. Use Green's Theorem to evaluate $\oint_C (4x^2 - y)dx + 4xdy$, where C is the circle $x^2 + y^2 = 16$. See Example 2 in Section 16.4.

3. If $\mathbf{F}(x, y, z) = 3x^2y\mathbf{i} - zy\mathbf{j} + xz^2\mathbf{k}$, find curl $\mathbf{F}$. See Example 1 in Section 16.5.

4. Using the methods in Section 16.5, determine whether the vector field

$$\mathbf{F}(x, y, z) = z^2y\mathbf{i} + 2x^2z\mathbf{j} - x^2yz\mathbf{k}$$

   is conservative.

5. If $\mathbf{F}(x, y, z) = z^2y\mathbf{i} + 2x^2z\mathbf{j} - x^2yz\mathbf{k}$, find div $\mathbf{F}$.

6. After reading through the examples of parametric surfaces in Section 16.6, explain why might you choose to describe a surface with parametric equations.

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Reading Assignment #11 (Sections 16.7-16.9)

Due Friday, May 3 at the beginning of class

1. Describe the relationship between surface integrals and surface area in your own words.

2. Evaluate $\iint_S (x^2+y^2+3z^2)\ dS$, where S is the part of the cone $z = 3 - \sqrt{x^2+y^2}$ above $z = 0$. See Example 2 in Section 16.7. Hint: Use polar coordinates.

3. Find the flux of the vector field $\mathbf{F}(x,y,z) = 4x\mathbf{i}+4y\mathbf{j}+4z\mathbf{k}$ across the sphere $x^2+y^2+z^2 = 2$. Use Example 4 in Section 16.7 for reference.

4. Use Stokes' Theorem to compute the integral $\iint_S \text{curl}\mathbf{F}\cdot d\mathbf{S}$, where

$$\mathbf{F}(x,y,z) = 3yz\mathbf{i} + xz\mathbf{j} + 2xy\mathbf{k}$$

   and S is the plane $3x+2y+z = 6$ in the first octant. See Example 2 in Section 16.8.

5. Read through the proof of the Divergence Theorem in Section 16.9. Prove equation 2 by assuming the region is type 2.

6. Using the Divergence Theorem, evaluate $\iint_S \mathbf{F}\cdot d\mathbf{S}$, where

$$\mathbf{F}(x,y,z) = (3x+z^7)\mathbf{i} + (y^2 - \sin^2(x)z^9)\mathbf{j} + (xz + xe^x)\mathbf{k}$$

   and S is the box $0 \le x \le 1, 0 \le y \le 4, 0 \le z \le 2$. See Example 2 in Section 16.9.