# Butterworth Filter and Sallen-Key Topology

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Contents



***Abstract*—This manual gives an idea about Butterworth filter and its use in signal processing. Sallen-Key topology is used to implement linear analog filter.**

## 1 Butterworth Filter: Sallen-Key Topology

1.1 Show that

$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{k\omega_c^2}{s^2 + \frac{\omega_c}{Q}s + \omega_c^2} \tag{1}$$

in Fig. 1.1, where

$$\omega_c = \frac{1}{\sqrt{R_1R_2C_1C_2}} \tag{2}$$

$$Q = \frac{R_1C_2 + R_2C_2 + R_1C_1(1-k)}{R_1R_2C_1C_2} \tag{3}$$

$$k = 1 + \frac{R_4}{R_3} \tag{4}$$

**Solution:** Because the op-amp is operating in non-inverting region,

$$V_0 = \left(1 + \frac{R_4}{R_3}\right)V_B \tag{5}$$

Applying KCL at Node A:

$$\frac{V_A - V_{in}}{R_1} + \frac{V_A - V_B}{R_2} + \frac{V_A - V_{out}}{1/C_2s} = 0 \tag{6}$$

Similarly at Node B: $\frac{V_B - V_A}{R_2} + \frac{V_B}{1/C_1s} = 0$

$$\implies V_A = (1 + R_2C_2s)V_B \tag{7}$$

Also,

$$k = \frac{R_3 + R_4}{R_3} \tag{8}$$

On solving the above equations for $\frac{V_o}{V_i}$ we obtain (1).

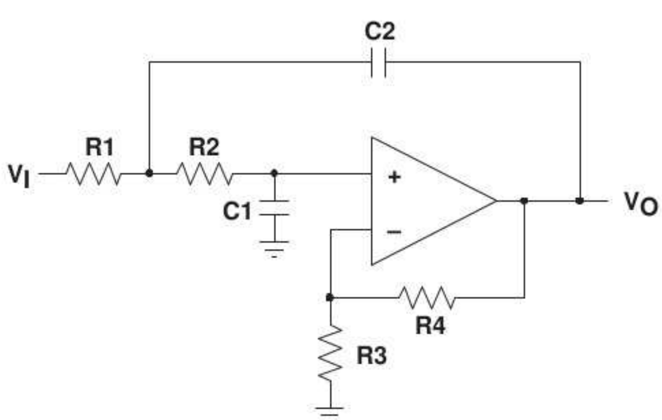


Fig. 1.1

1.2 If

$$\left|H(\jmath\omega)\right| = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^{2N}}}, \tag{9}$$

show that

$$N = 2, Q = \frac{1}{\sqrt{2}}. \tag{10}$$

1.3 Design a general $2^{nd}$ order butterworth low pass filter with cut-off frequency $f_c$ Hz .
**Solution:** The relevant parameters are

$$\omega_c = 2\pi f_c, \tag{11}$$

$$K = 3 - \frac{1}{Q} \tag{12}$$

1.4 Design a 4th order Butterworth filter with the following response

$$H(s) = \frac{2.57\omega_c^4}{(s^2 + 0.7654\omega_cs + \omega_c^2)(s^2 + 1.8478\omega_cs + \omega_c^2)} \tag{13}$$

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