CONTROL SYSTEMS
2018 Q.29 (EC SECTION)

K SRIKANTH
EE18BTECH11023

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QUESTION 2018 Q.29 (EC SECTION)

- The state equation and the output equation of a control system are given below :

$$\dot{X} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} X + \begin{bmatrix} 4 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} X$$

Then transfer function representation of the system is

(A) $\frac{(3s+5)}{(s^2+4s+6)}$

(B) $\frac{(3s-1.875)}{(s^2+4s+6)}$

(C) $\frac{(4s+1.5)}{(s^2+4s+6)}$

(D) $\frac{(6s+5)}{(s^2+4s+6)}$

SOLUTION

- From the given state space representation of the system, we can find matrices as

$$A = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix}$$

  when

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

  where $\;\mathrm{A, B, C, D}\;$ are matrices
  Then the transfer function can be find using

$$T(s) = C[(sI - A)^{-1}].B + D$$

SOLUTION

We can find the transfer function using

$$T(s) = C[(sI - A)^{-1}].B \tag{1}$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s+4 & -1.5 \\ -4 & s \end{bmatrix} \tag{2}$$

$$|sI - A| \quad = s(s+4) - (-4) \times (-1.5)$$

$$= s^2 + 4s + 6 \tag{3}$$

SOLUTION

$$Adj[sI - A] = \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

Hence

$$[sI - A]^{-1} = \frac{Adj[sI - A]}{|sI - A|} = \begin{bmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{(s+4)}{(s^2+4s+6)} \end{bmatrix} \tag{4}$$

$$[sI - A]^{-1}.B = \begin{bmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{(s+4)}{(s^2+4s+6)} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix} \tag{5}$$

SOLUTION

$$\therefore \qquad [sI - A]^{-1}.B = \begin{bmatrix} \frac{2s}{(s^2+4s+6)} \\ \frac{8}{(s^2+4s+6)} \end{bmatrix} \tag{6}$$

$$Substituting\, values\, of\, [sI - A]^{-1}.B\, and\, C\, in\, equation\, (1)$$

$$T(s) = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} \begin{bmatrix} \frac{2s}{(s^2+4s+6)} \\ \frac{8}{(s^2+4s+6)} \end{bmatrix} \tag{7}$$

SOLUTION

$$T(s) = \left[\ \frac{3s}{(s^2+4s+6)} + \frac{5}{(s^2+4s+6)}\ \right]$$

$$the\, transfer\, function\, representation\, of\, the\, system\, is$$

$$\therefore \qquad \mathbf{T(s)} = \left[\ \frac{3s+5}{(s^2+4s+6)}\ \right] \tag{8}$$