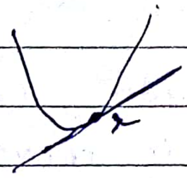
* Convex Optimisation (28/1/19) :-

- ~~$f: R^n \to R$~~ First order conditions :-

Fix x $\quad f(x) \geq f(x) + \nabla f^T(x)(y-x)$

- Taylor series:

$f(y) \approx f(x) + f'(x)(y-x)$



$$f(y) = f(x) + f'(x)(y-x) + f''(x)\frac{(y-x)^2}{2!} + \dots f^{(n)}(x)\frac{(y-x)^n}{n!}$$

- f is convex if & only if dom f is convex then

$f(x) \geq f(x) + \nabla f^T(x)(y-x)$ for all x.

Proof :-

$f(y) \geq f(x) + f'(x)(y-x)$.

$f(x) \geq f(z) + f'(y)(x-y)$

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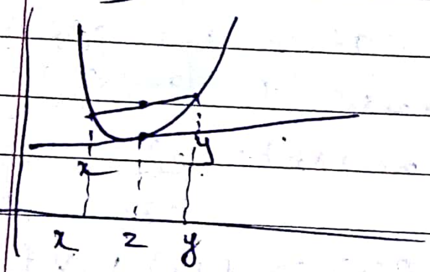

- Second order Condition :- (single variable fs)

- A twice differentiable f is convex if & only if dom f is convex, and $f''(x) \geq 0$ for all x.

ex: $f(x) = 1/x^2$ dom $f = R \setminus \{0\}$

$f''(x) = 0$ for all $x \in$ dom f ; but f^n is not convex.

- Convex $\Rightarrow$ Second order condition :-

Proof :-

$f(y) \geq f(x) + f'(x)(y-x)$

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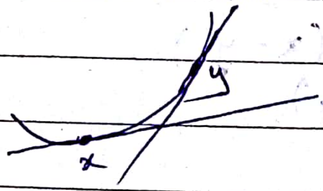

$(f'(x) - f'(y))(x-y) \geq 0$

$$\frac{f'(x) - f'(y)}{x-y} \geq 0.$$

Let $x \to y$; $\Rightarrow$ $f''(y) \geq 0$.

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* Second order condition $\Rightarrow$ convex.

- Taylor's series :-

$$f(y) = f(x) + f'(x)(y-x) + \frac{f''(z)\cdot(y-x)^2}{2!}$$

for some z,

$\Rightarrow f(y) \geq f(x) + f'(x)(y-x)$

$\Rightarrow$ convex.

$f(x_1, x_2, x_3, \ldots x_n)$ $\quad \nabla f = \begin{bmatrix} \frac{\partial f}{x_1} \\ \frac{\partial f}{x_2} \\ \vdots \\ \frac{\partial f}{x_n} \end{bmatrix}$

$$\nabla^2 f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} & \cdots & , \\ \frac{\partial^2 f}{\partial x_1 x_2} & \frac{\partial f}{\partial x_2^2} & \cdots & & , \\ & & & & , \\ \cdots & & & & \vdots \end{bmatrix}$$

Hession of f

- f is convex if & only if dom f is convex & $\nabla^2 f \geq 0$

$\tilde{f}'(t) = \nabla f(x+ty)y$ $\quad \tilde{f}'(t) = y\frac{\partial f}{\partial x_1}\Delta t + y_2 \frac{\partial f}{\partial x_2}\Delta t + \ldots + y_2 \frac{\partial f}{\partial x_n}\Delta t$.

$\tilde{f}''(t)$

$$= \sum_{i=1}^{n} y_i \frac{\partial f(x+ty)}{\partial x_i}$$

$$\tilde{f}''(t) = \underset{1\times n}{y^T} \underbrace{\nabla^2 f(x+ty)}_{n\times n} \underset{n\times 1}{y}$$

$\nabla^2 f$ is positive semi definite everywhere.

Q.) 1) $f(x) = ax^2 + bx + c$ $\quad$ 2) $f(x) = x^T Q x + b^T x + c$

3) $f(x) = x^\alpha$ $\quad$ 4) $f(x) = |x|^\alpha$

5) $f(x) = \log x$ $\quad$ 6) $f(x,y) = x^2/y$

} Check whether the f^{ns} are convex by taking 2^{nd} derative