1 Definitions

1. If $Y = f(Z)$ be monotonic,
$$p_Y(y)\,dy = p_Z(z)\,dz \tag{1.1.1}$$
2. The pdf of the $\chi^2(k)$ distribution is given by
$$p_X(x) = \frac{x^{\frac{k}{2}-1}e^{-\frac{x}{2}}}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)}u(x) \tag{1.2.1}$$
3. The Beta function is defined as
$$B\left(x, y\right) = \frac{\Gamma(x)\Gamma(y)}{\Gamma\left(x+y\right)} \tag{1.3.1}$$

2 Problems

1. Let $X_1 \sim \chi^2\left(m\right)$ and $X_2 \sim \chi^2\left(n\right)$ be independent. For
$$Z = \frac{X_1/m}{X_2/n}, \tag{2.1.1}$$
show that
$$F_Z(z) = E\left[F_{X_1}\left(\frac{mzX_2}{n}\right)\right] \tag{2.1.2}$$
2. Show that
$$p_Z(z) = E\left[mX_2 p_{X_1}\left(\frac{mzX_2}{n}\right)\right] \tag{2.2.1}$$
3. Show that
$$p_Z(z) = \frac{\left(\frac{m}{n}\right)^{m/2}}{B\left(\frac{m}{2},\frac{n}{2}\right)}x^{\frac{m}{2}-1}\left(1+\frac{m}{n}z\right)^{-\frac{m+n}{2}}u(z) \tag{2.3.1}$$
Z has an F distribution with (m, n) degrees of freedom.
4. Show that $Y = \frac{1}{Z}$ is monotonic.
5. Show that Y also has an F distribution with (n, m) degrees of freedom.
6. Find the pdf of $\frac{mZ}{mZ+n}$.