# Gate problems in DSP

*Abstract*—**These problems have been selected from GATE question papers and can be used for conducting tutorials in courses related to the course Digital Signal Processing in practice.**

1) If the impulse response of a discrete-time system is $h[n] = -5^n u[-n-1]$,then the system function $H(z)$ is equal to
   (A) $\frac{-z}{z-5}$ and the system is stable

   (B) $\frac{z}{z-5}$ and the system is stable

   (C) $\frac{-z}{z-5}$ and the system is unstable

   (D) $\frac{z}{z-5}$ and the system is unstable

2) A sequence $x(n)$ with the z-transform $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$ is applied as an input to a linear,time-invariant system with the impulse response $h(n) = 2\delta(n-3)$ where
$$\delta(n) = \begin{cases} 1, & \text{if n=0} \\ 0, & \text{otherwise} \end{cases} .$$
   The output at $n$=4 is

   (A) $-6$ (B) $0$ (C) $2$ (D) $-4$

   Data for ***Q.3-4*** are given below. Solve the problems and choose the correct answers.
   The system under consideration is an RC low-pass filter (RC-LPF) with R=1.0K$\Omega$ and C=1.0 $\mu$F

3) Let $H(f)$ denote the frequency response of the RC-LPF. Let $f_1$ be the highest frequency such that $0 \leq |f| \leq f_1, \frac{|H(f_1|}{H(0)} \geq 0.95$.Then $f_1$ (in HZ) is

   (A) $327.8$ (B) $163.9$ (C) $52.2$ (D) $104.4$

4) The impulse response $h[n]$ of a linear time-invariant system is given by $h[n] = u[n+3] + u[n-2] - 2u[n-7]$ where $u[n]$ is the unit step sequence.The above system is

   (A) Stable but not causal

   (B) Stable and Causal

   (C) Causal but unstable

   (D) Unstable and not Causal

5) The z-transform of a system is $H(z) = \frac{z}{z-0.2}$.If the ROC IS $|z| < 0.2$, then the impulse response of the system is

   (A) $(0.2)^n u[n]$ (C) $-(0.2)^n u[n]$

   (B) $(0.2)^n u[-n-1]$ (D) $-(0.2)^n u[-n-1]$

6) consider the sequence $x[n] = [4-j5 \;\; 1+j2 \;\; 4]$ The conjugate anti-symmetric part of the sequence is
   (A) $[-4-j2.5 \quad j2 \quad 4-j2.5]$

   (B) $[-j2.5 \quad 1 \quad j2.5]$

   (C) $[-j5 \quad j2 \quad 0]$

   (D) $[-4 \quad 1 \quad 4]$

7) A causal LTI system is described by the difference equation $2y[n] = ay[n-2] - 2x[n] + bx[n-1]$
   the system is stable only if
   (A) $|a| = 2, |b| < 2$

   (B) $|a| > 2, |b| > 2$

   (C) $|a| < 2$,any value of b

(D) $|b| < 2$, any value of a

8) The impulse responce $h[n]$ of a linear time invariant system is given as

$$h[n] = \begin{cases} -2\sqrt{2}, & \text{if n=1,-1} \\ 4\sqrt{2}, & \text{n=2,-2} \\ 0, & \text{otherwise}. \end{cases}$$

If the input to the above system is the sequence $e^{\frac{jpn}{4}}$,then the ouput is

(A) $4\sqrt{2}e^{\frac{jpn}{4}}$ (C) $4e^{\frac{jpn}{4}}$

(B) $4\sqrt{2}e^{\frac{-jpn}{4}}$ (D) $-4e^{\frac{jpn}{4}}$

9) The region of convergence of Z-transform of the sequence $(\frac{5}{6})^n u(n) - (\frac{6}{5})^n u(-n-1)$ must be

(A) $|z| < \frac{5}{6}$ (C) $\frac{5}{6} < |z| < \frac{5}{6}$

(B) $|z| > \frac{6}{5}$ (D) $\frac{6}{5} < |z| < \infty$

10) A signal $x(n) = sin(\omega_0 n + \phi)$ is the input to a LTI system frequency response $H(e^{j\omega})$.If the ouput of the system is $Ax(n-n_0)$, then the most general form of $\angle H(e^{j\omega})$ will be

(A) $-n_0\omega_0 + \beta$ for any arbitary real $\beta$

(B) $-n_0\omega_0 + 2\pi k$ for any arbitary integer k.

(C) $n_0\omega_0 + 2\pi k$ for any arbitary integer k.

(D) $-n_0\omega_0 + \phi$

11) A system with input $x[n]$ and the ouput $y[n]$ is given as $y[n] = (sin\frac{5}{6}\pi n)x[n]$.The system is

(A) Linear,stable and invertible

(B) non-linear,stable and non-invertible

(C) linear,stable and non-invertible

(D) linear,unstable and invertible

12) A 5-point sequence $x[n]$ is given as $x[-3] = 1, x[-2] = 1, x[-1] = 0, x[0] = 5, x[1] = 1$. Let $X(e^{j\omega})$ denote the discrete -time Fourier transform of $x[n]$. The value of $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$ is :

(A) 5 (B) $10\pi$ (C) $16\pi$ (D) $5 + j10\pi$

13) The $z$-transform $X[z]$ of a sequence $x[n]$ is given by $X[z] = \frac{0.5}{1-2z^{-1}}$.It is given that the region of convergence of $X[z]$ includes the unit circle. The value of $x[0]$ is:

(A) -0.5 (B) 0 (C) 0.25 (D) 0.5

14) A discrete time linear shift-invariant system has an impulse response $h[n]$ with $h[0] = 1, h[1] = -1, h[2] = -2$ and zero otherwise.The system is given an input sequence $x[n]$ with $x[0] = x[2] = 1$, and zero otherwise. The number of nonzero samples in the output sequence $y[n]$,and the value of $y[2]$ are, respectively

(A) 5,2 (C) 6,1

(B) 6,2 (D) 5,3

15) $\{x(n)\}$ is real-valued periodic sequence with a period N. $x(n)$ and $X(k)$ form N-point.Discrete Fourier Transform (DFT) pairs. The DFT $Y(k)$ of the sequence $y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)x(n+r)$

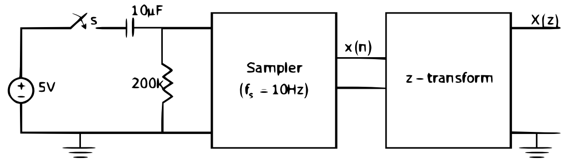
(A) $|X(k)|^2$

(B) $\frac{1}{N}\sum_{r=0}^{N-1} X(r) * X(k+r)$

(C) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)X(k+r)$

(D) 0

***DATA FOR Q.16 AND 17*** In the following network, the switch is closed at t=$0^-$ and the sampling starts from t=0. The sampling frequency is 10Hz.




16) The samples $x(n)$ at n=0,1,2,... given by
(A) $5(1-e^{-0.05n})$

(B) $5e^{-0.05n}$

(C) $5(1-e^{-5n})$

(D) $5e^{-5n}$

17) The expression and the region of convergence of the z-transform of the sampled signal are
(A) $\frac{5z}{z-e^{-5}}, |z| < e^{-5}$

(B) $\frac{5z}{z-e^{-0.05}}, |z| < e^{-0.05}$

(C) $\frac{5z}{z-e^{-5}}, |z| > e^{-0.05}$

(D) $\frac{5z}{z-e^{-5}}, |z| > e^{-5}$

18) The ROC of Z-transform of the discrete time sequence $x(n) = (\frac{1}{3})^n u(n) - (\frac{1}{2})^n u(-n-1)$ is

(A) $\frac{1}{3} < |z| < \frac{1}{2}$
(B) $|z| > \frac{1}{2}$
(C) $|z| < \frac{1}{3}$
(D) $2 < |z| < 3$

19) A system with transfer function $H(z)$ has impulse response h(x) defined as h(2)=1,h(3)=-1 and h(k)=0 otherwise.Consider the following statements.

S1: H(z) is a low pass filter

S2: H(z) is a FIR filter

which of the following is correct ?
(A) Only S2 is true

(B) Both S1 and S2 are false.

(C) Both S1 and S2 are true, and S2 is a reason for S1

(D) Both S1 and S2 are true, but S2 is not a reason for S1

20) Consider the z-transform $X(z) = 5z^2 + 4z^{-1} + 3; 0 < |z| < \infty$. The inverse z-transform $x[n]$
(A) $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$

(B) $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$

(C) $5u[n+2] + 3u[n] + 4u[n-1]$

(D) $5u[n-2] + 3u[n] + 4u[n+1]$

21) Two discrete time systems with impulse responses $h_1[n] = \delta[n-1]$ and $h_2[n] = \delta[n-2]$ are cascade. The overall impulse response of the cascaded system is

(A) $\delta[n-1] + \delta[n-2]$
(B) $\delta[n-4]$
(C) $\delta[n-3]$
(D) $\delta[n-1]\delta[n-2]$

22) The transfer function of a discrete time LTI system is given by $H(z) = \frac{2-\frac{3}{4}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$
Consider the following statements:
***S1:***The system is stable and causal for ROC:$|z| > \frac{1}{2}$
***S2:***The system is stable but not causal for ROC:$|z| < \frac{1}{4}$
***S3:***The system is neither stable nor causal for ROC:$\frac{1}{4} < |z| < \frac{1}{2}$
Which one of the following statements is valid?
(A) Both S1 and S2 are true.

(B) Both S2 and S3 are true.

(C) Both S1 and S3 are true.

(D) S1,S2 and S3 are all true.

23) A system is defined by its impulse response $h(n) = 2^n u(n-2)$. The system is
(A) stable and causal

(B) causal but not stable

(C) stable but not causal

(D) unstable and non-causal

24) Two systems $H_1(z)$ and $H_2(z)$ are connected in cascade as shown below.The overall output y(n) is the same as the input x(n) with a one unit delay. The transfer function of the second system $H_2(z)$ is

$$X(n) \longrightarrow \boxed{H_1(z) = \frac{1-0.4z^{-1}}{1-0.6z^{-1}}} \longrightarrow \boxed{H_2(z)} \longrightarrow y(n)$$

(A) $\frac{(1-0.6z^{-1})}{z^{-1}(1-0.4z^{-1})}$  (C) $\frac{z^{-1}(1-0.4z^{-1})}{(1-0.6z^{-1})}$

(B) $\frac{z^{-1}(1-0.6z^{-1})}{(1-0.4z^{-1})}$  (D) $\frac{(1-0.4z^{-1})}{z^{-1}(1-0.6z^{-1})}$

25) The first 6 points of the 8-point DFT of a real valued sequence are 5,1-j3,0,3-j4,0 and 3+j4. The last two points of the DFT are respectively

(A) 0,1-j3  (C) 1+j3,5

(B) 0,1+j3  (D) 1-j3,5

26) Let x[n]=x[-n]. Let X(z) be the z-transform of x[n]. If 0.5+j0.25 is a zero of X(z) then one of the following must be a zero of X(z).

(A) $0.5 - j0.25$  (C) $\frac{1}{0.5-j0.25}$

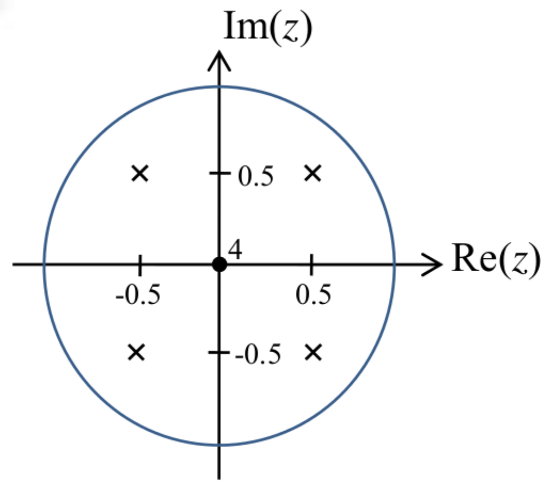
(B) $\frac{1}{0.5+j0.25}$  (D) $2 + j4$

27) The input-output relationship of a causal stable LTI system is given as $y[n] = \alpha y[n-1] + \beta x[n]$ If the impulse response $h[n]$ of this system satisfies the condition $\sum_{n=0}^{\infty} h[n] = 2$, the relationship between $\alpha$ and $\beta$ is

(A) $\alpha = 1 - \frac{\beta}{2}$  (C) $\alpha = 2\beta$

(B) $\alpha = 1 + \frac{\beta}{2}$  (D) $\alpha = -2\beta$

28) The pole-zero diagram of causal and stable discrete-time system is shown in figure. The zero at the origin has multiplicity 4. The impulse response of the system is $h[n]$. If $h[0] = 1$, we can conclude




(A) $h[n]$ is real for all n

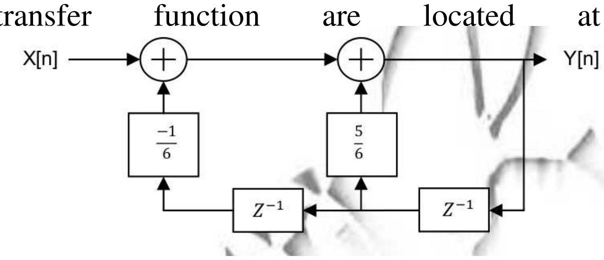
(B) $h[n]$ is purely imaginary for all n

(C) $h[n]$ is real for only even n

(D) $h[n]$ is purely imaginary for only odd n

29) Consider the signal $x[n] = 6\delta[n+2] + 3\delta[n+1] + 8\delta[n] + 7\delta[n-1] + 4\delta[n-2]$. If $X(e^{j\omega})$ is the discrete-time Fourier transform of $x[n]$. Then $\frac{1}{\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) sin^2(2\omega) d\omega$ is equal to __________

30) For the discrete-time shown in the figure, the poles of the system transfer function are located at




(A) $2, 3$  (C) $\frac{1}{2}, \frac{1}{3}$

(B) $\frac{1}{2}, 3$  (D) $2, \frac{1}{3}$

31) The DFT of vector $[a \ \ b \ \ c \ \ d]$ is the vector $[\alpha \ \ \beta \ \ \gamma \ \ \delta]$.Consider the product

$$[p \quad q \quad r \quad s] = [a \quad b \quad c \quad d] \begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}$$

The DFT of the vector $[p \quad q \quad r \quad s]$ is a scaled version of

(A) $[\alpha^2 \quad \beta^2 \quad \gamma^2 \quad \delta^2]$

(B) $[\sqrt{\alpha} \quad \sqrt{\beta} \quad \sqrt{\gamma} \quad \sqrt{\delta}]$

(C) $[\alpha+\beta \quad \beta+\delta \quad \gamma+\delta \quad \gamma+\alpha]$

(D) $[\alpha \quad \beta \quad \gamma \quad \delta]$

32) Two sequences $[a \quad b \quad c]$ and $[A \quad B \quad C]$ are related as

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{-1} & W_3^{-2} \\ 1 & W_3^{-2} & W_3^{-4} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$ Where

$W_3 = e^{j\frac{2\pi}{3}}$

If another sequence $[p \quad q \quad r]$ is derived as

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^1 & W_3^2 \\ 1 & W_3^2 & W_3^4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & W_3^2 & 0 \\ 0 & W_3^4 & 0 \end{bmatrix} \begin{bmatrix} \frac{A}{3} \\ \frac{B}{3} \\ \frac{C}{3} \end{bmatrix}$$

Then the relationship between the sequences $[p \quad q \quad r]$ and $[a \quad b \quad c]$

a) $[p \quad q \quad r] = [b \quad a \quad c]$

b) $[p \quad q \quad r] = [b \quad c \quad a]$

c) $[p \quad q \quad r] = [c \quad a \quad b]$

d) $[p \quad q \quad r] = [c \quad b \quad a]$

33) Let $h[n]$ be the impulse response of a discrete-time linear time invariant (LTI) filter. The impulse response is given by $h[0] = \frac{1}{3}; h[1] = \frac{1}{3}; h[2] = \frac{1}{3}$ and $h[n] = 0$ for $n < 0$ and $n > 0$. Let $H(\omega)$ be the discrete-time Fourier transform (DTFT) of $h[n]$,where $\omega$ is the normalized angular frequency in radians. Given that $H(\omega_0) = 0$ and $0 < \omega < \pi$, the value of $w_0$ (in radians) is equal to __________

34) A discrete-time signal $x[n] = \delta[n-3] + 2\delta[n-5]$ has $z-$transform $X(z)$. If $Y(z) = X(-z)$ is the $z-$ transform of another signal $y[n]$, then

(A) $y[n] = x[n]$

(B) $y[n] = x[-n]$

(C) $y[n] = -x[n]$

(D) $y[n] = -x[-n]$

35) The 4-point Discrete Fourier Transform (DFT) of a discrete time sequence 1,0,2,3 is

(A) $[0, -2+2j, 2, -2-2j]$

(B) $[2, 2+2j, 6, -2-2j]$

(C) $[6, 1-3j, 2, 1+3j]$

(D) $[6, -1+3j, 0, -1-3j]$