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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

1 STABILITY

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT

5 STATE SPACE ANALYSIS

5.0.1. A second-order LTI system is described by the following state equations

$$\frac{\partial x_1(t)}{\partial t} - x_2(t) = 0 \quad (5.0.1.1)$$

$$\frac{\partial x_2(t)}{\partial t} + 2x_1(t) + 3x_2(t) = r(t) \quad (5.0.1.2)$$

$$c(t) = x_1(t). \quad (5.0.1.3)$$

where $x_1(t)$ and $x_2(t)$ are the two state variables and $r(t)$ denotes the input. The output is $c(t)$. Express this in terms of the state space model.

Solution: From (??), (5.0.1.1)-(5.0.1.3) can be expressed as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (5.0.1.4)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \quad (5.0.1.5)$$

where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \quad (5.0.1.6)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (5.0.1.7)$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (5.0.1.8)$$

$$\mathbf{D} = 0 \quad (5.0.1.9)$$

5.0.2. Find the system transfer function $H(s)$.

Solution: From (??),

$$H(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}\mathbf{I} \quad (5.0.2.1)$$

$$= \frac{1}{s^2 + 3s + 2} \quad (5.0.2.2)$$

using the code in codes/ee18btech11031.py

5.0.3. Identify the damping type.

Solution: From (??) and

$$\omega = \sqrt{2}, \zeta = \frac{3}{2\sqrt{2}} > 1 \quad (5.0.3.1)$$

From Table ??, the system is overdamped.

5.0.4. Find and plot the unit step response for the system.

Solution:

$$Y(s) = U(s) * H(s) \quad (5.0.4.1)$$

$$= \frac{1}{s(s+1)(s+2)} \quad (5.0.4.2)$$

$$y(t) = \left(\frac{1}{2} - e^{-t} + \frac{e^{-2t}}{2}\right)u(t) \quad (5.0.4.3)$$

using the code in codes/ee18btech11031(2).py

The corresponding step response plot:

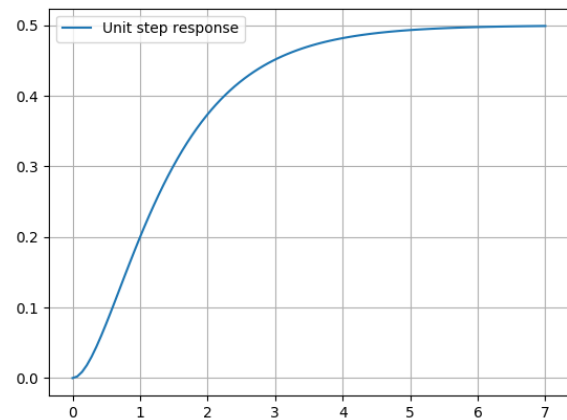


Fig. 5.0.4: (a)