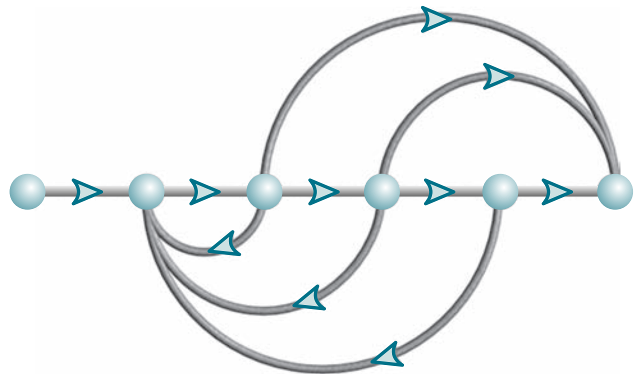
Frequency Response Techniques 10



Chapter Learning Outcomes

After completing this chapter the student will be able to:

- Define and plot the frequency response of a system (Section 10.1)
- Plot asymptotic approximations to the frequency response of a system (Section 10.2)
- Sketch a Nyquist diagram (Section 10.3–10.4)
- Use the Nyquist criterion to determine the stability of a system (Section 10.5)
- Find stability and gain and phase margins using Nyquist diagrams and Bode plots (Sections 10.6–10.7)
- Find the bandwidth, peak magnitude, and peak frequency of a closed-loop frequency response given the closed-loop time response parameters of peak time, settling time, and percent overshoot (Section 10.8)
- Find the closed-loop frequency response given the open-loop frequency response (Section 10.9)
- Find the closed-loop time response parameters of peak time, settling time, and percent overshoot given the open-loop frequency response (Section 10.10)

Case Study Learning Outcomes

You will be able to demonstrate your knowledge of the chapter objectives with a case study as follows:

- Given the antenna azimuth position control system shown on the front endpapers and using frequency response methods, you will be able to find the range of gain, K, for stability. You will also be able to find percent overshoot, settling time, peak time, and rise time, given K.

10.1 Introduction

The root locus method for transient design, steady-state design, and stability was covered in Chapters 8 and 9. In Chapter 8, we covered the simple case of design through gain adjustment, where a trade-off was made between a desired transient response and a desired steady-state error. In Chapter 9, the need for this trade-off was eliminated by using compensation networks so that transient and steady-state errors could be separately specified and designed. Further, a desired transient response no longer had to be on the original system's root locus.

This chapter and Chapter 11 present the design of feedback control systems through gain adjustment and compensation networks from another perspective—that of frequency response. The results of frequency response compensation techniques are not new or different from the results of root locus techniques.

Frequency response methods, developed by Nyquist and Bode in the 1930s, are older than the root locus method, which was discovered by Evans in 1948 (*Nyquist, 1932*; *Bode, 1945*). The older method, which is covered in this chapter, is not as intuitive as the root locus. However, frequency response yields a new vantage point from which to view feedback control systems. This technique has distinct advantages in the following situations:

1. When modeling transfer functions from physical data, as shown in Figure 10.1
2. When designing lead compensators to meet a steady-state error requirement and a transient response requirement
3. When finding the stability of nonlinear systems
4. In settling ambiguities when sketching a root locus

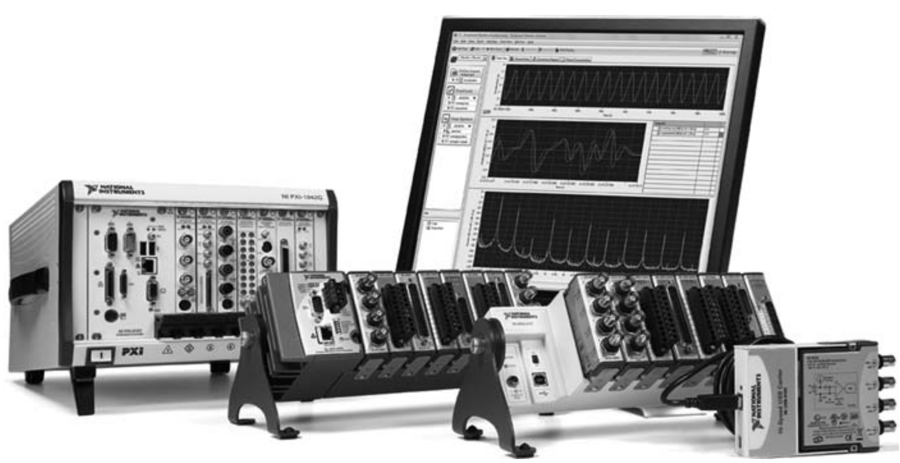


FIGURE 10.1 National Instruments PXI, Compact RIO, Compact DAQ, and USB hardware plantforms (shown from left to right) couple with NI LabVIEW software to provide stimulus and acquire signals from physical systems. NI LabVIEW can then be used to analyze data, determine the mathematical model, and prototype and deploy a controller for the physical system (Courtesy National Instruments © 2010).

We first discuss the concept of frequency response, define frequency response, derive analytical expressions for the frequency response, plot the frequency response, develop ways of sketching the frequency response, and then apply the concept to control system analysis and design.

The Concept of Frequency Response

In the steady state, sinusoidal inputs to a linear system generate sinusoidal responses of the same frequency. Even though these responses are of the same frequency as the input, they differ in amplitude and phase angle from the input. These differences are functions of frequency.

Before defining frequency response, let us look at a convenient representation of sinusoids. Sinusoids can be represented as complex numbers called *phasors*. The magnitude of the complex number is the amplitude of the sinusoid, and the angle of the complex number is the phase angle of the sinusoid. Thus, $M_1 \cos(\omega t + \phi_1)$ can be represented as $M_1 \angle \phi_1$ where the frequency, ω, is implicit.

Since a system causes both the amplitude and phase angle of the input to be changed, we can think of the system itself as represented by a complex number, defined so that the product of the input phasor and the system function yields the phasor representation of the output.

Consider the mechanical system of Figure 10.2(*a*). If the input force, $f(t)$, is sinusoidal, the steady-state output response, $x(t)$, of the system is also sinusoidal and at the same frequency as the input. In Figure 10.2(*b*) the input and output sinusoids are represented by complex numbers, or phasors, $M_i(\omega)\angle\phi_i(\omega)$ and $M_o(\omega)\angle\phi_o(\omega)$, respectively. Here the M's are the amplitudes of the sinusoids, and the ϕ's are the phase angles

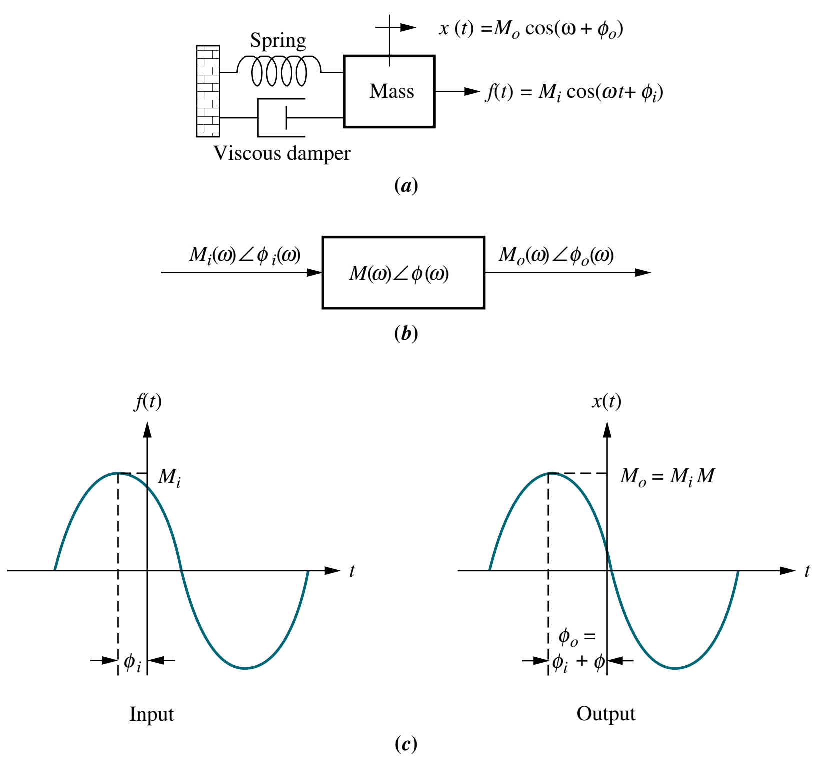


FIGURE 10.2 Sinusoidal frequency response: **a.** system; **b.** transfer function; **c.** input and output waveforms

of the sinusoids as shown in Figure 10.2(c). Assume that the system is represented by the complex number, $M(\omega)\angle\phi(\omega)$. The output steady-state sinusoid is found by multiplying the complex number representation of the input by the complex number representation of the system. Thus, the steady-state output sinusoid is

$$M_o(\omega)\angle\phi_o(\omega) = M_i(\omega)M(\omega)\angle[\phi_i(\omega) + \phi(\omega)] \tag{10.1}$$

From Eq. (10.1) we see that the system function is given by

$$M(\omega) = \frac{M_o(\omega)}{M_i(\omega)} \tag{10.2}$$

and

$$\phi(\omega) = \phi_o(\omega) - \phi_i(\omega) \tag{10.3}$$

Equations (10.2) and (10.3) form our definition of frequency response. We call $M(\omega)$ the *magnitude frequency response* and $\phi(\omega)$ the *phase frequency response*. The combination of the magnitude and phase frequency responses is called the *frequency response* and is $M(\omega)\angle\phi(\omega)$.

In other words, we define the magnitude frequency response to be the ratio of the output sinusoid's magnitude to the input sinusoid's magnitude. We define the phase response to be the difference in phase angle between the output and the input sinusoids. Both responses are a function of frequency and apply only to the steady-state sinusoidal response of the system.

Analytical Expressions for Frequency Response

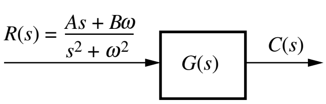


FIGURE 10.3 System with sinusoidal input

Now that we have defined frequency response, let us obtain the analytical expression for it (*Nilsson, 1990*). Later in the chapter, we will use this analytical expression to determine stability, transient response, and steady-state error. Figure 10.3 shows a system, $G(s)$, with the Laplace transform of a general sinusoid, $r(t) = A\cos\omega t + B\sin\omega t = \sqrt{A^2+B^2}\cos[\omega t - \tan^{-1}(B/A)]$ as the input. We can represent the input as a phasor in three ways: (1) in polar form, $M_i\angle\phi_i$, where $M_i = \sqrt{A^2+B^2}$ and $\phi_i = -\tan^{-1}(B/A)$; (2) in rectangular form, $A - jB$; and (3) using Euler's formula, $M_i e^{j\phi_i}$.

We now solve for the forced response portion of $C(s)$, from which we evaluate the frequency response. From Figure 10.3,

$$C(s) = \frac{As + B\omega}{(s^2 + \omega^2)}G(s) \tag{10.4}$$

We separate the forced solution from the transient solution by performing a partial-fraction expansion on Eq. (10.4). Thus,

$$\begin{aligned} C(s) &= \frac{As + B\omega}{(s + j\omega)(s - j\omega)}G(s) \\ &= \frac{K_1}{s + j\omega} + \frac{K_2}{s - j\omega} + \text{Partial fraction terms from } G(s) \end{aligned} \tag{10.5}$$

where

$$K_1 = \frac{As+B\omega}{s-j\omega}G(s)\bigg|_{s\to -j\omega} = \frac{1}{2}(A+jB)G(-j\omega) = \frac{1}{2}M_i e^{-j\phi_i} M_G e^{-j\phi_G} = \frac{M_i M_G}{2} e^{-j(\phi_i+\phi_G)} \tag{10.6a}$$

$$K_2 = \frac{As+B\omega}{s+j\omega}G(s)\bigg|_{s\to +j\omega} = \frac{1}{2}(A-jB)G(j\omega) = \frac{1}{2}M_i e^{j\phi_i} M_G e^{j\phi_G} = \frac{M_i M_G}{2} e^{j(\phi_i+\phi_G)} = K_1^* \tag{10.6b}$$

For Eqs. (10.6), K_1^* is the complex conjugate of K_1,

$$M_G = |G(j\omega)| \tag{10.7}$$

and

$$\phi_G = \text{angle of } G(j\omega) \tag{10.8}$$

The steady-state response is that portion of the partial-fraction expansion that comes from the input waveform's poles, or just the first two terms of Eq. (10.5). Hence, the sinusoidal steady-state output, $C_{ss}(s)$, is

$$C_{ss}(s) = \frac{K_1}{s+j\omega} + \frac{K_2}{s-j\omega} \tag{10.9}$$

Substituting Eqs. (10.6) into Eq. (10.9), we obtain

$$C_{ss}(s) = \frac{\frac{M_i M_G}{2} e^{-j(\phi_i+\phi_G)}}{s+j\omega} + \frac{\frac{M_i M_G}{2} e^{j(\phi_i+\phi_G)}}{s-j\omega} \tag{10.10}$$

Taking the inverse Laplace transformation, we obtain

$$c(t) = M_i M_G\left(\frac{e^{-j(\omega t+\phi_i+\phi_G)} + e^{j(\omega t+\phi_i+\phi_G)}}{2}\right) = M_i M_G \cos(\omega t + \phi_i + \phi_G) \tag{10.11}$$

which can be represented in phasor form as $M_o\angle\phi_o = (M_1\angle\phi_1)(M_G\angle\phi_G)$, where $M_G\angle\phi_G$ is the frequency response function. But from Eqs. (10.7) and (10.8), $M_G\angle\phi_G = G(j\omega)$. In other words, the frequency response of a system whose transfer function is $G(s)$ is

$$\boxed{G(j\omega) = G(s)|_{s\to j\omega}} \tag{10.12}$$

Plotting Frequency Response

$G(j\omega) = M_G(\omega) < \phi_G(\omega)$ can be plotted in several ways; two of them are (1) as a function of frequency, with separate magnitude and phase plots; and (2) as a polar plot, where the phasor length is the magnitude and the phasor angle is the phase. When plotting separate magnitude and phase plots, the magnitude curve can be plotted in decibels (dB) vs. log ω, where dB $= 20 \log M$.[1] The phase curve is plotted as phase angle vs. log ω. The motivation for these plots is shown in Section 10.2.

Using the concepts covered in Section 8.1, data for the plots also can be obtained using vectors on the s-plane drawn from the poles and zeros of $G(s)$ to the imaginary axis. Here the magnitude response at a particular frequency is the product of the vector lengths from the zeros of $G(s)$ divided by the product of the vector lengths from the poles of $G(s)$ drawn to points on the imaginary axis. The phase response is the sum of the angles from the zeros of $G(s)$ minus the sum of the angles from the poles of $G(s)$ drawn to points on the imaginary axis. Performing this operation for successive points along the imaginary axis yields the data for the frequency response. Remember, each point is equivalent to substituting that point, $s = j\omega_1$, into $G(s)$ and evaluating its value.

The plots also can be made from a computer program that calculates the frequency response. For example, the root locus program discussed in Appendix H at www.wiley.com/college/nise can be used with test points that are on the imaginary axis. The calculated K value at each frequency is the reciprocal of the scaled magnitude response, and the calculated angle is, directly, the phase angle response at that frequency.

The following example demonstrates how to obtain an analytical expression for frequency response and make a plot of the result.

Example 10.1

Frequency Response from The Transfer Function

PROBLEM: Find the analytical expression for the magnitude frequency response and the phase frequency response for a system $G(s) = 1/(s+2)$. Also, plot both the separate magnitude and phase diagrams and the polar plot.

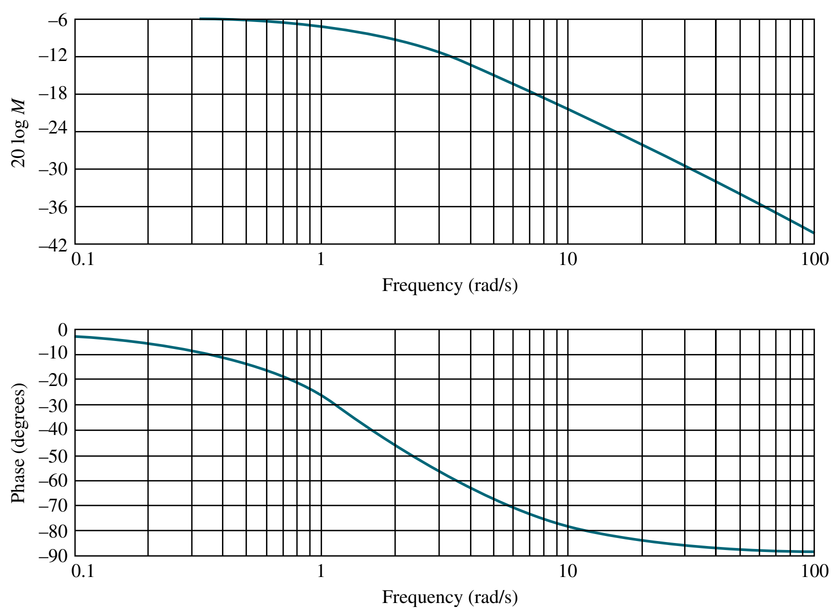


FIGURE 10.4 Frequency response plots for $G(s) = 1/(s+2)$: separate magnitude and phase diagrams.

[1] Throughout this book, "log" is used to mean $\log_{10}$, or logarithm to the base 10.

SOLUTION: First substitute $s = j\omega$ in the system function and obtain $G(j\omega) = 1/(j\omega + 2) = (2 - j\omega)/(\omega^2 + 4)$. The magnitude of this complex number, $|G(j\omega)| = M(\omega) = 1/\sqrt{(\omega^2 + 4)}$, is the magnitude frequency response. The phase angle of $G(j\omega)$, $\phi(\omega) = -\tan^{-1}(\omega/2)$, is the phase frequency response.

$G(j\omega)$ can be plotted in two ways: (1) in separate magnitude and phase plots and (2) in a polar plot. Figure 10.4 shows separate magnitude and phase diagrams, where the magnitude diagram is $20\log M(\omega) = 20\log(1/\sqrt{\omega^2 + 4})$ vs. $\log\omega$, and the phase diagram is $\phi(\omega) = -\tan^{-1}(\omega/2)$ vs. $\log\omega$. The polar plot, shown in Figure 10.5, is a plot of $M(\omega) < \phi(\omega) = 1/\sqrt{\omega^2 + 4} < -\tan^{-1}(\omega/2)$ for different ω.

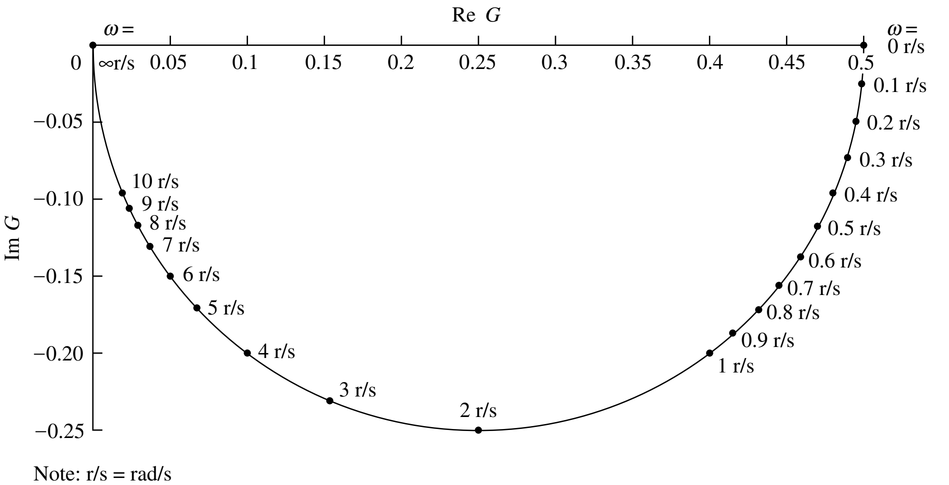


FIGURE 10.5 Frequency response plot for $G(s) = 1/(s + 2)$: polar plot

In the previous example, we plotted the separate magnitude and phase responses, as well as the polar plot, using the mathematical expression for the frequency response. Either of these frequency response presentations can also be obtained from the other. You should practice this conversion by looking at Figure 10.4 and obtaining Figure 10.5 using successive points. For example, at a frequency of 1 rad/s in Figure 10.4, the magnitude is approximately -7 dB, or $10^{-7/20} = 0.447$. The phase plot at 1 rad/s tells us that the phase is about -26°. Thus, on the polar plot a point of radius 0.447 at an angle of -26° is plotted and identified as 1 rad/s. Continuing in like manner for other frequencies in Figure 10.4, you can obtain Figure 10.5.

Similarly, Figure 10.4 can be obtained from Figure 10.5 by selecting a sequence of points in Figure 10.5 and translating them to separate magnitude and phase values. For example, drawing a vector from the origin to the point at 2 rad/s in Figure 10.5, we see that the magnitude is $20\log 0.35 = -9.12$ dB and the phase angle is about -45°. The magnitude and phase angle are then plotted at 2 rad/s in Figure 10.4 on the separate magnitude and phase curves.

Skill-Assessment Exercise 10.1

PROBLEM:

a. Find analytical expressions for the magnitude and phase responses of

$$G(s) = \frac{1}{(s+2)(s+4)}$$

b. Make plots of the log-magnitude and the phase, using log-frequency in rad/s as the ordinate.

c. Make a polar plot of the frequency response.

ANSWERS:

a. $M(\omega) = \dfrac{1}{\sqrt{(8-\omega^2)^2 + (6\omega)^2}}$; for $\omega \leq \sqrt{8}$: $\phi(\omega) = -\arctan\left(\dfrac{6\omega}{8-\omega^2}\right)$, for $\omega > \sqrt{8}$: $\phi(\omega) = -\left[\pi + \arctan\left(\dfrac{6\omega}{8-\omega^2}\right)\right]$

b. See the answer at www.wiley.com/college/nise.

c. See the answer at www.wiley.com/college/nise.

The complete solution is at www.wiley.com/college/nise.

In this section, we defined frequency response and saw how to obtain an analytical expression for the frequency response of a system simply by substituting $s = j\omega$ into $G(s)$. We also saw how to make a plot of $G(j\omega)$. The next section shows how to approximate the magnitude and phase plots in order to sketch them rapidly.

10.2 Asymptotic Approximations: Bode Plots

The log-magnitude and phase frequency response curves as functions of $\log \omega$ are called Bode plots or Bode diagrams. Sketching Bode plots can be simplified because they can be approximated as a sequence of straight lines. Straight-line approximations simplify the evaluation of the magnitude and phase frequency response.

Consider the following transfer function:

$$G(s) = \frac{K(s+z_1)(s+z_2)\cdots(s+z_k)}{s^m(s+p_1)(s+p_2)\cdots(s+p_n)} \tag{10.13}$$

The magnitude frequency response is the product of the magnitude frequency responses of each term, or

$$|G(j\omega)| = \left.\frac{K|(s+z_1)||(s+z_2)|\cdots|(s+z_k)|}{|s^m||(s+p_1)||(s+p_2)|\cdots|(s+p_n)|}\right|_{s\to j\omega} \tag{10.14}$$

Thus, if we know the magnitude response of each pole and zero term, we can find the total magnitude response. The process can be simplified by working with the logarithm of the magnitude since the zero terms' magnitude responses would be added and the pole terms' magnitude responses subtracted, rather than, respectively, multiplied or divided, to yield the logarithm of the total magnitude response. Converting the magnitude response into dB, we obtain

$$\begin{aligned} 20\log|G(j\omega)| &= 20\log K + 20\log|(s+z_1)| + 20\log|(s+z_2)| \\ &\quad + \cdots - 20\log|s^m| - 20\log|(s+p_1)| - \cdots|_{s\to j\omega} \end{aligned} \tag{10.15}$$

Thus, if we knew the response of each term, the algebraic sum would yield the total response in dB. Further, if we could make an approximation of each term that would consist only of straight lines, graphical addition of terms would be greatly simplified.

Before proceeding, let us look at the phase response. From Eq. (10.13), the phase frequency response is the *sum* of the phase frequency response curves of the zero terms minus the *sum* of the phase frequency response curves of the pole terms. Again, since the phase response is the sum of individual terms, straight-line approximations to these individual responses simplify graphical addition.

Let us now show how to approximate the frequency response of simple pole and zero terms by straight-line approximations. Later we show how to combine these responses to sketch the frequency response of more complicated functions. In subsequent sections, after a discussion of the Nyquist stability criterion, we learn how to use the Bode plots for the analysis and design of stability and transient response.

Bode Plots for $G(s) = (s + a)$

Consider a function, $G(s) = (s + a)$, for which we want to sketch separate logarithmic magnitude and phase response plots. Letting $s = j\omega$, we have

$$G(j\omega) = (j\omega + a) = a\left(j\frac{\omega}{a} + 1\right) \tag{10.16}$$

At low frequencies when ω approaches zero,

$$G(j\omega) \approx a \tag{10.17}$$

The magnitude response in dB is

$$20 \log M = 20 \log a \tag{10.18}$$

where $M = |G(j\omega)|$ and is a constant. Eq. (10.18) is shown plotted in Figure 10.6(a) from $\omega = 0.01a$ to a.

At high frequencies where $\omega \gg a$, Eq. (10.16) becomes

$$G(j\omega) \approx a\left(\frac{j\omega}{a}\right) = a\left(\frac{\omega}{a}\right)\angle 90^\circ = \omega\angle 90^\circ \tag{10.19}$$

The magnitude response in dB is

$$20 \log M = 20 \log a + 20 \log \frac{\omega}{a} = 20 \log \omega \tag{10.20}$$

where $a < \omega < \infty$. Notice from the middle term that the high-frequency approximation is equal to the low-frequency approximation when $\omega = a$, and increases for $\omega > a$.

If we plot dB, 20 log M, against $\log \omega$, Eq. (10.20) becomes a straight line:

$$y = 20x \tag{10.21}$$

where $y = 20 \log M$, and $x = \log \omega$. The line has a slope of 20 when plotted as dB vs. $\log \omega$.

Since each doubling of frequency causes 20 log ω to increase by 6 dB, the line rises at an equivalent slope of 6 dB/octave, where an *octave* is a doubling of frequency. This rise begins at $\omega = a$, where the low-frequency approximation equals the high-frequency approximation.

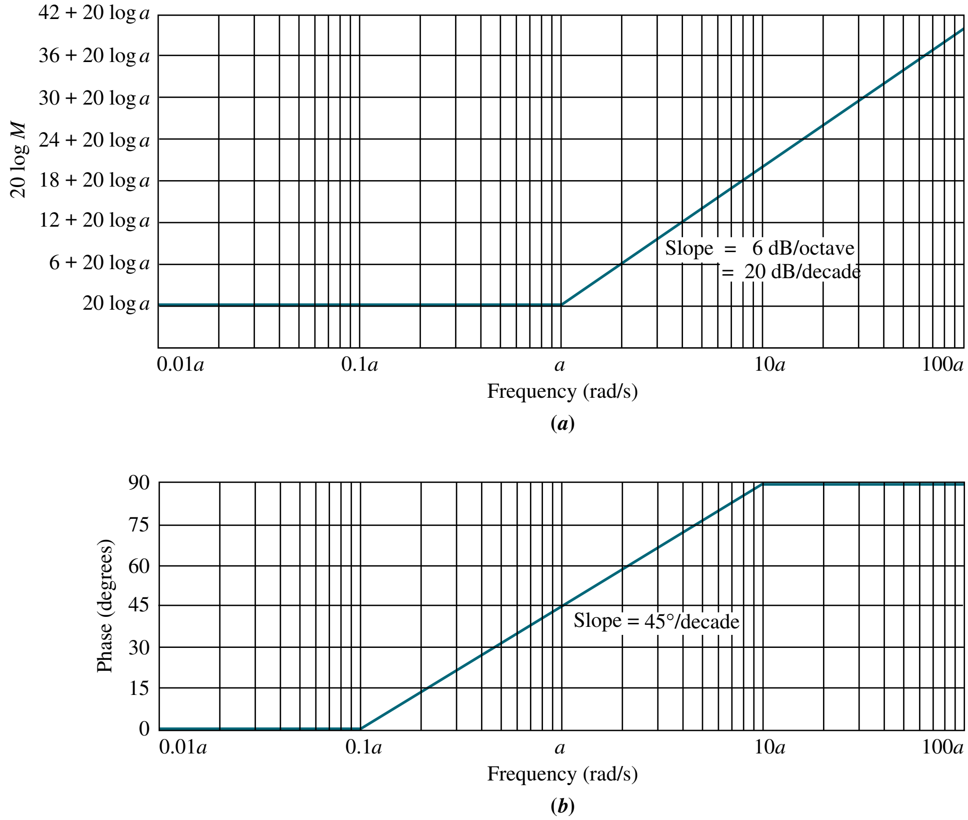


FIGURE 10.6 Bode plots of $(s + a)$: **a.** magnitude plot; **b.** phase plot

We call the straight-line approximations *asymptotes*. The low-frequency approximation is called the *low-frequency asymptote*, and the high-frequency approximation is called the *high-frequency asymptote*. The frequency, a, is called the *break frequency* because it is the break between the low- and the high-frequency asymptotes.

Many times it is convenient to draw the line over a decade rather than an octave, where a *decade* is 10 times the initial frequency. Over one decade, $20 \log \omega$ increases by 20 dB. Thus, a slope of 6 dB/octave is equivalent to a slope of 20 dB/decade. The plot is shown in Figure 10.6(a) from $\omega = 0.01a$ to $100a$.

Let us now turn to the phase response, which can be drawn as follows. At the break frequency, a, Eq. (10.16) shows the phase to be 45°. At low frequencies, Eq. (10.17) shows that the phase is 0°. At high frequencies, Eq. (10.19) shows that the phase is 90°. To draw the curve, start one decade $(1/10)$ below the break frequency, $0.1a$, with 0° phase, and draw a line of slope $+45°$ /decade passing through 45° at the break frequency and continuing to 90° one decade above the break frequency, $10a$. The resulting phase diagram is shown in Figure 10.6(b).

It is often convenient to *normalize* the magnitude and *scale* the frequency so that the log-magnitude plot will be 0 dB at a break frequency of unity. Normalizing and scaling helps in the following applications:

1. When comparing different first- or second-order frequency response plots, each plot will have the same low-frequency asymptote after normalization and the same break frequency after scaling.

2. When sketching the frequency response of a function such as Eq. (10.13), each factor in the numerator and denominator will have the same low-frequency asymptote after normalization. This common low-frequency asymptote makes it easier to add components to obtain the Bode plot.

To normalize $(s+a)$, we factor out the quantity a and form $a[(s/a)+1]$. The frequency is scaled by defining a new frequency variable, $s_1 = s/a$. Then the magnitude is divided by the quantity a to yield 0 dB at the break frequency. Hence, the normalized and scaled function is (s_1+1). To obtain the original frequency response, the magnitude and frequency are multiplied by the quantity a.

We now use the concepts of normalization and scaling to compare the asymptotic approximation to the actual magnitude and phase plot for $(s+a)$. Table 10.1 shows the comparison for the normalized and scaled frequency response of $(s+a)$. Notice that the actual magnitude curve is never greater than 3.01 dB from the asymptotes. This maximum difference occurs at the break frequency. The maximum difference for the phase curve is 5.71°, which occurs at the decades above and below the break frequency. For convenience, the data in Table 10.1 is plotted in Figures 10.7 and 10.8.

We now find the Bode plots for other common transfer functions.

TABLE 10.1 Asymptotic and actual normalized and scaled frequency response data for $(s+a)$

Frequency $\frac{\text{Frequency}}{a}$ (rad/s)	$20\log\frac{M}{a}$ (dB) Asymptotic	$20\log\frac{M}{a}$ (dB) Actual	Phase (degrees) Asymptotic	Phase (degrees) Actual
0.01	0	0.00	0.00	0.57
0.02	0	0.00	0.00	1.15
0.04	0	0.01	0.00	2.29
0.06	0	0.02	0.00	3.43
0.08	0	0.03	0.00	4.57
0.1	0	0.04	0.00	5.71
0.2	0	0.17	13.55	11.31
0.4	0	0.64	27.09	21.80
0.6	0	1.34	35.02	30.96
0.8	0	2.15	40.64	38.66
1	0	3.01	45.00	45.00
2	6	6.99	58.55	63.43
4	12	12.30	72.09	75.96
6	15.56	15.68	80.02	80.54
8	18	18.13	85.64	82.87
10	20	20.04	90.00	84.29
20	26.02	26.03	90.00	87.14
40	32.04	32.04	90.00	88.57
60	35.56	35.56	90.00	89.05
80	38.06	38.06	90.00	89.28
100	40	40.00	90.00	89.43

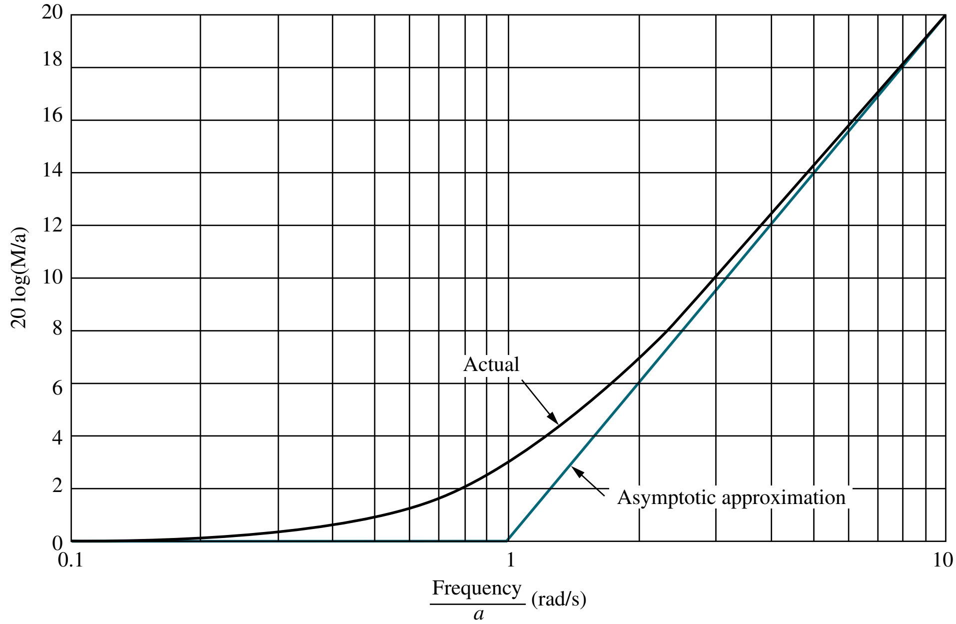


FIGURE 10.7 Asymptotic and actual normalized and scaled magnitude response of $(s + a)$

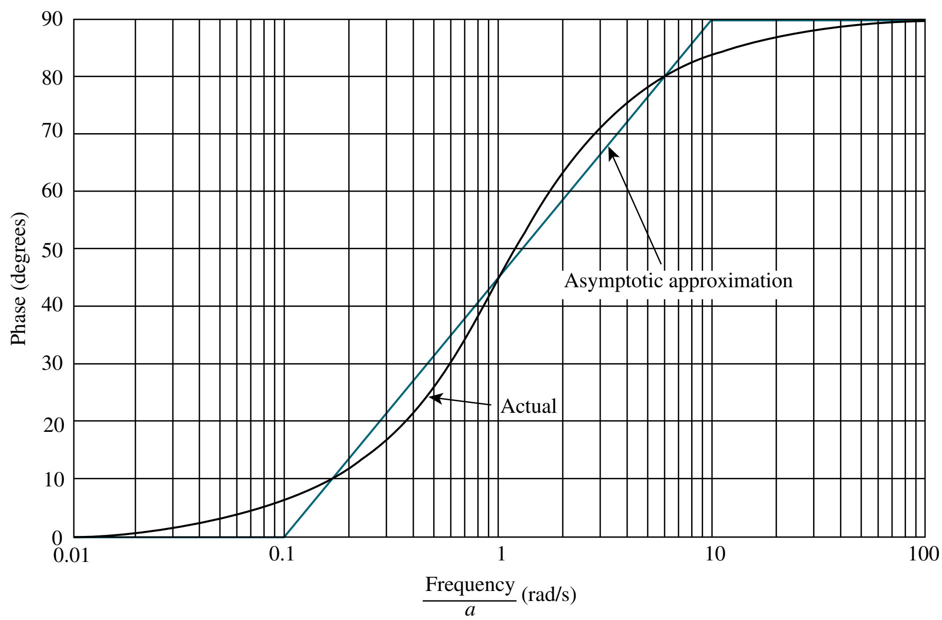


FIGURE 10.8 Asymptotic and actual normalized and scaled phase response of $(s + a)$

Bode Plots for $G(s) = 1/(s+a)$

Let us find the Bode plots for the transfer function

$$G(s) = \frac{1}{(s+a)} = \frac{1}{a\left(\frac{s}{a}+1\right)} \tag{10.22}$$

This function has a low-frequency asymptote of $20 \log (1/a)$, which is found by letting the frequency, s, approach zero. The Bode plot is constant until the break frequency, a rad/s, is reached. The plot is then approximated by the high-frequency asymptote found by letting s approach ∞. Thus, at high frequencies

$$G(j\omega) = \frac{1}{a\left(\dfrac{s}{a}\right)}\Bigg|_{s\to j\omega} = \frac{1}{a\left(\dfrac{j\omega}{a}\right)} = \frac{\dfrac{1}{a}}{\dfrac{\omega}{a}}\angle -90^\circ = \frac{1}{\omega}\angle -90^\circ \tag{10.23}$$

or, in dB,

$$20 \log M = 20 \log \frac{1}{a} - 20 \log \frac{\omega}{a} = -20 \log \omega \tag{10.24}$$

Notice from the middle term that the high-frequency approximation equals the low-frequency approximation when $\omega = a$, and decreases for $\omega > a$. This result is similar to Eq. (10.20), except the slope is negative rather than positive. The Bode log-magnitude diagram will decrease at a rate of 20 dB/decade rather than increase at a rate of 20 dB/decade after the break frequency.

The phase plot is the negative of the previous example since the function is the inverse. The phase begins at 0° and reaches -90° at high frequencies, going through -45° at the break frequency. Both the Bode normalized and scaled log-magnitude and phase plot are shown in Figure 10.9(d).

Bode Plots for $G(s) = s$

Our next function, $G(s) = s$, has only a high-frequency asymptote. Letting $s = j\omega$, the magnitude is $20 \log \omega$, which is the same as Eq. (10.20). Hence, the Bode magnitude plot is a straight line drawn with a $+20$ dB/decade slope passing through zero dB when $\omega = 1$. The phase plot, which is a constant $+90^\circ$, is shown with the magnitude plot in Figure 10.9(a).

Bode Plots for $G(s) = 1/s$

The frequency response of the inverse of the preceding function, $G(s) = 1/s$, is shown in Figure 10.9(b) and is a straight line with a -20 dB/decade slope passing through zero dB at $\omega = 1$. The Bode phase plot is equal to a constant -90°.

We have covered four functions that have first-order polynomials in s in the numerator or denominator. Before proceeding to second-order polynomials, let us

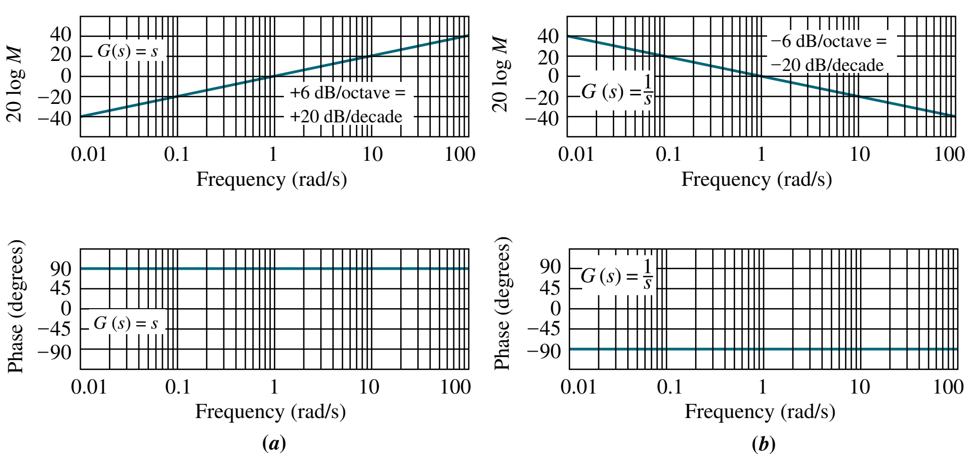


FIGURE 10.9 Normalized and scaled Bode plots for
a. $G(s) = s$;
b. $G(s) = 1/s$;
(*figure continues*)

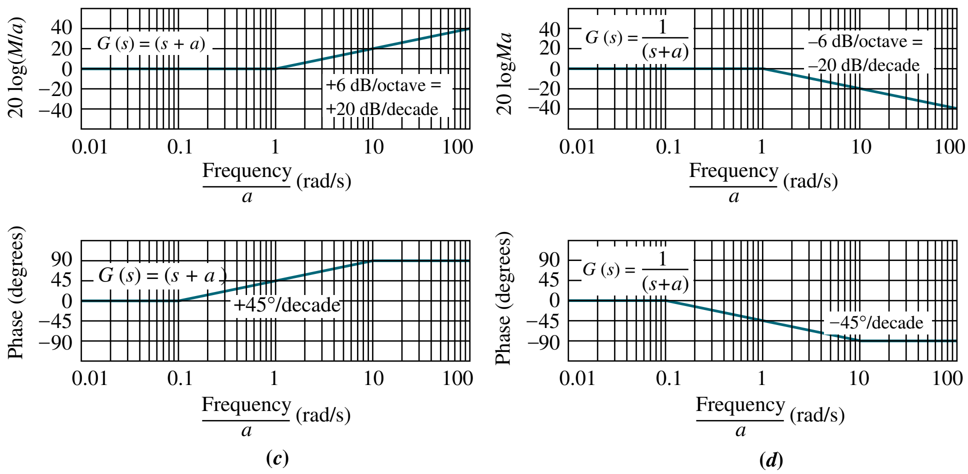


FIGURE 10.9 (*Continued*)
c. $G(s) = (s + a)$;
d. $G(s) = 1/(s + a)$

look at an example of drawing the Bode plots for a function that consists of the product of first-order polynomials in the numerator and denominator. The plots will be made by adding together the individual frequency response curves.

Example 10.2

Bode Plots for Ratio of First-Order Factors

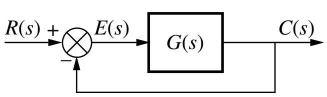


FIGURE 10.10 Closed-loop unity feedback system

PROBLEM: Draw the Bode plots for the system shown in Figure 10.10, where $G(s) = K(s + 3)/[s(s + 1)(s + 2)]$.

SOLUTION: We will make a Bode plot for the open-loop function $G(s) = K(s + 3)/[s(s + 1)(s + 2)]$. The Bode plot is the sum of the Bode plots for each first-order term. Thus, it is convenient to use the normalized plot for each of these terms so that the low-frequency asymptote of each term, except the pole at the origin, is at 0 dB, making it easier to add the components of the Bode plot. We rewrite $G(s)$ showing each term normalized to a low-frequency gain of unity. Hence,

$$G(s) = \frac{\frac{3}{2}K\left(\frac{s}{3} + 1\right)}{s(s + 1)\left(\frac{s}{2} + 1\right)} \tag{10.25}$$

Now determine that the break frequencies are at 1, 2, and 3. The magnitude plot should begin a decade below the lowest break frequency and extend a decade above the highest break frequency. Hence, we choose 0.1 radian to 100 radians, or three decades, as the extent of our plot.

At $\omega = 0.1$ the low-frequency value of the function is found from Eq. (10.25) using the low-frequency values for all of the $[(s/a) + 1]$ terms, (that is, $s = 0$) and the actual value for the s term in the denominator. Thus, $G(j0.1) \approx \frac{3}{2}K/0.1 = 15\,K$. The effect of K is to move the magnitude curve up (increasing K) or down (decreasing K) by the amount of 20 log K. K has no effect upon the phase curve. If we choose $K = 1$, the magnitude plot can be denormalized later for any value of K that is calculated or known.

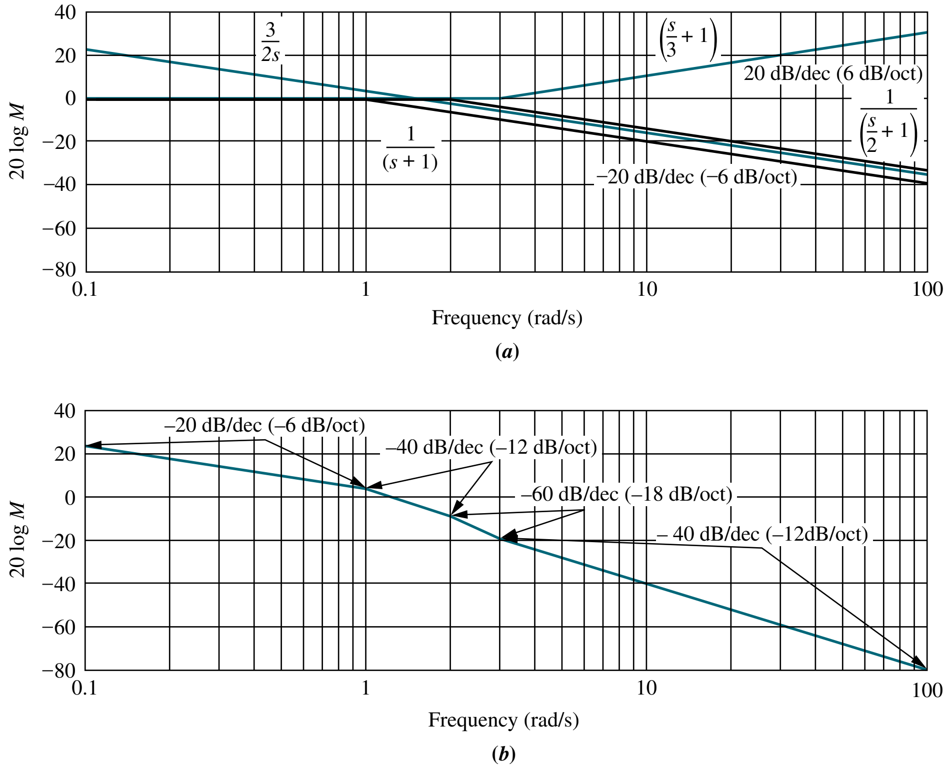


FIGURE 10.11 Bode log-magnitude plot for Example 10.2:
a. components;
b. composite

Figure 10.11(a) shows each component of the Bode log-magnitude frequency response. Summing the components yields the composite plot shown in Figure 10.11(b). The results are summarized in Table 10.2, which can be used to obtain the slopes. Each pole and zero is itemized in the first column. Reading across the table shows its contribution at each frequency. The last row is the sum of the slopes and correlates with Figure 10.11(b). The Bode magnitude plot for $K = 1$ starts at $\omega = 0.1$ with a value of $20 \log 15 = 23.52$ dB, and decreases immediately at a rate of -20 dB/decade, due to the s term in the denominator. At $\omega = 1$, the $(s + 1)$ term in the denominator begins its 20 dB/decade downward slope and causes an additional 20 dB/decade negative slope, or a total of -40 dB/decade. At $\omega = 2$, the term $[(s/2) + 1]$ begins its -20 dB/decade slope, adding yet another -20 dB/decade to the resultant plot, or a total of -60 dB/decade slope that continues until $\omega = 3$. At this frequency, the $[(s/3) + 1]$ term in the numerator begins its positive

TABLE 10.2 Bode magnitude plot: slope contribution from each pole and zero in Example 10.2

	Frequency (rad/s)			
Description	**0.1 (Start: Pole at 0)**	**1 (Start: Pole at −1)**	**2 (Start: Pole at −2)**	**3 (Start: Zero at −3)**
Pole at 0	−20	−20	−20	−20
Pole at −1	0	−20	−20	−20
Pole at −2	0	0	−20	−20
Zero at −3	0	0	0	20
Total slope (dB/dec)	−20	−40	−60	−40

20 dB/decade slope. The resultant magnitude plot, therefore, changes from a slope of -60 dB/decade to -40 dB/decade at $\omega = 3$, and continues at that slope since there are no other break frequencies.

The slopes are easily drawn by sketching straight-line segments decreasing by 20 dB over a decade. For example, the initial -20 dB/decade slope is drawn from 23.52 dB at $\omega = 0.1$, to 3.52 dB (a 20 dB decrease) at $\omega = 1$. The -40 dB/decade slope starting at $\omega = 1$ is drawn by sketching a line segment from 3.52 dB at $\omega = 1$, to -36.48 dB (a 40 dB decrease) at $\omega = 10$, and using only the portion from $\omega = 1$ to $\omega = 2$. The next slope of -60 dB/decade is drawn by first sketching a line segment from $\omega = 2$ to $\omega = 20$ (1 decade) that drops down by 60 dB, and using only that portion of the line from $\omega = 2$ to $\omega = 3$. The final slope is drawn by sketching a line segment from $\omega = 3$ to $\omega = 30$ (1 decade) that drops by 40 dB. This slope continues to the end of the plot.

Phase is handled similarly. However, the existence of breaks a decade below and a decade above the break frequency requires a little more bookkeeping. Table 10.3 shows the starting and stopping frequencies of the 45°/decade slope for

TABLE 10.3 Bode phase plot: slope contribution from each pole and zero in Example 10.2

	Frequency (rad/s)					
Description	**0.1 (Start: Pole at −1)**	**0.2 (Start: Pole at −2)**	**0.3 (Start: Pole at −3)**	**0 (End: Pole at −1)**	**20 (End: Pole at −2)**	**30 (End: Zero at −3)**
Pole at −1	−45	−45	−45	0		
Pole at −2		−45	−45	−45	0	
Zero at −3			45	45	45	0
Total slope (deg/dec)	−45	−90	−45	0	45	0

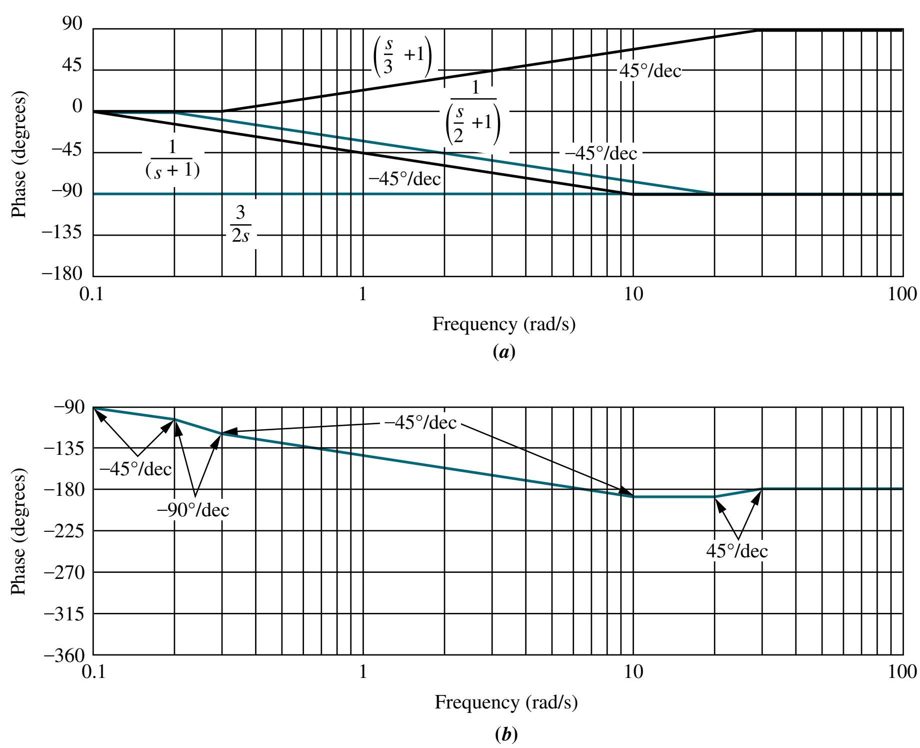


FIGURE 10.12 Bode phase plot for Example 10.2: **a.** components; **b.** composite

each of the poles and zeros. For example, reading across for the pole at -2, we see that the $-45°$ slope starts at a frequency of 0.2 and ends at 20. Filling in the rows for each pole and then summing the columns yields the slope portrait of the resulting phase plot. Looking at the row marked *Total slope*, we see that the phase plot will have a slope of $-45°$/decade from a frequency of 0.1 to 0.2. The slope will then increase to $-90°$/decade from 0.2 to 0.3. The slope will return to $-45°$/decade from 0.3 to 10 rad/s. A slope of 0 ensues from 10 to 20 rad/s, followed by a slope of $+45°$/decade from 20 to 30 rad/s. Finally, from 30 rad/s to infinity, the slope is $0°$/decade.

The resulting component and composite phase plots are shown in Figure 10.12. Since the pole at the origin yields a constant $-90°$ phase shift, the plot begins at $-90°$ and follows the slope portrait just described.

Bode Plots for $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$

Now that we have covered Bode plots for first-order systems, we turn to the Bode log-magnitude and phase plots for second-order polynomials in s. The second-order polynomial is of the form

$$G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = \omega_n^2\left(\frac{s^2}{\omega_n^2} + 2\zeta\frac{s}{\omega_n} + 1\right) \tag{10.26}$$

Unlike the first-order frequency response approximation, the difference between the asymptotic approximation and the actual frequency response can be great for some values of ζ. A correction to the Bode diagrams can be made to improve the accuracy. We first derive the asymptotic approximation and then show the difference between the asymptotic approximation and the actual frequency response curves.

At low frequencies, Eq. (10.26) becomes

$$G(s) \approx \omega_n^2 = \omega_n^2\angle 0° \tag{10.27}$$

The magnitude, M, in dB at low frequencies therefore is

$$20\log M = 20\log|G(j\omega)| = 20\log\omega_n^2 \tag{10.28}$$

At high frequencies,

$$G(s) \approx s^2 \tag{10.29}$$

or

$$G(j\omega) \approx -\omega^2 = \omega^2\angle 180° \tag{10.30}$$

The log-magnitude is

$$20\log M = 20\log|G(j\omega)| = 20\log\omega^2 = 40\log\omega \tag{10.31}$$

Equation (10.31) is a straight line with twice the slope of a first-order term (Eq. (10.20)). Its slope is 12 dB/octave, or 40 dB/decade.

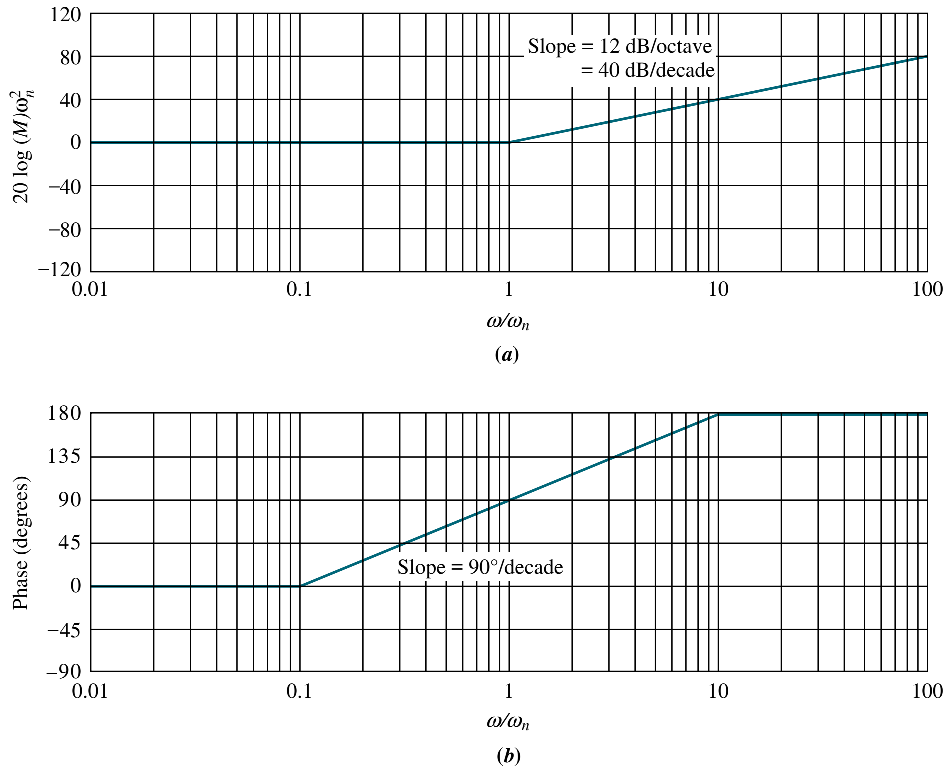


FIGURE 10.13 Bode asymptotes for normalized and scaled $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$: **a.** magnitude; **b.** phase

The low-frequency asymptote (Eq. (10.27)) and the high-frequency asymptote (Eq. (10.31)) are equal when $\omega = \omega_n$. Thus, ω_n is the break frequency for the second-order polynomial.

For convenience in representing systems with different ω_n, we normalize and scale our findings before drawing the asymptotes. Using the normalized and scaled term of Eq. (10.26), we normalize the magnitude, dividing by ω_n^2, and scale the frequency, dividing by ω_n. Thus, we plot $G(s_1)/\omega_n^2 = s_1^2 + 2\zeta s_1 + 1$, where $s_1 = s/\omega_n$. $G(s_1)$ has a low-frequency asymptote at 0 dB and a break frequency of 1 rad/s. Figure 10.13(a) shows the asymptotes for the normalized and scaled magnitude plot.

We now draw the phase plot. It is 0° at low frequencies (Eq. (10.27)) and 180° at high frequencies (Eq. (10.30)). To find the phase at the natural frequency, first evaluate $G(j\omega)$:

$$G(j\omega) = s^2 + 2\zeta\omega_n s + \omega_n^2|_{s\rightarrow j\omega} = (\omega_n^2 - \omega^2) + j2\zeta\omega_n\omega \tag{10.32}$$

Then find the function value at the natural frequency by substituting $\omega = \omega_n$. Since the result is $j2\zeta\omega_n^2$, the phase at the natural frequency is $+90°$. Figure 10.13(b) shows the phase plotted with frequency scaled by ω_n. The phase plot increases at a rate of $90°$/decade from 0.1 to 10 and passes through $90°$ at 1.

Corrections to Second-Order Bode Plots

Let us now examine the error between the actual response and the asymptotic approximation of the second-order polynomial. Whereas the first-order polynomial has a disparity of no more than 3.01 dB magnitude and 5.71° phase, the second-order function may have a greater disparity, which depends upon the value of ζ.

From Eq. (10.32), the actual magnitude and phase for $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$ are, respectively,

$$\boxed{M = \sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}} \tag{10.33}$$

$$\boxed{\text{Phase} = \tan^{-1}\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}} \tag{10.34}$$

These relationships are tabulated in Table 10.4 for a range of values of ζ and plotted in Figures 10.14 and 10.15 along with the asymptotic approximations for normalized

TABLE 10.4 Data for normalized and scaled log-magnitude and phase plots for $(s^2 + 2\zeta\omega_n s + \omega_n^2)$. Mag $= 20\log(M/\omega_n^2)$

Freq. $\frac{\omega}{\omega_n}$	**Mag (dB)** $\zeta = 0.1$	**Phase (deg)** $\zeta = 0.1$	**Mag (dB)** $\zeta = 0.2$	**Phase (deg)** $\zeta = 0.2$	**Mag (dB)** $\zeta = 0.3$	**Phase (deg)** $\zeta = 0.3$
0.10	−0.09	1.16	−0.08	2.31	−0.07	3.47
0.20	−0.35	2.39	−0.32	4.76	−0.29	7.13
0.30	−0.80	3.77	−0.74	7.51	−0.65	11.19
0.40	−1.48	5.44	−1.36	10.78	−1.17	15.95
0.50	−2.42	7.59	−2.20	14.93	−1.85	21.80
0.60	−3.73	10.62	−3.30	20.56	−2.68	29.36
0.70	−5.53	15.35	−4.70	28.77	−3.60	39.47
0.80	−8.09	23.96	−6.35	41.63	−4.44	53.13
0.90	−11.64	43.45	−7.81	62.18	−4.85	70.62
1.00	−13.98	90.00	−7.96	90.00	−4.44	90.00
1.10	−10.34	133.67	−6.24	115.51	−3.19	107.65
1.20	−6.00	151.39	−3.73	132.51	−1.48	121.43
1.30	−2.65	159.35	−1.27	143.00	0.35	131.50
1.40	0.00	163.74	0.92	149.74	2.11	138.81
1.50	2.18	166.50	2.84	154.36	3.75	144.25
1.60	4.04	168.41	4.54	157.69	5.26	148.39
1.70	5.67	169.80	6.06	160.21	6.64	151.65
1.80	7.12	170.87	7.43	162.18	7.91	154.26
1.90	8.42	171.72	8.69	163.77	9.09	156.41
2.00	9.62	172.41	9.84	165.07	10.19	158.20
3.00	18.09	175.71	18.16	171.47	18.28	167.32
4.00	23.53	176.95	23.57	173.91	23.63	170.91
5.00	27.61	177.61	27.63	175.24	27.67	172.87
6.00	30.89	178.04	30.90	176.08	30.93	174.13
7.00	33.63	178.33	33.64	176.66	33.66	175.00
8.00	35.99	178.55	36.00	177.09	36.01	175.64
9.00	38.06	178.71	38.07	177.42	38.08	176.14
10.00	39.91	178.84	39.92	177.69	39.93	176.53

(*table continues*)

TABLE 10.4 Data for normalized and scaled log-magnitude and phase plots for $(s^2 + 2\zeta\omega_n s + \omega_n^2)$. Mag $= 20\log(M/\omega_n^2)$ (*Continued*)

Freq. $\frac{\omega}{\omega_n}$	Mag (dB) $\zeta = 0.5$	Phase (deg) $\zeta = 0.5$	Mag (dB) $\zeta = 0.7$	Phase (deg) $\zeta = 0.7$	Mag (dB) $\zeta = 0.1$	Phase (deg) $\zeta = 0.1$
0.10	−0.04	5.77	0.00	8.05	0.09	11.42
0.20	−0.17	11.77	0.00	16.26	0.34	22.62
0.30	−0.37	18.25	0.02	24.78	0.75	33.40
0.40	−0.63	25.46	0.08	33.69	1.29	43.60
0.50	−0.90	33.69	0.22	43.03	1.94	53.13
0.60	−1.14	43.15	0.47	52.70	2.67	61.93
0.70	−1.25	53.92	0.87	62.51	3.46	69.98
0.80	−1.14	65.77	1.41	72.18	4.30	77.32
0.90	−0.73	78.08	2.11	81.42	5.15	83.97
1.00	0.00	90.00	2.92	90.00	6.02	90.00
1.10	0.98	100.81	3.83	97.77	6.89	95.45
1.20	2.13	110.14	4.79	104.68	7.75	100.39
1.30	3.36	117.96	5.78	110.76	8.60	104.86
1.40	4.60	124.44	6.78	116.10	9.43	108.92
1.50	5.81	129.81	7.76	120.76	10.24	112.62
1.60	6.98	134.27	8.72	124.85	11.03	115.99
1.70	8.10	138.03	9.66	128.45	11.80	119.07
1.80	9.17	141.22	10.56	131.63	12.55	121.89
1.90	10.18	143.95	11.43	134.46	13.27	124.48
2.00	11.14	146.31	12.26	136.97	13.98	126.87
3.00	18.63	159.44	19.12	152.30	20.00	143.13
4.00	23.82	165.07	24.09	159.53	24.61	151.93
5.00	27.79	168.23	27.96	163.74	28.30	157.38
6.00	31.01	170.27	31.12	166.50	31.36	161.08
7.00	33.72	171.70	33.80	168.46	33.98	163.74
8.00	36.06	172.76	36.12	169.92	36.26	165.75
9.00	38.12	173.58	38.17	171.05	38.28	167.32
10.00	39.96	174.23	40.00	171.95	40.09	168.58

magnitude and scaled frequency. In Figure 10.14, which is normalized to the square of the natural frequency, the normalized log-magnitude at the scaled natural frequency is $+20\log 2\zeta$. The student should verify that the actual magnitude at the unscaled natural frequency is $+20\log 2\zeta\omega_n^2$. Table 10.4 and Figures 10.14 and 10.15 can be used to improve accuracy when drawing Bode plots. For example, a magnitude correction of $+20\log 2\zeta$ can be made at the natural, or break, frequency on the Bode asymptotic plot.

Bode Plots for $G(s) = 1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$

Bode plots for $G(s) = 1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$ can be derived similarly to those for $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$. We find that the magnitude curve breaks at the natural frequency and decreases at a rate of -40 dB/decade. The phase plot is $0°$ at low

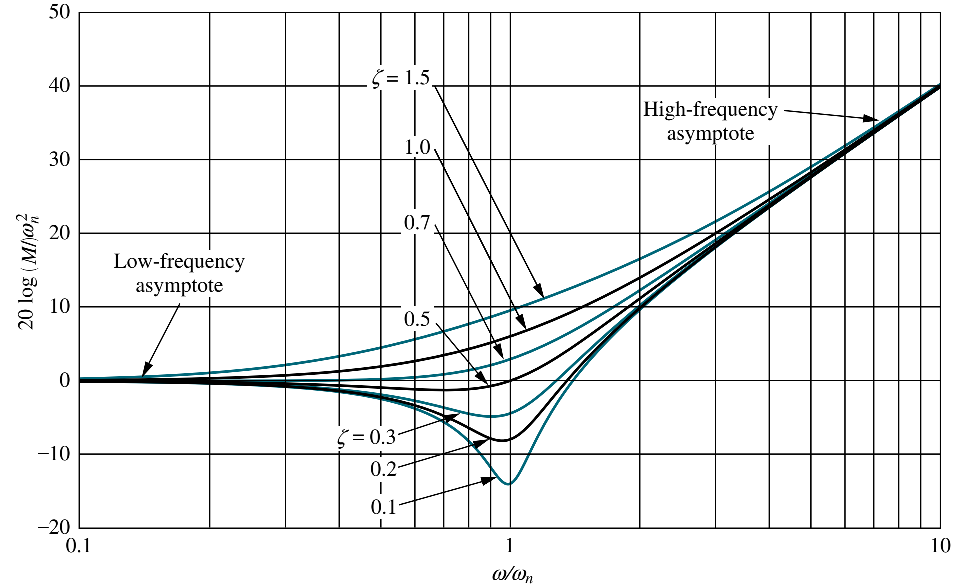


FIGURE 10.14 Normalized and scaled log-magnitude response for $(s^2 + 2\zeta\omega_n s + \omega_n^2)$

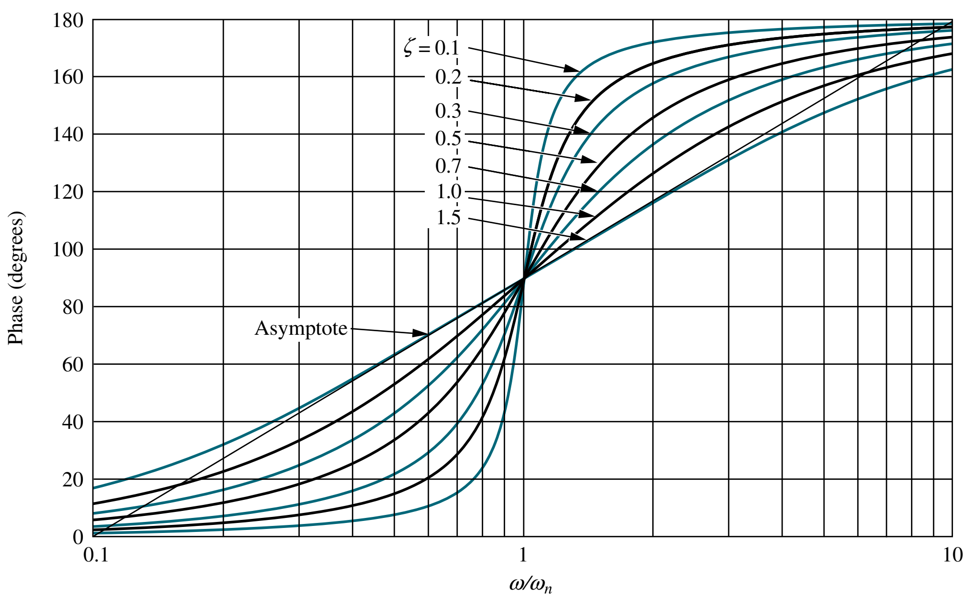


FIGURE 10.15 Scaled phase response for $(s^2 + 2\zeta\omega_n s + \omega_n^2)$

frequencies. At $0.1\omega_n$ it begins a decrease of $-90°$/decade and continues until $\omega = 10\omega_n$, where it levels off at $-180°$.

The exact frequency response also follows the same derivation as that of $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$. The results are summarized in Table 10.5, as well as Figures 10.16 and 10.17. The exact magnitude is the reciprocal of Eq. (10.33), and the exact phase is the negative of Eq. (10.34). The normalized magnitude at the scaled natural frequency is $-20 \log 2\zeta$, which can be used as a correction at the break frequency on the Bode asymptotic plot.

TABLE 10.5 Data for normalized and scaled log-magnitude and phase plots for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. Mag $= 20\log(M/\omega_n^2)$

Freq. $\frac{\omega}{\omega_n}$	Mag (dB) $\zeta = 0.1$	Phase (deg) $\zeta = 0.1$	Mag (dB) $\zeta = 0.2$	Phase (deg) $\zeta = 0.2$	Mag (dB) $\zeta = 0.3$	Phase (deg) $\zeta = 0.3$
0.10	0.09	−1.16	0.08	−2.31	0.07	−3.47
0.20	0.35	−2.39	0.32	−4.76	0.29	−7.13
0.30	0.80	−3.77	0.74	−7.51	0.65	−11.19
0.40	1.48	−5.44	1.36	−10.78	1.17	−15.95
0.50	2.42	−7.59	2.20	−14.93	1.85	−21.80
0.60	3.73	−10.62	3.30	−20.56	2.68	−29.36
0.70	5.53	−15.35	4.70	−28.77	3.60	−39.47
0.80	8.09	−23.96	6.35	−41.63	4.44	−53.13
0.90	11.64	−43.45	7.81	−62.18	4.85	−70.62
1.00	13.98	−90.00	7.96	−90.00	4.44	−90.00
1.10	10.34	−133.67	6.24	−115.51	3.19	−107.65
1.20	6.00	−151.39	3.73	−132.51	1.48	−121.43
1.30	2.65	−159.35	1.27	−143.00	−0.35	−131.50
1.40	0.00	−163.74	−0.92	−149.74	−2.11	−138.81
1.50	−2.18	−166.50	−2.84	−154.36	−3.75	−144.25
1.60	−4.04	−168.41	−4.54	−157.69	−5.26	−148.39
1.70	−5.67	−169.80	−6.06	−160.21	−6.64	−151.65
1.80	−7.12	−170.87	−7.43	−162.18	−7.91	−154.26
1.90	−8.42	−171.72	−8.69	−163.77	−9.09	−156.41
2.00	−9.62	−172.41	−9.84	−165.07	−10.19	−158.20
3.00	−18.09	−175.71	−18.16	−171.47	−18.28	−167.32
4.00	−23.53	−176.95	−23.57	−173.91	−23.63	−170.91
5.00	−27.61	−177.61	−27.63	−175.24	−27.67	−172.87
6.00	−30.89	−178.04	−30.90	−176.08	−30.93	−174.13
7.00	−33.63	−178.33	−33.64	−176.66	−33.66	−175.00
8.00	−35.99	−178.55	−36.00	−177.09	−36.01	−175.64
9.00	−38.06	−178.71	−38.07	−177.42	−38.08	−176.14
10.00	−39.91	−178.84	−39.92	−177.69	−39.93	−176.53

(*table continues*)

TABLE 10.5 Data for normalized and scaled log-magnitude and phase plots for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. Mag $= 20\log(M/\omega_n^2)$ (*Continued*)

Freq. $\frac{\omega}{\omega_n}$	**Mag (dB)** $\zeta = 0.5$	**Phase (deg)** $\zeta = 0.5$	**Mag (dB)** $\zeta = 0.7$	**Phase (deg)** $\zeta = 0.7$	**Mag (dB)** $\zeta = 0.1$	**Phase (deg)** $\zeta = 0.1$
0.10	0.04	−5.77	0.00	−8.05	−0.09	−11.42
0.20	0.17	−11.77	0.00	−16.26	−0.34	−22.62
0.30	0.37	−18.25	−0.02	−24.78	−0.75	−33.40
0.40	0.63	−25.46	−0.08	−33.69	−1.29	−43.60
0.50	0.90	−33.69	−0.22	−43.03	−1.94	−53.13
0.60	1.14	−43.15	−0.47	−52.70	−2.67	−61.93
0.70	1.25	−53.92	−0.87	−62.51	−3.46	−69.98
0.80	1.14	−65.77	−1.41	−72.18	−4.30	−77.32
0.90	0.73	−78.08	−2.11	−81.42	−5.15	−83.97
1.00	0.00	−90.00	−2.92	−90.00	−6.02	−90.00
1.10	−0.98	−100.81	−3.93	−97.77	−6.89	−95.45
1.20	−2.13	−110.14	−4.79	−104.68	−7.75	−100.39
1.30	−3.36	−117.96	−5.78	−110.76	−8.60	−104.86
1.40	−4.60	−124.44	−6.78	−116.10	−9.43	−108.92
1.50	−5.81	−129.81	−7.76	−120.76	−10.24	−112.62
1.60	−6.98	−134.27	−8.72	−124.85	−11.03	−115.99
1.70	−8.10	−138.03	−9.66	−128.45	−11.80	−119.07
1.80	−9.17	−141.22	−10.56	−131.63	−12.55	−121.89
1.90	−10.18	−143.95	−11.43	−134.46	−13.27	−124.48
2.00	−11.14	−146.31	−12.26	−136.97	−13.98	−126.87
3.00	−18.63	−159.44	−19.12	−152.30	−20.00	−143.13
4.00	−23.82	−165.07	−24.09	−159.53	−24.61	−151.93
5.00	−27.79	−168.23	−27.96	−163.74	−28.30	−157.38
6.00	−31.01	−170.27	−31.12	−166.50	−31.36	−161.08
7.00	−33.72	−171.70	−33.80	−168.46	−33.98	−163.74
8.00	−36.06	−172.76	−36.12	−169.92	−36.26	−165.75
9.00	−38.12	−173.58	−38.17	−171.05	−38.28	−167.32
10.00	−39.96	−174.23	−40.00	−171.95	−40.09	−168.58

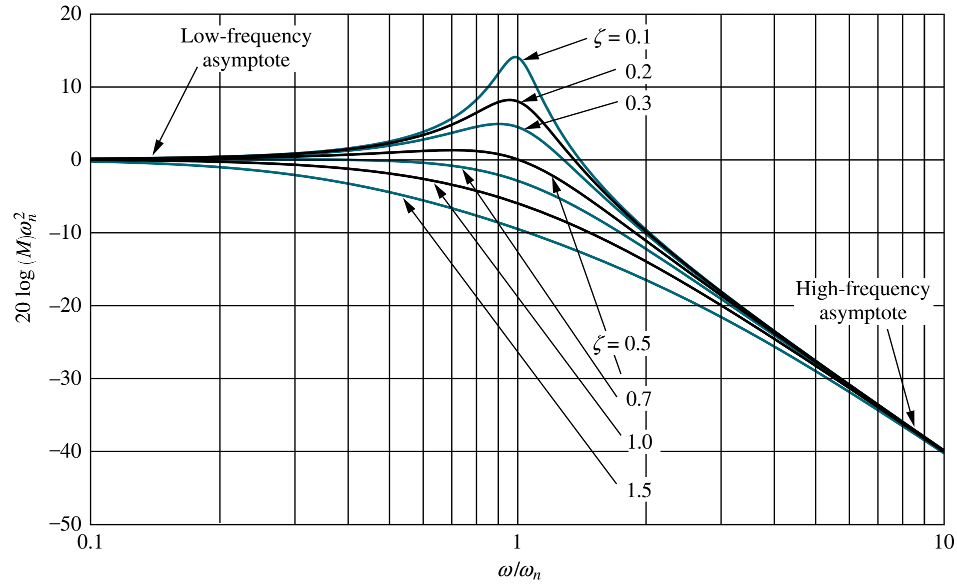


FIGURE 10.16 Normalized and scaled log-magnitude response for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$

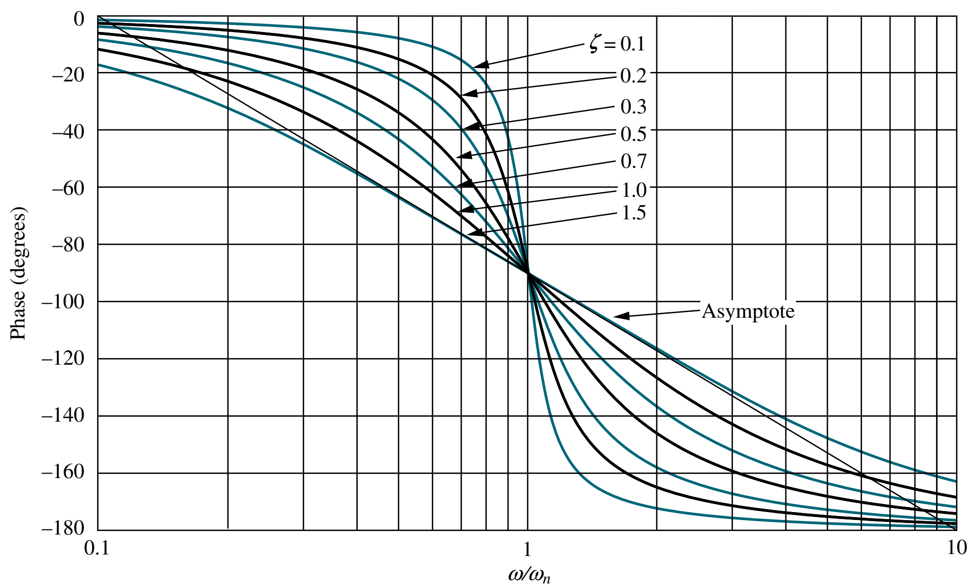


FIGURE 10.17 Scaled phase response for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$

Let us now look at an example of drawing Bode plots for transfer functions that contain second-order factors.

Example 10.3

Bode Plots for Ratio of First- and Second-Order Factors

PROBLEM: Draw the Bode log-magnitude and phase plots of $G(s)$ for the unity feedback system shown in Figure 10.10, where $G(s) = (s+3)/[(s+2)(s^2+2s+25)]$.

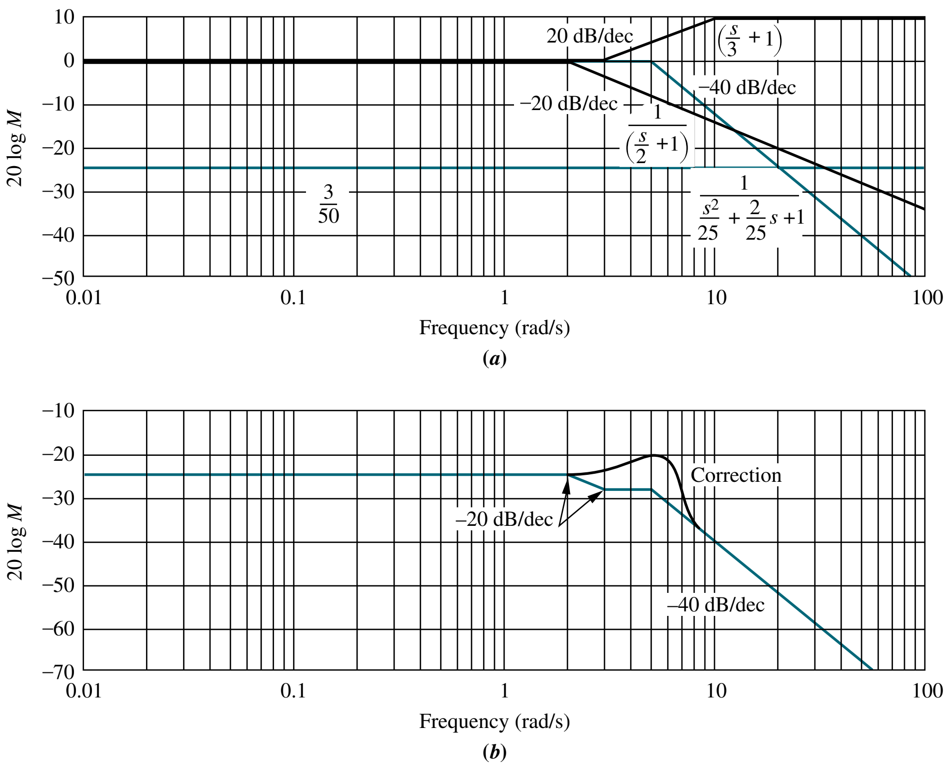


FIGURE 10.18 Bode magnitude plot for $G(s) = (s+3)/[(s+2)(s^2+2s+25)]$: **a.** components; **b.** composite

SOLUTION: We first convert $G(s)$ to show the normalized components that have unity low-frequency gain. The second-order term is normalized by factoring out ω_n^2, forming

$$\frac{s^2}{\omega_n^2}+\frac{2\zeta}{\omega_n}s+1 \tag{10.35}$$

Thus,

$$G(s)=\frac{3}{(2)(25)}\frac{\left(\frac{s}{3}+1\right)}{\left(\frac{s}{2}+1\right)\left(\frac{s^2}{25}+\frac{2}{25}s+1\right)}=\frac{3}{50}\frac{\left(\frac{s}{3}+1\right)}{\left(\frac{s}{2}+1\right)\left(\frac{s^2}{25}+\frac{2}{25}s+1\right)} \tag{10.36}$$

The Bode log-magnitude diagram is shown in Figure 10.18(b) and is the sum of the individual first- and second-order terms of $G(s)$ shown in Figure 10.18(a). We solve this problem by adding the slopes of these component parts, beginning and ending at the appropriate frequencies. The results are summarized in Table 10.6, which can be used to obtain the slopes. The low-frequency value for $G(s)$, found by

TABLE 10.6 Magnitude diagram slopes for Example 10.3

	Frequency (rad/s)			
Description	**0.01 (Start: Plot)**	**2 (Start: Pole at −2)**	**3 (Start: Zero at −3)**	**5 (Start: $\omega_n = 5$)**
Pole at −2	0	−20	−20	−20
Zero at −3	0	0	20	20
$\omega_n = 5$	0	0	0	−40
Total slope (dB/dec)	0	−20	0	−40

letting $s = 0$, is 3/50, or -24.44 dB. The Bode magnitude plot starts out at this value and continues until the first break frequency at 2 rad/s. Here the pole at -2 yields a -20 dB/decade slope downward until the next break at 3 rad/s. The zero at -3 causes an upward slope of $+20$ dB/decade, which, when added to the previous -20 dB/decade curve, gives a net slope of 0. At a frequency of 5 rad/s, the second-order term initiates a -40 dB/decade downward slope, which continues to infinity.

The correction to the log-magnitude curve due to the underdamped second-order term can be found by plotting a point $-20 \log 2\zeta$ above the asymptotes at the natural frequency. Since $\zeta = 0.2$ for the second-order term in the denominator of $G(s)$, the correction is 7.96 dB. Points close to the natural frequency can be corrected by taking the values from the curves of Figure 10.16.

TABLE 10.7 Phase diagram slopes for Example 10.3

	Frequency (rad/s)					
Description	**0.2 (Start: Pole at -2)**	**0.3 (Start: Zero at -3)**	**0.5 (Start: ω_n at -5)**	**20 (End: Pole at -2)**	**30 (End: Zero at -3)**	**50 (End: $\omega_n = 5$)**
Pole at -2	-45	-45	-45	0		
Zero at -3		45	45	45	0	
$\omega_n = 5$			-90	-90	-90	0
Total slope (dB/dec)	-45	0	-90	-45	-90	0

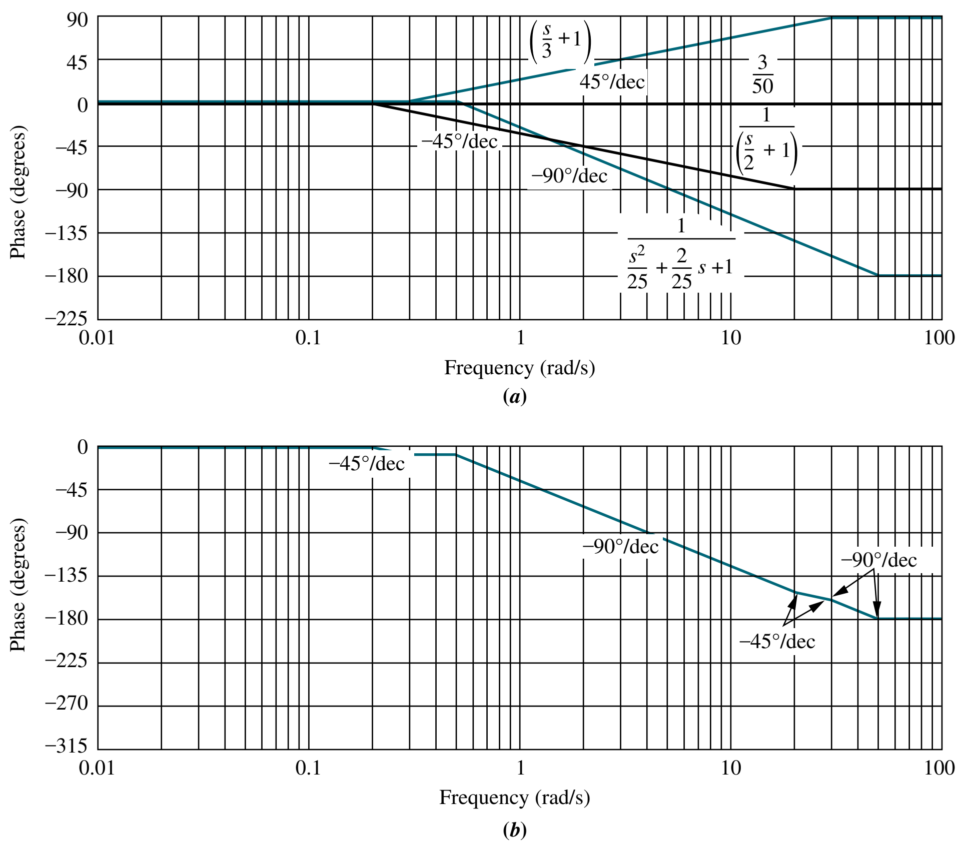


FIGURE 10.19 Bode phase plot for $G(s) = (s+3)/[(s+2)(s^2+2s+25)]$: **a.** components; **b.** composite

We now turn to the phase plot. Table 10.7 is formed to determine the progression of slopes on the phase diagram. The first-order pole at −2 yields a phase angle that starts at 0° and ends at −90° via a −45°/decade slope starting a decade below its break frequency and ending a decade above its break frequency. The first-order zero yields a phase angle that starts at 0° and ends at +90° via a +45°/decade slope starting a decade below its break frequency and ending a decade above its break frequency. The second-order poles yield a phase angle that starts at 0° and ends at −180° via a −90°/decade slope starting a decade below their natural frequency ($\omega_n = 5$) and ending a decade above their natural frequency. The slopes, shown in Figure 10.19(a), are summed over each frequency range, and the final Bode phase plot is shown in Figure 10.19(b).

MATLAB
ML

`Students who are using MATLAB should now run ch10p1 in Appendix B. You will learn how to use MATLAB to make Bode plots and list the points on the plots. This exercise solves Example 10.3 using MATLAB.`

Skill-Assessment Exercise 10.2

PROBLEM: Draw the Bode log-magnitude and phase plots for the system shown in Figure 10.10, where

$$G(s) = \frac{(s+20)}{(s+1)(s+7)(s+50)}$$

WileyPLUS
WPCS
Control Solutions

ANSWER: The complete solution is at www.wiley.com/college/nise.

TryIt 10.1

Use MATLAB, the Control System Toolbox, and the following statements to obtain the Bode plots for the system of Skill-Assessment Exercise 10.2

```
G=zpk([-20],[-1,-7,...
 -50],1)
bode(G);grid on
```

After the Bode plots appear, click on the curve and drag to read the coordinates.

In this section, we learned how to construct Bode log-magnitude and Bode phase plots. The Bode plots are separate magnitude and phase frequency response curves for a system, $G(s)$. In the next section, we develop the Nyquist criterion for stability, which makes use of the frequency response of a system. The Bode plots can then be used to determine the stability of a system.

10.3 Introduction to the Nyquist Criterion

The Nyquist criterion relates the stability of a closed-loop system to the open-loop frequency response and open-loop pole location. Thus, knowledge of the open-loop system's frequency response yields information about the stability of the closed-loop system. This concept is similar to the root locus, where we began with information about the open-loop system, its poles and zeros, and developed transient and stability information about the closed-loop system.

Although the Nyquist criterion will yield stability information at first, we will extend the concept to transient response and steady-state errors. Thus, frequency response techniques are an alternate approach to the root locus.

Derivation of the Nyquist Criterion

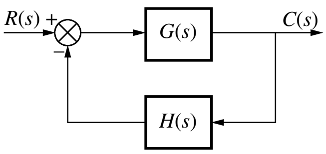


FIGURE 10.20 Closed-loop control system

Consider the system of Figure 10.20. The Nyquist criterion can tell us how many closed-loop poles are in the right half-plane. Before deriving the criterion, let us establish four important concepts that will be used during the derivation: (1) the relationship between the poles of $1 + G(s)H(s)$ and the poles of $G(s)H(s)$; (2) the relationship between the zeros of $1 + G(s)H(s)$ and the poles of the closed-loop transfer function, $T(s)$; (3) the concept of *mapping* points; and (4) the concept of mapping *contours*.

Letting

$$G(s) = \frac{N_G}{D_G} \tag{10.37a}$$

$$H(s) = \frac{N_H}{D_H} \tag{10.37b}$$

we find

$$G(s)H(s) = \frac{N_G N_H}{D_G D_H} \tag{10.38a}$$

$$1 + G(s)H(s) = 1 + \frac{N_G N_H}{D_G D_H} = \frac{D_G D_H + N_G N_H}{D_G D_H} \tag{10.38b}$$

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{N_G D_H}{D_G D_H + N_G N_H} \tag{10.38c}$$

From Eqs. (10.38), we conclude that (1) *the poles of* $1 + G(s)H(s)$ *are the same as the poles of* $G(s)H(s)$*, the open-loop system, and* (2) *the zeros of* $1 + G(s)H(s)$*are the same as the poles of* $T(s)$*, the closed-loop system.*

Next, let us define the term *mapping*. If we take a complex number on the s-plane and substitute it into a function, $F(s)$, another complex number results. This process is called *mapping*. For example, substituting $s = 4 + j3$ into the function $(s^2 + 2s + 1)$ yields $16 + j30$. We say that $4 + j3$ maps into $16 + j30$ through the function $(s^2 + 2s + 1)$.

Finally, we discuss the concept of mapping *contours*. Consider the collection of points, called a *contour*, shown in Figure 10.21 as contour A. Also, assume that

$$F(s) = \frac{(s - z_1)(s - z_2)\ldots}{(s - p_1)(s - p_2)\ldots} \tag{10.39}$$

Contour A can be mapped through $F(s)$ into contour B by substituting each point of contour A into the function $F(s)$ and plotting the resulting complex numbers. For example, point Q in Figure 10.21 maps into point Q' through the function $F(s)$.

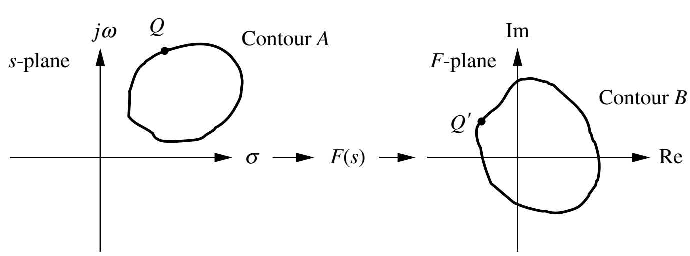


FIGURE 10.21 Mapping contour A through function $F(s)$ to contour B

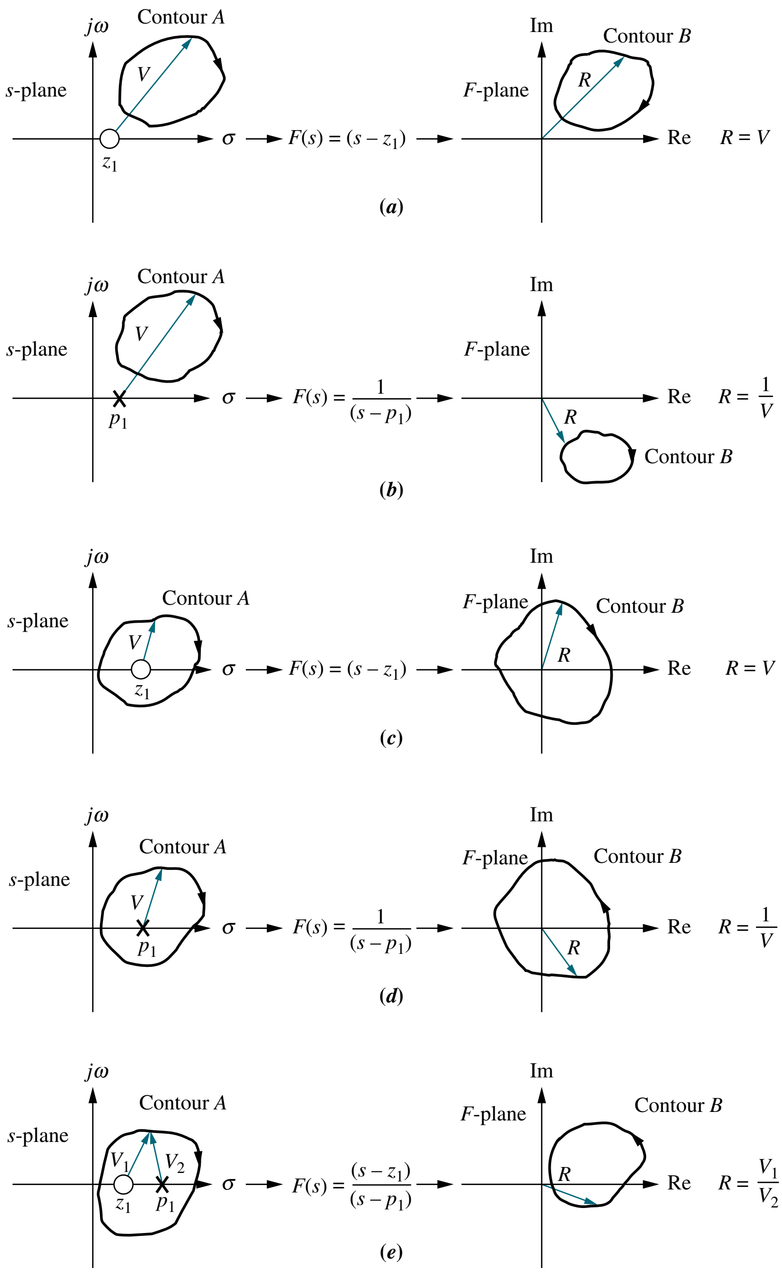


FIGURE 10.22 Examples of contour mapping

The vector approach to performing the calculation, covered in Section 8.1, can be used as an alternative. Some examples of contour mapping are shown in Figure 10.22 for some simple $F(s)$. The mapping of each point is defined by complex arithmetic, where the resulting complex number, R, is evaluated from the complex numbers represented by V, as shown in the last column of Figure 10.22. You should verify that if we assume a clockwise direction for mapping the points on contour A, then contour B maps in a clockwise direction if $F(s)$ in Figure 10.22 has just zeros or has just poles that are not encircled by the contour. The contour B maps in a counterclockwise direction if $F(s)$ has just poles that are encircled by the contour. Also, you should verify that if the pole or zero of $F(s)$ is enclosed by contour A, the

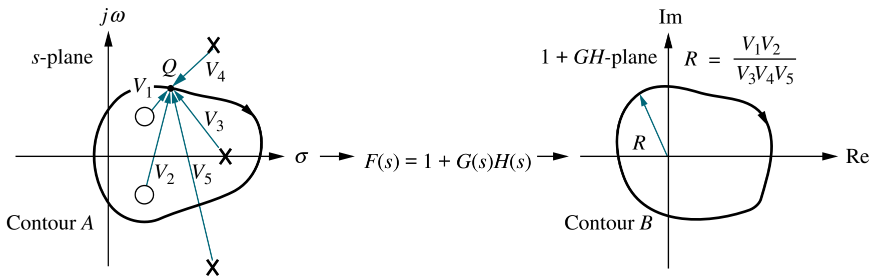


FIGURE 10.23 Vector representation of mapping

mapping encircles the origin. In the last case of Figure 10.22, the pole and zero rotation cancel, and the mapping does not encircle the origin.

Let us now begin the derivation of the Nyquist criterion for stability. We show that a unique relationship exists between the number of poles of $F(s)$ contained inside contour A, the number of zeros of $F(s)$ contained inside contour A, and the number of counterclockwise encirclements of the origin for the mapping of contour B. We then show how this interrelationship can be used to determine the stability of closed-loop systems. This method of determining stability is called the *Nyquist criterion*.

Let us first assume that $F(s) = 1 + G(s)H(s)$, with the picture of the poles and zeros of $1 + G(s)H(s)$ as shown in Figure 10.23 near contour A. Hence, $R = (V_1V_2)/(V_3V_4V_5)$. As each point Q of the contour A is substituted into $1 + G(s)H(s)$, a mapped point results on contour B. Assuming that $F(s) = 1 + G(s)H(s)$ has two zeros and three poles, each parenthetical term of Eq. (10.39) is a vector in Figure 10.23. As we move around contour A in a clockwise direction, each vector of Eq. (10.39) that lies inside contour A will appear to undergo a complete rotation, or a change in angle of 360°. On the other hand, each vector drawn from the poles and zeros of $1 + G(s)H(s)$ that exist outside contour A will appear to oscillate and return to its previous position, undergoing a net angular change of 0°.

Each pole or zero factor of $1 + G(s)H(s)$ whose vector undergoes a complete rotation around contour A must yield a change of 360° in the resultant, R, or a complete rotation of the mapping of contour B. If we move in a clockwise direction along contour A, each zero inside contour A yields a rotation in the clockwise direction, while each pole inside contour A yields a rotation in the counterclockwise direction since poles are in the denominator of Eq. (10.39).

Thus, $N = P - Z$, where N equals the number of counterclockwise rotations of contour B about the origin; P equals the number of poles of $1 + G(s)H(s)$ inside contour A, and Z equals the number of zeros of $1 + G(s)H(s)$ inside contour A.

Since the poles shown in Figure 10.23 are poles of $1 + G(s)H(s)$, we know from Eqs. (10.38) that they are also the poles of $G(s)H(s)$ and are known. But since *the zeros shown in Figure 10.23 are the zeros of* $1 + G(s)H(s)$, we know from Eqs. (10.38) that *they are also the poles of the closed-loop system and are not known*. Thus, P equals the number of enclosed open-loop poles, and Z equals the number of enclosed closed-loop poles. Hence, $N = P - Z$, or alternately, $Z = P - N$, tells us that the number of closed-loop poles inside the contour (which is the same as the zeros inside the contour) equals the number of open-loop poles of $G(s)H(s)$ inside the contour minus the number of counterclockwise rotations of the mapping about the origin.

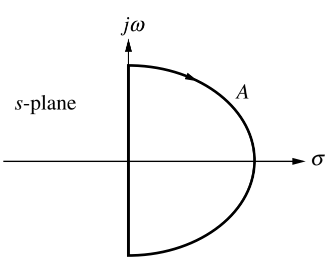


FIGURE 10.24 Contour enclosing right half-plane to determine stability

If we extend the contour to include the entire right half-plane, as shown in Figure 10.24, we can count the number of right–half-plane, closed-loop poles inside contour A and determine a system's stability. Since we can count the number of open-loop poles, P, inside the contour, which are the same as the right–half-plane poles of $G(s)H(s)$, the only problem remaining is how to obtain the mapping and find N.

Since all of the poles and zeros of $G(s)H(s)$ are known, what if we map through $G(s)H(s)$ instead of $1+G(s)H(s)$? The resulting contour is the same as a mapping through $1+G(s)H(s)$, except that it is translated one unit to the left; thus, we count rotations about -1 instead of rotations about the origin. Hence, the final statement of the Nyquist stability criterion is as follows:

If a contour, A, that encircles the entire right half-plane is mapped through $G(s)H(s)$, then *the number of closed-loop poles, Z, in the right half-plane equals the* number of open-loop poles, P, that are in the right half-plane minus the number of counterclockwise revolutions, N, around -1 of the mapping; that is, $Z = P - N$. The mapping is called the *Nyquist diagram*, or *Nyquist plot, of* $G(s)H(s)$.

We can now see why this method is classified as a frequency response technique. Around contour A in Figure 10.24, the mapping of the points on the $j\omega$-axis through the function $G(s)H(s)$ is the same as substituting $s = j\omega$ into $G(s)H(s)$ to form the frequency response function $G(j\omega)H(j\omega)$. We are thus finding the frequency response of $G(s)H(s)$ over that part of contour A on the positive $j\omega$-axis. In other words, part of the Nyquist diagram is the polar plot of the frequency response of $G(s)H(s)$.

Applying the Nyquist Criterion to Determine Stability

Before describing how to sketch a Nyquist diagram, let us look at some typical examples that use the Nyquist criterion to determine the stability of a system. These examples give us a perspective prior to engaging in the details of mapping. Figure 10.25(a) shows a contour A that does not enclose closed-loop poles, that is, the zeros of $1+G(s)H(s)$. The contour thus maps through $G(s)H(s)$ into a Nyquist diagram that does not encircle -1. Hence, $P = 0$, $N = 0$, and $Z = P - N = 0$. Since Z is the number of closed-loop poles inside contour A, which encircles the right half-plane, this system has no right–half-plane poles and is stable.

On the other hand, Figure 10.25(b) shows a contour A that, while it does not enclose open-loop poles, does generate two clockwise encirclements of -1. Thus, $P = 0$, $N = -2$, and the system is unstable; it has two closed-loop poles in the right half-plane since $Z = P - N = 2$. The two closed-loop poles are shown inside contour

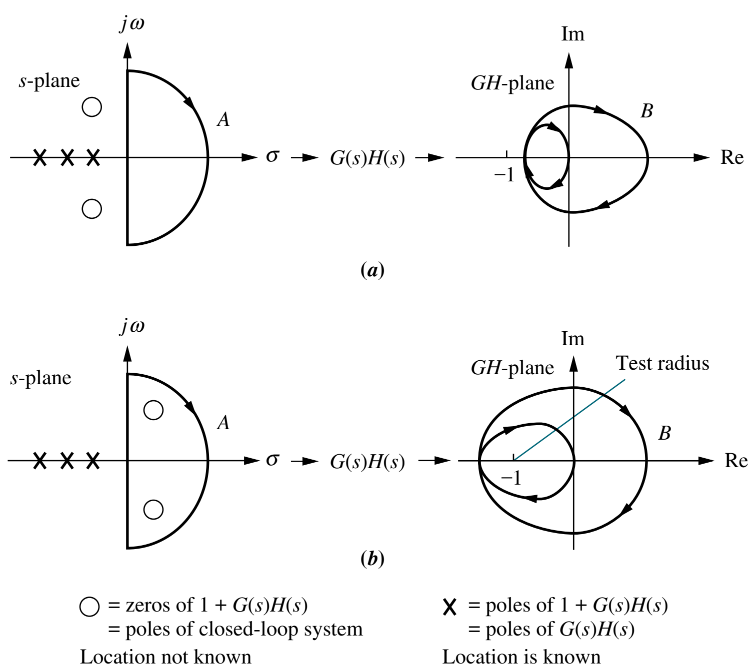


FIGURE 10.25 Mapping examples: **a.** Contour does not enclose closed-loop poles; **b.** contour does enclose closed-loop poles

A in Figure 10.25(b) as zeros of $1 + G(s)H(s)$. You should keep in mind that the existence of these poles is not known a priori.

In this example, notice that clockwise encirclements imply a negative value for N. The number of encirclements can be determined by drawing a test radius from -1 in any convenient direction and counting the number of times the Nyquist diagram crosses the test radius. Counterclockwise crossings are positive, and clockwise crossings are negative. For example, in Figure 10.25(b), contour B crosses the test radius twice in a clockwise direction. Hence, there are -2 encirclements of the point -1.

Before applying the Nyquist criterion to other examples in order to determine a system's stability, we must first gain experience in sketching Nyquist diagrams. The next section covers the development of this skill.

10.4 Sketching the Nyquist Diagram

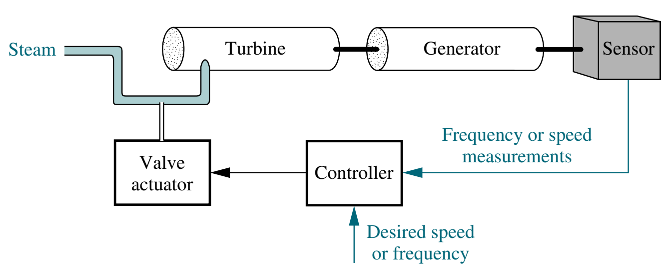
The contour that encloses the right half-plane can be mapped through the function $G(s)H(s)$ by substituting points along the contour into $G(s)H(s)$. The points along the positive extension of the imaginary axis yield the polar frequency response of $G(s)H(s)$. Approximations can be made to $G(s)H(s)$ for points around the infinite semicircle by assuming that the vectors originate at the origin. Thus, their length is infinite, and their angles are easily evaluated.

However, most of the time a simple sketch of the Nyquist diagram is all that is needed. A sketch can be obtained rapidly by looking at the vectors of $G(s)H(s)$ and their motion along the contour. In the examples that follow, we stress this rapid method for sketching the Nyquist diagram. However, the examples also include analytical expressions for $G(s)H(s)$ for each section of the contour to aid you in determining the shape of the Nyquist diagram.

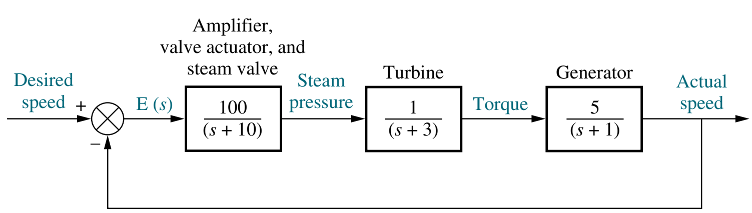
Example 10.4

Sketching a Nyquist Diagram

PROBLEM: Speed controls find wide application throughout industry and the home. Figure 10.26(a) shows one application: output frequency control of electrical



(a)



(b)

FIGURE 10.26
a. Turbine and generator; **b.** block diagram of speed control system for Example 10.4

power from a turbine and generator pair. By regulating the speed, the control system ensures that the generated frequency remains within tolerance. Deviations from the desired speed are sensed, and a steam valve is changed to compensate for the speed error. The system block diagram is shown in Figure 10.26(b). Sketch the Nyquist diagram for the system of Figure 10.26.

SOLUTION: Conceptually, the Nyquist diagram is plotted by substituting the points of the contour shown in Figure 10.27(a) into $G(s) = 500/[(s+1)(s+3)(s+10)]$. This process is equivalent to performing complex arithmetic using the vectors of $G(s)$ drawn to the points of the contour as shown in Figure 10.27(a) and (b). Each pole and zero term of $G(s)$ shown in Figure 10.26(b) is a vector in Figure 10.27(a) and (b). The resultant vector, R, found at any point along the contour is in general the product of the zero vectors divided by the product of the pole vectors (see Figure 10.27(c)). Thus, the magnitude of the resultant is the product of the zero lengths divided by the product of the pole lengths, and the angle of the resultant is the sum of the zero angles minus the sum of the pole angles.

As we move in a clockwise direction around the contour from point A to point C in Figure 10.27(a), the resultant angle goes from 0° to $-3 \times 90° = -270°$, or from A' to C' in Figure 10.27(c). Since the angles emanate from poles in the denominator of $G(s)$, the rotation or increase in angle is really a decrease in angle

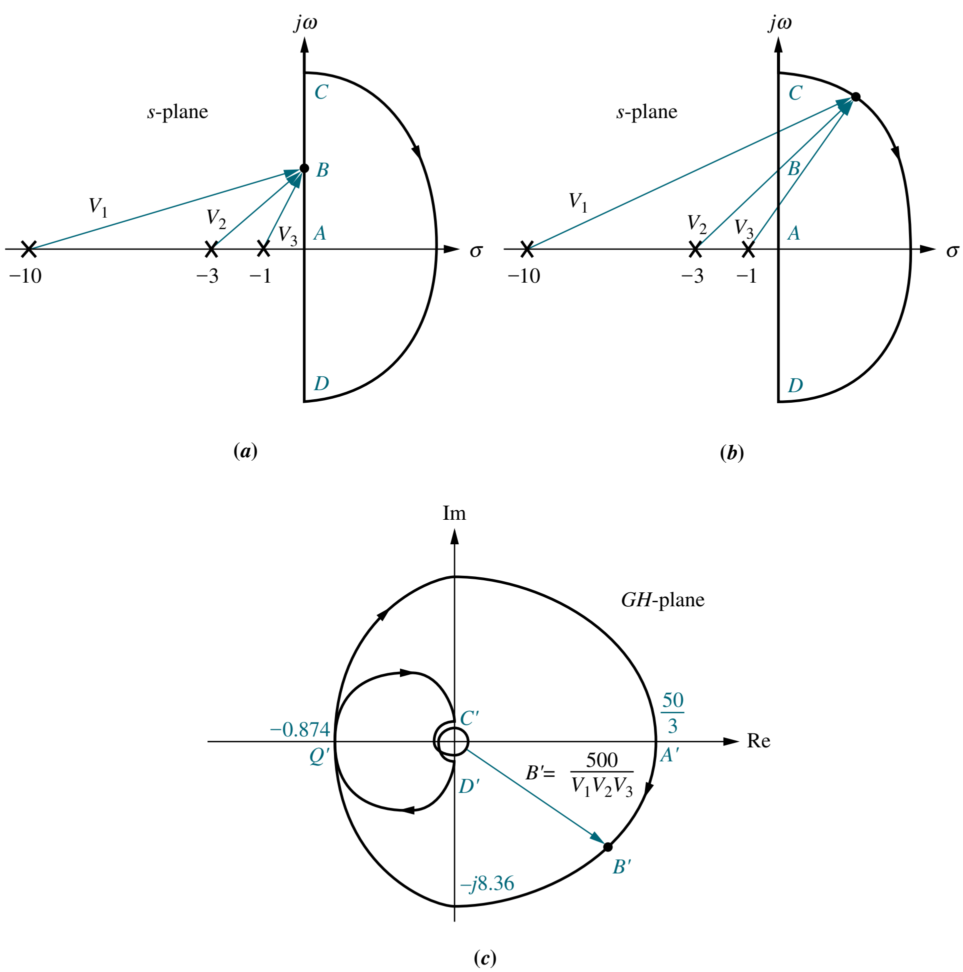


FIGURE 10.27 Vector evaluation of the Nyquist diagram for Example 10.4:
a. vectors on contour at low frequency;
b. vectors on contour around infinity;
c. Nyquist diagram

of the function $G(s)$; the poles gain 270° in a counterclockwise direction, which explains why the function loses 270°.

While the resultant moves from A' to C' in Figure 10.27(c), its magnitude changes as the product of the zero lengths divided by the product of the pole lengths. Thus, the resultant goes from a finite value at zero frequency (at point A of Figure 10.27(a), there are three finite pole lengths) to zero magnitude at infinite frequency at point C (at point C of Figure 10.27(a), there are three infinite pole lengths).

The mapping from point A to point C can also be explained analytically. From A to C the collection of points along the contour is imaginary. Hence, from A to C, $G(s) = G(j\omega)$, or from Figure 10.26(b),

$$G(j\omega) = \left.\frac{500}{(s+1)(s+3)(s+10)}\right|_{s\to j\omega} = \frac{500}{(-14\omega^2+30)+j(43\omega-\omega^3)} \tag{10.40}$$

Multiplying the numerator and denominator by the complex conjugate of the denominator, we obtain

$$G(j\omega) = 500\frac{(-14\omega^2+30)-j(43\omega-\omega^3)}{(-14\omega^2+30)^2+(43\omega-\omega^3)^2} \tag{10.41}$$

At zero frequency, $G(j\omega) = 500/30 = 50/3$. Thus, the Nyquist diagram starts at $50/3$ at an angle of 0°. As ω increases the real part remains positive, and the imaginary part remains negative. At $\omega = \sqrt{30/14}$, the real part becomes negative. At $\omega = \sqrt{43}$, the Nyquist diagram crosses the negative real axis since the imaginary term goes to zero. The real value at the axis crossing, point Q' in Figure 10.27(c), found by substituting into Eq. (10.41), is -0.874. Continuing toward $\omega = \infty$, the real part is negative, and the imaginary part is positive. At infinite frequency $G(j\omega) \approx 500j/\omega^3$, or approximately zero at 90°.

Around the infinite semicircle from point C to point D shown in Figure 10.27(b), the vectors rotate clockwise, each by 180°. Hence, the resultant undergoes a counterclockwise rotation of $3 \times 180°$, starting at point C' and ending at point D' of Figure 10.27(c). Analytically, we can see this by assuming that around the infinite semicircle, the vectors originate approximately at the origin and have infinite length. For any point on the s-plane, the value of $G(s)$ can be found by representing each complex number in polar form, as follows:

$$G(s) = \frac{500}{(R_{-1}e^{j\theta_{-1}})(R_{-3}e^{j\theta_{-3}})(R_{-10}e^{j\theta_{-10}})} \tag{10.42}$$

where R_{-i} is the magnitude of the complex number $(s+1)$, and θ_{-i} is the angle of the complex number $(s+i)$. Around the infinite semicircle, all R_{-i} are infinite, and we can use our assumption to approximate the angles as if the vectors originated at the origin. Thus, around the infinite semicircle,

$$G(s) = \frac{500}{\infty\angle(\theta_{-1}+\theta_{-3}+\theta_{-10})} = 0\angle-(\theta_{-1}+\theta_{-3}+\theta_{-10}) \tag{10.43}$$

At point C in Figure 10.27(b), the angles are all 90°. Hence, the resultant is $0\angle-270°$, shown as point C' in Figure 10.27(c). Similarly, at point D, $G(s) = 0\angle+270°$ and maps into point D'. You can select intermediate points to verify the spiral whose radius vector approaches zero at the origin, as shown in Figure 10.27(c).

The negative imaginary axis can be mapped by realizing that the real part of $G(j\omega)H(j\omega)$ is always an even function, whereas the imaginary part of $G(j\omega)H(j\omega)$ is an odd function. That is, the real part will not change sign when negative values of

ω are used, whereas the imaginary part will change sign. Thus, the mapping of the negative imaginary axis is a mirror image of the mapping of the positive imaginary axis. The mapping of the section of the contour from points D to A is drawn as a mirror image about the real axis of the mapping of points A to C.

In the previous example, there were no open-loop poles situated along the contour enclosing the right half-plane. If such poles exist, then a detour around the poles on the contour is required; otherwise, the mapping would go to infinity in an undetermined way, without angular information. Subsequently, a complete sketch of the Nyquist diagram could not be made, and the number of encirclements of -1 could not be found.

Let us assume a $G(s)H(s) = N(s)/sD(s)$ where $D(s)$ has imaginary roots. The s term in the denominator and the imaginary roots of $D(s)$ are poles of $G(s)H(s)$ that lie on the contour, as shown in Figure 10.28(a). To sketch the Nyquist diagram, the contour must detour around each open-loop pole lying on its path. The detour can be to the right of the pole, as shown in Figure 10.28(b), which makes it clear that each pole's vector rotates through $+180°$ as we move around the contour near that pole. This knowledge of the angular rotation of the poles on the contour permits us to complete the Nyquist diagram. Of course, our detour must carry us only an infinitesimal distance into the right half-plane, or else some closed-loop, right–half-plane poles will be excluded in the count.

We can also detour to the left of the open-loop poles. In this case, each pole rotates through an angle of $-180°$ as we detour around it. Again, the detour must be infinitesimally small, or else we might include some left–half-plane poles in the count. Let us look at an example.

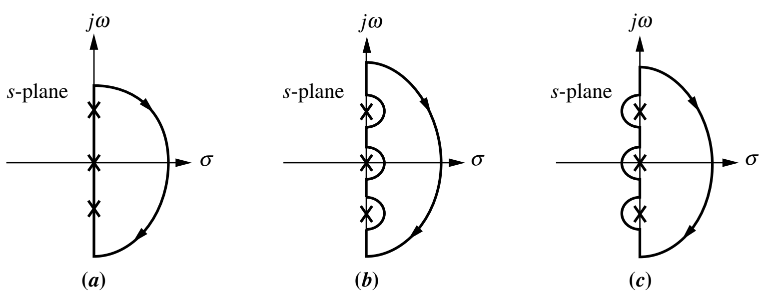


FIGURE 10.28 Detouring around open-loop poles: **a.** poles on contour; **b.** detour right; **c.** detour left

Example 10.5

Nyquist Diagram for Open-Loop Function with Poles on Contour

PROBLEM: Sketch the Nyquist diagram of the unity feedback system of Figure 10.10, where $G(s) = (s+2)/s^2$.

SOLUTION: The system's two poles at the origin are on the contour and must be bypassed, as shown in Figure 10.29(a). The mapping starts at point A and continues in a clockwise direction. Points A, B, C, D, E, and F of Figure 10.29(a) map respectively into points A', B', C', D', E', and F' of Figure 10.29(b).

At point A, the two open-loop poles at the origin contribute $2 \times 90° = 180°$, and the zero contributes $0°$. The total angle at point A is thus $-180°$. Close to the origin, the function is infinite in magnitude because of the close proximity to the

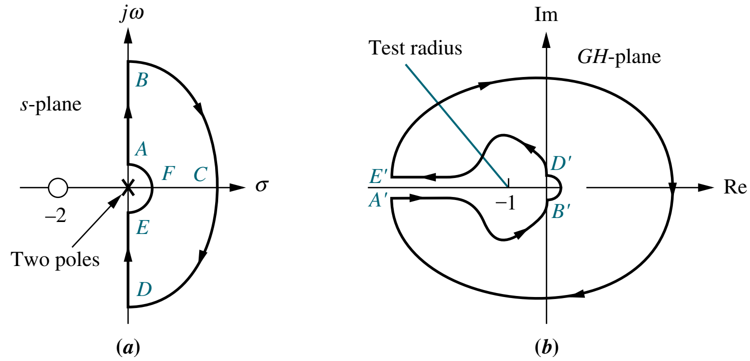


FIGURE 10.29 **a.** Contour for Example 10.5; **b.** Nyquist diagram for Example 10.5

two open-loop poles. Thus, point A maps into point A', located at infinity at an angle of $-180°$.

Moving from point A to point B along the contour yields a net change in angle of $+90°$ from the zero alone. The angles of the poles remain the same. Thus, the mapping changes by $+90°$ in the counterclockwise direction. The mapped vector goes from $-180°$ at A' to $-90°$ at B'. At the same time, the magnitude changes from infinity to zero since at point B there is one infinite length from the zero divided by two infinite lengths from the poles.

Alternately, the frequency response can be determined analytically from $G(j\omega) = (2 + j\omega)/(-\omega^2)$, considering ω going from 0 to ∞. At low frequencies, $G(j\omega) \approx 2/(-\omega^2)$, or $\infty \angle 180°$. At high frequencies, $G(j\omega) \approx j/(-\omega)$, or $0\angle -90°$. Also, the real and imaginary parts are always negative.

As we travel along the contour BCD, the function magnitude stays at zero (one infinite zero length divided by two infinite pole lengths). As the vectors move through BCD, the zero's vector and the two poles' vectors undergo changes of $-180°$ each. Thus, the mapped vector undergoes a net change of $+180°$, which is the angular change of the zero minus the sum of the angular changes of the poles $\{-180 - [2(-180)] = +180\}$. The mapping is shown as $B'\,C'\,D'$, where the resultant vector changes by $+180°$ with a magnitude of ϵ that approaches zero.

From the analytical point of view,

$$G(s) = \frac{R_{-2}\angle\theta_{-2}}{(R_0\angle\theta_0)(R_0\angle\theta_0)} \tag{10.44}$$

anywhere on the s-plane where $R_{-2}\angle\theta_{-2}$ is the vector from the zero at -2 to any point on the s-plane, and $R_0\angle\theta_0$ is the vector from a pole at the origin to any point on the s-plane. Around the infinite semicircle, all $R_{-i} = \infty$, and all angles can be approximated as if the vectors originated at the origin. Thus at point B, $G(s) = 0\angle -90°$ since all $\theta_{-i} = 90°$ in Eq. (10.44). At point C, all $R_{-i} = \infty$, and all $\theta_{-i} = 0°$ in Eq. (10.44). Thus, $G(s) = 0\angle 0°$. At point D, all $R_{-i} = \infty$, and all $\theta_{-i} = -90°$ in Eq. (10.44). Thus, $G(s) = 0\angle 90°$.

The mapping of the section of the contour from D to E is a mirror image of the mapping of A to B. The result is D' to E'.

Finally, over the section EFA, the resultant magnitude approaches infinity. The angle of the zero does not change, but each pole changes by $+180°$. This change yields a change in the function of $-2 \times 180° = -360°$. Thus, the mapping from E' to A' is shown as infinite in length and rotating $-360°$. Analytically, we can use Eq. (10.44) for the points along the contour EFA. At E,

$G(s) = (2\angle 0°)/[(\epsilon\angle -90°)(\epsilon\angle -90°)] = \infty\angle 180°$. At F, $G(s) = (2\angle 0°)/[(\epsilon\angle 0°)(\epsilon\angle 0°)] = \infty\angle 0°$. At A, $G(s) = (2\angle 0°)/[(\epsilon\angle 90°)(\epsilon\angle 90°)] = \infty\angle -180°$.

The Nyquist diagram is now complete, and a test radius drawn from -1 in Figure 10.29(b) shows one counterclockwise revolution, and one clockwise revolution, yielding zero encirclements.

MATLAB

ML

`Students who are using MATLAB should now run ch10p2 in Appendix B. You will learn how to use MATLAB to make a Nyquist plot and list the points on the plot. You will also learn how to specify a range for frequency. This exercise solves Example 10.5 using MATLAB.`

Skill-Assessment Exercise 10.3

PROBLEM: Sketch the Nyquist diagram for the system shown in Figure 10.10 where

$$G(s) = \frac{1}{(s+2)(s+4)}$$

Compare your sketch with the polar plot in Skill-Assessment Exercise 10.1(c).

ANSWER: The complete solution is located at www.wiley.com/college/nise.

In this section, we learned how to sketch a Nyquist diagram. We saw how to calculate the value of the intersection of the Nyquist diagram with the negative real axis. This intersection is important in determining the number of encirclements of -1. Also, we showed how to sketch the Nyquist diagram when open-loop poles exist on the contour; this case required detours around the poles. In the next section, we apply the Nyquist criterion to determine the stability of feedback control systems.

10.5 Stability via the Nyquist Diagram

We now use the Nyquist diagram to determine a system's stability, using the simple equation $Z = P - N$. The values of P, the number of open-loop poles of $G(s)H(s)$ enclosed by the contour, and N, the number of encirclements the Nyquist diagram makes about -1, are used to determine Z, the number of right–half-plane poles of the closed-loop system.

If the closed-loop system has a variable gain in the loop, one question we would like to ask is, "For what range of gain is the system stable?" This question, previously answered by the root locus method and the Routh-Hurwitz criterion, is now answered via the Nyquist criterion. The general approach is to set the loop gain equal to unity and draw the Nyquist diagram. Since gain is simply a multiplying factor, the effect of the gain is to multiply the resultant by a constant anywhere along the Nyquist diagram.

For example, consider Figure 10.30, which summarizes the Nyquist approach for a system with variable gain, K. As the gain is varied, we can visualize the Nyquist diagram in Figure 10.30(c) expanding (increased gain) or shrinking (decreased gain) like a balloon. This motion could move the Nyquist diagram past the -1 point, changing the stability picture. For this system, since $P = 2$, the critical point must be encircled by the Nyquist diagram to yield $N = 2$ and a stable system. A reduction in

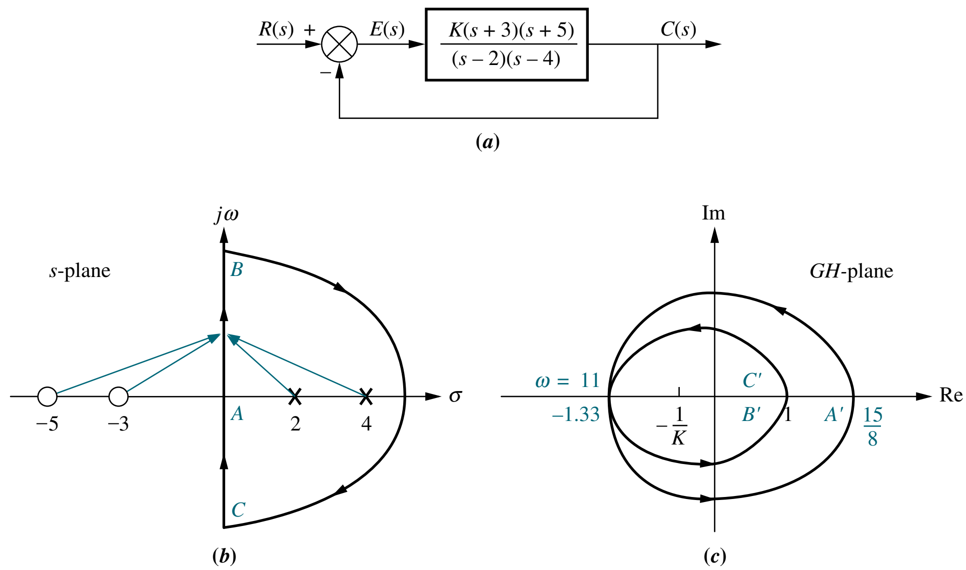


FIGURE 10.30 Demonstrating Nyquist stability: **a.** system; **b.** contour; **c.** Nyquist diagram

> **TryIt 10.2**
>
> Use MATLAB, the Control System Toolbox, and the following statements to plot the Nyquist diagram of the system shown in Figure 10.30(*a*).
>
> ```
> G=zpk([-3,-5],...
>  [2,4],1)
> nyquist(G)
> ```
>
> After the Nyquist diagram appears, click on the curve and drag to read the coordinates.

gain would place the critical point outside the Nyquist diagram where $N = 0$, yielding $Z = 2$, an unstable system.

From another perspective we can think of the Nyquist diagram as remaining stationary and the -1 point moving along the real axis. In order to do this, we set the gain to unity and position the critical point at $-1/K$ rather than -1. Thus, the critical point appears to move closer to the origin as K increases.

Finally, if the Nyquist diagram intersects the real axis at -1, then $G(j\omega)H(j\omega) = -1$. From root locus concepts, when $G(s)H(s) = -1$, the variable s is a closed-loop pole of the system. Thus, the frequency at which the Nyquist diagram intersects -1 is the same frequency at which the root locus crosses the $j\omega$-axis. Hence, the system is marginally stable if the Nyquist diagram intersects the real axis at -1.

In summary, then, if the open-loop system contains a variable gain, K, set $K = 1$ and sketch the Nyquist diagram. Consider the critical point to be at $-1/K$ rather than at -1. Adjust the value of K to yield stability, based upon the Nyquist criterion.

Example 10.6

Range of Gain for Stability via The Nyquist Criterion

PROBLEM: For the unity feedback system of Figure 10.10, where $G(s) = K/[s(s+3)(s+5)]$, find the range of gain, K, for stability, instability, and the value of gain for marginal stability. For marginal stability also find the frequency of oscillation. Use the Nyquist criterion.

SOLUTION: First set $K = 1$ and sketch the Nyquist diagram for the system, using the contour shown in Figure 10.31(*a*). For all points on the imaginary axis,

$$G(j\omega)H(j\omega) = \frac{K}{s(s+3)(s+5)}\bigg|_{\substack{K=1\\ s=j\omega}} = \frac{-8\omega^2 - j(15\omega - \omega^3)}{64\omega^4 + \omega^2(15-\omega^2)^2} \tag{10.45}$$

At $\omega = 0$, $G(j\omega)H(j\omega) = -0.0356 - j\infty$.

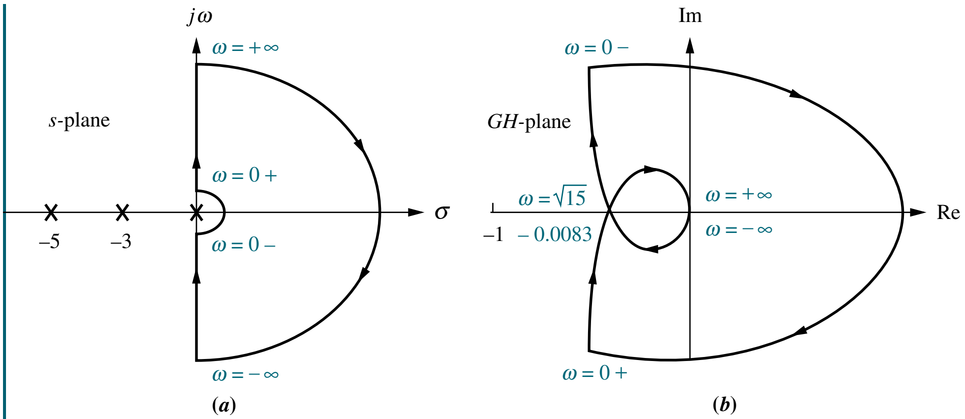


FIGURE 10.31
a. Contour for Example 10.6;
b. Nyquist diagram

Next find the point where the Nyquist diagram intersects the negative real axis. Setting the imaginary part of Eq. (10.45) equal to zero, we find $\omega = \sqrt{15}$. Substituting this value of ω back into Eq. (10.45) yields the real part of -0.0083. Finally, at $\omega = \infty$, $G(j\omega)H(j\omega) = G(s)H(s)|_{s\to j\infty} = 1/(j\infty)^3 = 0\angle -270^\circ$.

From the contour of Figure 10.31(a), $P = 0$; for stability N must then be equal to zero. From Figure 10.31(b), the system is stable if the critical point lies outside the contour ($N = 0$), so that $Z = P - N = 0$. Thus, K can be increased by $1/0.0083 = 120.5$ before the Nyquist diagram encircles -1. Hence, for stability, $K < 120.5$. For marginal stability $K = 120.5$. At this gain the Nyquist diagram intersects -1, and the frequency of oscillation is $\sqrt{15}$ rad/s.

Now that we have used the Nyquist diagram to determine stability, we can develop a simplified approach that uses only the mapping of the positive $j\omega$-axis.

Stability via Mapping Only the Positive $j\omega$-Axis

Once the stability of a system is determined by the Nyquist criterion, continued evaluation of the system can be simplified by using just the mapping of the positive $j\omega$-axis. This concept plays a major role in the next two sections, where we discuss stability margin and the implementation of the Nyquist criterion with Bode plots.

Consider the system shown in Figure 10.32, which is stable at low values of gain and unstable at high values of gain. Since the contour does not encircle open-loop

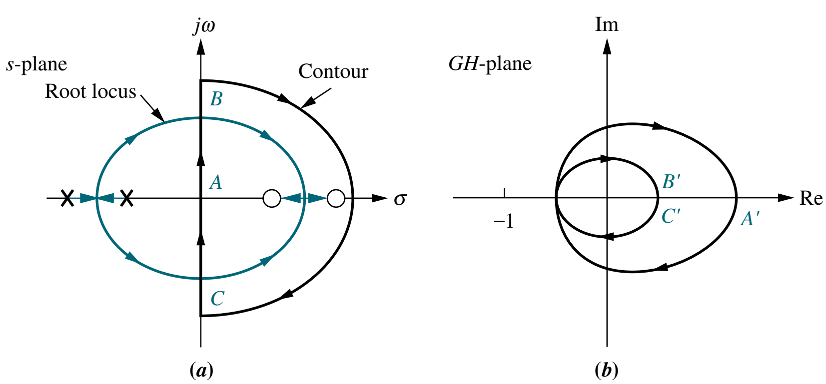


FIGURE 10.32
a. Contour and root locus of system that is stable for small gain and unstable for large gain;
b. Nyquist diagram

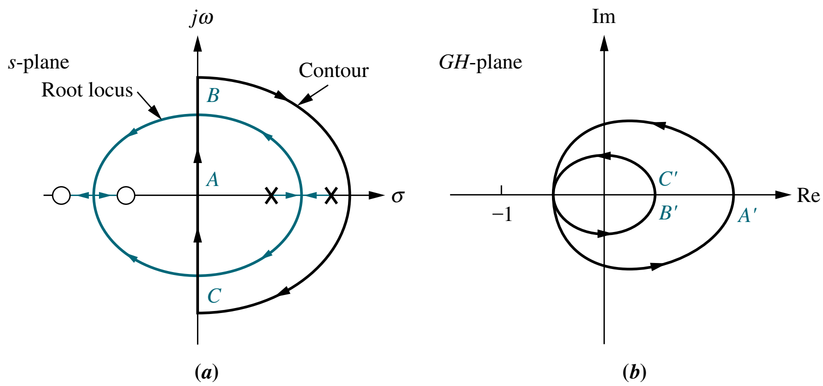


FIGURE 10.33 **a.** Contour and root locus of system that is unstable for small gain and stable for large gain; **b.** Nyquist diagram

poles, the Nyquist criterion tells us that we must have no encirclements of -1 for the system to be stable. We can see from the Nyquist diagram that the encirclements of the critical point can be determined from the mapping of the positive $j\omega$-axis alone. If the gain is small, the mapping will pass to the right of -1, and the system will be stable. If the gain is high, the mapping will pass to the left of -1, and the system will be unstable. Thus, this system is stable for the range of loop gain, K, that ensures that the *open-loop magnitude is less than unity at that frequency where the phase angle is* 180° (*or, equivalently*, -180°). This statement is thus an alternative to the Nyquist criterion for this system.

Now consider the system shown in Figure 10.33, which is unstable at low values of gain and stable at high values of gain. Since the contour encloses two open-loop poles, two counterclockwise encirclements of the critical point are required for stability. Thus, for this case the system is stable if the *open-loop magnitude is greater than unity at that frequency where the phase angle is* 180° (*or, equivalently*, -180°).

In summary, first determine stability from the Nyquist criterion and the Nyquist diagram. Next interpret the Nyquist criterion and determine whether the mapping of just the positive imaginary axis should have a gain of less than or greater than unity at 180°. If the Nyquist diagram crosses $\pm 180^\circ$ at multiple frequencies, determine the interpretation from the Nyquist criterion.

Example 10.7

Stability Design via Mapping Positive $j\omega$-Axis

PROBLEM: Find the range of gain for stability and instability, and the gain for marginal stability, for the unity feedback system shown in Figure 10.10, where $G(s) = K/[(s^2 + 2s + 2)(s + 2)]$. For marginal stability find the radian frequency of oscillation. Use the Nyquist criterion and the mapping of only the positive imaginary axis.

SOLUTION: Since the open-loop poles are only in the left–half-plane, the Nyquist criterion tells us that we want no encirclements of -1 for stability. Hence, a gain less than unity at $\pm 180^\circ$ is required. Begin by letting $K = 1$ and draw the portion of the contour along the positive imaginary axis as shown in Figure 10.34(a). In

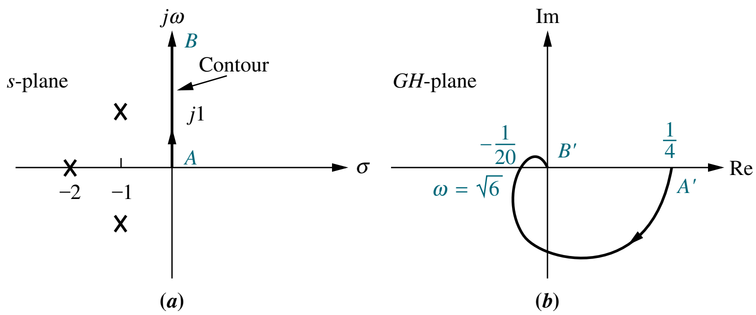


FIGURE 10.34 **a.** Portion of contour to be mapped for Example 10.7; **b.** Nyquist diagram of mapping of positive imaginary axis

Figure 10.34(b), the intersection with the negative real axis is found by letting $s = j\omega$ in $G(s)H(s)$, setting the imaginary part equal to zero to find the frequency, and then substituting the frequency into the real part of $G(j\omega)H(j\omega)$. Thus, for any point on the positive imaginary axis,

$$G(j\omega)H(j\omega) = \left.\frac{1}{(s^2+2s+2)(s+2)}\right|_{s\to j\omega} = \frac{4(1-\omega^2) - j\omega(6-\omega^2)}{16(1-\omega^2)^2 + \omega^2(6-\omega^2)^2} \tag{10.46}$$

Setting the imaginary part equal to zero, we find $\omega = \sqrt{6}$. Substituting this value back into Eq. (10.46) yields the real part, $-(1/20) = (1/20)\angle 180°$.

This closed-loop system is stable if the magnitude of the frequency response is less than unity at 180°. Hence, the system is stable for $K < 20$, unstable for $K > 20$, and marginally stable for $K = 20$. When the system is marginally stable, the radian frequency of oscillation is $\sqrt{6}$.

Skill-Assessment Exercise 10.4

WileyPLUS WPCS Control Solutions

PROBLEM: For the system shown in Figure 10.10, where

$$G(s) = \frac{K}{(s+2)(s+4)(s+6)}$$

do the following:

a. Plot the Nyquist diagram.

b. Use your Nyquist diagram to find the range of gain, K, for stability.

ANSWERS:

a. See the answer at www.wiley.com/college/nise.

b. Stable for $K < 480$

The complete solution is at www.wiley.com/college/nise.

10.6 Gain Margin and Phase Margin via the Nyquist Diagram

Now that we know how to sketch and interpret a Nyquist diagram to determine a closed-loop system's stability, let us extend our discussion to concepts that will eventually lead us to the design of transient response characteristics via frequency response techniques.

Using the Nyquist diagram, we define two quantitative measures of how stable a system is. These quantities are called *gain margin* and *phase margin*. Systems with greater gain and phase margins can withstand greater changes in system parameters before becoming unstable. In a sense, gain and phase margins can be qualitatively related to the root locus, in that systems whose poles are farther from the imaginary axis have a greater degree of stability.

In the last section, we discussed stability from the point of view of gain at 180° phase shift. This concept leads to the following definitions of gain margin and phase margin:

Gain margin, G_M. The gain margin is the change in open-loop gain, expressed in decibels (dB), required at 180° of phase shift to make the closed-loop system unstable.

Phase margin, Φ_M. The phase margin is the change in open-loop phase shift required at unity gain to make the closed-loop system unstable.

These two definitions are shown graphically on the Nyquist diagram in Figure 10.35.

Assume a system that is stable if there are no encirclements of -1. Using Figure 10.35, let us focus on the definition of gain margin. Here a gain difference between the Nyquist diagram's crossing of the real axis at $-1/a$ and the -1 critical point determines the proximity of the system to instability. Thus, if the gain of the system were multiplied by a units, the Nyquist diagram would intersect the critical point. We then say that the gain margin is a units, or, expressed in dB, $G_M = 20 \log a$. Notice that the gain margin is the reciprocal of the real-axis crossing expressed in dB.

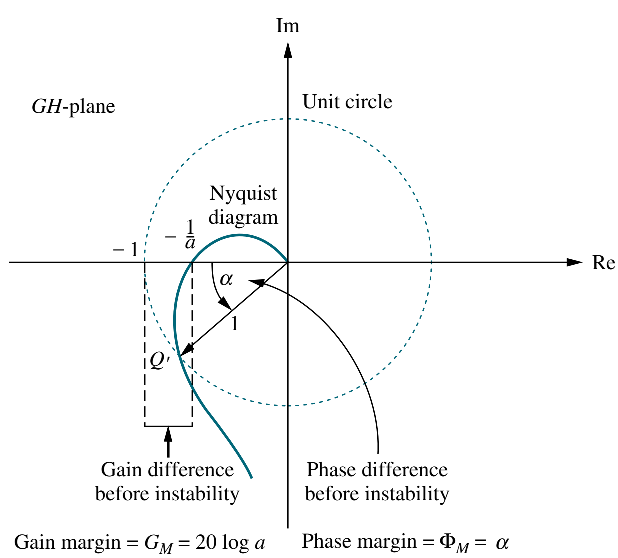


FIGURE 10.35 Nyquist diagram showing gain and phase margins

In Figure 10.35, we also see the phase margin graphically displayed. At point Q', where the gain is unity, a represents the system's proximity to instability. That is, at unity gain, if a phase shift of α degrees occurs, the system becomes unstable. Hence, the amount of phase margin is α. Later in the chapter, we show that phase margin can be related to the damping ratio. Thus, we will be able to relate frequency response characteristics to transient response characteristics as well as stability. We will also show that the calculations of gain and phase margins are more convenient if Bode plots are used rather than a Nyquist diagram, such as that shown in Figure 10.35.

For now let us look at an example that shows the calculation of the gain and phase margins.

Example 10.8

Finding Gain and Phase Margins

PROBLEM: Find the gain and phase margin for the system of Example 10.7 if $K = 6$.

SOLUTION: To find the gain margin, first find the frequency where the Nyquist diagram crosses the negative real axis. Finding $G(j\omega)H(j\omega)$, we have

$$\begin{aligned} G(j\omega)H(j\omega) &= \left.\frac{6}{(s^2+2s+2)(s+2)}\right|_{s\to j\omega} \\ &= \frac{6[4(1-\omega^2) - j\omega(6-\omega^2)]}{16(1-\omega^2)^2 + \omega^2(6-\omega^2)^2} \end{aligned} \tag{10.47}$$

The Nyquist diagram crosses the real axis at a frequency of $\sqrt{6}$ rad/s. The real part is calculated to be -0.3. Thus, the gain can be increased by $(1/0.3) = 3.33$ before the real part becomes -1. Hence, the gain margin is

$$G_M = 20 \log 3.33 = 10.45 \text{ dB} \tag{10.48}$$

To find the phase margin, find the frequency in Eq. (10.47) for which the magnitude is unity. As the problem stands, this calculation requires computational tools, such as a function solver or the program described in Appendix H.2. Later in the chapter we will simplify the process by using Bode plots. Eq. (10.47) has unity gain at a frequency of 1.253 rad/s. At this frequency, the phase angle is -112.3°. The difference between this angle and -180° is 67.7°, which is the phase margin.

MATLAB
ML

`Students who are using MATLAB should now run ch10p3 in Appendix B. You will learn how to use MATLAB to find gain margin, phase margin, zero dB frequency, and 180° frequency. This exercise solves Example 10.8 using MATLAB.`

Gui Tool
GUIT

`MATLAB's LTI Viewer, with the Nyquist diagram selected, is another method that may be used to find gain margin, phase margin, zero dB frequency, and 180° frequency. You are encouraged to study Appendix E, at www.wiley.com/college/nise, which contains a tutorial on the LTI Viewer as well as some examples. Example E.2 solves Example 10.8 using the LTI Viewer.`

Skill-Assessment Exercise 10.5

TryIt 10.3

Use MATLAB, the Control System Toolbox, and the following statements to find the gain and phase margins of $G(s)H(s) = 100/[(s+2)(s+4)(s+6)]$ using the Nyquist diagram.

```
G=zpk([],[-2,-4,-6],100)
nyquist(G)
```

After the Nyquist diagram appears:

1. Right-click in the graph area.
2. Select **Characteristics**.
3. Select **All Stability Margins**.
4. Let the mouse rest on the margin points to read the gain and phase margins.

WileyPLUS WPCS Control Solutions

PROBLEM: Find the gain margin and the 180° frequency for the problem in Skill-Assessment Exercise 10.4 if $K = 100$.

ANSWERS: Gain margin = 13.62 dB; 180° frequency = 6.63 rad/s

The complete solution is at www.wiley.com/college/nise.

In this section, we defined gain margin and phase margin and calculated them via the Nyquist diagram. In the next section, we show how to use Bode diagrams to implement the stability calculations performed in Sections 10.5 and 10.6 using the Nyquist diagram. We will see that the Bode plots reduce the time and simplify the calculations required to obtain results.

10.7 Stability, Gain Margin, and Phase Margin via Bode Plots

In this section, we determine stability, gain and phase margins, and the range of gain required for stability. All of these topics were covered previously in this chapter, using Nyquist diagrams as the tool. Now we use Bode plots to determine these characteristics. Bode plots are subsets of the complete Nyquist diagram but in another form. They are a viable alternative to Nyquist plots, since they are easily drawn without the aid of the computational devices or long calculations required for the Nyquist diagram and root locus. You should remember that all calculations applied to stability were derived from and based upon the Nyquist stability criterion. The Bode plots are an alternate way of visualizing and implementing the theoretical concepts.

Determining Stability

Let us look at an example and determine the stability of a system, implementing the Nyquist stability criterion using Bode plots. We will draw a Bode log-magnitude plot and then determine the value of gain that ensures that the magnitude is less than 0 dB (unity gain) at that frequency where the phase is ±180°.

Example 10.9

Range of Gain for Stability via Bode Plots

PROBLEM: Use Bode plots to determine the range of K within which the unity feedback system shown in Figure 10.10 is stable. Let $G(s) = K/[(s+2)(s+4)(s+5)]$.

SOLUTION: Since this system has all of its open-loop poles in the left–half-plane, the open-loop system is stable. Hence, from the discussion of Section 10.5, the closed-loop system will be stable if the frequency response has a gain less than unity when the phase is 180°.

Begin by sketching the Bode magnitude and phase diagrams shown in Figure 10.36. In Section 10.2, we summed normalized plots of each factor of $G(s)$ to create the Bode plot. We saw that at each break frequency, the slope of the resultant Bode plot changed by an amount equal to the new slope that was added. Table 10.6 demonstrates this observation. In this example, we use this fact to draw the Bode plots faster by avoiding the sketching of the response of each term.

The low-frequency gain of $G(s)H(s)$ is found by setting s to zero. Thus, the Bode magnitude plot starts at $K/40$. For convenience, let $K = 40$ so that the log-magnitude plot starts at 0 dB. At each break frequency, 2, 4, and 5, a 20 dB/decade increase in negative slope is drawn, yielding the log-magnitude plot shown in Figure 10.36.

The phase diagram begins at 0° until a decade below the first break frequency of 2 rad/s. At 0.2 rad/s the curve decreases at a rate of $-45°$/decade, decreasing an additional 45°/decade at each subsequent frequency (0.4 and 0.5 rad/s) a decade below each break. At a decade above each break frequency, the slopes are reduced by 45°/decade at each frequency.

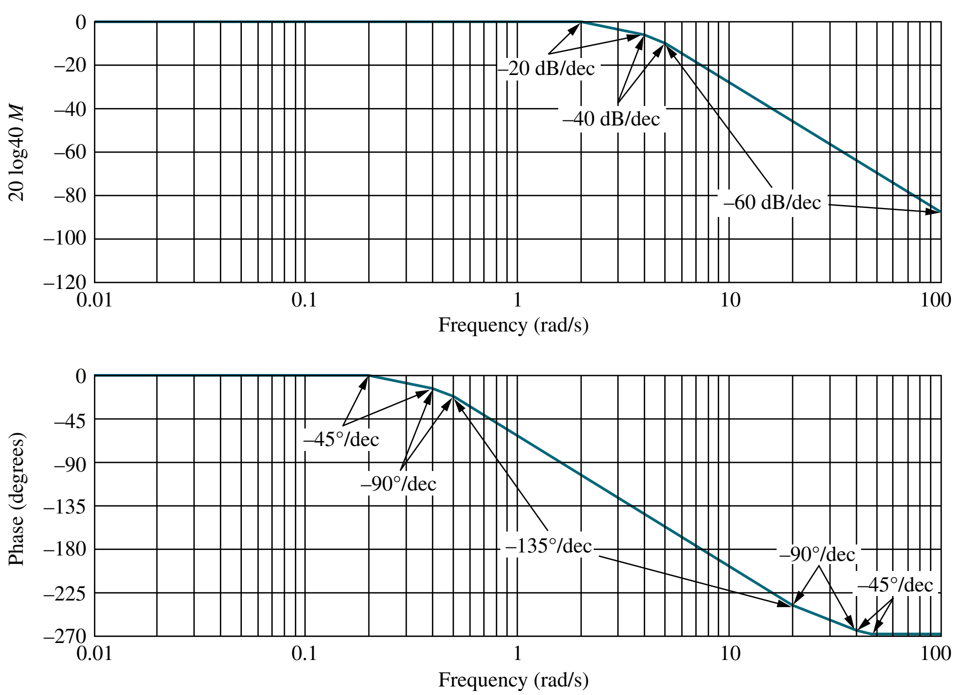


FIGURE 10.36 Bode log-magnitude and phase diagrams for the system of Example 10.9

The Nyquist criterion for this example tells us that we want zero encirclements of -1 for stability. Thus, we recognize that the Bode log-magnitude plot must be less than unity when the Bode phase plot is 180°. Accordingly, we see that at a frequency of 7 rad/s, when the phase plot is $-180°$, the magnitude plot is -20 dB. Therefore, an increase in gain of $+20$ dB is possible before the system becomes unstable. Since the gain plot was scaled for a gain of 40, $+20$ dB (a gain of 10) represents the required increase in gain above 40. Hence, the gain for instability is $40 \times 10 = 400$. The final result is $0 < K < 400$ for stability.

This result, obtained by approximating the frequency response by Bode asymptotes, can be compared to the result obtained from the actual frequency response, which yields a gain of 378 at a frequency of 6.16 rad/s.

MATLAB
ML

`Students who are using MATLAB should now run ch10p4 in Appendix B. You will learn how to use MATLAB to find the range of gain for stability via frequency response methods. This exercise solves Example 10.9 using MATLAB.`

Evaluating Gain and Phase Margins

Next we show how to evaluate the gain and phase margins by using Bode plots (Figure 10.37). The gain margin is found by using the phase plot to find the frequency, ω_{G_M}, where the phase angle is 180°. At this frequency, we look at the magnitude plot to determine the gain margin, G_M, which is the gain required to raise the magnitude curve to 0 dB. To illustrate, in the previous example with $K = 40$, the gain margin was found to be 20 dB.

The phase margin is found by using the magnitude curve to find the frequency, ω_{Φ_M}, where the gain is 0 dB. On the phase curve at that frequency, the phase margin, ϕ_M, is the difference between the phase value and 180°.

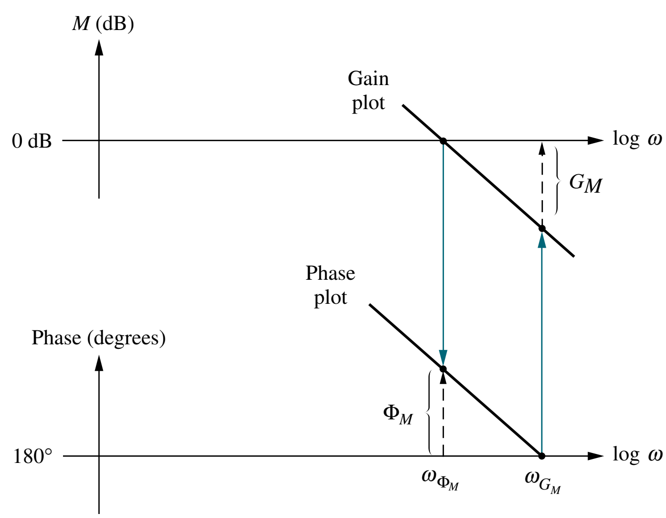


FIGURE 10.37 Gain and phase margins on the Bode diagrams

Example 10.10

Gain and Phase Margins from Bode Plots

PROBLEM: If $K = 200$ in the system of Example 10.9, find the gain margin and the phase margin.

SOLUTION: The Bode plot in Figure 10.36 is scaled to a gain of 40. If $K = 200$ (five times as great), the magnitude plot would be $20 \log 5 = 13.98$ dB higher.

To find the gain margin, look at the phase plot and find the frequency where the phase is 180°. At this frequency, determine from the magnitude plot how much the gain can be increased before reaching 0 dB. In Figure 10.36, the phase angle is 180° at approximately 7 rad/s. On the magnitude plot, the gain is $-20 + 13.98 = -6.02$ dB. Thus, the gain margin is 6.02 dB.

To find the phase margin, we look on the magnitude plot for the frequency where the gain is 0 dB. At this frequency, we look on the phase plot to find the difference between the phase and 180°. This difference is the phase margin. Again, remembering that the magnitude plot of Figure 10.36 is 13.98 dB lower than the actual plot, the 0 dB crossing (−13.98 dB for the normalized plot shown in Figure 10.36) occurs at 5.5 rad/s. At this frequency the phase angle is −165°. Thus, the phase margin is $-165° - (-180°) = 15°$.

Gui Tool
GUIT

MATLAB's LTI Viewer, with Bode plots selected, is another method that may be used to find gain margin, phase margin, zero dB frequency, and 180° frequency. You are encouraged to study Appendix E at www.wiley.com/college/nise, which contains a tutorial on the LTI Viewer as well as some examples. Example E.3 solves Example 10.10 using the LTI Viewer.

Skill-Assessment Exercise 10.6

PROBLEM: For the system shown in Figure 10.10, where

$$G(s) = \frac{K}{(s+5)(s+20)(s+50)}$$

do the following:

a. Draw the Bode log-magnitude and phase plots.

b. Find the range of K for stability from your Bode plots.

c. Evaluate gain margin, phase margin, zero dB frequency, and 180° frequency from your Bode plots for $K = 10,000$.

ANSWERS:

a. See the answer at www.wiley.com/college/nise.

b. $K < 96,270$

c. Gain margin = 19.67 dB, phase margin = 92.9°, zero dB frequency = 7.74 rad/s, and 180° frequency = 36.7 rad/s

The complete solution is at www.wiley.com/college/nise.

TryIt 10.4

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 10.6(c) using Bode plots.

```
G=zpk([],...
 [-5,-20,-50],10000)
bode(G)
grid on
```

After the Bode plot appears:

1. Right-click in the graph area.
2. Select **Characteristics**.
3. Select **All Stability Margins**.
4. Let the mouse rest on the margin points to read the gain and phase margins.

We have seen that the open-loop frequency response curves can be used not only to determine whether a system is stable but to calculate the range of loop gain that will ensure stability. We have also seen how to calculate the gain margin and the phase margin from the Bode diagrams.

Is it then possible to parallel the root locus technique and analyze and design systems for transient response using frequency response methods? We will begin to explore the answer in the next section.

10.8 Relation Between Closed-Loop Transient and Closed-Loop Frequency Responses

Damping Ratio and Closed-Loop Frequency Response

In this section, we will show that a relationship exists between a system's transient response and its closed-loop frequency response. In particular, consider the second-order feedback control system of Figure 10.38, which we have been using since Chapter 4, where we derived relationships between the closed-loop transient response and the poles of the closed-loop transfer function,

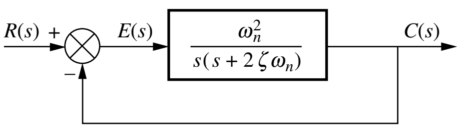


FIGURE 10.38 Second-order closed-loop system

$$\frac{C(s)}{R(s)} = T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{10.49}$$

We now derive relationships between the transient response of Eq. (10.49) and characteristics of its frequency response. We define these characteristics and relate them to damping ratio, natural frequency, settling time, peak time, and rise time. In Section 10.10, we will show how to use the frequency response of the open-loop transfer function

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)} \tag{10.50}$$

shown in Figure 10.38, to obtain the same transient response characteristics.

Let us now find the frequency response of Eq. (10.49), define characteristics of this response, and relate these characteristics to the transient response. Substituting $s = j\omega$ into Eq. (10.49), we evaluate the magnitude of the closed-loop frequency response as

$$M = |T(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2}} \tag{10.51}$$

A representative sketch of the log plot of Eq. (10.51) is shown in Figure 10.39.

We now show that a relationship exists between the peak value of the closed-loop magnitude response and the damping ratio. Squaring Eq. (10.51), differentiating with respect to ω^2, and setting the derivative equal to zero yields the maximum value of M, M_p, where

$$\boxed{M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}}} \tag{10.52}$$

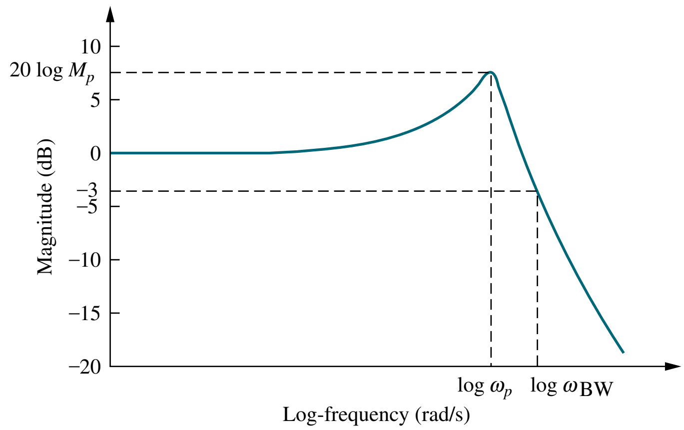


FIGURE 10.39 Representative log-magnitude plot of Eq. (10.51)

at a frequency, ω_p, of

$$\boxed{\omega_p = \omega_n\sqrt{1-2\zeta^2}} \tag{10.53}$$

Since ζ is related to percent overshoot, we can plot M_p vs. percent overshoot. The result is shown in Figure 10.40.

Equation (10.52) shows that the maximum magnitude on the frequency response curve is directly related to the damping ratio and, hence, the percent overshoot. Also notice from Eq. (10.53) that the peak frequency, ω_p, is not the natural frequency. However, for low values of damping ratio, we can assume that the peak occurs at the natural frequency. Finally, notice that there will not be a peak at frequencies above zero if $\zeta > 0.707$. This limiting value of ζ for peaking on the magnitude response curve should not be confused with overshoot on the step response, where there is overshoot for $0 < \zeta < 1$.

Response Speed and Closed-Loop Frequency Response

Another relationship between the frequency response and time response is between the speed of the time response (as measured by settling time, peak time, and rise time) and the *bandwidth* of the closed-loop frequency response, which is defined here as the frequency, ω_{BW}, at which the magnitude response curve is 3 dB down from its value at zero frequency (see Figure 10.39).

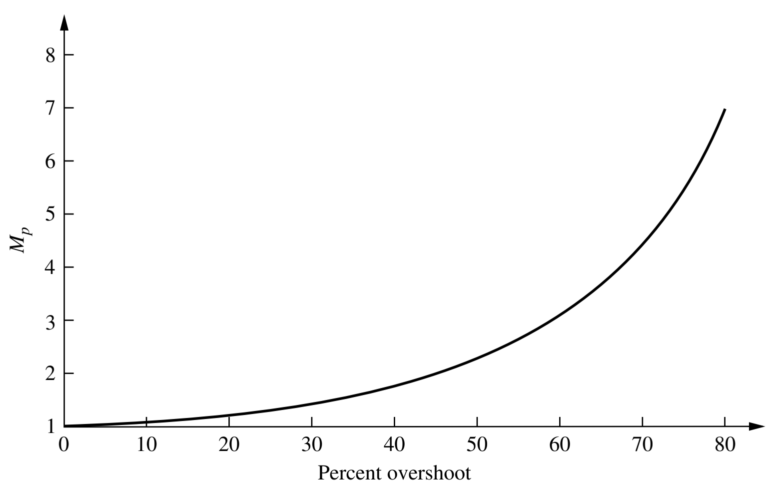


FIGURE 10.40 Closed-loop frequency response peak vs. percent overshoot for a two-pole system

The bandwidth of a two-pole system can be found by finding that frequency for which $M = 1/\sqrt{2}$ (that is, -3 dB) in Eq.(10.51). The derivation is left as an exercise for the student. The result is

$$\boxed{\omega_{BW} = \omega_n \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}} \tag{10.54}$$

To relate ω_{BW} to settling time, we substitute $\omega_n = 4/T_s\zeta$ into Eq. (10.54) and obtain

$$\boxed{\omega_{BW} = \frac{4}{T_s\zeta} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}} \tag{10.55}$$

Similarly, since, $\omega_n = \pi/(T_p\sqrt{1 - \zeta^2})$,

$$\boxed{\omega_{BW} = \frac{\pi}{T_p\sqrt{1 - \zeta^2}} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}} \tag{10.56}$$

To relate the bandwidth to rise time, T_r, we use Figure 4.16, knowing the desired ζ and T_r. For example, assume $\zeta = 0.4$ and $T_r = 0.2$ second. Using Figure 4.16, the ordinate $T_r\omega_n = 1.463$, from which $\omega_n = 1.463/0.2 = 7.315$ rad/s. Using Eq. (10.54), $\omega_{BW} = 10.05$ rad/s. Normalized plots of Eqs. (10.55) and (10.56) and the relationship between bandwidth normalized by rise time and damping ratio are shown in Figure 10.41.

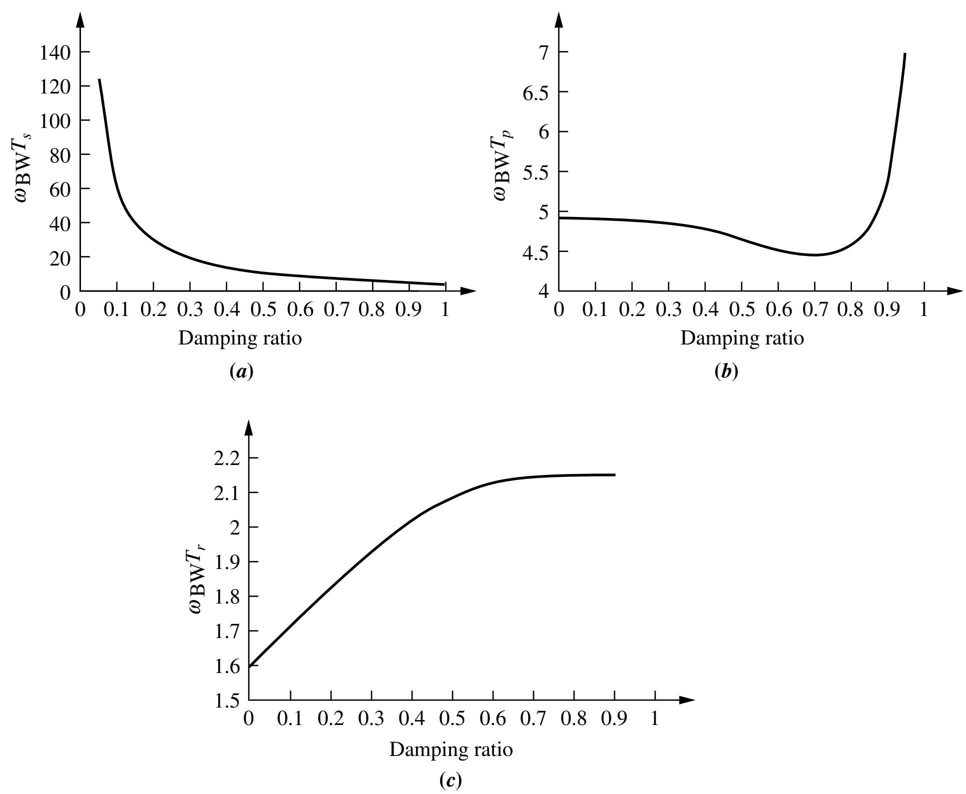


FIGURE 10.41 Normalized bandwidth vs. damping ratio for **a.** settling time; **b.** peak time; **c.** rise time

Skill-Assessment Exercise 10.7

PROBLEM: Find the closed-loop bandwidth required for 20% overshoot and 2-seconds settling time.

ANSWER: $\omega_{BW} = 5.79$ rad/s

The complete solution is at www.wiley.com/college/nise.

In this section, we related the closed-loop transient response to the closed-loop frequency response via bandwidth. We continue by relating the closed-loop frequency response to the open-loop frequency response and explaining the impetus.

10.9 Relation Between Closed- and Open-Loop Frequency Responses

At this point, we do not have an easy way of finding the closed-loop frequency response from which we could determine M_p and thus the transient response.[2] As we have seen, we are equipped to rapidly sketch the open-loop frequency response but not the closed-loop frequency response. However, if the open-loop response is related to the closed-loop response, we can combine the ease of sketching the open-loop response with the transient response information contained in the closed-loop response.

Constant *M* Circles and Constant *N* Circles

Consider a unity feedback system whose closed-loop transfer function is

$$T(s) = \frac{G(s)}{1 + G(s)} \tag{10.57}$$

The frequency response of this closed-loop function is

$$T(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)} \tag{10.58}$$

Since $G(j\omega)$ is a complex number, let $G(j\omega) = P(\omega) + jQ(\omega)$ in Eq. (10.58), which yields

$$T(j\omega) = \frac{P(\omega) + jQ(\omega)}{[(P(\omega) + 1) + jQ(\omega)]} \tag{10.59}$$

Therefore,

$$M^2 = |T^2(j\omega)| = \frac{P^2(\omega) + Q^2(\omega)}{[(P(\omega) + 1)^2 + Q^2(\omega)]} \tag{10.60}$$

Eq. (10.60) can be put into the form

$$\left(P + \frac{M^2}{M^2 - 1}\right)^2 + Q^2 = \frac{M^2}{(M^2 - 1)^2} \tag{10.61}$$

[2] At the end of this subsection, we will see how to use MATLAB to obtain closed-loop frequency responses.

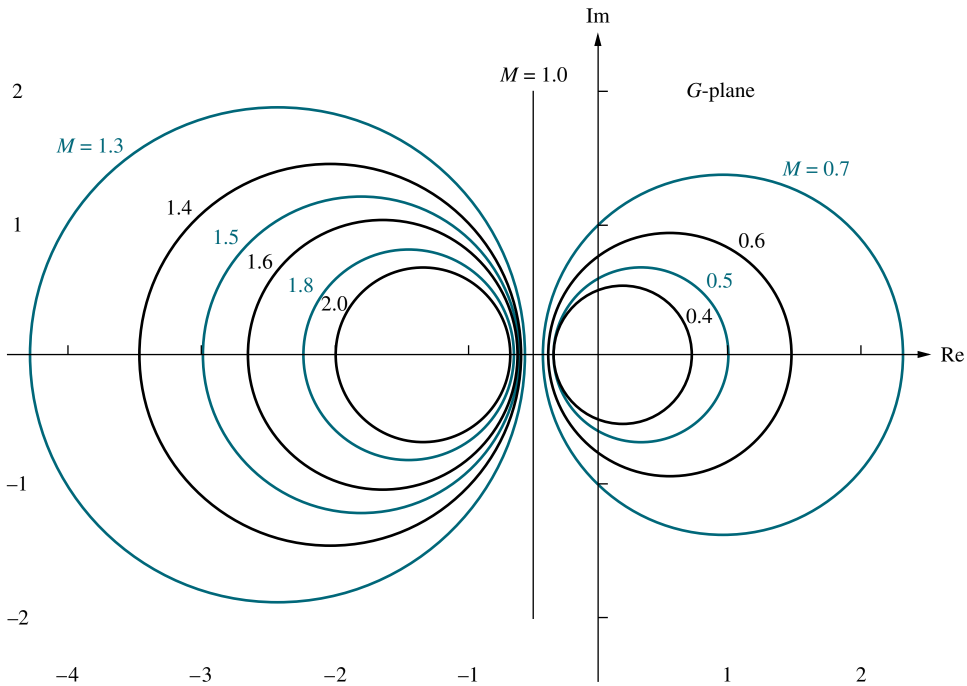


FIGURE 10.42 Constant M circles

which is the equation of a circle of radius $M/(M^2-1)$ centered at $[-M^2/(M^2-1),\ 0]$. These circles, shown plotted in Figure 10.42 for various values of M, are called *constant M circles* and are the locus of the closed-loop magnitude frequency response for unity feedback systems. Thus, if the polar frequency response of an open-loop function, $G(s)$, is plotted and superimposed on top of the constant M circles, the closed-loop magnitude frequency response is determined by each intersection of this polar plot with the constant M circles.

Before demonstrating the use of the constant M circles with an example, let us go through a similar development for the closed-loop phase plot, the constant N circles. From Eq. (10.59), the phase angle, ϕ, of the closed-loop response is

$$\begin{aligned}\phi &= \tan^{-1}\frac{Q(\omega)}{P(\omega)} - \tan^{-1}\frac{Q(\omega)}{P(\omega)+1} \\ &= \tan^{-1}\frac{\dfrac{Q(\omega)}{P(\omega)} - \dfrac{Q(\omega)}{P(\omega)+1}}{1+\dfrac{Q(\omega)}{P(\omega)}\left(\dfrac{Q(\omega)}{P(\omega)+1}\right)}\end{aligned} \tag{10.62}$$

after using $\tan(\alpha-\beta) = (\tan\alpha - \tan\beta)/(1+\tan\alpha\tan\beta)$. Dropping the functional notation,

$$\tan\phi = N = \frac{Q}{P^2+P+Q^2} \tag{10.63}$$

Equation (10.63) can be put into the form of a circle,

$$\left(P+\frac{1}{2}\right)^2 + \left(Q-\frac{1}{2N}\right)^2 = \frac{N^2+1}{4N^2} \tag{10.64}$$

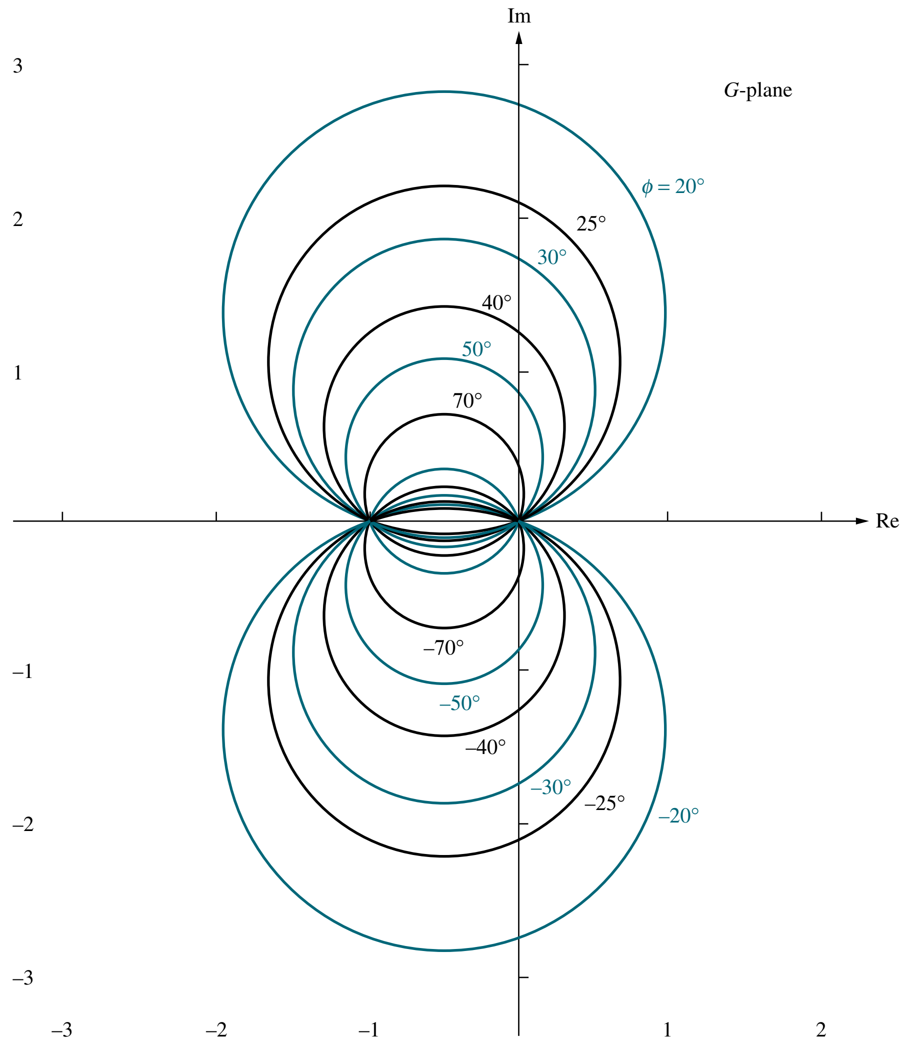


FIGURE 10.43 Constant N circles

which is plotted in Figure 10.43 for various values of N. The circles of this plot are called *constant N circles*. Superimposing a unity feedback, open-loop frequency response over the constant N circles yields the closed-loop phase response of the system. Let us now look at an example of the use of the constant M and N circles.

Example 10.11

Closed-Loop Frequency Response from Open-Loop Frequency Response

PROBLEM: Find the closed-loop frequency response of the unity feedback system shown in Figure 10.10, where $G(s) = 50/[s(s+3)(s+6)]$, using the constant M circles, N circles, and the open-loop polar frequency response curve.

SOLUTION: First evaluate the open-loop frequency function and make a polar frequency response plot superimposed over the constant M and N circles. The

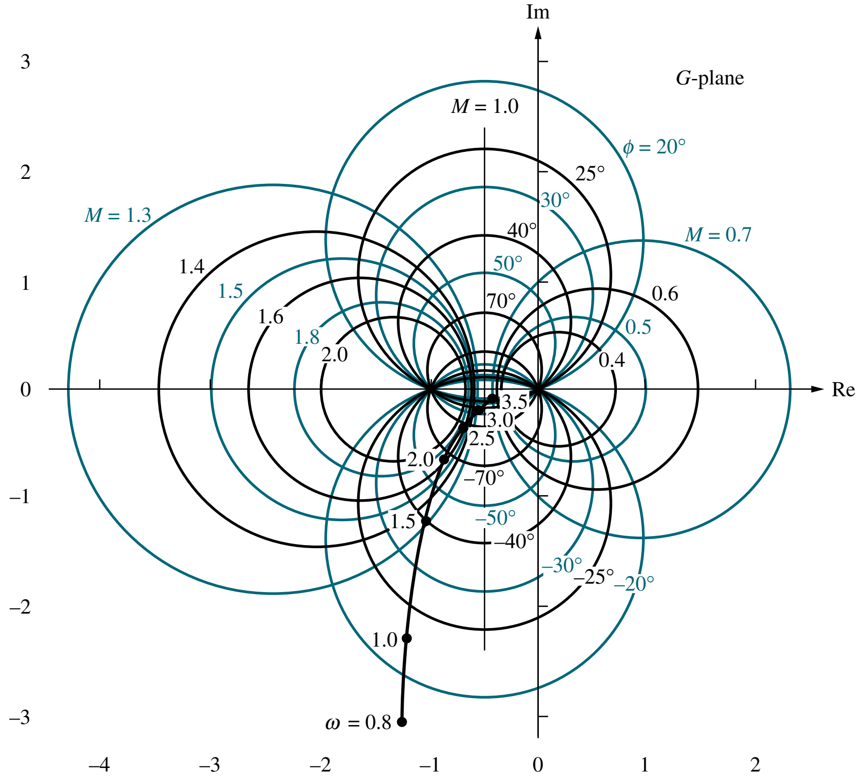


FIGURE 10.44 Nyquist diagram for Example 10.11 and constant M and N circles

open-loop frequency function is

$$G(j\omega) = \frac{50}{-9\omega^2 + j(18\omega - \omega^3)} \tag{10.65}$$

from which the magnitude, $|G(j\omega)|$, and phase, $\angle G(j\omega)$, can be found and plotted. The polar plot of the open-loop frequency response (Nyquist diagram) is shown superimposed over the M and N circles in Figure 10.44.

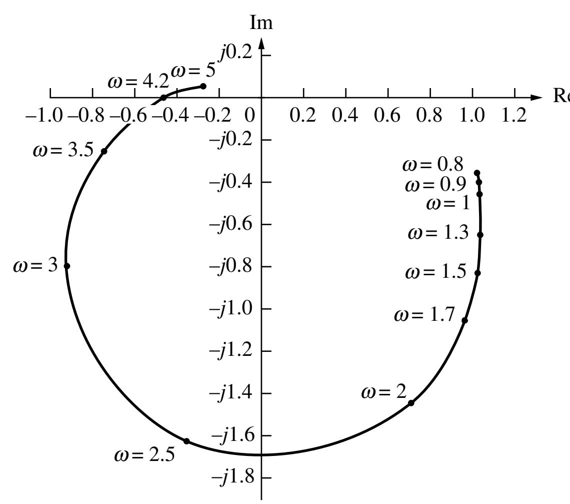


FIGURE 10.45 Closed-loop frequency response for Example 10.11

The closed-loop magnitude frequency response can now be obtained by finding the intersection of each point of the Nyquist plot with the M circles, while the closed-loop phase response can be obtained by finding the intersection of each point of the Nyquist plot with the N circles. The result is shown in Figure 10.45.[3]

```
Students who are using MATLAB should now run ch10p5 in Appendix B.
You will learn how to use MATLAB to find the closed-loop frequency
response. This exercise solves Example 10.11 using MATLAB.
```

MATLAB
ML

Nichols Charts

A disadvantage of using the M and N circles is that changes of gain in the open-loop transfer function, $G(s)$, cannot be handled easily. For example, in the Bode plot, a gain change is handled by moving the Bode magnitude curve up or down an amount equal to the gain change in dB. Since the M and N circles are not dB plots, changes in gain require each point of $G(j\omega)$ to be multiplied in length by the increase or decrease in gain.

Another presentation of the M and N circles, called a *Nichols chart*, displays the constant M circles in dB, so that changes in gain are as simple to handle as in the Bode plot. A Nichols chart is shown in Figure 10.46. The chart is a plot of open-loop magnitude in dB vs. open-loop phase angle in degrees. Every point on the M circles can be transferred to the Nichols chart. Each point on the constant M circles is represented by magnitude and angle (polar coordinates). Converting the magnitude to dB, we can transfer the point to the Nichols chart, using the polar coordinates with magnitude in dB plotted as the ordinate, and the phase angle plotted as the abscissa. Similarly, the N circles also can be transferred to the Nichols chart.

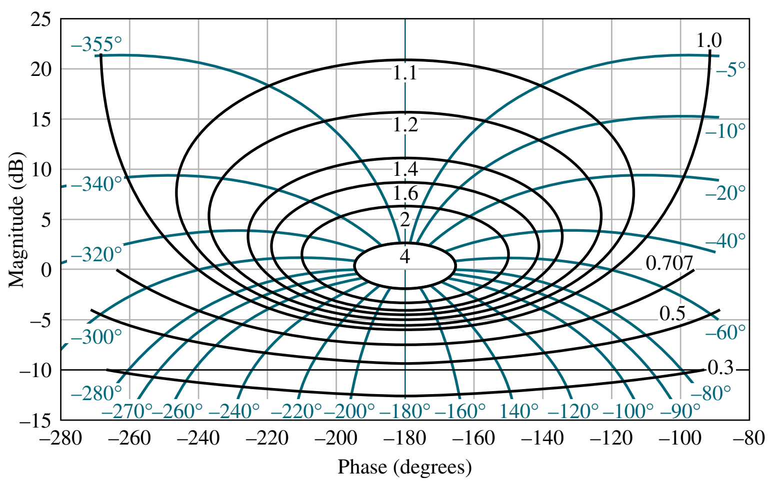


FIGURE 10.46 Nichols chart

[3] You are cautioned not to use the *closed-loop* polar plot for the Nyquist criterion. The closed-loop frequency response, however, can be used to determine the closed-loop transient response, as discussed in Section 10.8.

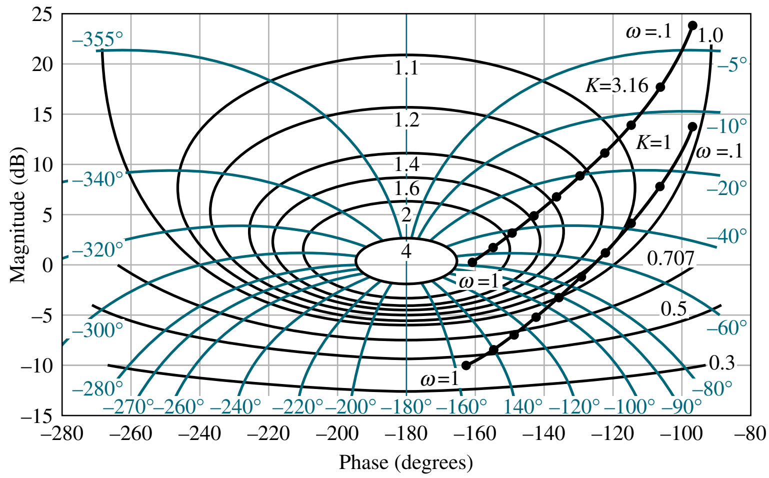


FIGURE 10.47 Nichols chart with frequency response for $G(s) = K/[s(s+1)(s+2)]$ superimposed. Values for $K = 1$ and $K = 3.16$ are shown

For example, assume the function

$$G(s) = \frac{K}{s(s+1)(s+2)} \tag{10.66}$$

Superimposing the frequency response of $G(s)$ on the Nichols chart by plotting magnitude in dB vs. phase angle for a range of frequencies from 0.1 to 1 rad/s, we obtain the plot in Figure 10.47 for $K = 1$. If the gain is increased by 10 dB, simply raise the curve for $K = 1$ by 10 dB and obtain the curve for $K = 3.16(10\,\text{dB})$. The intersection of the plots of $G(j\omega)$ with the Nichols chart yields the frequency response of the closed-loop system.

MATLAB
ML

Students who are using MATLAB should now run ch10p6 in Appendix B. You will learn how to use MATLAB to make a Nichols plot. This exercise makes a Nichols plot of $G(s) = 1/[s(s+1)(s+2)]$ using MATLAB.

Gui Tool
GUIT

MATLAB's LTI Viewer is an alternative method of obtaining the Nichols chart. You are encouraged to study Appendix E at www.wiley.com/college/nise, which contains a tutorial on the LTI Viewer as well as some examples. Example E.4 shows how to obtain Figure 10.47 using the LTI Viewer.

Skill-Assessment Exercise 10.8

TryIt 10.5

Use MATLAB, the Control System Toolbox, and the following statements to make a Nichols chart of the system given in Skill-Assessment Exercise 10.8

```
G=zpk([],...
 [-5,-20,-50],8000)
nichols(G)
grid on
```

PROBLEM: Given the system shown in Figure 10.10, where

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Control Solutions

$$G(s) = \frac{8000}{(s+5)(s+20)(s+50)}$$

plot the closed-loop log-magnitude and phase frequency response plots using the following methods:

a. M and N circles

b. Nichols chart

ANSWER: The complete solution is at www.wiley.com/college/nise.

10.10 Relation Between Closed-Loop Transient and Open-Loop Frequency Responses

Damping Ratio From *M* Circles

We can use the results of Example 10.11 to estimate the transient response characteristics of the system. We can find the peak of the closed-loop frequency response by finding the maximum M curve tangent to the open-loop frequency response. Then we can find the damping ratio, ζ, and subsequently the percent overshoot, via Eq. (10.52). The following example demonstrates the use of the open-loop frequency response and the M circles to find the damping ratio or, equivalently, the percent overshoot.

Example 10.12

Percent Overshoot from Open-Loop Frequency Response

PROBLEM: Find the damping ratio and the percent overshoot expected from the system of Example 10.11, using the open-loop frequency response and the M circles.

SOLUTION: Equation (10.52) shows that there is a unique relationship between the closed-loop system's damping ratio and the peak value, M_P, of the closed-loop system's magnitude frequency plot. From Figure 10.44, we see that the Nyquist diagram is tangent to the 1.8 M circle. We see that this is the maximum value for the closed-loop frequency response. Thus, $M_p = 1.8$.

We can solve for ζ by rearranging Eq. (10.52) into the following form:

$$\zeta^4 - \zeta^2 + (1/4M_p^2) = 0 \tag{10.67}$$

Since $M_p = 1.8$, then $\zeta = 0.29$ and 0.96. From Eq. (10.53), a damping ratio larger than 0.707 yields no peak above zero frequency. Thus, we select $\zeta = 0.29$, which is equivalent to 38.6% overshoot. Care must be taken, however, to be sure we can make a second-order approximation when associating the value of percent overshoot to the value of ζ. A computer simulation of the step response shows 36% overshoot.

So far in this section, we have tied together the system's transient response and the peak value of the closed-loop frequency response as obtained from the open-loop frequency response. We used the Nyquist plots and the M and N circles to obtain the closed-loop transient response. Another association exists between the open-loop frequency response and the closed-loop transient response that is easily implemented with the Bode plots, which are easier to draw than the Nyquist plots.

Damping Ratio from Phase Margin

Let us now derive the relationship between the phase margin and the damping ratio. This relationship will enable us to evaluate the percent overshoot from the phase margin found from the open-loop frequency response.

Consider a unity feedback system whose open-loop function

$$G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)} \tag{10.68}$$

yields the typical second-order, closed-loop transfer function

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{10.69}$$

In order to evaluate the phase margin, we first find the frequency for which $|G(j\omega)| = 1$. Hence,

$$|G(j\omega)| = \frac{\omega_n^2}{|-\omega^2 + j2\zeta\omega_n\omega|} = 1 \tag{10.70}$$

The frequency, ω_1, that satisfies Eq. (10.70) is

$$\omega_1 = \omega_n\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}} \tag{10.71}$$

The phase angle of $G(j\omega)$ at this frequency is

$$\begin{aligned} \angle G(j\omega) &= -90 - \tan^{-1}\frac{\omega_1}{2\zeta\omega_n} \\ &= -90 - \tan^{-1}\frac{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4 + 1}}}{2\zeta} \end{aligned} \tag{10.72}$$

The difference between the angle of Eq. (10.72) and $-180°$ is the phase margin, ϕ_M. Thus,

$$\boxed{\begin{aligned} \Phi_M &= 90 - \tan^{-1}\frac{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}{2\zeta} \\ &= \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}} \end{aligned}} \tag{10.73}$$

Equation (10.73), plotted in Figure 10.48, shows the relationship between phase margin and damping ratio.

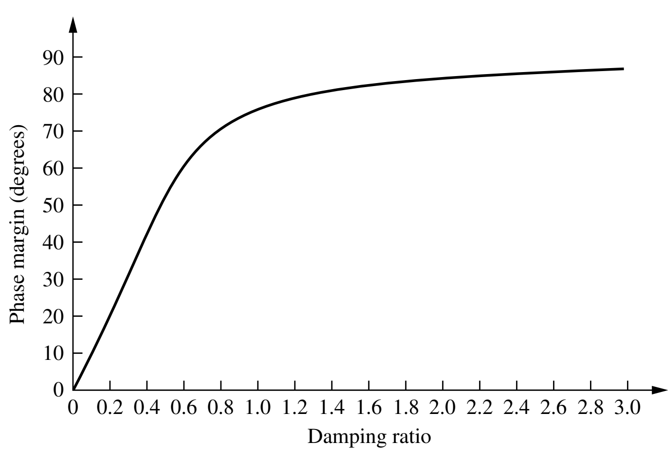

FIGURE 10.48 Phase margin vs. damping ratio

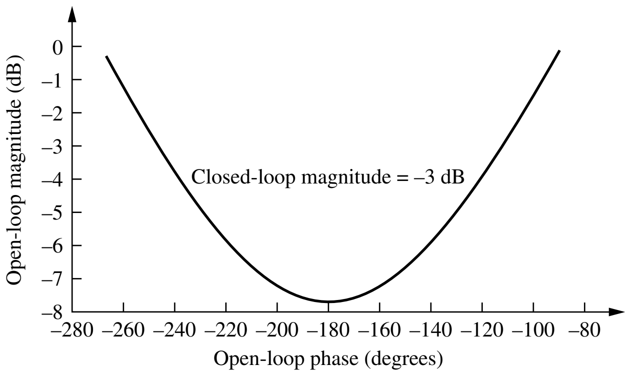

FIGURE 10.49 Open-loop gain vs. open-loop phase angle for −3 dB closed-loop gain

As an example, Eq. (10.53) tells us that there is no peak frequency if $\zeta = 0.707$. Hence, there is no peak to the closed-loop magnitude frequency response curve for this value of damping ratio and larger. Thus, from Figure 10.48, a phase margin of $65.52^\circ(\zeta = 0.707)$ or larger is required from the *open-loop* frequency response to ensure there is no peaking in the *closed-loop* frequency response.

Response Speed from Open-Loop Frequency Response

Equations (10.55) and (10.56) relate the closed-loop bandwidth to the desired settling or peak time and the damping ratio. We now show that the closed-loop bandwidth can be estimated from the open-loop frequency response. From the Nichols chart in Figure 10.46, we see the relationship between the open-loop gain and the closed-loop gain. The $M = 0.707(-3\text{ dB})$ curve, replotted in Figure 10.49 for clarity, shows the open-loop gain when the closed-loop gain is −3 dB, which typically occurs at ω_{BW} if the low-frequency closed-loop gain is 0 dB. We can approximate Figure 10.49 by saying that the closed-loop bandwidth, ω_{BW} (the frequency at which the closed-loop magnitude response is −3dB), equals the frequency at which the open-loop magnitude response is between −6 and −7.5 dB if the open-loop phase response is between -135° and -225°. Then, using a second-order system approximation, Eqs. (10.55) and (10.56) can be used, along with the desired damping ratio, ζ, to find settling time and peak time, respectively. Let us look at an example.

Example 10.13

Settling and Peak Times from Open-Loop Frequency Response

PROBLEM: Given the system of Figure 10.50(a) and the Bode diagrams of Figure 10.50(b), estimate the settling time and peak time.

SOLUTION: Using Figure 10.50(b), we estimate the closed-loop bandwidth by finding the frequency where the open-loop magnitude response is in the range of −6 to −7.5 dB if the phase response is in the range of -135° to -225°. Since Figure 10.50(b) shows −6 to −7.5 dB at approximately 3.7 rad/s with a phase response in the stated region, $\omega_{\text{BW}} \cong 3.7$ rad/s.

Next find ζ via the phase margin. From Figure 10.50(b), the phase margin is found by first finding the frequency at which the magnitude plot is 0 dB. At this frequency, 2.2 rad/s, the phase is about -145°. Hence, the phase margin is approximately $(-145^\circ - (-180^\circ)) = 35^\circ$. Using Figure 10.48, $\zeta = 0.32$. Finally, using Eqs. (10.55) and (10.56), with the values of ω_{BW} and ζ just found, $T_s = 4.86$

seconds and $T_p = 129$ seconds. Checking the analysis with a computer simulation shows $T_s = 5.5$ seconds, and $T_p = 1.43$ seconds.

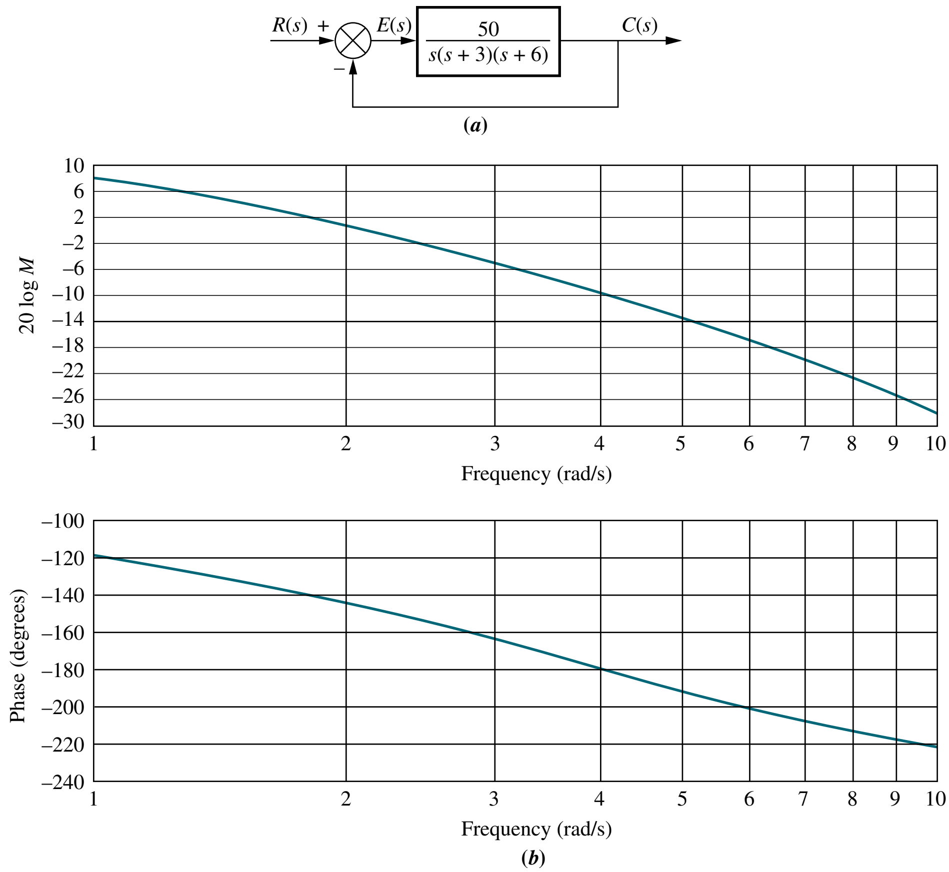

FIGURE 10.50 **a.** Block diagram; **b.** Bode diagrams for system of Example 10.13

Skill-Assessment Exercise 10.9

PROBLEM: Using the open-loop frequency response for the system in Figure 10.10, where

$$G(s) = \frac{100}{s(s+5)}$$

estimate the percent overshoot, settling time, and peak time for the closed-loop step response.

ANSWER: $\%OS = 44\%$, $T_s = 1.64$ s, and $T_P = 0.33$ s

The complete solution is at www.wiley.com/college/nise.

10.11 Steady-State Error Characteristics from Frequency Response

In this section, we show how to use Bode diagrams to find the values of the static error constants for equivalent unity feedback systems: K_p for a Type 0 system, K_v for a Type 1 system, and K_a for a Type 2 system. The results will be obtained from unnormalized and unscaled Bode log-magnitude plots.

Position Constant

To find K_p, consider the following Type 0 system:

$$G(s) = K\frac{\prod_{i=1}^{n}(s+z_i)}{\prod_{i=1}^{m}(s+p_i)} \tag{10.74}$$

A typical unnormalized and unscaled Bode log-magnitude plot is shown in Figure 10.51(*a*). The initial value is

$$20\log M = 20\log K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} \tag{10.75}$$

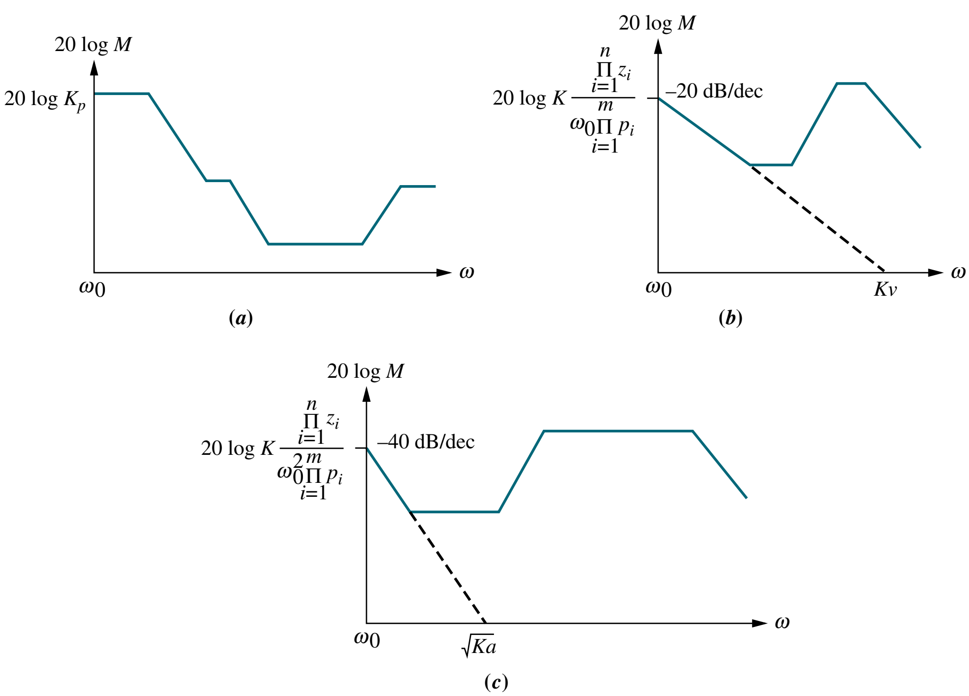

FIGURE 10.51 Typical unnormalized and unscaled Bode log-magnitude plots showing the value of static error constants: **a.** Type 0; **b.** Type 1; **c.** Type 2

But for this system

$$K_p = K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} \tag{10.76}$$

which is the same as the value of the low-frequency axis. Thus, for an unnormalized and unscaled Bode log-magnitude plot, the low-frequency magnitude is 20 log K_p for a Type 0 system.

Velocity Constant

To find K_v for a Type 1 system, consider the following open-loop transfer function of a Type 1 system:

$$G(s) = K\frac{\prod_{i=1}^{n}(s+z_i)}{s\prod_{i=1}^{m}(s+p_i)} \tag{10.77}$$

A typical unnormalized and unscaled Bode log-magnitude diagram is shown in Figure 10.51(b) for this Type 1 system. The Bode plot starts at

$$20\log M = 20\log K\frac{\prod_{i=1}^{n} z_i}{\omega_0\prod_{i=1}^{m} p_i} \tag{10.78}$$

The initial −20 dB/decade slope can be thought of as originating from a function,

$$G'(s) = K\frac{\prod_{i=1}^{n} z_i}{s\prod_{i=1}^{m} p_i} \tag{10.79}$$

$G'(s)$ intersects the frequency axis when

$$\omega = K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} \tag{10.80}$$

But for the original system (Eq. (10.77)),

$$K_v = K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} \tag{10.81}$$

which is the same as the frequency-axis intercept, Eq. (10.80). Thus, we can find K_v by extending the initial −20 dB/decade slope to the frequency axis on an unnormalized and unscaled Bode diagram. The intersection with the frequency axis is K_v.

Acceleration Constant

To find K_a for a Type 2 system, consider the following:

$$G(s) = K\frac{\prod_{i=1}^{n}(s+z_i)}{s^2\prod_{i=1}^{m}(s+p_i)} \tag{10.82}$$

A typical unnormalized and unscaled Bode plot for a Type 2 system is shown in Figure 10.51(c). The Bode plot starts at

$$20\log M = 20\log K\frac{\prod_{i=1}^{n} z_i}{\omega_0^2\prod_{i=1}^{m} p_i} \tag{10.83}$$

The initial -40 dB/decade slope can be thought of as coming from a function,

$$G'(s) = K\frac{\prod_{i=1}^{n} z_i}{s^2\prod_{i=1}^{m} p_i} \tag{10.84}$$

$G'(s)$ intersects the frequency axis when

$$\omega = \sqrt{K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i}} \tag{10.85}$$

But for the original system (Eq. (10.82)),

$$K_a = K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} \tag{10.86}$$

Thus, the initial -40 dB/decade slope intersects the frequency axis at $\sqrt{K_a}$.

Example 10.14

Static Error Constants from Bode Plots

PROBLEM: For each unnormalized and unscaled Bode log-magnitude plot shown in Figure 10.52,

a. Find the system type.

b. Find the value of the appropriate static error constant.

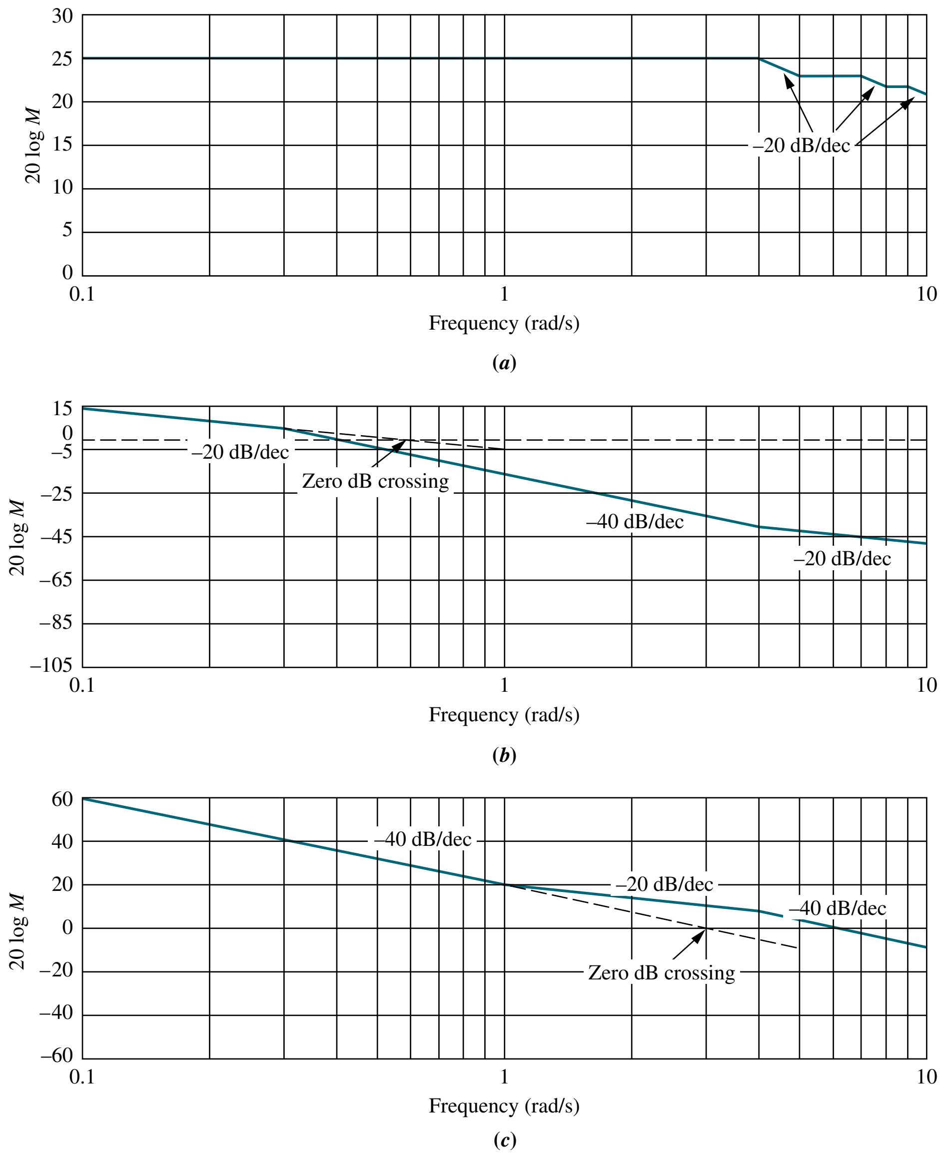

FIGURE 10.52 Bode log-magnitude plots for Example 10.14

SOLUTION: Figure 10.52(a) is a Type 0 system since the initial slope is zero. The value of K_p is given by the low-frequency asymptote value. Thus, $20 \log K_p = 25$, or $K_p = 17.78$.

Figure 10.52(b) is a Type 1 system since the initial slope is -20 dB/decade. The value of K_v is the value of the frequency that the initial slope intersects at the zero dB crossing of the frequency axis. Hence, $K_v = 0.55$.

Figure 10.52(c) is a Type 2 system since the initial slope is -40 dB/decade. The value of $\sqrt{K_a}$ is the value of the frequency that the initial slope intersects at the zero dB crossing of the frequency axis. Hence, $K_a = 3^2 = 9$.

Skill-Assessment Exercise 10.10

PROBLEM: Find the static error constants for a stable unity feedback system whose open-loop transfer function has the Bode magnitude plot shown in Figure 10.53.

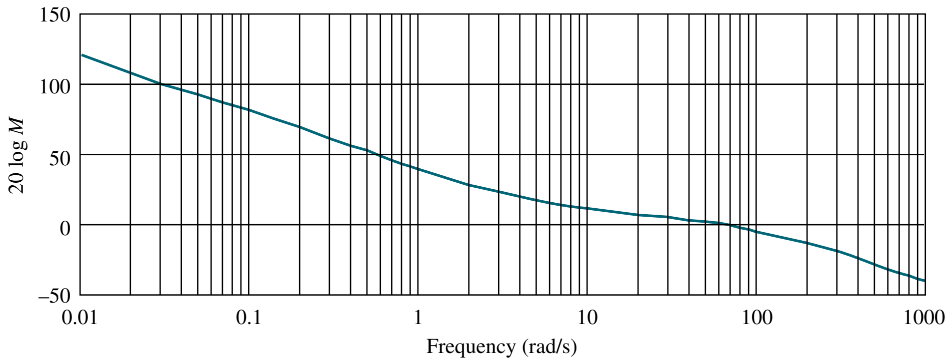

FIGURE 10.53 Bode log-magnitude plot for Skill-Assessment Exercise 10.10

ANSWERS: $K_p = \infty,\ K_v = \infty,\ K_a = 90.25$

The complete solution is www.wiley.com/college/nise.

10.12 Systems with Time Delay

Time delay occurs in control systems when there is a delay between the commanded response and the start of the output response. For example, consider a heating system that operates by heating water for pipeline distribution to radiators at distant locations. Since the hot water must flow through the line, the radiators will not begin to get hot until after a specified time delay. In other words, the time between the command for more heat and the commencement of the rise in temperature at a distant location along the pipeline is the time delay. Notice that this is not the same as the transient response or the time it takes the temperature to rise to the desired level. During the time delay, nothing is occurring at the output.

Modeling Time Delay

Assume that an input, $R(s)$, to a system, $G(s)$, yields an output, $C(s)$. If another system, $G'(s)$, delays the output by T seconds, the output response is $c(t-T)$. From Table 2.2, Item 5, the Laplace transform of $c(t-T)$ is $e^{-sT}C(s)$. Thus, for the system without delay, $C(s) = R(s)G(s)$, and for the system with delay, $e^{-sT}C(s) = R(s)G'(s)$. Dividing these two equations, $G'(s)/G(s) = e^{-sT}$. Thus, a system with time delay T can be represented in terms of an equivalent system without time delay as follows:

$$\boxed{G'(s) = e^{-sT}G(s)} \tag{10.87}$$

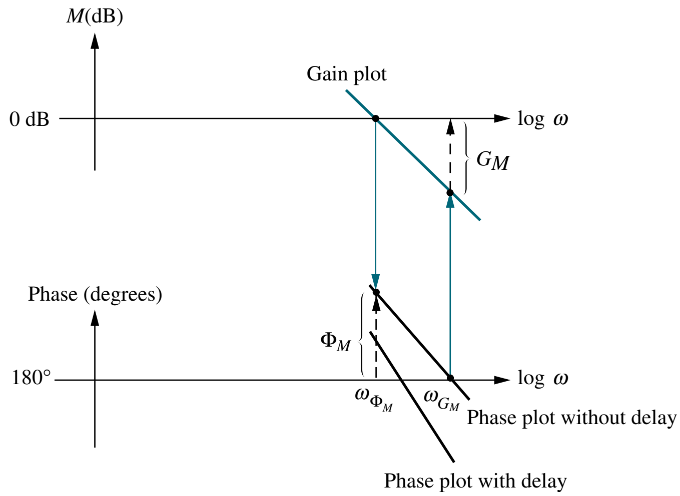

FIGURE 10.54 Effect of delay upon frequency response

The effect of introducing time delay into a system can also be seen from the perspective of the frequency response by substituting $s = j\omega$ in Eq. (10.87). Hence,

$$\boxed{G'(j\omega) = e^{-j\omega T}G(j\omega) = |G(j\omega)|\angle\{-\omega T + \angle G(j\omega)\}} \qquad (10.88)$$

In other words, the time delay does not affect the magnitude frequency response curve of $G(j\omega)$, but it does subtract a linearly increasing phase shift, ωT, from the phase frequency response plot of $G(j\omega)$.

The typical effect of adding time delay can be seen in Figure 10.54. Assume that the gain and phase margins as well as the gain- and phase-margin frequencies shown in the figure apply to the system without delay. From the figure, we see that the reduction in phase shift caused by the delay reduces the phase margin. Using a second-order approximation, this reduction in phase margin yields a reduced damping ratio for the closed-loop system and a more oscillatory response. The reduction of phase also leads to a reduced gain-margin frequency. From the magnitude curve, we can see that a reduced gain-margin frequency leads to reduced gain margin, thus moving the system closer to instability.

An example of plotting frequency response curves for systems with delay follows.

Example 10.15

Frequency Response Plots of a System with Time Delay

PROBLEM: Plot the frequency response for the system $G(s) = K/[s(s+1)(s+10)]$ if there is a time delay of 1 second through the system. Use the Bode plots.

SOLUTION: Since the magnitude curve is not affected by the delay, it can be plotted by the methods previously covered in the chapter and is shown in Figure 10.55(a) for $K = 1$.

The phase plot, however, is affected by the delay. Figure 10.55(b) shows the result. First draw the phase plot for the delay, $e^{-j\omega T} = 1\angle -\omega T = 1\angle -\omega$, since $T = 1$ from the problem statement. Next draw the phase plot of the system, $G(j\omega)$,

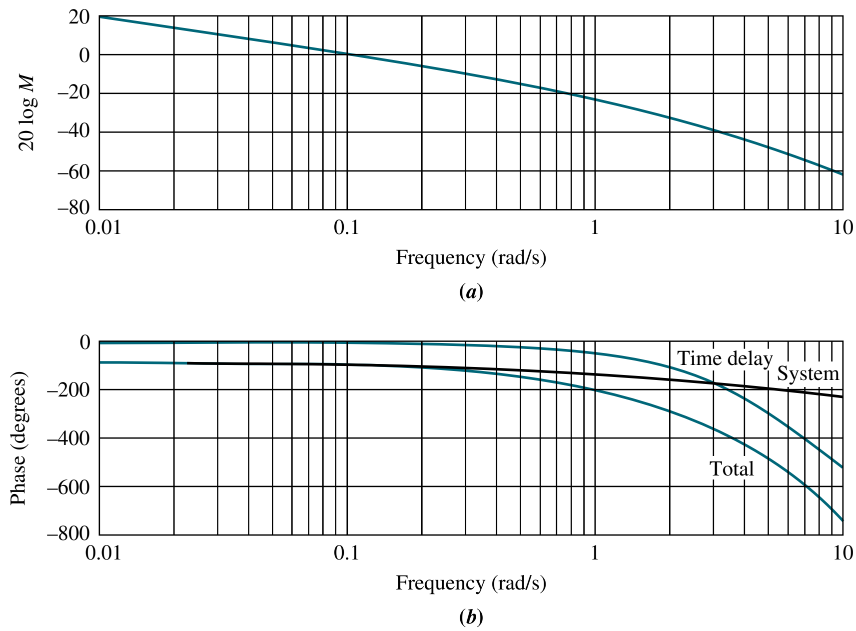

FIGURE 10.55 Frequency response plots for $G(s) = K/[s(s+1)(s+10)]$ with a delay of 1 second and $K = 1$: **a.** magnitude plot; **b.** phase plot

using the methods previously covered. Finally, add the two phase curves together to obtain the total phase response for $e^{-j\omega T}G(j\omega)$. Be sure to use consistent units for the phase angles of $G(j\omega)$ and the delay; either degrees or radians.

Notice that the delay yields a decreased phase margin, since at any frequency the phase angle is more negative. Using a second-order approximation, this decrease in phase margin implies a lower damping ratio and a more oscillatory response for the closed-loop system.

Further, there is a decrease in the gain-margin frequency. On the magnitude curve, note that a reduction in the gain-margin frequency shows up as reduced gain margin, thus moving the system closer to instability.

MATLAB
ML

`Students who are using MATLAB should now run ch10p7 in Appendix B. You will learn how to use MATLAB to include time delay on Bode plots. You will also use MATLAB to make multiple plots on one graph and label the plots. This exercise solves Example 10.15 using MATLAB.`

Let us now use the results of Example 10.15 to design stability and analyze transient response and compare the results to the system without time delay.

Example 10.16

Range of Gain for Stability for System with Time Delay

PROBLEM: The open-loop system with time delay in Example 10.15 is used in a unity feedback configuration. Do the following:

a. Find the range of gain, K, to yield stability. Use Bode plots and frequency response techniques.

b. Repeat Part **a** for the system without time delay.

SOLUTION:

a. From Figure 10.55, the phase angle is -180° at a frequency of 0.81 rad/s for the system with time delay, marked "Total" on the phase plot. At this frequency, the magnitude curve is at -20.39 dB. Thus, K can be raised from its current value of unity to $10^{20.39/20} = 10.46$. Hence, the system is stable for $0 < K \leq 10.46$.

b. If we use the phase curve without delay, marked "System," -180° occurs at a frequency of 3.16 rad/s, and K can be raised 40.84 dB or 110.2. Thus, without delay the system is stable for $0 < K \leq 110.2$, an order of magnitude larger.

Example 10.17

Percent Overshoot for System with Time Delay

PROBLEM: The open-loop system with time delay in Example 10.15 is used in a unity feedback configuration. Do the following:

a. Estimate the percent overshoot if $K = 5$. Use Bode plots and frequency response techniques.

b. Repeat Part **a** for the system without time delay.

SOLUTION:

a. Since $K = 5$, the magnitude curve of Figure 10.55 is raised by 13.98 dB. The zero dB crossing then occurs at a frequency of 0.47 rad/s with a phase angle of -145°, as seen from the phase plot marked "Total." Therefore, the phase margin is $(-145^\circ - (-180^\circ)) = 35^\circ$. Assuming a second-order approximation and using Eq. (10.73) or Figure 10.48, we find $\zeta = 0.33$. From Eq. (4.38), $\%OS = 33\%$. The time response, Figure 10.56(a), shows a 38% overshoot instead of the predicted 33%. Notice the time delay at the start of the curve.

b. The zero dB crossing occurs at a frequency of 0.47 rad/s with a phase angle of -118°, as seen from the phase plot marked "System." Therefore, the phase

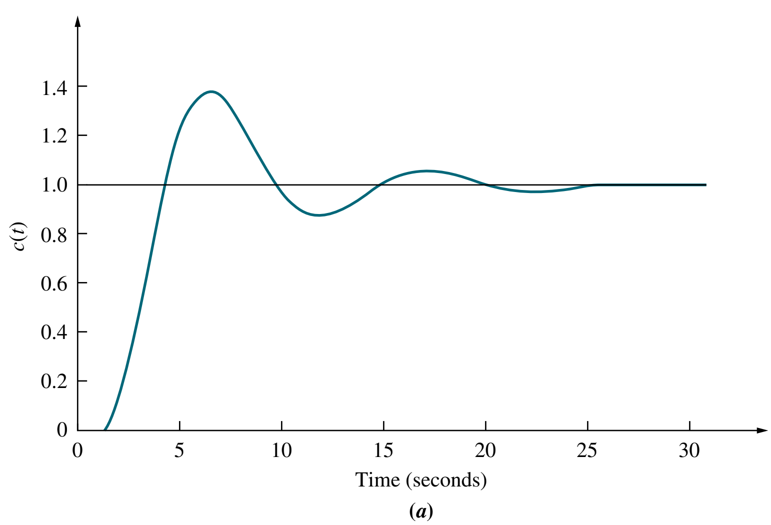

FIGURE 10.56 Step response for closed-loop system with $G(s) = 5/[s(s+1)(s+10)]$: **a.** with a 1-second delay; (*figure continues*)

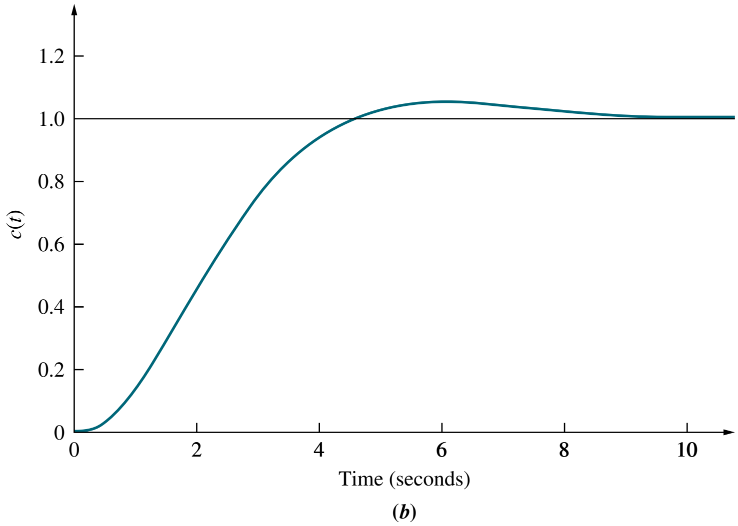

FIGURE 10.56 (*Continued*)
b. without delay

margin is $(-118° - (-180°)) = 62°$. Assuming a second-order approximation and using Eq. (10.73) or Figure 10.48, we find $\zeta = 0.64$. From Eq. (4.38), $\%OS = 7.3\%$. The time response is shown in Figure 10.56(b). Notice that the system without delay has less overshoot and a smaller settling time.

Skill-Assessment Exercise 10.11

PROBLEM: For the system shown in Figure 10.10, where

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$$G(s) = \frac{10}{s(s+1)}$$

find the phase margin if there is a delay in the forward path of

a. 0 s
b. 0.1 s
c. 3 s

ANSWERS:

a. 18.0°
b. 0.35°
c. −151.41°

The complete solution is at www.wiley.com/college/nise.

TryIt 10.6

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 10.11. For each part of the problem let $d =$ the specified delay.

```
G=zpk([ ],[0,-1],10)
d=0
[numGd,denGd]=pade...
 (d,12)
Gd=tf(numGd,denGd)
Ge=G*Gd
bode(Ge)
grid on
```

After the Bode diagrams appear:

1. Right-click in the graph area.
2. Select **Characteristics.**
3. Select **All Stability Margins.**
4. Let the mouse rest on the margin point on the phase plot to read the phase margin.

In summary, then, systems with time delay can be handled using previously described frequency response techniques if the phase response is adjusted to reflect the time delay. Typically, time delay reduces gain and phase margins, resulting in increased percent overshoot or instability in the closed-loop response.

10.13 Obtaining Transfer Functions Experimentally

In Chapter 4, we discussed how to obtain the transfer function of a system through step-response testing. In this section, we show how to obtain the transfer function using sinusoidal frequency response data.

The analytical determination of a system's transfer function can be difficult. Individual component values may not be known, or the internal configuration of the system may not be accessible. In such cases, the frequency response of the system, from input to output, can be obtained experimentally and used to determine the transfer function. To obtain a frequency response plot experimentally, we use a sinusoidal force or signal generator at the input to the system and measure the output steady-state sinusoid amplitude and phase angle (see Figure 10.2). Repeating this process at a number of frequencies yields data for a frequency response plot. Referring to Figure 10.2(b), the amplitude response is $M(\omega) = M_o(\omega)/M_i(\omega)$, and the phase response is $\phi(\omega) = \phi_o(\omega) - \phi_i(\omega)$. Once the frequency response is obtained, the transfer function of the system can be estimated from the break frequencies and slopes. Frequency response methods can yield a more refined estimate of the transfer function than the transient response techniques covered in Chapter 4.

Bode plots are a convenient presentation of the frequency response data for the purpose of estimating the transfer function. These plots allow parts of the transfer function to be determined and extracted, leading the way to further refinements to find the remaining parts of the transfer function.

Although experience and intuition are invaluable in the process, the following steps are still offered as a guideline:

1. Look at the Bode magnitude and phase plots and estimate the pole-zero configuration of the system. Look at the initial slope on the magnitude plot to determine system type. Look at phase excursions to get an idea of the difference between the number of poles and the number of zeros.
2. See if portions of the magnitude and phase curves represent obvious first- or second-order pole or zero frequency response plots.
3. See if there is any telltale peaking or depressions in the magnitude response plot that indicate an underdamped second-order pole or zero, respectively.
4. If any pole or zero responses can be identified, overlay appropriate ± 20 or ± 40 dB/decade lines on the magnitude curve or $\pm 45^\circ$/decade lines on the phase curve and estimate the break frequencies. For second-order poles or zeros, estimate the damping ratio and natural frequency from the standard curves given in Section 10.2.
5. Form a transfer function of unity gain using the poles and zeros found. Obtain the frequency response of this transfer function and subtract this response from the previous frequency response (*Franklin*, *1991*). You now have a frequency response of reduced complexity from which to begin the process again to extract more of the system's poles and zeros. A computer program such as MATLAB is of invaluable help for this step.

Let us demonstrate.

Example 10.18

Transfer Function from Bode Plots

PROBLEM: Find the transfer function of the subsystem whose Bode plots are shown in Figure 10.57.

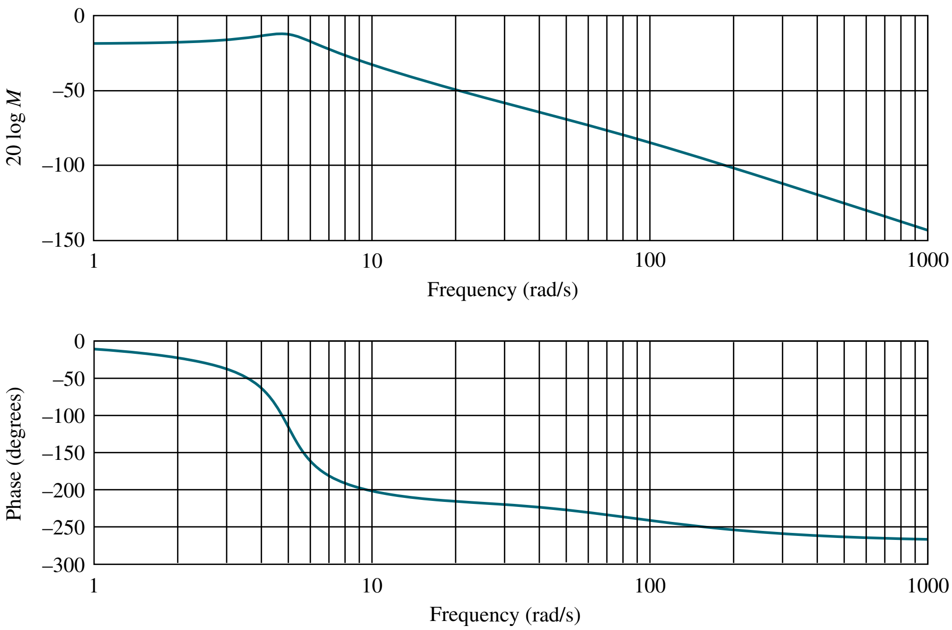

FIGURE 10.57 Bode plots for subsystem with undetermined transfer function

SOLUTION: Let us first extract the underdamped poles that we suspect, based on the peaking in the magnitude curve. We estimate the natural frequency to be near the peak frequency, or approximately 5 rad/s. From Figure 10.57, we see a peak of about 6.5 dB, which translates into a damping ratio of about $\zeta = 0.24$ using Eq. (10.52). The unity gain second-order function is thus $G_1(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2) = 25/(s^2 + 2.4s + 25)$. The frequency response plot of this function is made and subtracted from the previous Bode plots to yield the response in Figure 10.58.

Overlaying a -20 dB/decade line on the magnitude response and a $-45°$/decade line on the phase response, we detect a final pole. From the phase response, we estimate the break frequency at 90 rad/s. Subtracting the response of $G_2(s) = 90/(s + 90)$ from the previous response yields the response in Figure 10.59.

Figure 10.59 has a magnitude and phase curve similar to that generated by a lag function. We draw a -20 dB/decade line and fit it to the curves. The break frequencies are read from the figure as 9 and 30 rad/s. A unity gain transfer function containing a pole at -9 and a zero at -30 is $G_3(s) = 0.3(s + 30)/(s + 9)$. Upon subtraction of $G_1(s)G_2(s)G_3(s)$, we find the magnitude frequency response flat ± 1 dB and the phase response flat at $-3° \pm 5°$. We thus conclude that we are finished extracting dynamic transfer functions. The low-frequency, or dc, value of the original curve is -19 dB, or 0.11. Our estimate of the subsystem's transfer function

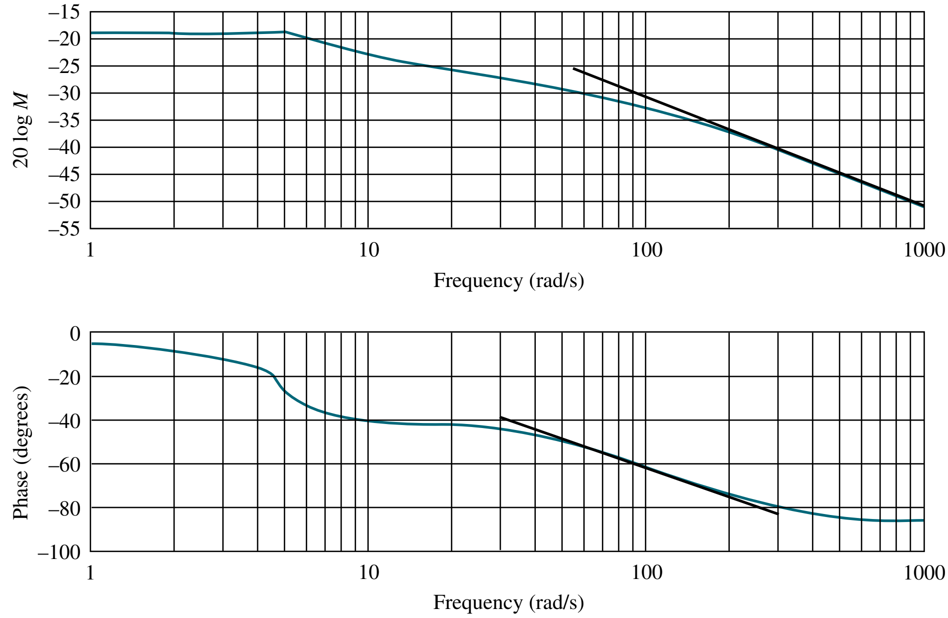

FIGURE 10.58 Original Bode plots minus response of $G_1(s) = 25/(s^2 + 2.4s + 25)$

is $G(s) = 0.11G_1(s)G_2(s)G_3(s)$, or

$$
\begin{aligned}
G(s) &= 0.11\left(\frac{25}{s^2 + 2.4s + 25}\right)\left(90\frac{1}{s + 90}\right)\left(0.3\frac{s + 30}{s + 9}\right) \\
&= 74.25\frac{s + 30}{(s + 9)(s + 90)(s^2 + 2.4s + 25)}
\end{aligned}
\tag{10.89}
$$

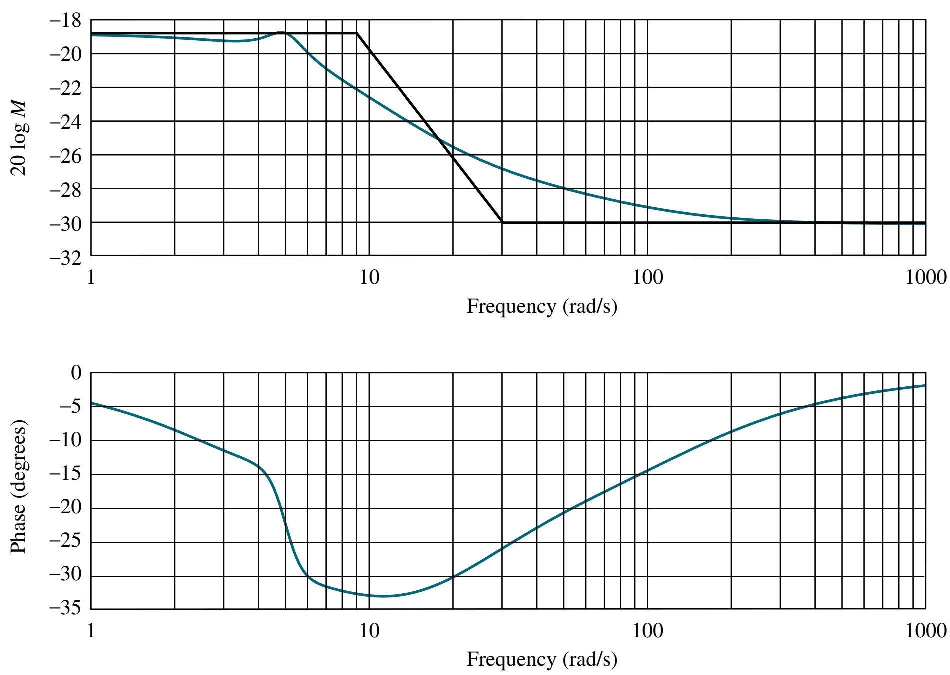

FIGURE 10.59 Original Bode plot minus response of $G_1(s)G_2(s) = [25/(s^2 + 2.4s + 25)]$ $[90/(s + 90)]$

It is interesting to note that the original curve was obtained from the function

$$G(s) = 70\frac{s+20}{(s+7)(s+70)(s^2+2s+25)} \tag{10.90}$$

MATLAB
ML

`Students who are using MATLAB should now run ch10p8 in Appendix B. You will learn how to use MATLAB to subtract Bode plots for the purpose of estimating transfer functions through sinusoidal testing. This exercise solves a portion of Example 10.18 using MATLAB.`

Skill-Assessment Exercise 10.12

PROBLEM: Estimate $G(s)$, whose Bode log-magnitude and phase plots are shown in Figure 10.60.

ANSWER: $G(s) = \dfrac{30(s+5)}{s(s+20)}$

The complete solution is at www.wiley.com/college/nise.

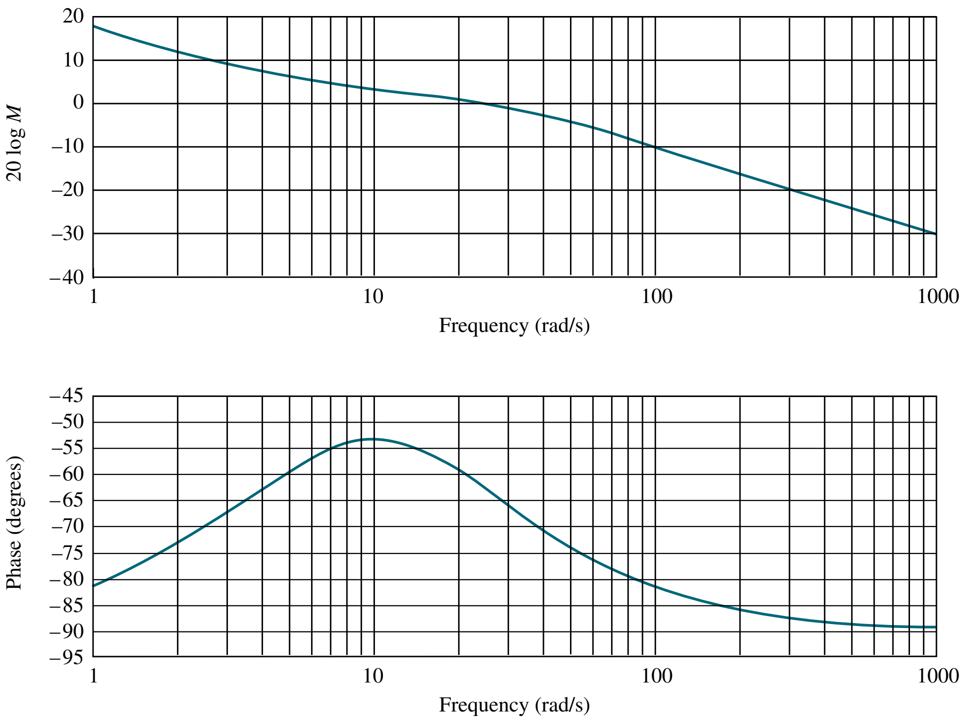

FIGURE 10.60 Bode plots for Skill-Assessment Exercise 10.12

In this chapter, we derived the relationships between time response performance and the frequency responses of the open- and closed-loop systems. The methods derived, although yielding a different perspective, are simply alternatives to the root locus and steady-state error analyses previously covered.

Case Study

Antenna Control: Stability Design and Transient Performance

Design
D

Our ongoing antenna position control system serves now as an example that summarizes the major objectives of the chapter. The case study demonstrates the use of frequency response methods to find the range of gain for stability and to design a value of gain to meet a percent overshoot requirement for the closed-loop step response.

PROBLEM: Given the antenna azimuth position control system shown on the front endpapers, Configuration 1, use frequency response techniques to find the following:

a. The range of preamplifier gain, K, required for stability

b. Percent overshoot if the preamplifier gain is set to 30

c. The estimated settling time

d. The estimated peak time

e. The estimated rise time

SOLUTION: Using the block diagram (Configuration 1) shown on the front endpapers and performing block diagram reduction yields the loop gain, $G(s)H(s)$, as

$$G(s)H(s) = \frac{6.63K}{s(s+1.71)(s+100)} = \frac{0.0388K}{s\left(\frac{s}{1.71}+1\right)\left(\frac{s}{100}+1\right)} \tag{10.91}$$

Letting $K = 1$, we have the magnitude and phase frequency response plots shown in Figure 10.61.

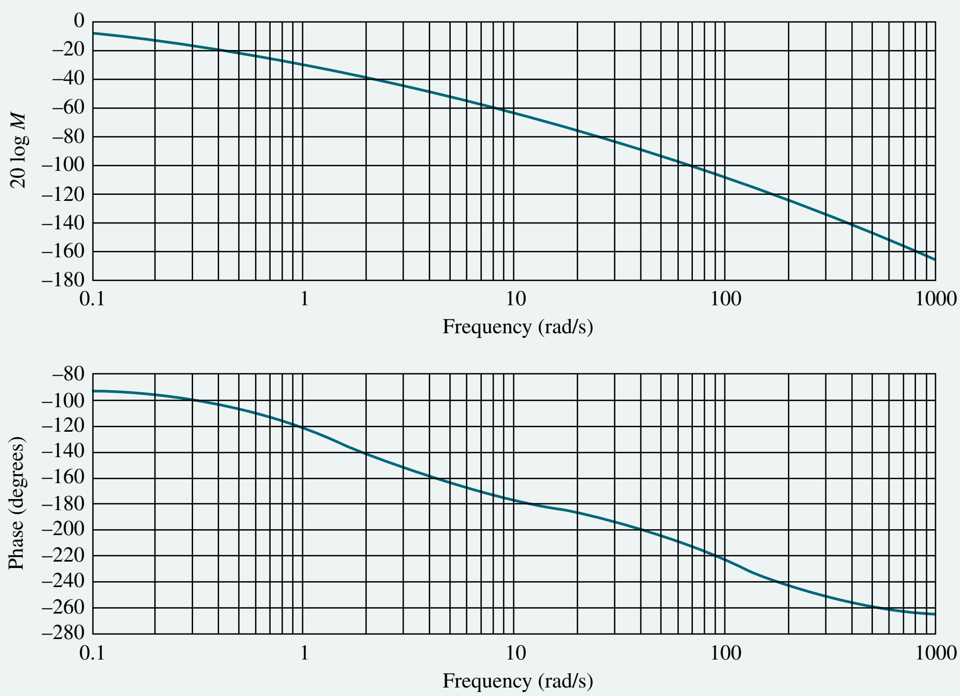

FIGURE 10.61 Open-loop frequency response plots for the antenna control system ($K = 1$)

a. In order to find the range of K for stability, we notice from Figure 10.61 that the phase response is $-180°$ at $\omega = 13.1$ rad/s. At this frequency, the magnitude plot is -68.41 dB. The gain, K, can be raised by 68.41 dB. Thus, $K = 2633$ will cause the system to be marginally stable. Hence, the system is stable if $0 < K < 2633$.

b. To find the percent overshoot if $K = 30$, we first make a second-order approximation and assume that the second-order transient response equations relating percent overshoot, damping ratio, and phase margin are true for this system. In other words, we assume that Eq. (10.73), which relates damping ratio to phase margin, is valid. If $K = 30$, the magnitude curve of Figure 10.61 is moved up by $20 \log 30 = 29.54$ dB. Therefore, the adjusted magnitude curve goes through zero dB at $\omega = 1$. At this frequency, the phase angle is $-120.9°$, yielding a phase margin of $59.1°$. Using Eq. (10.73) or Figure 10.48, $\zeta = 0.6$, or 9.48% overshoot. A computer simulation shows 10%.

c. To estimate the settling time, we make a second-order approximation and use Eq. (10.55). Since $K = 30$ (29.54 dB), the open-loop magnitude response is -7 dB when the normalized magnitude response of Figure 10.61 is -36.54 dB. Thus, the estimated bandwidth is 1.8 rad/s. Using Eq. (10.55), $T_s = 4.25$ seconds. A computer simulation shows a settling time of about 4.4 seconds.

d. Using the estimated bandwidth found in **c.** along with Eq. (10.56), and the damping ratio found in **a.** we estimate the peak time to be 2.5 seconds. A computer simulation shows a peak time of 2.8 seconds.

e. To estimate the rise time, we use Figure 4.16 and find that the normalized rise time for a damping ratio of 0.6 is 1.854. Using Eq. (10.54), the estimated bandwidth found in **c**, and $\zeta = 0.6$, we find $\omega_n = 1.57$. Using the normalized rise time and ω_n, we find $T_r = 1.854/1.57 = 1.18$ seconds. A simulation shows a rise time of 1.2 seconds.

CHALLENGE: You are now given a problem to test your knowledge of this chapter's objectives. You are given the antenna azimuth position control system shown on the front endpapers, Configuration 3. Record the block diagram parameters in the table shown on the front endpapers for Configuration 3 for use in subsequent case study challenge problems. Using frequency response methods, do the following:

a. Find the range of gain for stability.

b. Find the percent overshoot for a step input if the gain, K, equals 3.

c. Repeat Parts **a.** and **b.** using MATLAB.

MATLAB
ML

Summary

Frequency response methods are an alternative to the root locus for analyzing and designing feedback control systems. Frequency response techniques can be used more effectively than transient response to model physical systems in the laboratory. On the other hand, the root locus is more directly related to the time response.

The input to a physical system can be sinusoidally varying with known frequency, amplitude, and phase angle. The system's output, which is also sinusoidal in the steady state, can then be measured for amplitude and phase angle at different frequencies. From this data the magnitude frequency response of the system, which is the ratio of the output amplitude to the input amplitude, can be plotted and used in place of an analytically obtained magnitude frequency response. Similarly, we can obtain the phase response by finding the difference between the output phase angle and the input phase angle at different frequencies.

The frequency response of a system can be represented either as a polar plot or as separate magnitude and phase diagrams. As a polar plot, the magnitude response is the length of a vector drawn from the origin to a point on the curve, whereas the phase response is the angle of that vector. In the polar plot, frequency is implicit and is represented by each point on the polar curve. The polar plot of $G(s)H(s)$ is known as a *Nyquist diagram*.

Separate magnitude and phase diagrams, sometimes referred to as *Bode plots*, present the data with frequency explicitly enumerated along the abscissa. The magnitude curve can be a plot of log-magnitude versus log-frequency. The other graph is a plot of phase angle versus log-frequency. An advantage of Bode plots over the Nyquist diagram is that they can easily be drawn using asymptotic approximations to the actual curve.

The Nyquist criterion sets forth the theoretical foundation from which the frequency response can be used to determine a system's stability. Using the Nyquist criterion and Nyquist diagram, or the Nyquist criterion and Bode plots, we can determine a system's stability.

Frequency response methods give us not only stability information but also transient response information. By defining such frequency response quantities as gain margin and phase margin, the transient response can be analyzed or designed. *Gain margin* is the amount that the gain of a system can be increased before instability occurs if the phase angle is constant at 180°. *Phase margin* is the amount that the phase angle can be changed before instability occurs if the gain is held at unity.

While the open-loop frequency response leads to the results for stability and transient response just described, other design tools relate the closed-loop frequency response peak and bandwidth to the transient response. Since the closed-loop response is not as easy to obtain as the open-loop response because of the unavailability of the closed-loop poles, we use graphical aids in order to obtain the closed-loop frequency response from the open-loop frequency response. These graphical aids are the M and N circles and the Nichols chart. By superimposing the open-loop frequency response over the M and N circles or the Nichols chart, we are able to obtain the closed-loop frequency response and then analyze and design for transient response.

Today, with the availability of computers and appropriate software, frequency response plots can be obtained without relying on the graphical techniques described in this chapter. The program used for the root locus calculations and described in Appendix H.2 is one such program. MATLAB is another.

We concluded the chapter discussion by showing how to obtain a reasonable estimate of a transfer function using its frequency response, which can be obtained experimentally. Obtaining transfer functions this way yields more accuracy than transient response testing.

This chapter primarily has examined *analysis* of feedback control systems via frequency response techniques. We developed the relationships between frequency response and both stability and transient response. In the next chapter, we apply the concepts to the *design* of feedback control systems, using the Bode plots.

Review Questions

1. Name four advantages of frequency response techniques over the root locus.
2. Define frequency response as applied to a physical system.
3. Name two ways to plot the frequency response.
4. Briefly describe how to obtain the frequency response analytically.
5. Define Bode plots.
6. Each pole of a system contributes how much of a slope to the Bode magnitude plot?
7. A system with only four poles and no zeros would exhibit what value of slope at high frequencies in a Bode magnitude plot?
8. A system with four poles and two zeros would exhibit what value of slope at high frequencies in a Bode magnitude plot?
9. Describe the asymptotic phase response of a system with a single pole at -2.
10. What is the major difference between Bode magnitude plots for first-order systems and for second-order systems?
11. For a system with three poles at -4, what is the maximum difference between the asymptotic approximation and the actual magnitude response?
12. Briefly state the Nyquist criterion.
13. What does the Nyquist criterion tell us?
14. What is a Nyquist diagram?
15. Why is the Nyquist criterion called a frequency response method?
16. When sketching a Nyquist diagram, what must be done with open-loop poles on the imaginary axis?
17. What simplification to the Nyquist criterion can we usually make for systems that are open-loop stable?
18. What simplification to the Nyquist criterion can we usually make for systems that are open-loop unstable?
19. Define gain margin.
20. Define phase margin.
21. Name two different frequency response characteristics that can be used to determine a system's transient response.
22. Name three different methods of finding the closed-loop frequency response from the open-loop transfer function.
23. Briefly explain how to find the static error constant from the Bode magnitude plot.
24. Describe the change in the open-loop frequency response magnitude plot if time delay is added to the plant.
25. If the phase response of a pure time delay were plotted on a linear phase versus linear frequency plot, what would be the shape of the curve?
26. When successively extracting component transfer functions from experimental frequency response data, how do you know when you are finished?

Problems

1. Find analytical expressions for the magnitude and phase response for each $G(s)$ below. [Section: 10.1]

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   a. $G(s) = \dfrac{1}{s(s+2)(s+4)}$

   b. $G(s) = \dfrac{(s+5)}{(s+2)(s+4)}$

   c. $G(s) = \dfrac{(s+3)(s+5)}{s(s+2)(s+4)}$

2. For each function in Problem 1, make a plot of the log-magnitude and the phase, using log-frequency in rad/s as the ordinate. Do not use asymptotic approximations. [Section: 10.1]
3. For each function in Problem 1, make a polar plot of the frequency response. [Section: 10.1]
4. For each function in Problem 1, sketch the Bode asymptotic magnitude and asymptotic phase plots. Compare your results with your answers to Problem 1. [Section: 10.2]
5. Sketch the Nyquist diagram for each of the systems in Figure P10.1. [Section: 10.4]
6. Draw the polar plot from the separate magnitude and phase curves shown in Figure P10.2. [Section: 10.1]

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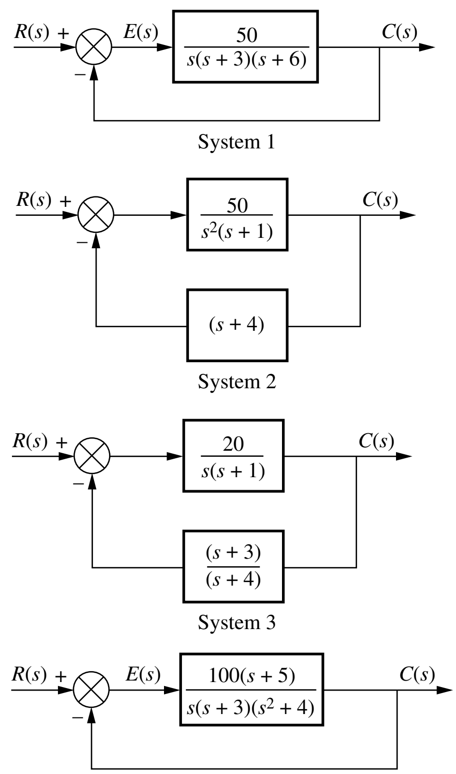

FIGURE P10.1

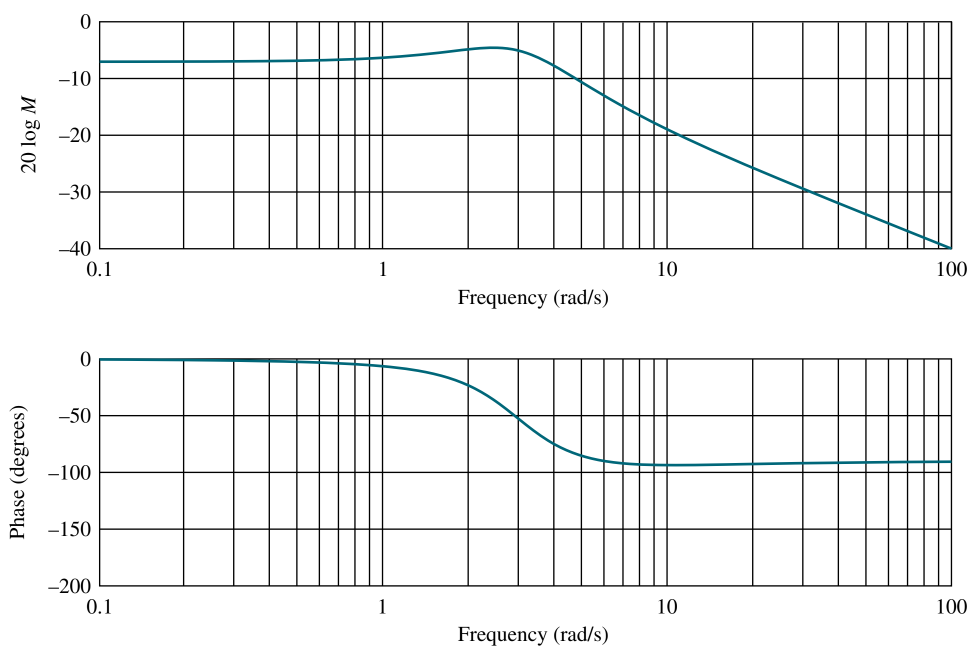

FIGURE P10.2

7. Draw the separate magnitude and phase curves from the polar plot shown in Figure P10.3. [Section: 10.1]

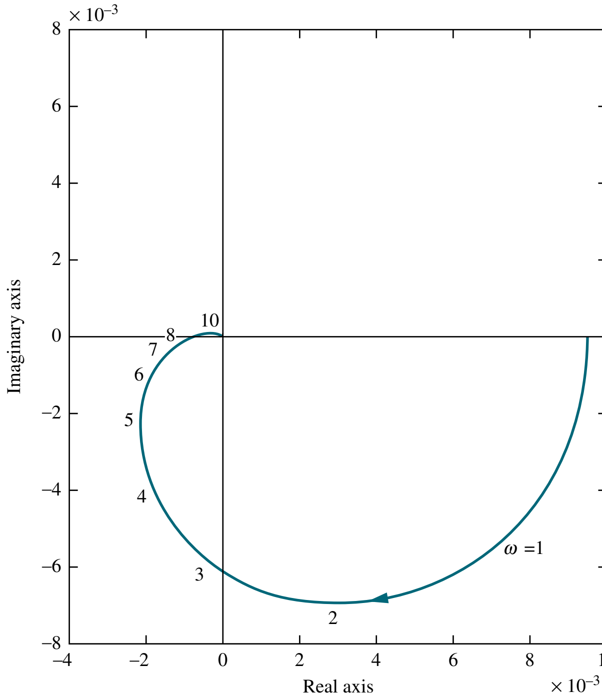

FIGURE P10.3

8. Write a program in MATLAB that will do the following:

MATLAB ML

a. Plot the Nyquist diagram of a system

b. Display the real-axis crossing value and frequency

Apply your program to a unity feedback system with

$$G(s) = \frac{K(s+5)}{(s^2+6s+100)(s^2+4s+25)}$$

9. Using the Nyquist criterion, find out whether each system of Problem 5 is stable. [Section: 10.3]

10. Using the Nyquist criterion, find the range of K for stability for each of the systems in Figure P10.4. [Section: 10.3]

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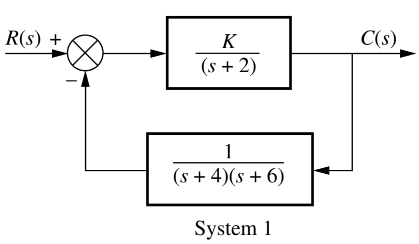

FIGURE P10.4 (*figure continues*)

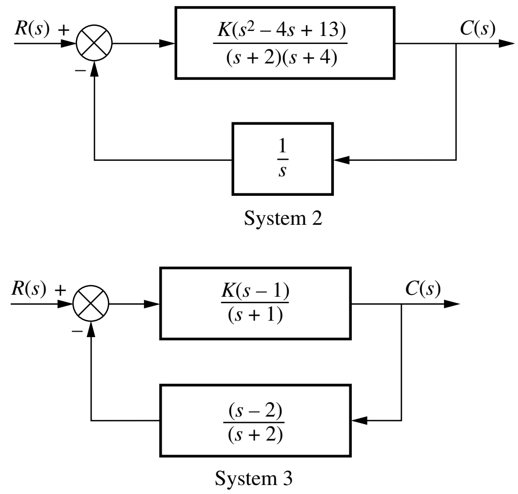

FIGURE P10.4 (*Continued*)

11. For each system of Problem 10, find the gain margin and phase margin if the value of K in each part of Problem 10 is [Section: 10.6]

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a. $K = 1000$

b. $K = 100$

c. $K = 0.1$

12. Write a program in MATLAB that will do the following:

MATLAB ML

a. Allow a value of gain, K, to be entered from the keyboard

b. Display the Bode plots of a system for the entered value of K

c. Calculate and display the gain and phase margin for the entered value of K

Test your program on a unity feedback system with $G(s) = K/[s(s+3)(s+12)]$.

13. Use MATLAB's LTI Viewer to find the gain margin, phase margin, zero dB frequency, and 180° frequency for a unity feedback system with

Gui Tool GUIT

$$G(s) = \frac{8000}{(s+6)(s+20)(s+35)}$$

Use the following methods:

a. The Nyquist diagram

b. Bode plots

14. Derive Eq. (10.54), the closed-loop bandwidth in terms of ζ and ω_n of a two-pole system. [Section: 10.8]

15. For each closed-loop system with the following performance characteristics, find the closed-loop bandwidth: [Section: 10.8]

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a. $\zeta = 0.2,\ T_s = 3$ seconds

b. $\zeta = 0.2,\ T_p = 3$ seconds

c. $T_s = 4$ seconds, $T_p = 2$ seconds

d. $\zeta = 0.3,\ T_r = 4$ seconds

16. Consider the unity feedback system of Figure 10.10. For each $G(s)$ that follows, use the M and N circles to make a plot of the closed-loop frequency response: [Section: 10.9]

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a. $G(s) = \dfrac{10}{s(s+1)(s+2)}$

b. $G(s) = \dfrac{1000}{(s+3)(s+4)(s+5)(s+6)}$

c. $G(s) = \dfrac{50(s+3)}{s(s+2)(s+4)}$

17. Repeat Problem 16, using the Nichols chart in place of the M and N circles. [Section: 10.9]

18. Using the results of Problem 16, estimate the percent overshoot that can be expected in the step response for each system shown. [Section: 10.10]

19. Use the results of Problem 17 to estimate the percent overshoot if the gain term in the numerator of the forward path of each part of the problem is respectively changed as follows: [Section: 10.10]

a. From 10 to 30

b. From 1000 to 2500

c. From 50 to 75

20. Write a program in MATLAB that will do the following:

MATLAB
ML

a. Allow a value of gain, K, to be entered from the keyboard

b. Display the closed-loop magnitude and phase frequency response plots of a unity feedback system with an open-loop transfer function, $KG(s)$

c. Calculate and display the peak magnitude, frequency of the peak magnitude, and bandwidth for the closed-loop frequency response and the entered value of K

Test your program on the system of Figure P10.5 for $K = 40$.

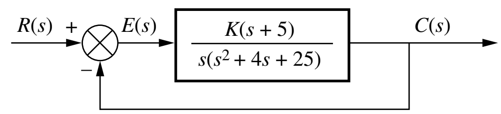

FIGURE P10.5

21. Use MATLAB's LTI Viewer with the Nichols plot to find the gain margin, phase margin, zero dB frequency, and 180° frequency for a unity feedback system with the forward-path transfer function

Gui Tool
GUIT

$$G(s) = \frac{5(s+6)}{s(s^2+4s+15)}$$

22. Write a program in MATLAB that will do the following:

MATLAB
ML

a. Make a Nichols plot of an open-loop transfer function

b. Allow the user to read the Nichols plot display and enter the value of M_p

c. Make closed-loop magnitude and phase plots

d. Display the expected values of percent overshoot, settling time, and peak time

e. Plot the closed-loop step response

Test your program on a unity feedback system with the forward-path transfer function

$$G(s) = \frac{5(s+6)}{s(s^2+4s+15)}$$

and explain any discrepancies.

23. Using Bode plots, estimate the transient response of the systems in Figure P10.6. [Section: 10.10]

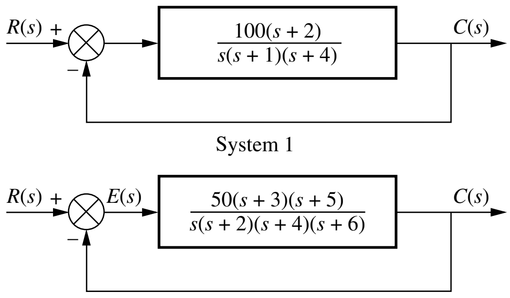

FIGURE P10.6

24. For the system of Figure P10.5, do the following: [Section: 10.10]

a. Plot the Bode magnitude and phase plots.

b. Assuming a second-order approximation, estimate the transient response of the system if $K = 40$.

c. `Use MATLAB or any other program to check your assumptions by simulating the step response of the system.`

MATLAB ML

25. The Bode plots for a plant, $G(s)$, used in a unity feedback system are shown in Figure P10.7. Do the following:

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a. Find the gain margin, phase margin, zero dB frequency, 180° frequency, and the closed-loop bandwidth.

b. Use your results in Part **a** to estimate the damping ratio, percent overshoot, settling time, and peak time.

26. `Write a program in MATLAB that will use an open-loop transfer function, G(s), to do the following:`

MATLAB ML

a. `Make a Bode plot`

b. `Use frequency response methods to estimate the percent overshoot, settling time, and peak time`

c. `Plot the closed-loop step response`

`Test your program by comparing the results to those obtained for the systems of Problem 23.`

27. The open-loop frequency response shown in Figure P10.8 was experimentally obtained from a unity feedback system. Estimate the percent overshoot and steady-state error of the closed-loop system. [Sections: 10.10, 10.11]

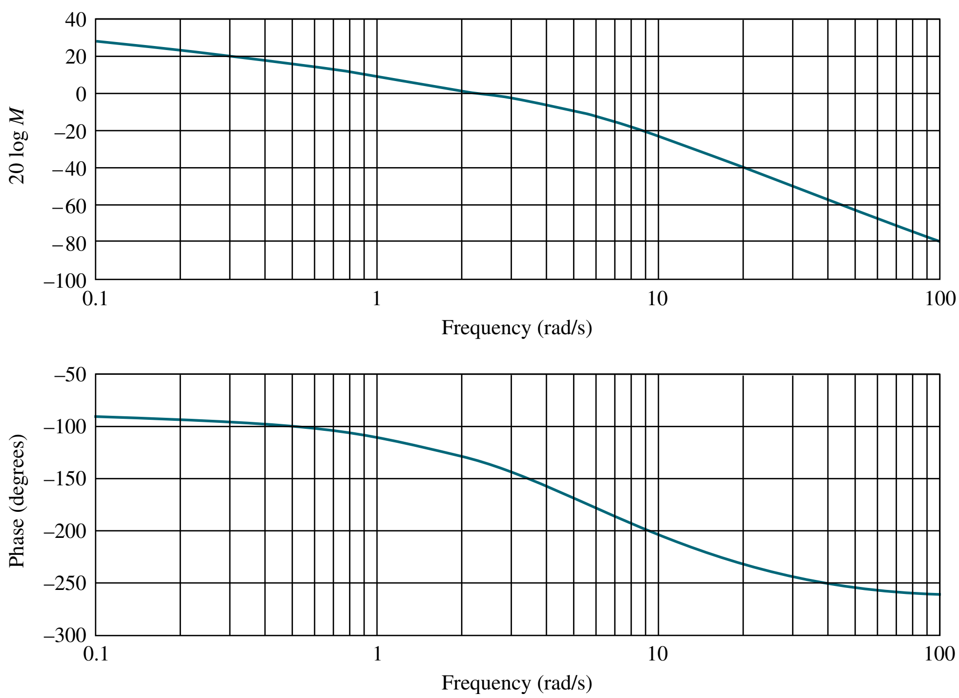

FIGURE P10.7

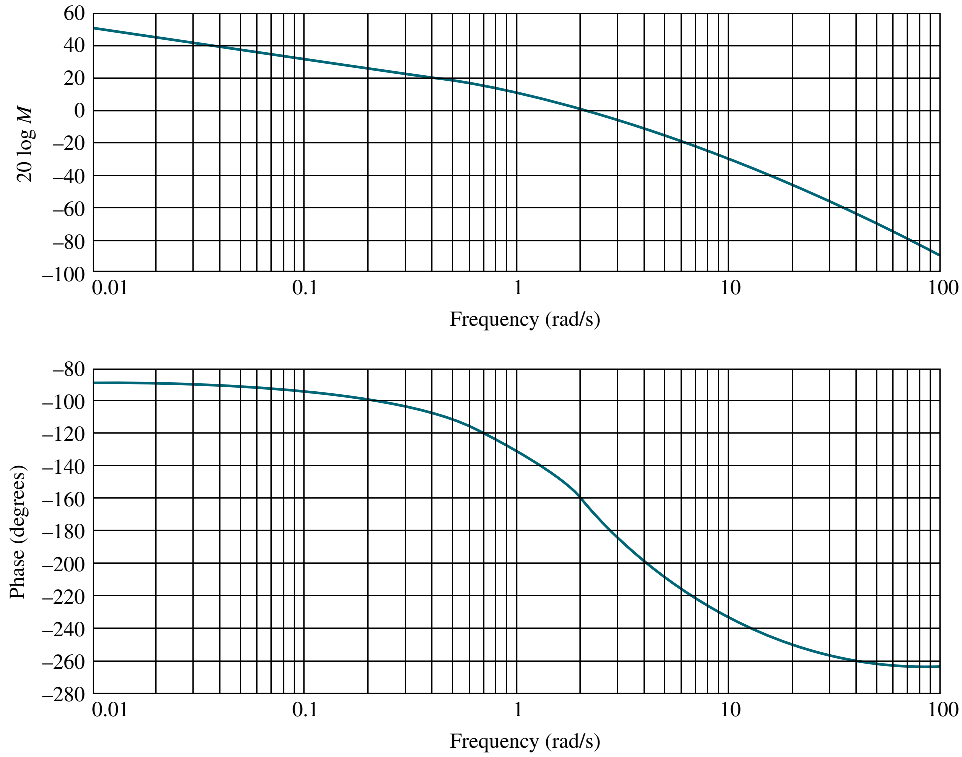

FIGURE P10.8

28. Consider the system in Figure P10.9. [Section: 10.12]

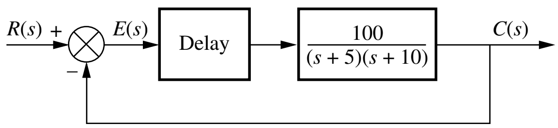

FIGURE P10.9

a. Find the phase margin if the system is stable for time delays of 0, 0.1, 0.2, 0.5, and 1 second.

b. Find the gain margin if the system is stable for each of the time delays given in Part **a**.

c. For what time delays mentioned in Part **a** is the system stable?

d. For each time delay that makes the system unstable, how much reduction in gain is required for the system to be stable?

29. Given a unity feedback system with the forward-path transfer function

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$$G(s) = \frac{K}{(s+1)(s+3)(s+6)}$$

and a delay of 0.5 second, find the range of gain, K, to yield stability. Use Bode plots and frequency response techniques. [Section: 10.12]

30. Given a unity feedback system with the forward-path transfer function

$$G(s) = \frac{K}{s(s+3)(s+12)}$$

and a delay of 0.5 second, make a second-order approximation and estimate the percent overshoot if $K = 40$. Use Bode plots and frequency response techniques. [Section: 10.12]

31. `Use the MATLAB function pade(T,n) to model the delay in Problem 30. Obtain the unit step response and evaluate your second-order approximation in Problem 30.`

MATLAB ML

32. For the Bode plots shown in Figure P10.10, determine the transfer function by hand or via MATLAB. [Section: 10.13]

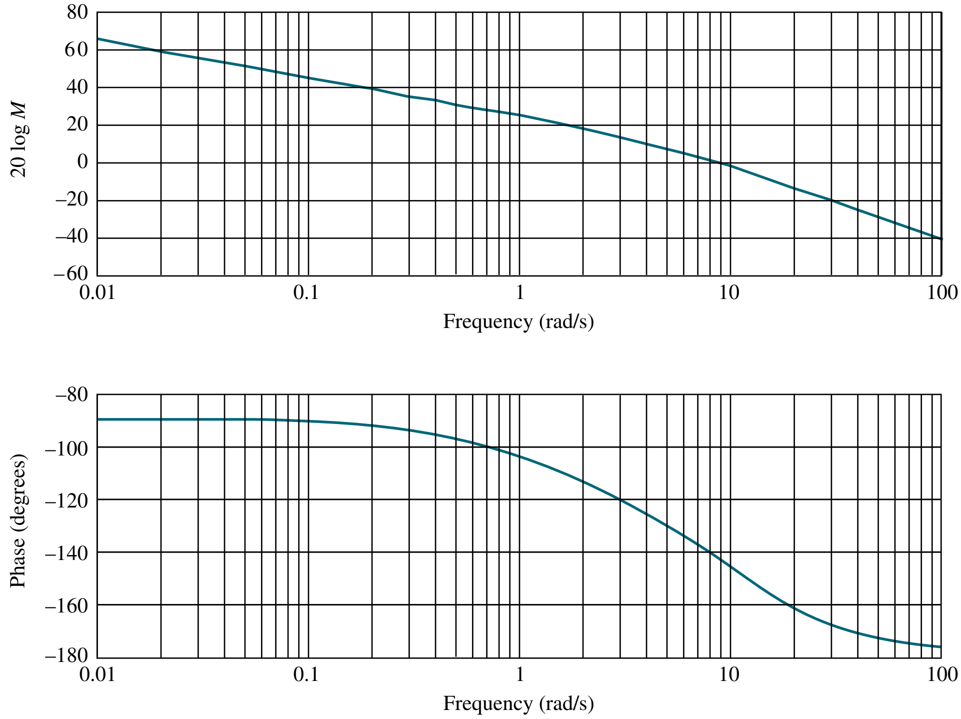

FIGURE P10.10

33. Repeat Problem 32 for the Bode plots shown in Figure P10.11. [Section: 10.13]

34. An overhead crane consists of a horizontally moving trolley of mass m_T dragging a load of mass m_L, which dangles from its bottom surface at the end of a rope of fixed length, L. The position of the trolley is controlled in the feedback configuration shown in Figure 10.20. Here,

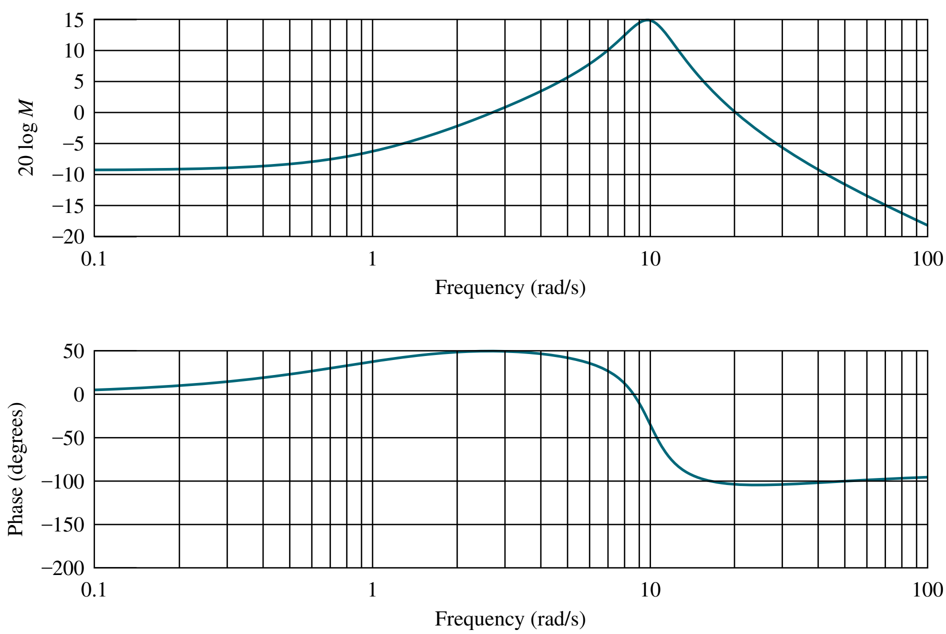

FIGURE P10.11

$G(s) = KP(s)$, $H = 1$, and

$$P(s) = \frac{X_T(s)}{F_T(s)} = \frac{1}{m_T} \frac{s^2 + \omega_0^2}{s^2(s^2 + a\omega_0^2)}$$

The input is $f_T(t)$, the input force applied to the trolley. The output is $x_T(t)$, the trolley displacement. Also, $\omega_0 = \sqrt{\frac{g}{L}}$ and $a = (m_L + m_T)/m_T$ (*Marttinen, 1990*) Make a qualitative Bode plot of the system assuming $a > 1$.

35. A room's temperature can be controlled by varying the radiator power. In a specific room, the transfer function from indoor radiator power, $\dot{Q}$, to room temperature, T in °C is (*Thomas, 2005*)

$$P(s) = \frac{T(s)}{\dot{Q}(s)} = \frac{(1 \times 10^{-6})s^2 + (1.314 \times 10^{-9})s + (2.66 \times 10^{-13})}{s^3 + 0.00163\,s^2 + (5.272 \times 10^{-7})s + (3.538 \times 10^{-11})}$$

The system is controlled in the closed-loop configuration shown in Figure 10.20 with $G(s) = KP(s)$, $H = 1$.

a. Draw the corresponding Nyquist diagram for $K = 1$.

b. Obtain the gain and phase margins.

c. Find the range of K for the closed-loop stability. Compare your result with that of Problem 61, Chapter 6.

36. The open-loop dynamics from dc voltage armature to angular position of a robotic manipulator joint is given by $P(s) = \frac{48500}{s^2 + 2.89s}$ (*Low, 2005*).

a. Draw by hand a Bode plot using asymptotic approximations for magnitude and phase.

b. `Use MATLAB to plot the exact Bode plot and compare with your sketch from Part` **a**. MATLAB ML

37. Problem 49, Chapter 8 discusses a magnetic levitation system with a plant transfer function $P(s) = -\frac{1300}{s^2 - 860^2}$ (*Galvão, 2003*). Assume that the plant is in cascade with an $M(s)$ and that the system will be controlled by the loop shown in Figure 10.20, where $G(s) = M(s)P(s)$ and $H = 1$. For each $M(s)$ that follows, draw the Nyquist diagram when $K = 1$, and find the range of closed-loop stability for $K > 0$.

a. $M(s) = -K$

b. $M(s) = -\frac{K(s + 200)}{s + 1000}$

c. Compare your results with those obtained in Problem 49, Chapter 8.

38. The simplified and linearized model for the transfer function of a certain bicycle from steer angle (δ) to roll angle (φ) is given by (*Åstrom, 2005*)

$$P(s) = \frac{\varphi(s)}{\delta(s)} = \frac{10(s + 25)}{s^2 + 25}$$

Assume the rider can be represented by a gain K, and that the closed-loop system is shown in Figure 10.20 with $G(s) = KP(s)$ and $H = 1$. Use the Nyquist stability criterion to find the range of K for closed-loop stability.

39. The control of the radial pickup position of a digital versatile disk (DVD) was discussed in Problem 48, Chapter 9. There, the open-loop transfer function from coil input voltage to radial pickup position was given as (*Bittanti, 2002*)

$$P(s) = \frac{0.63}{\left(1 + \frac{0.36}{305.4}s + \frac{s^2}{305.4^2}\right)\left(1 + \frac{0.04}{248.2}s + \frac{s^2}{248.2^2}\right)}$$

Assume the plant is in cascade with a controller,

$$M(s) = \frac{0.5(s + 1.63)}{s(s + 0.27)}$$

and in the closed-loop configuration shown in Figure 10.20, where $G(s) = M(s)P(s)$ and $H = 1$. Do the following:

a. Draw the open-loop frequency response in a Nichols chart.

b. Predict the system's response to a unit step input. Calculate the $\%OS$, c_{final}, and T_s.

c. `Verify the results of Part` **b** `using MATLAB simulations.` MATLAB ML

40. The Soft Arm, used to feed people with disabilities, was discussed in Problem 57 in Chapter 6. Assuming the system block diagram shown in Figure P10.12, use frequency response techniques to determine the following (*Kara, 1992*):

a. Gain margin, phase margin, zero dB frequency, and 180° frequency

b. Is the system stable? Why?

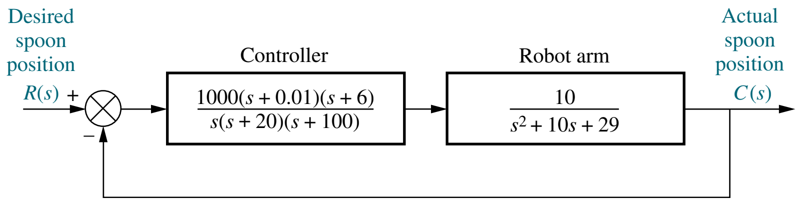

FIGURE P10.12 Soft Arm position control system block diagram

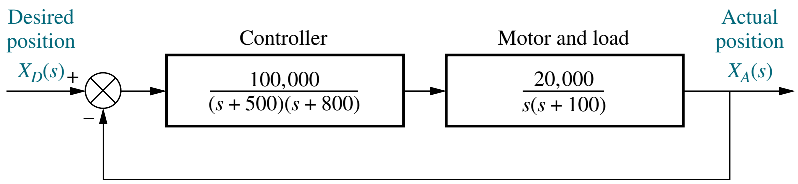

FIGURE P10.13 Floppy disk drive block diagram

41. A floppy disk drive was discussed in Problem 57 in Chapter 8. Assuming the system block diagram shown in Figure P10.13, use frequency response techniques to determine the following:

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a. Gain margin, phase margin, zero dB frequency, 180° frequency, and closed-loop bandwidth

b. Percent overshoot, settling time, and peak time

c. `Use MATLAB to simulate the closed-loop step response and compare the results to those obtained in Part` **b.**

MATLAB **ML**

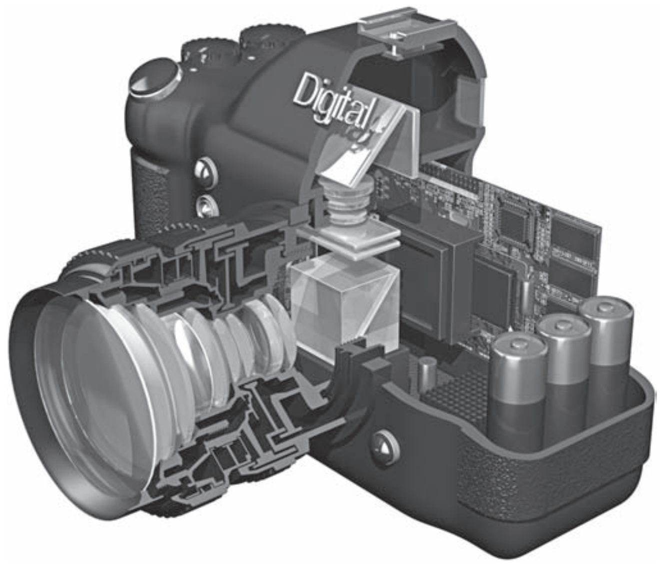
42. Industrial robots, such as that shown in Figure P10.14, require accurate models for design of high performance. Many transfer function models for industrial robots assume interconnected rigid bodies with the drive-torque source modeled as a pure gain, or first-order system. Since the motions associated with the robot are connected to the drives through flexible linkages rather than rigid linkages, past modeling does not explain the resonances observed. An accurate, small-motion, linearized model has been developed that takes into consideration the flexible drive. The transfer function

$$G(s) = 999.12\frac{(s^2+8.94s+44.7^2)}{(s+20.7)(s^2+34.858s+60.1^2)}$$

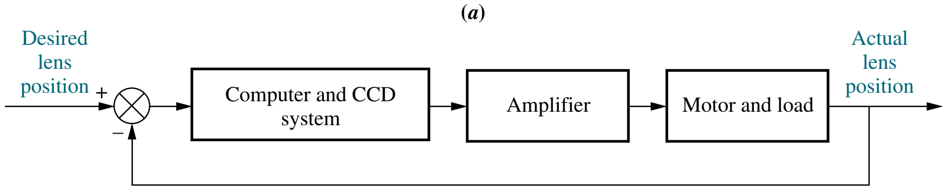
relates the angular velocity of the robot base to electrical current commands (*Good*, *1985*). Make a Bode plot of the frequency response and identify the resonant frequencies.

FIGURE P10.14 Robot performing construction of computer memory units (© Michael Rosenfield/Science Faction/ © Corbis).

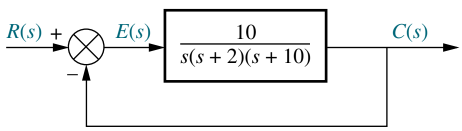
43. The charge-coupled device (CCD) that is used in video movie cameras to convert images into electrical signals can be used as part of an automatic focusing system in cameras. Automatic focusing can be implemented by focusing the center of the image on a charge-coupled device array through two lenses. The separation of the two images on the CCD is related to the focus. The camera senses the separation, and a computer drives the lens and



(*a*)



(*b*)



(*c*)

FIGURE P10.15 **a.** A cutaway view of a digital camera showing parts of the CCD automatic focusing system (© Stephen Sweet/iStockphoto); **b.** functional block diagram; **c.** block diagram

focuses the image. The automatic focus system is a position control, where the desired position of the lens is an input selected by pointing the camera at the subject. The output is the actual position of the lens. The camera in Figure P10.15(*a*) uses a CCD automatic focusing system. Figure P10.15(*b*) shows the automatic focusing feature represented as a position control system. Assuming the simplified model shown in Figure P10.15(*c*), draw the Bode plots and estimate the percent overshoot for a step input.

44. A ship's roll can be stabilized with a control system. A voltage applied to the fins' actuators creates a roll torque that is applied to the ship. The ship, in response to the roll torque, yields a roll angle. Assuming the block diagram for the roll control system shown in Figure P10.16, determine the gain and phase margins for the system.

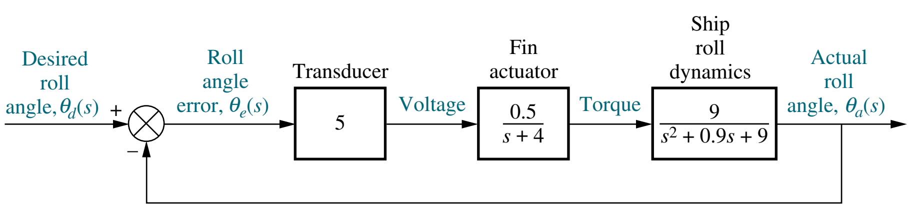


FIGURE P10.16 Block diagram of a ship's roll-stabilizing system

45. The linearized model of a particular network link working under TCP/IP and controlled using a random early detection (RED) algorithm can be described by Figure 10.20 where $G(s) = M(s)P(s)$, $H = 1$, and (*Hollot, 2001*)

$$M(s) = \frac{0.005L}{s + 0.005}; \; P(s) = \frac{140625e^{-0.1s}}{(s + 2.67)(s + 10)}$$

a. Plot the Nichols chart for $L = 1$. Is the system closed-loop stable?

b. Find the range of L for closed-loop stability.

c. Use the Nichols chart to predict $\%OS$ and T_s for $L = 0.95$. Make a hand sketch of the expected unit step response.

d. Verify Part **c** with a Simulink unit step response simulation. Simulink SL

46. In the TCP/IP network link of Problem 45, let $L = 0.8$, but assume that the amount of delay is an unknown variable.

a. Plot the Nyquist diagram of the system for zero delay, and obtain the phase margin.

b. Find the maximum delay allowed for closed-loop stability.

47. Thermal flutter of the Hubble Space Telescope (HST) produces errors for the pointing control system. Thermal flutter of the solar arrays occurs when the spacecraft passes from sunlight to darkness and when the spacecraft is in daylight. In passing from daylight to darkness, an end-to-end bending oscillation of frequency f_1 rad/s is experienced. Such oscillations interfere with the pointing control system of the HST. A filter with the transfer function

$$G_f(s) = \frac{1.96(s^2 + s + 0.25)(s^2 + 1.26s + 9.87)}{(s^2 + 0.015s + 0.57)(s^2 + 0.083s + 17.2)}$$

is proposed to be placed in cascade with the PID controller to reduce the bending (*Wie, 1992*).

a. Obtain the frequency response of the filter and estimate the bending frequencies that will be reduced.

b. Explain why this filter will reduce the bending oscillations if these oscillations are thought to be disturbances at the output of the control system.

48. An experimental holographic media storage system uses a flexible photopolymer disk. During rotation, the disk tilts, making information retrieval difficult. A system that compensates for the tilt has been developed. For this, a laser beam is focused on the disk surface and disk variations are measured through reflection. A mirror is in turn adjusted to align with the disk and makes information retrieval possible. The system can be represented by a unity feedback system in which a controller with transfer function

$$G_C(s) = \frac{78.575(s + 436)^2}{(s + 132)(s + 8030)}$$

and a plant

$$P(s) = \frac{1.163 \times 10^8}{s^3 + 962.5s^2 + 5.958 \times 10^5 s + 1.16 \times 10^8}$$

form an open loop transmission $L(s) = G_c(s)P(s)$ (*Kim, 2009*).

a. Use MATLAB to obtain the system's Nyquist diagram. Find out if the system is stable. MATLAB ML

b. Find the system's phase margin.

c. Use the value of phase margin obtained in **b.** to calculate the expected system's overshoot to a step input.

d. Simulate the system's response to a unit step input and verify the %OS calculated in **c.**

49. The design of cruise control systems in heavy vehicles such as big rigs is especially challenging due to the extreme variations in payload. A typical frequency response for the transfer function from fuel mass flow to vehicle speed is shown in Figure P10.17.

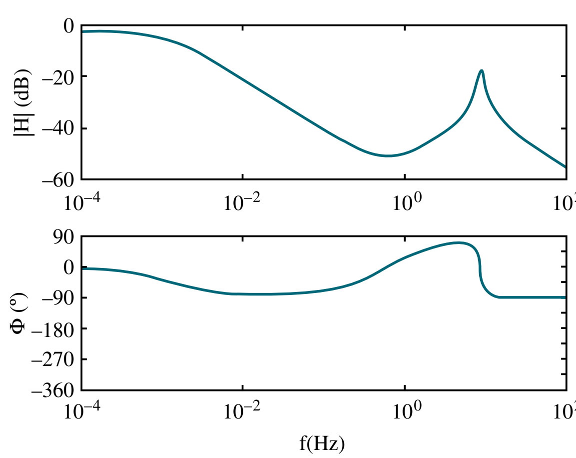


FIGURE P10.17

This response includes the dynamics of the engine, the gear box, the propulsion shaft, the differential, the drive shafts, the chassis, the payload, and tire dynamics. Assume that the system is controlled in a closed-loop, unity-feedback loop using a proportional compensator (*van der Zalm*, *2008*).

a. Make a plot of the Nyquist diagram that corresponds to the Bode plot of Figure P10.17.

b. Assuming there are no open-loop poles in the right half-plane, find out if the system is closed-loop stable when the proportional gain $K = 1$.

c. Find the range of positive K for which the system is closed-loop stable.

50. Use LabVIEW with the Control Design and Simulation Module, and MathScript RT Module and modify the CDEx Nyquist Analysis.vi to obtain the range of K for stability using the Nyquist plot for any system you enter. In addition, design a LabVIEW VI that will accept as an input the polynomial numerator and polynomial denominator of an open-loop transfer function and obtain a Nyquist plot for a value of $K = 10,000$. Your VI will also display the following as generated from the Nyquist plot: (1) gain margin, (2) phase margin, (3) zero dB frequency, and (4) 180 degrees frequency. Use the system and results of Skill-Assessment Exercise 10.6 to test your VIs.

51. Use LabVIEW with the Control Design and Simulation Module, and MathScript RT Module to build a VI that will accept an open-loop transfer function, plot the Bode diagram, and plot the closed-loop step response. Your VI will also use the CD Parametric Time Response.vi to display (1) rise time, (2) peak time, (3) settling time, (4) percent overshoot, (5) steady-state value, and (6) peak value. Use the system in Skill-Assessment Exercise 10.9 to test your VI. Compare the results obtained from your VI with those obtained in Skill-Assessment Exercise 10.9.

52. The block diagram of a cascade system used to control water level in a steam generator of a nuclear power plant (*Wang, 2009*) was presented in Figure P.6.19. In that system, the level controller, $G_{LC}(s)$, is the *master* controller and the feed-water flow controller, $G_{FC}(s)$, is the slave controller. Consider that the inner feedback loop is replaced by its equivalent transfer function, $G_{WX}(s)$.

Using numerical values in (*Wang, 2009*) and (*Bhambhani, 2008*) the transfer functions with a 1 second pure delay are:

$$G_{fw}(s) = \frac{2 \cdot e^{-\tau s}}{s(T_1 s + 1)} = \frac{2 \cdot e^{-s}}{s(25s + 1)};$$

$$G_{WX}(s) = \frac{(4s + 1)}{3(3.333s + 1)};$$

$$G_{LC}(s) = K_{P_{LC}} + K_{D_{LC}} s = 1.5(10s + 1).$$

Use MATLAB or any other program to:

a. Obtain Bode magnitude and phase plots for this system using a fifth-order Padé approximation (available in MATLAB). Note on these plots, if applicable, the gain and phase margins.

b. Plot the response of the system, $c(t)$, to a unit step input, $r(t) = u(t)$. Note on the $c(t)$ curve the rise time, T_r, the settling time, T_s, the final value of the output, and, if applicable, the percent overshoot, $\%OS$, and mid peak time, T_p.

c. Repeat the above two steps for a pure delay of 1.5 seconds.

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

53. High-speed rail pantograph. Problem 21 in Chapter 1 discusses active control of a pantograph mechanism for high-speed rail systems. In Problem 79(a), Chapter 5, you found the block diagram for the active pantograph control system. In Chapter 8, Problem 72, you designed the gain to yield a closed-loop step response with 30% overshoot. A plot of the step response should have shown a settling time greater than 0.5 second as well as a high-frequency oscillation superimposed over the step response. In Chapter 9, Problem 55, we reduced the settling time to about 0.3 second, reduced the step response steady-state error to zero, and eliminated the high-frequency oscillations by using a notch filter (*O'Connor, 1997*). Using the equivalent forward transfer function found in Chapter 5 cascaded with the notch filter specified in Chapter 9, do the following using frequency response techniques:

a. Plot the Bode plots for a total equivalent gain of 1 and find the gain margin, phase margin, and 180° frequency.

b. Find the range of K for stability.

c. Compare your answer to Part **b** with your answer to Problem 67, Chapter 6. Explain any differences.

54. Control of HIV/AIDS. The linearized model for an HIV/AIDS patient treated with RTIs was obtained in Chapter 6 as (*Craig, 2004*);

$$P(s) = \frac{Y(s)}{U_1(s)} = \frac{-520s - 10.3844}{s^3 + 2.6817s^2 + 0.11s + 0.0126}$$

a. Consider this plant in the feedback configuration in Figure 10.20 with $G(s) = P(s)$ and $H(s) = 1$. Obtain the Nyquist diagram. Evaluate the system for closed-loop stability.

b. Consider this plant in the feedback configuration in Figure 10.20 with $G(s) = -P(s)$ and $H(s) = 1$. Obtain the Nyquist diagram. Evaluate the system for closed-loop stability. Obtain the gain and phase margins.

55. Hybrid vehicle. In Problem 8.74 we used MATLAB to plot the root locus for the speed control of an HEV rearranged as a unity-feedback system, as shown in Figure P7.34 (*Preitl, 2007*). The plant and compensator were given by

MATLAB
ML

$$G(s) = \frac{K(s + 0.6)}{(s + 0.5858)(s + 0.0163)}$$

and we found that $K = 0.78$, resulted in a critically damped system.

a. Use MATLAB or any other program to plot

i. The Bode magnitude and phase plots for that system, and

ii. The response of the system, $c(t)$, to a step input, $r(t) = 4\,u(t)$. Note on the $c(t)$ curve the rise time, T_r, and settling time, T_s, as well as the final value of the output.

b. Now add an integral gain to the controller, such that the plant and compensator transfer function becomes

$$G(s) = \frac{K_1(s + Z_c)(s + 0.6)}{s(s + 0.5858)(s + 0.0163)}$$

where $K_1 = 0.78$ and $Z_c = \dfrac{K_2}{K_1} = 0.4$. Use MATLAB or any other program to do the following:

i. Plot the Bode magnitude and phase plots for this case.

ii. Obtain the response of the system to a step input, $r(t) = 4\,u(t)$. Plot $c(t)$ and note on it the rise time, T_r, percent overshoot, $\%OS$, peak time, T_p, and settling time, T_s.

c. Does the response obtained in **a.** or **b.** resemble a second-order overdamped, critically damped, or underdamped response? Explain.

Cyber Exploration Laboratory

Experiment 10.1

Objective To examine the relationships between open-loop frequency response and stability, open-loop frequency response and closed-loop transient response, and the effect of additional closed-loop poles and zeros upon the ability to predict closed-loop transient response

Minimum Required Software Packages MATLAB, and the Control System Toolbox

Prelab

1. Sketch the Nyquist diagram for a unity negative feedback system with a forward transfer function of $G(s) = \dfrac{K}{s(s+2)(s+10)}$. From your Nyquist plot, determine the range of gain, K, for stability.

2. Find the phase margins required for second-order closed-loop step responses with the following percent overshoots: 5%, 10%, 20%, 30%.

Lab

1. Using the SISO Design Tool, produce the following plots simultaneously for the system of Prelab 1: root locus, Nyquist diagram, and step response. Make plots for the following values of K: 50, 100, the value for marginal stability found in Prelab 1, and a value above that found for marginal stability. Use the zoom tools when required to produce an illustrative plot. Finally, change the gain by grabbing and moving the closed-loop poles along the root locus and note the changes in the Nyquist diagram and step response.
2. Using the SISO Design Tool, produce Bode plots and closed-loop step responses for a unity negative feedback system with a forward transfer function of $G(s) = \frac{K}{s(s+10)^2}$. Produce these plots for each value of phase margin found in Prelab 2. Adjust the gain to arrive at the desired phase margin by grabbing the Bode magnitude curve and moving it up or down. Observe the effects, if any, upon the Bode phase plot. For each case, record the value of gain and the location of the closed-loop poles.
3. Repeat Lab 2 for $G(s) = \frac{K}{s(s+10)}$.

Postlab

1. Make a table showing calculated and actual values for the range of gain for stability as found in Prelab 1 and Lab 1.
2. Make a table from the data obtained in Lab 2 itemizing phase margin, percent overshoot, and the location of the closed-loop poles.
3. Make a table from the data obtained in Lab 3 itemizing phase margin, percent overshoot, and the location of the closed-loop poles.
4. For each Postlab task 1 to 3, explain any discrepancies between the actual values obtained and those expected.

Experiment 10.2

Objective To use LabVIEW and Nichols charts to determine the closed-loop time response performance.

Minimum Required Software Packages LabVIEW, Control Design and Simulation Module, MathScript RT Module, and MATLAB

Prelab

1. Assume a unity-feedback system with a forward-path transfer function, $G(s) = \frac{100}{s(s+5)}$. Use MATLAB or any method to determine gain and phase margins. In addition, find the percent overshoot, settling time, and peak time of the closed-loop step response.
2. Design a LabVIEW VI that will create a Nichols chart. Adjust the Nichols chart's scale to estimate gain and phase margins. Then, prompt the user to enter the values of

gain and phase margin found from the Nichols chart. In response, your VI will produce the percent overshoot, settling time, and peak time of the closed-loop step response.

Lab Run your VI for the system given in the Prelab. Test your VI with other systems of your choice.

Postlab Compare the closed-loop performance calculated in the Prelab with those produced by your VI.

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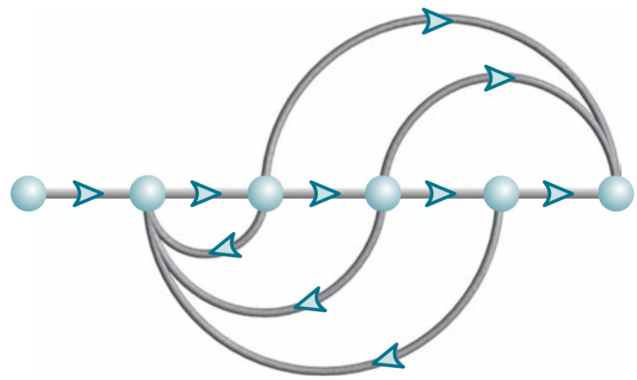
Design via Frequency Response

11



Chapter Learning Outcomes

After completing this chapter the student will be able to:

- Use frequency response techniques to adjust the gain to meet a transient response specification (Sections 11.1–11.2)
- Use frequency response techniques to design cascade compensators to improve the steady-state error (Section 11.3)
- Use frequency response techniques to design cascade compensators to improve the transient response (Section 11.4)
- Use frequency response techniques to design cascade compensators to improve both the steady-state error and the transient response (Section 11.5)

Case Study Learning Outcomes

You will be able to demonstrate your knowledge of the chapter objectives with case studies as follows:

- Given the antenna azimuth position control system shown on the front endpapers, you will be able to use frequency response techniques to design the gain to meet a transient response specification.
- Given the antenna azimuth position control system shown on the front endpapers, you will be able to use frequency response techniques to design a cascade compensator to meet both transient and steady-state error specifications.

11.1 Introduction

In Chapter 8, we designed the transient response of a control system by adjusting the gain along the root locus. The design process consisted of finding the transient response specification on the root locus, setting the gain accordingly, and settling for the resulting steady-state error. The disadvantage of design by gain adjustment is that only the transient response and steady-state error represented by points along the root locus are available.

In order to meet transient response specifications represented by points not on the root locus and, independently, steady-state error requirements, we designed cascade compensators in Chapter 9. In this chapter, we use Bode plots to parallel the root locus design process from Chapters 8 and 9.

Let us begin by drawing some general comparisons between root locus and frequency response design.

Stability and transient response design via gain adjustment. Frequency response design methods, unlike root locus methods, can be implemented conveniently without a computer or other tool except for testing the design. We can easily draw Bode plots using asymptotic approximations and read the gain from the plots. Root locus requires repeated trials to find the desired design point from which the gain can be obtained. For example, in designing gain to meet a percent overshoot requirement, root locus requires the search of a radial line for the point where the open-loop transfer function yields an angle of 180°. To evaluate the range of gain for stability, root locus requires a search of the $j\omega$-axis for 180°. Of course, if one uses a computer program, such as MATLAB, the computational disadvantage of root locus vanishes.

Transient response design via cascade compensation. Frequency response methods are not as intuitive as the root locus, and it is something of an art to design cascade compensation with the methods of this chapter. With root locus, we can identify a specific point as having a desired transient response characteristic. We can then design cascade compensation to operate at that point and meet the transient response specifications. In Chapter 10, we learned that phase margin is related to percent overshoot (Eq. (10.73)) and bandwidth is related to both damping ratio and settling time or peak time (Eqs. (10.55) and (10.56)). These equations are rather complicated. When we design cascade compensation using frequency response methods to improve the transient response, we strive to reshape the open-loop transfer function's frequency response to meet both the phase-margin requirement (percent overshoot) and the bandwidth requirement (settling or peak time). There is no easy way to relate all the requirements prior to the reshaping task. Thus, the reshaping of the open-loop transfer function's frequency response can lead to several trials until all transient response requirements are met.

Steady-state error design via cascade compensation. An advantage of using frequency design techniques is the ability to design derivative compensation, such as lead compensation, to speed up the system and at the same time build in a desired steady-state error requirement that can be met by the lead compensator alone. Recall that in using root locus there are an infinite number of possible solutions to the design of a lead compensator. One of the differences between these solutions is the steady-state error. We must make numerous tries to arrive at the solution that yields the required steady-state error performance. With frequency response techniques, we build the steady-state error requirement right into the design of the lead compensator.

You are encouraged to reflect on the advantages and disadvantages of root locus and frequency response techniques as you progress through this chapter. Let us take a closer look at frequency response design.

When designing via frequency response methods, we use the concepts of stability, transient response, and steady-state error that we learned in Chapter 10. First, the Nyquist criterion tells us how to determine if a system is stable. Typically, an open-loop stable system is stable in closed-loop if the open-loop magnitude frequency response has a gain of less than 0 dB at the frequency where the phase frequency response is 180°. Second, percent overshoot is reduced by increasing the phase margin, and the speed of the response is increased by increasing the bandwidth. Finally, steady-state error is improved by increasing the low-frequency magnitude responses, even if the high-frequency magnitude response is attenuated.

These, then, are the basic facts underlying our design for stability, transient response, and steady-state error using frequency response methods, where the Nyquist criterion and the Nyquist diagram compose the underlying theory behind the design process. Thus, even though we use the Bode plots for ease in obtaining the frequency response, the design process can be verified with the Nyquist diagram when questions arise about interpreting the Bode plots. In particular, when the structure of the system is changed with additional compensator poles and zeros, the Nyquist diagram can offer a valuable perspective.

The emphasis in this chapter is on the design of lag, lead, and lag-lead compensation. General design concepts are presented first, followed by step-by-step procedures. These procedures are only suggestions, and you are encouraged to develop other procedures to arrive at the same goals. Although the concepts in general apply to the design of PI, PD, and PID controllers, in the interest of brevity, detailed procedures and examples will not be presented. You are encouraged to extrapolate the concepts and designs covered and apply them to problems involving PI, PD, and PID compensation presented at the end of this chapter. Finally, the compensators developed in this chapter can be implemented with the realizations discussed in Section 9.6.

11.2 Transient Response via Gain Adjustment

Let us begin our discussion of design via frequency response methods by discussing the link between phase margin, transient response, and gain. In Section 10.10, the relationship between damping ratio (equivalently percent overshoot) and phase margin was derived for $G(s) = \omega_n^2/s(s + 2\zeta\omega_n)$. Thus, if we can vary the phase margin, we can vary the percent overshoot. Looking at Figure 11.1, we see that if we desire a phase margin, Φ_M, represented by CD, we would have to raise the magnitude curve by AB. Thus, a simple gain adjustment can be used to design phase margin and, hence, percent overshoot.

We now outline a procedure by which we can determine the gain to meet a percent overshoot requirement using the open-loop frequency response and assuming dominant second-order closed-loop poles.

Design Procedure

1. Draw the Bode magnitude and phase plots for a convenient value of gain.
2. Using Eqs. (4.39) and (10.73), determine the required phase margin from the percent overshoot.

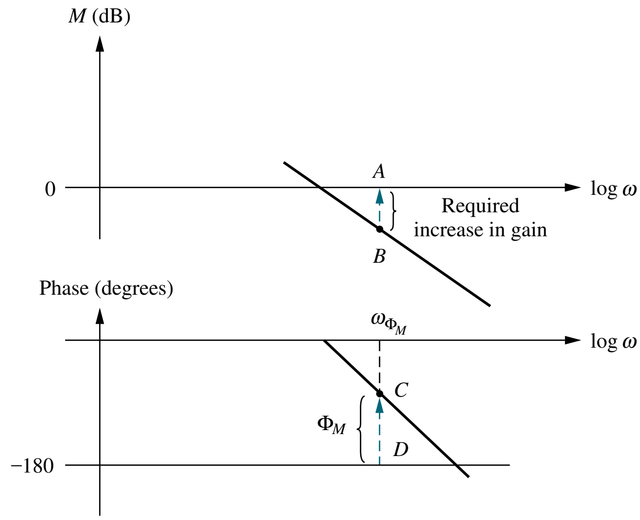


FIGURE 11.1 Bode plots showing gain adjustment for a desired phase margin

3. Find the frequency, ω_{Φ_M}, on the Bode phase diagram that yields the desired phase margin, CD, as shown on Figure 11.1.
4. Change the gain by an amount AB to force the magnitude curve to go through 0 dB at ω_{Φ_M}. The amount of gain adjustment is the additional gain needed to produce the required phase margin.

We now look at an example of designing the gain of a third-order system for percent overshoot.

Example 11.1

Transient Response Design via Gain Adjustment

Design
D

PROBLEM: For the position control system shown in Figure 11.2, find the value of preamplifier gain, K, to yield a 9.5% overshoot in the transient response for a step input. Use only frequency response methods.

SOLUTION: We will now follow the previously described gain adjustment design procedure.

1. Choose $K = 3.6$ to start the magnitude plot at 0 dB at $\omega = 0.1$ in Figure 11.3.
2. Using Eq. (4.39), a 9.5% overshoot implies $\zeta = 0.6$ for the closed-loop dominant poles. Equation (10.73) yields a 59.2° phase margin for a damping ratio of 0.6.

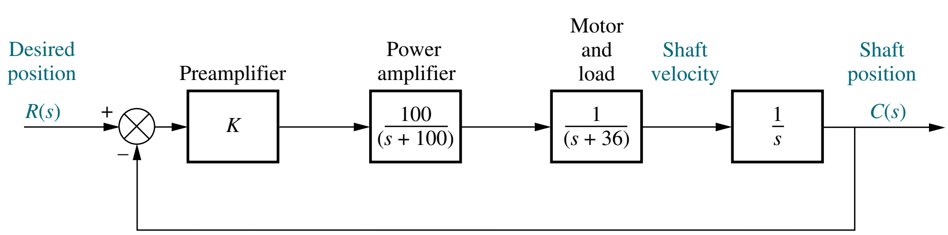


FIGURE 11.2 System for Example 11.1

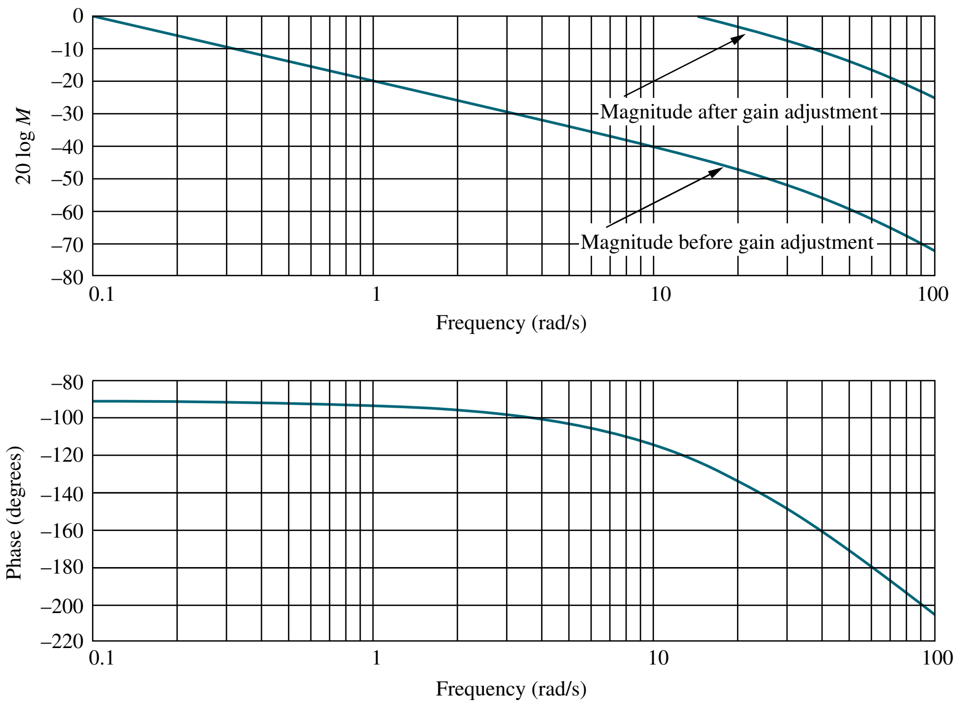


FIGURE 11.3 Bode magnitude and phase plots for Example 11.1

3. Locate on the phase plot the frequency that yields a 59.2° phase margin. This frequency is found where the phase angle is the difference between $-180°$ and $59.2°$, or $-120.8°$. The value of the phase-margin frequency is 14.8 rad/s.
4. At a frequency of 14.8 rad/s on the magnitude plot, the gain is found to be -44.2 dB. This magnitude has to be raised to 0 dB to yield the required phase margin. Since the log-magnitude plot was drawn for $K = 3.6$, a 44.2 dB increase, or $K = 3.6 \times 162.2 = 583.9$, would yield the required phase margin for 9.48% overshoot.

The gain-adjusted open-loop transfer function is

$$G(s) = \frac{58{,}390}{s(s+36)(s+100)} \tag{11.1}$$

Table 11.1 summarizes a computer simulation of the gain-compensated system.

TABLE 11.1 Characteristic of gain-compensated system of Example 11.1

Parameter	Proposed specification	Actual value
K_v	—	16.22
Phase margin	59.2°	59.2°
Phase-margin frequency	—	14.8 rad/s
Percent overshoot	9.5	10
Peak time	—	0.18 second

MATLAB **ML**

`Students who are using MATLAB should now run ch11p1 in Appendix B. You will learn how to use MATLAB to design a gain to meet a percent overshoot specification using Bode plots. This exercise solves Example 11.1 using MATLAB.`

Skill-Assessment Exercise 11.1

WileyPLUS
WPCS
Control Solutions

PROBLEM: For a unity feedback system with a forward transfer function

$$G(s) = \frac{K}{s(s+50)(s+120)}$$

use frequency response techniques to find the value of gain, K, to yield a closed-loop step response with 20% overshoot.

ANSWER: $K = 194{,}200$

The complete solution is located at www.wiley.com/college/nise.

TryIt 11.1

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 11.1.

```
pos=20
z=(-log(pos/100))/...
 (sqrt(pi^2+...
 log(pos/100)^2))
Pm=atan(2*z/...
 (sqrt(-2*z^2+...
 sqrt(1+4*z^4))))*...
 (180/pi)
G=zpk([],...
 [0.-50,-120],1)
sisotool
```

In the SISOTOOL Window:

1. Select **Import** . . . in the **File** menu.
2. Click on **G** in the **System Data Window** and click **Browse** . . .
3. In the **Model Import Window** select radio button Workspace and select **G** in **Available Models**. Click Import, then Close.
4. Click Ok in the **System Data Window.**
5. Right-click in the Bode graph area and be sure all selections under **Show** are checked.
6. Grab the stability margin point in the magnitude diagram and raise the magnitude curve until the phase curve shows the phase margin calculated by the program and shown in the MATLAB **Command Window** as Pm.
7. Right-click in the Bode plot area, select **Edit Compensator** . . . and read the gain under **Compensator** in the resulting window.

In this section, we paralleled our work in Chapter 8 with a discussion of transient response design through gain adjustment. In the next three sections, we parallel the root locus compensator design in Chapter 9 and discuss the design of lag, lead, and lag-lead compensation via Bode diagrams.

11.3 Lag Compensation

In Chapter 9, we used the root locus to design lag networks and PI controllers. Recall that these compensators permitted us to design for steady-state error without appreciably affecting the transient response. In this section, we provide a parallel development using the Bode diagrams.

Visualizing Lag Compensation

The function of the lag compensator as seen on Bode diagrams is to (1) improve the static error constant by increasing only the low-frequency gain without any resulting instability, and (2) increase the phase margin of the system to yield the desired transient response. These concepts are illustrated in Figure 11.4.

The uncompensated system is unstable since the gain at 180° is greater than 0 dB. The lag compensator, while not changing the low-frequency gain, does reduce

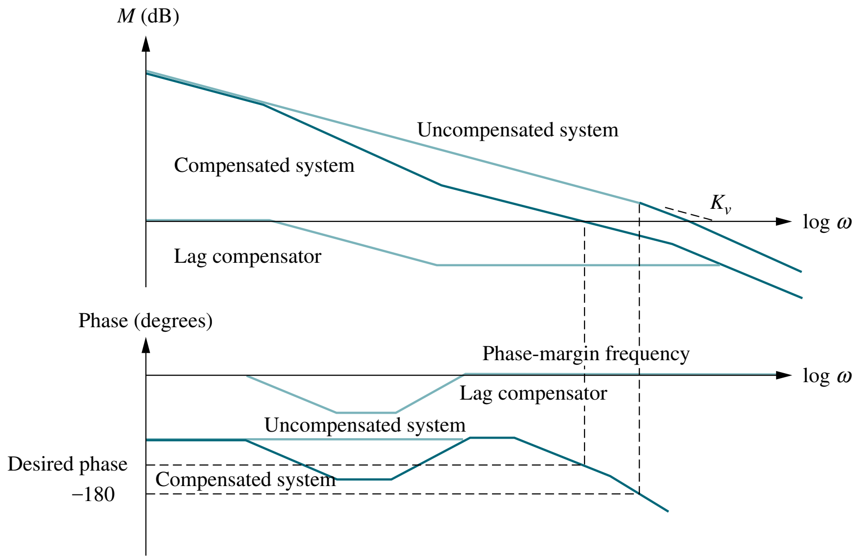


FIGURE 11.4 Visualizing lag compensation

the high-frequency gain.[1] Thus, the low-frequency gain of the system can be made high to yield a large K_v without creating instability. This stabilizing effect of the lag network comes about because the gain at 180° of phase is reduced below 0 dB. Through judicious design, the magnitude curve can be reshaped, as shown in Figure 11.4, to go through 0 dB at the desired phase margin. Thus, both K_v and the desired transient response can be obtained. We now enumerate a design procedure.

Design Procedure

1. Set the gain, K, to the value that satisfies the steady-state error specification and plot the Bode magnitude and phase diagrams for this value of gain.
2. Find the frequency where the phase margin is 5° to 12° greater than the phase margin that yields the desired transient response (*Ogata, 1990*). This step compensates for the fact that the phase of the lag compensator may still contribute anywhere from −5° to − 12° of phase at the phase-margin frequency.
3. Select a lag compensator whose magnitude response yields a composite Bode magnitude diagram that goes through 0 dB at the frequency found in Step 2 as follows: Draw the compensator's high-frequency asymptote to yield 0 dB for the compensated system at the frequency found in Step 2. Thus, if the gain at the frequency found in Step 2 is $20 \log K_{PM}$, then the compensator's high-frequency asymptote will be set at $-20 \log K_{PM}$; select the upper break frequency to be 1 decade below the frequency found in Step 2;[2] select the low-frequency asymptote to be at 0 dB; connect the compensator's high- and low-frequency asymptotes with a −20 dB/decade line to locate the lower break frequency.
4. Reset the system gain, K, to compensate for any attenuation in the lag network in order to keep the static error constant the same as that found in Step 1.

---

[1] The name *lag compensator* comes from the fact that the typical phase angle response for the compensator, as shown in Figure 11.4, is always negative, or *lagging* in phase angle.

[2] This value of break frequency ensures that there will be only −5° to − 12° phase contribution from the compensator at the frequency found in Step 2.

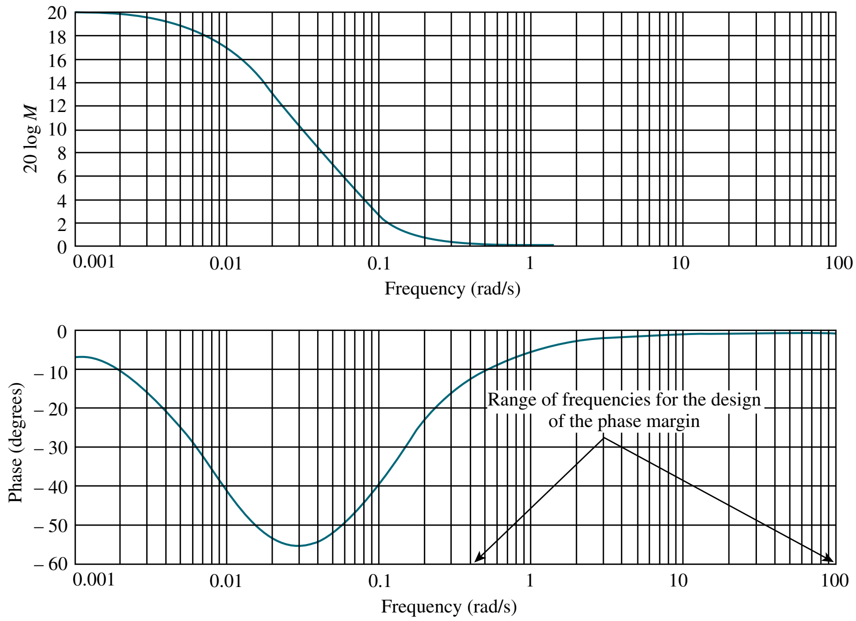


FIGURE 11.5 Frequency response plots of a lag compensator, $G_c(s) = (s + 0.1)/(s + 0.01)$

From these steps, you see that we are relying upon the initial gain setting to meet the steady-state requirements and then relying upon the lag compensator's -20 dB/decade slope to meet the transient response requirement by setting the 0 dB crossing of the magnitude plot.

The transfer function of the lag compensator is

$$\boxed{G_c(s) = \frac{s + \dfrac{1}{T}}{s + \dfrac{1}{\alpha T}}} \tag{11.2}$$

where $\alpha > 1$.

Figure 11.5 shows the frequency response curves for the lag compensator. The range of high frequencies shown in the phase plot is where we will design our phase margin. This region is after the second break frequency of the lag compensator, where we can rely on the attenuation characteristics of the lag network to reduce the total open-loop gain to unity at the phase-margin frequency. Further, in this region the phase response of the compensator will have minimal effect on our design of the phase margin. Since there is still some effect, approximately 5° to 12°, we will add this amount to our phase margin to compensate for the phase response of the lag compensator (see Step 2).

Example 11.2

Lag Compensation Design

Design
D

PROBLEM: Given the system of Figure 11.2, use Bode diagrams to design a lag compensator to yield a tenfold improvement in steady-state error over the gain-compensated system while keeping the percent overshoot at 9.5%.

SOLUTION: We will follow the previously described lag compensation design procedure.

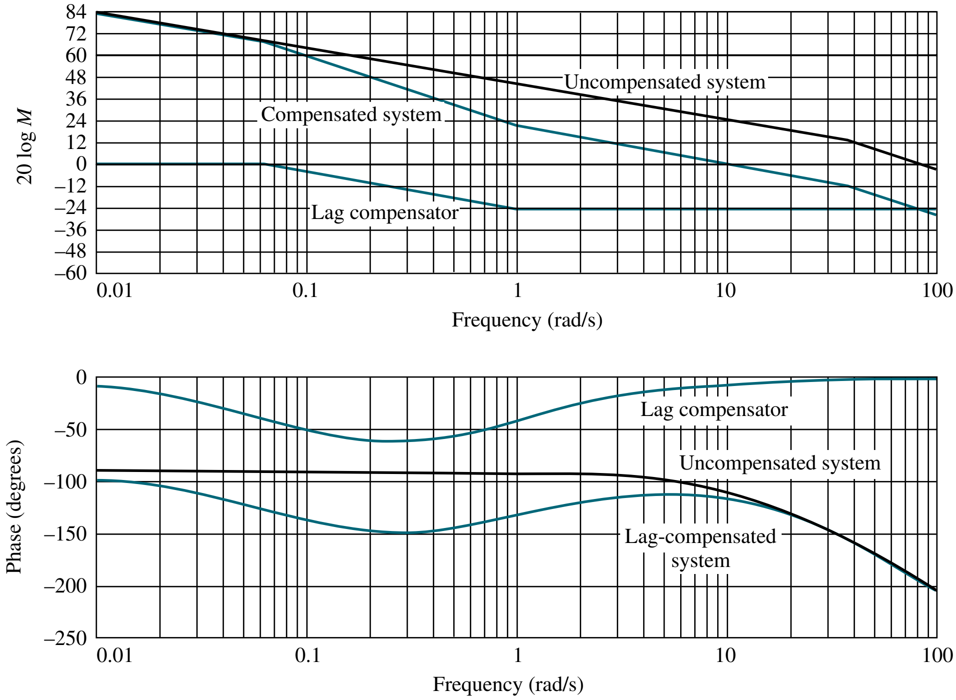


FIGURE 11.6 Bode plots for Example 11.2.

1. From Example 11.1 a gain, K, of 583.9 yields a 9.5% overshoot. Thus, for this system, $K_v = 16.22$. For a tenfold improvement in steady-state error, K_v must increase by a factor of 10, or $K_v = 162.2$. Therefore, the value of K in Figure 11.2 equals 5839, and the open-loop transfer function is

$$G(s) = \frac{583{,}900}{s(s+36)(s+100)} \tag{11.3}$$

The Bode plots for $K = 5839$ are shown in Figure 11.6.

2. The phase margin required for a 9.5% overshoot ($\zeta = 0.6$) is found from Eq. (10.73) to be 59.2°. We increase this value of phase margin by 10° to 69.2° in order to compensate for the phase angle contribution of the lag compensator. Now find the frequency where the phase margin is 69.2°. This frequency occurs at a phase angle of $-180° + 69.2° = -110.8°$ and is 9.8 rad/s. At this frequency, the magnitude plot must go through 0 dB. The magnitude at 9.8 rad/s is now +24 dB (exact, that is, nonasymptotic). Thus, the lag compensator must provide −24 dB attenuation at 9.8 rad/s.

3.&4. We now design the compensator. First draw the high-frequency asymptote at −24 dB. Arbitrarily select the higher break frequency to be about one decade below the phase-margin frequency, or 0.98 rad/s. Starting at the intersection of this frequency with the lag compensator's high-frequency asymptote, draw a −20 dB/decade line until 0 dB is reached. The compensator must have a dc gain of unity to retain the value of K_v that we have already designed by setting $K = 5839$. The lower break frequency is found to be 0.062 rad/s. Hence, the lag compensator's transfer function is

$$G_c(s) = \frac{0.063(s+0.98)}{(s+0.062)} \tag{11.4}$$

where the gain of the compensator is 0.063 to yield a dc gain of unity.

The compensated system's forward transfer function is thus

$$G(s)G_c(s) = \frac{36{,}786(s+0.98)}{s(s+36)(s+100)(s+0.062)} \tag{11.5}$$

The characteristics of the compensated system, found from a simulation and exact frequency response plots, are summarized in Table 11.2.

TABLE 11.2 Characteristics of the lag-compensated system of Example 11.2

Parameter	Proposed specification	Actual value
K_v	162.2	161.5
Phase margin	59.2°	62°
Phase-margin frequency	—	11 rad/s
Percent overshoot	9.5	10
Peak time	—	0.25 second

MATLAB

ML

`Students who are using MATLAB should now run ch11p2 in Appendix B. You will learn how to use MATLAB to design a lag compensator. You will enter the value of gain to meet the steady-state error requirement as well as the desired percent overshoot. MATLAB then designs a lag compensator using Bode plots, evaluates` $K_v$`, and generates a closed-loop step response. This exercise solves Example 11.2 using MATLAB.`

Skill-Assessment Exercise 11.2

PROBLEM: Design a lag compensator for the system in Skill-Assessment Exercise 11.1 that will improve the steady-state error tenfold, while still operating with 20% overshoot.

ANSWER:

$$G_{\text{lag}}(s) = \frac{0.0691(s+2.04)}{(s+0.141)}; \quad G(s) = \frac{1{,}942{,}000}{s(s+50)(s+120)}$$

The complete solution is at www.wiley.com/college/nise.

TryIt 11.2

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 11.2.

```
pos=20
Ts=0.2
z=(-log(pos/100))/(sqrt(pi^2+log(pos/100)^2))
Pm=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi)
Wbw=(4/(Ts*z))*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2))
K=1942000
G=zpk([],[0,-50,-120],K)
sisotool(G,1)
```

(*TryIt continues*)

(*TryIt Continued*)
When the **SISO Design for SISO Design Task Window** appears:

1. Right-click on the Bode plot area and select **Grid.**
2. Note the phase margin shown in the MATLAB **Command Window.**
3. Using the Bode phase plot, estimate the frequency at which the phase margin from Step 2 occurs.
4. On the **SISO Design for SISO Design Task Window toolbar**, click on the red zero.
5. Place the zero of the compensator by clicking on the gain plot at a frequency that is 1/10 that found in Step 3.
6. On the **SISO Design for SISO Design Task Window toolbar**, click on the red pole.
7. Place the pole of the compensator by clicking on the gain plot to the left of the compensator zero.
8. Grab the pole with the mouse and move it until the phase plot shows a P.M. equal to that found in Step 2.
9. Right-click in the Bode plot area and select **Edit Compensator . . .**
10. Read the lag compensator in the **Control and Estimation Tools Manager Window.**

In this section, we showed how to design a lag compensator to improve the steady-state error while keeping the transient response relatively unaffected. We next discuss how to improve the transient response using frequency response methods.

11.4 Lead Compensation

For second-order systems, we derived the relationship between phase margin and percent overshoot as well as the relationship between closed-loop bandwidth and other time-domain specifications, such as settling time, peak time, and rise time. When we designed the lag network to improve the steady-state error, we wanted a minimal effect on the phase diagram in order to yield an imperceptible change in the transient response. However, in designing lead compensators via Bode plots, we want to change the phase diagram, increasing the phase margin to reduce the percent overshoot, and increasing the gain crossover to realize a faster transient response.

Visualizing Lead Compensation

The lead compensator increases the bandwidth by increasing the gain crossover frequency. At the same time, the phase diagram is raised at higher frequencies. The result is a larger phase margin and a higher phase-margin frequency. In the time domain, lower percent overshoots (larger phase margins) with smaller peak times (higher phase-margin frequencies) are the results. The concepts are shown in Figure 11.7.

The uncompensated system has a small phase margin (B) and a low phase-margin frequency (A). Using a phase lead compensator, the phase angle plot (compensated system) is raised for higher frequencies.[3] At the same time, the gain crossover frequency in the magnitude plot is increased from A rad/s to C rad/s. These effects yield a larger phase margin (D), a higher phase-margin frequency (C), and a larger bandwidth.

One advantage of the frequency response technique over the root locus is that we can implement a steady-state error requirement and then design a transient response. This specification of transient response with the constraint of a steady-state error is easier to implement with the frequency response technique than with the root locus. Notice that the initial slope, which determines the steady-state error, is not affected by the design for the transient response.

---

[3] The name *lead compensator* comes from the fact that the typical phase angle response shown in Figure 11.7 is always positive, or *leading* in phase angle.

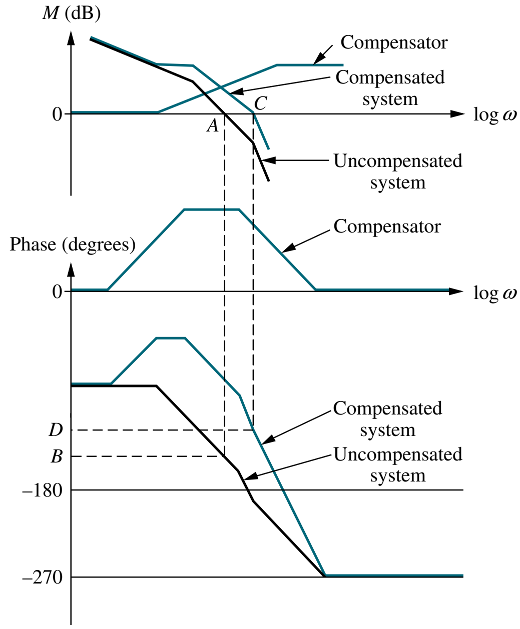


FIGURE 11.7 Visualizing lead compensation

Lead Compensator Frequency Response

Let us first look at the frequency response characteristics of a lead network and derive some valuable relationships that will help us in the design process. Figure 11.8 shows plots of the lead network

$$G_c(s) = \frac{1}{\beta}\frac{s+\dfrac{1}{T}}{s+\dfrac{1}{\beta T}} \tag{11.6}$$

for various values of β, where $\beta < 1$. Notice that the peaks of the phase curve vary in maximum angle and in the frequency at which the maximum occurs. The dc gain of the compensator is set to unity with the coefficient $1/\beta$, in order not to change the dc gain designed for the static error constant when the compensator is inserted into the system.

In order to design a lead compensator and change both the phase margin and phase-margin frequency, it is helpful to have an analytical expression for the maximum value of phase and the frequency at which the maximum value of phase occurs, as shown in Figure 11.8.

From Eq. (11.6) the phase angle of the lead compensator, ϕ_c, is

$$\phi_c = \tan^{-1}\omega T - \tan^{-1}\omega\beta T \tag{11.7}$$

Differentiating with respect to ω, we obtain

$$\frac{d\phi_c}{d\omega} = \frac{T}{1+(\omega T)^2} - \frac{\beta T}{1+(\omega\beta T)^2} \tag{11.8}$$

Setting Eq. (11.8) equal to zero, we find that the frequency, ω_{max}, at which the maximum phase angle, ϕ_{max}, occurs is

$$\omega_{max} = \frac{1}{T\sqrt{\beta}} \tag{11.9}$$

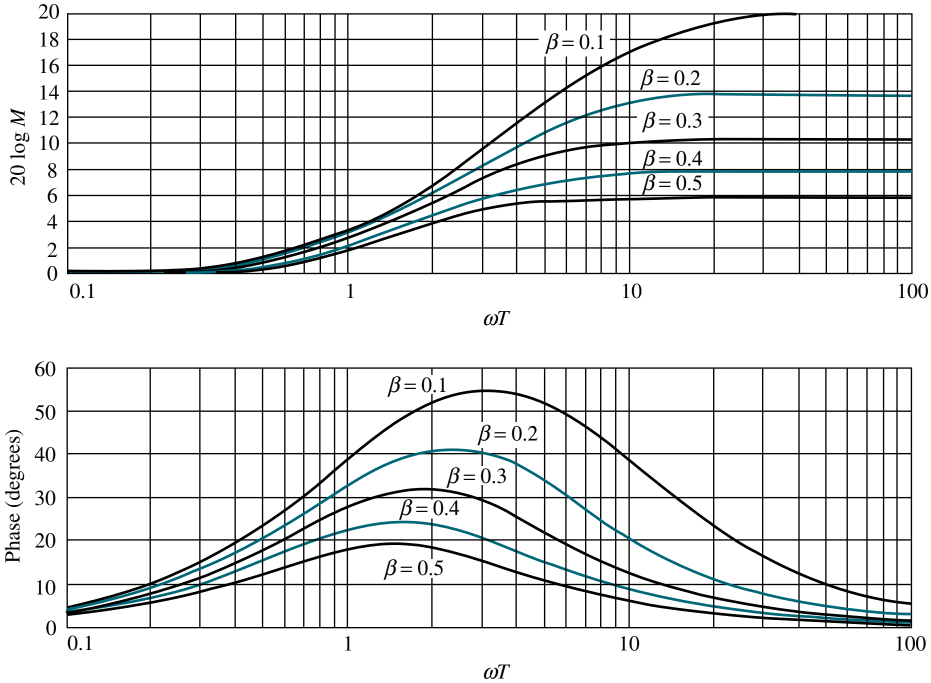


FIGURE 11.8 Frequency response of a lead compensator, $G_c(s) = [1/\beta][(s + 1/T)/(s + 1/\beta T)]$

Substituting Eq. (11.9) into Eq. (11.6) with $s = j\omega_{\max}$,

$$G_c(j\omega_{\max}) = \frac{1}{\beta}\frac{j\omega_{\max} + \dfrac{1}{T}}{j\omega_{\max} + \dfrac{1}{\beta T}} = \frac{j\dfrac{1}{\sqrt{\beta}} + 1}{j\sqrt{\beta} + 1} \tag{11.10}$$

Making use of $\tan(\phi_1 - \phi_2) = (\tan\phi_1 - \tan\phi_2)/(1 + \tan\phi_1 \tan\phi_2)$, the maximum phase shift of the compensator, $\phi_{\max}$, is

$$\boxed{\phi_{\max} = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}} = \sin^{-1}\frac{1-\beta}{1+\beta}} \tag{11.11}$$

and the compensator's magnitude at $\omega_{\max}$ is

$$\boxed{|G_c(j\omega_{\max})| = \frac{1}{\sqrt{\beta}}} \tag{11.12}$$

We are now ready to enumerate a design procedure.

Design Procedure

1. Find the closed-loop bandwidth required to meet the settling time, peak time, or rise time requirement (see Eqs. (10.54) through (10.56)).
2. Since the lead compensator has negligible effect at low frequencies, set the gain, K, of the uncompensated system to the value that satisfies the steady-state error requirement.

3. Plot the Bode magnitude and phase diagrams for this value of gain and determine the uncompensated system's phase margin.
4. Find the phase margin to meet the damping ratio or percent overshoot requirement. Then evaluate the additional phase contribution required from the compensator.[4]
5. Determine the value of β (see Eqs. (11.6) and (11.11)) from the lead compensator's required phase contribution.
6. Determine the compensator's magnitude at the peak of the phase curve (Eq. (11.12)).
7. Determine the new phase-margin frequency by finding where the uncompensated system's magnitude curve is the negative of the lead compensator's magnitude at the peak of the compensator's phase curve.
8. Design the lead compensator's break frequencies, using Eqs. (11.6) and (11.9) to find T and the break frequencies.
9. Reset the system gain to compensate for the lead compensator's gain.
10. Check the bandwidth to be sure the speed requirement in Step 1 has been met.
11. Simulate to be sure all requirements are met.
12. Redesign if necessary to meet requirements.

From these steps, we see that we are increasing both the amount of phase margin (improving percent overshoot) and the gain crossover frequency (increasing the speed). Now that we have enumerated a procedure with which we can design a lead compensator to improve the transient response, let us demonstrate.

Example 11.3

Design

D

Lead Compensation Design

PROBLEM: Given the system of Figure 11.2, design a lead compensator to yield a 20% overshoot and $K_v = 40$, with a peak time of 0.1 second.

SOLUTION: The uncompensated system is $G(s) = 100K/[s(s+36)(s+100)]$. We will follow the outlined procedure.

1. We first look at the closed-loop bandwidth needed to meet the speed requirement imposed by $T_p = 0.1$ second. From Eq. (10.56), with $T_p = 0.1$ second and $\zeta = 0.456$ (i.e., 20% overshoot), a closed-loop bandwidth of 46.6 rad/s is required.
2. In order to meet the specification of $K_v = 40$, K must be set at 1440, yielding $G(s) = 144{,}000/[s(s+36)(s+100)]$.
3. The uncompensated system's frequency response plots for $K = 1440$ are shown in Figure 11.9.
4. A 20% overshoot implies a phase margin of 48.1°. The uncompensated system with $K = 1440$ has a phase margin of 34° at a phase-margin frequency

[4] We know that the phase-margin frequency will be increased after the insertion of the compensator. At this new phase-margin frequency, the system's phase will be smaller than originally estimated, as seen by comparing points B and D in Figure 11.7. Hence, an additional phase should be added to that provided by the lead compensator to correct for the phase reduction caused by the original system.

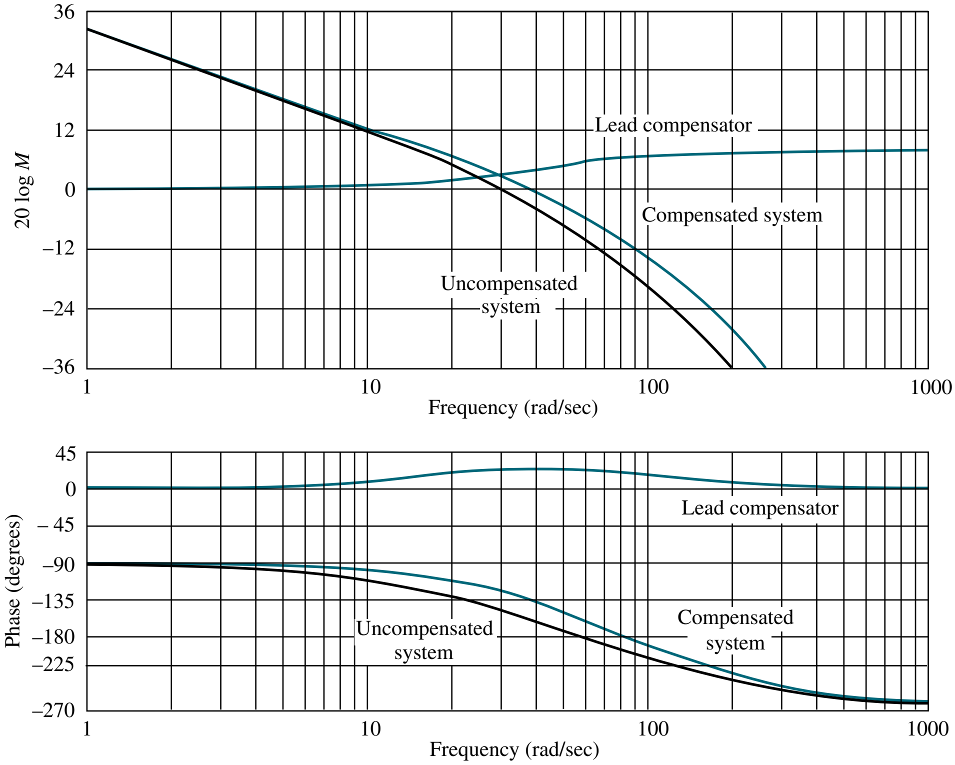


FIGURE 11.9 Bode plots for lead compensation in Example 11.3

of 29.6. To increase the phase margin, we insert a lead network that adds enough phase to yield a 48.1° phase margin. Since we know that the lead network will also increase the phase-margin frequency, we add a correction factor to compensate for the lower uncompensated system's phase angle at this higher phase-margin frequency. Since we do not know the higher phase-margin frequency, we assume a correction factor of 10°. Thus, the total phase contribution required from the compensator is $48.1° - 34° + 10° = 24.1°$. In summary, our compensated system should have a phase margin of 48.1° with a bandwidth of 46.6 rad/s. If the system's characteristics are not acceptable after the design, then a redesign with a different correction factor may be necessary.

5. Using Eq. (11.11), $\beta = 0.42$ for $\phi_{max} = 24.1°$.
6. From Eq. (11.12), the lead compensator's magnitude is 3.76 dB at ω_{max}.
7. If we select ω_{max} to be the new phase-margin frequency, the uncompensated system's magnitude at this frequency must be -3.76 dB to yield a 0 dB crossover at ω_{max} for the compensated system. The uncompensated system passes through -3.76 dB at $\omega_{max} = 39$ rad/s. This frequency is thus the new phase-margin frequency.
8. We now find the lead compensator's break frequencies. From Eq. (11.9), $1/T = 25.3$ and $1/\beta T = 60.2$.
9. Hence, the compensator is given by

$$G_c(s) = \frac{1}{\beta}\frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} = 2.38\frac{s + 25.3}{s + 60.2} \tag{11.13}$$

where 2.38 is the gain required to keep the dc gain of the compensator at unity so that $K_v = 40$ after the compensator is inserted.

The final, compensated open-loop transfer function is then

$$G_c(s)G(s) = \frac{342{,}600(s+25.3)}{s(s+36)(s+100)(s+60.2)} \tag{11.14}$$

10. From Figure 11.9, the lead-compensated open-loop magnitude response is -7 dB at approximately 68.8 rad/s. Thus, we estimate the closed-loop bandwidth to be 68.8 rad/s. Since this bandwidth exceeds the requirement of 46.6 rad/s, we assume the peak time specification is met. This conclusion about the peak time is based upon a second-order and asymptotic approximation that will be checked via simulation.

11. Figure 11.9 summarizes the design and shows the effect of the compensation. Final results, obtained from a simulation and the actual (nonasymptotic) frequency response, are shown in Table 11.3. Notice the increase in phase margin, phase-margin frequency, and closed-loop bandwidth after the lead compensator was added to the gain-adjusted system. The peak time and the steady-state error requirements have been met, although the phase margin is less than that proposed and the percent overshoot is 2.6% larger than proposed. Finally, if the performance is not acceptable, a redesign is necessary.

TABLE 11.3 Characteristic of the lead-compensated system of Example 11.3

Parameter	Proposed specification	Actual gain-compensated value	Actual lead-compensated value
K_v	40	40	40
Phase margin	48.1°	34°	45.5°
Phase-margin frequency	—	29.6 rad/s	39 rad/s
Closed-loop bandwidth	46.6 rad/s	50 rad/s	68.8 rad/s
Percent overshoot	20	37	22.6
Peak time	0.1 second	0.1 second	0.075 second

MATLAB

ML

Students who are using MATLAB should now run ch11p3 in Appendix B. You will learn how to use MATLAB to design a lead compensator. You will enter the desired percent overshoot, peak time, and K_v. MATLAB then designs a lead compensator using Bode plots, evaluates K_v, and generates a closed-loop step response. This exercise solves Example 11.3 using MATLAB.

Skill-Assessment Exercise 11.3

WileyPLUS

WPCS

Control Solutions

PROBLEM: Design a lead compensator for the system in Skill-Assessment Exercise 11.1 to meet the following specifications: $\%OS = 20\%$, $T_s = 0.2$ s and $K_v = 50$.

ANSWER: $G_{\text{lead}}(s) = \dfrac{2.27(s+33.2)}{(s+75.4)}$; $G(s) = \dfrac{300{,}000}{s(s+50)(s+120)}$

The complete solution is at www.wiley.com/college/nise.

TryIt 11.3

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 11.3.

```
pos=20
Ts=0.2
z=(-log(pos/100))/(sqrt(pi^2+log(pos/100)^2))
Pm=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi)
Wbw=(4/(Ts*z))*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2))
K=50*50*120
G=zpk([],[0,-50,-120],K)
sisotool(G,1)
```

When the **SISO Design for SISO Design Task Window** appears:

1. Right-click on the Bode plot area and select **Grid.**
2. Note the phase margin and bandwidth shown in the MATLAB **Command Window.**
3. On the **SISO Design for SISO Design Task Window toolbar,** click on the red pole.
4. Place the pole of the compensator by clicking on the gain plot at a frequency that is to the right of the desired bandwidth found in Step 2.
5. On the **SISO Design for SISO Design Task Window toolbar,** click on the red zero.
6. Place the zero of the compensator by clicking on the gain plot to the left of the desired bandwidth.
7. Reshape the Bode plots: alternately grab the pole and the zero with the mouse and alternately move them along the phase plot until the phase plot show a P.M. equal to that found in Step 2 and a phase-margin frequency close to the bandwidth found in Step 2.
8. Right-click in the Bode plot area and select **Edit Compensator** . . .
9. Read the lead compensator in the **Control and Estimation Tools Manager Window.**

Keep in mind that the previous examples were designs for third-order systems and must be simulated to ensure the desired transient results. In the next section, we look at lag-lead compensation to improve steady-state error and transient response.

11.5 Lag-Lead Compensation

In Section 9.4, using root locus, we designed lag-lead compensation to improve the transient response and steady-state error. Figure 11.10 is an example of a system to which lag-lead compensation can be applied. In this section we repeat the design, using frequency response techniques. One method is to design the lag compensation to lower the high-frequency gain, stabilize the system, and improve the steady-state error and then design a lead compensator to meet the phase-margin requirements. Let us look at another method.

Section 9.6 describes a passive lag-lead network that can be used in place of separate lag and lead networks. It may be more economical to use a single, passive network that performs both tasks, since the buffer amplifier that separates the lag network from the lead network may be eliminated. In this section, we emphasize lag-lead design, using a single, passive lag-lead network.

The transfer function of a single, passive lag-lead network is

$$G_c(s) = G_{\text{Lead}}(s)G_{\text{Lag}}(s) = \left(\frac{s+\dfrac{1}{T_1}}{s+\dfrac{\gamma}{T_1}}\right)\left(\frac{s+\dfrac{1}{T_2}}{s+\dfrac{1}{\gamma T_2}}\right) \tag{11.15}$$

(*a*) (*b*)

FIGURE 11.10 **a.** The National Advanced Driving Simulator at the University of Iowa; **b.** test driving the simulator with its realistic graphics (Katharina Bosse/laif/Redux Pictures.)

where $\gamma > 1$. The first term in parentheses produces the lead compensation, and the second term in parentheses produces the lag compensation. The constraint that we must follow here is that the single value γ replaces the quantity α for the lag network in Eq. (11.2) and the quantity β for the lead network in Eq. (11.6). For our design, α and β must be reciprocals of each other. An example of the frequency response of the passive lag-lead is shown in Figure 11.11.

We are now ready to enumerate a design procedure.

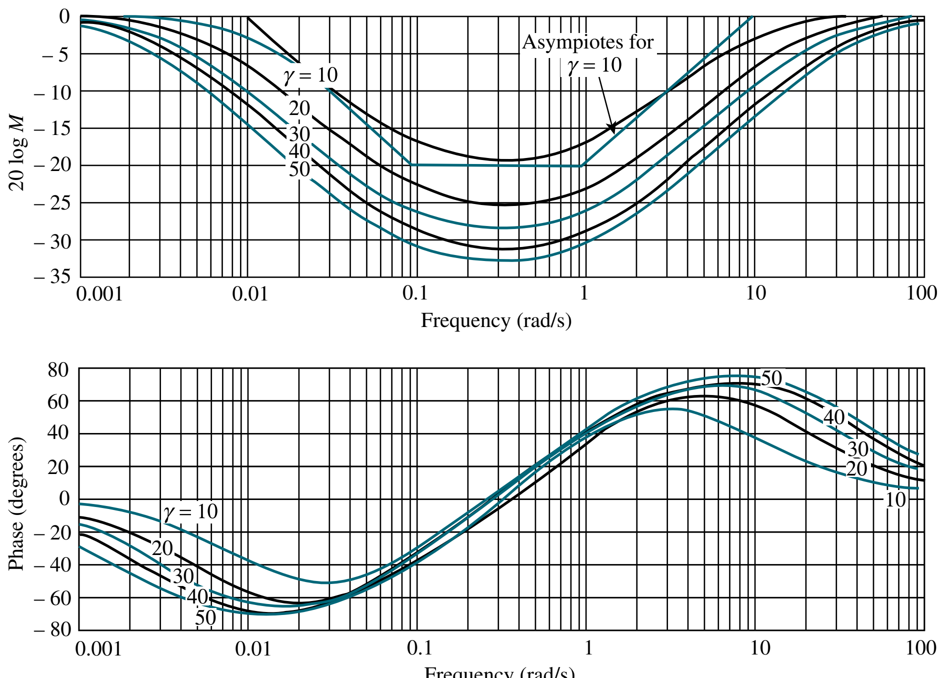


FIGURE 11.11 Sample frequency response curves for a lag-lead compensator, $G_c(s) = [(s+1)(s+0.1)]/\left[(s+\gamma)\left(s+\frac{0.1}{\gamma}\right)\right]$

Design Procedure

1. Using a second-order approximation, find the closed-loop bandwidth required to meet the settling time, peak time, or rise time requirement (see Eqs. (10.55) and (10.56)).
2. Set the gain, K, to the value required by the steady-state error specification.
3. Plot the Bode magnitude and phase diagrams for this value of gain.
4. Using a second-order approximation, calculate the phase margin to meet the damping ratio or percent overshoot requirement, using Eq. (10.73).
5. Select a new phase-margin frequency near ω_{BW}.
6. At the new phase-margin frequency, determine the additional amount of phase lead required to meet the phase-margin requirement. Add a small contribution that will be required after the addition of the lag compensator.
7. Design the lag compensator by selecting the higher break frequency one decade below the new phase-margin frequency. The design of the lag compensator is not critical, and any design for the proper phase margin will be relegated to the lead compensator. The lag compensator simply provides stabilization of the system with the gain required for the steady-state error specification. Find the value of γ from the lead compensator's requirements. Using the phase required from the lead compensator, the phase response curve of Figure 11.8 can be used to find the value of $\gamma = 1/\beta$. This value, along with the previously found lag's upper break frequency, allows us to find the lag's lower break frequency.
8. Design the lead compensator. Using the value of γ from the lag compensator design and the value assumed for the new phase-margin frequency, find the lower and upper break frequency for the lead compensator, using Eq. (11.9) and solving for T.
9. Check the bandwidth to be sure the speed requirement in Step 1 has been met.
10. Redesign if phase-margin or transient specifications are not met, as shown by analysis or simulation.

Let us demonstrate the procedure with an example.

Example 11.4

Lag-Lead Compensation Design

PROBLEM: Given a unity feedback system where $G(s) = K/[s(s+1)(s+4)]$, design a passive lag-lead compensator using Bode diagrams to yield a 13.25% overshoot, a peak time of 2 seconds, and $K_v = 12$.

SOLUTION: We will follow the steps previously mentioned in this section for lag-lead design.

1. The bandwidth required for a 2-seconds peak time is 2.29 rad/s.
2. In order to meet the steady-state error requirement, $K_v = 12$, the value of K is 48.
3. The Bode plots for the uncompensated system with $K = 48$ are shown in Figure 11.12. We can see that the system is unstable.
4. The required phase margin to yield a 13.25% overshoot is 55°.

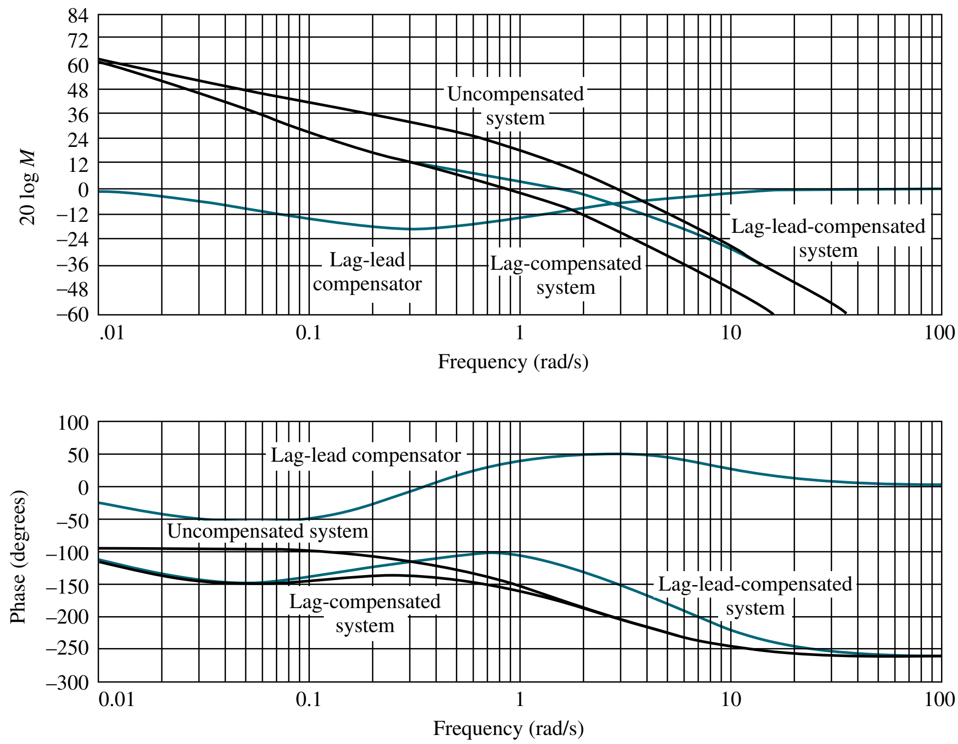


FIGURE 11.12 Bode plots for lag-lead compensation in Example 11.4

5. Let us select $\omega = 1.8$ rad/s as the new phase-margin frequency.
6. At this frequency, the uncompensated phase is -176° and would require, if we add a -5° contribution from the lag compensator, a 56° contribution from the lead portion of the compensator.
7. The design of the lag compensator is next. The lag compensator allows us to keep the gain of 48 required for $K_v = 12$ and not have to lower the gain to stabilize the system. As long as the lag compensator stabilizes the system, the design parameters are not critical since the phase margin will be designed with the lead compensator. Thus, choose the lag compensator so that its phase response will have minimal effect at the new phase-margin frequency. Let us choose the lag compensator's higher break frequency to be 1 decade below the new phase-margin frequency, at 0.18 rad/s. Since we need to add 56° of phase shift with the lead compensator at $\omega = 1.8$ rad/s, we estimate from Figure 11.8 that, if $\gamma = 10.6$ (since $\gamma = 1/\beta$, $\beta = 0.094$), we can obtain about 56° of phase shift from the lead compensator. Thus with $\gamma = 10.6$ and a new phase-margin frequency of $\omega = 1.8$ rad/s, the transfer function of the lag compensator is

$$G_{\text{lag}}(s) = \frac{1}{\gamma}\frac{\left(s + \dfrac{1}{T_2}\right)}{\left(s + \dfrac{1}{\gamma T_2}\right)} = \frac{1}{10.6}\,\frac{(s + 0.183)}{(s + 0.0172)} \tag{11.16}$$

where the gain term, $1/\gamma$, keeps the dc gain of the lag compensator at 0 dB. The lag-compensated system's open-loop transfer function is

$$G_{\text{lag-comp}}(s) = \frac{4.53(s+0.183)}{s(s+1)(s+4)(s+0.0172)} \tag{11.17}$$

8. Now we design the lead compensator. At $\omega = 1.8$, the lag-compensated system has a phase angle of 180°. Using the values of $\omega_{\max} = 1.8$ and $\beta = 0.094$, Eq. (11.9) yields the lower break, $1/T_1 = 0.56$ rad/s. The higher break is then $1/\beta T_1 = 5.96$ rad/s. The lead compensator is

$$G_{\text{lead}}(s) = \gamma \frac{\left(s + \dfrac{1}{T_1}\right)}{\left(s + \dfrac{\gamma}{T_1}\right)} = 10.6\frac{(s+0.56)}{(s+5.96)} \tag{11.18}$$

The lag-lead-compensated system's open-loop transfer function is

$$G_{\text{lag-lead-comp}}(s) = \frac{48(s+0.183)(s+0.56)}{s(s+1)(s+4)(s+0.0172)(s+5.96)} \tag{11.19}$$

9. Now check the bandwidth. The closed-loop bandwidth is equal to that frequency where the open-loop magnitude response is approximately −7 dB. From Figure 11.12, the magnitude is −7 dB at approximately 3 rad/s. This bandwidth exceeds that required to meet the peak time requirement.

The design is now checked with a simulation to obtain actual performance values. Table 11.4 summarizes the system's characteristics. The peak time requirement is also met. Again, if the requirements were not met, a redesign would be necessary.

TABLE 11.4 Characteristics of gain-compensated system of Example 11.4

Parameter	Proposed specification	Actual value
K_v	12	12
Phase margin	55°	59.3°
Phase-margin frequency	—	1.63 rad/s
Closed-loop bandwidth	2.29 rad/s	3 rad/s
Percent overshoot	13.25	10.2
Peak time	2.0 seconds	1.61 seconds

MATLAB

ML

Students who are using MATLAB should now run ch11p4 in Appendix B. You will learn how to use MATLAB to design a lag-lead compensator. You will enter the desired percent overshoot, peak time, and K_v. MATLAB then designs a lag-lead compensator using Bode plots, evaluates K_v, and generates a closed-loop step response. This exercise solves Example 11.4 using MATLAB.

For a final example, we include the design of a lag-lead compensator using a Nichols chart. Recall from Chapter 10 that the Nichols chart contains a presentation of both the open-loop frequency response and the closed-loop frequency response. The axes of the Nichols chart are the open-loop magnitude and phase (y and x axis, respectively). The open-loop frequency response is plotted using the coordinates of the Nichols chart at each frequency. The open-loop plot is overlaying a grid that yields the closed-loop magnitude and phase. Thus, we have a presentation of both the open- and closed-loop responses. Thus, a design can be implemented that reshapes the Nichols plot to meet both open- and closed-loop frequency specifications.

From a Nichols chart, we can see simultaneously the following frequency response specifications that are used to design a desired time response: (1) phase margin, (2) gain margin, (3) closed-loop bandwidth, and (4) closed-loop peak amplitude.

In the following example, we first specify the following: (1) maximum allowable percent overshoot, (2) maximum allowable peak time, and (3) minimum allowable static error constant. We first design the lead compensator to meet the transient requirements followed by the lag compensator design to meet the steady-state error requirement. Although calculations could be made by hand, we will use MATLAB and SISOTOOL to make and shape the Nichols plot.

Let us first outline the steps that we will take in the example:

1. Calculate the damping ratio from the percent overshoot requirement using Eq. (4.39)
2. Calculate the peak amplitude, M_p, of the closed-loop response using Eq. (10.52) and the damping ratio found in (1).
3. Calculate the minimum closed-loop bandwidth to meet the peak time requirement using Eq. (10.56), with peak time and the damping ratio from (1).
4. Plot the open-loop response on the Nichols chart.
5. Raise the open-loop gain until the open-loop plot is tangent to the required closed-loop magnitude curve, yielding the proper M_p.
6. Place the lead zero at this point of tangency and the lead pole at a higher frequency. Zeros and poles are added in SISOTOOL by clicking either one on the tool bar and then clicking the position on the open-loop frequency response curve where you desire to add the zero or pole.
7. Adjust the positions of the lead zero and pole until the open-loop frequency response plot is tangent to the same M_p curve, but at the approximate frequency found in (3). This yields the proper closed-loop peak and proper bandwidth to yield the desired percent overshoot and peak time, respectively.
8. Evaluate the open-loop transfer function, which is the product of the plant and the lead compensator, and determine the static error constant.
9. If the static error constant is lower than required, a lag compensator must now be designed. Determine how much improvement in the static error constant is required.
10. Recalling that the lag pole is at a frequency below that of the lag zero, place a lag pole and zero at frequencies below the lead compensator and adjust to yield the desired improvement in static error constant. As an example, recall from Eq. (9.5) that the improvement in static error constant for a Type 1 system is equal to the ratio of the lag zero value divided by the lag pole value. Readjust the gain if necessary.

Example 11.5

MATLAB
ML

Gui Tool
GUIT

Lag-Lead Design Using the Nichols Chart, MATLAB, and SISOTOOL

PROBLEM: Design a lag-lead compensator for the plant, $G(s) = \dfrac{K}{s(s+5)(s+10)}$, to meet the following requirements: (1) a maximum of 20% overshoot, (2) a peak time of no more than 0.5 seconds, (3) a static error constant of no less than 6.

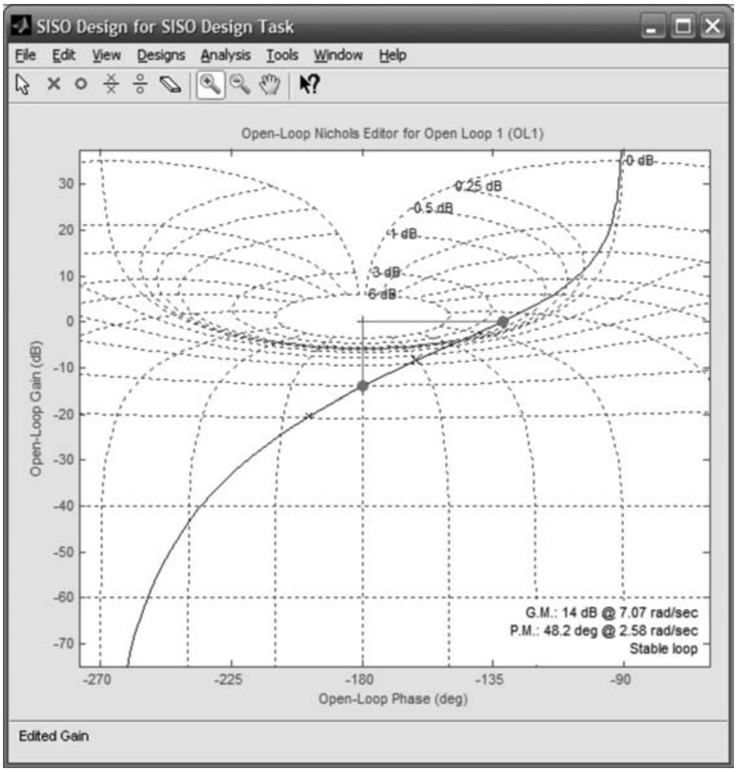


FIGURE 11.13 Nichols chart after gain adjustment

SOLUTION: We follow the steps enumerated immediately above,

1. Using Eq. (4.39), $\zeta = 0.456$ for 20% overshoot.
2. Using Eq. (10.52), $M_p = 1.23 = 1.81$ dB for $\zeta = 0.456$.
3. Using Eq. (10.56), $\omega_{BW} = 9.3$ r/s for $\zeta = 0.456$ and $T_p = 0.5$.
4. Plot the open-loop frequency response curve on the Nichols chart for $K = 1$.
5. Raise the open-loop frequency response curve until it is tangent to the closed-loop peak of 1.81 dB curve as shown in Figure 11.13. The frequency at the tangent point is approximately 3 r/s, which can be found by letting your mouse rest on the point of tangency. On the menu bar, select **Designs/Edit Compensator** . . . and find the gain added to the plant. Thus, the plant is now $G(s) = \dfrac{150}{s(s+5)(s+10)}$. The gain-adjusted closed-loop step response is shown in Figure 11.14. Notice that the peak time is about 1 second and must be decreased.
6. Place the lead zero at this point of tangency and the lead pole at a higher frequency.
7. Adjust the positions of the lead zero and pole until the open-loop frequency response plot is tangent to the same M_p curve, but at the approximate frequency found in **3**.
8. Checking **Designs/Edit Compensator** . . . shows $G(s)G_{\text{lead}}(s) = \dfrac{1286(s+1.4)}{s(s+5)(s+10)(s+12)}$, which yields a $K_v = 3$.

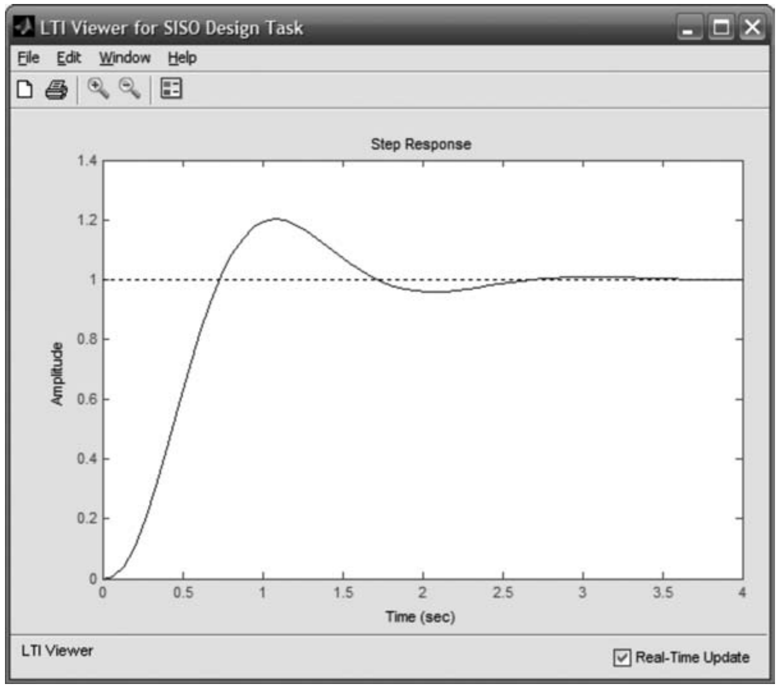


FIGURE 11.14 Gain-adjusted closed-loop step response

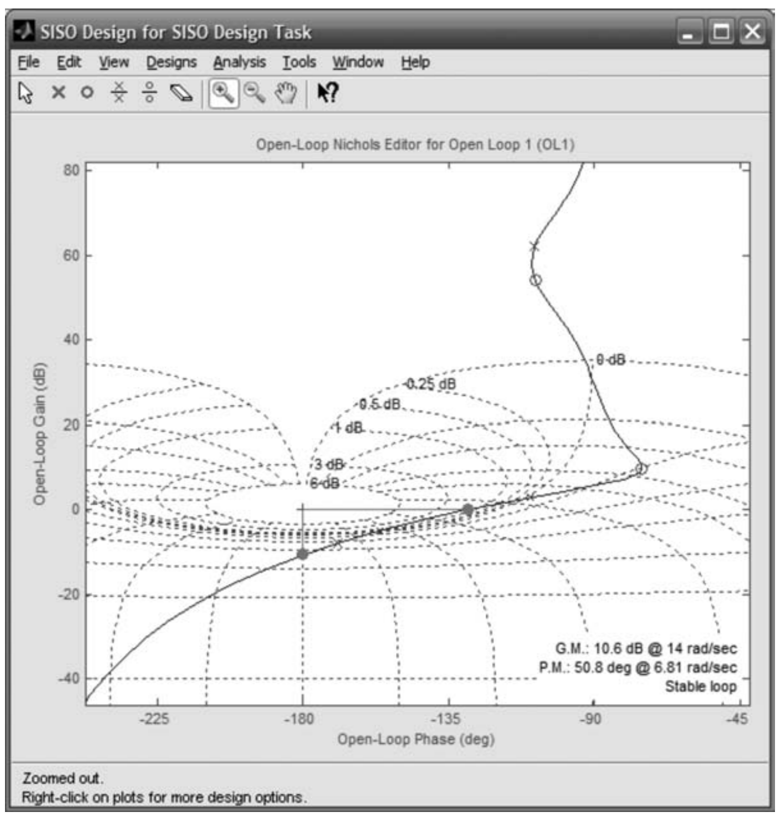


FIGURE 11.15 Nichols chart after lag-lead compensation

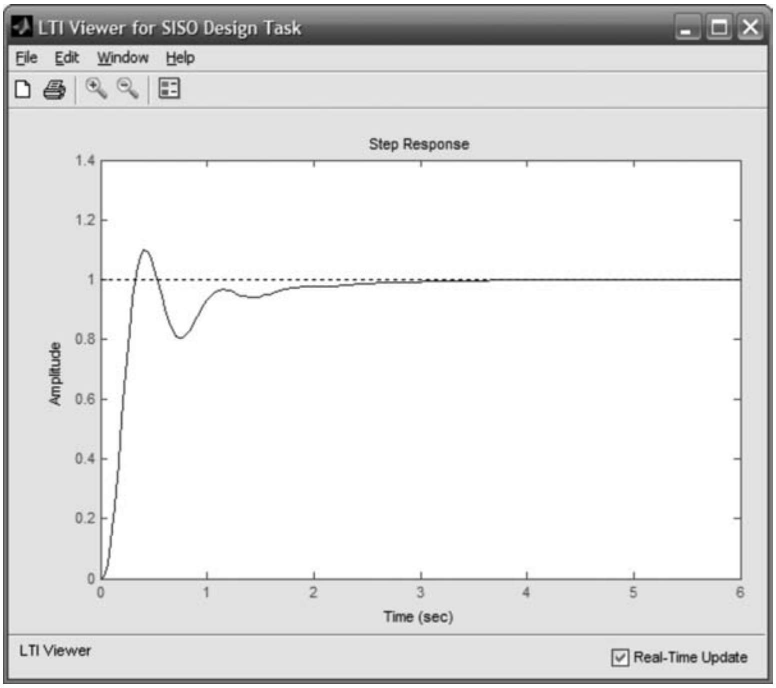


FIGURE 11.16 Lag-lead compensated closed-loop step response

9. We now add lag compensation to improve the static error constant by at least 2.
10. Now add a lag pole at -0.004 and a lag zero at -0.008. Readjust the gain to yield the same tangency as after the insertion of the lead. The final forward path is found to be $G(s)G_{\text{lead}}(s)G_{\text{lag}}(s) = \dfrac{1381(s+1.4)(s+0.008)}{s(s+5)(s+10)(s+12)(s+0.004)}$. The final Nichols chart is shown in Figure 11.15 and the compensated time response is shown in Figure 11.16. Notice that the time response has the expected slow climb to the final value that is typical of lag compensation. If your design requirements require a faster climb to the final response, then redesign the system with a larger bandwidth or attempt a design only with lead compensation. A problem at the end of the chapter provides the opportunity for practice.

Skill-Assessment Exercise 11.4

PROBLEM: Design a lag-lead compensator for a unity feedback system with the forward-path transfer function

$$G(s) = \frac{K}{s(s+8)(s+30)}$$

to meet the following specifications: $\%OS = 10\%$, $T_p = 0.6\,\text{s}$, and $K_v = 10$. Use frequency response techniques.

ANSWER: $G_{\text{lag}}(s) = 0.456\dfrac{(s+0.602)}{(s+0.275)}$; $G_{\text{lead}}(s) = 2.19\dfrac{(s+4.07)}{(s+8.93)}$; $K = 2400$.

The complete solution is at www.wiley.com/college/nise.

Case Studies

Our ongoing antenna azimuth position control system serves now as an example to summarize the major objectives of the chapter. The following cases demonstrate the use of frequency response methods to (1) design a value of gain to meet a percent overshoot requirement for the closed-loop step response and (2) design cascade compensation to meet both transient and steady-state error requirements.

Antenna Control: Gain Design

Design D

PROBLEM: Given the antenna azimuth position control system shown on the front endpapers, Configuration 1, use frequency response techniques to do the following:

a. Find the preamplifier gain required for a closed-loop response of 20% overshoot for a step input.

b. Estimate the settling time.

SOLUTION: The block diagram for the control system is shown on the inside front cover (Configuration 1). The loop gain, after block diagram reduction, is

$$G(s) = \frac{6.63K}{s(s+1.71)(s+100)} = \frac{0.0388K}{s\left(\frac{s}{1.71}+1\right)\left(\frac{s}{100}+1\right)} \tag{11.20}$$

Letting $K = 1$, the magnitude and phase frequency response plots are shown in Figure 10.61.

a. To find K to yield a 20% overshoot, we first make a second-order approximation and assume that the second-order transient response equations relating percent overshoot, damping ratio, and phase margin are true for this system. Thus, a 20% overshoot implies a damping ratio of 0.456. Using Eq. (10.73), this damping ratio implies a phase margin of 48.1°. The phase angle should therefore be $(-180^\circ + 48.1^\circ) = -131.9^\circ$. The phase angle is -131.9° at $\omega = 1.49$ rad/s, where the gain is -34.1 dB. Thus $K = 34.1$ dB $= 50.7$ for a 20% overshoot. Since the system is third-order, the second-order approximation should be checked. A computer simulation shows a 20% overshoot for the step response.

b. Adjusting the magnitude plot of Figure 10.61 for $K = 50.7$, we find -7 dB at $\omega = 2.5$ rad/s, which yields a closed-loop bandwidth of 2.5 rad/s. Using Eq. (10.55) with $\zeta = 0.456$ and $\omega_{\text{BW}} = 2.5$, we find $T_s = 4.63$ seconds. A computer simulation shows a settling time of approximately 5 seconds.

CHALLENGE: We now give you a problem to test your knowledge of this chapter's objectives. You are given the antenna azimuth position control system shown on the inside front cover (Configuration 3). Using frequency response methods do the following:

a. Find the value of K to yield 25% overshoot for a step input.

b. Repeat Part **a** using MATLAB.

MATLAB **ML**

Design **D**

Antenna Control: Cascade Compensation Design

PROBLEM: Given the antenna azimuth position control system block diagram shown on the front endpapers, Configuration 1, use frequency response techniques and design cascade compensation for a closed-loop response of 20% overshoot for a step input, a fivefold improvement in steady-state error over the gain-compensated system operating at 20% overshoot, and a settling time of 3.5 seconds.

SOLUTION: Following the lag-lead design procedure, we first determine the value of gain, K, required to meet the steady-state error requirement.

1. Using Eq. (10.55) with $\zeta = 0.456$, and $T_s = 3.5$ seconds, the required bandwidth is 3.3 rad/s.

2. From the preceding case study, the gain-compensated system's open-loop transfer function was, for $K = 50.7$,

$$G(s)H(s) = \frac{6.63K}{s(s+1.71)(s+100)} = \frac{336.14}{s(s+1.71)(s+100)} \tag{11.21}$$

This function yields $K_v = 1.97$. If $K = 254$, then $K_v = 9.85$, a fivefold improvement.

3. The frequency response curves of Figure 10.61, which are plotted for $K = 1$, will be used for the solution.

4. Using a second-order approximation, a 20% overshoot requires a phase margin of 48.1°.

5. Select $\omega = 3$ rad/s to be the new phase-margin frequency.

6. The phase angle at the selected phase-margin frequency is $-152°$. This is a phase margin of 28°. Allowing for a 5° contribution from the lag compensator, the lead compensator must contribute $(48.1° - 28° + 5°) = 25.1°$.

7. The design of the lag compensator now follows. Choose the lag compensator upper break one decade below the new phase-margin frequency, or 0.3 rad/s. Figure 11.8 says that we can obtain 25.1° phase shift from the lead if $\beta = 0.4$ or $\gamma = 1/\beta = 2.5$. Thus, the lower break for the lag is at $1/(\gamma T) = 0.3/2.5 = 0.12$ rad/s.

Hence,

$$G_{\text{lag}}(s) = 0.4\frac{(s+0.3)}{(s+0.12)} \tag{11.22}$$

8. Finally, design the lead compensator. Using Eq. (11.9), we have

$$T = \frac{1}{\omega_{\max}\sqrt{\beta}} = \frac{1}{3\sqrt{0.4}} = 0.527 \tag{11.23}$$

Therefore the lead compensator lower break frequency is $1/T = 1.9$ rad/s, and the upper break frequency is $1/(\beta T) = 4.75$ rad/s. Thus, the lag-lead-compensated forward path is

$$G_{\text{lag-lead-comp}}(s) = \frac{(6.63)(254)(s+0.3)(s+1.9)}{s(s+1.71)(s+100)(s+0.12)(s+4.75)} \tag{11.24}$$

9. A plot of the open-loop frequency response for the lag-lead-compensated system shows −7 dB at 5.3 rad/s. Thus, the bandwidth meets the design requirements for settling time. A simulation of the compensated system shows a 20% overshoot and a settling time of approximately 3.2 seconds, compared to a 20% overshoot for the uncompensated system and a settling time of approximately 5 seconds. K_v for the compensated system is 9.85 compared to the uncompensated system value of 1.97.

CHALLENGE: We now give you a problem to test your knowledge of this chapter's objectives. You are given the antenna azimuth position control system shown on the front endpapers (Configuration 3). Using frequency response methods, do the following:

a. Design a lag-lead compensator to yield a 15% overshoot and $K_v = 20$. In order to speed up the system, the compensated system's phase-margin frequency will be set to 4.6 times the phase-margin frequency of the uncompensated system.

MATLAB
ML

b. `Repeat Part a using MATLAB.`

Summary

This chapter covered the design of feedback control systems using frequency response techniques. We learned how to design by gain adjustment as well as cascaded lag, lead, and lag-lead compensation. Time response characteristics were related to the phase margin, phase-margin frequency, and bandwidth.

Design by gain adjustment consisted of adjusting the gain to meet a phase-margin specification. We located the phase-margin frequency and adjusted the gain to 0 dB.

A lag compensator is basically a low-pass filter. The low-frequency gain can be raised to improve the steady-state error, and the high-frequency gain is reduced to yield stability. Lag compensation consists of setting the gain to meet the steady-state error requirement and then reducing the high-frequency gain to create stability and meet the phase-margin requirement for the transient response.

A lead compensator is basically a high-pass filter. The lead compensator increases the high-frequency gain while keeping the low-frequency gain the same. Thus, the steady-state error can be designed first. At the same time, the lead compensator increases the phase angle at high frequencies. The effect is to produce a faster, stable system since the uncompensated phase margin now occurs at a higher frequency.

A lag-lead compensator combines the advantages of both the lag and the lead compensator. First, the lag compensator is designed to yield the proper steady-state error with improved stability. Next, the lead compensator is designed to speed up the transient response. If a single network is used as the lag-lead, additional design considerations are applied so that the ratio of the lag zero to the lag pole is the same as the ratio of the lead pole to the lead zero.

In the next chapter, we return to state space and develop methods to design desired transient and steady-state error characteristics.

Review Questions

1. What major advantage does compensator design by frequency response have over root locus design?
2. How is gain adjustment related to the transient response on the Bode diagrams?
3. Briefly explain how a lag network allows the low-frequency gain to be increased to improve steady-state error without having the system become unstable.
4. From the Bode plot perspective, briefly explain how the lag network does not appreciably affect the speed of the transient response.
5. Why is the phase margin increased above that desired when designing a lag compensator?
6. Compare the following for uncompensated and lag-compensated systems designed to yield the same transient response: low-frequency gain, phase-margin frequency, gain curve value around the phase-margin frequency, and phase curve values around the phase-margin frequency.
7. From the Bode diagram viewpoint, briefly explain how a lead network increases the speed of the transient response.
8. Based upon your answer to Question 7, explain why lead networks do not cause instability.
9. Why is a correction factor added to the phase margin required to meet the transient response?
10. When designing a lag-lead network, what difference is there in the design of the lag portion as compared to a separate lag compensator?

Problems

1. Design the value of gain, K, for a gain margin of 10 dB in the unity feedback system of Figure P11.1 if [Section: 11.2]

   a. $G(s) = \dfrac{K}{(s+4)(s+10)(s+15)}$

   b. $G(s) = \dfrac{K}{s(s+4)(s+10)}$

   c. $G(s) = \dfrac{K(s+2)}{s(s+4)(s+6)(s+10)}$

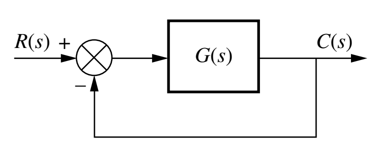


FIGURE P11.1

2. For each of the systems in Problem 1, design the gain, K, for a phase margin of 40°. [Section: 11.2]
3. Given the unity feedback system of Figure P11.1, use frequency response methods to determine the value of gain, K, to yield a step response with a 20% overshoot if [Section: 11.2]

a. $G(s) = \dfrac{K}{s(s+8)(s+15)}$

b. $G(s) = \dfrac{K(s+4)}{s(s+8)(s+10)(s+15)}$

c. $G(s) = \dfrac{K(s+2)(s+7)}{s(s+6)(s+8)(s+10)(s+15)}$

4. Given the unity feedback system of Figure P11.1 with

$$G(s) = \frac{K(s+20)(s+25)}{s(s+6)(s+9)(s+14)}$$

do the following: [Section: 11.2]

a. Use frequency response methods to determine the value of gain, K, to yield a step response with a 15% overshoot. Make any required second-order approximations.

b. Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K.

MATLAB ML

5. The unity feedback system of Figure P11.1 with

WileyPLUS WPCS Control Solutions

$$G(s) = \frac{K}{s(s+7)}$$

is operating with 15% overshoot. Using frequency response techniques, design a compensator to yield $K_v = 50$ with the phase-margin frequency and phase margin remaining approximately the same as in the uncompensated system. [Section: 11.3]

6. Given the unity feedback system of Figure P11.1 with

$$G(s) = \frac{K(s+10)(s+11)}{s(s+3)(s+6)(s+9)}$$

do the following: [Section: 11.3]

a. Use frequency response methods to design a lag compensator to yield $K_v = 1000$ and 15% overshoot for the step response. Make any required second-order approximations.

b. Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K and lag compensator.

MATLAB ML

7. The unity feedback system shown in Figure P11.1 with

$$G(s) = \frac{K}{(s+2)(s+5)(s+7)}$$

is operating with 15% overshoot. Using frequency response methods, design a compensator to yield a five-fold improvement in steady-state error without appreciably changing the transient response. [Section: 11.3]

8. Design a lag compensator so that the system of Figure P11.1 where

$$G(s) = \frac{K(s+4)}{(s+2)(s+6)(s+8)}$$

operates with a 45° phase margin and a static error constant of 100. [Section: 11.3]

9. Design a PI controller for the system of Figure 11.2 that will yield zero steady-state error for a ramp input and a 9.48% overshoot for a step input. [Section: 11.3]

10. For the system of Problem 6, do the following: [Section: 11.3]

a. Use frequency response methods to find the gain, K, required to yield about 15% overshoot. Make any required second-order approximations.

b. Use frequency response methods to design a PI compensator to yield zero steady-state error for a ramp input without appreciably changing the transient response characteristics designed in Part **a**.

c. Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K and PI compensator.

MATLAB ML

11. Write a MATLAB program that will design a PI controller assuming a second-order approximation as follows:

MATLAB ML

a. Allow the user to input from the keyboard the desired percent overshoot

b. Design a PI controller and gain to yield zero steady-state error for a closed-loop step response as well as meet the percent overshoot specification

c. Display the compensated closed-loop step response

Test your program on

$$G(s) = \frac{K}{(s+5)(s+10)}$$

and 25% overshoot.

12. Design a compensator for the unity feedback system of Figure P11.1 with

WileyPLUS WPCS Control Solutions

$$G(s) = \frac{K}{s(s+3)(s+15)(s+20)}$$

to yield a $K_v = 4$ and a phase margin of 40°. [Section: 11.4]

13. Consider the unity feedback system of Figure P11.1 with

$$G(s) = \frac{K}{s(s+5)(s+20)}$$

The uncompensated system has about 55% overshoot and a peak time of 0.5 second when $K_v = 10$. Do the following: [Section: 11.4]

a. Use frequency response methods to design a lead compensator to reduce the percent overshoot to 10%, while keeping the peak time and steady-state error about the same or less. Make any required second-order approximations.

b. Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K.

MATLAB ML

14. The unity feedback system of Figure P11.1 with

$$G(s) = \frac{K(s+5)}{(s+2)(s+6)(s+10)}$$

is operating with 20% overshoot. [Section: 11.4]

a. Find the settling time.

b. Find K_p.

c. Find the phase margin and the phase-margin frequency.

d. Using frequency response techniques, design a compensator that will yield a threefold improvement in K_p and a twofold reduction in settling time while keeping the overshoot at 20%.

15. Repeat the design of Example 11.3 in the text using a PD controller. [Section: 11.4]

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16. Repeat Problem 13 using a PD compensator. [Section: 11.4]

17. Write a MATLAB program that will design a lead compensator assuming second-order approximations as follows:

MATLAB ML

a. Allow the user to input from the keyboard the desired percent overshoot, peak time, and gain required to meet a steady-state error specification

b. Display the gain-compensated Bode plot

c. Calculate the required phase margin and bandwidth

d. Display the pole, zero, and gain of the lead compensator

e. Display the compensated Bode plot

f. Output the step response of the lead-compensated system to test your second-order approximation

Test your program on a unity feedback system where

$$G(s) = \frac{K(s+1)}{s(s+2)(s+6)}$$

and the following specifications are to be met: percent overshoot = 10%, peak time = 0.1 second, and $K_v = 30$.

18. Repeat Problem 17 for a PD controller.

MATLAB ML

19. Use frequency response methods to design a lag-lead compensator for a unity feedback system where [Section: 11.4]

$$G(s) = \frac{K(s+7)}{s(s+5)(s+15)}$$

and the following specifications are to be met: percent overshoot = 15%, settling time = 0.1 second, and $K_v = 1000$.

20. Write a MATLAB program that will design a lag-lead compensator assuming second-order approximations as follows: [Section: 11.5]

MATLAB ML

a. Allow the user to input from the keyboard the desired percent overshoot, settling time, and gain required to meet a steady-state error specification

b. Display the gain-compensated Bode plot

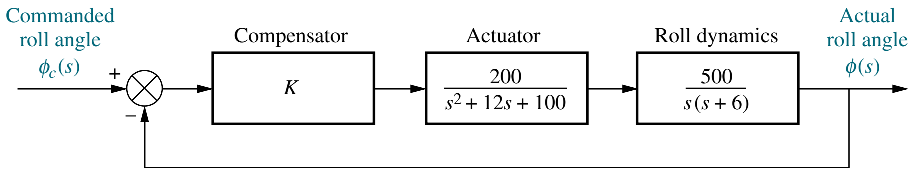

FIGURE P11.2 Towed-vehicle roll control

c. Calculate the required phase margin and bandwidth

d. Display the poles, zeros, and the gain of the lag-lead compensator

e. Display the lag-lead-compensated Bode plot

f. Display the step response of the lag-lead compensated system to test your second-order approximation

Use your program to do Problem 19.

21. Given a unity feedback system with

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$$G(s) = \frac{K}{s(s+2)(s+5)}$$

design a PID controller to yield zero steady-state error for a ramp input, as well as a 20% overshoot, and a peak time less than 2 seconds for a step input. Use only frequency response methods. [Section: 11.5]

22. A unity feedback system has

MATLAB ML

$$G(s) = \frac{K}{s(s+3)(s+6)}$$

If this system has an associated 0.5 second delay, use MATLAB to design the value of K for 20% overshoot. Make any necessary second-order approximations, but test your assumptions by simulating your design. The delay can be represented by cascading the MATLAB function padé(T,n) with $G(s)$, where T is the delay in seconds and n is the order of the Pade approximation (use 5). Write the program to do the following:

a. Accept your value of percent overshoot from the keyboard

b. Display the Bode plot for $K = 1$

c. Calculate the required phase margin and find the phase-margin frequency and the magnitude at the phase-margin frequency

d. Calculate and display the value of K

DESIGN PROBLEMS

23. Aircraft are sometimes used to tow other vehicles. A roll control system for such an aircraft was discussed in Problem 58 in Chapter 6. If Figure P11.2 represents the roll control system, use only frequency response techniques to do the following (*Cochran, 1992*):

a. Find the value of gain, K, to yield a closed-loop step response with 10% overshoot.

b. Estimate peak time and settling time using the gain-compensated frequency response.

c. Use MATLAB to simulate your system. Compare the results of the simulation with the requirements in Part **a** and your estimation of performance in Part **b**.

MATLAB ML

24. The model for a specific linearized TCP/IP computer network queue working under a random early detection (RED) algorithm has been modeled using the block diagram of Figure P11.1, where $G(s) = M(s)P(s)$, with

$$M(s) = \frac{0.005L}{s+0.005}$$

and

$$P(s) = \frac{140{,}625e^{-0.1s}}{(s+2.67)(s+10)}$$

Also, L is a parameter to be varied (*Hollot, 2001*).

a. Adjust L to obtain a 15% overshoot in the transient response for step inputs.

b. Verify Part **a** with a Simulink unit step response simulation.

Simulink SL

25. An electric ventricular assist device (EVAD) that helps pump blood concurrently to a defective natural heart in sick patients can be shown to have a transfer function

$$G(s) = \frac{P_{ao}(s)}{E_m(s)} = \frac{1361}{s^2 + 69s + 70.85}$$

The input, $E_m(s)$, is the motor's armature voltage, and the output is $P_{ao}(s)$, the aortic blood pressure (*Tasch*, *1990*). The EVAD will be controlled in the closed-loop configuration shown in Figure P11.1.

a. Design a phase lag compensator to achieve a tenfold improvement in the steady-state error to step inputs without appreciably affecting the transient response of the uncompensated system.

b. `Use MATLAB to simulate the uncompensated and compensated systems for a unit step input.` MATLAB ML

26. A Tower Trainer 60 Unmanned Aerial Vehicle has a transfer function

$$P(s) = \frac{h(s)}{\delta_e(s)} = \frac{-34.16s^3 - 144.4s^2 + 7047s + 557.2}{s^5 + 13.18s^4 + 95.93s^3 + 14.61s^2 + 31.94s}$$

where $\delta_e(s)$ is the elevator angle and $h(s)$ is the change in altitude (*Barkana*, *2005*).

a. Assuming the airplane is controlled in the closed-loop configuration of Figure P11.1 with $G(s) = KP(s)$, find the value of K that will result in a 30° phase margin.

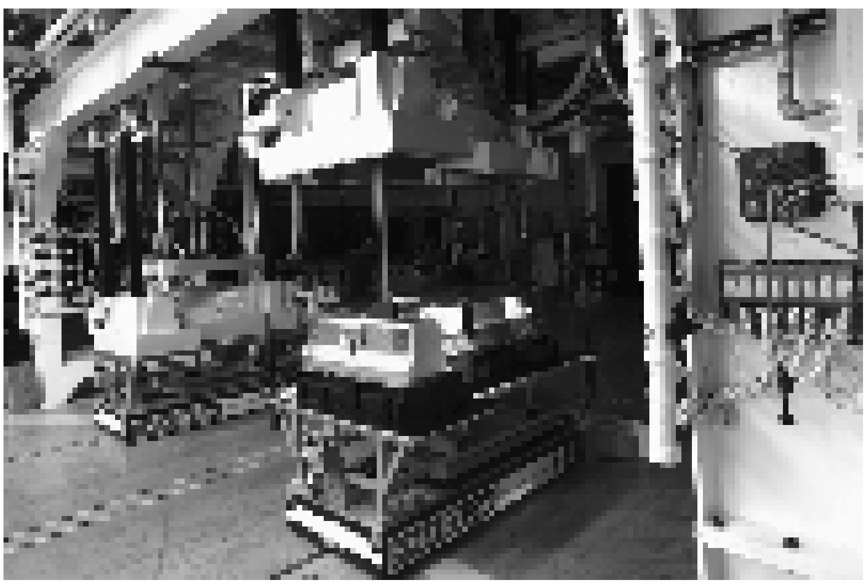
b. For the value of K calculated in Part **a**, obtain the corresponding gain margin.

c. Obtain estimates for the system's $\%OS$ and settling times T_s for step inputs.

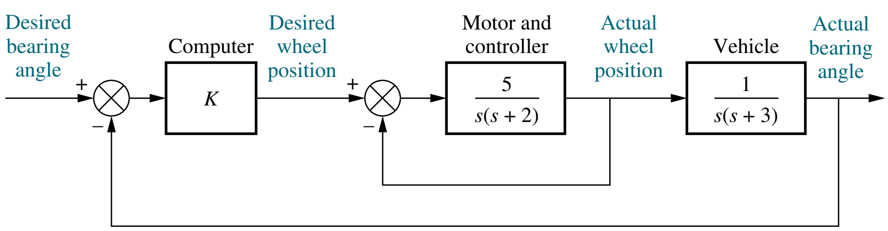
d. `Simulate the step response of the system using MATLAB.` MATLAB ML

e. Explain the simulation results and discuss any inaccuracies in the estimates obtained in Part **c**.

27. Self-guided vehicles, such as that shown in Figure P11.3(*a*), are used in factories to transport products from station to station. One method of construction



(*a*)



(*b*)

FIGURE P11.3 **a.** Automated guided carts in the final assembly area of lithium-ion batteries for Chevrolet Volt™ electric vehicles (Rebecca Cook/Rueters/©Corbis); **b.** simplified block diagram of a guided cart

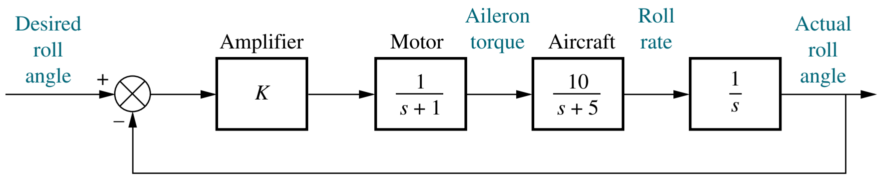


FIGURE P11.4

is to embed a wire in the floor to provide guidance. Another method is to use an onboard computer and a laser scanning device. Bar-coded reflective devices at known locations allow the system to determine the vehicle's angular position. This system allows the vehicle to travel anywhere, including between buildings (*Stefanides, 1987*). Figure P11.3(*b*) shows a simplified block diagram of the vehicle's bearing control system. For 11% overshoot, K is set equal to 2. Design a lag compensator using frequency response techniques to improve the steady-state error by a factor of 30 over that of the uncompensated system.

28. An aircraft roll control system is shown in Figure P11.4. The torque on the aileron generates a roll rate. The resulting roll angle is then controlled through a feedback system as shown. Design a lead compensator for a 60° phase margin and $K_v = 5$.

29. The transfer function from applied force to arm displacement for the arm of a hard disk drive has been identified as

$$G(s) = \frac{X(s)}{F(s)} = \frac{3.3333 \times 10^4}{s^2}$$

The position of the arm will be controlled using the feedback loop shown in Figure P11.1 (*Yan, 2003*).

a. Design a lead compensator to achieve closed-loop stability with a transient response of 16% overshoot and a settling time of 2 msec for a step input.

b. `Verify your design through MATLAB simulations.` MATLAB **ML**

30. A pitch axis attitude control system utilizing a momentum wheel was the subject of Problem 61 in Chapter 8. In that problem, the compensator is shown as a PI compensator. We want to replace the PI compensator with a lag-lead compensator to improve both transient and steady-state error performance. The block diagram for the pitch axis attitude control is shown in Figure P11.5, where $\theta_c(s)$ is a commanded pitch angle and $\theta(s)$ is the actual pitch angle of the spacecraft. If $\tau = 23$ seconds and $I_z = 963$ 1 in-lb-s^2, do the following (*Piper, 1992*): WileyPLUS **WPCS** Control Solutions

a. Design a lag-lead compensator and find $G_c(s)$ and K to yield a system with the following performance specifications: percent overshoot = 20%, settling time = 10 seconds, $K_v = 200$. Make any required second-order approximations.

b. `Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K and lag-lead compensator.` MATLAB **ML**

31. For the heat exchange system described in Problem 36, Chapter 9 (*Smith, 2002*):

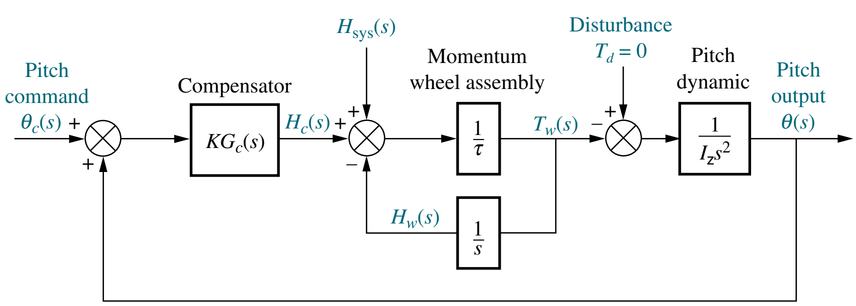


FIGURE P11.5

a. Design a passive lag-lead compensator to achieve 5% steady-state error with a transient response of 10% overshoot and a settling time of 60 seconds for step inputs.

b. `Use MATLAB to simulate and verify your design.` MATLAB ML

32. Active front steering is used in front-steering four-wheel cars to control the yaw rate of the vehicle as a function of changes in wheel-steering commands. For a certain car, and under certain conditions, it has been shown that the transfer function from steering wheel angle to yaw rate is given by (*Zhang*, *2008*):

$$P(s) = \frac{28.4s + 119.7}{s^2 + 7.15s + 14.7}$$

The system is controlled in a unity-feedback configuration.

a. Use the Nichols chart and follow the procedure of Example 11.5 to design a lag-lead compensator such that the system has zero steady-state error for a step input. The bandwidth of the closed-loop system must be $\omega_B = 10$ rad/sec. Let the open-loop magnitude response peak be less than 1 dB and the steady-state error constant $K_v = 20$.

b. Relax the bandwidth requirement to $\omega_B \geq 10$ rad/sec. Design the system for a steady-state error of zero for a step input. Let the open-loop magnitude response peak be less than 1 dB and $K_v = 20$ using only a lead compensator.

c. `Simulate the step response of both designs using MATLAB.` MATLAB ML

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

33. **High-speed rail pantograph.** Problem 21 in Chapter 1 discusses active control of a pantograph mechanism for high-speed rail systems. In Problem 79(a), Chapter 5, you found the block diagram for the active pantograph control system. In Chapter 8, Problem 72, you designed the gain to yield a closed-loop step response with 38% overshoot. A plot of the step response should have shown a settling time greater than 0.5 second as well as a high-frequency oscillation superimposed over the step response. In Chapter 9, Problem 55, we reduced the settling time to about 0.3 second, reduced the step response steady-state error to zero, and eliminated the high-frequency oscillations using a notch filter (*O'Connor*, *1997*). Using the equivalent forward transfer function found in Chapter 5 cascaded with the notch filter specified in Chapter 9, design, using frequency response techniques, a lag-lead compensator to meet the following specifications:

a. At least 35° phase margin

b. A maximum of 10% steady-state error for the closed-loop step response

c. At least 35 rad/s bandwidth

34. **Control of HIV/AIDS.** In Chapter 6, the model for an HIV/AIDS patient treated with RTIs was linearized and shown to be

$$P(s) = \frac{Y(s)}{U_1(s)} = \frac{-520s - 10.3844}{s^3 + 2.6817s^2 + 0.11s + 0.0126}$$
$$= \frac{-520(s + 0.02)}{(s + 2.2644)(s^2 + 0.04s + 0.0048)}$$

It is assumed here that the patient will be treated and monitored using the closed-loop configuration shown in Figure P11.1 Since the plant has a negative dc gain, assume for simplicity that $G(s) = G_c(s)P(s)$ and $G_c(0) < 0$. Assume also that the specifications for the design are (1) zero steady-state error for step inputs, (2) overdamped time-domain response, and (3) settling time $T_s \approx 100$ days (*Craig*, *2004*).

a. The overdamped specification requires a $\Phi_M \approx 90°$. Find the corresponding bandwidth required to satisfy the settling time requirement.

b. The zero steady-state error specification implies that the open-loop transfer function must be augmented to Type 1. The -0.02 zero of the plant adds too much phase lead at low frequencies, and the complex conjugate poles, if left uncompensated within the loop, result in undesired oscillations in the time domain. Thus, as an initial approach to compensation for this system we can try

$$G_c(s) = \frac{-K(s^2 + 0.04s + 0.0048)}{s(s + 0.02)}$$

For $K = 1$, make a Bode plot of the resulting system. Obtain the value of K necessary to achieve the design demands. Check for closed-loop stability.

c. Simulate the unit step response of the system using MATLAB. Adjust K to achieve the desired response.

MATLAB ML

35. Hybrid vehicle. In Part **b** of Problem 10.55 we used a proportional-plus-integral (PI) speed controller that resulted in an overshoot of 20% and a settling time, $T_s = 3.92$ seconds (*Preitl, 2007*).

a. Now assume that the system specifications require a steady-state error of zero for a step input, a ramp input steady-state error $\leq 2\%$, a $\%OS \leq 4.32\%$, and a settling time ≤ 4 seconds. One way to achieve these requirements is to cancel the PI-controller's zero, Z_I, with the real pole of the uncompensated system closest to the origin (located at -0.0163). Assuming exact cancellation is possible, the plant and controller transfer function becomes

$$G(s) = \frac{K(s+0.6)}{s(s+0.5858)}$$

Design the system to meet the requirements. You may use the following steps:

i. Set the gain, K, to the value required by the steady-state error specifications. Plot the Bode magnitude and phase diagrams.

ii. Calculate the required phase margin to meet the damping ratio or equivalently the $\%OS$ requirement, using Eq. (10.73). If the phase margin found from the Bode plot obtained in Step **i** is greater than the required value, simulate the system to check whether the settling time is less than 4 seconds and whether the requirement of a $\%OS \leq 4.32\%$ has been met. Redesign if the simulation shows that the $\%OS$ and/or the steady-state error requirements have not been met. If all requirements are met, you have completed the design.

b. In most cases, perfect pole-zero cancellation is not possible. Assume that you want to check what happens if the PI-controller's zero changes by $\pm$ 20%, e.g., if Z_I moves to:

$$\text{Case 1: } -0.01304$$

or to

$$\text{Case 2: } -0.01956.$$

The plant and controller transfer function in these cases will be, respectively:

$$\text{Case 1: } G(s) = \frac{K(s+0.6)(s+0.01304)}{s(s+0.0163)(s+0.5858)}$$

$$\text{Case 2: } G(s) = \frac{K(s+0.6)(s+0.01956)}{s(s+0.0163)(s+0.5858)}$$

Set K in each case to the value required by the steady-state error specifications and plot the Bode magnitude and phase diagrams. Simulate the closed-loop step response for each of the three locations of Z_I: pole/zero cancellation, Case 1, and Case 2, given in the problem.

Do the responses obtained resemble a second-order overdamped, critically damped, or underdamped response? Is there a need to add a derivative mode?

Cyber Exploration Laboratory

Experiment 11.1

Objectives To design a PID controller using MATLAB's SISO Design Tool. To see the effect of a PI and a PD controller on the magnitude and phase responses at each step of the design of a PID controller.

Minimum Required Software Packages MATLAB, and the Control System Toolbox

Prelab

1. What is the phase margin required for 12% overshoot?
2. What is the bandwidth required for 12% overshoot and a peak time of 2 seconds?

3. Given a unity feedback system with $G(s) = \dfrac{K}{s(s+1)(s+4)}$, what is the gain, K, required to yield the phase margin found in Prelab 1? What is the phase-margin frequency?
4. Design a PI controller to yield a phase margin 5° more than that found in Prelab 1.
5. Complete the design of a PID controller for the system of Prelab 3.

Lab

1. Using MATLAB's SISO Design Tool, set up the system of Prelab 3 and display the open-loop Bode plots and the closed-loop step response.
2. Drag the Bode magnitude plot in a vertical direction until the phase margin found in Prelab 1 is obtained. Record the gain K, the phase margin, the phase-margin frequency, the percent overshoot, and the peak time. Move the magnitude curve up and down and note the effect upon the phase curve, the phase margin, and the phase-margin frequency.
3. Design the PI controller by adding a pole at the origin and a zero one decade below the phase-margin frequency found in Lab 2. Readjust the gain to yield a phase margin 5° higher than that found in Prelab 1. Record the gain K, the phase margin, the phase-margin frequency, the percent overshoot, and the peak time. Move the zero back and forth in the vicinity of its current location and note the effect on the magnitude and phase curve. Move the magnitude curve up and down and note its effect on the phase curve, the phase margin, and the phase-margin frequency.
4. Design the PD portion of the PID controller by first adjusting the magnitude curve to yield a phase-margin frequency slightly below the bandwidth calculated in Prelab 2. Add a zero to the system and move it until you obtain the phase margin calculated in Prelab 1. Move the zero and note its effect. Move the magnitude curve and note its effect.

Postlab

1. Compare the Prelab PID design with that obtained via the SISO Design Tool. In particular, compare the gain K, the phase margin, the phase-margin frequency, the percent overshoot, and the peak time.
2. For the uncompensated system, describe the effect of changing gain on the phase curve, the phase margin, and the phase-margin frequency.
3. For the PI-compensated system, describe the effect of changing gain on the phase curve, the phase margin, and the phase-margin frequency. Repeat for changes in the zero location.
4. For the PID-compensated system, describe the effect of changing gain on the phase curve, the phase margin, and the phase-margin frequency. Repeat for changes in the PD zero location.

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