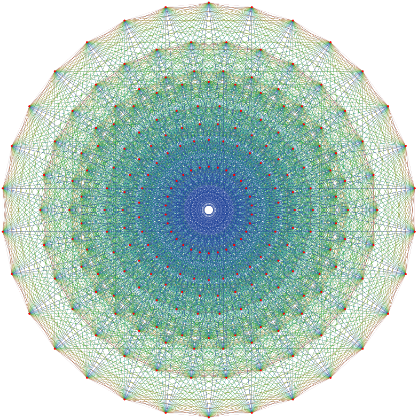

Photo Credit: Claudio Rocchini

What makes something beautiful?

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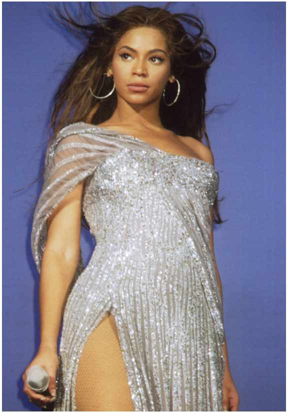

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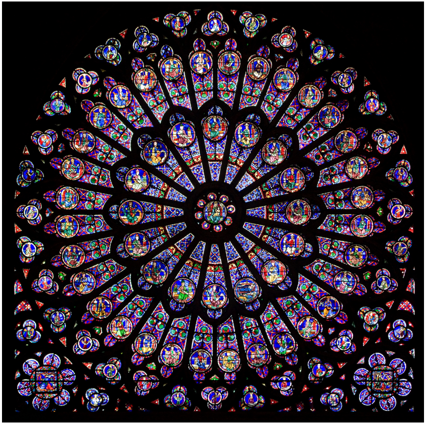


Photo Credit: Julie Anne Workman

What makes something beautiful?

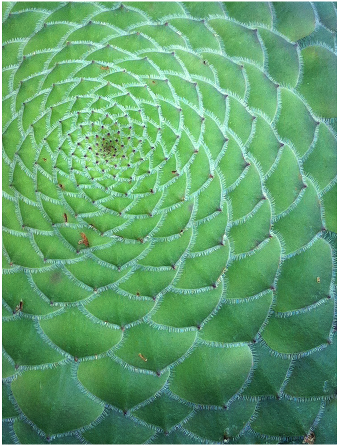


Photo Credit: Max Ronnersjö

What makes something beautiful?

Photo Credit: Gage Skidmore

What makes something beautiful?

There's no one definition of beauty, but there are certain common traits that humans gravitate towards.

One of those traits is **symmetry**.

Why study symmetry?

Formally, symmetry is a property of *invariance* (or stability) with respect to some transformation. This may mean a geometric transformation (reflection, rotation, scaling), or it may relate to change with respect to the passage of time, change in temperature, etc.

Emily Noether proved that there is a correspondence between symmetries of physical systems and conservation laws in physics.

Studying symmetry may lead to answers to fundamental questions regarding the shape of the universe.

The language of symmetry

Different fields of mathematics (algebra, analysis, differential geometry, topology, etc.) frame symmetry in different language.

In algebra, symmetry is most often encoded in the language of *groups*. At Miami, this is typically first introduced in MTH 421.

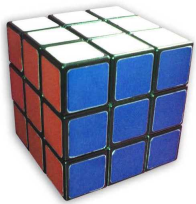
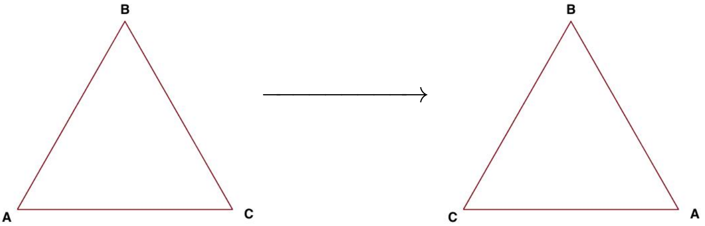


Photo Credit: Mike Gonzalez

Symmetries

A **rigid motion** of the plane is a transformation that preserves distance. A rigid motion of the plane is either a rotation, reflection, translation, or a glide reflection. We call these rigid motions **symmetries**.

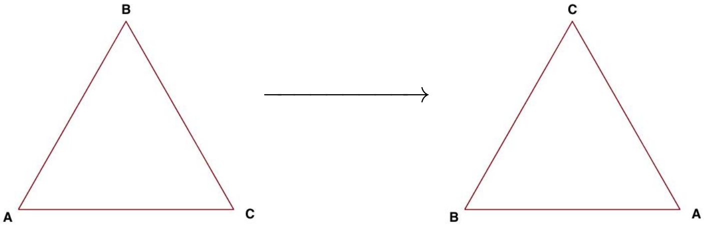
An equilateral triangle has two types of symmetries: **Reflections**



Symmetries

A **rigid motion** of the plane is a transformation that preserves distance. A rigid motion of the plane is either a rotation, reflection, translation, or a glide reflection. We call these rigid motions **symmetries**.

An equilateral triangle has two types of symmetries: **Rotations**

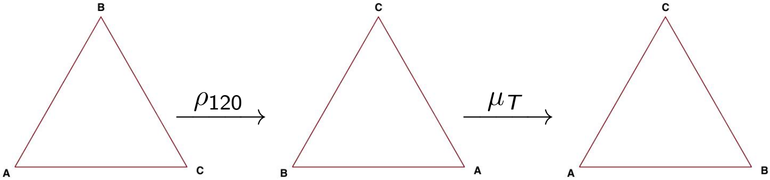


Symmetries

In total, there are 6 symmetries of an equilateral triangle:

- 3 reflections: μ_T, μ_L, μ_R (through the Top, Left, and Right vertex, respectively)
- 3 rotations (counterclockwise): $\rho_0, \rho_{120}, \rho_{240}$.

We can compose symmetries as functions. The composition $\mu_T \circ \rho_{120}$ is:



This is the same as μ_L. In fact, the composition of *any* two symmetries of an equilateral triangle is another symmetry.

Symmetries

We can construct a multiplication table (really a composition table) for the symmetries of an equilateral triangle.

	ρ_0	ρ_{120}	ρ_{240}	μ_T	μ_L	μ_R
ρ_0	ρ_0	ρ_{120}	ρ_{240}	μ_T	μ_L	μ_R
ρ_{120}	ρ_{120}	ρ_{240}	ρ_0	μ_R	μ_T	μ_L
ρ_{240}	ρ_{240}	ρ_0	ρ_{120}	μ_L	μ_R	μ_T
μ_T	μ_T	μ_L	μ_R	ρ_0	ρ_{120}	ρ_{240}
μ_L	μ_L	μ_R	μ_T	ρ_{240}	ρ_0	ρ_{120}
μ_R	μ_R	μ_T	μ_L	ρ_{120}	ρ_{240}	ρ_0

The group D_3

What we've constructed is the *group of symmetries of an equilateral triangle*, called D_3. It has four key properties:

- The composition of two symmetries is another symmetry.
- The operation of composition is associative.
- There is an identity element.
- Every symmetry has an *inverse symmetry* that "undoes" the operation.

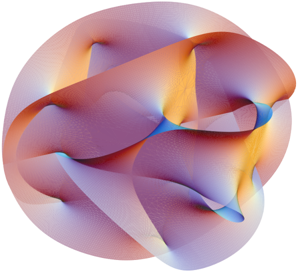
This same construction for any *regular* polygon. The group of symmetries of a regular n-gon has $2n$ elements (n rotations and n reflections), and is denoted D_n.

The Monster

Groups come in lots of different flavors.

In the mid-to-late 70s, mathematicians discovered a "simple" group called the Monster that contains approximately 8×10^{53} elements.

This group has connections to the study of string theory.



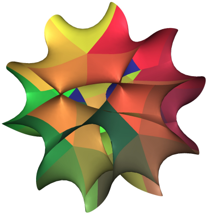


Image Credit: Andrew J. Hanson

Why study symmetry?

Symmetry is ubiquitous in mathematics and science. Here are just a few examples of where this study might be useful (outside of pure mathematics).

- Coding theory (especially RSA, which is why using a credit card online is secure...for now).
- Crystallography (from Wikipedia: "the experimental science of determining the arrangement of atoms in crystalline solids").
- One can discuss groups of transformations in the plane (or in 3D space), the so-called Orthogonal Groups. These are necessary for computer applications, such a video games.

Studying symmetry

Mathematicians study group theory in a variety of ways.

- Group theorists study groups themselves.
- Representation theorists study the spaces on which groups act (things *like* the polygon example, but usually more intricate).
- Invariant theorists study how other groups act on various algebraic objects, like polynomials.

One major area of study in my subfield of algebra is *quantum symmetry*.

Questions for me? Want to talk about this more? Send me an email: gaddisj@miamioh.edu.