Generators and Relations of the Brauer Group of an Elliptic Curve

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Abstract

Let k be a field of characteristic different from 2 or 3 and let E be an elliptic curve over k. Assume additionally that k contains a primitive third root of unity ρ. The Brauer group of E is an important invariant of the elliptic curve. Its elements are given by Morita equivalence classes of Azumaya algebras. By the Merkurjev-Suslin theorem every element in the three torsion of the Brauer group can also be written as a tensor product of symbol algebras over $k(E)$. These are algebras with presentation

$$(f,g)_{3,k(E)} = k \left\langle x, y : x^3 = \alpha, y^3 = \beta, xy = \rho yx \right\rangle$$

In my talk, I will describe an algorithm to calculate generators and relations of the three torsion of the Brauer group of E in terms of these tensor products.