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1 Phase Margin 1

Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 PHASE MARGIN

1.1. Find the Phase Margin of $G(s)$ in degrees where

$$G(s) = \frac{2}{(s+1)(s+2)} \quad (1.1.1)$$

Solution: Phase Margin: It is the difference between phase of the system and -180° at the gain crossover frequency, (the gain crossover frequency being the frequency at which the open-loop gain first reaches 1).

Phase Margin is given by,

$$P.M = \phi - \angle G(j\omega)|_{\omega=\omega_{pc}} = \phi + 180^\circ \quad (1.1.2)$$

where,

$$\phi = \angle G(j\omega)|_{\omega=\omega_{gc}} \quad (1.1.3)$$

ω_{pc} is the Phase crossover frequency (The frequency at which the phase of open-loop transfer function reaches -180°).

ω_{gc} is the Gain crossover frequency (The frequency at which the gain of the open-loop transfer function reaches 1).

Given,

$$G(s) = \frac{2}{(s+1)(s+2)} \quad (1.1.4)$$

$$G(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)} \quad (1.1.5)$$

We can find magnitude and phase as

$$|G(j\omega)| = \frac{2}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})} \quad (1.1.6)$$

$$\angle G(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) \quad (1.1.7)$$

We know that, Gain in dB = 0 at $\omega = \omega_{gc}$

$$20\log_{10}|G(j\omega_{gc})| = 0 \quad (1.1.8)$$

$$|G(j\omega_{gc})| = 1 \quad (1.1.9)$$

$$\frac{2}{(\sqrt{\omega_{gc}^2+1})(\sqrt{\omega_{gc}^2+4})} = 1 \quad (1.1.10)$$

Solving we get,

$$\omega_{gc}^2(\omega_{gc}^2+5) = 0 \quad (1.1.11)$$

$$\Rightarrow \omega_{gc} = 0, +j\sqrt{5}, -j\sqrt{5} \quad (1.1.12)$$

As frequency is a real quantity

Hence, $\omega_{gc} \neq \text{Imaginary}$

$$\therefore \omega_{gc} = 0 \quad (1.1.13)$$

From (1.1.7) and (1.1.3)

$$\phi = \angle G(j\omega_{gc}) = -\tan^{-1}(\omega_{gc}) - \tan^{-1}\left(\frac{\omega_{gc}}{2}\right) \quad (1.1.14)$$

$$\Rightarrow \phi = 0^\circ \quad (1.1.15)$$

$$\therefore P.M = 180^\circ + 0^\circ = 180^\circ \quad (1.1.16)$$

1.2. We can verify the above result using phase plot. The following code plots Fig(1.3)

```
codes/ee18btech11017.py
```

1.3. The Phase plot is as shown, We can observe

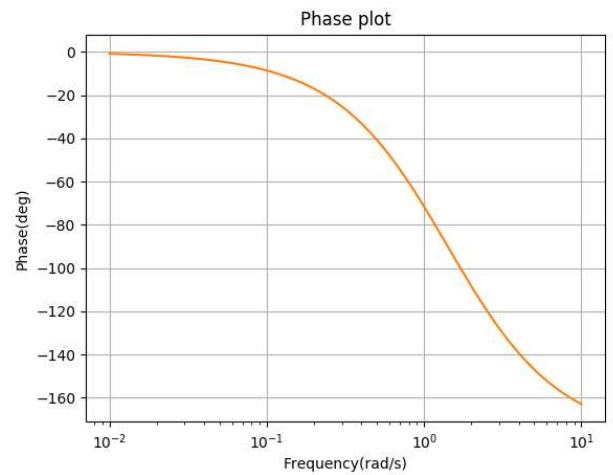


Fig. 1.3

that at $\omega_{gc} = 0$, $\phi = 0^\circ$

$$\therefore P.M = 180^\circ \quad (1.3.1)$$