Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

0.1.

$$\dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t) \quad (0.1.1)$$

$$y(t) = \boldsymbol{C}x(t) + Du(t) \quad (0.1.2)$$

Taking Laplace transform on both sides we have the following equations

$$s\boldsymbol{I}X(s) - x(0) = \boldsymbol{A}X(s) + \boldsymbol{B}U(s) \quad (0.1.3)$$

$$(s\boldsymbol{I} - \boldsymbol{A})X(s) = \boldsymbol{B}U(s) + x(0) \quad (0.1.4)$$

$$X(s) = (s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}U(s) + (s\boldsymbol{I} - \boldsymbol{A})^{-1}x(0) \quad (0.1.5)$$

and

$$Y(s) = \boldsymbol{C}X(s) + D\boldsymbol{I}U(s) \quad (0.1.6)$$

substituting 0.1.1 in this equation gives

$$Y(s) = (\boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + D\boldsymbol{I})U(s) + \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}x(0) \quad (0.1.7)$$

Assuming initially system was at rest i.e x(0)=0

$$H(s) = \frac{Y(s)}{U(s)} = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + D\boldsymbol{I} \quad (0.1.8)$$

Given,D=0 and $\boldsymbol{B} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \quad (0.1.9)$$

Now,

$$\frac{Y(s)}{U(s)} = \frac{\frac{Y(s)}{X(s)}}{\frac{U(s)}{X(s)}} \quad (0.1.10)$$

We get

$$\frac{Y(s)}{X(s)} = 1 \quad (0.1.11)$$

and

$$\frac{U(s)}{X(s)} = s^3 + 3s^2 + 2s + 1 \quad (0.1.12)$$

giving

$$U(s) = s^3X(s) + 3s^2X(s) + 2sX(s) + X(s) \quad (0.1.13)$$

so equation 0.1.13 can be written as

$$\begin{pmatrix} sX(s) \\ s^2X(s) \\ s^3X(s) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \begin{pmatrix} X(s) \\ s(s) \\ s^2X(s) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} U \quad (0.1.14)$$

So $\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$

$$Y = X_1(s) = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} X(s) \\ sX(s) \\ s^2X(s) \end{pmatrix} \quad (0.1.15)$$

$$\boldsymbol{C} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \quad (0.1.16)$$

1 Stability

1.1 Second order System

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot