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**Abstract—**This manual provides applications of Complex Analysis in Electrical Engineering.

Using the inverse fourier transform relationship, show that

$$F(t)e^{-\sigma t} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{j\omega t}}{s(1-s^2)} d\omega \quad (7)$$

## 1 THE INVERSE Z TRANSFORM

**Problem 1.** Show that  $z^n$  is analytic everywhere for  $n \geq 0$ .

**Problem 6.** Show that

$$F(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{e^{st}}{s(1-s^2)} ds \quad (8)$$

**Problem 2.** Show that for  $C : z = Re^{j\theta}, 0 < \theta < 2\pi$ ,

$$\oint_C \frac{dz}{z^n} = \begin{cases} 2\pi j & n = 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

**Problem 7.** Let

$$F(t) = \begin{cases} \frac{1}{2\pi j} \oint_{C_1} \frac{e^{st}}{s(1-s^2)} ds & t > 0 \\ \frac{1}{2\pi j} \oint_{C_2} \frac{e^{st}}{s(1-s^2)} ds & t < 0 \end{cases} \quad (9)$$

**Definition 1.1.** The Z transform of  $x(n)$  is defined as

$$X(z) = \sum_{k=-\infty}^{\infty} x(k)z^{-k} \quad (2)$$

where  $C_1, C_2$  are the closed contours on the left and right respectively as shown in Fig. 7. Find  $F(t)$  given that the ROC of  $\frac{M(s)}{s}$  is  $0 < \text{Re}(s) < 1$ .

**Problem 3.** Show that

$$\frac{1}{2\pi j} \oint_C X(z)z^{n-1} dz = \sum_{k=-\infty}^{\infty} x(k) \oint_C z^{n-k-1} dz \quad (3)$$

$$= x(n) \quad (4)$$

**Problem 4.** The Z transform of  $x(n)$  is given by

$$X(z) = \frac{z^{20}}{\left(z - \frac{1}{2}\right)(z-2)^5\left(z + \frac{5}{2}\right)^2(z+3)} \quad (5)$$

Also, it is known that  $X(z)$  is analytic for  $|z| = 1$ . Find  $x(-18)$ .

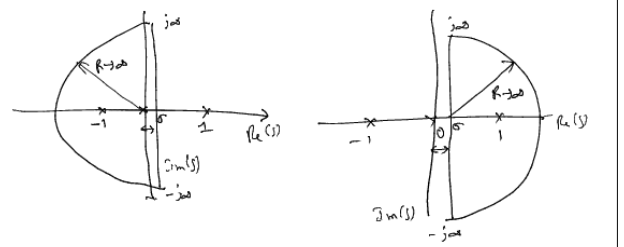


Fig. 7

## 2 THE LAPLACE TRANSFORM

**Problem 5.** Let

$$\int_{-\infty}^{\infty} F(t)e^{-st} dt = \frac{1}{s(1-s^2)} = \frac{M(s)}{s}, \quad s = \sigma + j\omega \quad (6)$$

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## 3 THE GIL-PELAEZ INTEGRAL

**Problem 8.** Let

$$F(t) = \frac{1}{2} - \frac{1}{2\pi j} \times \text{c.p.v.} \int_{-\infty}^{\infty} \frac{e^{-j\omega t}}{\omega(1+\omega^2)} d\omega, \quad (10)$$

where c.p.v. denotes the Cauchy Principal Value. Use the contour in Fig. 8 to evaluate  $F(t), t < 0$  by showing that

1)

$$\lim_{r \rightarrow 0} \int_{C_r} \frac{e^{-jzt}}{z(1+z^2)} dz = -j\pi,$$

$$C_r : z = re^{j\theta}, 0 < \theta < \pi \quad (11)$$

where  $C_r$  is in the clockwise direction.

2)

$$\lim_{\substack{r \rightarrow 0 \\ R \rightarrow \infty}} \int_{-R}^{-r} \frac{e^{-j\omega t}}{\omega(1+\omega^2)} d\omega + \int_r^R \frac{e^{-j\omega t}}{\omega(1+\omega^2)} d\omega$$

$$= j\pi + \oint_C \frac{e^{-jzt}}{z(1+z^2)} dz \quad (12)$$

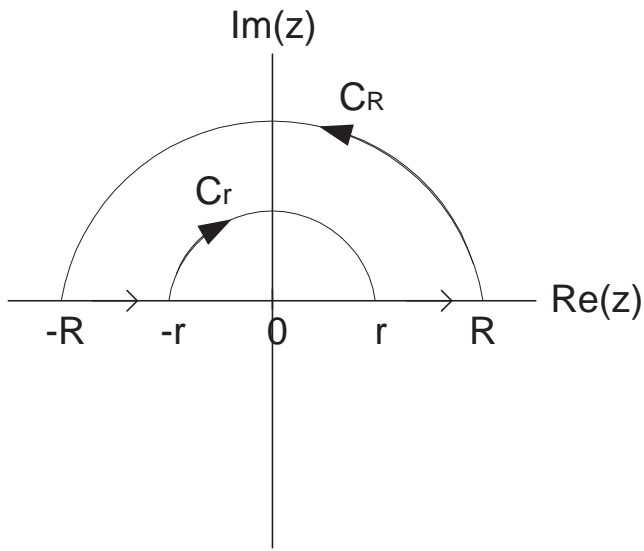


Fig. 8

**Problem 9.** Using a suitable contour, evaluate  $F(t), t > 0$ .