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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

0.1. State the general model of a state space system specifying the dimensions of the matrices and vectors.

Solution: The model is given by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (0.1.1)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \quad (0.1.2)$$

with parameters listed in Table 0.1.

TABLE 0.1

0.2. Find the transfer function $\mathbf{H}(s)$ for the general system.

Solution: Taking Laplace transform on both sides we have the following equations

$$s\mathbf{I}\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s) \quad (0.2.1)$$

$$(\mathbf{sI} - \mathbf{A})\mathbf{X}(s) = \mathbf{B}\mathbf{U}(s) + \mathbf{x}(0) \quad (0.2.2)$$

$$\mathbf{X}(s) = (\mathbf{sI} - \mathbf{A})^{-1}\mathbf{B}\mathbf{U}(s) + (\mathbf{sI} - \mathbf{A})^{-1}\mathbf{x}(0) \quad (0.2.3)$$

and

$$\mathbf{Y}(s) = \mathbf{C}\mathbf{X}(s) + \mathbf{D}\mathbf{I}\mathbf{U}(s) \quad (0.2.4)$$

Substituting from (0.2.3) in the above,

$$\mathbf{Y}(s) = (\mathbf{C}(\mathbf{sI} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}\mathbf{I})\mathbf{U}(s) + \mathbf{C}(\mathbf{sI} - \mathbf{A})^{-1}\mathbf{x}(0) \quad (0.2.5)$$

0.3. Find $H(s)$ for a SISO (single input single output) system.

Solution:

$$H(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(\mathbf{sI} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}\mathbf{I} \quad (0.3.1)$$

0.4. Given

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \quad (0.4.1)$$

$$D = 0 \quad (0.4.2)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (0.4.3)$$

find \mathbf{A} and \mathbf{C} such that the state-space realization is in *controllable canonical form*.

Solution:

$$\therefore \frac{Y(s)}{U(s)} = \frac{Y(s)}{X(s)} \times \frac{X(s)}{U(s)}, \quad (0.4.4)$$

letting

$$\frac{Y(s)}{X(s)} = 1, \quad (0.4.5)$$

results in

$$\frac{U(s)}{X(s)} = s^3 + 3s^2 + 2s + 1 \quad (0.4.6)$$

giving

$$U(s) = s^3 X(s) + 3s^2 X(s) + 2s X(s) + X(s) \quad (0.4.7)$$

so equation 0.1.13 can be written as

$$\begin{pmatrix} sX(s) \\ s^2 X(s) \\ s^3 X(s) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \begin{pmatrix} X(s) \\ sX(s) \\ s^2 X(s) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} U \quad (0.4.8)$$

So

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \quad (0.4.9)$$

$$\mathbf{Y} = \mathbf{X}(s) = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} X(s) \\ sX(s) \\ s^2 X(s) \end{pmatrix} \quad (0.4.10)$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \quad (0.4.11)$$

0.5. Obtain \mathbf{A} and \mathbf{C} so that the state-space realization is in *observable canonical form*.

0.6. Find the eigenvalues of \mathbf{A} and the roots of the characteristic equation of $H(s)$ using a python code.

Solution: The code

codes/ee18btech11006/eigen.py provides the necessary solution as

The eigenvalues are:

$-2.32471796+0.j$, $-0.33764102+0.56227951j$,
 $-0.33764102-0.56227951j$

Roots of the the polynomial:

$-2.32471796+0.j$, $-0.33764102+0.56227951j$,
 $-0.33764102-0.56227951j]$

1 STABILITY

1.1 Second order System

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT