Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

2 Routh Hurwitz Criterion

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4 Nyquist Plot

5 State Space Analysis

5.0.1. A second-order LTI system is described by the following state equations:

$$\frac{\partial x_1(t)}{\partial t} - x_2(t) = 0 \tag{5.0.1.1}$$

$$\frac{\partial x_2(t)}{\partial t} + 2x_1(t) + 3x_2(t) = r(t) \tag{5.0.1.2}$$

where $x_1(t)$ and $x_2(t)$ are the two state variables and r(t) denotes the input. The output c(t) = $x_1(t)$.

5.0.2. Identify the type of system employing the state space model.
Solution: Any state space model is represented by:
State Equation:

$$\dot{X} = AX + BU \tag{5.0.2.1}$$

Output equation:

$$Y = CX + DU \tag{5.0.2.2}$$

Comparing these with (5.0.1.1) and (5.0.1.2) we get the state matrices as,

$$A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \tag{5.0.2.3}$$

$$B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{5.0.2.4}$$

$$C = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{5.0.2.5}$$

$$D = 0 \tag{5.0.2.6}$$

5.0.3. Solving for the system transfer function $H(s)$.
Solution: The transfer function for the state space model is given by:

$$H(s) = C(sI - A)^{-1}B + D \tag{5.0.3.1}$$

$$= \frac{\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}}{s(s+2\beta)+\alpha} \tag{5.0.3.2}$$

$$\implies H(s) = \frac{1}{s^2+3s+2} \tag{5.0.3.3}$$

Therefore, poles of transfer function are given by:

$$s = -1 \tag{5.0.3.4}$$

$$s = -2 \tag{5.0.3.5}$$

Since the poles are negative real and distinct, the system is an OVERDAMPED SYSTEM.

5.0.4. Natural response and comparison plot.
Solution: Natural response:

$$h(t) = L^{-1}(H(s)) = e^{-t} - e^{-2t} \tag{5.0.4.1}$$

Solution: Comparison plot:
Consider the system with same natural frequency , but different damping ratios. The figure below shows a comparative plot of natural response between all the three types of systems.