Octave for Mathematics

G V V Sharma*

Problem 1. *For* $x \in \mathbf{R}, x \neq 0, x \neq 1,$ *let* $f_0(x) = \frac{1}{1-x}$ *and* $f_{n+1}(x) = f_0(f_n(x)), n = 0, 1, \ldots.$ *Then find the value of* $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$.

Solution:

1) Manual Solution :

On analysing the problem ,It is found that :

$$f_0(x) = \frac{1}{1-x} \tag{1}$$

$n = 0$

$$\Rightarrow f_{n+1}(x) = f_1(x) = f_0(f_0(x)) = \frac{1}{1 - \frac{1}{1-x}} = \frac{1-x}{-x}$$

Similarly $n = 1$,

$$\Rightarrow f_{n+1}(x) = f_2(x) = f_0(f_1(x)) = \frac{1}{1 - \frac{1-x}{-x}} = x$$

Similarly $n = 2$,

$$\Rightarrow f_{n+1}(x) = f_3(x) = f_0(f_2(x)) = \frac{1}{1-x} = f_0(x)$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502205 India e-mail: gadepall@iith.ac.in.

Also at $n = 3$ We get,

$$f_{n+1}(x) = f_4(x) = f_0(f_3(x)) = \frac{1}{1 - \frac{1}{1-x}} = \frac{1-x}{-x} = f_1(x)$$

Thus ,it can be concluded that the function repeats in the similar manner for other values of n also.

So,

$$f_{100}(x) = f_1(x) \tag{2}$$

$$\Rightarrow f_{100}(3) = f_1(3) = \frac{1-3}{-3} = \frac{2}{3} \tag{3}$$

$$\Rightarrow f_1\left(\frac{2}{3}\right) = \frac{1-\frac{2}{3}}{-\frac{2}{3}} = \frac{-1}{2} \tag{4}$$

$$\Rightarrow f_2\left(\frac{3}{2}\right) = \frac{3}{2} \tag{5}$$

$$\Rightarrow f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = \frac{2}{3} + \frac{-1}{2} + \frac{3}{2} = \frac{5}{3} \tag{6}$$

2) Octave Code :

This code gives the required function in terms of x for calculating $f_1(x), f_2(x), f_{100}(x)$:

```
function fn=recr(n,x)


if(rem(n,3)==1)

fn=1./(1-x);            %f0


elseif(rem(n,3)==2)

fn=recr(1,recr(1,x));   %f1

```

```
end;

endfunction;
```

This code executes the function of previous code to find the value of required functions :

```
clear;

close;

clc;


a=recr(1,recr(100,3))   %f100 which is equal to f1

b=recr(1,recr(1,2/3))   %f1

c=recr(1,recr(2,3/2))   %f2


Sum=a+b+c;              %Gives the require result
```

3) Explaination :

```
function fn=recr(n,x)
```

This statement defines the function. To be able to use this function we save the file with the same name as that of function Here it is **(recr)** so we save it as **recr.m**.

Here **fn** is the return value.

```
if(rem(n,3)==1)
fn=1./(1-x);            %f0


elseif(rem(n,3)==2)
fn=recr(1,recr(1,x));   %f1
```

Here, $f_0(x) = \frac{1}{1-x}$ and $f_1(x) = f_0(f_0(x)) = \frac{1}{1-\frac{1}{1-x}} = \frac{1-x}{-x}$

```
a=recr(1,recr(100,3))   %f100 which is equal to f1
b=recr(1,recr(1,2/3))   %f1
c=recr(1,recr(2,3/2))   %f2
```

In these statements **a,b,c** give the value of composite functions

$$f_{100}(x), f_1(x), f_2(x)$$

at repective values of **x**.

```
Sum=a+b+c;              %Gives the require result
```

The **Sum** function gives the value of the sum of the composite functions (that is the reqired result).

Problem 2. *If* $P = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}, A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ *and* $Q = PAP^T$, *find* $P^T Q^{2015} P$.

Solution:

We Know that : $PP^T = I$

Given $Q = PAP^T$

Hence,

$$P^T Q^{2015} P = P^T[(PAP^T)(PAP^T)(PAP^T)(PAP^T)\ldots\ldots\ldots_{(2015\ terms)}]P$$

$$=(P^T P)A(P^T P)A(P^T P)A(P^T P)A\ldots\ldots\ldots_{(2015\ terms)}$$

$$= (I)A(I)A(I)A(I)A(I)A\ldots\ldots\ldots_{(2015\ times)}$$

$$= A^{2015}$$

Since, A=$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $A^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ $A^3 = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ $A^4 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$ Similarly,

$$A^{2015} = \begin{pmatrix} 1 & 2015 \\ 0 & 1 \end{pmatrix}$$

Hence,

$$P^T Q^{2015} P = \begin{pmatrix} 1 & 2015 \\ 0 & 1 \end{pmatrix}$$

```
clear;

close;

clc;

P=[sqrt(3)/2,1/2;-1/2,sqrt(3)/2];
```

This is how we create matrix in octave. Rows are separated by (;) and columns by (,).

```
A=[1,1;0,1];
```

```
Q=P*A*(P');
```

This creates the [Q] matrix , matrices are multiplied by a (*) symbol ,[P'] represents transpose of [P] matrix.

```
ANS=(P')*(Q^2015)*P;
```

Problem 3. *Evaluate* $\sum_{r=1}^{15} r^2 \frac{\binom{15}{r}}{\binom{15}{r-1}}$.

Solution:

0.1 Naive and Unoptimised solution using Octave

To evaluate the sum expression, expand it term by term and add the values together. We can

```
# prob3_naive.m

sum = 0;

for r = 1:15
        sum += r^2 * nCr(15,r) / nCr(15,r-1);
end;

sum
```

```
# nCr.m

function c = nCr (n,r)
```

```
c = factorial(n)./factorial(r)./factorial(n-r);
```

```
sum = 680
```

0.2 Explaination of Naive and Unoptimised solution using Octave

‘Binomial Coeffecients’ are just the ‘No of Combinations’:

$$\binom{n}{r} = {}^{n}C_{r}$$

To make the ‘combinations’ functionality, create a function named `nCr`. Since the name of the function is `nCr`, the name of its file will be `nCr.m`. The function will take two inputs

```
function c = nCr (n,r)
```

So, in the body of the function, just calculate `c` according to formula:

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

```
c = factorial(n)./factorial(r)./factorial(n-r);
```

Now, to actually calculate the summation, create a program, say, `prob3.m`

```
sum = 0;
```

Now we will run a loop to repetatively add the calculated values of each term of summation $\sum_{r=1}^{15} r^2 \frac{\binom{15}{r}}{\binom{15}{r-1}}$ to the variable `sum`.

Here, the index variable will be `r`. Obviously, `r` will run from `1` to `15`.

```
for r = 1:15
```

```
sum += r^2 * nCr(15,r) / nCr(15,r-1);
```

```
sum
```

Which will give output `sum = 680` on the screen.

0.3 Faster and Optimised solution using Octave

Simplifying the solution:

$$
\begin{aligned}
\sum_{r=1}^{15} r^2 \frac{\binom{15}{r}}{\binom{15}{r-1}} &= \sum_{r=1}^{15} r^2 \ \frac{15!}{r! \ (15-r)!} \ \frac{(r-1)! \ (15-r+1)!}{15!} \\
&= \sum_{r=1}^{15} r^2 \ \frac{(r-1)!}{r! \ (r-1)!} \ \frac{(16-r)! \ (15-r)!}{(15-r)!} \\
&= \sum_{r=1}^{15} r^2 \ \frac{(16-r)!}{r} \\
&= \sum_{r=1}^{15} (16r - r^2) \\
&= 16 \sum_{r=1}^{15} r \ - \ \sum_{r=1}^{15} r^2 \\
&= 16 \frac{r \ (r+1)}{2} \ - \ \frac{r \ (r+1) \ (2r+1)}{6} \\
&= \frac{(48r^2 + 48r) \ - \ (2r^3 + 3r^2 + r)}{6} \\
&= \frac{-2r^3 + 45r^2 + 47r}{6}
\end{aligned}
$$

Now we just need this simple code to get the answer:

```
# prob3_optimised.m


r = 15;

sum = (-2*r^3 + 45*r^2 + 47*r)./6;

sum
```

Which will give output `sum = 680` on the screen.

0.4 Visualisation of Problem

To visualise the problem, analyse the summation $\sum_{r=1}^{15} r^2 \frac{\binom{15}{r}}{\binom{15}{r-1}}$ numberically and theorectically.

Observe what are the values of each term and how is the total sum growing as summation progresses from `r = 1` to `r = 15`.

From above discussion:

	r^{th} term	partial sum upto r^{th} term
Nummerical	$r^2 \frac{\binom{15}{r}}{\binom{15}{r-1}}$	$\sum_{r=1}^{15} r^2 \frac{\binom{15}{r}}{\binom{15}{r-1}}$
Theorectical	$16r - r^2$	$\frac{-2r^3+45r^2+47r}{6}$

```
# visualise.m


hold all;


x = 1:15;

```

```
term = x.*16 - x.^2;
sum = (-2*x.^3 + 45*x.^2 + x.*47)./6;

plot(x, term, ':or;Value of "r"th term (theoretical);');
plot(x, sum, ':sr;Partial Sum upto "r"th term (theoretical);');

for r = 1:15
        term(r) = r^2 * nCr(15,r) / nCr(15,r-1);
end;

for r = 1:15
        temp = 0;
        for i = 1:r
                temp += term(i);
        end;
        sum(r) = temp;
end;

plot(x, term, 'ob;Value of "r"th term (numerical);', "markersize", 10);
plot(x, sum, 'sb;Partial Sum upto "r"th term (numerical);', "markersize", 10);

legend('location','northoutside');
axis([0 16 0 700]);
xlabel('Index of summation "r"');
ylabel('Value');
grid;
set(gca, "xtick", 1:15);
set(gca, "ytick", 0:100:700);
```

```
daspect([1,50]);


print('graph.eps', '-depsc', '-S720');
```

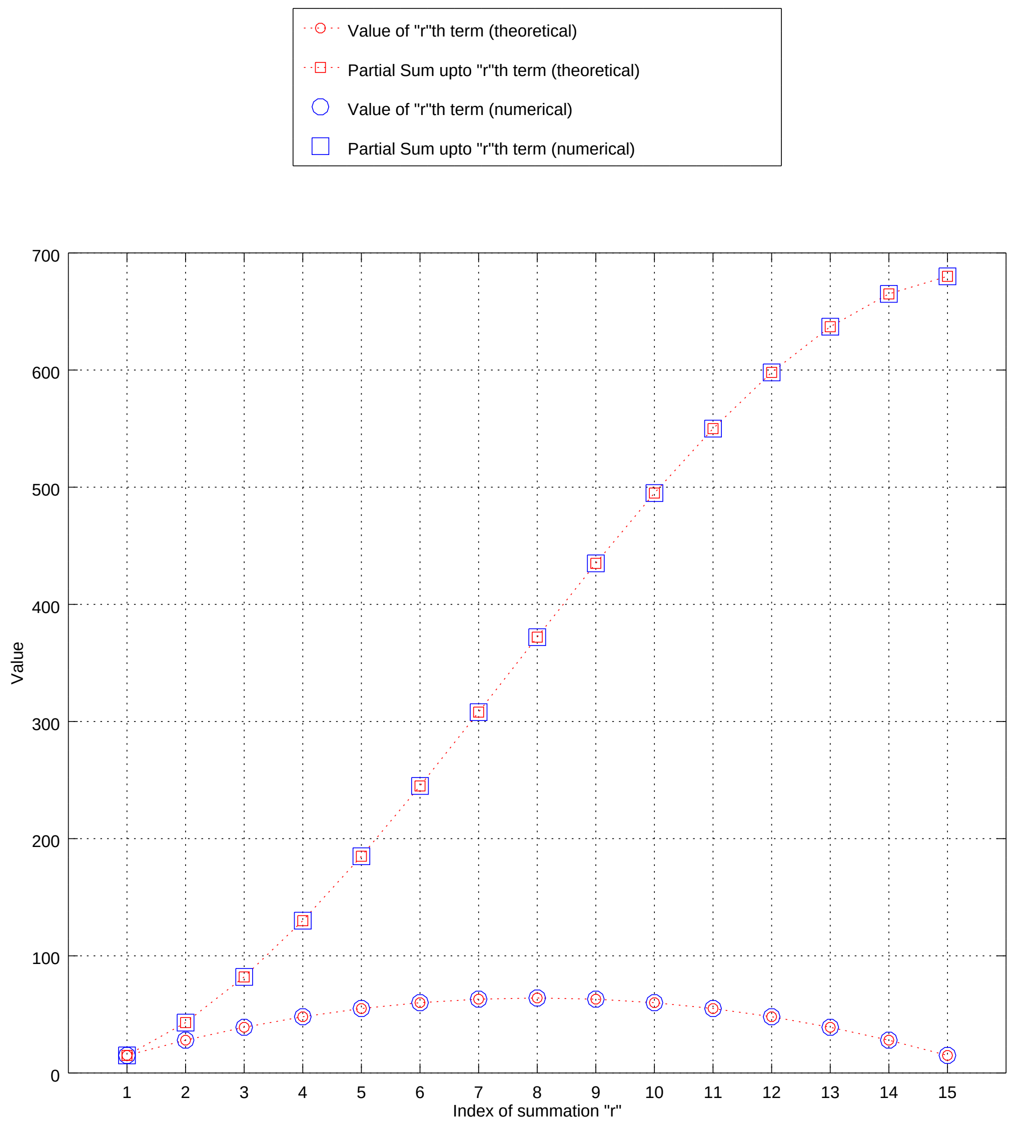


Fig. 1: The behaviour of the summation $\sum_{r=1}^{15} r^2 \frac{\binom{15}{r}}{\binom{15}{r-1}}$

Problem 4. *If* $\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x} = e^3$, *find a.*

Solution:

If there is a limit

$$\lim_{x\to\infty} f(x)^{g(x)}$$

where

$$f(x) \to 1 \quad and \quad g(x) \to \infty$$

then the limit reduces to

$$\lim_{x\to\infty} e^{[f(x)-1]*g(x)} \tag{7}$$

So, according to the question

$\lim_{x\to\infty} (1+\frac{a}{x}-\frac{4}{x^2})^{2x} \Rightarrow \lim_{x\to\infty} e^{(\frac{a}{x}-\frac{4}{x^2})(2x)} \Rightarrow \lim_{x\to\infty} e^{(2a-\frac{8}{x})}$

$taking \quad lim \quad x \to \infty$

$\Rightarrow e^{2a} = e^3$ [given]

$So \quad a = 3/2$

Now by putting the value of a in the question

$\Rightarrow \lim_{x\to\infty} (1+\frac{3}{2x}-\frac{4}{x^2})^{2x} = e^3$ when $x \to \infty$

```
clear;

close;

clc;

x=linspace(-100,2000,101);

y=(1.+(3./(2.*x))-(4./(x.^2))).^(2.*x);

plot(x,y);
```

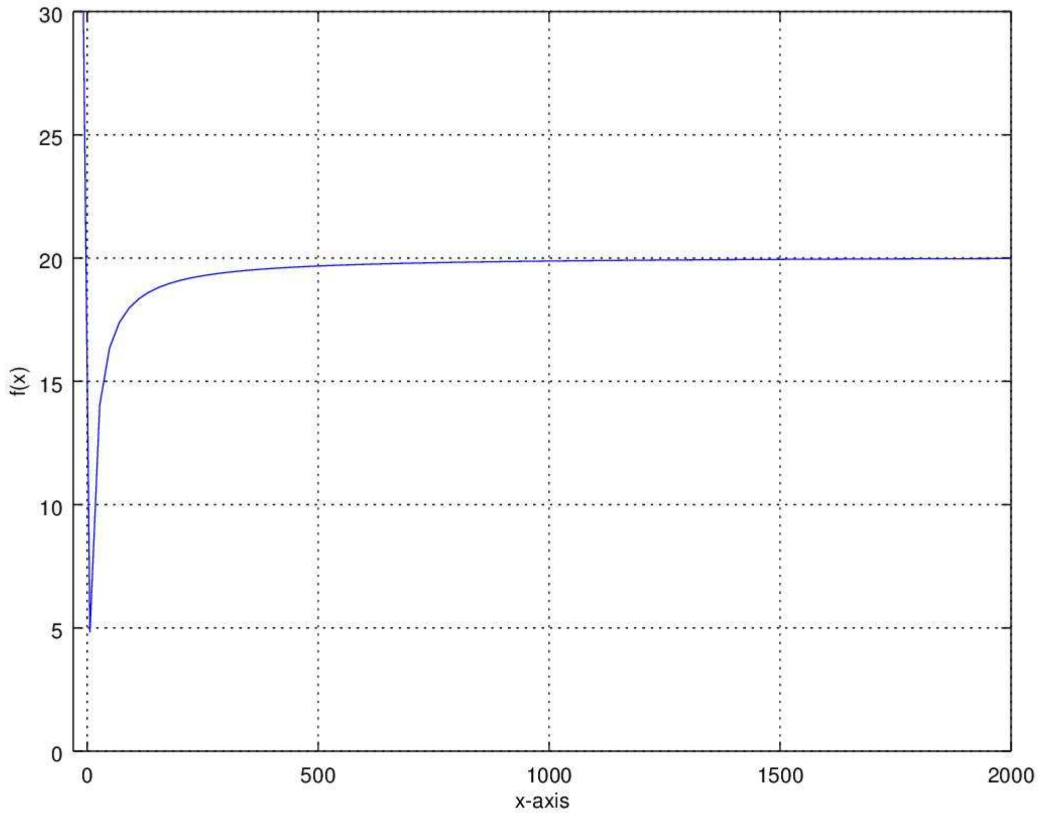


Fig. 2: Quiz 1:f(x)→ e^3 *when* $x \to \infty$

```
axis([-30 2000 -30 30]);

grid
```

Problem 5. *The function*

$$f(x) = \begin{cases} -x & x < 1 \\ a + \cos^{-1}(x+b) & 1 \leq x \leq 2 \end{cases} \tag{8}$$

is known to be differentiable at $x = 1$. *What is the value of* $\frac{a}{b}$*?*

Solution:

$$for \quad x < 1, \qquad f(x) = -x \tag{9}$$

$$\lim_{x\to 1^-} f'(x) = -1 \tag{10}$$

$$for \quad 1 \le x \le 2, \qquad f(x) = a + \cos^{-1}(x+b) \tag{11}$$

$$\lim_{x\to 1^+} f'(x) = 0 - \frac{1}{\sqrt{1-(x+b)^2}} \tag{12}$$

Since the function is differentiable at x=1,

$$-\frac{1}{\sqrt{1-(x+b)^2}} = -1 \tag{13}$$

$$1-(x+b)^2 = 1 \tag{14}$$

$$-(x+b)^2 = 0 \tag{15}$$

$$b = -x \tag{16}$$

$$b = -1 \qquad As \quad x = 1 \tag{17}$$

The fact that a differentiable function is also a continuous function, we know that

$$\lim_{x\to 1^+} \quad a + \cos^{-1}(x+b) = \lim_{x\to 1^-}(-x) \tag{18}$$

$$a + \frac{\pi}{2} = -1 \tag{19}$$

$$a = -1 - \frac{\pi}{2} \tag{20}$$

$$The \quad value \quad of \quad c = \frac{a}{b} \tag{21}$$

$$c = \frac{-1-\frac{\pi}{2}}{-1} \tag{22}$$

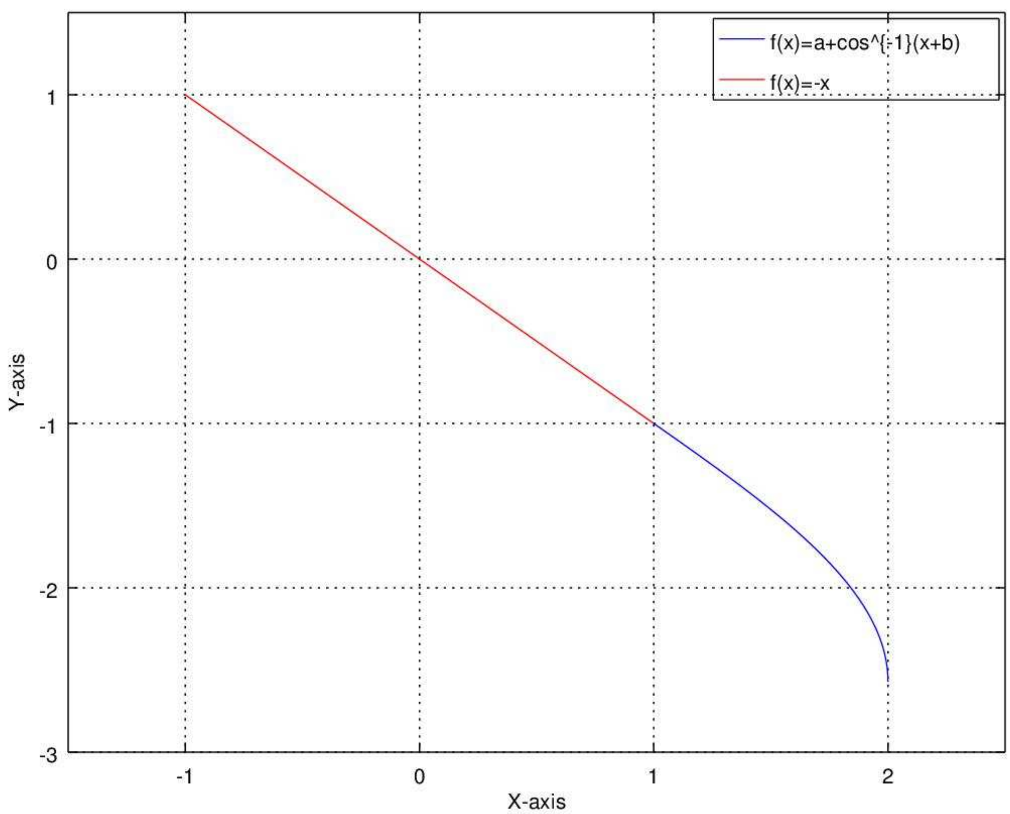
$$c = 1 + \frac{\pi}{2} \tag{23}$$

```
clear;\\

close;\\

clc;\\

x1=1;\\

x2=linspace(-1,1,1000);\\
```

```
x3=linspace(1,2,1000);\\

b=-x1;\\

a=-x1-acos(b+x1);\\

c=a./b\\

y=-x2;\\

z=a+acos(b+x3);\\

plot(x3,z,"b",x2,y,"r");\\

axis([-1.5 2.5 -3 1.5]);\\

grid\\

xlabel('X-axis')\\

ylabel('Y-axis')\\

legend('f(x)=a+cos^{-1}(x+b)','f(x)=-x');
```

GRAPH FROM OCTAVE:

We put the values of 'a' and 'b' in $f(x)$ then the graph is smooth at $x = 1$, which proves that $f(x)$ is differentiable $x = 1$.



Problem 6. *The tangent at point P, for the curve $x = 4t^2 + 3, y = 8t^3 - 1$, with parameter $t \in \mathbf{R}$, meets the curve again at Q. Find the coordinates of Q.*

Solution:

$$P(4t^2 + 3, \quad 8t^3 - 1) \text{ let } Q(4t_1{}^2 + 3, \quad 8t_1{}^3 - 1) \text{ at P, } \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{24t^2}{8t} = 3t \ .$$

$$\text{tangent at P is } y8t^3 + 1 = 3t(x4t^2 3)$$

$$\text{Q will satisfy it } 8t_1{}^3 8t^3 = 3t(t_1{}^2 4t^2)$$

$$8(t1t)(t_1{}^2 + t_1 t + t^2) = 3t.4(t_1 t)(t_1 + t)$$

$$2(t_1{}^2 + t_1 t + t^2) = 3t(t_1 + t).$$

$$2(t_1) + 2t_1 t + 2t^2 = 3tt_1 + 3t^2$$

```
clear;
close;
clc;
x = linspace(-100,100,1000);
z=(x - 3 ).^(3./2) - 1;
y=(9./2)*x - 28 ;
plot(x,y,"r",x,z,"b")
axis([-30 30 -10 50]);
xlabel('X-Axis');
ylabel('Y-Axis');
grid
```

Fig. 3: Quiz 1:CODE FOR OCTAVE

$2{t_1}^2 t t_1 t t^2 = 0$ $(t_1 t)(2t_1 + t) = 0$ $t_1 = \frac{-t}{2}$ The x-coordinate varies as $t^2 + 3$ and the y-coordinate varies as $t^{(}3) - 1$.

Problem 7. *Find the minimum distance of a point on the curve $y = x^2 - 4$ from the origin.*

Solution:

Step:1 Choose a general point p(x,y) on curve.

Step:2 Write distance d from point p(x,y) to the origin.

$$d = \sqrt{((x-0)^2 + (y-0)^2)} \tag{24}$$

Step:3 Using the concepts of minima calculate the shortest distance.

$$d^2 = ((x-0)^2 + (y-0)^2) \tag{25}$$

$$D = d^2 = x^2 + y^2 \tag{26}$$

putting value of y from equation of curve and diffrentiating.

$$\frac{d}{dx}(D) = 2x(2x^2 - 7) \tag{27}$$

Step:4 Since derivative vanishes at x=0 and $\pm\sqrt{\frac{7}{2}}$ but the double derivative is positive at $x = \pm\sqrt{\frac{7}{2}}$ therefore minima occurs at $x = \pm\sqrt{\frac{7}{2}}$

$$\tfrac{d^2}{dx^2}(D) = 2(6x - 7) \quad is \quad > 0 \quad at \quad x = +\sqrt{\tfrac{7}{2}}$$

Step:5 Calculate the min distance by putiing $x = \pm\sqrt{\frac{7}{2}}$ in eqn(1).

Step:6 **Ans:1.9365** at point $p = (x = \pm\sqrt{\frac{7}{2}}, -\frac{1}{2})$.

```
1.close;

2.clear;

3.clc;

4.x=linspace(-10,10,1000);

5.y=x.^2-4;

6.z=sqrt(14).*x-7.5;

7.m=(-x)./(sqrt(14));

8.d=sqrt(x.^2+y.^2);

9.min(d)

10.plot(x,y,x,z,x,m);

11.grid

12.xlabel('x-axis')

13.ylabel('y-axis')
```

Problem 8. *Sketch the region*

$$A = \left\{(x, y)\,|y \geq x^2 - 5x + 4, x + y \geq 1, y \leq 0\right\}. \tag{28}$$

Problem 9. *A variable line drawn through the intersection of the lines* $\frac{x}{3} + \frac{y}{4} = 1$ *and* $\frac{x}{4} + \frac{y}{3} = 1$ *meets the coordinate axes at A and* $B, A \neq B$*. Sketch the locus of the midpoint of AB.*

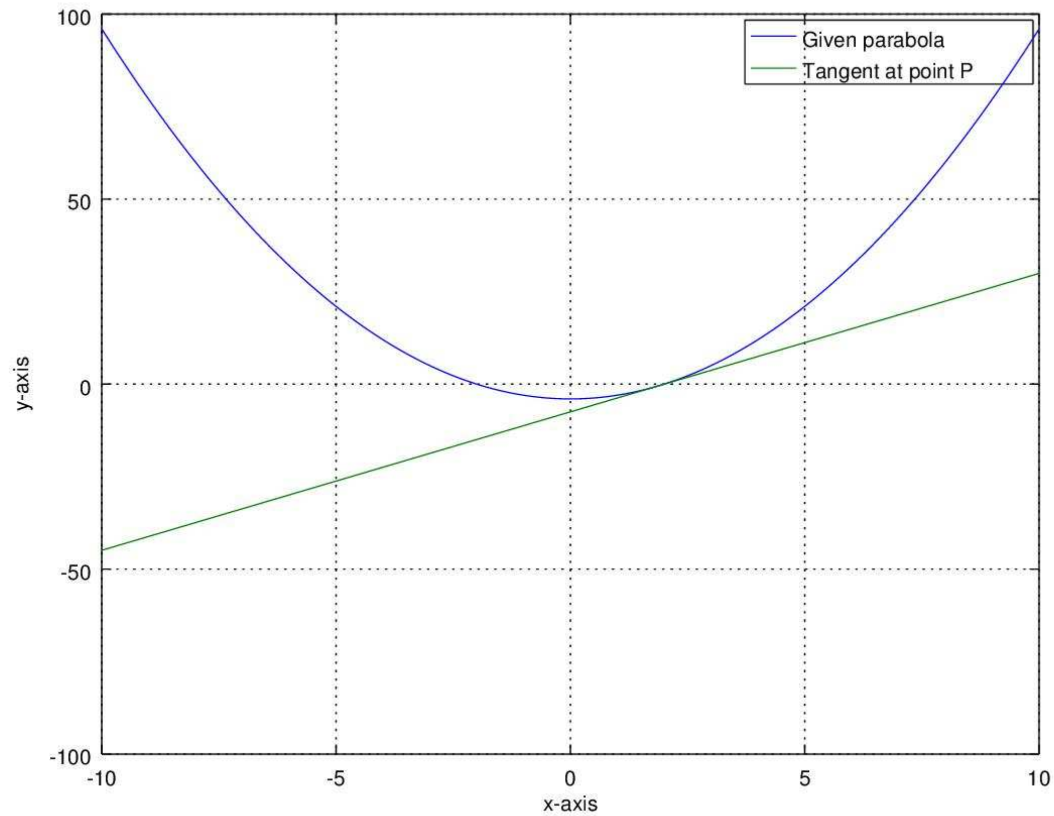


Fig. 4: Graph of $y = x^2 - 4$ and its tangent at p.

Solution:

rewrite equation $x/3 + y/4 = 1$ and$x/4 + y/3 = 1$ into $4x + 3y = 12$ and $3x + 4y = 12$

then point of intersection of these lines

$$= 3x + 4y - 12 - (4x + 3y - 12)$$

$$= -x + y = 0$$

$$x = y$$

put $x = y$ in $4x + 3y = 12$

$$4x + 3x = 12$$

$$x = 12/7$$

point of intersection $(12/7, 12/7)$

Also given line pass through point of intersection cut x-axis at A so point(a,0) and y-axis at

B(0,b)

let m be slope of line

then equation of line

$$(y - 12/7) = m(x - 12/7)$$

put A(a,0) in the above line because A(a,0) lies on line

$$-12/7 = m(a - 12/7)$$

$$(a = 12/7(1 - 1/m)$$

we get A(12/7(1-1/m),0)

now, put B(0,b) in

$$(y - 12/7) = m(x - 12/7)$$

we get

$$(b - 12/7) = m(-12/7)$$

we get B(0,12/7(1-m))

now we have A(12/7(1-1/m),0) and B(0,12/7(1-m))

$$midpointofABi = (a + 0/2, 0 + b/2)$$

$$= (a/2, b/2);$$

$$= (12/7(1 - 1/m/2, 12/7(1 - m)/2);$$

$$= (6/7(1 - 1/m), 6/7(1 - m));$$

let $h = 6/7(1 - 1/m)$ and $k = 6/7(1 - m)$

from $k = 6/7(1 - m)$

we get $m = 1 - (7k/6)$

put $m = 1 - (7k/6)$ in $h = 6/7(1 - 1/m)$

we get

$h = 6/7(1 - 1/(1 - 7k/6))$

by further simplifing

$h = (-6/6 - 7k)k$

put $x = h$ and $y = k$

$x = (-6/6 - 7k)k$

this is locus of midpoint AB

1 Octave code

Listing 1: prob9.m

```
clear;

close;

clc;

x=linspace(-50,50,100);

y1=(-6./(6-7.*x)).*x;

y2=(24./7)-x;

plot(x,y1,'r',6/7,6/7);

legend('6.*x-7.*x.*y-6.*y=0');

hold on;

plot(x,y2,'g');

legend(7.*x+7.*y-24);

xlabel(x-axis);

ylabel(y-axis);

hold on;

hold off;
```

```
grid;

print('prob9.eps','-deps');
```

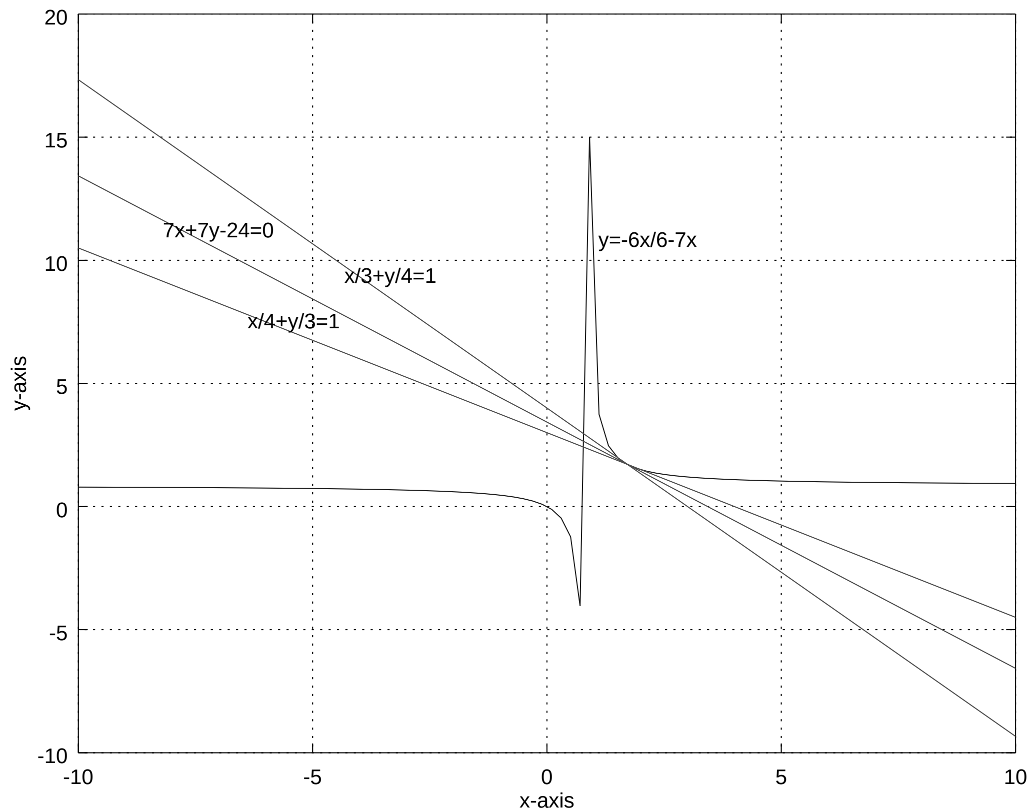


Fig. 5: sketch of a locus of midpoint

‘

Problem 10. *The point* $(2, 1)$ *is translated parallel to the line* $L : x - y = 4$ *by* $2\sqrt{3}$ *units to yield the point Q. If Q lies in the 3rd quadrant, sketch the line passing through Q and* $\perp$ *L.*

Problem 11. *A circle passes through* $(-2, 4)$ *and touches the* $y-$*axis at* $(0, 2)$*. Find out which of the following lines represents the diameter of the circle.*

1) $4x + 5y - 6 = 0$

2) $2x - 3y + 10 = 0$

3) $3x + 4y - 3 = 0$

4) $5x + 2y + 4 = 0$

Problem 12. *The eccentricity of a hyperbola satisfies the equation* $9e^2 - 18e + 5 = 0$. $(5, 0)$

is a focus and the corresponding directrix is $5x = 9$*. Plot the hyperbola.*

Solution:

1.1 knowledge about Hyperbola

1. standard equation of hyperbola is :

$x^2/a^2 - y^2/b^2 = 1$;

2. eccentricity of Hyperbola is: $e^2 = (a^2 + b^2)/a^2$;

3. focus is at (ae,0);

4. $e > 1$

1.2 value of eccentricity(e)

$$9e^2 - 18e + 5 = 0,$$

$$9e^2 - 15e - 3e + 5 = 0$$

$$(3e - 1)(3e - 5) = 0$$

or, $e = 1/3$ or $e = 5/3$

since , hyperbola has $e > 1$, so here, e=5/3;

1.3 value of a and b

given, focus = (5,0);

we know that focus = (ae,0);

so, $ae = 5$

since $e = 5/3$

so, $a = 3$

now, putting the value of a and e in

$$e^2 = (a^2 + b^2)/a^2$$

we get $b = 4$

1.4 Graph

so, the standard equation to be plotted is:

$$x^2/a^2 - y^2/b^2 = 1$$

putting $a = 3$ and $b = 4$;

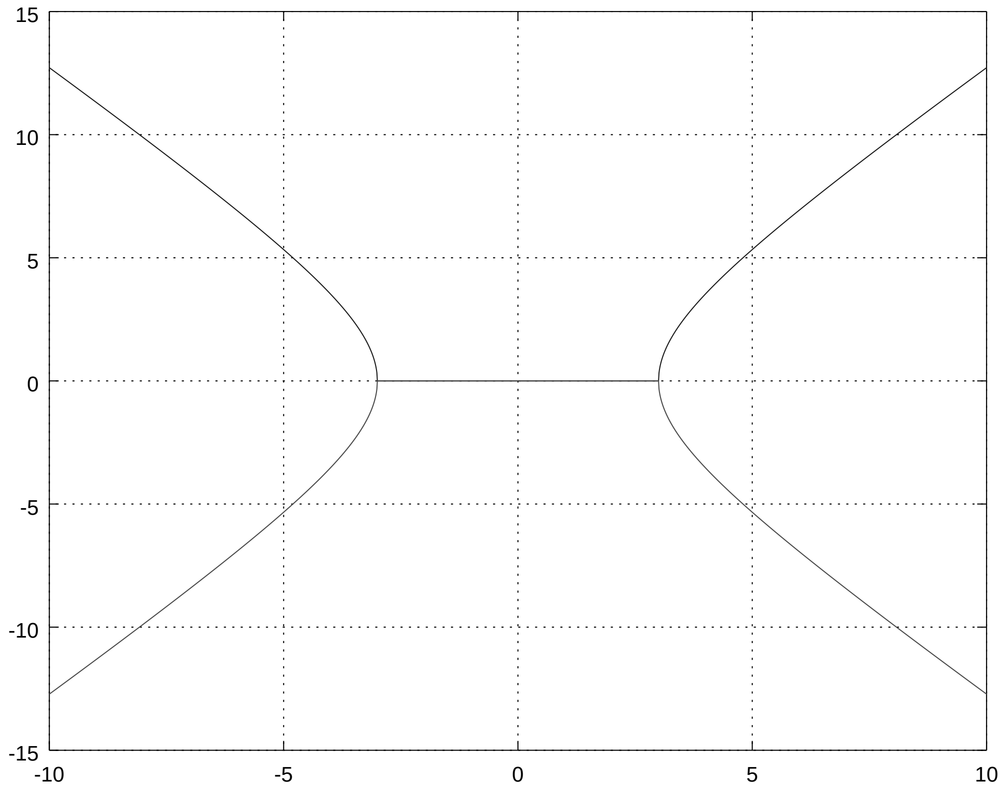
$$x^2/9 - y^2/16 = 1$$

$$or, y = +sqrt(16(x^2/9 - 1));$$

$$y = -sqrt(16(x^2/9 - 1));$$

```
x=-10:0.01:10;


y1=sqrt(16*((x.^2)/9-1));

y2=-sqrt(16*((x.^2)/9-1));

plot(x,y1,x,y2)


grid;

print('graph.eps','-deps');
```

1.6 required graph is :



focus of this hyperbola is at (5,0);

Directrix of this hyperbola is :

$$9x = 5;$$

Problem 13. *Sketch the ellipse* $\frac{x^2}{27} + \frac{y^2}{3} = 1$.

Solution:

$$\frac{x^2}{27} + \frac{y^2}{3} = 1$$

$$y = 3(\sqrt{1 - \frac{x^2}{27}})$$

Octave code for this graph

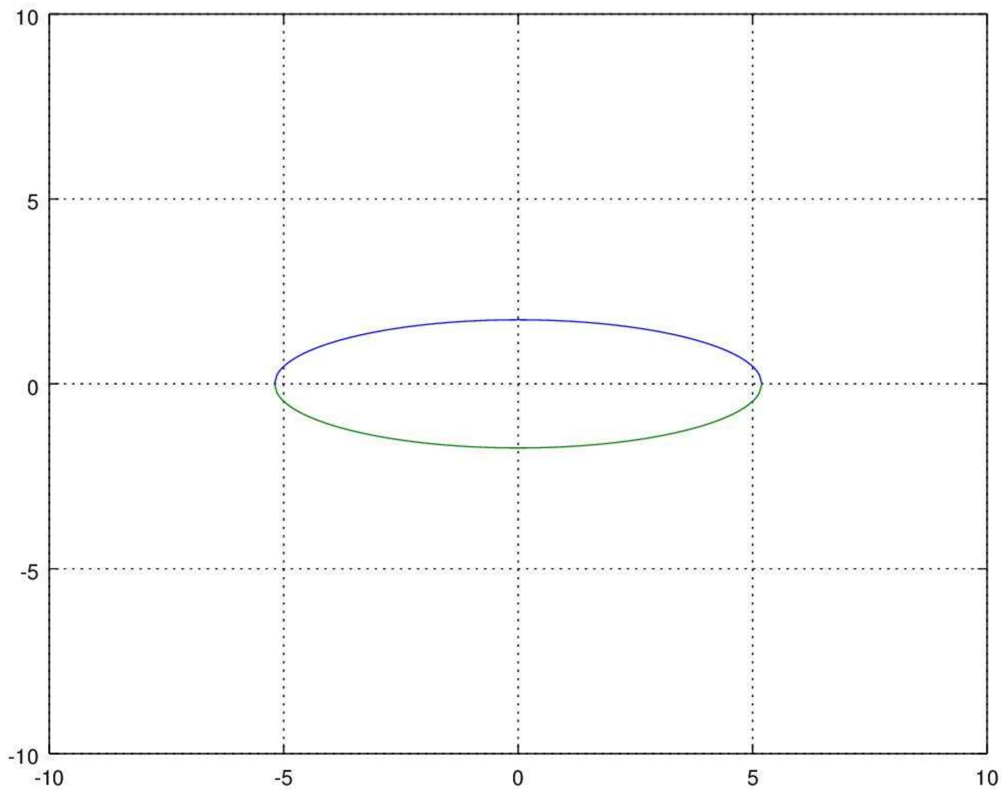


Fig. 6: Graph of ellipse $\dfrac{x^2}{27} + \dfrac{y^2}{3} = 1$

```
clear;

close;

clc;


x=linspace (-3*sqrt(3),3*sqrt(3), 200) ;

y=(3.0*(1-((x.*x)./27.0))).^0.5 ;

z=-(3.0*(1-((x.*x)./27.0))).^0.5 ;


plot (x,y,x,z) ;

grid;

axis([-10 10 -10 10])
```

Problem 14. *Find the minimum and maximum values of* $4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x, x \in \mathbf{R}$.

Problem 15. *Find the solution of the equation* $\sqrt{2x+1} - \sqrt{2x-1} = 1, x \geq \frac{1}{2}$.

Solution:

1.7 solving the equation

$$\sqrt{(2x+1)}\text{-}\sqrt{(2x-1)} = 1$$

squaring both sides:

$$2x + 1 + 2x - 1 - 2\sqrt{(4x^2 - 1)} = 1$$

$$\text{or,}4x - 2\sqrt{(4x^2 - 1)} = 1$$

$$\text{or,}4x - 1 = 2\sqrt{(4x^2 - 1)}$$

squaring both sides:

$$16x^2 + 1 - 8x = 16x^2 - 4$$

$$\text{or,}8x - 5 = 0$$

$$\text{or,}x = 5/8$$

1.8 Graph

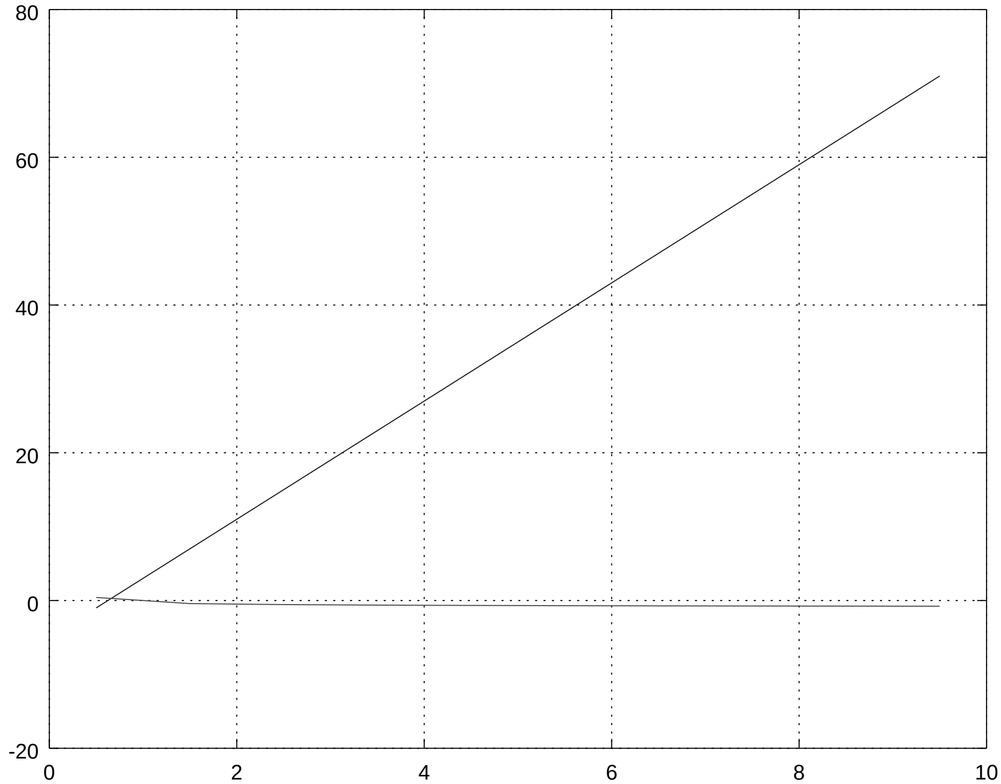
so, the standard equation to be plotted is:

$$x = 5/8$$

and $\sqrt{(2x+1)}\text{-}\sqrt{(2x-1)} = 1$

```
x=1/2:10;

y1=8x-5;

y2=sqrt{(2x+1)}-sqrt{(2x-1)}-1;

plot(x,y1,x,y2)

grid

print('mani1.eps','-deps')
```

1.10 required graph is :



Problem 16. *Let $z = 1 + a\imath, a > 0$ be a complex number such that z^3 is a real number. Find $\sum_{k=0}^{11} z^k$.*

Solution:

$$z^3 = (1 + ai)^3$$

$$= 1 + a^3 i^3 + 3ai(1 + ai)$$

$$= 1 - a^3 i + 3ai - 3a^2$$

$$= (1 - 3a^2) + (3a - a^3)i$$

We know that z^3 is a real number.Hence

$$Im(z) = 0$$

$$\Rightarrow 3a - a^3 = 0$$

$$\Rightarrow a = -\sqrt{3}, a = 0, a = \sqrt{3}$$

$\Rightarrow a = \sqrt{3}$ is the only true value. Hence $z = 1 + \sqrt{3}i = 2e^{\frac{i\pi}{3}}$

$$\begin{aligned} \sum_{k=0}^{11} z^k &= \frac{1(z^{12}-1)}{z-1}(G.P) \\ &= \frac{2^{12}e^{\frac{i12\pi}{3}}}{1+\sqrt{3}i-1} \\ \textbf{Answer:} \quad &= \frac{2^{12}}{\sqrt{3}i} \end{aligned}$$

```
clear;

close;

clc;




a=linspace(-2,2.1,10000);

i=sqrt(-1);


z=1+(a*i);

m=(1-3*a.^2)+(3*a-a.^3)*i;

x=0

k=imag(m);

plot(a,k,'ro',x,a)


f = @(k) z^k;

sum(f([0:11]))

ans = -9.0949e-13 - 2.3642e+03i
```

$$ans \approx -2364$$

Problem 17. $A = \begin{pmatrix} -4 & -1 \\ 3 & 1 \end{pmatrix}$. *Find the determinant of* $A^{2016} - 2A^{2015} - A^{2014}$.

Problem 18. *Find the solutions of the following equations*

$$n^2 - 3n - 108 = 0$$

$$n^2 + 5n - 84 = 0$$

$$n^2 + 2n - 80 = 0$$

$$n^2 + n - 110 = 0$$

Which of these satisfy $\frac{{}^{n+2}C_6}{{}^{n-2}P_2} = 11$*?*

Problem 19. *Sketch*

$$f(x) = \begin{cases} \frac{2x^2}{a} & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ \frac{2b^2-4b}{x^3} & \sqrt{2} \leq x < \infty \end{cases}$$

for (a, b) *equal to*

1) $\left(\sqrt{2}, 1 - \sqrt{3}\right)$
2) $\left(-\sqrt{2}, 1 + \sqrt{3}\right)$
3) $\left(\sqrt{2}, -1 + \sqrt{3}\right)$
4) $\left(-\sqrt{2}, 1 - \sqrt{3}\right)$

In which case is $f(x)$ *continuous?*

Solution:

To solve this problem we plot the graph of $f(x)$ for all the four cases using GNU Octave.

GRAPHS

$$f(x) = \frac{2x^2}{a}$$

$$f(x) = a$$

$$f(x) = \frac{2b^2-4b}{x^3}$$

$f(x)$ is a continuous function when $a = \sqrt{2}$ and $b = 1 - \sqrt{3}$.

$f(x)$ is a discontinuous function when $a = -\sqrt{2}$ and $b = 1 + \sqrt{3}$.

It is discountinuous at $x = \sqrt{2}$.

$f(x)$ is a discontinuous function when $a = \sqrt{2}$ and $b = -1 + \sqrt{3}$.

It is discountinuous at $x = \sqrt{2}$.

$f(x)$ is a discontinuous function when $a = -\sqrt{2}$ and $b = 1 - \sqrt{3}$.

It is discountinuous at $x = \sqrt{2}$.

GNU Octave's CODE

```
clear;

close;

clc;


a = sqrt(2); b = 1 - sqrt(3);

x1 = linspace(0, 1, 1000);

x2 = linspace(1, sqrt(2), 1000);

x3 = linspace(sqrt(2), 5, 100000);


y1 = 2.*(x1.^2)./a;

y2 = a;

y3 = (2*(b^2) - 4*b)./(x3.^3);


plot(x1, y1, "2", x2, y2, "5", x3, y3, "3");

grid minor;
```

```
title("Problem 19\n
     1.)  a = sqrt(2); b = 1 - sqrt(3)\n
     f(x) is Continuous");
xlabel("x");
ylabel("f(x)");
```

```
clear;
close;
clc;

a = -sqrt(2); b = 1 + sqrt(3);
x1 = linspace(0, 1, 1000);
x2 = linspace(1, sqrt(2), 1000);
x3 = linspace(sqrt(2), 5, 100000);

y1 = 2.*(x1.^2)./a;
y2 = a;
y3 = (2*(b^2) - 4*b)./(x3.^3);

plot(x1, y1, "2", x2, y2, "5", x3, y3, "3");
grid minor;
title("Problem 19\n
     2.)  a = -sqrt(2); b = 1 + sqrt(3)\n
     f(x) is Discontinuous");
xlabel("x");
ylabel("f(x)");
```

1.13 For 3)

```
clear;

close;

clc;


a = sqrt(2); b = -1 + sqrt(3);

x1 = linspace(0, 1, 1000);

x2 = linspace(1, sqrt(2), 1000);

x3 = linspace(sqrt(2), 5, 100000);


y1 = 2.*(x1.^2)./a;

y2 = a;

y3 = (2*(b^2) - 4*b)./(x3.^3);


plot(x1, y1, "2", x2, y2, "5", x3, y3, "3");

grid minor;

title("Problem 19\n

    3.)  a = sqrt(2); b = -1 + sqrt(3)\n

    f(x) is Discontinuous");

xlabel("x");

ylabel("f(x)");
```

```
clear;

close;

clc;


a = -sqrt(2); b = 1 - sqrt(3);

x1 = linspace(0, 1, 1000);

x2 = linspace(1, sqrt(2), 1000);
```

```
x3 = linspace(sqrt(2), 5, 100000);


y1 = 2.*(x1.^2)./a;

y2 = a;

y3 = (2*(b^2) - 4*b)./(x3.^3);


plot(x1, y1, "2", x2, y2, "5", x3, y3, "3");

grid minor;

title("Problem 19\n

    4.)  a = -sqrt(2); b = 1 - sqrt(3)\n

    f(x) is Discontinuous");

xlabel("x");

ylabel("f(x)");
```

Problem 20. *Sketch* $f(x) = \sin^4 x + \cos^4 x$. *Find the intervals within* $(0, \pi)$ *when it is increasing.*

Solution:

Finding the first-derivative of the given equation gives

$$y' = -2sin(2x)cos(2x)$$

Plotting region of first-derivative i.e $y' = -2sin(2x)cos(2x)$

Observing the preceding graph , the interval in which graph

$$y' >= 0$$

gives

$$[0.79, 1.57] and [2.36, 3.14]$$

Finally plotting the given equation $f(x) = sin^4(x) + cos^4(x)$

and then shadding the region where graph increases

```
x=linspace(0,0.79,10000);
z=linspace(0.79,1.57,10000);
t=linspace(1.57,2.36,10000);
s=linspace(2.36,3.14,10000);


y=sin(x).^4+cos(x).^4;
u=sin(z).^4+cos(z).^4;
v=sin(t).^4+cos(t).^4;
w=sin(s).^4+cos(s).^4;


plot (x,y,"3",z,u,"3",t,v,"3",s,w,"3");
hold on;
area(z,u,"facecolor","green");
area(s,w,"facecolor","green");
grid minor;
xlabel('0<x<pi');
ylabel('Y-axis');
title('y=sin^4(x)+cos^4(x)');
legend(sprintf('Green shaded region where \n Graph
is increasing'),'location','northeastoutside');
```

```
clear;
close;
clc;
x=linspace(0,pi,10000);
y=-2*sin(2*x).*cos(2*x);



plot (x,y);
```

```
hold on;
y1=y;
y1(y1<0)=0;
area(x,y1);
grid minor;
xlabel('0<x<pi');
ylabel('Y-axis');
title('y'' = -2sin(2x)cos(2x)');
legend('Region where Derivative >=0');
```

Problem 21. *The reflected line is given by* $y + 2x = 1$. *The surface is given by* $7x - y + 1 = 0$. *Which of the following is the incident line?*

1) $41x - 38y + 38 = 0$

2) $41x + 25y - 25 = 0$

3) $41x + 38y - 38 = 0$

4) $41x - 25y + 25 = 0$

Solution:

We have ,

$$y = 1 - 2x$$

This is the reflected line .

$$z = 7x + 1$$

This is the surface along which the line is reflected .

The point of intersection of the above lines is (0,1).

Angle between the given line is given

$$\theta = \tan^{-1} \frac{(m1 - m2)}{(1 + m1 * m2)} \tag{29}$$

where m1, m2 are slope of surface and reflected line respectively.

$$\theta = \tan^{-1} \frac{(7 - (-2))}{(1 + 7 * (-2)))} \tag{30}$$

$$\theta = \tan^{-1} \frac{-9}{13} \tag{31}$$

The slope of the incident line can be found by reversing the direction of the angle along the surface.

$$\theta = \tan^{-1} \frac{(m1 - tan\theta)}{(1 + m1 * tan\theta)} \tag{32}$$

$$m = \frac{(7 - \frac{9}{13})}{(1 + 7 * \frac{9}{13})} \tag{33}$$

$$m = \frac{(91 - 9)}{(63 + 13)} \tag{34}$$

$$m = 41/38 \tag{35}$$

Since m is the slope and 1 is the intercept and thus in slope form equation of line is y=mx+1. Thus the equation of the incident line is

$$y = \frac{41}{38}x \quad + 1$$

$$a.)38y = 41x + 38$$

Code for Octave

```
close;
clc;
clear;
x=linspace(-3,3,5);
```

```
y=1-2*x;
z=7*x+1;
thetha=atan((7-(-2))./(1+7*(-2)));
m2=(7+tan(thetha))./(1-7*tan(thetha));
k=m2*x+1;
plot(x,y,"r",x,z,"b",x,k,"g");
grid
```

Problem 22. *The lines $x - y = 1$ and $2x + y = 3$ intersect at O. A circle with centre at point O passes through the point $(-1, 1)$. Sketch the following lines*

1) $4x + y - 3 = 0$
2) $x + 4y + 3 = 0$
3) $3x - y - 4 = 0$
4) $x - 3y - 4 = 0$

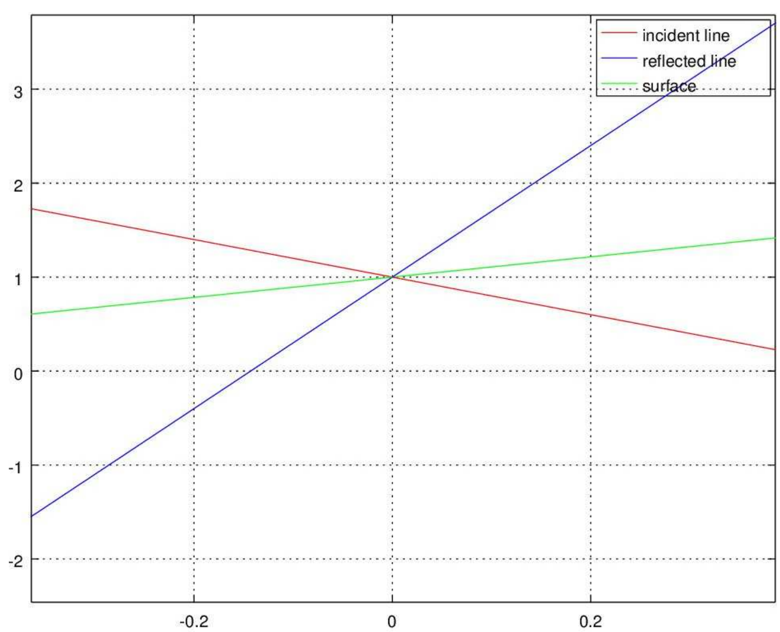


Fig. 7: GRAPH FOR QUESTION 21

Which of these is a tangent to the circle? At what point?

Solution:

3 How to solve the problem

The following steps are required to solve the problem:

STEP-1 Find the intersection of lines $x - y = 1$ and $2x + y = 3$ to find the point O.

STEP-2 From the equation of a circle ,find it's radius as it passes through (-1,1).

STEP-3 Plot the circle along with the given lines to check the tangency of lines.

STEP-4 If the line appear to be tangent solve it with the circle's equation to find the required point.

4 STEP-1

4.1 Solving problem manually

The lines $x - y = 1$ and $2x + y = 3$ intersect at O . We solve the lines to find the point O.

$$x - y = 1........(1)$$

$$2x + y = 3.......(2)$$

By adding (1) and(2), we have

$$3x = 4$$

$$x = \frac{4}{3}$$

On substituting x in (1) ,we have

$$\frac{4}{3} - y = 1$$

$$y = \frac{1}{3}$$

Therefore

$$O(x, y) = (\frac{4}{3}, \frac{1}{3})$$

4.2 *Visualising the point with octave*

4.2.1 *script:*

1.clear;

2.close;

3.clc;

4.x=linspace(-2,2);

5.y=x-1;

6.z=3-2*x;

7.plot(x,y,’g’,x,z,’r’)

8.axis(”equal”)

9.grid

5 STEP-2

Since the circle passes through $(-1, 1)$ and $(\frac{4}{3}, \frac{1}{3})$,we can find it’s radius from the standard equation of a circle.

The equation of a circle having centre (h,k) and radius a is given by

$$(x - h)^2 + (y - k)^2 = a^2$$

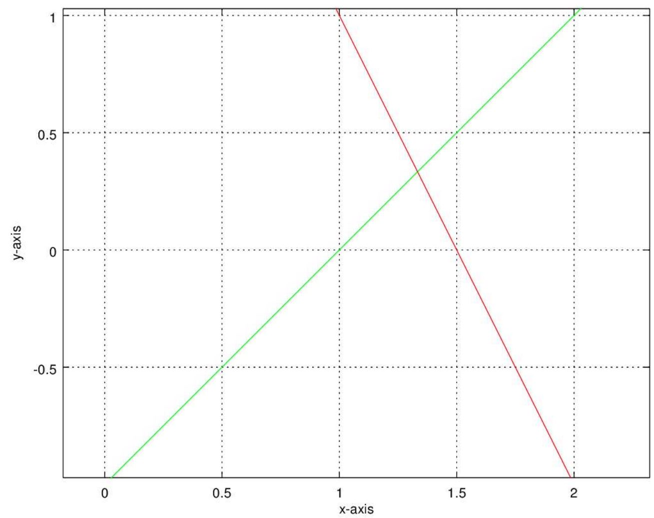


Fig. 8: Intersection of 2 lines

Since the centr of circle is $(\frac{4}{3}, \frac{1}{3})$.Therefore,

$$(h, k) = (\frac{4}{3}, \frac{1}{3})$$

So the new equation of circle will be

$$(x - \frac{4}{3})^2 + (y - \frac{1}{3})^2 = a^2$$

Now the equation of circle given , we find radius by substituting (-1,1) in the equation .So we have

$$(-1 - \frac{4}{3})^2 + (1 - \frac{1}{3})^2 = a^2$$

$$a^2 = \frac{53}{9}$$

$$a = \sqrt{\frac{53}{9}}$$

Therefore the equation of circle is given by

$$(x - \frac{4}{3})^2 + (y - \frac{1}{3})^2 = \frac{53}{9}$$

6 STEP-3

The following is the graph with circles and lines plotted with help of octave.

6.0.2 *script:*

clear;

close;

clc;

x=linspace(-4*pi,4*pi,1000000);

y=$((53/9)^{0.5} - (x - (4/3)).^{2}).^{0.5} + 1/3$;

z=$-((53/9)^{0.5} - (x - (4/3)).^{2}).^{0.5} + 1/3$;

r=3-4*x;

s=(3-x)./4;

t=3*x-4;

u=(x-4)./3;

plot(x,y,x,z,x,r,x,s,x,t,x,u)

axis(”equal”)

grid

The circle with given 4 lines

1)$4x + y - 3 = 0$

2)$x + 4y + 3 = 0$

3)$3x - y - 4 = 0$

4)$x - 3y - 4 = 0$

7 Conclusion

Since the lines intersect the circle ,they are not tangent to it and no point of tangency exist.

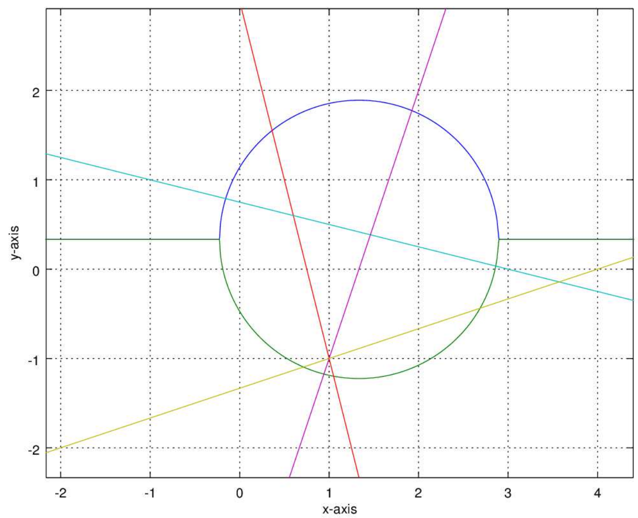


Fig. 9: Circle with four lines

Problem 23. *P and Q are distinct points on the parabola $y^2 = 4x$, with parameters t and t_1 respectively. The normal at P passes through Q. Find the minimum value of t_1^2.*

Solution:

Step 1. Parametric equation of normal to given parabola is $y = -xt + 2t + t^3$ —— eq(1)

Step 2. Since normal passes through point $Q(t_1)$.

therefore, on putting in equation (1) it gives $2t_1 + tt_1^2 = 2t + t^3$ ———eq(2)

Step 3. On solving the equ (2) it gives and $t_1 = \left(t + \frac{2}{t}\right)$

Step 4. Let $z = t_1^2$

i.e $z = t^2 + \frac{4}{t^2} + 4$ ——eq(3)

Step 5. Plot the first derivative of z Step 6.graph of z is symmetric about y-axis

On moving towards y-axis and infinity on x-axis z increases.

So from graph at t=1.4142 z has minimum value

Step 7. Therefore on putting value of t in eq(3) Gives the minimum value of t_1^2 as 8.012.

8 Code

```
x=linspace(0,10,10000);
y1=2*sqrt(x);
y2=-2*sqrt(x);
plot(x,y1,"3",x,y2,"3");
grid minor;
title('y^2=4*x');
```

```
t=linspace(0.5,10,1000);
t1 = t + 2./t;
z = t.^2+4./t.^2+4;
plot(t, z);
grid minor;
title('z=t^2 +4/t^2+4');
```

```
t=linspace(0.5,10,1000);
z = 2*t-8./t.^3;
plot(t, z);
grid minor;
title('z''=2t-8/t^3')
```

Problem 24. *The transverse axis of a hyperbola is along the major axis of the conic* $\frac{x^2}{3}+\frac{y^2}{4} = 1$. *The vertices of the hyperbola are at the foci of this conic. The eccentricity of the hyperbola is* $\frac{3}{2}$. *Which of the points* $(0,2),\left(\sqrt{5},2\sqrt{2}\right),\left(\sqrt{10},2\sqrt{3}\right),\left(5,2\sqrt{3}\right)$*, lie on the Hyperbola?*

Problem 25. *Find the minimum value of* $\tan A + \tan B$*, given that* $A + B = \frac{\pi}{6}, A > 0, B > 0$.

Solution:

Given,

$$A + B = \frac{\pi}{6} \tag{36}$$

$$\tan A + \tan B$$

$$= \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}$$

$$= \frac{\sin(A + B)}{\cos A \cos B}$$

$$= \frac{1}{2 \cos A \cos B}$$

$$= \frac{1}{\cos(A + B) + \cos(A - B)}$$

But $A + B = \frac{\pi}{6}$ [from (1)]

$$= \frac{1}{\frac{\sqrt{3}}{2} + \cos(A - B)}$$

$$= \frac{2}{\sqrt{3} + 2 \cos(A - B)}$$

$$\tan A + \tan B = \frac{2}{\sqrt{3} + 2 \cos(A - B)}$$

Since we want to minimize $\tan A + \tan B$, we need to maximize the denominator (i.e $\sqrt{3} + 2 \cos(A - B)$).

$\sqrt{3} + 2 \cos(A - B)$ is maximum if $\cos(A - B)$ is 1

When $A > 0$ and $B > 0$, $\cos(A - B) = 1$ only when $A - B = 0$

So,

$$A = B$$

But $A + B = \frac{\pi}{6}$ [from (1)]

$$A = \frac{\pi}{12}$$

Hence $\tan A + \tan B$ attains minimum when $A = B = \frac{\pi}{12}$

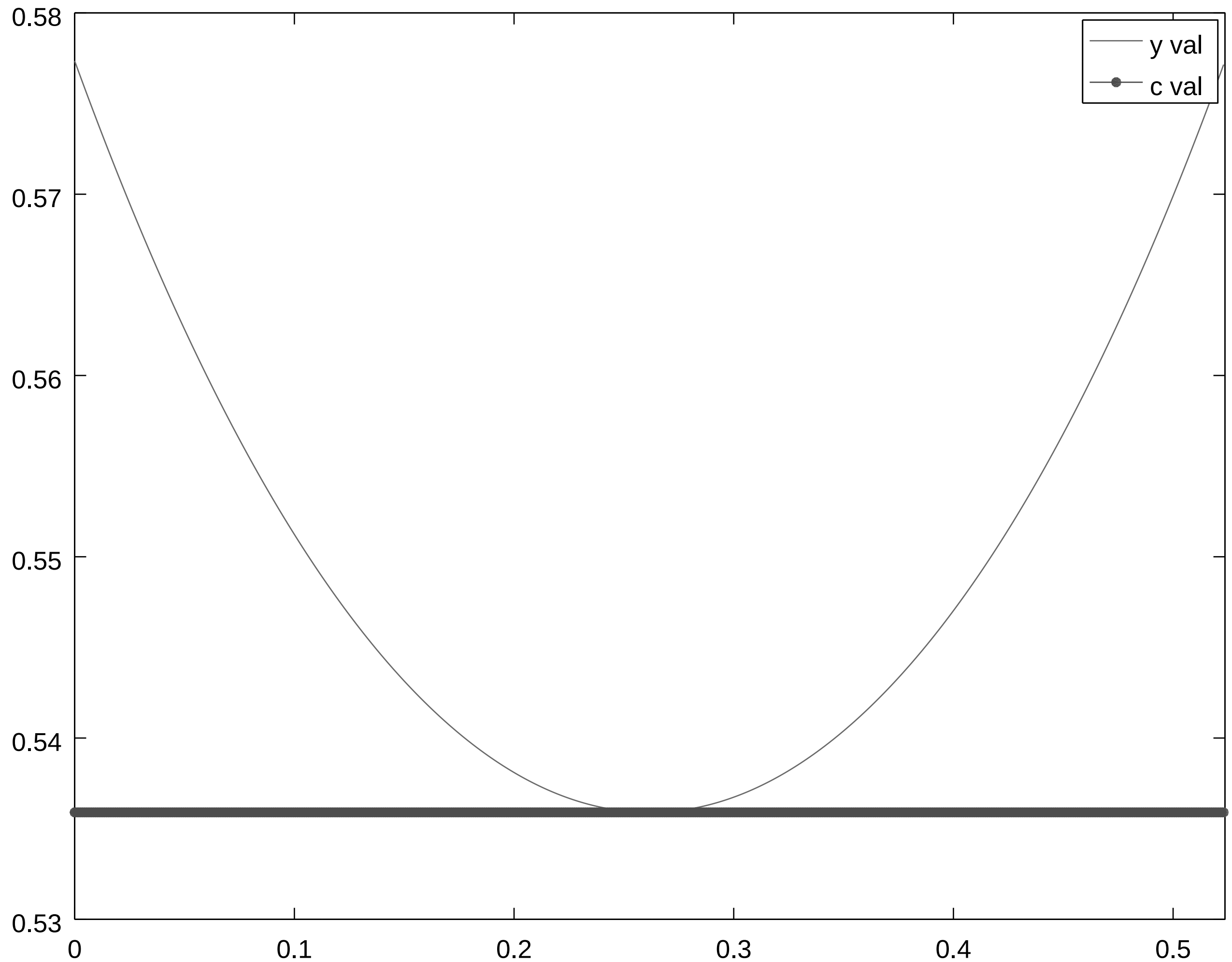


Fig. 10: Graph of $\tan A + \tan B$ v/s A

```
clear;

close;

clc;


a = 0:0.000001:pi/6;

b = (pi./6)-a;


y = tan(a) + tan(b);


min(y)


plot(a,y);

print('problem.eps','-deps');
```

Upon running the above cod in octave, the output will be **0.53590**

Problem 26. *Find θ for which $\frac{2+3\imath sin\theta}{1-2\imath\sin\theta}$ is purely imaginary.*

Solution:

Consider a complex number , z $z = \frac{2+3\imath\sin\theta}{1-2\imath\sin\theta}$

Multiply and divide by

$$(1 + 2\imath\sin\theta)$$

$$z = \frac{2 + 3\iota\sin\theta}{1 - 2\iota\sin\theta} * \frac{1 + 2\iota\sin\theta}{1 + 2\iota\sin\theta}$$

$$z = \frac{(2 + 3\iota\sin\theta)(1 + 2\iota\sin\theta)}{1 + 4(\sin\theta)^2}$$

$$z = \frac{(2 - 6\,(\sin\theta)^2) + 7\iota\sin\theta}{1 + 4\,(\sin\theta)^2}$$

For purely imaginary z, Re(z)=0

$$2 - 6\,(\sin\theta)^2 = 0$$

$$\sin\theta = \pm\frac{1}{\sqrt{3}}$$

$$\theta = \arcsin\pm\left(\frac{1}{\sqrt{3}}\right)$$

$$\theta = \pm 0.6154 \quad radians$$

$$Graph \quad of \quad Re(z) \quad w.r.t \quad \theta \quad is \quad given \quad by:$$

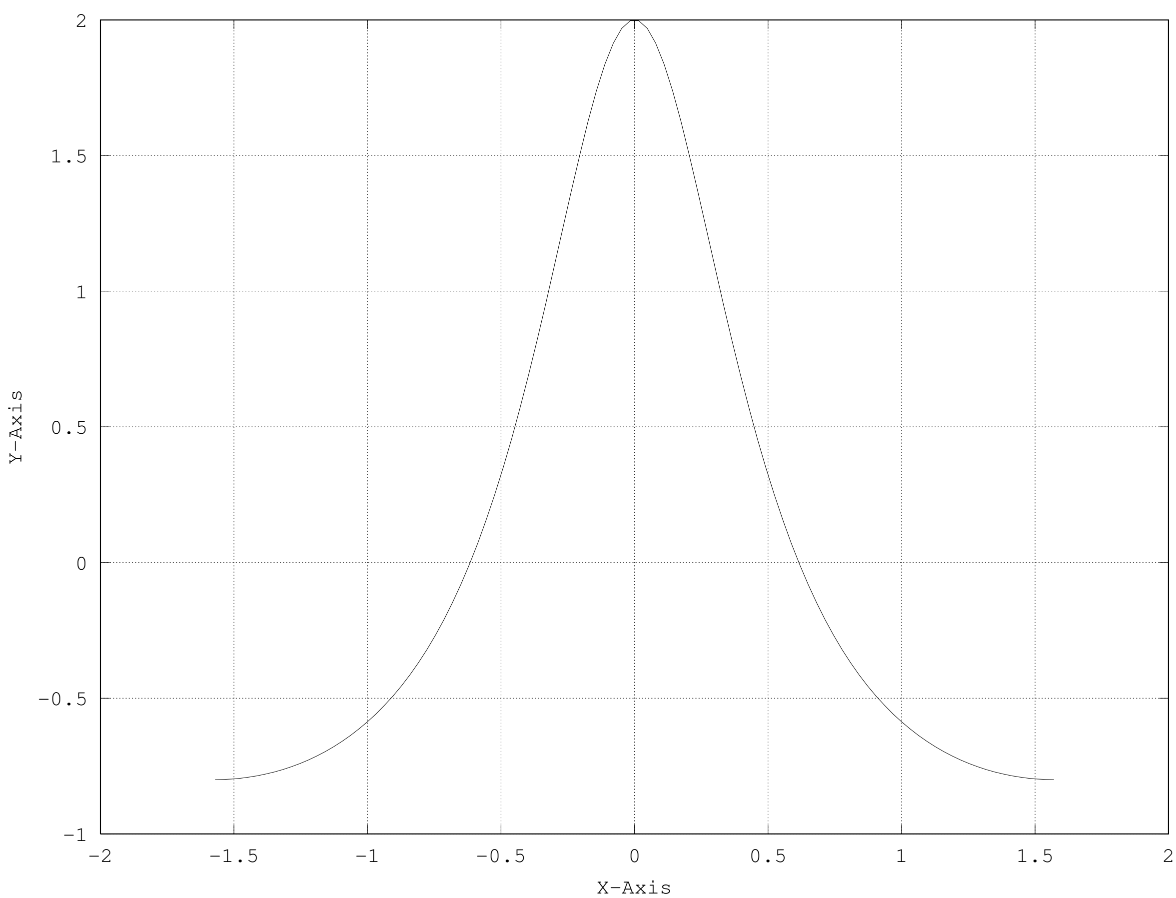


Fig. 11: fig26.1

Octave Code

```
clear;

close;

clc;


x=linspace(-pi/2,pi/2,100);

a=(2-6*((sin(x)).^2))./(1+4*((sin(x)).^2));

plot(x,a)

grid

xlabel('X-Axis')

ylabel('Y-Axis')

x=asin(1/(3.^(1/2)))

print  fig.eps
```

Problem 27. *Find the sum of all the solutions of*

$$\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$$

Solution:

Following are the conditions when the given equation satisfies;

Case 1:-

$$\left(x^2 - 5x + 5\right) = 1$$

that is;

$$\Rightarrow \left(x^2 - 5x + 4\right) = 0$$

$$\Rightarrow x^2 - x - 4x + 4 = 0$$

$$\Rightarrow x\,(x-1) - 4\,(x-1) = 0$$

$$\Rightarrow (x-1)\,(x-4) = 0$$

$$\Rightarrow x = 1, 4$$

$$(seeFig.1)$$

Case 2:-

$$\left(x^2 - 5x + 5\right) = -1$$

$$with \quad even \quad power.$$

that is;

$$\Rightarrow \left(x^2 - 5x + 6\right) = 0$$

$$\Rightarrow x^2 - 3x - 2x + 6 = 0$$

$$\Rightarrow x(x-3) - 2(x-3) = 0$$

$$\Rightarrow (x-3)(x-2) = 0$$

$$\Rightarrow x = 3, 2$$

$$(see Fig.2)$$

note:- x =3 is rejected,as on putting x =3 in equation power is not even

Case 3:-

$$\left(x^2 + 4x - 60\right) = 0, with \quad non \quad zero \quad base$$

that is;

$$\Rightarrow x^2 + 10x - 6x - 60 = 0$$

$$\Rightarrow x(x+10) - 6(x+10) = 0$$

$$\Rightarrow (x + 10)(x - 6) = 0$$

$$\Rightarrow x = -10, 6$$

$$(seeFig.3)$$

Conclusion:- Hence the required solutions are

$$x = 1, 4, 2, 6, -10$$

"And the required sum is 3"

OCTAVE CODE

clear;

close;

clc;

$$x = linspace(-10, 10, 1000)$$

$$y = \left(x^2 - 5x + 4\right);$$

$$plot(x, y);$$

grid

10

OCTAVE CODE

clear;

close;

clc;

$x = linspace\,(-10, 10, 1000)$

$y = \left(x^2 - 5x + 6\right);$

$plot\,(x, y)\,;$

grid

11

OCTAVE CODE

clear;

close;

clc;

$x = linspace\,(-10, 10, 1000)$

$y = \left(x^2 + 4x - 60\right);$

$plot\,(x, y)\,;$

grid

Problem 28. *The sum of the first 10 terms of the series* $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots$ *is* $\frac{16}{5}m$*. Find m.*

Solution:

$$S = \left(\frac{4}{5}\right)^2 \left(2^2 + 3^2 + 4^2 + ...11^2\right)$$

we know that $: \left(1^2 + 2^2 + 3^2 + 4^2 + ...n^2\right) = \left(\frac{n(n+1)(2n+1)}{6}\right)$

$$\rightarrow so \quad by \quad this \quad we \quad have;$$

$$= \frac{16}{25}\left(\frac{11.12.23}{6} - 1\right)$$

$$\rightarrow \frac{16}{25} * 505 \;\; = \frac{16}{5} * 101$$

$$hence \quad m = 101$$

Octave code:

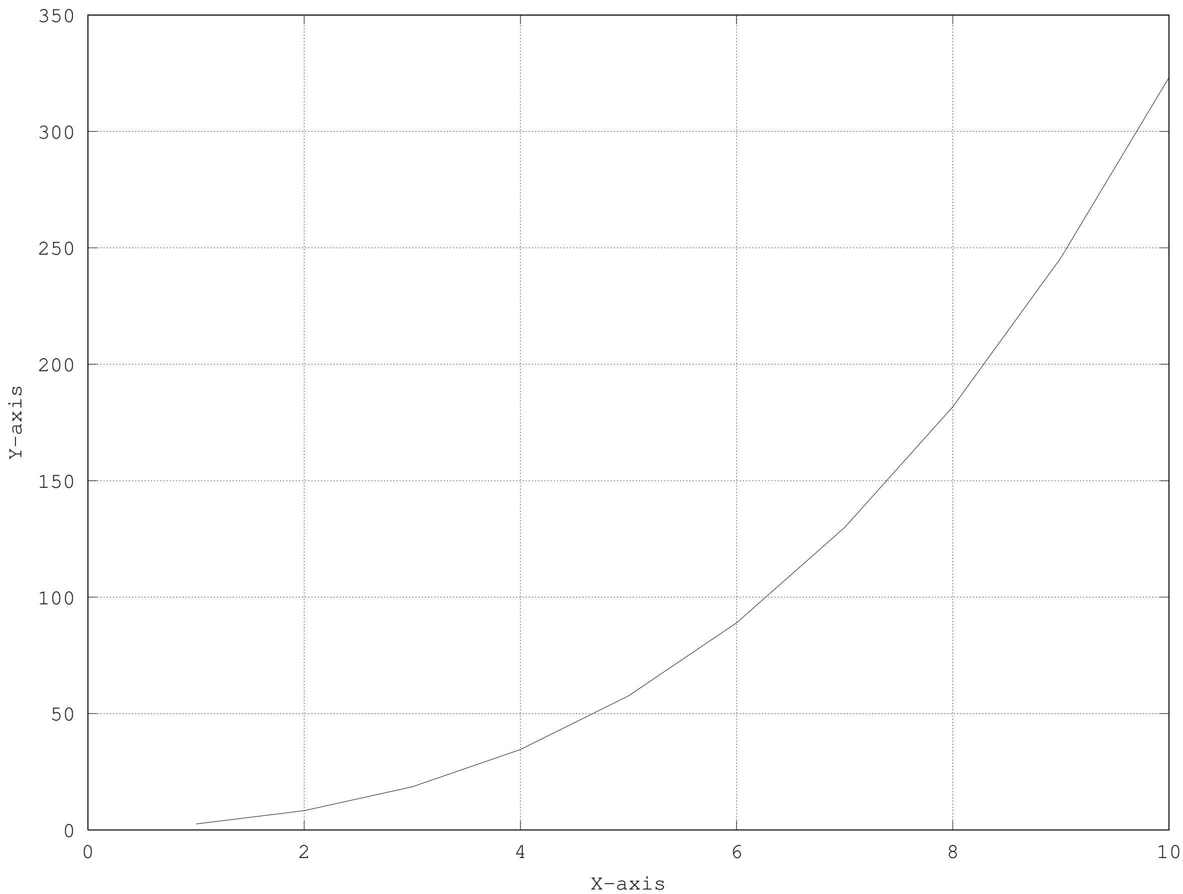


Fig. 12: fig28.1

```
clear;
close;
clc;


for n=1:10,
for k= 1:n,
x(k)= (4.*(1+k)./5).^2;
end
s(n)= sum(x);
end
plot(1:10,s)
xlabel('X-axis');
```

```
ylabel('Y-axis');

grid

print  figure.eps
```

Problem 29. $p = \lim_{x \to 0+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$. *Find* $\log p$.

Problem 30. $f(x) = \left|\log 2 - \sin x\right|, x \in \mathbf{R}$ *and* $g(x) = f(f(x))$. *Which of the following is true?*

1) g is not differntiable at $x = 0$

2) $g'(0) = \cos(\log 2)$

3) $g'(0) = -\cos(\log 2)$

4) g is differentiable at $x = 0$ *and* $g'(0) = -\sin(\log 2)$.

Solution:

Option-1

Theoritical Keys To Know: If the graph is continuous with no sharp point at x=0 then it can be said as differentiable at that point

Also for $g(x) = |log2 - sin|log2 - sinx||$ to be differentiable Left Hand Derivative=Right Hand Derivative at that point.

Octave Commands:

Listing 2: prob30a.m

```
clear;

clc;

close;

x=linspace (-1,1,500);
```

```
z=abs(log(2)-sin(abs(log(2)-sin(x))));

plot (x,z);
```

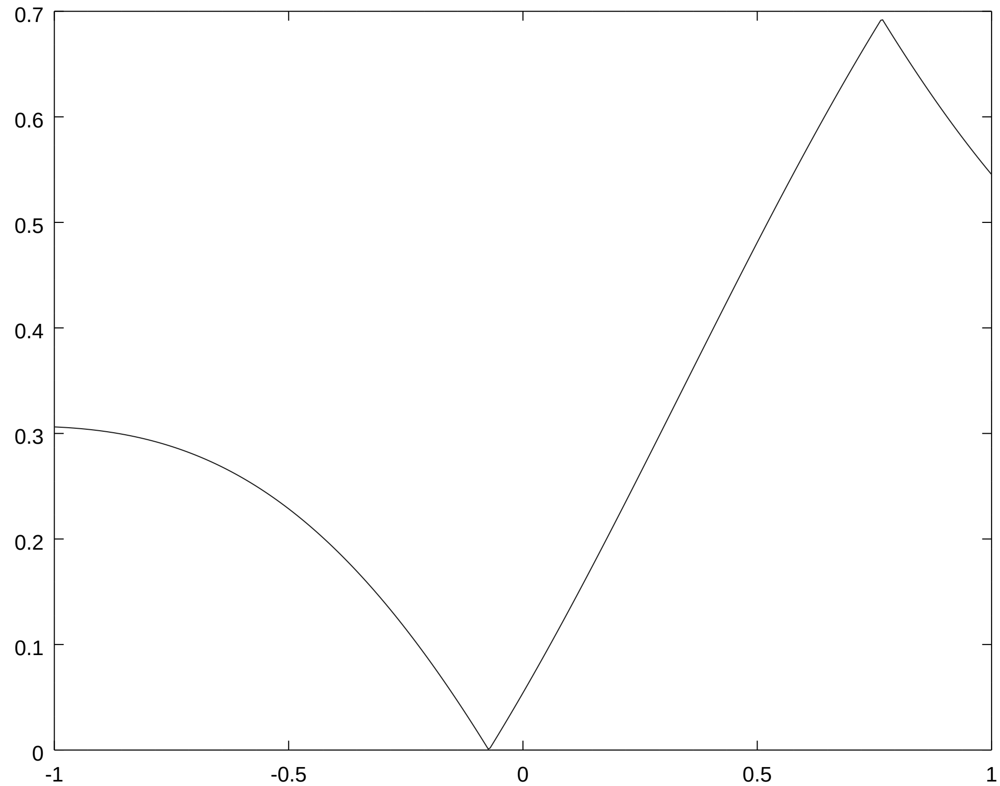


Fig. 13: Graph is continuous with no sharp point at x=0

Since the graph is continuous with no sharp point at x=0 thus it can be said as differentiable at x=0 ; thus option **one is wrong.**

Option 2

Theoritical Keys To Know: First way is to calculate $g'(x)|_{x=0}$ by differentiating directly if we know g(x) is differentiable

and the another way is by first principal if

Left Hand Derivative and Right Hand Derivative and equating to each other(can be done by graphs using octave).

Octave Commands: We will plot graph for LHD and RHD and cos(log2) at check at x=0 for their values.

$$RHD = g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$LHD = g'(x) = \lim_{h \to 0} \frac{g(x-h) - g(x)}{-h}$$

where $g(x) = |log2 - sin|log2 - sinx||$.

Listing 3: prob30b.m

```
clear;

clc;

close;

x=linspace (-10,10,500);

h=10.^(-30);

z=abs(log(2)-sin(abs(log(2)-sin(x+(h)))))
  -abs(log(2)-sin(abs(log(2)-sin(x)))))./(h);


u=cos(log(2));


t=-abs(log(2)-sin(abs(log(2)-sin(x-(h)))))
   +abs(log(2)-sin(abs(log(2)-sin(x)))))./(h);


plot(x,z,x,u,x,t);
```

From the graph it's clear that LHD=RHD=cos(log2). Thus the **answer is option2**

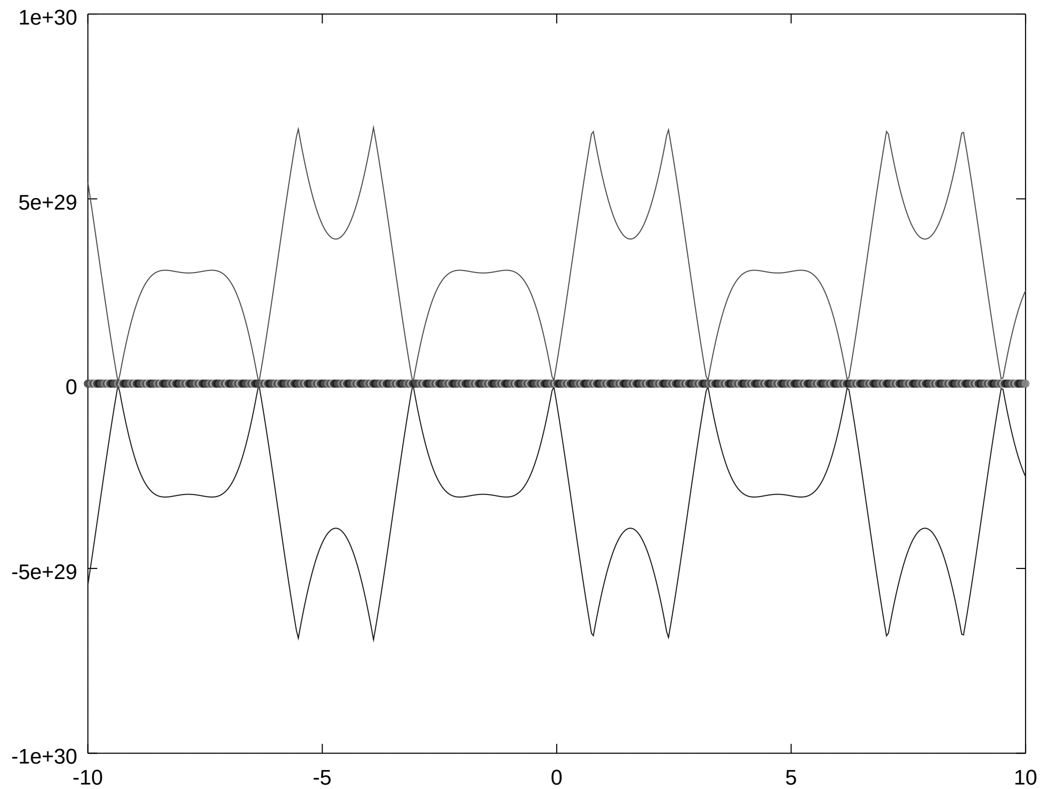


Fig. 14: As h tends to zero LHD equals RHD equals to cos(log2) at x=0

Manual Solution

Right Hand Derivative at x=0

$$g'(0^+) = \lim_{h\to 0^+} \frac{|log2 - sin|log2 - sin(0+h)|| - |log2 - sin|log2 - sin(0)||}{h}$$

$$g'(0^+) = \lim_{h\to 0^+} \frac{|log2 - sin|log2 - sin(h)|| - |log2 - sin|log2||}{h}$$

Using L Hospital Rule we get(as it is of the for 0/0)

$$g'(0^+) = \lim_{h\to 0^+} -cos|log2 - sinh| * -cosh$$

$$g'(0^+) = cos(log2)$$

Left Hand Derivative at x=0

$$g'(0^-) = \lim_{h \to 0^-} \frac{|log2 - sin|log2 - sin(0-h)|| - |log2 - sin|log2 - sin(0)||}{-h}$$

$$g'(0^-) = \lim_{h \to 0^-} \frac{|log2 - sin|log2 - sin(-h)|| - |log2 - sin|log2||}{-h}$$

Using L Hospital Rule we get(as it is of the for 0/0)

$$g'(0^-) = \lim_{h \to 0^-} -1 * -cos|log2 + sinh| * cosh$$

$$g'(0^-) = cos(log2)$$

as LHD=RHD thus function is differentiable at x=0 and the value of function is cos(log2).

Conclusion

From graphical approach by using octave ; with the help
of theoritical knowledge as well as by manually solving the

problem it's clear that g(x) is differentiable with

$$g'(x)|_{x=0} = cos(log2).$$

Problem 31. *Consider*

$$f(x) = \tan^{-1}\sqrt{\left(\frac{1+\sin x}{1-\sin x}\right)}, x \in \left(0, \frac{\pi}{2}\right)$$

Sketch the normal to $f(x)$ at $x = \frac{\pi}{6}$. Does it pass through any of the points $(0,0), \left(0, \frac{2\pi}{3}\right), \left(\frac{\pi}{6}, 0\right), \left(\frac{\pi}{4}, 0\right)$?

Solution:

$$f(x) = tan^{-1}\sqrt{\frac{1+\sin(x)}{1-\sin(x)}}$$

$$f(x) = \pi/4 + x/2$$

Differentiating $f(x)$, we get

$$dy/dx = 1/2$$

Slope for the normal

$$m = -1/(dy/dx)$$

So, $m = -2$. Equation of the normal is given by

$$y - y. = m(x - x.)$$

$$x. = \pi/6, \quad y. = \pi/3$$

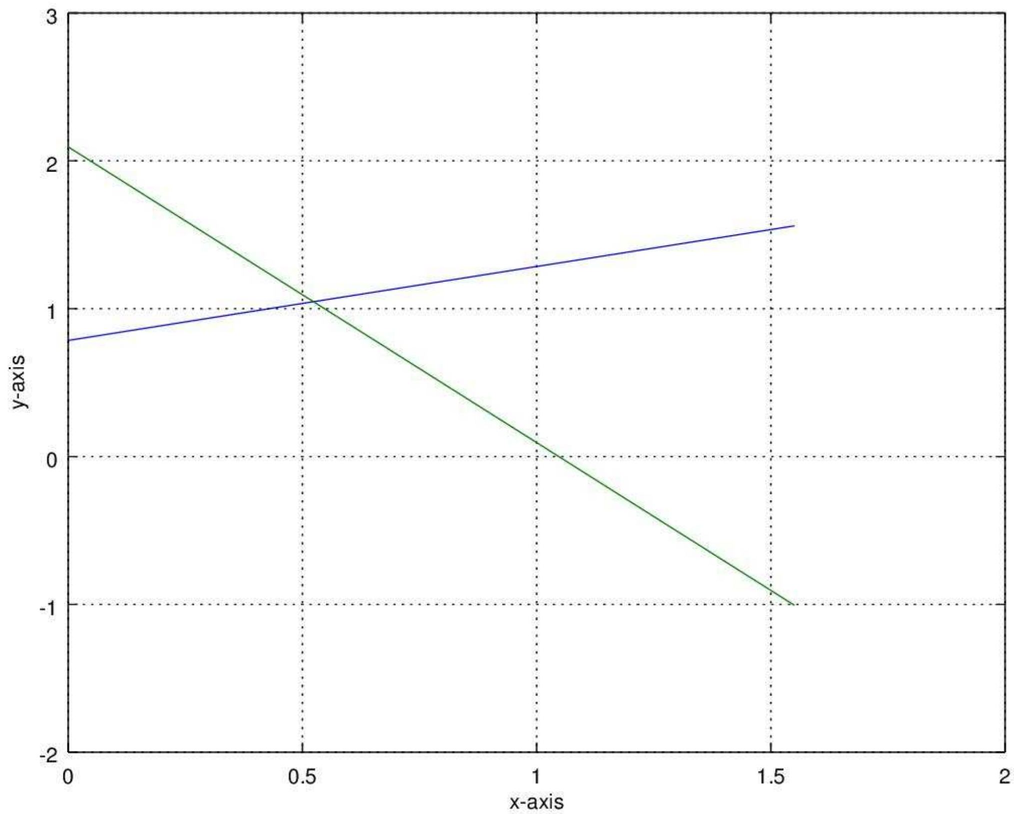
$$y - \pi/3 = m(x - \pi/6)$$

where, $(x., y.)$ is a point on the graph. So, equation of normal is

$$y + 2x = 2\pi/3$$

CODE IN OCTAVE

```
clc;

clear;

close;


x=0:0.05:pi/2;

y=atan(sqrt((1+sin(x))./(1-sin(x))));

z=2*pi/3-2*x;


plot(x,y,x,z);

grid;
```



a) From the equation and graph it is clear that it passes through (0, 2π/3):

Problem 32. *Sketch* $\frac{\{(n+1)(n+2)...(3n)\}^{\frac{1}{n}}}{n^{2n}}$ *and verify if its limit at* $n \to \infty$ *is* $\frac{18}{e^4}, \frac{27}{e^2}, \frac{9}{e^2}$ *or* $3\log 3 - 2$.

Problem 33. *Sketch the region*

$$\left\{(x, y) : y^2 \geq 2x, x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\right\}$$

Solution:

First draw graph of :

$$y^2 = 2x$$

Shade the portion which covers

$$y^2 \geq 2x$$

Octave code for this is:

ERROR:
ioerror
OFFENDING COMMAND:
--nostringval--
STACK:
-mark-
-mark-
-mark-