

Butterworth Filter and Sallen-Key Topology

Dheeraj Racha and Utkarsh Bhura *

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Abstract—This manual gives an idea about Butterworth filter and its use in signal processing. Sallen-Key topology is used to implement linear analog filter.

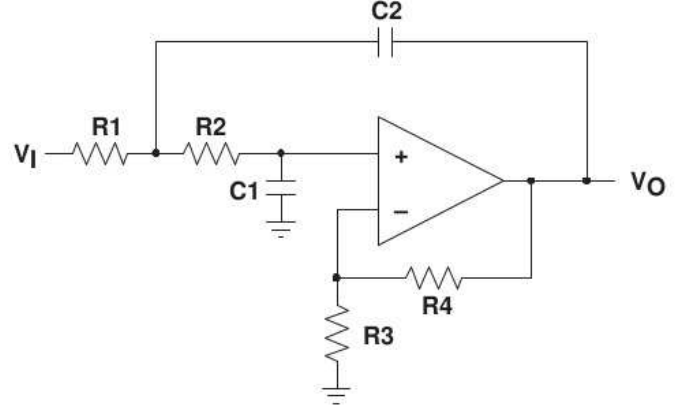


Fig. 1.1

1 BUTTERWORTH FILTER: SALLEN-KEY TOPOLOGY

1.1 Show that

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{k\omega_c^2}{s^2 + \frac{\omega_c}{Q}s + \omega_c^2} \quad (1)$$

in Fig. 1.1, where

$$\omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad (2)$$

$$Q = \frac{R_1 C_2 + R_2 C_2 + R_1 C_1 (1 - k)}{R_1 R_2 C_1 C_2} \quad (3)$$

$$k = 1 + \frac{R_4}{R_3} \quad (4)$$

Solution: Because the op-amp is operating in non-inverting region,

$$V_o = \left(1 + \frac{R_4}{R_3}\right) V_B \quad (5)$$

Applying KCL at Node A:

$$\frac{V_A - V_{in}}{R_1} + \frac{V_A - V_B}{R_2} + \frac{V_A - V_{out}}{1/C_2 s} = 0 \quad (6)$$

Similarly at Node B: $\frac{V_B - V_A}{R_2} + \frac{V_B}{1/C_1 s} = 0$

$$\Rightarrow V_A = (1 + R_2 C_2 s) V_B \quad (7)$$

Also,

$$k = \frac{R_3 + R_4}{R_3} \quad (8)$$

On solving the above equations for $\frac{V_o}{V_i}$ we obtain (1).

1.2 If

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}}}, \quad (9)$$

show that

$$N = 2, Q = \frac{1}{\sqrt{2}}. \quad (10)$$

1.3 Design a general 2nd order butterworth low pass filter with cut-off frequency f_c Hz .

Solution: $\omega_c = 2\pi f_c$

In (4.1.5), $Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 C_2 + R_2 C_2 + R_1 C_1 (1 - k)}$
 $\omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$. And $K = 3 - \frac{1}{Q} = 1 + \frac{R_4}{R_3}$.
 Let $R_1 = R_2 = R$ and $C_1 = C_2 = C$. And for Butterworth Filter $Q = 0.707$.

Therefore $H(s) = \frac{1.586\omega_c^2}{s^2 + 1.414\omega_c s + \omega_c^2}$
 and $R_4 = 0.586R_3$

1.4 Design a 4th order Butterworth filter with the

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All solutions in this manual is released under GNU GPL. Free and open source.

following response

$$H(S) = \frac{2.57\omega_c^4}{(s^2 + 0.7654\omega_c s + \omega_c^2)(s^2 + 1.8478\omega_c s + \omega_c^2)} \quad (11)$$