1 Axioms of Probability

1.1 Definitions

1. For any event $A, 0 \leq \Pr(A) \leq 1$.
2. $A \cup B \triangleq A + B$.
3. $A \cap B \triangleq AB$.
4. The null and complete event are $\phi = 0, S = 1$.
5. If $AB = 0, \Pr(A + B) = \Pr(A) + \Pr(B)$.
6. $(A + B)' = A'B'$

1.2 Problems

Prove the following:

1. $$A = AB + AB' \tag{1.2.1.1}$$
2. $$\Pr(A) = \Pr(AB) + \Pr(AB') \tag{1.2.2.1}$$
3. $$A + B = B + AB' \tag{1.2.3.1}$$
4. $$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{1.2.4.1}$$

2 Binomial Distribution

2.1 Definitions

1. The mean of X is defined as
$$E(X) = \sum_k k p_X(k) \tag{2.1.1.1}$$
2. The Z transform of X is defined as
$$M_X(z) = E\left(z^{-X}\right) = \sum_{k=-\infty}^{\infty} z^{-k} p_X(k) \tag{2.1.2.1}$$

3. There is a one to one relationship betwwen the pmf and its Z transform.
4. If If X_1 and X_2 are independent,

$$E\left[f\left(X_1\right)g\left(X_2\right)\right] = E\left[f\left(X_1\right)\right]E\left[g\left(X_2\right)\right] \tag{2.1.4.1}$$

5. For a Bernoulli random variable X, the pmf is

$$p_X(n) = \begin{cases} p & k = 1 \\ 1 - p & k = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.1.5.1}$$

6. X_i are said to be i.i.d (independent and identically distributed) if they are independent and have the same pmf.

2.2 Problem

1. Find the Z-transform for X, given that X is a Bernoulli random variable with parameter p.
2. If X_1 and X_2 are independent, and

$$Y = X_1 + X_2, \tag{2.2.2.1}$$

show that

$$M_Y(z) = M_{X_1}(z)M_{X_2}(z) \tag{2.2.2.2}$$

3. Find the Z-transform of Y, given that X_i are i.i.d Bernoulli random variables with parameter p.
4. Find the pmf of Y.
5. Find the pmf of

$$Y = \sum_{i=1}^{N} X_i, \tag{2.2.5.1}$$

where X_i are i.i.d.