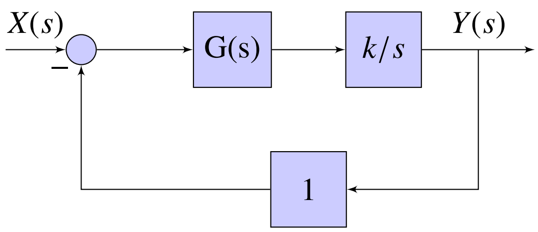

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

2 Routh Hurwitz Criterion

2.1. Consider a unity feedback system as shown in the figure,shown with an integral compensator k/s and open-loop transfer function

$$G(s) = \frac{1}{s^2 + 3s + 2} \tag{2.1.1}$$

where k greater than 0. The positive value of k for which there are two poles of unity feedback system on jω axis is equal to——(rounded off to two decimal places)

Solution: The transfer function for negative feedback is given by

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{2.1.2}$$

where H(s) = 1 for unity feedback system and G(s) is net forward open loop gain

$$G(s) = \left[\frac{1}{s^2 + 3s + 2}\right]\left[\frac{k}{s}\right] = \frac{k}{s^3 + 3s^2 + 2s} \tag{2.1.3}$$

Characteristic equation is..,

$$1 + G(s)H(s) = 0 \tag{2.1.4}$$

$$=> 1 + \left[\frac{k}{s^3 + 3s^2 + 2s}\right] = 0 \tag{2.1.5}$$

$$=> s^3 + 3s^2 + 2s + k = 0 \tag{2.1.6}$$

The Routh hurwitz criterion:- This criterion is based on arranging the coefficients of characteristic equation into an array called Routh array. For any characteristic equation q(s),

$$q(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + ... + a_{n-1} s + a_n = 0 \tag{2.1.7}$$

Routh array can be constructed as follows..,

$$\begin{pmatrix} s^n \\ s^{n-1} \\ s^{n-2} \\ \vdots \end{pmatrix} \begin{pmatrix} a_0 & a_2 & a_4 & \cdots \\ a_1 & a_3 & a_5 & \cdots \\ b_1 & b_2 & b_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \ldots \end{pmatrix}$$

where

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \tag{2.1.8}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \tag{2.1.9}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \tag{2.1.10}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1} \tag{2.1.11}$$

For poles to lie on imaginary axis any one entire row of hurwitz matrix should be zero. Constructing the routh array for the characteristic equation obtained in equation(2.1.4)

$$s^3 + 3s^2 + 2s + k = 0 \tag{2.1.12}$$

$$\begin{pmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & k \\ \frac{6-k}{3} & 0 \\ k & 0 \end{pmatrix} \tag{2.1.13}$$

For poles on jω axis any one of the row should be zero.

$$\frac{6-k}{3} = 0 \ or \ k = 0 \tag{2.1.14}$$

But given k greater than 0 ...

$$6 - k = 0 \tag{2.1.15}$$

$$k = 6 \tag{2.1.16}$$

3 Compensators

4 Nyquist Plot