EE5603:Conceention Inequalities

J. Balasubramaniam† and G V V Sharma*

1 Convergence

1.1 Definitions

• **Review** • the mc diamid's inequality from alot like the hoeffding's inequality

• can we express the difference $V = g(X^n) - E[g(X^n)]$as a sum of V_i where V_i are such that:

(i)$V = \Sigma_{1=1}^n V_i$

(ii)V_i depend only on X^i

(iii)given X^{i-1},there exist function u, L_isuch that

$$[u_i - L_i \leqslant c_i]L_i \leqslant V_i u_i \tag{1.1}$$

Recall $u_i = sup_{\lambda' \epsilon x} E[g(x^n)|x^{1-1}, \lambda^1] - E[g(x^n)|x^{1-1}]$

$L_i = inf_{\lambda \epsilon x} E[g(x^n)|x^{1-1}, \lambda] - E[g(x^n)|x^{1-1}]$

$u_i - L_i = sup_{\lambda' \epsilon x} E[g(x^n)|x^{1-1}, x'] - E[g(x^n)|x^{1-1}]$

$inf_{\lambda \epsilon x} E[g(x^n)|x^{1-1}, x] - E[g(x^n)|x^{1-1}]$

$= sub_{\lambda \epsilon x} sup_{\lambda \epsilon x} E[g(x^n)|x^{1-1} x] - E[g(x^n)|x^{1-1}, x]$

$= sub_{\lambda \epsilon x} sup_{\lambda \epsilon x} \int [g(x^n|x^{1-1}, x) - g(x^n|x^{1-1}, x^1)]$

$$dpx_{i+1}^n \tag{1.2}$$

$\leqslant sup_{\lambda \epsilon x} sup_{\lambda \epsilon x} \int |g(x^n|x^{1-1}, x) - g(x^n|x^{1-1}, x^1)| dpX_{1-1}^n (since \int f - g \leqslant \int |f - g|)$

$\leqslant c_i$ (from bounded differences property

$\therefore u_i - L_i \leqslant c_i$ or $L_i \leqslant u_i \leqslant L_i + c_i$

† The author is with the Department of Mathematics, IIT Hyderabad. *The author is with the Department of Electrical Engineering, IIT, Hyderabad 502285 India e-mail: {jbala,gadepall}@iith.ac.in. All material in the manuscript is released under GNU GPL. Free to use for all.

observation: mc diarmid's inequality is a poerful result since we only require,

(i)independence of RVs...$x_1 ... x_u$,
(ii)$g(x^n)$ to satisty bounded difference property

note that we did not impore any restrions on the distributions of x_i

• **Eform-stein inequality:**let

$$X_1, \dots, X_n \tag{1.3}$$

be indepentent RVs. let $\delta : x^n \to |R$ be a square integrable function. let $z = f(x_1 ... x_n,)$

$$Var(z) \leqslant \Sigma_{1=1}^n E(z - E^i(z))^2 \tag{1.4}$$

$$E_i(z) = E(f(x^n)|x^1]. \tag{1.5}$$

To dos
• $\triangle_i = E_i(z) - E_{1=1}(z)$

•$Z - E[z] = \Sigma_{1=1}^n \; \triangle_i$

•if$E^i(z) = \int f(x_1 ... x_i - 1, x_{i+1} ... x_n)$

$dp(x_i) E_i[E^i(z)] = E_{i-1(z)}$