* Convex Optimization (25/1/19) :-

- A function f is convex if :
  i) dom. of f is convex, and
  ii) $f(\theta x_1 + (1-\theta)x_2) \le \theta f(x_1) + (1-\theta) f(x_2)$
  for any $0 \le \theta \le 1$.

- $f(x) = ax + b \Rightarrow$ convex $f^n$

$f(x) = a^T x + b$

$f(x) = ax^2 + bx + c$ if $a \ge 0 \Rightarrow$ convex $f^n$.

$f(x) = ||x||_p = \sqrt[p]{\sum_{i=1}^{n} |x_i|^p}$

$||A||_F = \sqrt{\sum_{i,j} |a_{ij}|^2}$ ; $||A|| = \sigma_{max}(A)$ } convex

$||A||_* = \sum_{i=1}^{n} \sigma_i(A)$

* Checking for convexity :-

- Extended value expression.

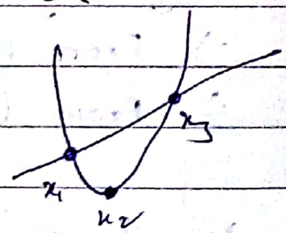
$$\tilde{f}(x) = \begin{cases} & \text{if } x \in \text{dom } f \\ \infty & \text{if } x \notin \text{dom } f \end{cases}$$

Extension preserves convexity

$f(\theta x_1 + (1-\theta)x_2) \le \theta f(x_1) + (1-\theta) f(x_2)$

1) Definition : $f(\theta x_1 + (1-\theta)x_2) \le \theta f(x_1) + (1-\theta) f(x_2)$ $\quad 0 \le \theta \le 1$

$$\frac{f(x_3) - f(x_1)}{x_3 - x_1} \ge \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$




(or) $\frac{f(z) - f(x)}{z - x}$ is monotonically non-decreasing as a $f^n$ of y.

$x_1 \le x_2 \le x_3$.

- A single variable convex $f^n$ f is always continous in the interior of the domain.

Q.) Exercise : Use monotonicity of slope.

- $f: R^n \to R$

A function f is convex if & only if the restriction of f to any line is convex.

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- Fix $x$ & $y$, consider $f(x+ty)$, $f(tx+(1-t)y)$
- $f(x) = x^T Q x + b^T x + c$
- $f(x+ty) = t(b^T y + Q x^T Q y) + x^T Q x + b^T x + c$

$y^T Q y \geq 0$ for every $y \Rightarrow P \geq 0$

2) First order Conditions :-

- If $f$ is convex differentiable.

$$\lim_{y \to x} \frac{f(y) - f(x)}{y - x} \; ; \; \text{Gradient} = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \partial x_n \end{bmatrix}_{n \times 1}$$

$f(x_1, x_2, \ldots x_n)$

Along the direction $u$, the rate of change is $\nabla f^T u$

If $u$ is along $\nabla f$, then the rate of increase is the highest. If $u$ is opposite of $\nabla f$ then the rate of increase is lowest.

$$\nabla f = \begin{bmatrix} 2x_1 / x_2 \\ -x_1^2 / x_2^2 \end{bmatrix} \quad \text{for } f(x_1, x_2) = \frac{x_1^2}{x_2}$$

- $f$ is convex if & only if $f(x) \geq \underbrace{f(x) + \nabla f^T(x)(y-x}_{\text{tangent to the surface at } x}$

$f$ is single variable :

$$f(y) \geq f(x) + f'(x)(y-x).$$

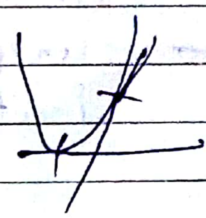
$$\frac{f(z) - f(x)}{z - x} \geq \frac{f(y) - f(x)}{y - x}$$

$$= f'(x)$$

- $f(y) \geq f(x) + f'(x)(y-x)$

$f(x) \geq f(y) + f'(y)(x-y)$

$0 \geq -f'(x) + f'(y)(x-y)$



* Construct : $f(tx + (1-t)y) \geq f(y) +$ $(tx$

$\tilde{f}(t) = f(tx + (1-t)y)$.

$\tilde{f}(t) \geq \tilde{f}(0) + f'(0)(t-0)$ for every $t$

$\tilde{f}(1) \geq \tilde{f}(0) + f'(0)(1)$; $f(x) \geq f(y) +$

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