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Abstract—This manual provides applications of Complex Analysis in Electrical Engineering.

Using the inverse fourier transform relationship, show that

$$F(t)e^{-\sigma t} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{j\omega t}}{s(1-s^2)} d\omega \quad (7)$$

1 THE INVERSE Z TRANSFORM

Problem 1. Show that z^n is analytic everywhere for $n \geq 0$.

Problem 6. Show that

$$F(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{e^{st}}{s(1-s^2)} ds \quad (8)$$

Problem 2. Show that for $C : z = Re^{j\theta}, 0 < \theta < 2\pi$,

$$\oint_C \frac{dz}{z^n} = \begin{cases} 2\pi j & n = 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Problem 7. Let

$$F(t) = \begin{cases} \frac{1}{2\pi j} \oint_{C_1} \frac{e^{st}}{s(1-s^2)} ds & t > 0 \\ \frac{1}{2\pi j} \oint_{C_2} \frac{e^{st}}{s(1-s^2)} ds & t < 0 \end{cases} \quad (9)$$

Definition 1.1. The Z transform of $x(n)$ is defined as

$$X(z) = \sum_{k=-\infty}^{\infty} x(k)z^{-k} \quad (2)$$

where C_1, C_2 are the closed contours on the left and right respectively as shown in Figs. 7.1 and 7.2. Find $F(t)$ given that the ROC of $\frac{M(s)}{s}$ is $0 < \text{Re}(s) < 1$.

Problem 3. Show that

$$\frac{1}{2\pi j} \oint_C X(z)z^{n-1} dz = \sum_{k=-\infty}^{\infty} x(k) \oint_C z^{n-k-1} dz \quad (3)$$

$$= x(n) \quad (4)$$

Problem 4. The Z transform of $x(n)$ is given by

$$X(z) = \frac{z^{20}}{(z - \frac{1}{2})(z - 2)^5(z + \frac{5}{2})^2(z + 3)} \quad (5)$$

Also, it is known that $X(z)$ is analytic for $|z| = 1$. Find $x(-18)$.

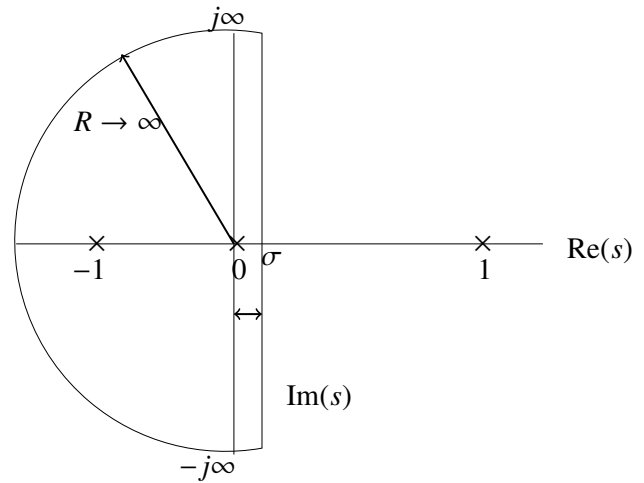


Fig. 7.1

2 THE LAPLACE TRANSFORM

Problem 5. Let

$$\int_{-\infty}^{\infty} F(t)e^{-st} dt = \frac{1}{s(1-s^2)} = \frac{M(s)}{s}, \quad s = \sigma + j\omega \quad (6)$$

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3 THE GIL-PELAEZ INTEGRAL

Problem 8. Let

$$F(t) = \frac{1}{2} - \frac{1}{2\pi j} \times \text{c.p.v.} \int_{-\infty}^{\infty} \frac{e^{-j\omega t}}{\omega(1+\omega^2)} d\omega, \quad (10)$$

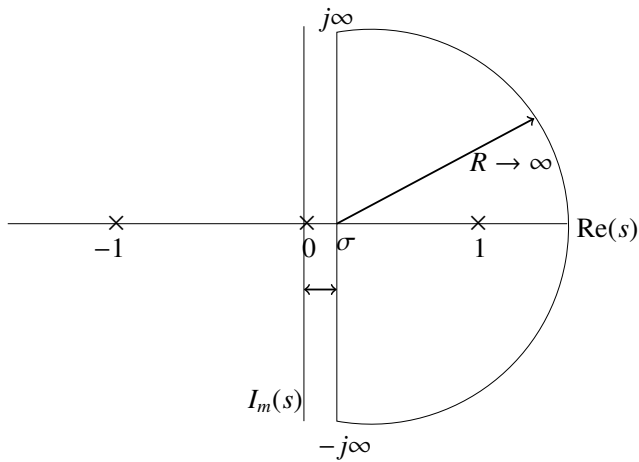


Fig. 7.2

where c.p.v. denotes the Cauchy Principal Value. Use the contour in Fig. 8 to evaluate $F(t), t < 0$ by showing that

- 1) $\lim_{r \rightarrow 0} \int_{C_r} \frac{e^{-jzt}}{z(1+z^2)} dz = -j\pi,$
 $C_r : z = re^{j\theta}, 0 < \theta < \pi$
 where C_r is in the clockwise direction.
- 2) $\lim_{\substack{r \rightarrow 0 \\ R \rightarrow \infty}} \int_{-R}^{-r} \frac{e^{-j\omega t}}{\omega(1+\omega^2)} d\omega + \int_r^R \frac{e^{-j\omega t}}{\omega(1+\omega^2)} d\omega$
 $= j\pi + \oint_C \frac{e^{-jzt}}{z(1+z^2)} dz$

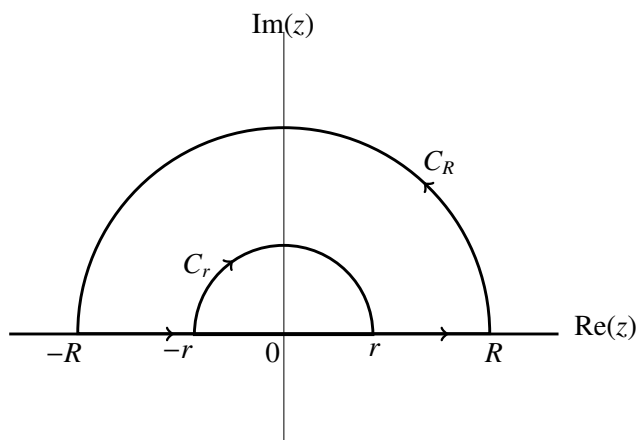


Fig. 8

$z = c$ is defined as

$$\text{Res}_{z=c} f(z) = \text{Res}(f, c) = \frac{1}{2\pi i} \oint_{\Gamma} f(z) dz \quad (11)$$

where c lies inside Γ .

Problem 10. Find the inverse Z transform of

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2} \quad (12)$$

Problem 11. Find the inverse Z transform of

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} \quad |z| < \frac{1}{2} \quad (13)$$

Problem 12. Let

$$X(z) = \frac{1 - a^2}{(1 - az)(1 - az^{-1})}, \quad \text{ROC} : a < |z| < \frac{1}{a}, \quad (14)$$

$0 < a < 1$. Determine $x(n)$

Problem 9. Using a suitable contour, evaluate $F(t), t > 0$.

Definition 3.1. The residue of a function $f(z)$ at