1 Definitions

1. Fig. 1.1.1 shows a Markovs chain with 5 states. Transition from one state to another happens over time. s_0 and s_4 are absorbing states.

$$p + q = 1 \tag{1.1.1}$$

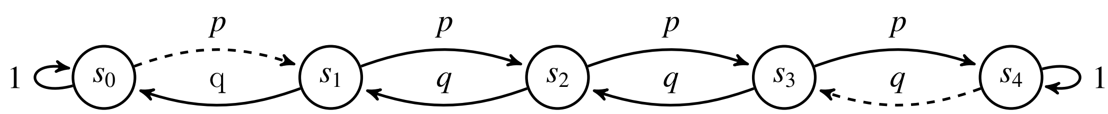

Fig. 1.1.1

2. At time instant n,

$$\mathbf{p}^{(n)} = \begin{pmatrix} P_0^{(n)} \\ P_1^{(n)} \\ P_2^{(n)} \\ P_3^{(n)} \\ P_4^{(n)} \end{pmatrix} \tag{1.2.1}$$

   where $P_i^{(n)}$ are defined to be the *stationary* probabilities.
3. $P_{i|j}$ is defined as the *transition* probability of going to state i from state j.
4. For a matrix $\mathbf{A}$, let

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x}. \tag{1.4.1}$$

   Then λ is a scalar defined to be the eigenvalue of $\mathbf{A}$ and $\mathbf{x}$ is the corresponding eigenvector.

2 Problems

1. Let

$$P_0^{(n+1)} = P_0^{(n)} \tag{2.1.1}$$
$$P_1^{(n+1)} = pP_0^{(n)} + qP_2^{(n)} \tag{2.1.2}$$
$$P_2^{(n+1)} = pP_1^{(n)} + qP_3^{(n)} \tag{2.1.3}$$
$$P_3^{(n+1)} = pP_2^{(n)} + qP_4^{(n)} \tag{2.1.4}$$
$$P_4^{(n+1)} = P_4^{(n)} \tag{2.1.5}$$

   Find the matrix $\mathbf{P}$ such that $\mathbf{p^{(n+1)}} = \mathbf{Pp^{(n)}}$
2. Show that 1 is an eigen value of $\mathbf{P}$.
3. Show that

$$P_2^{(n+1)} = pP_1^{(n)} + qP_3^{(n)} \tag{2.3.1}$$

4. If $P_0 = 1, P_N = 0$ and

$$P_i = pP_{i-1} + qP_{i+1} \tag{2.4.1}$$

   show that

$$P_i = \frac{1 - \left(\frac{p}{q}\right)^i}{1 - \left(\frac{p}{q}\right)^N}, 0 \leq i \leq N-1 \tag{2.4.2}$$

5. Find the angle between the two lines $2x = 3y = -z$ and $6x = -y = -4z$.