EE5603:Conceention Inequalities

J. Balasubramaniam† and G V V Sharma*

1 Convergence

1.1 Definitions

Reviw: Bounded differences property: if $f : X^n \rightarrow |R$,there exist a non negative ci for all i between —and n such that,

$$sup|f(x_1...x_i....x_n) - f(x_1...x_i', ...x_n)| \tag{1.1}$$

$$x_1...x_n, x_i' \epsilon x \tag{1.2}$$

mc diarmid's inequality:if

$$x^n(x_1....x_n) \tag{1.3}$$

is a set of in indep. RVs, bounded differences property, then for any $t > 0$.

$$p(g(x^n) - E[g(x^n)] \geqslant t) \leqslant exp(\frac{-2t^2}{\Sigma_{1=1}^n ci^2}] \tag{1.4}$$

$$P[g(x^n) - E[g(x^n)] \leqslant -t] \leqslant exp[\frac{-2t^2}{\Sigma_{1=1}^n (i^2)} \tag{1.5}$$

Proof onthine :find Vi such that

(i) V=$g(x^n) - E(g(x^n)] = \Sigma_{1=1} Vi$

(ii) Vi dependo only on Xi

(iii) given X^{1-1}, there exist u_i, L_i such that $L_i \leqslant V_i \leqslant U_i and u_i - L_i \leqslant ci$

if

$$V_i = E[g(x^n)|X^i] - E[g(x^n)|X^{1-1})] \tag{1.6}$$

$$\Sigma_{1=1}^n Vi = E[g(X^n)|X^n] - E[g(X^n)|X^{n-1} + \tag{1.7}$$

$$E[g(X^n)|X^{n-1} - E[g(X^n)|X^{n-1}] + ... \tag{1.8}$$

$$E[g(X^n)|X^1] - E[g(X^n)] \tag{1.9}$$

$$= E[g(X^n)|X^n] - E[g(X^n)] \tag{1.10}$$

$$= g(X^n) - E[g(X^n)] \tag{1.11}$$

$$= V \tag{1.12}$$

- $E[g(X^n)|X^1]$

$$\int g(X^n).f_{x1+1}(Xi + 1)....f_{xn}(X_n). \tag{1.13}$$

$$dx_{i+1}...dX_n \tag{1.14}$$

It follow that Vi satisties property(ii)

- $Li = int_{x\epsilon x}[E[g(x^n)|X^{1-1}, X] - E[g(X^n)|X^{1-1}]$

$ui = sup_{x\epsilon x}[E(g(x^n)|X^{1-1}, X^1] - [g(X^n)|X^{1-1}]$

from the defin of int, sup (or) Vi ,

$L_i \leqslant V_i \leqslant\leqslant u_i$ | recall: $V_i = E[g(X^n)|X^i] - E[g(X^n)|X^{1-1}]$

- show that $u_i - L_i \leqslant c_i$

† The author is with the Department of Mathematics, IIT Hyderabad. *The author is with the Department of Electrical Engineering, IIT, Hyderabad 502285 India e-mail: {jbala,gadepall}@iith.ac.in.