

# GATE problems in Complex Analysis

**Abstract—This manual provides applications of Complex Analysis in Electrical Engineering.**

1. The contour  $C$  given below is on the complex plane  $z = x + jy$ . Find the value of the integral  $\frac{1}{\pi j} \oint_C \frac{dz}{z^2 - 1}$ .

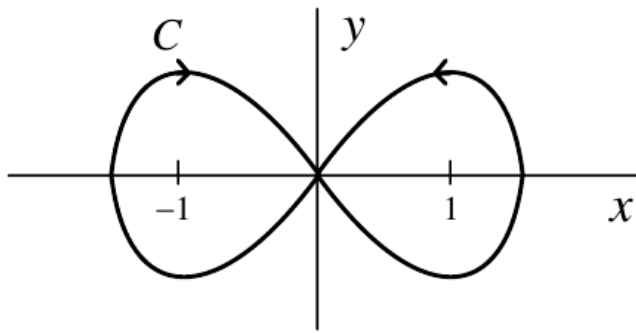


Fig. 1

2. An integral  $I$  is given by

$$I = \oint_C \frac{z^2 - 1}{z^2 + 1} e^z dz. \quad (1)$$

If  $C : |z| = 3$ , find the value of  $I$ .

3. Consider contour integration performed over  $C : |z| = 1$  in the anticlockwise direction. Which of the following is NOT true?
- The residue of  $\frac{z}{z^2 - 1}$  at  $z = 1$  is  $\frac{1}{2}$ .
  - $\oint_C z^2 dz = 0$
  - $\frac{1}{2\pi j} \oint_C \frac{dz}{z} = 1$ .
  - $\bar{z}$  is an analytic function.

4. Find the value of

$$\oint_C \frac{z^2 - z + 4j}{z + 2j} dz, \quad C : |z| = 3. \quad (2)$$

5. Given

$$f(z) = \frac{1}{z + 1} - \frac{2}{z + 3}. \quad (3)$$

If  $|z + 1| = 1$ , find  $\frac{1}{2\pi j} \oint_C f(z) dz$ .

6. Find

$$\oint_C \frac{-3z + 4}{z^2 + 4z + 5} dz, \quad C : |z| = 1 \quad (4)$$

7. Find the residues of a complex function

$$X(z) = \frac{1 - 12z}{z(z - 1)(z - 2)} \quad (5)$$

at its poles.

8. If  $f(z) = c + 0 + c_1 z^{-1}$ , find

$$\oint_C \frac{1 + f(z)}{z} dz, \quad C : |z| = 1 \quad (6)$$

9. Find the residue of the function

$$f(z) = \frac{1}{(z + 2)^2 (z - 2)^2} \quad (7)$$

at  $z = 2$ .

10. If the semi-circular contour  $D$  of radius 2 is as shown in the figure, then find

$$\oint_D \frac{dz}{z^2 + 4} \quad (8)$$

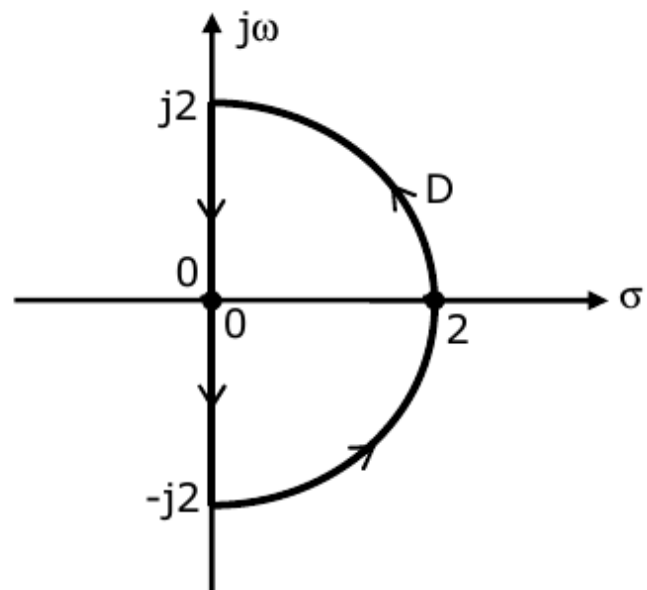


Fig. 10

11. Find

$$\oint_{|z-j|=2} \frac{1}{z^2-1} ds \quad (9)$$