EE5603:Conceention Inequalities

J. Balasubramaniam[†] and G V V Sharma[*]

1 Convergence

1.1 Definitions

• **Moteration:** Tocome up with bound on function of independent RVs.

• First analyze function that satisty the [bounded difference property].

Review:if AC is a sat of real numbers , then $M\epsilon|R$ is called as upper bound of A if $x \leqslant m$ for every $x\epsilon A$.

• The deast upper bound of A is called the supermum of A.

• if AC|R is a set of realnumbers, then $m\epsilon|R$ is calleda lower bound of A if $x \geqslant m$ for evens $x\epsilon A$.

• The greatest lower bound of A is called infimum of A

• if sub $A\epsilon A$, then maximum of A= subA.

• if int $A\epsilon A$,then minimum of A= int A

.

Allernatirty , M=sub A if and only if

•M is aupper bound of A.

• For every $\mu < m$, then exists an $x\epsilon A$such that $x > \mu$.

•$A = \frac{1}{n} : n\epsilon N$

int A=0, sup A=1=max.A

minimum does not exist.

† The author is with the Department of Mathematics, IIT Hyderabad. *The author is with the Department of Electrical Engineering, IIT, Hyderabad 502285 India e-mail: {jbala,gadepall}@iith.ac.in.

• These notons carry over to functions when desind on therang of valus taken by the function

if $f : A \rightarrow |R$, int f=int f(x):$x\epsilon A$

$$supf = supf(x) : x\epsilon A \tag{1.1}$$

• Ex: f(x)= $x[0 \leqslant]\lambda 1[0x = 1]$

$int_[0, 1]f = min_[0, 1]$ f=0

sup f=1

2 Bounded differences property

let x be a set and f:$x^n \rightarrow |R$. if there exist non negetive for all i($1 \leqslant i \leqslant n$), such that

$$sup|f(x_1...x_i...x_n) - f(x_1...x_i^1...x_n)| \tag{2.1}$$

$$x_1...x_n...'\epsilon x \tag{2.2}$$

3 MC Diarmid's inequality

let

$$x^n = (x_1...x_n) \tag{3.1}$$

be in collection of ni independent RVs and $g : X^n \rightarrow |R$ that has the bounded differences property, then for any $t > 0$

$$P[g(X^n) - E[g(x^n)] \geqslant t] \leqslant exp[\frac{-2t^2}{\sum_{1=1}^{n}(i^2)}] \tag{3.2}$$

$$P[g(x^n) - E[g(x^n)] \leqslant -t] \leqslant exp[\frac{-2t^2}{\sum_{1=1}^{n}(i^2)} \tag{3.3}$$

Proof onthive:• note that this bound looks a lot like the hoeffding's inequality.

• let V=$g(x^n) - E[g(x^n)]$

- if V=$\Sigma_{1=1}^{n} V_i$ such that each element of the sum V_i is bounded and the length of the bounded interval is ci,

we can resat to our previous approach to prove the hoeffding's inequality.

$$P(g(x^n) - E[g(x^n)] \geqslant t) = p(v \geqslant t) \quad (3.4)$$

$$= p(e^{sv} \geqslant e^{st}) \quad (3.5)$$

$$\leqslant e^{-st\cdot} E[e^{sv}] \quad (3.6)$$

$$= e^{st} E[e^{s} \Sigma_{1=1}^{n} Vi] \quad (3.7)$$

- Vi should also have the property that it depends only on X^i.

$$= e^{st} E[E[e^{s} \Sigma_{1=1}^{n} Vi | X^{n-1}]] \quad (3.8)$$

$$= e^{-st} E[e^{s} \Sigma_{1=1}^{n-1} Xi.E[e^{sv_n} | X^{n-1}]] \quad (3.9)$$

•Vi should be such that given X^{1-1},there exist Liand ui such that $Li \leqslant Vi \leqslant ui$ and $ui - Li \leqslant ci$

$$\leqslant e^{st} e \frac{s^2.cn^2}{8} E[e^{s} \Sigma_{1=1}^{n-1} Vi] \quad (3.10)$$

$$= e^{-s2}.e \frac{s^2 \Sigma_{1=1}^{n} ci^2}{8} \quad (3.11)$$

(due to hoeffding's lemma).