Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot

5 State Space Analysis

5.0.1. A second-order LTI system is described by the following state equations:

$$\frac{\partial x_1(t)}{\partial t} - x_2(t) = 0 \tag{5.0.1.1}$$

$$\frac{\partial x_2(t)}{\partial t} + 2x_1(t) + 3x_2(t) = r(t) \tag{5.0.1.2}$$

where $x_1(t)$ and $x_2(t)$ are the two state variables and r(t) denotes the input. The output c(t) = $x_1(t)$.

5.0.2. Identify the type of system employing the state space model.
Solution: Any state space model is represented by:
State Equation:

$$\dot{X} = AX + BU \tag{5.0.2.1}$$

Output equation:

$$Y = CX + DU \tag{5.0.2.2}$$

Comparing these with (5.0.1.1) and (5.0.1.2) we get the state matrices as,

$$A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \tag{5.0.2.3}$$

$$B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{5.0.2.4}$$

$$C = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{5.0.2.5}$$

$$D = 0 \tag{5.0.2.6}$$

5.0.3. Solving for the system transfer function $H(s)$.
Solution: The transfer function for the state space model is given by:

$$H(s) = C(sI - A)^{-1}B + D \tag{5.0.3.1}$$

$$= \frac{\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}}{s(s+2\beta)+\alpha} \tag{5.0.3.2}$$

$$\implies H(s) = \frac{1}{s^2+3s+2} \tag{5.0.3.3}$$

Therefore, poles of transfer function are given by:

$$s = -1 \tag{5.0.3.4}$$

$$s = -2 \tag{5.0.3.5}$$

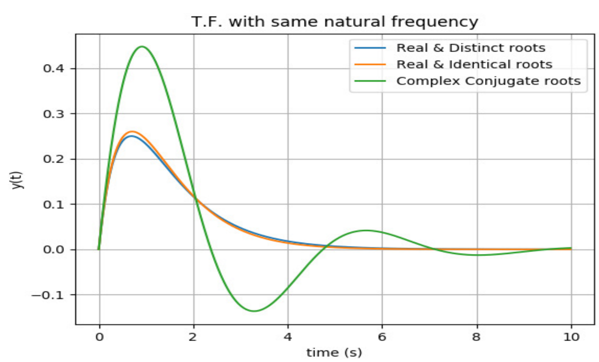
Since the poles are negative real and distinct, the system is an OVERDAMPED SYSTEM.

5.0.4. Perform the calculations to obtain the output as well as transfer function from state space matrices:
Solution: The following code:
https://github.com/neildhami18/EE2227/blob/master/codes

5.0.5. Natural response and comparison plot.
Solution: Natural response:

$$h(t) = L^{-1}(H(s)) = e^{-t} - e^{-2t} \tag{5.0.5.1}$$

Solution: Comparison plot:
Consider the system with same natural frequency , but different damping ratios. The figure below shows a comparative plot of natural response between all the three types of systems.


T.F. with same natural frequency
Real & Distinct roots
Real & Identical roots
Complex Conjugate roots
y(t)
0.4
0.3
0.2
0.1
0.0
−0.1
0
2
4
6
8
10
time (s)