IIT Hyderabad
Department of Mathematics
MA 101 - Calculus I

22, November 2010 **End Semester** 180 mins **50** marks

1. Give examples of the following, with a brief justification. (**7** marks)
   (a) A function $f(x, y)$ and a point (x_0, y_0) such that (x_0, y_0) is a saddle point.
   (b) A function $f(x, y)$ such that $\nabla f = 9x^2y^2\hat{\imath} + 6x^3y\hat{\jmath}$.
   (c) A function $f(x, y)$ such that at the point $(3, 1)$ the direction of steepest increase of f is along the vector $\bar{u} = \hat{\imath} + 6\hat{\jmath}$.
   (d) A power series such that it is convergent only at $x = 1$.
   (e) A function $f(x)$ that is nowhere piecewise monotonic.
   (f) A sequence of functions $\{f_n\}$ defined everywhere (i.e., for all $x \in \mathbb{R}$) but whose functional series $\sum_{n=1}^{\infty} f_n(x)$ does **not** converge anywhere (i.e., for any $x \in \mathbb{R}$).
   (g) A sequence of functions $\{f_n\}$ to show that the converse of the following statement is **not true**: "*If a sequence of continuous functions $\{f_n\}$ converges uniformly to f, then f is continuous.*"
2. Use total differential to estimate $(1.95)^4(3.04)^3(0.97)^5$. (**2** marks)
3. Show by definition that $f(x, y) = xy^2$ is differentiable. (**3** marks)
4. Determine the nature of the critical points of $f(x, y) = 9x^2 + 4y^2 - 12xy + 7$. (**3** marks)
5. A firm has Rs. 250,000 to spend on labour and raw materials. The output of the firm is αxy where α is a constant and x, y are the quantity of labour and raw materials consumed. If the unit price of hiring labour is Rs. 5000 and the unit price of raw materials is 2500, find the ratio of x to y that maximizes output. (**3** marks)
6. Find the point at which $w = xyz$ attains its maximum w.r.to the constraints $x + y + z = 30$ and $x + y = z$. (**4** marks)
7. Discuss the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{4^n n^p}$ w.r.to both x and an arbitrary constant p. (**4** marks)
8. Expand in Fourier series the function $f(x) = \dfrac{\pi^2}{12} - \dfrac{x^2}{4}$ in the interval $[-\pi, \pi]$ and hence show that $\displaystyle\frac{\pi^2}{12} = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$. (**4** marks)
9. Show using Taylor's series that $e^{i\theta} = \sin\theta + i\ \cos\theta$. (**4** marks)
10. Show that every line normal to the surface of a sphere passes through its center. (**4** marks)

ALL THE BEST