Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

0.1.

$$\dot{x(t)} = Ax(t) + Bu(t) \tag{0.1.1}$$
$$y(t) = Cx(t) + Du(t) \tag{0.1.2}$$

Taking Laplace transform on both sides we have the following equations

$$sIX(s) = AX(s) + BU(s) \tag{0.1.3}$$
$$(sI - A)X(s) = BU(s) \tag{0.1.4}$$
$$X(s) = (sI - A)^{-1}BU(s) \tag{0.1.5}$$

and

$$Y(s) = CX(s) + DIU(s) \tag{0.1.6}$$

substituting 0.1.1 in this equation gives

$$Y(s) = (C(sI - A)^{-1}B + DI)U(s) \tag{0.1.7}$$

$$H(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + DI \tag{0.1.8}$$

In the above question D=0, so our transfer function becomes

$$C(sI - A)^{-1}B = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{0.1.9}$$

Now,

$$\frac{Y(s)}{U(s)} = \frac{\frac{Y(s)}{X_1(s)}}{\frac{U(s)}{X_1(s)}} \tag{0.1.10}$$

We get

$$\frac{Y(s)}{X_1(s)} = 1 \tag{0.1.11}$$

and

$$\frac{U(s)}{X_1(s)} = s^3 + 3s^2 + 2s + 1 \tag{0.1.12}$$

giving

$$U(s) = s^3X_1(s) + 3s^2X_1(s) + 2sX_1(s) + X_1(s) \tag{0.1.13}$$

so equation 0.1.13 can be written as

$$\begin{bmatrix} sX_1(s) \\ s^2X_1(s) \\ s^3X_1(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} X_1(s) \\ sX_1(s) \\ s^2X1(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U \tag{0.1.14}$$

So A=$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$

and

$$\tag{0.1.15}$$

$$Y = X_1(s) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} X_1(s) \\ sX_1(s) \\ s^2X_1(s) \end{bmatrix} \tag{0.1.16}$$

$$\tag{0.1.17}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{0.1.18}$$

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot