EE608 Adaptive Signal Processing
Problem Set 3

1) Consider the state space model

$$X(k+1) = F(k)X(k) + G(k)u(k), x(0) = x_0$$

with $E[X_0] = 0, E[u(k)] = 0, E[x_0 {x_0}^T] = \Pi_0, E[u(k)u^T(j)] = Q(k)\delta(k-j)$, and for $j \geq k, E[x(k)u^T(j)] = 0$. Let $\Pi(k) = E[x(k)x^T(k)]$.

(a) Show that

$$\Pi(k+1) = F(k)\Pi(k)F^T(k) + G(k)Q(k)G^T(k)$$

(b) Show that

$$\Pi(k) - P(k|k-1) \geq 0$$

Give an interpresentation to this result.

2) Verify the matrix inversion identity

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}$$

3) Show that the filtered error covariance matrix is given by

$$P(k|k) = P(k|k-1) - P(k|k-1)H^T {R_\epsilon}^{-1}(k)HP(k|k-1)$$

4) **Measurement Update and Time Update in Kalman Filtering**
Let the filtered estimate be $\hat{x}(k|k) \stackrel{\text{def}}{=} \mathcal{P}(k)|\{y(0), ...y(k)\}]$. Using the kalman Filter (one step predictoer) equations developed in class and associated manipulations, show that the filtered and one step predicted estimate can be computed through the following equations.
Measurement Update

$$\begin{aligned}
\hat{x}(k|k) &= \hat{x}(k|(k-1) + K_f(k)[y(k) - H\hat{x}(k|k-1)], \hat{x}(0|-1) = 0 \\
k_f(k) &= P(k|k-1)H^T R\epsilon^{-1}(k) \\
&= P(k|k)H^T R^{-1} \\
R_\epsilon(k) &= R + HP(k|k-1)H^T \\
P(k|k) &= P(k|k-1) - P(k|k-1)H^T {R_\epsilon}^{-1} HP(k|k-1), P(0|-1) = 0
\end{aligned}$$

Time Update

$$\begin{aligned}
\hat{x}(k+1|k) &= F\hat{x}(k|k) \\
P(k+1|k) &= FP(k|k)F^T + GQG^T
\end{aligned}$$

Note, form the above equation it should be obvious as to why the computation of the filtered estimate from the one step predicted estimate is referred to as the Measurement Update, while the computation of the one step predicted estimate from the filtered estimate is referred to as the Time Update.