

Abstract—A collection of problems from JEE papers are available in this document. These problems should be solved using Optimization techniques. Verify your solutions using *cvxpy*.

1. Let \mathbf{P} be the point on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - (8 \ 0) \mathbf{x} = 0 \quad (1)$$

which is at a minimum distance from the centre \mathbf{C} of the circle

$$\mathbf{x}^T \mathbf{x} + (0 \ 12) \mathbf{x} = 1 \quad (2)$$

Find the equation of the circle passing through \mathbf{C} and having its centre at \mathbf{P} .

2. Let \mathbf{P} be a point on the parabola

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + (0 \ 4) \mathbf{x} = 0 \quad (3)$$

Given that the distance of \mathbf{P} from the centre of the circle

$$\mathbf{x}^T \mathbf{x} + \begin{pmatrix} 6 \\ 0 \end{pmatrix} \mathbf{x} + 8 = 0 \quad (4)$$

is minimum. Find the equation of the tangent to the parabola at \mathbf{P} .

3. Find the eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half the distance between its foci.
4. \mathbf{P} and \mathbf{Q} are two distinct points on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - (4 \ 0) \mathbf{x} = 0 \quad (5)$$

with parameters t and t_1 respectively. If the normal at \mathbf{P} passes through \mathbf{Q} , then find the minimum value of t_1^2 using a descent algorithm.

5. A tangent at a point on the ellipse

$$\mathbf{x}^T \mathbf{V} \mathbf{x} = 51 \quad (6)$$

where

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 27 \end{pmatrix} \quad (7)$$

meets the coordinate axes at \mathbf{A} and \mathbf{B} . If \mathbf{O} be the origin, find the minimum area of $\triangle OAB$.

6. Find the shortest distance between the line

$$(1 \ -1) \mathbf{x} = 0 \quad (8)$$

and the curve

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - (1 \ 0) \mathbf{x} + 2 = 0 \quad (9)$$

7. Let S be the set of all complex numbers z satisfying $|z - 2 + j| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0 - 1|}$ is the maximum of the set $\left\{ \frac{1}{|z - 1|} : z \in S \right\}$, find the principal argument of

$$\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2j} \quad (10)$$

8. Let $\omega \neq 1$ be a cube root of unity. Find the minimum of the set

$$|a + b\omega + c\omega^2|, \quad (11)$$

where a, b, c are distinct nonzero integers.

9. Let

$$\mathbf{M} = \begin{pmatrix} \sin^4 \theta & -1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{pmatrix} = \alpha \mathbf{I} + \beta \mathbf{M}^{-1} \quad (12)$$

where α, β are real functions of θ and \mathbf{I} is the identity matrix. If

$$\alpha^* = \min_{\theta} \alpha(\theta) \quad (13)$$

$$\beta^* = \min_{\theta} \beta(\theta), \quad (14)$$

find $\alpha^* + \beta^*$.

10. Find the minimum value of

$$\cos(P + Q) \cos(Q + R) \cos(R + P) \quad (15)$$

in $\triangle PQR$.

11. Find the minimum value of α for which

$$4\alpha x^2 + \frac{1}{x} \geq 1, x > 0. \quad (16)$$

12. Let

$$S = S_1 \cap S_2 \cap S_3, \quad (17)$$

where

$$S_1 = \{z \in \mathbb{C} : |z| < 4\} \quad (18)$$

$$S_2 = \left\{ z \in \mathbb{C} : \Im \left[\frac{z - 1 + j\sqrt{3}}{1 - j\sqrt{3}} \right] \right\}, \quad (19)$$

$$S_3 = \{z \in \mathbb{C} : \Re(z) > 0\} \quad (20)$$

Find

$$\min_{z \in S} |1 - 3j - z| \quad (21)$$

13. A line

$$L : (m - 1)\mathbf{x} = -3 \quad (22)$$

passes through

$$\mathbf{E} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (23)$$

and

$$\mathbf{x} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - (16 \ 0)\mathbf{x} = 0, 0 \leq (0 \ 1)\mathbf{x} \leq 6 \quad (24)$$

at the point \mathbf{F} . Find m such that the area of $\triangle EFG$ is maximum.

14. If $|z - 3 - 2j| \leq 2$, find

$$\min_z |2z - 6 + 5j| \quad (25)$$

15. Find

$$\max_z \left| \text{Arg} \left(\frac{1}{1 - z} \right) \right| \quad (26)$$

$$s.t. \quad |z| = 1, z \neq 1. \quad (27)$$

16. Find the maximum value of the function

$$f(x) = 2x^3 - 15x^2 + 36x - 48 \quad (28)$$

on the set

$$A = \{x : x^2 + 20 \leq 9x\} \quad (29)$$

17. Find the minimum distance of a point on the curve

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + (0 - 1)\mathbf{x} = 4 \quad (30)$$

from the origin.

18. Find

$$\min_z \left| z + \frac{1}{2} \right| \quad (31)$$

$$s.t. \quad |z| \geq 2 \quad (32)$$

19. Find the minimum value of

$$\tan A + \tan B \quad (33)$$

$$s.t. \quad A + B = 6, \quad (34)$$

$$A, B \geq 0 \quad (35)$$

20. Show that

$$\begin{aligned} & \sin A_1 + \sin A_2 + \dots + \sin A_n \\ & \leq n \sin \left(\frac{A_1 + A_2 + \dots + A_n}{n} \right) \\ & 0 < A_i < \pi, i = 1, 2, \dots, n, n \geq 1 \end{aligned} \quad (36)$$

21. Let $\mathbf{F}_1, \mathbf{F}_2$ be the foci of the standard ellipse with parameters a and b . If \mathbf{P} be any point on the ellipse, find the maximum area of $\triangle PF_1F_2$.

22. A circle $\|\mathbf{x}\| = 1$ intersects the X -axis at \mathbf{P} and \mathbf{Q} . Another circle with centre \mathbf{Q} intersects this circle above the X -axis at \mathbf{R} and the line segment PQ at \mathbf{S} . Find the maximum area of $\triangle QSR$.

23. Let \mathbf{M} be a fixed point in the first quadrant. A line through \mathbf{M} intersects the positive axes at \mathbf{P}, \mathbf{Q} respectively. If \mathbf{O} be the origin, find the minimum area of $\triangle OPQ$.