भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology
Hyderabad

Complex Analysis in Electrical Engineering

Teaching Learning Center
IIT Hyderabad

G V V Sharma*

***Abstract*—This manual provides applications of Complex Analysis in Electrical Engineering.**

1 The Inverse Z Transform

Problem 1. Show that z^n is analytic everywhere for $n \geq 0$.

Problem 2. Show that for $C : z = Re^{\jmath\theta}, 0 < \theta < 2\pi$,

$$\oint_C \frac{dz}{z^n} = \begin{cases} 2\pi\jmath & n = 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Definition 1.1. The Z transform of $x(n)$ is defined as

$$X(z) = \sum_{k=-\infty}^{\infty} x(k) z^{-k} \tag{2}$$

Problem 3. Show that

$$\frac{1}{2\pi\jmath} \oint_C X(z) z^{n-1}\, dz = \sum_{k=-\infty}^{\infty} x(k) \oint_C z^{n-k-1}\, dz \tag{3}$$

$$= x(n) \tag{4}$$

Problem 4. The Z transform of $x(n)$ is given by

$$X(z) = \frac{z^{20}}{\left(z - \frac{1}{2}\right)(z-2)^5 \left(z + \frac{5}{2}\right)^2 (z+3)} \tag{5}$$

Also, it is known that $X(z)$ is analytic for $|z| = 1$. Find $x(-18)$.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502205 India e-mail: gadepall@iith.ac.in.