

## CONTENTS

# 1 Phase Margin 1

**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

## 1 PHASE MARGIN

1.1. Find the Phase Margin of  $G(s)$  in degrees where

$$G(s) = \frac{2}{(s+1)(s+2)} \quad (1.1.1)$$

**Solution: Phase Margin:** It is the difference between phase of the system and  $-180^\circ$  at the gain crossover frequency, (the gain crossover frequency being the frequency at which the open-loop gain first reaches 1).

Phase Margin is given by,

$$P.M = \phi - \angle G(j\omega)|_{\omega=\omega_{pc}} = \phi + 180^\circ \quad (1.1.2)$$

where,

$$\phi = \angle G(j\omega)|_{\omega=\omega_{gc}} \quad (1.1.3)$$

$\omega_{pc}$  is the Phase crossover frequency (The frequency at which the phase of open-loop transfer function reaches  $-180^\circ$ ).

$\omega_{gc}$  is the Gain crossover frequency (The frequency at which the gain of the open-loop transfer function reaches 1).

Given,

$$G(s) = \frac{2}{(s+1)(s+2)} \quad (1.1.4)$$

$$G(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)} \quad (1.1.5)$$

We can find magnitude and phase as

$$|G(j\omega)| = \frac{2}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})} \quad (1.1.6)$$

$$\angle G(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) \quad (1.1.7)$$

We know that, Gain in dB = 0 at  $\omega = \omega_{gc}$

$$20\log_{10}|G(j\omega_{gc})| = 0 \quad (1.1.8)$$

$$|G(j\omega_{gc})| = 1 \quad (1.1.9)$$

$$\frac{2}{(\sqrt{\omega_{gc}^2+1})(\sqrt{\omega_{gc}^2+4})} = 1 \quad (1.1.10)$$

Solving we get,

$$\omega_{gc}^2(\omega_{gc}^2+5) = 0 \quad (1.1.11)$$

$$\Rightarrow \omega_{gc} = 0, +j\sqrt{5}, -j\sqrt{5} \quad (1.1.12)$$

As frequency is a real quantity

Hence,  $\omega_{gc} \neq \text{Imaginary}$

$$\therefore \omega_{gc} = 0 \quad (1.1.13)$$

From (1.1.3) and (1.1.7)

$$\phi = \angle G(j\omega_{gc}) = -\tan^{-1}(\omega_{gc}) - \tan^{-1}\left(\frac{\omega_{gc}}{2}\right) \quad (1.1.14)$$

$$\Rightarrow \phi = 0^\circ \quad (1.1.15)$$

$$\therefore P.M = 180^\circ + 0^\circ = 180^\circ \quad (1.1.16)$$

1.2. We can verify the above result using phase plot. The following code plots Fig(1.3)

```
codes/ee18btech11017.py
```

1.3. The Phase plot is as shown, We can observe

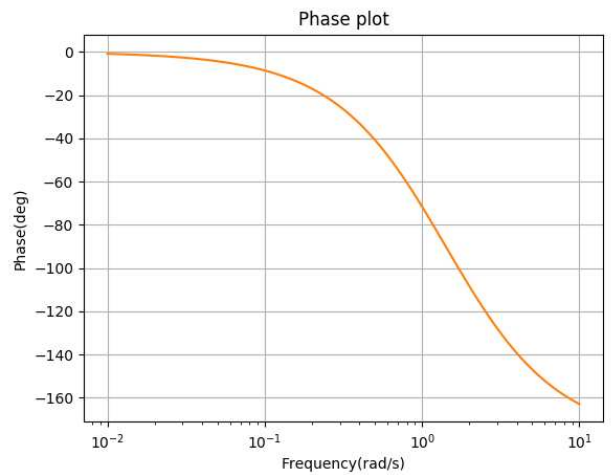


Fig. 1.3

that at  $\omega_{gc} = 0$ ,  $\phi = 0^\circ$

$$\therefore P.M = 180^\circ \quad (1.3.1)$$

1.4. **Application:** Phase margin is measure of stability in closed-loop, dynamic-control systems.(i.e, For stability of a system both gain margin and phase margin should be positive.)

1.5. **Example:**Phase Margin is a measure of stability.

Consider a standard second order system with

$$G(s)H(s) = \frac{\omega_n^2}{s(s + 2\delta\omega_n)} \quad (1.5.1)$$

Substituting,  $s=j\omega$

$$G(j\omega)H(j\omega) = \frac{\omega_n^2}{(j\omega)(j\omega + 2\delta\omega_n)} \quad (1.5.2)$$

$$(1.5.3)$$

At the gain cross over frequency  $\omega_{gc}$ ,  $|G(j\omega)H(j\omega)| = 1$

$$\frac{\omega_n^2}{\omega_{gc} \sqrt{\omega_{gc}^2 + 4\delta^2\omega_n^2}} = 1 \quad (1.5.4)$$

$$\Rightarrow \omega_{gc}^4 + 4\delta^2\omega_{gc}^2\omega_n^2 - \omega_n^4 = 0 \quad (1.5.5)$$

Solving we get,

$$\omega_{gc}^2 = \omega_n^2 \sqrt{4\delta^4 + 1} - 2\delta^2 \quad (1.5.6)$$

Phase Margin is given by,

$$P.M = 180^\circ + (-90^\circ - \tan^{-1}(\frac{\omega_{gc}}{2\delta\omega_n})) \quad (1.5.7)$$

Further simplifying we get

$$P.M = \tan^{-1}(\frac{2\delta\omega_n}{\omega_{gc}}) \quad (1.5.8)$$

We can observe from (1.5.8) for P.M to be negative

$$1. \delta < 0, \omega_n > 0$$

$$2. \delta > 0, \omega_n < 0$$

For both the cases the pole in the Equation(1.5.1) lie on right half of s-plane .

So, the system is unstable.