Control systems

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Problem

Consider the transfer function $G(s) = \frac{2}{(s+1)(s+2)}$. The Phase Margin of $G(s)$ in degrees is ________

Solution

Gain Margin: The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable. It is usually expressed as a magnitude in dB.

Gain Margin $= \frac{1}{|G(j\omega)|}$ at $\omega = \omega_{pc}$

ω_{pc} =phase crossover frequency (The frequency at which at which phase becomes -180°

Phase Margin: Phase margin refers to the amount of phase, which can be increased or decreased without making the system unstable. It is usually expressed as a phase in degrees.

Phase margin $=\phi - \angle(G(j\omega)|_{\omega=\omega_{pc}} = 180^\circ + \phi$
Where , $\phi = \angle G(j\omega)_{\omega=\omega_{gc}}$

$\omega_{gc} =$ The Gain crossover frequency (frequency where Gain becomes 0)
Given, $G(s) = \frac{2}{(s+1)(s+2)}$

$$\implies G(j\omega) = \frac{2}{(j\omega+1)(j\omega+2)}$$

$$\implies |G(j\omega)| = \frac{2}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})}$$

$$\implies \angle G(j\omega) = -tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{2})$$

To find find gain margin we need find $\angle G(j\omega)$

$$\angle(G(j\omega)|_{\omega=\omega_{pc}} = -180^{\circ}\text{=-tan}^{-1}(\omega) - tan^{-1}(\frac{\omega}{2})$$

$$\implies \omega_{pc} = \infty \implies \text{Gain margin} = \infty$$

We have to find phase margin, which is calculated over the gain cross over frequency (ω_{gc})
To find ω_{gc},
We know , Gain=0 at $\omega = \omega_{gc}$

$$\implies log_{10}|G(j\omega)| = 0 \text{ at } \omega = \omega_{gc}$$

$$\implies |G(j\omega_{gc})| = 1$$

So, $\frac{2}{(\sqrt{\omega_{gc}^2+1})(\sqrt{\omega_{gc}^2+4})} = 1$

$$\implies (\omega_{gc}^2 + 1)(\omega_{gc}^2 + 4) = 4$$

$$\implies \omega_{gc}^4 + 5\omega_{gc}^2 + 4 = 4$$

$$\implies \omega_{gc}^2(\omega_{gc}^2 + 5) = 0$$

$\therefore \omega_{gc} = 0, +j\sqrt{5}, -j\sqrt{5}$

As frequency is a real quantity
Hence, $\omega_{gc} \neq$ Imaginary
So, $\omega_{gc} = 0$

$$\therefore \angle G(j\omega_{gc}) = -tan^{-1}(0) - tan^{-1}(0) = 0$$

$$\implies \phi = 0^{\circ}$$

$$\therefore PhaseMargin = 180^{\circ} + 0^{\circ}$$

$\therefore$ **Phase Margin = 180°**

we can verify the phase margin by bode plot