Quiz 13 AI1110

1 Definitions

1. A function g is said to be convex if
$$g(\lambda x_1 + (1-\lambda)x_2) \le \lambda g(x_1) + (1-\lambda) g(x_2) \tag{1.1.1}$$
2. g is convex if and only if
$$g''(x) \ge 0 \tag{1.2.1}$$
3. The *information* associated with an event E is defined as
$$I(E) = -\log_2(p(E)) \tag{1.3.1}$$
4. The *entropy* of a random variable X is given by
$$H(X) = -E\left[\log_2(p(X))\right] \tag{1.4.1}$$
5. For a *bernoulli* random variable $Y \in \{0,1\}$, the p.m.f is given by
$$p_Y(k) = \begin{cases} p & k = 0 \\ 1-p & k = 1 \end{cases} \tag{1.5.1}$$

2 Problems

1. Show that $\log\left(\frac{1}{x}\right)$ is convex.
2. Show that
$$H(Y) = -p\log_2 p - (1-p)\log_2(1-p) \tag{2.2.1}$$
and find the maximum value of $H(Y)$.
3. Let $X \in \{x_1, x_2\}$ and
$$\lambda = q = p_X(X = x_1) \tag{2.3.1}$$
in (1.1.1). Show that
$$E[g(X)] \le g[E(X)]. \tag{2.3.2}$$
4. Using (2.3.2), show that
$$H(Y) \le 1 \tag{2.4.1}$$