A Linear-Programming Approach for Throughput Maximization of Uplink Multiclass VSG CDMA in Rayleigh Fading

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***Abstract*—This letter presents a new efficient technique to solve a throughput maximization problem on the uplink of a variable spreading gain code division multiple access (VSG-CDMA) system operated in an independent Rayleigh fading environment. A new linear set of constrains is found that enables a linear-programming solution to the problem of maximizing the overall average throughput in a multiclass system while maintaining the QoS constraints for all users.**

***Index Terms*—Linear programming, Rayleigh fading, throughput maximization, variable spreading gain code division multiple access (VSG CDMA).**

I. Introduction

THIS LETTER focuses on the throughput maximization problem on the uplink of a multi-class variable spreading gain code division multiple access (VSG CDMA) wireless network. Recently, the issue on throughput maximization for multiclass CDMA has received a considerable research interest, and several maximization techniques have been proposed [1]–[4]. Generally, the constraints of these techniques are represented by signal-to-interference-plus-noise ratio (SINR) [1]–[3] or outage probability [4]. Due to the fact that existing expressions of the constraints are nonlinear, previous optimization techniques are based on either nonlinear programming [1], [2] or analytical differentiation methods [3], [4]. However, it is understood that the computational complexity of nonlinear programming rises in accordance with the number of classes or users in the system [1], while the differentiation techniques are limited to some simplified CDMA system models [3].

In this letter, we propose an efficient technique to tackle a throughput maximization problem for a multiclass VSG CDMA. In contrast to the literature, the proposed technique can be easily solved by basic linear-programming methods, e.g., the simplex method [5]. This results in a significant reduction in required computational complexity.

Generally, there are two types of power control in CDMA networks, namely, fast and slow power control. The major difference of two strategies is on the frequency of the feedbacks communicating between the base station and its mobile users. Though fast power control seems superior in terms of rapid tracking and reporting channel measurements, however, it could be impossible to obtain correct measurements of channel variations on the time-scale required for fast feedback, especially when the processing and propagation delays are taken into account [7]. In practice, slow power control is therefore more reasonable to feedback the mobiles accurate measurements so that the power control process is able to catch up the channel variations [8]. Therefore, in this letter, we assume slow power control in a Rayleigh fading environment. We note that the assumption on slow power control is not only widely used in analyzing the system models of CDMA wireless communications, e.g., [1]–[4], but also well matched to Rayleigh fading channel [9].

The rest of this letter is organized as follows. In Section II, we present a model of a multirate VSG CDMA BPSK-modulated system. In Section III, a new linear technique for the throughput maximization problem is formulated. In Section IV, the proposed technique is extended to the multicell case. Section V illustrates the numerical results, and Section VI concludes this letter.

II. System Model

Consider a multirate VSG CDMA BPSK-based system with M different classes (rates) operated in the additive white Gaussian noise (AWGN) frequency nonselective Rayleigh fading channel. Let $m = 1, 2, \ldots, M$ be the normalized rate of class-m users (normalized to class-1 traffic, which is considered as the basic rate) and x_m be the total number of class-m users. Then, the SINR experienced by the reference user of class-n (user number 1) is a random variable that can be defined as

$$\mathrm{SINR}_n = \frac{G_n h_{1,n} s_n}{\sum\limits_{m=1}^{M} \sum\limits_{\substack{x=1, m\neq n \\ x=2, m=n}}^{x_m} \chi_{x,m} h_{x,m} s_m + G N_0} \tag{1}$$

where $G_n = G/n$,, and G is the total spreading bandwidth (normalized to the basic rate). s_n is the local mean-power of a class-n signal, and N_0 is the power spectral density of the additive white noise. In this letter, slow power control is assumed so that the average (over fading) powers received from the same class of traffics are identical [8]. $h_{1,n}$ is the channel gain of the reference signal, while $h_{x,m}$ represents the channel gain of the xth class-m interfering signal. It is assumed that all $\{h_{x,m} : x = 1, \ldots, x_m; m = 1, 2, \ldots, M\}$ are independent and exponentially distributed random variables that model the

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Rayleigh fading. Note that $\chi_{x,m}$ in (1) is a binary random variable representing the activity factor, where

$$\chi_{x,m} = \begin{cases} 1, & \text{if the user is active} \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

In order to derive an expression for class-n outage probability, notice from (1) that when we condition on the random variables $\{\chi_{x,m}, h_{x,m,j}, d_r(r,\alpha)\}$, then for any $z > 0$, we have

$$\Pr(\mathrm{SINR}_n < z | \{\chi_{x,m}, h_{x,m}\}) = 1 - e^{-z\left(1/G_n s_n \sum_{m=1}^{M} \sum_{\substack{x=1, m\neq n \\ x=2, m=n}}^{x_m} \chi_{x,m} h_{x,m} s_m + (nN_0/s_n)\right)} \quad (3)$$

which follows the fact that $h_{1,n}$ is an exponentially distributed random variable (a consequence of Rayleigh fading).

Now since different signals are independent, we obtain from (3) when we average out the independent random variables $\{\chi_{x,m}, h_{x,m}\}$

$$\Pr(\mathrm{SINR}_n < \zeta_n) = 1 - \left\{ e^{-(ns_1/s_n)/(E_b/N_0)\zeta_n} V^{x_n - 1}(n\zeta_n) \cdot \prod_{m=1, m\neq n}^{M} V^{x_m}\left(\frac{ns_m}{s_n}\zeta_n\right) \right\}; \quad n = 1, \ldots, M \quad (4)$$

where $E_b/N_0 = s_1/N_0$ is the bit-energy-to-noise ratio of the basic rate (class-1). ζ_n is the targeted class-n SINR, and

$$V(z) = E\left[e^{-(z/G)\chi_{x,m} h_{x,m}}\right] = 1 - q_m + q_m\left(\frac{1}{1+\frac{z}{G}}\right) \quad (5)$$

where $q_m = \Pr\{\chi_{x,m} = 1\}$, is the activity probability of a class-m user.

In Section III, we show that (4) can be used to provide linear constrains that facilitate applying the linear-programming technique to the optimization problem.

III. Throughput Maximization

This section introduces a new linear-programming approach maximizing the average overall throughput while satisfying the constraints of all users. Regardless of the blocking probability at the call admission layer, we formulate this problem as

$$\max_{x_1, x_2, \ldots x_M \geq 0} \sum_{m=1}^{M} \varpi_m q_m x_m \quad (6)$$

subject to

$$\Pr(\mathrm{SINR}_m < \zeta_m) \leq \epsilon_m; \quad m = 1, 2, \ldots, M \quad (7)$$

where $\Pr(\mathrm{SINR}_m < \zeta_m)$ are given in (4), and ε_m is the threshold of class-m outage probability. ϖ_m is an arbitrary positive-number that represents the weight of class-m traffic.

Considering the constraints in (7), we have found that it is possible in the Rayleigh fading channel to transform the non-linear constraints into a set of linear inequalities by taking the natural logarithm of both sides and afterward rearranging in a matrix form. This leads us to present the throughput maximization problem as a linear-programming problem by

$$\max_{x_1, x_2, \ldots, x_M \geq 0} \sum_{m=1}^{M} \varpi_m q_m x_m \quad (8)$$

subject to

$$\mathbf{Ax} \leq \boldsymbol{\varsigma} \quad (9)$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_M]^T$, $\boldsymbol{\varsigma} = [\varsigma_1, \varsigma_2, \ldots, \varsigma_M]^T$ with

$$\varsigma_m = -\ln\left[(1-\varepsilon_m)V(m\zeta_m)\right] - \frac{\frac{ms_1}{s_m}}{\frac{E_b}{N_0}}\zeta_m \quad (10)$$

and $\mathbf{A} = [\upsilon_{mn}]_{M\times M}$ is the coefficient matrix with elements $\upsilon_{mn} = -\ln[V((ms_n/s_m)\zeta_m)]; n, m = 1, 2, \ldots, M$. Note that the proposed formula in (8) can be simply solved by any linear-programming methods such as the simplex method [5].

IV. Multicell Consideration

Here we extend the proposed maximization technique to a multicell VSG CDMA scenario. Let us consider the inter-cell interferences from the first tier of an hexagonal cellular network that is well approximated by the circular model [6].

Recall from (1) that the expression of class-n SINR for multicell is modified by adding $\sum_{j=1}^{6}\sum_{m=1}^{M}\sum_{x=1}^{x_m} \chi_{x,m,j} h_{x,m,j} \mathcal{S}_{x,m,j}$ directly into the denominator. Let $\mathcal{S}_{x,m,j} = \mathcal{P}_{x,m,j} d^{-\beta}$ be the interfering power from the xth user of class-m in the jth cell with $\mathcal{P}_{x,m,j}$ being the transmit power and $d = \sqrt{r^2 + D^2 - 2Dr\cos\alpha}$ being the distance between the interfering user and the targeted base station. β is path loss exponent [10]. $D = \sqrt{3}$ (for CDMA and normalized by the cell radius, R) is the distance between the centers of the co-channel cells. r is the propagation path of the interfering user in respect to its serving cell. $\alpha \epsilon (0, 2\pi)$ is the angle between the distance D and path r.

For the sake of perfect power control, class-m power received at the base station is equal to s_m for every cell [8]; then $\mathcal{P}_{x,m,j} = s_m r^{\beta}$. This leads to $\mathcal{S}_{x,m,j} = s_m d_r^{-\beta}(r,\alpha)$, where $d_r(r,\alpha) = d/r = \sqrt{1 + (D/r)^2 - 2(D/r)\cos\alpha}$. Accordingly, class-$n$ outage probability for multicell is given by

$$\Pr(\mathrm{SINR}_n < \zeta_n) = 1 - \left\{ e^{-(ns_1/s_n)/(E_b/N_0)\zeta_n} V^{x_n-1}(n\zeta_n) \cdot \left[\prod_{m=1, m\neq n}^{M} V^{x_m}\left(\frac{ns_m}{s_n}\zeta_n\right)\right] \left[\prod_{m=1}^{M} W^{x_m}\left(\frac{ns_m}{s_n}\zeta_n\right)\right]^6 \right\} \quad (11)$$

where $V(z)$ is given in (5), and

$$W(z) = 1 - q_m + q_m\left[\frac{1}{\pi}\int_0^{2\pi}\int_0^1 \frac{r\,dr\,d\alpha}{1 + \frac{z}{G}d_r^{-\beta}(r,\alpha)}\right] \quad (12)$$

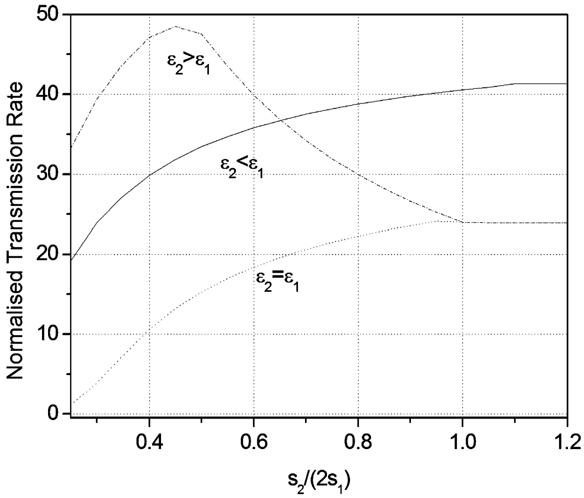

Fig. 1. Throughput versus normalized power ratio $(s_2/2s_1)$ of a dual-class VSG CDMA with $G = 500$, targeted SINR $(\zeta) = 2$ dB, $E_b/N_o = 20$ dB, the normalized class-2 rate $(m) = 2$.

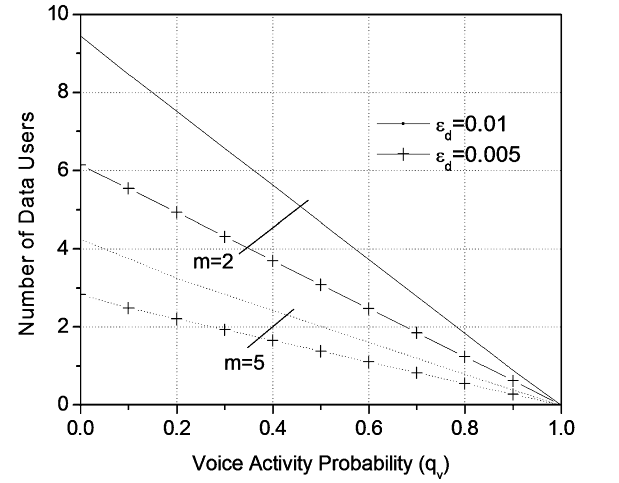

Fig. 3. Number of data users versus voice activity probability (q_v), $G = 500$, targeted SINR $(\zeta) = 2$ dB, $E_b/N_o = 20$ dB, the normalized data rate = 2 and 5 ($m = 2, 5$), and maximum outage probability of data users = 0.01 and 0.005 ($\varepsilon_d = 0.01, 0.005$).

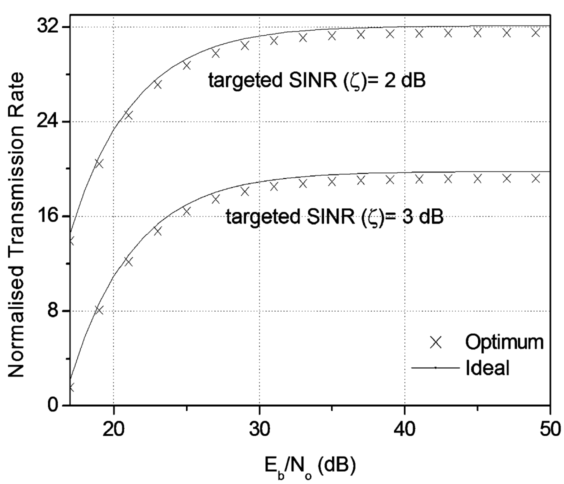

Fig. 2. Optimum and ideal throughput of the dual class VSG CDMA system versus E_b/N_0 at $G = 500$, targeted $\mathrm{SINR} = 2$ and 3 dB ($\zeta = 2, 3$ dB), maximum outage probability $(\varepsilon) = 0.01$, and the normalized class-2 rate $(m) = 2$.

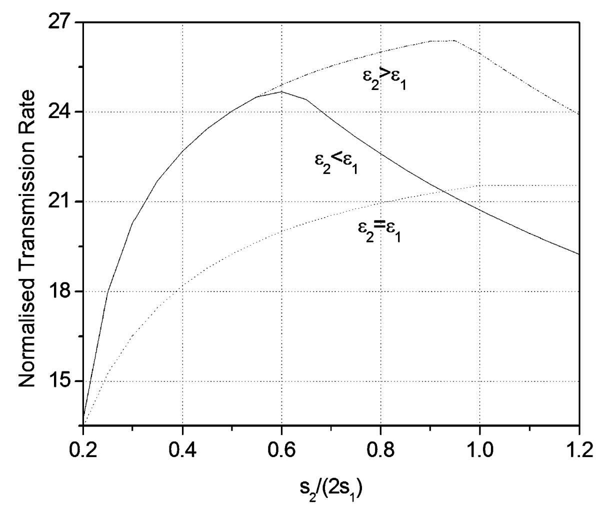

Fig. 4. Multicell: Throughput versus normalized power ratio $(s_2/2s_1)$ of a dual-class VSG CDMA with $G = 500$, targeted SINR $(\zeta) = 2$ dB, $E_b/N_o =$ 20 dB, the normalized class-2 rate $(m) = 2$.

which is achieved by taking the expectation values on the random variables $\{\chi_{x,m,j}, h_{x,m,j}, d_r(r,\alpha)\}$.

Obtained by the same procedure as (8), the linear optimization formula for multicell is similar to (8), but the components $[v_{mn}]_{M\times M}$ in matrix $\mathbf{A}$ are modified to $v_{mn} = -\ln[V((ms_n/s_m)\zeta_m)] - 6\ln[W((ms_n/s_m)\zeta_m)]$.

V. Numerical Results

This section can be outlined as follows. Fig. 1 investigates the effect of varying the relative power levels on the maximum throughput of a dual class VSG CDMA system. Fig. 2 compares the optimum throughput with the nominal throughput (or so-called ideal throughput), while Fig. 3 is concerned with determining an optimum number of data users in an integrated voice and data VSG CDMA system. Lastly, the selection of optimum powers for multicell is shown in Fig. 4.

It is very common in the literature of VSG CDMA (e.g., [11] and [12]) to assume that class-m power is m times higher than the power of the basic rate, i.e., $s_m = ms_1$; then the normalized power ratio is $(s_m/ms_1) = 1$. Nevertheless, this assumption lacks any proof of efficiency; therefore, it is of theoretical and practical interest to examine the impact of power selection on the maximum throughput.

Fig. 1 shows the optimum throughput against the normalized power ratio $(s_2/2s_1)$ of a dual-class VSG with $m = 2$. We observe that the throughput is optimized in the case of $\varepsilon_2 > \varepsilon_1$ at $(s_2/2s_1) \approx 0.5$. On the other hand, the selection of $(s_2/2s_1) = 1$ maximizes the throughput when $\varepsilon_2 \leq \varepsilon_1$.

It is worth concluding that, to achieve the maximum throughput, the optimum power ratio cannot be simply assumed to be 1, but it rather depends on the system parameters; i.e., the optimum ratio becomes smaller against larger high-rate QoS requirements, whereas it increases along with rising spreading gain. However, the graphical illustration concerning the impact of spreading gain on power selection is omitted due to page limitation.

The comparison between the optimum and the nominal throughput (ideal throughput) is shown in Fig. 2. Owing to the fact that the constraint of the proposed formula in (8) is accurately derived from the cumulative distribution function of SINR, it is difficult to compare with the other formulas in the literature, which are generally based on simplified forms of SINR. For this reason, it is very useful to compare the optimum throughput with the so-called *nominal throughput,* which is commonly known as a reference of the ideal throughput [13], [14].

Let $\Gamma = \max[\varpi_1 x_{1,\max}, \varpi_2 x_{2,\max}, \ldots, \varpi_M x_{M,\max}]$ be the "*nominal throughput*," which is regarded as the maximum feasible throughput. ϖ_m is the reward per class-m transmission. $x_{n,\max}$ is the maximum number of class-n users that can be accommodated into the system by the whole capacity without any violation of class-n constraint. Accordingly, nominal throughput is considered as the maximum value of $\varpi_m x_{m,\max}$; $m = 1, 2, \ldots, M$, which is always more than the optimum throughput $\Gamma_{opt} = \sum_{m=1}^{M} \varpi_m x_{m,opt}$, where $x_{1,opt}, x_{2,opt}, \ldots x_{M,opt}$ are selected by linear-programming optimization.

In order to find the expression of $x_{m,\max}$, recall (4) and let $x_n = 0$ for $n \neq m$; then, we obtain the class-m outage probability, which is applicable in the case that there are only class-m users existing in the system, as $\Pr(\mathrm{SINR}_m < \zeta_m) = 1 - [\exp(-(ms_1/s_m)/(E_b/N_0)\zeta_m)]V^{x_m - 1}(m\zeta_m) \leq \varepsilon_m$. Setting $\Pr(\mathrm{SINR}_m < \zeta_m) = \varepsilon_m$ and taking natural logarithm on both sides of the equation, we express $x_{m,\max}$ as

$$x_{m,\max} = \frac{\ln(1-\varepsilon_m) + \frac{\left(\frac{ms_1}{s_m}\right)}{\left(\frac{E_b}{N_0}\right)}\zeta_m}{\ln(V(m\zeta_m))} + 1. \tag{13}$$

Here we replace the reward function (ϖ) with the transmission rate normalized by the basic rate (class-1 rate). Then, the nominal throughput is also represented in terms of *normalized transmission rate* as well as the optimum throughput. From Fig. 2, it is found that the optimum throughput is in close proximity (less than 1%) to the ideal (nominal) throughput in every channel condition (E_b/N_0) and targeted SINR (ζ).

Consider an integrated voice and data VSG CDMA system. We fix the number of voice users at 20 and obtain the optimum number of data users by (8). Note that although the existence of voice users is kept constant, the number of voice interferences is a random variable because of the activity probability. Assume the power ratio is unity and the transmission speed of data is twice as voice. Let activity probability of voice users be $q_v \in [0, 1]$, whereas every data user is always active. From Fig. 3, the optimum data users turn down linearly against the increasing q_v. These curves are very useful for the system administrators to choose the optimum number of admitted data users in accordance with voice loads, required outage probabilities, and data rates.

Fig. 4 studies the effect of the optimum power selection on the throughput in a multicell scale. Assume a BPSK-modulated dual-class VSG CDMA system, and let $G = 500$, $E_b/N_0 = 20$ dB, targeted SINR $(\zeta) = 2$ dB, and $\beta = 2$. In the figure, the optimum power ratios are achieved at 0.6 for $\varepsilon_1 > \varepsilon_2$ and 0.9 for $\varepsilon_1 < \varepsilon_2$, respectively. At $\varepsilon_1 > \varepsilon_2$, the throughput improves by 30% at the optimum point compared with $(s_2/2s_1) = 1$, whereas there is a minor improvement for $\varepsilon_1 < \varepsilon_2$. Meanwhile, on $\varepsilon_1 = \varepsilon_2$ curve, the throughput increases exponentially when $(s_2/2s_1) < 1$ and becomes saturated at $(s_2/2s_1) \geq 1$.

VI. Summary

We have proposed a new linear-programming technique to find the optimum throughput of a multiclass VSG CDMA system in an independent Rayleigh fading channel. In contrast to previous techniques that were based on nonlinear objective functions, we have found that it is possible in a frequency nonselective Rayleigh fading channel to transform the usual nonlinear set of constraints into a linear set. This has led to a great simplification to the optimization problem and facilitates applying the computationally efficient linear-programming techniques to optimize the average overall throughput of a multiclass VSG CDMA system while bounding the outage probability of each user. Based on the numerical results, the maximum throughput is achieved equivalently to the ideal in every channel condition and targeted SINR. It is also found that the optimum power ratio is not always equal to unity but varies according to spreading gain, rate ratio, noise condition (E_b/N_0), and targeted SINR. In addition, the proposed technique can be employed as a handy formula to select the optimum number of data users in voice and data integrated VSG CDMA systems.

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