

## CONTENTS

# 1 Phase Margin 1

**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

## 1 PHASE MARGIN

1.1. Find the Phase Margin of  $G(s)$  in degrees where

$$G(s) = \frac{2}{(s+1)(s+2)} \quad (1.1.1)$$

**Solution: Phase Margin:** It is the difference between phase of the system and  $-180^\circ$  at the gain crossover frequency, (the gain crossover frequency being the frequency at which the open-loop gain first reaches 1).

Phase Margin is given by,

$$P.M = \phi - \angle G(j\omega)|_{\omega=\omega_{pc}} = \phi + 180^\circ \quad (1.1.2)$$

where,

$$\phi = \angle G(j\omega)|_{\omega=\omega_{gc}} \quad (1.1.3)$$

$\omega_{pc}$  is the Phase crossover frequency (The frequency at which the phase of open-loop transfer function reaches  $-180^\circ$ ).

$\omega_{gc}$  is the Gain crossover frequency (The frequency at which the gain of the open-loop transfer function reaches 1).

Given,

$$G(s) = \frac{2}{(s+1)(s+2)} \quad (1.1.4)$$

$$G(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)} \quad (1.1.5)$$

We can find magnitude and phase as

$$|G(j\omega)| = \frac{2}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})} \quad (1.1.6)$$

$$\angle G(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) \quad (1.1.7)$$

We know that, Gain in dB = 0 at  $\omega = \omega_{gc}$

$$20\log_{10}|G(j\omega_{gc})| = 0 \quad (1.1.8)$$

$$|G(j\omega_{gc})| = 1 \quad (1.1.9)$$

$$\frac{2}{(\sqrt{\omega_{gc}^2+1})(\sqrt{\omega_{gc}^2+4})} = 1 \quad (1.1.10)$$

Solving we get,

$$\omega_{gc}^2(\omega_{gc}^2+5) = 0 \quad (1.1.11)$$

$$\Rightarrow \omega_{gc} = 0, +j\sqrt{5}, -j\sqrt{5} \quad (1.1.12)$$

As frequency is a real quantity

Hence,  $\omega_{gc} \neq \text{Imaginary}$

$$\therefore \omega_{gc} = 0 \quad (1.1.13)$$

From (1.1.3) and (1.1.7)

$$\phi = \angle G(j\omega_{gc}) = -\tan^{-1}(\omega_{gc}) - \tan^{-1}\left(\frac{\omega_{gc}}{2}\right) \quad (1.1.14)$$

$$\Rightarrow \phi = 0^\circ \quad (1.1.15)$$

$$\therefore P.M = 180^\circ + 0^\circ = 180^\circ \quad (1.1.16)$$

1.2. We can verify the above result using phase plot. The following code plots Fig(1.3)

```
codes/ee18btech11017.py
```

1.3. The Phase plot is as shown, We can observe

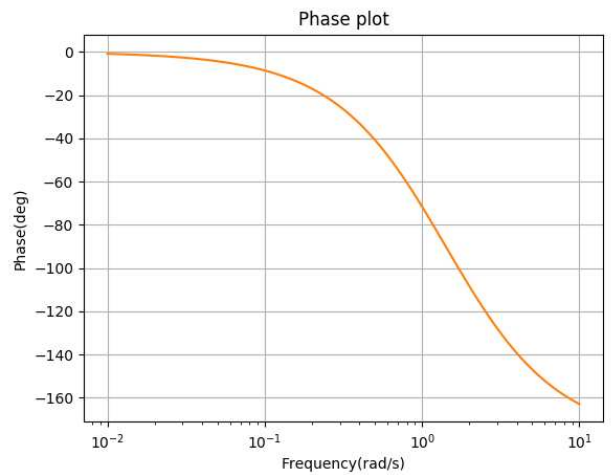


Fig. 1.3

that at  $\omega_{gc} = 0$ ,  $\phi = 0^\circ$

$$\therefore P.M = 180^\circ \quad (1.3.1)$$

1.4. **Application:** Phase margin is measure of stability in closed-loop, dynamic-control systems.(i.e, For stability of a system both gain margin and phase margin should be positive.)

1.5. **Example:**Phase Margin is a measure of stability.

Consider a unity negative feedback system with open loop gain ,

$$G(s) = \frac{10000}{s(s+10)^2} \quad (1.5.1)$$

We can find mgnitude and phase as

$$|G(j\omega)| = \frac{10^4}{\omega \sqrt{(\omega^2 + 10^2)^2}} \quad (1.5.2)$$

$$\angle G(j\omega) = -90^\circ - 2\tan^{-1} \frac{\omega}{10} \quad (1.5.3)$$

At the gain cross over frequency  $\omega_{gc}, |G(j\omega)| = 1$

$$\frac{10^4}{\omega_{gc} \sqrt{(\omega_{gc}^2 + 10^2)^2}} = 1 \quad (1.5.4)$$

$$(1.5.5)$$

Simplifying we get,

$$\omega_{gc}^3 + 10^2 \omega_{gc} - 10^4 = 0 \quad (1.5.6)$$

$$\Rightarrow \omega_{gc} = 20, -10 + 20j, -10 - 20j \quad (1.5.7)$$

As frequency is a real quantity

Hence,  $\omega_{gc} \neq \text{Imaginary}$

$$\therefore \omega_{gc} = 20 \quad (1.5.8)$$

From (1.5.3) and (1.5.8) we get,

$$P.M = 180^\circ - 90^\circ - 2\tan^{-1}\left(\frac{\omega_{gc}}{10}\right) \quad (1.5.9)$$

$$\Rightarrow P.M = -36.9^\circ \quad (1.5.10)$$

We can verify the above result using phase plot.The following code plots Fig(1.5)

codes/ee18btech11017.py

The Phase plot is as shown, We can observe that at  $\omega_{gc} = 20$  ,  $\phi = -216.86^\circ$

$$\therefore P.M = 180^\circ + -216.86^\circ = -36.9^\circ \quad (1.5.11)$$

$$(1.5.12)$$

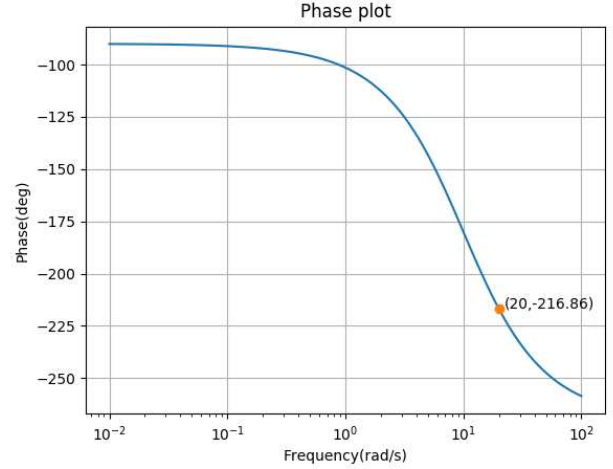


Fig. 1.5

As Phase Margin is negative . Therefore the closed loop system is unstable . We can verify closed loop stability using Routh-Hurwitz criterion.

1.6. Verifying the above example using Routh-Hurwitz criterion.

Let  $T(s)$  be Closed loop transfer function ,

$$T(s) = \frac{N(s)}{D(s)} = \frac{G(s)}{1 + G(s)} \quad (1.6.1)$$

The characteristic equation is

$$D(s) = 0 \quad (1.6.2)$$

$$1 + G(s) = 0 \quad (1.6.3)$$

$$\Rightarrow s^3 + 20s^2 + 100s + 10000 = 0 \quad (1.6.4)$$

Constructing routh array for (1.6.4)...

$$\begin{array}{c|ccc} s^3 & 1 & 100 & 0 \\ s^2 & 20 & 10000 & 0 \\ s & -400 & 0 & 0 \end{array} \quad (1.6.5)$$

$$\begin{array}{c|ccc} s^3 & 1 & 100 & 0 \\ s^2 & 20 & 10000 & 0 \\ s & -400 & 0 & 0 \\ s^0 & 10000 & 0 & 0 \end{array} \quad (1.6.6)$$

There are two sign changes in the first column

of the routh array. So, two poles lie on right half of s-plane.

Therefore, the system is unstable.

The following code generates routh array.

```
codes/RouthHurwitz.py
```