# EE5603:Conceention Inequalities

J. Balasubramaniam† and G V V Sharma*

## 1 Convergence

### 1.1 Definitions

## 2 benntl's inequality

Let

$$X_1, \ldots, X_n \tag{2.1}$$

be independent RVS with finite vaniance and $xi \leqslant bfarb > 0$ almost surely for $i \leqslant n$ . let $v = \Sigma_{1=1}^{n} Exi^2$. if,

S= $\Sigma_{1=1}^{N} xi - Exi$,then for any $\lambda > 0$

$$logE[e^{\lambda s} \leqslant n.log(1 + \frac{v}{nb^2}\phi(\lambda b)) \leqslant \frac{v}{b^2}\phi(\lambda b) \tag{2.2}$$

where

$$\phi(u) = e^u - u - 1., u\varepsilon R \tag{2.3}$$

**proof:** • let no assume b=1 →
1

• Observe that $u^2$ $.\phi(u)$ is a non decreacing function → 2

from $1or2$ we can write

$$(\lambda xi)^2.\phi(\lambda xi) \leqslant \lambda^2.\phi(\lambda) \tag{2.4}$$

$$\Rightarrow \phi(\lambda xi) \leqslant xi^2\phi(\lambda) \tag{2.5}$$

$$e^{\lambda xi} - \lambda xi - 1 \leqslant xi^2\phi(\lambda) \tag{2.6}$$

$$E[e^{\lambda xi} - \lambda xi - 1] \leqslant E[xi^2].\phi(\lambda). \tag{2.7}$$

$$E[e^{\lambda xi}] \leqslant E[\lambda xi] + 1 + E[xi^2].\phi(\lambda) \tag{2.8}$$

$$logE[e^{\lambda xi}] \leqslant log[E[\lambda xi] + 1 + E[xi^2]\phi(\lambda)] \tag{2.9}$$

$$logE[e^{\lambda xi} - \lambda E[xi] \leqslant log(1 + E[\lambda xi] + E[xi^2].\phi(\lambda) \tag{2.10}$$

$$\Sigma_{1=1}^{n} log[\frac{E[e^{\lambda xi}}{e^E \lambda Ei] \leqslant}]log(1 + E[\lambda xi] + E[xi^2]\phi(\lambda)\lambda E[xi]] \tag{2.11}$$

$$\psi_s(\lambda) \leqslant \Sigma_{1=1}^{n} \tag{2.12}$$

observe that log() is a concare function i.e.

$$f(\lambda x + (1 - \lambda).y) \geqslant \lambda.f(x) + (1 - \lambda).f(y) - 3 \tag{2.13}$$

$$\Sigma_{1=1}^{n}[1 + \lambda.E[xi] + E[xi^2].\phi(\lambda)] = \tag{2.14}$$

$$\eta.\Sigma_{1=1}^{n}\frac{1}{n}.log[1 + \lambda E[xi] + E[xi^2].\phi(\lambda)] \tag{2.15}$$

from-3

$$\eta.\Sigma_{1=1}^{n} log[1 + \lambda.E[xi] + E[xi^2]\phi(\lambda)] \leqslant \tag{2.16}$$

$$\eta.log(\frac{1}{n}.\Sigma_{1=1}^{n}(1 + \lambda E[xi] + E[xi] + E[xi^2].\phi(\lambda))) \tag{2.17}$$

$$= \eta.log(1 + \Sigma_{1=1}^{n}\lambda\frac{E[xi]}{n} + \frac{\vartheta}{n}\phi(\lambda) \tag{2.18}$$

Appling 4 and 2b give us

$$\psi_s(\lambda) \leqslant \eta[log(1 + [\Sigma_{1=1}^{n}\frac{\lambda.E[xi]}{n} + \frac{\vartheta}{n}.\phi(\lambda))\frac{1}{n}\Sigma_{1=1}^{n}\lambda E[xi]] \tag{2.19}$$

observe log (1+x)⩽x $x \geqslant 0$.this helps us show

$$\eta.[log(1 + \Sigma_{1=1}^{n}\lambda\frac{E[xi]}{n} + \frac{\vartheta}{n}\phi(\lambda)) - \frac{1}{n}\Sigma_{1=1}^{n}\lambda E[x] \tag{2.20}$$

† The author is with the Department of Mathematics, IIT Hyderabad. *The author is with the Department of Electrical Engineering, IIT, Hyderabad 502285 India e-mail: {jbala,gadepall}@iith.ac.in. 

$$\leqslant n[\Sigma_{1=1}^{n}\frac{\lambda.E[xi}{n} + \frac{\vartheta}{n}.\phi(\lambda) - \frac{1}{n}\Sigma_{1=1}\lambda E[xi]] \quad (2.21)$$

$$= \vartheta.\phi(\lambda) \quad (2.22)$$