

Autonomous Navigation

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CONTENTS

1	Installation	1
2	Least Squares	1
3	Singular Value Decomposition	1
4	Vector Calculus	2

1 INSTALLATION

1.1. Create virtual environment

```
sudo pip install virtualenv
mkdir ugv
virtualenv ugv
source ugv /bin/activate
```

1.2. Install required packages

1.3. Download abhishek.zip from

```
https://iith-my.
sharepoint.com/:u:/g
/personal/gadepall_
ee_iith_ac_in/EaPt9
rhAVxJKs5IKTHyK30sBD
5ttt1dtDrT-kwupl-
CaVQ?e=w6O0ca
```

1.4. Unzip

```
unzip abhishek.zip
mv abhishek ugv/
```

2 LEAST SQUARES

Consider the lines

$$L_1: \mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad (4.1)$$

$$L_2: \mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad (4.2)$$

2.1. If the two lines intersect, show that

$$\mathbf{M} \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} = \mathbf{x}_2 - \mathbf{x}_1 \quad (2.1.1)$$

where

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \mathbf{m}_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \mathbf{m}_2 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}. \quad (2.1.2)$$

$$\mathbf{M} = \begin{pmatrix} \mathbf{m}_1 & \mathbf{m}_2 \end{pmatrix} \quad (2.1.3)$$

2.2. Find the rank of the augmented matrix in (2.1.1). Show that the lines in (4.2),(4.2) do not intersect.

2.3. Let

$$\mathbf{A} = \mathbf{x}_1 + \lambda_1 \mathbf{m}_1 \quad (2.3.1)$$

$$\mathbf{B} = \mathbf{x}_2 + \lambda_2 \mathbf{m}_2 \quad (2.3.2)$$

be the closest points on L_1 and L_2 respectively. Then the shortest distance between two skew lines will be the length of line perpendicular to both the lines L_1, L_2 and passing through A and B . Show that

$$\mathbf{M}^T (\mathbf{A} - \mathbf{B}) = 0 \quad (2.3.3)$$

From (2.3.2) and (2.1.3)

$$\mathbf{A} - \mathbf{B} = \mathbf{x}_1 - \mathbf{x}_2 + \mathbf{M} \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} \quad (2.3.4)$$

2.4. Show that

$$\mathbf{M}^T \mathbf{M} \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} = \mathbf{M}^T (\mathbf{x}_2 - \mathbf{x}_1) \quad (2.4.1)$$

2.5. Obtain λ_1 and λ_2 .

2.6. Show that the distance between the lines is

$$\frac{3}{\sqrt{2}} \quad (2.6.1)$$

3 SINGULAR VALUE DECOMPOSITION

3.1. Find $\mathbf{M}^T \mathbf{M}$ and $\mathbf{M} \mathbf{M}^T$.

3.2. Obtain the eigen decomposition

$$\mathbf{M}^T \mathbf{M} = \mathbf{P}_1 \mathbf{D}_1 \mathbf{P}_1^T \quad (3.2.1)$$

and

$$\mathbf{M} \mathbf{M}^T = \mathbf{P}_2 \mathbf{D}_2 \mathbf{P}_2^T \quad (3.2.2)$$

3.3. Perform the QR decompositions

$$\mathbf{P}_1 = \mathbf{U} \mathbf{R}_1 \mathbf{P}_2 = \mathbf{V} \mathbf{R}_2 \quad (3.3.1)$$

3.4. The singular value decomposition is the given by

$$\mathbf{M} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T, \quad (3.4.1)$$

where $\mathbf{\Sigma}$ has the same shape as \mathbf{M} and

$$\mathbf{\Sigma} = \begin{pmatrix} \mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \quad (3.4.2)$$

3.5. Let

$$\mathbf{b} = \mathbf{x}_2 - \mathbf{x}_1 \quad (3.5.1)$$

$$\mathbf{y} = \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} \quad (3.5.2)$$

(2.1.1) can then be expressed as

$$\mathbf{U} \mathbf{\Sigma} \mathbf{V}^T \mathbf{y} = \mathbf{b} \quad (3.5.3)$$

$$\implies \mathbf{y} = \mathbf{V} \mathbf{\Sigma}^{-1} \mathbf{U}^T \mathbf{b} \quad (3.5.4)$$

where $\mathbf{\Sigma}^{-1}$ is obtained by inverting only the non-zero elements of $\mathbf{\Sigma}$.

4 VECTOR CALCULUS

4.1. *Definition:* Let $\mathbf{x} \in \mathbb{R}^2$, $f(\mathbf{x}) \in \mathbb{R}$. Then,

$$\frac{df(\mathbf{x})}{d\mathbf{x}} = \begin{pmatrix} \frac{\partial f(\mathbf{x})}{\partial x_1} \\ \frac{\partial f(\mathbf{x})}{\partial x_2} \end{pmatrix} \quad (4.1.1)$$

4.2. Show that

$$\begin{aligned} \frac{d(\mathbf{a}^T \mathbf{y})}{d\mathbf{y}} &= \mathbf{a} \\ \frac{d(\mathbf{y}^T \mathbf{A} \mathbf{y})}{d\mathbf{x}} &= 2\mathbf{A} \mathbf{y} \end{aligned} \quad (4.2.1)$$

4.3. Show that

$$f(\mathbf{y}) = \|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{b}\|^2 + \|\mathbf{M} \mathbf{y}\|^2 \quad (4.3.1)$$

$$- 2\mathbf{b}^T \mathbf{M} \mathbf{y} \quad (4.3.2)$$

4.4. Show that

$$\frac{d(f(\mathbf{y}))}{d\mathbf{y}} = \mathbf{0} \quad (4.4.1)$$

yields (2.4.1).

4.5. Verify if $\mathbf{M}^T \mathbf{M} \geq 0$.