1 Definitions

1. The pmf for a Bernoulli r.v. X_i is given by

$$p_{X_i}(k) = \begin{cases} p_0 & k = 1 \\ 1 - p_0 & k = 0 \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$

2. The pmf for a Binomial Random variable

$$X = \sum_{i=1}^{n} X_i \tag{1.2}$$

is given by

$$p_X(X = k) = \binom{n}{k} p_0^k (1 - p_0)^{n-k}, \quad 0 \le k \le n \tag{1.3}$$

where X_i are i.i.d.

3. Let $Z \sim \mathcal{N}(0, 1)$. Then the Q function is defined as,

$$Q(z) = \Pr(Z > z), \quad z \ge 0 \tag{1.4}$$

4. The MGF of Y is defined as

$$M_Y(s) = E\left(e^{-sY}\right) \tag{1.5}$$

5. The MGF of Z is

$$M_Z(s) = \frac{1}{2} e^{\frac{1}{2}s^2} \tag{1.6}$$

6.

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e \tag{1.7}$$

2 Problems

1. Show that the mean and variance of X_i are $\mu = p_0$ and $\sigma^2 = p(1 - p)$.
2. Show that the mean and variance of

$$Y_i = \frac{X_i - \mu}{\sigma} \tag{2.1}$$

are 0 and 1 respectively.

3. Show that the mean and variance of

$$Y = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} Y_i \tag{2.2}$$

are 0 and 1 respectively.

4. Show that

$$M_Y(s) = \left[E\left(e^{-\frac{sY_i}{\sqrt{n}}} \right) \right]^n \tag{2.3}$$

5. Show that

$$E\left(e^{-\frac{sY_i}{\sqrt{n}}} \right) = 1 + \frac{s^2}{2n} + \frac{1}{n} R(s,n) \tag{2.4}$$

where $R(s,n)$ is an infinite series.

6. Show that

$$\lim_{n\to\infty} Y = Z \tag{2.5}$$

This is known as the *Central Limit Theorem*.

7. Let

$$p = \frac{1}{n} \sum_{i=1}^{n} X_i \tag{2.6}$$

Show that

$$E(p) = \mu = p_0 \tag{2.7}$$

$$var(p) = \frac{\sigma^2}{n} = \frac{p_0(1-p_0)}{n} \tag{2.8}$$

8. Show that

$$\lim_{n\to\infty} \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = Z \tag{2.9}$$

9. Find z if

$$\Pr(|Z| < z) = 1 - \alpha \tag{2.10}$$

10. Show that

$$p_0 - \sqrt{\frac{p_0(1-p_0)}{n}} Q^{-1}\left(\frac{\alpha}{2}\right) < p < p_0 + \sqrt{\frac{p_0(1-p_0)}{n}} Q^{-1}\left(\frac{\alpha}{2}\right) \tag{2.11}$$

11. Among 4000 newborns, 2080 are male. Find the $1 - \alpha = 0.99$ confidence interval of the probability that a male child is born.