Butterworth Filter and Sallen-Key Topology

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Contents

***Abstract*—This manual gives an idea about Butterworth filter and its use in signal processing. Sallen-Key topology is used to implement linear analog filter.**

1 Introduction

1.1 What is a Butterworth Filter?
Solution: It is a low pass filter designed to have a linear magnitude response in pass band. So, it is also referred to as a maximally flat magnitude filter. Even though it does not provide the sharp cut-off response, it is often considered as the all-round filter which is used in many applications.

2 Frequency Response

2.1 Write a mathematical expression for the magnitude response of this filter.
Solution:

$$|H_n(j\omega)| = \frac{V_o}{V_i} = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^{2N}}}$$

where ω_c *is the cut-off/corner frequency, N is the order of the filter or number of reactive elements in a passive filter.*

2.2 Plot the frequency response of an ideal Butterworth filter. Write your inference.
Solution: Refer to Fig.2.2 for frequency response.

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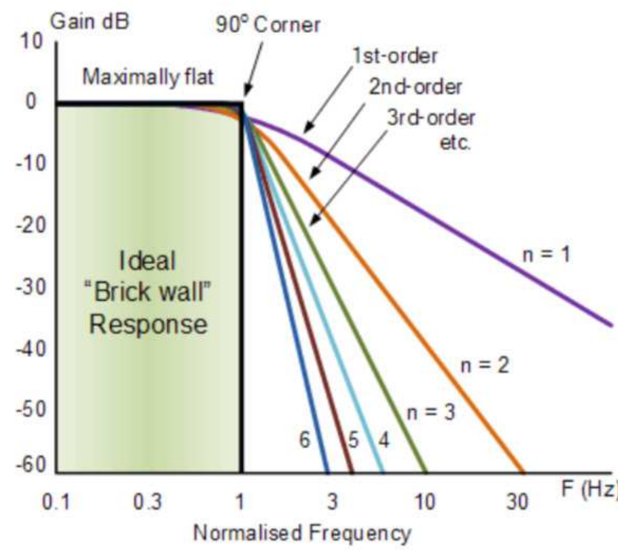

Fig. 2.2

The Butterworth filter achieves pass band flatness at the expense of a wide transition band as the filter changes from the pass band to the stop band.

3 Filter Implementation using Circuits

3.1 What are the Filter topologies used to implement this filter.
Solution: There are several different filter topologies available to implement a linear analog filter. The most often used topology for a passive realisation is Cauer topology and the most often used topology for an active realisation is Sallen–Key topology.

3.2 Describe briefly Cauer and Sallen-Key topology.
Solution: (i) *Cauer Topology*: The Cauer topology uses passive components (shunt capacitors and series inductors) to implement a linear analog filter. The Butterworth filter having a given transfer function can be realised

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using a Cauer 1-form. Fig shown below is an example of Cauer topology.

(ii) *Sallen-Key Topology*: The Sallen–Key

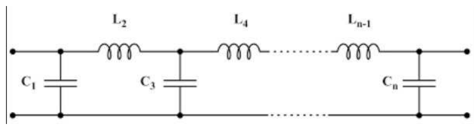

Fig. 3.2

topology uses active and passive components (op-amps operating in non-inverting region, resistors, and capacitors) to implement a linear analog filter. Each Sallen–Key stage implements a conjugate pair of poles; the overall filter is implemented by cascading all stages in series. Fig shown below is an example of Sallen-Key topology.

****NOTE**: We will limit our discussion to

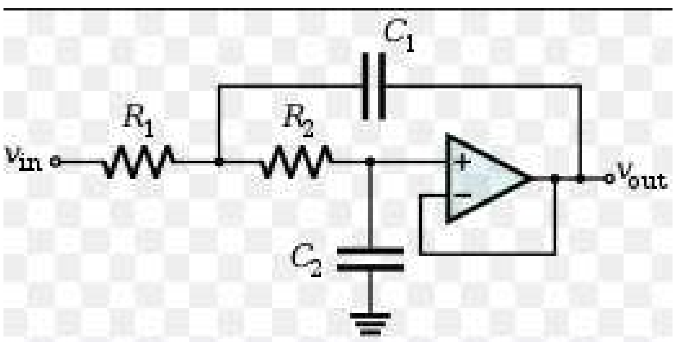

Fig. 3.2: A second order sallen-key filter

Sallen-Key topology only.

4 Sallen-Key Circuit Analysis

4.1 Derive the expression for transfer function of a general second order sallen-key filter.

Solution: Because the op-amp is operating in

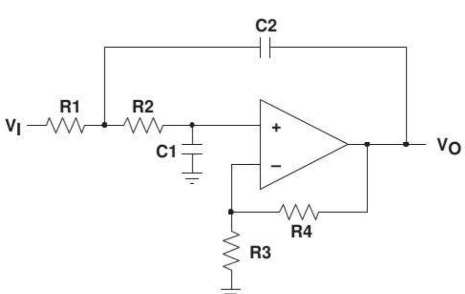

Fig. 4.1

non-inverting region,

$$V_0 = (1 + \frac{R_4}{R_3})V_B \tag{4.1.1}$$

Applying KCL at Node A:

$$\frac{V_A - V_{in}}{R_1} + \frac{V_A - V_B}{R_2} + \frac{V_A - V_{out}}{1/C_2 s} = 0 \tag{4.1.2}$$

Similarly at Node B: $\frac{V_B - V_A}{R_2} + \frac{V_B}{1/C_1 s} = 0$

$$\implies V_A = (1 + R_2C_2s)V_B \tag{4.1.3}$$

Also,

$$k = \frac{R_3 + R_4}{R_3} \tag{4.1.4}$$

On solving the above equations for $\frac{V_o}{V_i}$. we have,

$$\frac{V_o}{V_i} = \frac{\frac{k}{R_1R_2C_1C_2}}{s^2 + s\frac{R_1C_2 + R_2C_2 + R_1C_1(1-k)}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}} \tag{4.1.5}$$

4.2 Design a general 2^{nd} order butterworth low pass filter with cut-off frequency f_c Hz .

Solution: $\omega_c = 2\pi f_c$

In (4.1.5),$Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1C_2 + R_2C_2 + R_1C_1(1-k)}$

$\omega_c = \frac{1}{\sqrt{R_1R_2C_1C_2}}$. And $K = 3 - \frac{1}{Q} = 1 + \frac{R_4}{R_3}$.

Let $R_1 = R_2 = R$ and $C_1 = C_2 = C$. And for Butterworth Filter Q = 0.707.

Therefore $H(s) = \frac{1.586\omega_c^2}{s^2 + 1.414\omega_c s + \omega_c^2}$

and $R_4 = 0.586R_3$

4.3 Design a 4th order Butterworth filter with cut-off frequency 1 kHz.

Solution: Cascading two 2^{nd} order butterworth filters realised with sallen-key topology

we get,

$\omega_c = 2000\pi$ rad/sec

$$H(S) = \frac{2.57\omega_c^4}{(s^2 + 0.7654\omega_c s + \omega_c^2)(s^2 + 1.8478\omega_c s + \omega_c^2)}$$