1 Definitions

1. The pdf of $X \sim \mathcal{N}\left(\mu, \sigma^2\right)$ is defined as

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{-} \frac{(x-\mu)^2}{2\sigma^2} \quad -\infty < x < \infty \tag{1.1.1}$$

2. The Jacobian matrix transforming R, Θ to X_1, X_2 is defined as

$$\mathbf{J} = \begin{pmatrix} \frac{\partial X_1}{\partial R} & \frac{\partial X_1}{\partial \Theta} \\ \frac{\partial X_2}{\partial R} & \frac{\partial X_2}{\partial \Theta} \end{pmatrix} \tag{1.2.1}$$

3.

$$p_{R,\Theta}(r, \theta) = p_{X_1,X_2}(x_1, x_2) \left|\mathbf{J}\right| \tag{1.3.1}$$

4. The marginal distribution

$$p_R(r) = \int_0^{2\pi} p_{R,\Theta}(r, \theta)\, d\theta \tag{1.4.1}$$

5. The Laplace transform of $p_Y(y)$ is given by

$$M_Y(s) = E\left(e^{-sY}\right) \tag{1.5.1}$$

6. The unit step function is defined as

$$u(y) = \begin{cases} 1 & y \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{1.6.1}$$

2 Problems

1. Let $X_1, X_2 \in N \sim (0, 1)$ be i.i.d. Find $p_{X_1,X_2}(x_1, x_2)$.
2. Let

$$X_1 = R \cos \Theta \tag{2.2.1}$$
$$X_2 = R \sin \Theta \tag{2.2.2}$$

   Find $p_{R,\theta}(r, \theta)$.
3. Find $p_R(r)$.
4. Find $p_\Theta(\theta)$.

5. Find the distribution of

$$Y = X_1^2 + X_2^2 \tag{2.5.1}$$

6. Find the Laplace transform of $e^{-y}u(y)$
7. Find the Laplace transform of $p_{X_1^2}(x_1)$.
8. Find the Laplace transform of $p_Y(y)$ using (2.5.1).
9. Find the distribution of Y.