Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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5.1. The state equation and the output equation of a control system are given below :

$$\dot{\mathbf{X}} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} \mathbf{U} \tag{5.1.1}$$

$$\mathbf{Y} = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \mathbf{X} \tag{5.1.2}$$

Then transfer function representation of the system is

5.2. **Solution:** when

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU} \tag{5.2.1}$$

$$\mathbf{Y} = \mathbf{CX} + \mathbf{DU} \tag{5.2.2}$$

Compare the results of star configuration with the results of delta configuration to verify the accuracy of your calculation. where A,B,C,D are matrices. Then the transfer function can be find using

$$T(s) = \mathbf{C}\left[(sI - \mathbf{A})^{-1}\right].\mathbf{B} + \mathbf{D} \tag{5.2.3}$$

From the given state space representation of the system, we can find matrices as

$$\mathbf{A} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \tag{5.2.4}$$

$$\mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5.2.5}$$

$$\mathbf{C} = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \tag{5.2.6}$$

We can find the transfer function using

$$T(s) = \mathbf{C}\left[(sI - \mathbf{A})^{-1}\right].\mathbf{B} + \mathbf{D} \tag{5.2.7}$$

$$(sI - \mathbf{A}) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \tag{5.2.8}$$

$$(sI - \mathbf{A}) = \begin{pmatrix} s-4 & -1.5 \\ 4 & s \end{pmatrix} \tag{5.2.9}$$

$$|sI - \mathbf{A}| = s(s+4) - (-4)\times(-1.5) \tag{5.2.10}$$

$$|sI - \mathbf{A}| = s^2 + 4s + 6 \tag{5.2.11}$$

and from (5.2.9)

$$Adj\,[sI - \mathbf{A}] = \begin{pmatrix} s & -1.5 \\ 4 & s+4 \end{pmatrix} \tag{5.2.12}$$

$$[sI - \mathbf{A}]^{-1} = \frac{Adj[sI - \mathbf{A}]}{|sI - \mathbf{A}|} \tag{5.2.13}$$

$$= \begin{pmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{s+4}{(s^2+4s+6)} \end{pmatrix}$$

$$[sI - \mathbf{A}]^{-1}.\mathbf{B} = \begin{pmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{s+4}{(s^2+4s+6)} \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5.2.14}$$

$$\therefore \qquad [sI - \mathbf{A}]^{-1}.\mathbf{B} = \begin{pmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{pmatrix} \tag{5.2.15}$$

$Substituting\ the\ values\ of\ [sI - \mathbf{A}]^{-1}.\mathbf{B}\ and\ \mathbf{C}\ in\ equation(1.2.7)$

$$T(s) = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \begin{pmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{pmatrix} \tag{5.2.16}$$

$$T(s) = \left(\frac{6s}{(s^2+4s+6)} + \frac{10}{(s^2+4s+6)}\right) \tag{5.2.17}$$

the transfer function representation of the system is

$$\therefore \qquad \mathbf{T(s)} = \left(\frac{6s+10}{(s^2+4s+6)}\right) \tag{5.2.18}$$

verify the answer with python code https://github.com/srikanth2001/EE2227-control-systems/tree/master/codes

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