Mathematics through OPAMP

G V S S Praneeth Varma, CH. Gajendranath Choudhury and G V V Sharma

Contents

***Abstract*—This manual shows how to use an OPAMP for implementing mathematical functions.**

1 Linear Combination

Problem 1. In Fig. 1, the current entering the + and − terminals of the opamp is 0. The voltages at both terminals is v. Show that

$$y = k_1x_1 - k_2x_2, \quad k_1, k_2 > 0 \tag{1}$$

where x_1, x_2 are the inputs. Find the values of k_1 and k_2.

Solution: Using node analysis,

$$\frac{v - x_2}{R_2} + \frac{v - y}{R_f} = 0 \tag{2}$$

$$\frac{v - x_1}{R_1} + \frac{v}{R_3} = 0 \tag{3}$$

resulting in

$$v\left(\frac{1}{R_2} + \frac{1}{R_f}\right) = \frac{x_2}{R_2} + \frac{y}{R_f} \tag{4}$$

$$\frac{x_1}{R_1} = v\left(\frac{1}{R_1} + \frac{1}{R_3}\right) \tag{5}$$

The authors are with the Department of Electrical Engineering, IIT, Hyderabad 502285 India e-mail: {gadepall}@iith.ac.in.

Simplifying,

$$\frac{x_1}{R_1}\left(\frac{1}{R_2} + \frac{1}{R_f}\right) = \frac{x_2}{R_2}\left(\frac{1}{R_1} + \frac{1}{R_3}\right) + \frac{y}{R_f}\left(\frac{1}{R_1} + \frac{1}{R_3}\right) \tag{6}$$

$$\implies y = \frac{-\frac{x_2}{R_2}\left(\frac{1}{R_1} + \frac{1}{R_3}\right) + \frac{x_1}{R_1}\left(\frac{1}{R_2} + \frac{1}{R_f}\right)}{\frac{1}{R_f}\left(\frac{1}{R_1} + \frac{1}{R_3}\right)} \tag{7}$$

$$= \underbrace{\frac{R_3\left(R_2 + R_f\right)}{R_1\left(R_1 + R_3\right)}}_{k_1} x_1 - \underbrace{\frac{R_f}{R_2}}_{k_2} x_2 \tag{8}$$

$$= k_1x_1 - k_2x_2 \tag{9}$$

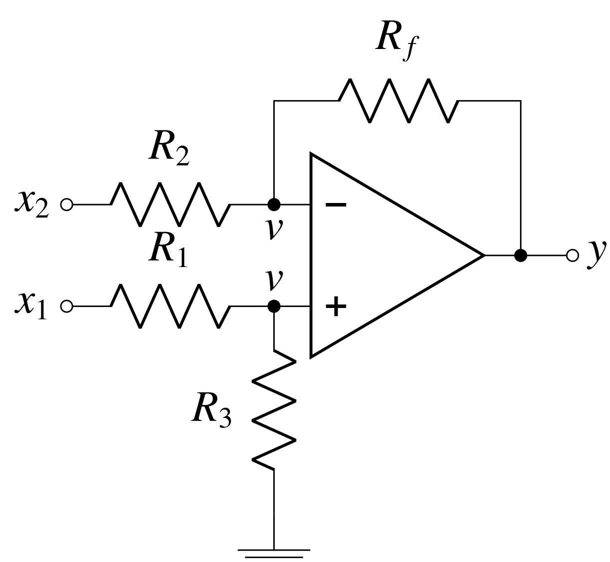

Fig. 1: $y = k_1x_1 - k_2x_2$.

Problem 2. Design a circuit for

$$y = kx, \quad k > 0 \tag{10}$$

Problem 3. Design a circuit for

$$y = -kx, \quad k > 0 \tag{11}$$

2 Differentiation and Integration

Problem 4. Design a circuit for

$$x = -k\frac{dy}{dt}, k > 0 \tag{12}$$

and obtain an expression for k.

Solution: Fig. 2 provides the solution which is explained below. Using node analysis in the s domain,

$$\frac{V(s) - Y(s)}{R} + sCV(s) - X(s) = 0 \tag{13}$$

Since $v = 0$, the above equation results in

$$Y(s) = -sCRX(s) \tag{14}$$

$$\implies y(t) = -\underbrace{RC}_{k}\frac{dx}{dt} \tag{15}$$

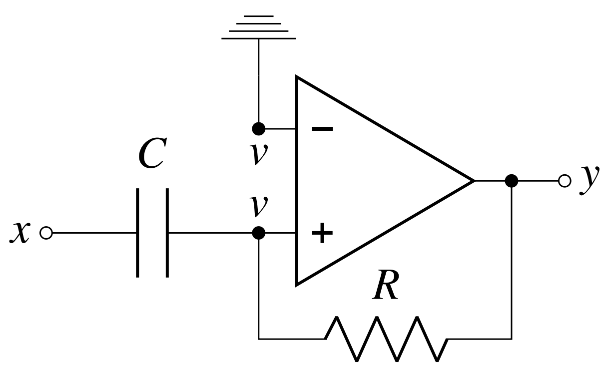

Fig. 2: $y = -k\frac{dx}{dt}$.

Problem 5. Modify the circuit in Problem 4 to obtain

$$y = k\int x(t)\,dt \tag{16}$$

Is $k > 0$?.

Problem 6. How will you obtain

$$y = k\frac{dx}{dt}, \quad k > 0? \tag{17}$$

3 Practical Verification

In the following, $1k\Omega$ and $2k\Omega$ resistances are available.

Problem 7. Verify your circuit in Problem 1 for $x_1 = 0.5V, x_2 = 1V, y = -1.1.V$.

Problem 8. Verify your circuit for (10) for $x = 0.5V$ and $y = 1.5V$. You will have to choose the resistances appropriately.

Problem 9. Verify your circuit for (11) if $x = 1V, y = -2V$.

Problem 10. A triangular wave with $V_{pp} = 1V$ and frequency 100 Hz is given as the input in Problem 4. What is the output for $C = 1\mu F$ and $R = 10k\Omega$? Verify your result.