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**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

## 1 STABILITY

## 2 ROUTH HURWITZ CRITERION

## 3 COMPENSATORS

## 4 NYQUIST PLOT

## 5 STATE SPACE ANALYSIS

5.0.1. A second-order LTI system is described by the following state equations:

$$\frac{\partial x_1(t)}{\partial t} - x_2(t) = 0 \quad (5.0.1.1)$$

$$\frac{\partial x_2(t)}{\partial t} + 2x_1(t) + 3x_2(t) = r(t) \quad (5.0.1.2)$$

where  $x_1(t)$  and  $x_2(t)$  are the two state variables and  $r(t)$  denotes the input. The output  $c(t) = x_1(t)$ .

5.0.2. Identify the type of system employing the state space model.

**Solution:** Any state space model is represented by:

State Equation:

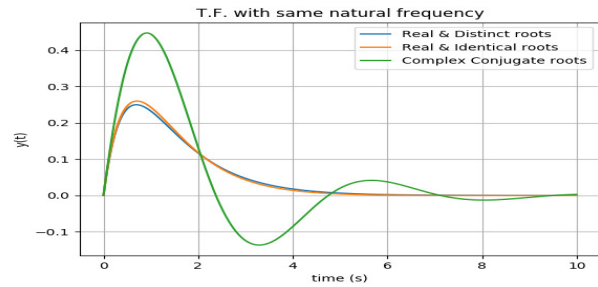
$$\dot{X} = AX + BU \quad (5.0.2.1)$$

Output equation:

$$Y = CX + DU \quad (5.0.2.2)$$

Comparing these with (5.0.1.1) and (5.0.1.2) we get the state matrices as,

$$A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \quad (5.0.2.3)$$



$$B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (5.0.2.4)$$

$$C = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (5.0.2.5)$$

$$D = 0 \quad (5.0.2.6)$$

5.0.3. Solving for the system transfer function  $H(s)$ .

**Solution:** The transfer function for the state space model is given by:

$$H(s) = C(sI - A)^{-1}B + D \quad (5.0.3.1)$$

$$= \frac{\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{s(s+2\beta) + \alpha} \quad (5.0.3.2)$$

$$\Rightarrow H(s) = \frac{1}{s^2 + 3s + 2} \quad (5.0.3.3)$$

Therefore, poles of transfer function are given by:

$$s = -1 \quad (5.0.3.4)$$

$$s = -2 \quad (5.0.3.5)$$

Since the poles are negative real and distinct, the system is an OVERDAMPED SYSTEM.

5.0.4. Natural response and comparison plot.

**Solution:** Natural response:

$$h(t) = L^{-1}(H(s)) = e^{-t} - e^{-2t} \quad (5.0.4.1)$$

**Solution:** Comparison plot:

Consider the system with same natural frequency, but different damping ratios. The figure below shows a comparative plot of natural response between all the three types of systems.