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1 Phase Margin 1

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Phase Margin

1.1. Find the Phase Margin of $G(s)$ in degrees where

$$G(s) = \frac{2}{(s+1)(s+2)} \tag{1.1.1}$$

Solution: Phase Margin:It is the difference between phase of the system and $-180°$ at the gain crossover frequency,(the gain crossover frequency being the frequency at which the open-loop gain first reaches 1).
Phase Margin is given by,

$$P.M = \phi - \angle G(j\omega)|_{\omega=\omega_{pc}} = \phi + 180° \tag{1.1.2}$$

where,

$$\phi = \angle G(j\omega)|_{\omega=\omega_{gc}} \tag{1.1.3}$$

ω_{pc} is the Phase crossover frequency (The frequency at which the phase of open-loop transfer function reaches -180°).
ω_{gc} is the Gain crossover frequency (The frequency at which the gain of the open-loop transfer fuction reaches 1).
Given,

$$G(s) = \frac{2}{(s+1)(s+2)} \tag{1.1.4}$$

$$G(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)} \tag{1.1.5}$$

We can find magnitude and phase as

$$|G(j\omega)| = \frac{2}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})} \tag{1.1.6}$$

$$\angle G(j\omega) = -tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{2}) \tag{1.1.7}$$

We know that, Gain in dB = 0 at $\omega = \omega_{gc}$

$$20log_{10}|G(j\omega_{gc})| = 0 \tag{1.1.8}$$

$$|G(j\omega_{gc})| = 1 \tag{1.1.9}$$

$$\frac{2}{(\sqrt{\omega_{gc}^2+1})(\sqrt{\omega_{gc}^2+4})} = 1 \tag{1.1.10}$$

Solving we get,

$$\omega_{gc}^2(\omega_{gc}^2+5) = 0 \tag{1.1.11}$$

$$=> \omega_{gc} = 0, +j\sqrt{5}, -j\sqrt{5} \tag{1.1.12}$$

As frequency is a real quantity
Hence, $\omega_{gc} \neq$ Imaginary

$$\therefore \omega_{gc} = 0 \tag{1.1.13}$$

From (1.1.3) and (1.1.7)

$$\phi = \angle G(j\omega_{gc}) = -tan^{-1}(\omega_{gc}) - tan^{-1}(\frac{\omega_{gc}}{2}) \tag{1.1.14}$$

$$=> \phi = 0° \tag{1.1.15}$$

$$\therefore P.M = 180° + 0° = 180° \tag{1.1.16}$$

1.2. We can verify the above result using phase plot.The following code plots Fig(1.3)

```
codes/ee18btech11017.py
```

1.3. The Phase plot is as shown, We can observe

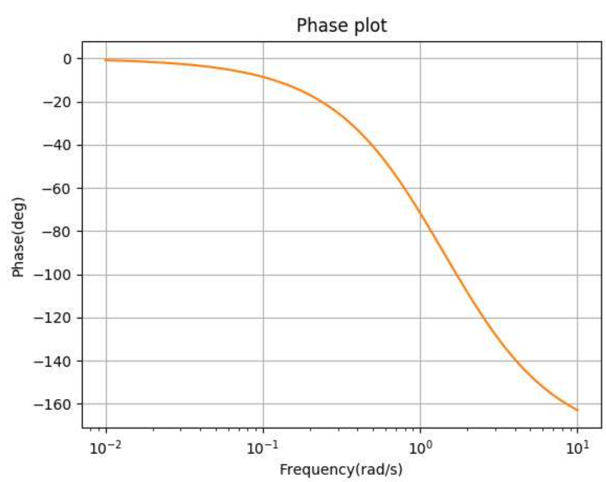

Fig. 1.3

that at $\omega_{gc} = 0$, $\phi = 0°$

$$\therefore P.M = 180° \tag{1.3.1}$$

1.4. **Application:** Phase margin is measure of stability in closed-loop, dynamic-control systems.(i.e, For stability of a system both gain margin and phase margin should be positive.)

1.5. **Example:**Phase Margin is a measure of stability.
Consider a standard second order system with

$$G(s)H(s) = \frac{\omega_n^2}{s(s+2\delta\omega_n)} \tag{1.5.1}$$

Sunstituting,s=jω

$$G(j\omega)H(j\omega) = \frac{\omega_n^2}{(j\omega)(j\omega+2\delta\omega_n)} \tag{1.5.2}$$

$$\tag{1.5.3}$$

At the gain cross over frequency ω_{gc},$|G(j\omega)H(j\omega)| = 1$

$$\frac{\omega_n^2}{\omega_{gc}\sqrt{\omega_{gc}^2 + 4\delta^2\omega_n^2}} = 1 \tag{1.5.4}$$

$$==> \omega_{gc}^4 + 4\delta_2\omega_{gc}^2\omega_n^2 - \omega_n^4 = 0 \tag{1.5.5}$$

Solving we get,

$$\omega_{gc}^2 = \omega_n^2\sqrt{4\delta^4+1} - 2\delta^2 \tag{1.5.6}$$

Phase Margin is given by,

$$P.M = 180^\circ + (-90^\circ - tan^{-1}(\frac{\omega_{gc}}{2\delta\omega_n})) \tag{1.5.7}$$

Further simplifying we get

$$P.M = tan^{-1}(\frac{2\delta\omega_n}{\omega_{gc}}) \tag{1.5.8}$$

We can observe from (1.5.8) for P.M to be negative
1.$\delta < 0$,$\omega_n > 0$
2.$\delta > 0$,$\omega_n < 0$
For both the cases the pole in the Equation(1.5.1) lie on right half of s-plane . So,the system is unstable.