Quiz 10 AI1110

1 Definitions

1. The pdf of an *exponential* distribution is given by

$$p_X(x) = e^{-x}u(x) \tag{1.1.1}$$

where $u(\cdot)$ is the unit step function.

2 Problems

1. Find $F_X(x)$.
2. Let X and Y be iid exponential. Find $F_{XY}(z,z)$
3. Show that

$$\Pr(X \le z, X > Y) = \int_{x=-\infty}^{z}\int_{y=-\infty}^{x} p_{X,Y}(x,y)\,dxdy \tag{2.3.1}$$

$$= \frac{e^{-2z}}{2} - e^{-z} \tag{2.3.2}$$

4. Find

$$\Pr(Y \le z, X \le Y) \tag{2.4.1}$$

5. Let

$$Z = \max(X,Y) \tag{2.5.1}$$

Show that

$$F_Z(z) = \Pr(X \le z, X > Y) + \Pr(Y \le z, X \le Y) \tag{2.5.2}$$

6. Find the pdf of Z.
7. Find the pdf of $W = \min(X,Y)$.
8. Find $F_W(vZ)$, where v is a constant.
9. Find $E[F_W(vZ)]$.
10. Find the pdf of $V = \frac{W}{Z}$.