EE608 Adaptive Signal Processing

Problem Set 6

1) *An alternate aspect of the convergence analysis for the RLS algorithm*
   Suppose there exists a true model for $d(n)$ of the form

$$d(n) = X^T(n)W_o + v(n)$$

   where $v(n)$ is some unknown error, and W_o the true weight vector which is also unknown.

   The question we want to investigate is whether $W(n) \to W_o$ in the mean and the mean square sense. Note this is different from what we did in class where we were considering the convergence of $W(n)$ to W_* the Wiener optimal solution.

   In this problem we shall only investigate the convergence of $W(n) to W_o$ in the mean. Also consider the RLS algorithm based on

$$W(n) = [\sum_{k=0}^{n} X(k)X^T(k)]^{-1}[\sum_{k=0}^{n} X(k)d(k)]$$

   (a) Show that, if $v(i) \perp\!\!\!\perp X(k)$for $k \leq i$, then $E[W(n)] = W_o$.
   (b) In a deterministic sense, show that $W(n) \to W_o$ if $\lim_{n\to\infty} \frac{1}{n}\sum_{k=0}^{n} X(k)v(k) = 0$.

2) Start with the following non-recursive equations and derive the RLS algorithm having the necessary initial condition $(P(0) = \delta^{-1})I$.

$$W(n) = [\delta I + \sum_{i=1}^{n} X(i)X^T(i)]^{-1}[\sum_{i=1}^{n} X(i)d(i)]$$

$$P(n) = [\delta I + \sum_{i=1}^{n} X(i)X^T(i)]^{-1}$$

3) Simulate the normalized LMS algorithm and compare with the LMS algorithm. For this you should use the
   (a) data generated by the model in the previous problem.
   (b) generate the data for an AR model

$$x(n) + a_1x(n-1)a_2x(n-2)a_3x(n-3) + a_4x(n-4) = v(n)$$

   where the AR parameters correspond to the four poles $\lambda_1 = 0.8, \lambda_2 = 0.6 + j0.5, \lambda_3 = 0.6 - j0.5, \lambda_4 = 0.65$. We assume that that $v(n)$ is zero mean, unit intensity white sequence. You will need to determine the AR parameters for the given pole locations. Note, you should generate at least 100 data sets, each data set having large enough points so that you can get a reasonable assessment of the mean squared error. The length of the data set will depend on how many iterations are required for convergence

   Use the following form of the normalized LMS algorithm.

$$W(n+1) = W(n) + \frac{aX(n+1)}{c + X^T(n+1)X(n+1)}(d(n) - X^T(n+1)W(n)), W(0) = 0$$

   You should use the following for the purpose of comparison and validation

   (a) Learning curve (i.e mean square error curve)
   (b) Convergent values of $W(n)$

(c) Whiteness of the error

4) Simulate the RLS algorithm using
   (a) The data generated during the simulations for the LMS and N-LMS algorithms in the earlier home works.
   (b) Compare the results with those obtained for the LMS and N-LMS algorithms (rate of convergence, excess mean square error, weight values, etc.)
5) Apply the above RLS algorithm for the noise cancellation problem. For this use the data that was used in the earlier noise cancellation simulation. Compare your noise cancellation results using LMS, NLMS and RLS.
6) **Divergence of RLS Algorithm**
   (a) Simulate the divergence in the RLS algorithm. One way to achieve this is to is to round-off every variable to, say, the third or second significant digit, at the end of each iteration (if divergence still does not occur, you may have to round-off to first significant digit).This way, we artificially introduce round-off errors and examine its accumulation effect.
   (b) For the the procedure of part (a) which simulates divergence, now implement the reinitialization scheme for the RLS algorithm and check whether it is able to overcome the problem of divergence. Thus, in the reinitialization algorithm too, you will be rounding off each variable to third or fourth significant digit.