Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

0.1. State the general model of a state space system specifying the dimensions of the matrices and vectors.

Solution: The model is given by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{0.1.1}$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \tag{0.1.2}$$

with parameters listed in Table 0.1.

TABLE 0.1

0.2. Find the transfer function $\mathbf{H}(s)$ for the general system.

Solution: Taking Laplace transform on both sides we have the following equations

$$s\mathbf{I}X(s) - x(0) = \mathbf{A}X(s) + \mathbf{B}U(s) \tag{0.2.1}$$
$$(s\mathbf{I} - \mathbf{A})X(s) = \mathbf{B}U(s) + x(0) \tag{0.2.2}$$
$$X(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}U(s) + (s\mathbf{I} - \mathbf{A})^{-1}x(0) \tag{0.2.3}$$

and

$$Y(s) = \mathbf{C}X(s) + D\mathbf{I}U(s) \tag{0.2.4}$$

Substituting from (0.2.3) in the above,

$$Y(s) = (\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D\mathbf{I})U(s) + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}x(0) \tag{0.2.5}$$

0.3. Find $H(s)$ for a SISO (single input single output) system.

Solution:

$$H(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D\mathbf{I} \tag{0.3.1}$$

0.4. Given

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{0.4.1}$$
$$D = 0 \tag{0.4.2}$$
$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{0.4.3}$$

find $\mathbf{A}$ and $\mathbf{C}$ such that the state-space realization is in *controllable canonical form*.

Solution:

$$\because \frac{Y(s)}{U(s)} = \frac{Y(s)}{X(s)} \times \frac{X(s)}{U(s)}, \tag{0.4.4}$$

letting

$$\frac{Y(s)}{X(s)} = 1, \tag{0.4.5}$$

results in

$$\frac{U(s)}{X(s)} = s^3 + 3s^2 + 2s + 1 \tag{0.4.6}$$

giving

$$U(s) = s^3X(s) + 3s^2X(s) + 2sX(s) + X(s) \tag{0.4.7}$$

so equation 0.1.13 can be written as

$$\begin{pmatrix} sX(s) \\ s^2X(s) \\ s^3X(s) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \begin{pmatrix} X(s) \\ sX(s) \\ s^2X(s) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} U \tag{0.4.8}$$

So

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \tag{0.4.9}$$

$$Y = X(s) = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} X(s) \\ sX(s) \\ s^2X(s) \end{pmatrix} \tag{0.4.10}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \tag{0.4.11}$$

0.5. Obtain $\mathbf{A}$ and $\mathbf{C}$ so that the state-space realization in in *observable canonical form*.

0.6. Find the eigenvalues of $\mathbf{A}$ and the roots of the characteristic equation of $H(s)$ using a python code.

Solution: The code eigen.py

provides the necessary solution as

The eigenvalues are:

-2.32471796+0.j , -0.33764102+0.56227951j,
-0.33764102-0.56227951j
Roots of the the polynomial:

-2.32471796+0.j , -0.33764102+0.56227951j,
-0.33764102-0.56227951j

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot