# EE5603:Conceention Inequalities

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## 1 Convergence

### 1.1 Definitions

show that

## 2 Review

$$u_i - L_i \leqslant c_i, u_i = sup_{\lambda' \epsilon x} E[g(x^n)|x^{1=1}, \lambda'] - E[g(x^{1=1}] \tag{2.1}$$

$$L_i = inf_{\lambda \epsilon x} E[g(x^n)|x^{1=1}, x] - E[g(x^n)|x^{1=1}] \tag{2.2}$$

$$u_i - L_i = sup_{\lambda \epsilon x} sup_{\lambda \epsilon x} E[g(x^n)|x^{1=1} - E[g(x^n)|x^{1=1}, x' \tag{2.3}$$

## 3 Efrom-stein inquality

let

$$X_1, \dots, X_n \tag{3.1}$$

be independent RVs,let $f : x^n \to |R$ be a square integrable, $z = f(x_1....x_n)$.

$$Var(z) \leqslant \Sigma_{1=1}^n E[(z - E^i(z))^2] dif\vartheta \tag{3.2}$$

$$\bullet E_i(z) = E[f(x_i...x_n)|x^i]; E_\circ = E \tag{3.3}$$

$$\bullet E^i(z) = \int_{\lambda_i \epsilon x} f(x_1...x_i, ...x_n) dp(xi) \tag{3.4}$$

$$\bullet if \Delta_i = E_i(z) - E_{i-1}(z), \Sigma_{1=1}^n \Delta_i = z - E(z). \to 1 \tag{3.5}$$

$$\bullet Var(z) = E[(z - E(z))^2](fromdefin) \tag{3.6}$$

$$= E[(\Sigma_{1=1}^n \Delta_i)^2](from1) \tag{3.7}$$

$$= E[\Sigma_{1=1}^n \Delta_i^2 + 2\Sigma_{j>i} \Delta_i \Delta_j] \tag{3.8}$$

$$E[\Sigma_{1=1}^n \Delta_i^2] + 2.\Sigma_{j>i} E[\Delta_j \Delta_i] \tag{3.9}$$

**Claim:**if $j > i, E_i \Delta_i = 0$

$$E_i \Delta_j = E_i[E_j(z) - E_{j-1}(z)] \tag{3.10}$$

$$= E[E_j(z) - E_{j-1}(z)|x^i] \tag{3.11}$$

$$= E_i(z) - E_i(z) \leftarrow (since j > i) \tag{3.12}$$

$$= 0 \tag{3.13}$$

if$E_i \Delta_j = 0 j > i$,then $2\Sigma_{j>i} E[\Delta_i \Delta_j] =? = 0$

$$\Rightarrow Var(z) = E(\Sigma_{1=1}^n \Delta_i^2) \to 2 \tag{3.14}$$

• Recall. $E_i[E^i(z)] = E_{i-1}(z)$

$$E^i(z) = \int_{\lambda_i \epsilon x} f(x_1...x_{i-1}, xi, x_{i-1}...x_n) dp(xi) \tag{3.15}$$

$$E^i(z) = \int_{\lambda_{i+1} \epsilon x^{n-1}} f(x_1...x_{i-1}, x_{i+1}...x_n) d.p(x_{i+1}^n) \tag{3.16}$$

$$\Rightarrow E_i[E^i(z)] = \int_{x_{i+1} \epsilon x^{n-i}} \int_{x_i \epsilon \chi} f(x_1...x_i, ...x_{i-1}...x_n).dp(x_i) \tag{3.17}$$

$$= \int_{x_i \epsilon x^{n-(i-1)}} f(x_1...x_{i-1}, x_i, ...x_n).dp(x_i^n) \tag{3.18}$$

$$= E_{i-1}(z) \to 3 \tag{3.19}$$

$$\Delta_i = E_i(z) - E_{i-1}(z) \tag{3.20}$$

$$\Delta_i = E_i[z - E^i(z)]. \tag{3.21}$$

$$\Delta_i^2 = [Ei[z - E^i(z)]]^2. \tag{3.22}$$

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**jensun's inequality** if g is aconvex function and x is a random variable then

$$E[f(x)] \geqslant f(Ex) \tag{3.23}$$

Apply this result to $\Delta_i$

$$E[\Delta_i^2] \geqslant (E[\Delta_i])^2 or \tag{3.24}$$

$$(E[\Delta_i])^2 \leqslant E[\Delta_i^2] :\to 4 \tag{3.25}$$

Recall Var $= E[\Sigma_{1=1}^{n} \Delta_i^2]$