EE608 Adaptive Signal Processing
Problem Set 2

1) **Computing the Sample Covariance Matrix**
Let $Y(k)$ be a discrete time stochastic process, and $\{y(k)\}_0^{N-1}$ be a realization of this process. A typical problem in practice is to compute the second order statistics of the process.
The procedure for computing the mean is well known; in fact it is the answer obtained earlier using the maximum likelihood estimation procedure under Gaussian assumption, namely

$$\hat{m}_N = \frac{1}{N} \sum_{0}^{N-1} Y(k)$$

To avoid a new notation we use the *computer programming definition*

$Y(k) := Y(k) - \hat{m}_N \rightarrow y(k) := y(k) - \hat{m}_N$

Now we look for procedures for computing the covariance of Y(k) using $\{y(k)\}_0^{N-1}$.
(a) Let

$$\bar{r}_N(k) = \frac{1}{N-k}[\sum_{j=0}^{N-k-1} y(j+k)y(j)], k = 0, 1, ...N-1$$

show that
i. $\bar{r}_N(k)$ is an unbiased estimate of the true covariance parameter $r(k) = E[(j+k)Y(j)]$
ii. the covariance matrix constructed using $r_N(k)$ will not be non-negative definite. Because of this problem one looks for alternative methods for computing the sample covariance matrix.

(b) Alternately let the sample covariance matrix be defined as

$$r_N(k) = \frac{1}{N-k}[\sum_{j=0}^{N-k-1} y(j+k)y(j)], k = 0, 1, ...N-1$$

i. Show that $r_N(k)$ is not an unbiased estimate of the true covariance parameter; nevertheless it is an asymptotically unbiased estimate.
ii. Show that the covariance matrix constructed using $\bar{r}_N(k)$ is non-negative definite.
(Hint: Express the sample covariance matrix as a product of data matrix with its transpose).

2) Let $x(\cdot)$ and $y(\cdot)$ be discrete time, jointly stationary zero mean processes with $E[x(n)y(m)] = r_{xy}(n-m), E[y(n)y(m)] = r_{yy}(n-m)$ Show that the parameters involved in computing the following linear least-squares estimates are identical.
(a) $\hat{E}[x(n)|y(n-1), ...y(n-M)]$
(b) $\hat{E}[x(n+N)|y(n+N-1), ...y(n+N-M)]$

*Note:*This simple result shall be used often.

3) **Determining $\hat{x}(k|n)$ using Levinson's Algorithm**

   In class we developed the Levinson's Algorithm for the one step prediction problem and for generating the innovations. In this problem you are required to develop a recursive scheme for computing $\hat{x}(k|n)$ in the framework of Levinson's algorithm.
   Show that

   $\hat{x}(k|n+1) = \hat{x}(k|n) + \Gamma(k,n)\epsilon(n)$
   Determine the expression for the gain $\Gamma(k,n)$ and an expression for recursively computing it. The above is referred to as the measurement update equation.

4) In the Levinson Algorithm derive the expression

$$p(n) = r(0) + \sum_{i-0}^{n-1} a_n(n-i)r(i-n)$$

   where $p(n)$ is the innovations (error) variance

$$p(n) = E[\{y(n) - \hat{y}(n|n-1)\}^2]$$

5) Let $y(\cdot)$ be a zero mean process with

$$E[y(k+m)y(m)] = (\frac{1}{4})^{|k|}$$

   and $x(\cdot)$ be another process with

$$E[y(k+m)y(m)] = \begin{cases} (2/3)^k & k \geq 0 \\ \\ (1/3)^k & k \leq 0 \end{cases}$$

   $E[x(k+m)x(m)] = (\frac{1}{2})^{|k|}$

   (a) Find the optimal filter for computing the linear least squares estimate of $x(n+N), N > 0$, based on $\{y(k), k \leq n\}$.

6) **Simulation:**Consider the sample values of a stationary process y($\cdot$) on the home page for the **ASP course:**in http: //sharada.ee.iitb.ac.in/ ee608/, file name is levinson.dat

   (a) Determine the first 50 covariance lags $r(n)$ $0 \leq n \leq 50$.
   (b) Using Levinson's algorithm determine an appropriate AR model. Comment on the goodness of this model. If there are any deficiencies or problems give reasons for them.