1 Definitions

1. Let $X(t)$ be a random process. The *autocorrelation function* is then defined as

$$R_X(\tau) = E\left[X(t)X^*(t+\tau)\right] \tag{1.1}$$

2. The *Fourier* transform of $g(t)$ is defined as

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-\jmath 2\pi f t}\, dt \tag{1.2}$$

3. The *power spectral density* of $X(t)$ is defined as

$$R_X(\tau) \xleftrightarrow{\mathcal{F}} S_X(f) \tag{1.3}$$

4.

$$g(t) \xleftrightarrow{\mathcal{F}} G(f) \tag{1.4}$$

$$\implies G(t) \xleftrightarrow{\mathcal{F}} g(-f) \tag{1.5}$$

5.

$$g(at) \xleftrightarrow{\mathcal{F}} \frac{1}{|a|}G\left(\frac{f}{|a|}\right) \tag{1.6}$$

6.

$$\mathrm{rect}\,(t) = \begin{cases} 1 & t \in \left(-\frac{1}{2}, \frac{1}{2}\right) \\ 0 & \text{otherwise} \end{cases} \tag{1.7}$$

$$\mathrm{sinc}\,(f) = \frac{\sin(\pi f)}{\pi f} \tag{1.8}$$

7. If

$$Y(t) = X(t) * h(t), \tag{1.9}$$

$$S_Y(f) = |H(f)|^2 S_X(f) \tag{1.10}$$

8.

$$\triangle\left(\frac{t}{2}\right) = \begin{cases} 1 - |t| & t \in (-1, 1) \\ 0 & \text{otherwise} \end{cases} \tag{1.11}$$

9.

$$g_1(t) * g_2(t) \overset{\mathcal{F}}{\longleftrightarrow} G_1(f)G_2(f) \tag{1.12}$$

10. The *Dirac delta* function is defined as

$$\delta(t) = 0, t \neq 0 \tag{1.13}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \tag{1.14}$$

2 Problems

1. Show that

$$\text{rect}\,(t) \overset{\mathcal{F}}{\longleftrightarrow} \text{sinc}\,(f) \tag{2.1}$$

2. Show that

$$g(t-T) \overset{\mathcal{F}}{\longleftrightarrow} G(f)e^{-\jmath 2\pi f T} \tag{2.2}$$

3. Let

$$Y = \int_0^T X(t) \tag{2.3}$$

Show that

$$Y = Y(t)|_{t=T} \tag{2.4}$$

where

$$Y(t) = X(t) * h(t) \tag{2.5}$$

$$h(t) = \text{rect}\left(\frac{t-\frac{T}{2}}{T}\right) \tag{2.6}$$

4. Show that

$$\text{rect}\,(t) * \text{rect}\,(t) = \triangle\left(\frac{t}{2}\right) \tag{2.7}$$

5. Show that

$$\triangle\left(\frac{t}{2}\right) \overset{\mathcal{F}}{\longleftrightarrow} \text{sinc}^2(f) \tag{2.8}$$

6. Show that

$$E\left(Y^2\right) = R_Y(0) = \int_{-\infty}^{\infty} S_Y(f)\, df \tag{2.9}$$

7. Show that

$$E\left(Y^2\right) = T \int_{-T}^{T} \triangle\left(\frac{\tau}{T}\right) R_X(\tau)\, d\tau \tag{2.10}$$

8. Show that

$$\delta(t) \overset{\mathcal{F}}{\longleftrightarrow} 1 \tag{2.11}$$

9. Show that

$$e^{-a|t|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2a}{a^2 + (\jmath 2\pi f)^2} \tag{2.12}$$

10. For

$$T = 10, E\left[X(t)\right] = 8, R_X\left(\tau\right) = 64 + e^{-2|\tau|}, \tag{2.13}$$

find the mean and variance of Y.