

**IIT Hyderabad**  
**Department of Mathematics**  
**MA 101 - Calculus I**

21, October 2009

**Quiz II**

90 mins

**25** marks

1. Give examples of functions with the following properties: (7 marks)

- (a) A function  $f(x, y)$  whose domain is  $\{(x, y) \in \mathbb{R}^2 | x \neq 0\}$  and whose range is  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ .
- (b) A function  $f(x)$  and a point  $x_0$  such that  $\frac{df}{dx}(x_0) = 0$  but  $x_0$  is not an extreme point.
- (c) A function  $f(x)$  and a point  $x_0$  such that  $x_0$  is an extreme point but  $\frac{df}{dx}(x_0)$  does not exist.
- (d) A function  $f(x)$  and a point  $x_0$  such that  $\frac{df}{dx}$  is continuous at  $x_0$  but not differentiable at  $x_0$ .
- (e) A function  $f(x, y)$  such that  $\frac{\partial f}{\partial x}$  exists at  $(0, 0)$  but  $f$  is not continuous at  $(0, 0)$ .
- (f) A function  $f(x, y)$  such that  $f_{xy}(0, 0) \neq f_{yx}(0, 0)$ .
- (g) A function  $f(x, y)$  such that  $f(0, 0)$  is defined but  $\nabla f(0, 0)$  is not.

**NB:** Questions 2 - 10 carry **2** marks each.

2. Find the domain, range and level curves of  $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$ .
3. Show that  $\lim_{(x, y) \rightarrow (0, 0)} \frac{y^2 - 2x}{y^2 + 2x}$  does not exist.
4. Find the unit vector along which  $4x^2 + 9y^2$  increases most rapidly at the point  $(2, 1)$ .
5. Find a number such that the following function is continuous at the origin:

$$f(x, y) = \begin{cases} \frac{-2xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ c, & (x, y) = (0, 0) \end{cases}.$$

6. Examine the function  $y = 2 \sin x + \cos 2x$  for maxima and minima.
7. Let  $z = f(x, y)$  with  $x = r \cos \theta$  and  $y = r \sin \theta$ . Also writing  $z = g(r, \theta) = f(r \cos \theta, r \sin \theta)$ , show that

$$\left(\frac{\partial g}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial g}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2.$$

8. Determine the nature of the critical points of  $f(x, y) = 2x^3 - 24xy + 16y^3$ .
9. How much wood is contained in the sides of a rectangular box with sides of inside measurements 1.5m, 1.3m and 2m, if the thickness of the wood making up the sides is 3cm?
10. A silo is in the shape of a cylinder topped with a cone (much like a circus tent). If the radius of each is 6m, and the total surface area is  $200 \text{ m}^2$  (excluding the base), what are the heights of the cylinder and the cone that maximize the volume enclosed by the silo?

**ALL THE BEST**