QoE-Aware Scheduling for Sigmoid Optimization in Wireless Networks

Gilsoo Lee, Hongseok Kim, Yunhee Cho, and Seung-Hwan Lee

Abstract—This letter proposes a quality-of-experience (QoE)aware scheduler that maximizes the average number of satisfied users. QoE is different from quality-of-service (QoS) in that QoE is determined by users' subjective satisfaction while QoS is objectively measured. Hence, users' satisfaction needs to be incorporated into the scheduler to better understand QoE maximization. In this regard we propose a simple but effective user participation scheme exploiting one-bit feedback that indicates user's satisfaction upon the end of service. Then, user-centric QoE function is derived, which serves our objective function. Assuming that the QoE function can be successfully modeled by a sigmoid function we solve the non-convex sigmoid optimization. We show that, interestingly, adding non-trivial fairness constraints for starvation prevention can convexify the problem, and we obtain the unique optimal solution. By comparing the case without user participation we show that user participation can substantially improve the total QoE, e.g., nearly 100% improvement of the low 10 percentile users and 40% improvement of the low 50 percentile users.

Index Terms—Quality-of-experience, resource management, scheduling, nonconvex optimization, cellular system.

I. Introduction

ITH the evolution to 5G wireless network and high performance smart devices, a wide range of real-time video or voice applications is prevalent; video mobile traffic rapidly grows and generates 7.4 exabytes while total mobile traffic is expected to reach 11.2 exabytes by 2017 [1]. Hence, supporting high-definition video or voice services over the cellular networks while satisfying users satisfaction is one of the most important concerns that mobile network operators commonly have.

As for video or voice services, users' subjective satisfaction is measured by quality-of-experience (QoE) rather than other objective metrics such as the quality-of-service (QoS) [2]. By nature QoE varies from person to person; both technical/environmental factors as well as users characteristics affect the QoE [3]. Hence the *subjective* property of the QoE makes it hard to quantify unless users are in the control loop. In this regard, we envisage a simple and realizable method where each

Manuscript received March 21, 2014; revised July 22, 2014; accepted August 17, 2014. Date of publication September 4, 2014; date of current version November 7, 2014. This work was supported in part by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT and Future Planning under Grant NRF-2014R1A1A1006551 and the ICT R&D program of MSIP/IITP [14-000-04-001, Development of 5G Mobile Communication Technologies for Hyper-connected smart services. The associate editor coordinating the review of this paper and approving it for publication was D. W. K. Ng.

G. Lee and H. Kim are with the Department of Electronic Engineering, Sogang University, Seoul 121-742, Korea (e-mail: gilsoolee@sogang.ac.kr; hongseok@sogang.ac.kr).

Y. Cho and S.-H. Lee are with Electronics and Telecommunications Research Institute (ETRI), Daejeon 305-700, Korea (e-mail: yhcho@etri.re.kr; lsh@etri.re.kr).

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Digital Object Identifier 10.1109/LCOMM.2014.2354409

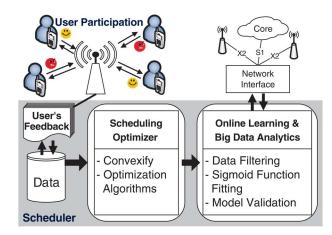


Fig. 1. User participation-based QoE assessment framework.

user sends a single bit feedback that indicates the *satisfaction* or *dissatisfaction* upon the end of the service. The framework is shown in Fig. 1. It is expected that statistical exploitation of user feedbacks might be used to quantify the *individual* QoE, which is the online-computed version of the mean opinion score (MOS) that is made from offline experiments so far.

Typically, the level of satisfaction, (a.k.a. *utility*) upon using best effort service is modeled by a concave function [4], [5]. However, the utility corresponding to real-time services including video streaming are usually modeled by a sigmoid function [4], [6]. The authors of [7] formulated a general problem using a nonconvex utility function and proposed a suboptimal distributed rate allocation algorithm. The authors of [6] tried to solve non-convex optimization problem onto transmission rates when users have sigmoid utility functions. Our work is different from the literature in two folds; the subjective QoE functions are derived from user's feedbacks, and the service rates, not the signal to interference plus noise ratio, are the variables in the QoE functions.

The main contributions of this letter are summarized as follows. We propose a model to derive the individual user's QoE function from users' online feedback upon users' participation. Hence users are in the control loop in optimizing QoE. Then, we prove that our non-convex sigmoid optimization problem can be convexified by imposing non-trivial *fairness constraints* that guarantee the minimum QoE. Based on our model we show that users' participation can significantly improve the average QoE as well as edge users' QoE. In addition, adding fairness constraints improves the performance of low-QoE users with negligible performance degradation of high-QoE users.

II. SYSTEM MODEL

A. Assumptions

We consider a general framework and do not restrict the scheduler to specific applications. Our scheduler runs periodically, e.g., every time frame. For simplicity, we first consider the data rate (or service rate) is the main factor that affects the satisfaction.

Definition 1 (One-Bit Feedback): We define the one-bit feedback of user i as

$$\mathbf{1}_{\{x_i\}} = \begin{cases} 1 & \text{if } x_i \ge X_i \\ 0 & \text{if } x_i < X_i \end{cases}$$

where x_i is the data rate of user $i \in \{1, \ldots, n\}$, X_i is the threshold above which user i subjectively feels satisfied. Hence, $\mathbf{1}_{\{x_i\}}$ indicates the satisfaction of user i when served by data rate x_i . While multi-bit feedback can be considered, one-bit feedback is used because the simplest feedback encourages the users' active participation.

B. Problem Formulation

We assume that the threshold X_i is a random variable. Note that users are typically unaware of the thresholds above which they are satisfied. Indeed users do not care the video delivery technologies such as video encoding rate, but only feel that the quality is satisfactory or not from their own perspective implying the uncertainty of thresholds. Furthermore, the satisfaction threshold may depend on, for example, the type of videos (movie, sports or news), video encoding method, etc, which the scheduler is not aware of and thus contribute to making X_i random. Note that X_i for $i=1,\ldots,n$ are not necessarily i.i.d. since users may have different characteristics.

Our problem is to maximize the average number of satisfied users with the power constraint. Let $x_i = \log_2(1 + \mathrm{SNR}_i(\mathbf{p}))$ where SNR_i is the signal to noise ratio of user i, and \mathbf{p} is the vector of transmit power p_i for user i. We mainly focus on a single cell scenario considering the complexity of nonconvex optimization under intercell interference. We assume that users are scheduled either temporally fair (TDMA) or equal bandwidth sharing (OFDMA) in resource blocks, and our scheduling solely focuses on power allocation. Then we have $\mathrm{SNR}_i = (g_i p_i)/N_0$ where g_i is the channel gain of user i, and N_0 is the noise power.

Note that the average number of satisfied users equals to the expectation of the sum of the indicator functions. Then, our problem is to find $\mathbf{x} = (x_1, \dots, x_n)$ that maximizes the total QoE function $Q(\mathbf{x})$,

$$\max_{\mathbf{x}} \quad Q(\mathbf{x}) = E\left[\sum_{i=1}^{n} \mathbf{1}_{\{x_i\}}\right]$$
s.t
$$x_i = \log_2\left(1 + \mathsf{SNR}_i(\mathbf{p})\right)$$

$$\sum_{i=1}^{n} p_i \le P_{\max}, \quad p_i \ge 0$$
(2)

where P_{max} is the maximum transmission power of a BS.

C. Motivation for Our Objective Function

The QoE function of user i is defined by the expectation of the indicator functions, which is given by a cumulative distribution function (CDF) of X_i because $E[\mathbf{1}_{\{x_i\}}] = 1 \times P(X_i \leq x_i) + 0 \times P(X_i > x_i) = F_{X_i}(x_i)$.

It is well known that concave approximation is valid for the utility of data service, but not for the video services. In fact, a sigmoid function better fits for video. In this regard, we assume that the QoE function, derived from the feedbacks, can be successfully modeled by a sigmoid function for analytical tractability. We will validate this assumption in our future research by performing extensive experiments about video applications. Hereafter we focus on the QoE gain by exploiting users' online feedbacks.

Definition 2 (User-Centric QoE Function): For user i, we define the user-centric QoE function as a sigmoid function, which is given by

$$F_{X_i}(x_i) = \frac{e^{\theta_i(x_i - \mu_i)}}{1 + e^{\theta_i(x_i - \mu_i)}}$$
(3)

where θ_i and μ_i are parameters that reflect the characteristics of user i; θ_i determines the randomness of the satisfaction threshold.

For example, if θ_i is high, the CDF looks like a step function, and thus a user with high θ_i is relatively firmed in their satisfaction. μ_i represents the average of the threshold. A user with high μ_i can be considered as demanding while the user with low μ_i is easily satisfied. We assume that based on techniques such as machine learning or big data analytics the scheduler can successfully estimate θ_i and μ_i for each user.

Our problem is then equivalent to

$$\max_{\mathbf{x}} \quad \sum_{i=1}^{n} F_{X_i}(x_i)$$
s.t
$$x_i = \log_2 (1 + \text{SNR}_i(\mathbf{p}))$$

$$\sum_{i=1}^{n} p_i \le P_{\text{max}}, \quad p_i \ge 0.$$
 (4)

To quantify the benefit of user participation, we consider the case where users do not feedback their satisfactions. Thus, the scheduler can only have a *typical* CDF, which is assumed to be obtained offline and common for all users.

Definition 3 (User-Agnostic QoE Function): When individual feedbacks are unavailable, the scheduler has a common QoE function for all users.

Lemma 1: The user-agnostic QoE function for a typical user is given by

$$E\left[1_{\{x_I\}}\right] = E\left[F_{X_I}(x_I)\right] \tag{5}$$

where I is a random variable denoting a typical user.

Proof: Since the user-agnostic QoE function is obtained for all users without user-specific information, it can be expressed by $E[1_{\{x_I\}}] = E[E[1_{\{x_I\}}|I]]$ from the law of iterated expectations. Since $E[1_{\{x_I\}}|I] = F_{X_I}(x_I)$, it completes the proof.

Note that when user feedback are unavailable, the scheduler needs to use (5) instead of (4). However, the actual scheduling performance *experienced by users* still needs to be evaluated by $Q(\mathbf{x})$ in (1).

Lemma 2: The total QoE obtained using the user-centric QoE function is greater than or equal to the total QoE obtained using the user-agnostic QoE function.

Proof: Let \mathbf{x}_{UC} and \mathbf{x}_{UA} are scheduling vectors that maximize each problem consisting of either user-centric or user-agnostic QoE function. Since \mathbf{x}_{UC} is the optimal solution that maximizes $Q(\mathbf{x})$, we have $Q(\mathbf{x}_{\mathrm{UC}}) \geq Q(\mathbf{x})$ for any $\mathbf{x} \neq \mathbf{x}_{\mathrm{UC}}$ where a set of feasible $\mathbf{x} \in \mathbb{R}^n$ includes \mathbf{x}_{UA} .

Note that \mathbf{x}_{UA} provides a lower bound of total QoE. The difference of total QoEs with \mathbf{x}_{UC} and \mathbf{x}_{UA} measures the virtue of users' participation in improving QoE.³ The problem

¹The feedback can be transmitted via application layer upon the end of the service.

²Research about a multicell scenario remains as a future work.

³In this letter we consider two corner cases: when all users participate or none participates. As one might expect, when only partial users participate in the feedback, the gain becomes less compared to the case of all users' participation.

(4) is non-convex optimization in general since the objective functions are sigmoid.

III. PROBLEM CONVEXIFICATION AND SOLUTIONS

A. Problem Convexification

In this section, we prove that the non-convex optimization problem (4) becomes a convex optimization by adding fairness constraints, without which users can be starved in sigmoid optimization [8].

Proposition 1: Suppose that $F_{X_i}(x_i)$ is given by (3). Then, there exists $\exists \kappa_i < 0.5$ such that $\sum_{i=1}^n F_{X_i}(x_i)$ can become concave if $F_{X_i}(x_i) \geq \kappa_i$, for $i=1,\ldots,n$.

Proof: We first show that $F_{X_i}(x_i) = 0$.

concave if $F_{X_i}(x_i) \ge \kappa_i$ for $\exists \kappa_i < 0.5$. Recall that (3) is the function of x_i . Substituting (2) into (3) makes (3) becomes the function of transmission power p_i and is given by

$$H_i(p_i) = \frac{(1 + \beta_i p_i)^{\theta_i}}{\alpha_i + (1 + \beta_i p_i)^{\theta_i}}$$
 (6)

where $\alpha_i = e^{\theta_i \mu_i}$ and $\beta_i = g_i/N_0$. The second-order derivatives of (6) is given in (7), shown at the bottom of the page. If $H_i''(p_i) \leq 0$, then $H_i(p_i)$ is concave. When $\theta_i \leq 1$, $H_i(p_i)$ is concave for all p_i . When $\theta_i > 1$, if the numerators of (7) is not positive,

$$(\theta_i - 1) \left(\alpha_i + (1 + \beta_i p_i)^{\theta_i}\right) - 2\theta_i (1 + \beta_i p_i)^{\theta_i} \le 0,$$
 (8)

then $H_i(p_i)$ become concave. From (8), we have $p_i \ge$ $(e^{\mu_i + (1/\theta_i) \ln((\theta_i - 1)/(\theta_i + 1))} - 1)/\beta_i = p_{i,\min}$, which is equivalently to

$$x_i \ge \mu_i + \frac{1}{\theta_i} \ln \left(\frac{\theta_i - 1}{\theta_i + 1} \right).$$
 (9)

When (9) is satisfied, the user-centric QoE function is

$$F_{X_i}(x_i) = \frac{e^{\theta_i(x_i - \mu_i)}}{1 + e^{\theta_i(x_i - \mu_i)}} \ge \frac{\theta_i - 1}{2\theta_i}.$$
 (10)

Thus, when $F_{X_i}(x_i)$ is restricted by $F_{X_i}(x_i) \ge \kappa_i$ such that $\kappa_i = ((\theta_i - 1)/(2\theta_i)) < 0.5, F_{X_i}(x_i)$ can be transformed to concave.

Next, we show that our new objective function

$$H(\mathbf{p}) = \sum_{i=1}^{n} H_i(p_i) \tag{11}$$

is concave. The Hessian of (11) is given by

$$\nabla^2 H = \operatorname{diag}\left(H_1''(p_1), \dots, H_n''(p_n)\right)$$

where $\operatorname{diag}(\cdot)$ means a diagonal matrix. Since all $H_i(p_i)$ are concave for $p_i \ge p_{i,\min}$, $H_i''(p_i) \le 0$. Hence, $\nabla^2 H$ is negative semi-definite, and H is concave when $p_i \ge p_{i,\min}$ is satisfied for all $i = 1, \ldots, n$.

After convexification, our problem with the maximum transmission power constraint and the fairness constraints with respect to **p** becomes

$$\max_{\mathbf{p}} \quad \sum_{i=1}^{n} H_i(p_i)$$
s.t
$$\sum_{i=1}^{n} p_i \le P_{\max}$$

$$p_i \ge p_{i,\min}, \quad i = 1, \dots, n.$$
(12)

The problem (12) has a concave objective function with linear constraints, so (12) is a convex optimization problem. Note that the problem (12) has a unique optimal transmission power p^* since $H(\mathbf{p})$ is strictly concave in \mathbf{p} .

To readily handle the problem, we transform variables: $\tilde{p}_i =$ $p_i - p_{i,\min}$, $H_i(p_i) = \tilde{H_i}(\tilde{p}_i + p_{i,\min}) = \tilde{H}_i(\tilde{p}_i)$, and $\tilde{P}_{\max} = P_{\max} - \sum_{i=1}^n p_{i,\min}$.

B. Solution

From our maximization problem, the Lagrangian is

$$L(\tilde{p}_i, \lambda) = \sum_{i=1}^n \tilde{H}_i(\tilde{p}_i) - \lambda_0 \left(\sum_{i=1}^n \tilde{p}_i - \tilde{P}_{\max} \right) + \sum_{i=1}^n \lambda_i \tilde{p}_i.$$
 (13)

From the Karush-Kuhn-Tucker (KKT) conditions, the optimal $\lambda_i^*, i = 0, \dots, n$ should satisfy

$$\lambda_i^* \ge 0, \quad i = 0, \dots, n \tag{14}$$

$$\lambda_0^* \left(\sum_{i=1}^n \tilde{p}_i^* - \tilde{P}_{\text{max}} \right) = 0 \tag{15}$$

$$\lambda_i^* \tilde{p}_i^* = 0, \quad i = 1, \dots, n \tag{16}$$

$$\lambda_{i}^{*}\tilde{p}_{i}^{*} \stackrel{=}{=} 0, \quad i = 1, \dots, n$$

$$\frac{\partial L}{\partial \tilde{p}_{i}} = \frac{\partial}{\partial \tilde{p}_{i}} \tilde{H}_{i} \left(\tilde{p}_{i}^{*} \right) - \lambda_{0}^{*} + \lambda_{i}^{*} = 0, \quad i = 1, \dots, n. \quad (17)$$

Inserting (17) into (14) gives us

$$\lambda_i^* = \lambda_0^* - \frac{\partial}{\partial \tilde{p}_i} \tilde{H}_i(\tilde{p}_i^*) \ge 0. \tag{18}$$

Then, combining (16) and (18) gives us the solution such as

$$\tilde{p}_i^* \left(\lambda_0^* \right) = \max \left\{ \left(\frac{\partial}{\partial \tilde{p}_i} \tilde{H}_i \right)^{-1} \left(\lambda_0^* \right), 0 \right\}. \tag{19}$$

IV. ALGORITHMS

A. Convex Optimization Algorithm

We propose convex optimization algorithm (COA) that yields the optimal solution of the convex optimization problem. Our algorithm is based on the bisection method. The initial value of λ_{\min} and λ_{\max} are set as zero and a large number, respectively. Then, the algorithm checks whether power constraint is satisfied or not with initial λ_0 . To satisfy (15), the sum of all allocated transmission power for all users should be same to power budget of the BS. Once λ_0^* is found, then optimal \tilde{p}_i^* can be calculated using the result of (19). The algorithm

$$H_{i}''(p_{i}) = \frac{\theta_{i}\alpha_{i}\beta_{i}^{2}(1+\beta_{i}p_{i})^{\theta_{i}-2}\left(\alpha_{i}+(1+\beta_{i}p_{i})^{\theta_{i}}\right)\times\left\{(\theta_{i}-1)\left(\alpha_{i}+(1+\beta_{i}p_{i})^{\theta_{i}}\right)-2\theta_{i}(1+\beta_{i}p_{i})^{\theta_{i}}\right\}}{\left(\alpha_{i}+(1+\beta_{i}p_{i})^{\theta_{i}}\right)^{4}}$$
(7)

is summarized below along with a general algorithm in Section IV-B to save the space.

Convex (General) Optimization Algorithm

```
1: Set initial \lambda_{\min} and \lambda_{\max}.
2: do
3:
       \lambda_0 = (\lambda_{\min} + \lambda_{\max})/2.
4:
       Convex Optimization Algorithm:
5:
           For each user i, solve (18) and (19).
           Check the constraint: D = \sum_{i=1}^{n} \tilde{p}_i(\lambda_0) - \tilde{P}_{\max}
6:
        (General Optimization Algorithm:)
7:
8:
           For each user i, solve (20).
           Check the constraint: D = \sum_{i=1}^{n} p_i(\lambda_0) - P_{\text{max}}
9:
         If D \geq 0, \lambda_{\min} = \lambda_0.
10:
         If D < 0, \lambda_{\text{max}} = \lambda_0.
11:
12: while D > \epsilon.
```

B. General Optimization Algorithm

For the non-convex problem, general optimization algorithm (GOA) is proposed to derive a solution of a non-convex problem, which is not necessarily optimal. Let us now consider a baseline when the scheduler only has a typical QoE function (5) for all users. Let $F_X(x)$ denote a typical QoE function for notational simplicity. We solve the problem that aims to maximize $\sum_{i=1}^n F_X(x_i)$ with the same constraints shown in the problem (4). Note that this objective function is not exactly sigmoid since it is the average of user-centric QoE functions. Since the problem is non-convex, we exploit the necessary condition using Lagrange method. Let the Lagrangian be $L(\mathbf{x},\lambda) = \sum_{i=1}^n F_X(x_i) - \lambda_0(\sum_{i=1}^n p_i(x_i) - P_{\max})$. Then, the necessary condition of the optimality is

$$\frac{\partial}{\partial x_i} F_X(x_i) - \lambda_0 \frac{\partial}{\partial x_i} p_i(x_i) = 0.$$
 (20)

Note that the number of x_i that satisfies the above condition is not necessarily one; it can be multiple or even zero depending on λ_0 , g_i , θ_i , and μ_i . λ_0 that satisfies the constraint can be found iteratively by using the bisection method.

V. NUMERICAL RESULTS

We verify the proposed optimization algorithms through extensive simulations. We assume that users are uniformly distributed in a cell with 600 m radius. The maximum transmit power of the BS is 43 dBm. We use the ITU Pedestrian B path loss model ($PL_{\rm [dB]} = -16.62 - 37.6\log_{10}d_{\rm [m]}$) with 2.3 GHz carrier frequency. The total bandwidth is 10 MHz, and users are assumed to share the the bandwidth equally. The power spectral density of thermal noise is -174 dBm/Hz.

Fig. 2 shows CDF of 1000 users obtained through 20 iterations of 50 user case, and results are summarized in Table I. In Fig. 2, we compare the distributions of QoE that reflect three different cases: 1) with user-agnostic QoE function solved by GOA (UA-GO), 2) with user-centric QoE function solved by GOA (UC-GO), and 3) with user-centric QoE function solved by COA (UC-CO). We use $\theta_i=2$, $\kappa_i=0.25$ from (10), and μ_i are uniformly distributed from 6.5 to 7.5 for $i=1,\ldots,n$ for user-centric QoE functions.

First we compare UA-GO and UC-GO. It shows that UC-GO outperforms UA-GO, which implies that user's participation improves the total QoE significantly. Furthermore, it enhances the average QoE of low 50 percentile users by 40%. The performance improvement is even more substantial for for low

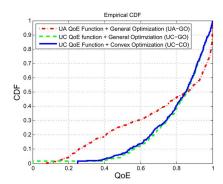


Fig. 2. The CDF of QoE in the case of 50 users.

TABLE I SIMULATION RESULTS

	Avg. QoE	Avg. QoE	Avg. QoE	Bad QoE
	(all users)	(low 50%)	(low 10%)	users ratio
UA-GO	0.7059	0.4538	0.1752	29.1%
UC-GO	0.7826	0.6355	0.3413	8.9%
UC-CO	0.7786	0.6296	0.3586	10.4%

10 percentile users: almost 100% improvement, which shows that user participation significantly improves the edge users' QoE. Then, we compare UC-GO and UC-CO. If QoE less than 0.5 is considered *bad*, the results show that using feedback information can decrease the percentage of bad QoE users from 29.1% to 8.9%. Moreover, thanks to the fairness constraint in UC-CO, there is no zero-QoE user whereas 2.1% users experience zero QoE in UC-GO.

VI. CONCLUSION

In this letter we proposed a QoE-aware scheduling framework to maximize the average number of satisfied users. In doing this we assume that users can send a single bit feedback to indicate their satisfaction levels, and thus users are in the control loop of the QoE maximization. The feedbacks can be used to make user-centric QoE functions. Since users may suffer from starvation, we added a non-trivial fairness constraint, and we proved that, interestingly, the fairness constraints convexified the original problem. Our results confirmed that exploiting users' feedback can significantly improve the total QoE in comparison to a baseline where users do not feedback online. For future work, we will investigate the impact of users' intentional no feedback or malicious feedback in a game theoretical framework.

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