EE5603:Conceention Inequalities

J. Balasubramaniam[†] and G V V Sharma[*]

1 Convergence

1.1 Definitions

2 Basice into to measure theroetic probability:

•$(\Omega, \int, \rho)$ the probility triplet

Ω : set of proible ontcones w.

$\int$: σ-algebra defined on Ω that satsti the following axiome

3 Axionss

A.1. $\Omega \epsilon$

A.2.if A $\epsilon \int, A^c \epsilon$

A.3.if A $\epsilon, B\epsilon, then A U B \epsilon$

From A.2,(OR) A.3 we can show that if A ε $\int$,B ϵ $\int$

then,A$\bigcap$B ϵ $\int$

proof : $A^c \epsilon \int, B^c \epsilon \int$ (from A.2)

$\Rightarrow A^c \cup B^c \varepsilon \int$(from A.3)

$\Rightarrow (A^c \cup B^c)^c \epsilon \int$ (from A.2)

we know $A \cup B = (A^c \cup B^c)^c$

$\therefore A \cap B \epsilon \int$.

P:A probalitity measure defined on that satises the following axiome

† The author is with the Department of Mathematics, IIT Hyderabad. *The author is with the Department of Electrical Engineering, IIT, Hyderabad 502285 India e-mail: {jbala,gadepall}@iith.ac.in. All material in the manuscript is released under GNU GPL. Free to use for all.

P.1:$\Pr(A) \geqslant 0$ for all Aε

P.2:$\Pr(A_1 \cup A_2) = \Pr(A_1) + Pr(A_2)$ for distint sets (A_1, A_2)

P.3:$\Pr(\Omega) = 1$

A random variable x map Ω to and is $\int$ -measurable.

for any ε, w:X(W)$\leq$ ε ε ϵ

4 Recall defin of a.s. conesgence:

$$\Pr(line|x_n = x) = 1 \tag{4.1}$$

can be interpreted as

$$\Pr(w : x_n(w) = x(w)) = 1 \tag{4.2}$$

ex: Ω=a,b,c,d
σ-algebra =Ω, $\emptyset$,a,b,c,d

$$\bullet F_x(x) = \Pr w : x(w) \leqslant x \tag{4.3}$$

$$= \Pr(X \leq \lambda) \tag{4.4}$$

$$\bullet F_x(\lambda) = \int_{-\propto}^{x} f_x(t)dt \tag{4.5}$$

• Review /prowe basic ineqlities:
• markor inequality: For a non- negative Rv x,and for any $\varepsilon > 0$

$$\Pr(x \geqslant \varepsilon) \leqslant \frac{EX}{\varepsilon} \tag{4.6}$$

$$Ex = \int_0^{\infty} \Pr(x \geqslant t)dt \tag{4.7}$$

$$= \int_0^\infty \int_t^\infty f_x(x) d_x dt \tag{4.8}$$

$$= \int_0^\infty t.f_x(t) dt \tag{4.9}$$

5 DISCRETC CASE:

$x = nn\epsilon IN, \Pr(n)$

$$E[x] = \Sigma_{n=1}^{\infty} n.\Pr(n) = 1.\Pr(1) + 2p(2).... \tag{5.1}$$

n=1 $\Updownarrow \rightarrow p(x \geqslant 1)$

$$= 1.\Pr(1) + 1.\Pr(2).... \tag{5.2}$$

$$\tag{5.3}$$

$$= \Sigma_{m=0}^{\infty} \Pr(x \geqslant m) \tag{5.4}$$

$$\tag{5.5}$$

E[x]=$\int_{-\infty}^{\infty} x f_x(\lambda) dx$=$\int_0^\infty x.f_x(\lambda) dx (\therefore x \geqslant 0)$

$$= \int_0^\varepsilon x.f_x(\lambda).dx. + \int_\varepsilon^\infty x.f_x(\lambda) dx \tag{5.6}$$

$$\geqslant \int_\varepsilon^\infty x.f_x(\lambda) dx \tag{5.7}$$

$$\geqslant \int_\varepsilon^\infty \varepsilon.f_x(\lambda) dx \tag{5.8}$$

(since x is non negetive)

$$= \varepsilon \int_\varepsilon^\infty f_x(\lambda) dx \tag{5.9}$$

$$= \varepsilon.\Pr(x \geqslant \varepsilon) \tag{5.10}$$

$$i.e E[X] \geqslant \varepsilon.\Pr(x \geqslant \varepsilon) \tag{5.11}$$

$$Or \Pr(x \geqslant \varepsilon) \leqslant \frac{EX}{\varepsilon}; \varepsilon > 0 \tag{5.12}$$

6 CHEBYSHER INEQNALITY:

For any random variable x,and for any $\varepsilon > 0$,

$$\Pr[1x - E[x]| \geqslant \varepsilon] \leqslant \frac{Var(x)}{\varepsilon^2} \tag{6.1}$$

if $\phi(\lambda)$ is stricthy monitrmic and incaring,

$$\Pr(x \geqslant \varepsilon) = \Pr(\phi(x) \geqslant \phi(\varepsilon)). \tag{6.2}$$

using this result and lelting

$$Y = |x - E[x]| \tag{6.3}$$

$\Pr(y \geqslant \varepsilon) = \Pr(\phi(Y) \geqslant \phi(c))$ Where $\phi(\lambda) = x^2$,

$$= \Pr[(x - E[x])^2 \geqslant \varepsilon^2] \tag{6.4}$$

Cheorrly $\phi(y)$ is non-negative and the μI is Valid.

$$\Rightarrow \Pr(y \geqslant \varepsilon) \leqslant \frac{E(x - Ex)}{\varepsilon^2} \tag{6.5}$$