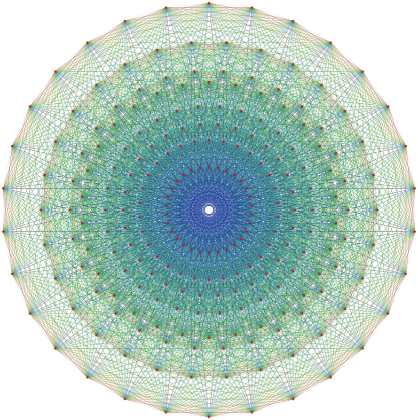

Photo Credit: Claudio Rocchini

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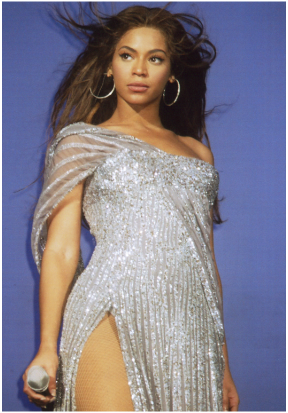

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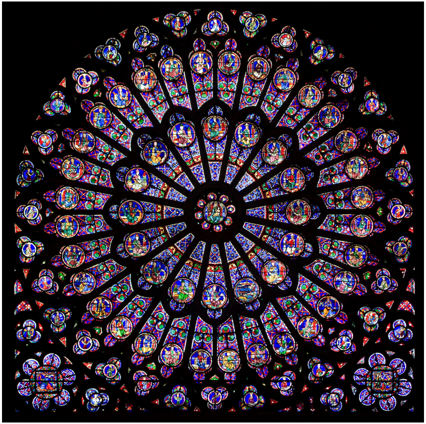

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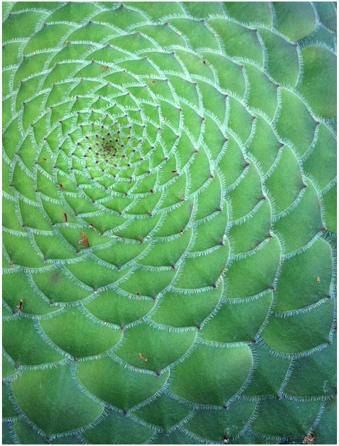

Photo Credit: Max Ronnersjö

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Studying symmetry may lead to answers to fundamental questions regarding the shape of the universe.

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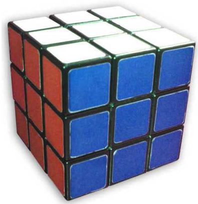

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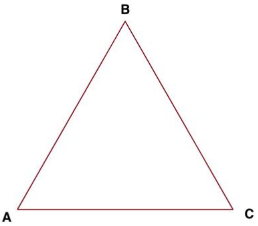

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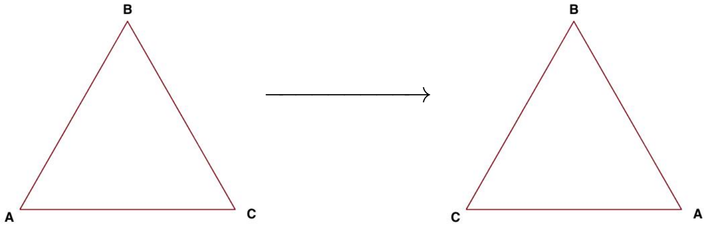

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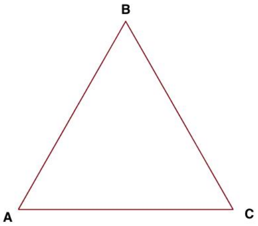

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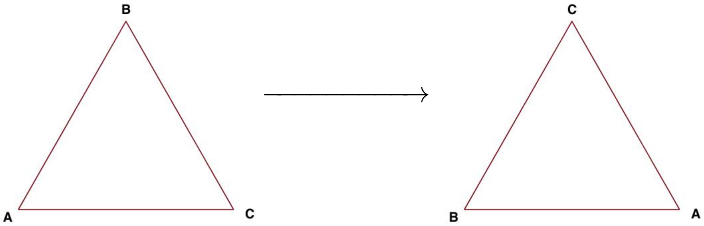

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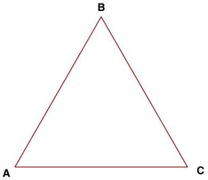

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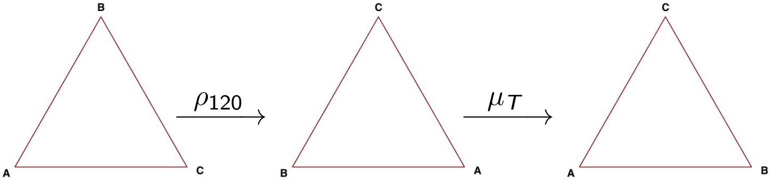

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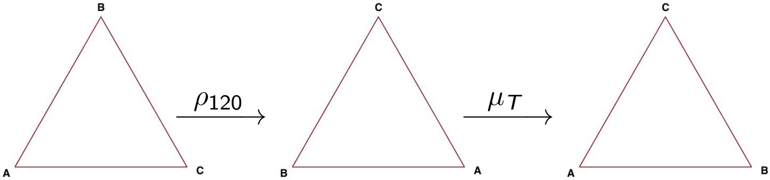

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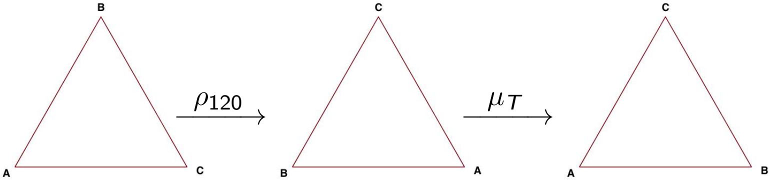

This is the same as μ_L. In fact, the composition of *any* two symmetries of an equilateral triangle is another symmetry.

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	ρ_0	ρ_{120}	ρ_{240}	μ_T	μ_L	μ_R
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ρ_{120}	ρ_{120}	ρ_{240}	ρ_0	μ_R	μ_T	μ_L
ρ_{240}	ρ_{240}	ρ_0	ρ_{120}	μ_L	μ_R	μ_T
μ_T	μ_T	μ_L	μ_R	ρ_0	ρ_{120}	ρ_{240}
μ_L	μ_L	μ_R	μ_T	ρ_{240}	ρ_0	ρ_{120}
μ_R	μ_R	μ_T	μ_L	ρ_{120}	ρ_{240}	ρ_0

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This same construction for any *regular* polygon. The group of symmetries of a regular n-gon has $2n$ elements (n rotations and n reflections), and is denoted D_n.

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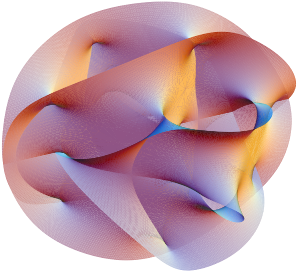
This group has connections to the study of string theory.



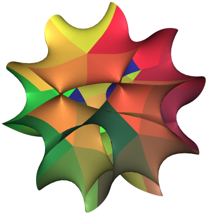


Image Credit: Andrew J. Hanson

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- One can discuss groups of transformations in the plane (or in 3D space), the so-called Orthogonal Groups. These are necessary for computer applications, such a video games.

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Questions for me? Want to talk about this more? Send me an email: gaddisj@miamioh.edu.