Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

1 Stability

1.1. The state equation and the output equation of a control system are given below :

$$\dot{X} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} X + \begin{bmatrix} 4 \\ 0 \end{bmatrix} U \tag{1.1.1}$$

$$Y = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} X \tag{1.1.2}$$

Then transfer function representation of the system is

1.2. **Solution:** when

$$\dot{X} = AX + BU \tag{1.2.1}$$

$$Y = CX + DU \tag{1.2.2}$$

where A, B, C, D are matrices
Then the transfer function can be find using

$$T(s) = C[(sI - A)^{-1}].B + D \tag{1.2.3}$$

From the given state space representation of the system, we can find matrices as

$$A = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} \tag{1.2.4}$$

$$B = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \tag{1.2.5}$$

$$C = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} \tag{1.2.6}$$

We can find the transfer function using

$$T(s) = C[(sI - A)^{-1}].B \tag{1.2.7}$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} \tag{1.2.8}$$

$$(sI - A) = \begin{bmatrix} s+4 & -1.5 \\ -4 & s \end{bmatrix} \tag{1.2.9}$$

$$|sI - A| = s(s+4) - (-4) \times (-1.5) \tag{1.2.10}$$

$$= s^2 + 4s + 6 \tag{1.2.11}$$

$$Adj[sI - A] = \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix} \tag{1.2.12}$$

Hence

$$[sI - A]^{-1} = \frac{Adj[sI - A]}{|sI - A|} \tag{1.2.13}$$

$$= \begin{bmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{(s+4)}{(s^2+4s+6)} \end{bmatrix} \tag{1.2.14}$$

$$[sI - A]^{-1}.B = \begin{bmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{(s+4)}{(s^2+4s+6)} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix} \tag{1.2.15}$$

$$\therefore \quad [sI - A]^{-1}.B = \begin{bmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{bmatrix} \tag{1.2.16}$$

$$Substituting\ the\ values\ of\ [sI - A]^{-1}.B \quad and\ C\ in\ equation\ (1.2.7)$$

$$T(s) = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} \begin{bmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{bmatrix} \tag{1.2.17}$$

$$T(s) = \begin{bmatrix} \frac{6s}{(s^2+4s+6)} + \frac{10}{(s^2+4s+6)} \end{bmatrix} \tag{1.2.18}$$

the transfer function representation of the system is

$$\therefore \quad \mathbf{T(s)} = \begin{bmatrix} \frac{6s+10}{(s^2+4s+6)} \end{bmatrix} \tag{1.2.19}$$

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot