# EE608 Adaptive Signal Processing

Problem Set 7

1) As shown in class the decision feedback equalizer is obtained by finding $\alpha_k$ and $\beta_k$ such that $E[\mid I_k - \hat{I}_k \mid^2]$ is minimized

   subject to

$$\hat{I}_k = \sum_{k=-M_1}^{0} \alpha_k y(k-n) + \sum_{k=1}^{M_2} \beta_k I_{k-n}$$

   Show that $\{\alpha_k, \beta_k\}$ are obtained by solving

$$\sum_{j=-M_1}^{0} \alpha_j r(l_1 - j) + \sum_{j=1}^{M_2} \beta_j q^*(j - l_1) = q^*(-l_1)$$

$$\sum_{j=-M_1}^{0} \alpha_j q(-j + l_2) 1(L + j - l_2) + \beta_{l_2} = 0$$

   where $M_1 \leq l_1 \leq 0$ and $1 \leq l_1 \leq M_2$.
   In the above $q(\cdot)$ is the impulse response sequence such that the preprocessed received signal (preprocessed in the sense that the received signal has gone through a matched filter and a whitening filter) is given by

$$y(k) = \sum_{n=0}^{L} q(n) I_{k-n} + \eta(k)$$

   where we assume that $I_n \perp\!\!\!\perp I_m$ for $m \neq n$, and $E[I_n I_m^*] = \delta_{m,n}$. Also $I_n \perp\!\!\!\perp \eta(k)$ for all n and k, and $E_\eta(k)\eta(l) = \sigma_n \delta_{k,l}$.

2) Derive the update equation for the the training based LMS Algorithm for adaptive equalization, namely

   $W(n+1) = W(n) + \mu Y^*(n)[I(n) - Y^T(n)W(n)], W(0) = 0$

3) Derive the CM algorithm of Goddard for the case for

   $J_{12} = (|\hat{I}_k| - D)^2$
   Derive the weight update equation as well as the carrier tracking equation.
   **Simulations**

4) **LMS algorithm for channel equalization using training signals:**
   Consider the following block diagram for the adaptive channel equalizer using training sequences.

   In the above we are assuming that the transmitted symbols are binary (bpsk) with zero mean $b_n = \pm 1$. These are passed through a channel with impulse response

$$h(n) = \begin{cases} \frac{1}{2}[1 + cos(\frac{2\pi}{F}(n-2))] & n = 1,2,3 \\ 0 & otherwise \end{cases}$$

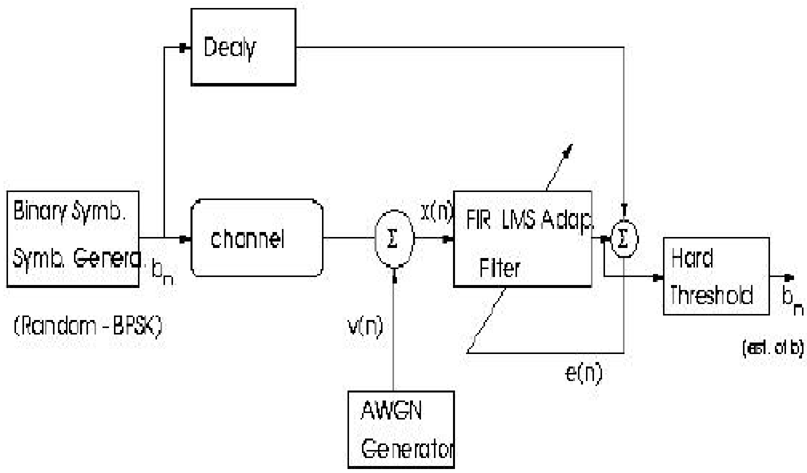


Figure 1: For problem no. 4

and corrupted by additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2$. The delayed symbols that are fed at the output of the adaptive filter act as training signals.The channel impulse response is an 3-point FIR filter with a raised cosine type of structure.Parameter F controls the eigen value spread of $R = E[X(n)X^T(n)$.The structure of this problem is very similar to the one given in Haykin.

Note:

$$x(n) = \sum_{k=1}^{3} h(k)b_{n-k} + v(n)$$

You will need two random number generators; one for the symbols b n and the other for the AWGN.For AWGN assume $\sigma^2 = 0.01$. For $b_n$ use a binary random number generator where 1 and -1 are generated with equal probability. You could do this by using a uniform random number generator, uniform over [-0.5, 0.5]. Whenever you get a negative number, set it to -1 and whenever you get a positive number set it to +1.

(a) Experiment with different number of tap weights in the FIR LMS adaptive filter. For example you could consider 7, 9, or 11 tap weights.
(b) Experiment with different variance for AWGN.
(c) Estimate the amount of delay you need based on the number of tap weights you are using.
(d) Run the experiment for two vales of $F$, say $F = 3$ and $F = 3.2$.
(e) Plot the mean square error curves for each case.
(f) Give a table for the values for $W(\infty)$.
(g) At the output of the adaptive filter have a hard threshold unit to classify the output as 1 or -1. This is done since we know that the transmitted symbols were 1 or -1. Now compute the percentage bits that are in error at convergence (this will give you the bit error rate (BER)).

5) Repeat the above channel equalization problem for the following cases:

(a) The channel is no longer as given by the cosine function. But, now let the channel be modeled by an FIR filter of length 10, and each FIR coefficient be of unit magnitude. This way we will be considering the performance of the equalizer under a narrow band channel.
(b) The data symbols are not binary (BPSK) but are QPSK. You can generalize the method used for generating BPSK to a method for generating QPSK. Plot the bit error rate. Do this for both

the cases: (a) the FIR channel specified above, and (b) any other narrow band channel of your choice. Your channel could be FIR, IIR or non-linear.