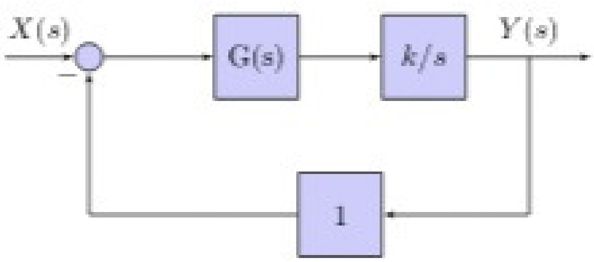

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

2 Routh Hurwitz Criterion

2.1. Consider a unity feedback system as shown in the figure,shown with an integral compensator k/s and open-loop transfer function

$$G(s) = \frac{1}{s^2+3s+2} \tag{2.1.1}$$

where k greater than 0. The positive value of k for which there are two poles of unity feedback system on jω axis is equal to——(rounded off to two decimal places) **Solution:** The transfer function for negative feedback is given by

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)H(s)} \tag{2.1.2}$$

where H(s) = 1 for unity feedback system and G(s) is net forward open loop gain

$$G(s) = \left[\frac{1}{s^2+3s+2}\right]\left[\frac{k}{s}\right] = \frac{k}{s^3+3s^2+2s} \tag{2.1.3}$$

Characteristic equation is..,

$$1+G(s)H(s) = 0 \tag{2.1.4}$$

$$=> 1+\left[\frac{k}{s^3+3s^2+2s}\right] = 0 \tag{2.1.5}$$

$$=> s^3+3s^2+2s+k = 0 \tag{2.1.6}$$

The Routh hurwitz criterion:- This criterion is based on arranging the coefficients of characteristic equation into an array called Routh array. For any characteristic equation q(s),

$$q(s) = a_0s^n + a_1s^{n-1} + ... + a_{n-1}s + a_n = 0 \tag{2.1.7}$$

Routh array can be constructed as follows..,

$$\begin{pmatrix} s^n \\ s^{n-1} \\ s^{n-2} \\ \vdots \end{pmatrix}\begin{pmatrix} a_0 & a_2 & a_4 & \cdots & \\ a_1 & a_3 & a_5 & \cdots & \\ b_1 & b_2 & b_3 & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \vdots\dots \end{pmatrix}$$

where

$$b_1 = \frac{a_1a_2 - a_0a_3}{a_1} \tag{2.1.8}$$

$$b_2 = \frac{a_1a_4 - a_0a_5}{a_1} \tag{2.1.9}$$

$$c_1 = \frac{b_1a_3 - a_1b_2}{b_1} \tag{2.1.10}$$

$$c_2 = \frac{b_1a_5 - a_1b_3}{b_1} \tag{2.1.11}$$

For poles to lie on imaginary axis any one entire row of hurwitz matrix should be zero. Constructing the routh array for the characteristic equation obtained in equation(2.1.4)

$$s^3+3s^2+2s+k = 0 \tag{2.1.12}$$

$$\begin{pmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & k \\ \frac{6-k}{3} & 0 \\ k & 0 \end{pmatrix} \tag{2.1.13}$$

For poles on jω axis any one of the row should be zero.

$$\frac{6-k}{3} = 0 \ or \ k = 0 \tag{2.1.14}$$

But given k greater than 0 ...

$$6-k = 0 \tag{2.1.15}$$

$$k = 6 \tag{2.1.16}$$

2.2. Repeat the above using the determinant method. **Solution:** Determinent method:
$$\begin{pmatrix} a_0 & a_2 & a_4 & \cdots & \\ a_1 & a_3 & a_5 & \cdots & \\ 0 & a_0 & a_2 \cdots & & \\ 0 & a_1 & a_3 \cdots & & \\ \vdots & \vdots & \vdots & \ddots & \vdots \cdots \end{pmatrix}$$

$$D1 = |a_0| \tag{2.2.1}$$

$$D2 = \begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix} \tag{2.2.2}$$

$$D3 = \begin{vmatrix} a_0 & a_2 & a_4 \\ a_1 & a_3 & a_5 \\ 0 & a_0 & a_2 \end{vmatrix} \tag{2.2.3}$$

and so on... If atleast any one of the Determinents are zero then the poles lie on imaginary axes. So,For the given question the routh array is obtained from equation(2.1.4)

$$D1 = 1 \tag{2.2.4}$$

$$D2 = \begin{vmatrix} 1 & 2 \\ 3 & k \end{vmatrix} \qquad = k - 6 \tag{2.2.5}$$

$$D3 = \begin{vmatrix} 1 & 2 & 0 \\ 3 & k & 0 \\ 0 & 1 & 2 \end{vmatrix} \qquad = 2k - 12 \tag{2.2.6}$$

D2 = k-6 =0 for the poles to lie on imaginary axis

$$k - 6 = 0 \tag{2.2.7}$$

$$k = 6 \tag{2.2.8}$$

2.3. Verify your answer using a python code for both the determinant method as well as the tabular method. **Solution:** The following code codes/ee18btech11005/routhhurwitztabular.py

2.4. Why the sign change in routh array determine stability? **Solution:** For the system to be stable all coefficients should lie on left half of s-plane. Because if any pole is in right half of s-plane then there will be a component in output that increases without bound,causing system to be unstable. All the coefficients in the characteristic equation should be positive.This is necessary condition but not sufficient.Because it may have poles on right half of s plane. Poles are the roots of the characteristic equation.

- A system is stable if all of its characteristic modes go to finite value as t goes to infinity.It is possible only if all the poles are on the left half of s plane. The characteristic equation should have negative roots only. So the first column should always be greater than zero.That means no sign changes.
- A system is unstable if its characteristic modes are not bounded. Then the characteristic equation will also have roots in the right side of s-plane.That means it has sign changes.

3 Compensators

4 Nyquist Plot