## Control Systems

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Abstract-This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.

Download python codes using

svn co https://github.com/gadepall/school/trunk/ control/codes

tion of

$$\dot{X} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} X + \begin{bmatrix} 4 \\ 0 \end{bmatrix} U \tag{5.1.1}$$

$$Y = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} X \tag{5.1.2}$$

of the

$$\dot{X} = AX + BU \tag{5.2.1}$$

$$Y = CX + DU (5.2.2)$$

where A, B, C, D are matrices Then the transfer function can be find using

$$T(s) = C[(sI - A)^{-1}].B + D (5.2.3)$$

From the given state space representation of the system, we can find matrices as

$$A = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}$$
 (5.2.4)

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(5.2.19)

$$B = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \tag{5.2.5}$$

$$C = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix}$$
 (5.2.6)

$$T(s) = \left[ \frac{6s}{(s^2 + 4s + 6)} + \frac{10}{(s^2 + 4s + 6)} \right]$$
 (5.2.18)

the transfer function representation of the system is

.  $T(s) = \left[ \frac{6s+10}{(s^2+4s+6)} \right]$ 

**6** Nyquist Plot

7 Compensators8 Phase Margin

We can find the transfer function using

$$T(s) = C[(sI - A)^{-1}].B (5.2.7)$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}$$
 (5.2.8) 5.1 Controllability and Observability 5.2 Second Order System

$$(sI - A) = \begin{bmatrix} s + 4 & -1.5 \\ -4 & s \end{bmatrix}$$
 (5.2.9)

$$|sI - A| = s(s + 4) - (-4) \times (-1.5)$$
 (5.2.10)

$$|sI - A| = s^2 + 4s + 6 (5.2.11)$$

and from (1.2.9)

$$Adj[sI - A] = \begin{bmatrix} s & -1.5 \\ 4 & s + 4 \end{bmatrix}$$
 (5.2.12)

Hence

$$[sI - A]^{-1} = \frac{Adj[sI - A]}{|sI - A|}$$
 (5.2.13)

$$= \begin{bmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{(s+4)}{(s^2+4s+6)} \end{bmatrix}$$
 (5.2.14)

$$[sI - A]^{-1}.B = \begin{bmatrix} \frac{s}{(s^2 + 4s + 6)} & \frac{-1.5}{(s^2 + 4s + 6)} \\ \frac{4}{(s^2 + 4s + 6)} & \frac{(s + 4)}{(s^2 + 4s + 6)} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$
(5.2.15)

$$[sI - A]^{-1}.B = \begin{bmatrix} \frac{4s}{(s^2 + 4s + 6)} \\ \frac{16}{(s^2 + 4s + 6)} \end{bmatrix} (5.2.16)$$

Substituting the values of  $[sI - A]^{-1}.B$ and C in equation (1.2.7)

$$T(s) = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} \begin{bmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{bmatrix}$$
 (5.2.17)