* Convex Optimisation :- (18/1/19)

- Convex Sets :-

$A \in S^n$ is positive semi-definite if $x^T A x \geq 0$ for every $x \in R^n$.

Set of all positive definite A is denoted S^n_+.

$A_1, A_2, \ldots A_K \in S^n_+$

$$x^T \left(\sum_{i=1}^{k} \theta_i A_i \right) x = \sum_{i=1}^{k} \theta_i x^T A_i x.$$

* S^2_+ $\begin{bmatrix} x & y \\ z & z \end{bmatrix}$ is in S^n_+ iff $\begin{cases} x \geq 0, y \geq 0 \\ xy - z^2 \geq 0 \end{cases}$

Consider the following subset of R^3

$S = \{(x, y, z) : x \geq 0, y \geq 0, xy \geq z^2\}$

S is convex.

$f : R^3 \to S^2$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \to \begin{bmatrix} x & y \\ z & z \end{bmatrix}$$

* **Matrix Inequalities** :-

A	B
Ψ	Ψ
S^n	S^n

We say that $A \leq B$ if $B - A$ is positive semi-definite

* **Linear matrix Inequalities** :-

- $A_1, A_2, \ldots, A_K \in S^n_+$

$\{(x_1, x_2, \ldots) : x_1 A_1 + x_2 A_2 + \ldots + x_k A_k \geq 0\} = S$

S is convex. | S is the inverse image of S^n_+

$$\begin{bmatrix} x_1 \\ \vdots \\ x_k \end{bmatrix} \to x_1 A_1 + x_2 A_2 \ldots + x_k A_k$$

$$R^k \to S^{k}$$

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* **Ellipsoids** :-

$$\{x : x^T P^{-1} x \leq 1\} \quad \text{where } P \in S^n_{++}$$

$$\{x : \|x\|_2 \leq 1\} \quad \{x : x^T x \leq 1\}.$$

$$P^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow x_1^2 + 2x_2^2 \leq 1$$

A is positive definite if $x^T A x > 0$ whenever $x \neq 0$.

set of all positive definite matrices S^n_{++}

$$S = \{(x_1, x_2, x_3) : x_1^2 + x_2^2 \leq x_3^2\}, x_3 \geq 0\}$$

$$S = \{(x, z) : x^T x \leq z^2, \quad z \geq 0)$$

$$S = \{x : x^T x \leq (c^T x)^2, \quad c^T x \geq 0\}$$

$$\begin{pmatrix} x \\ z \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ c^T x \end{pmatrix}$$

$$S = \{1, 2, 3, \ldots\ldots n\}$$

$P = \{p \in R^n$ that are valid probability vectors$\}$

$= \{p \in R^n : p \geq 0, \ 1^T p = 1\} \Rightarrow$ Polyhedron.

$$1^T(\theta \cdot \bar{P_1} + (1-\theta)\bar{P_2}) = \theta 1^T P_1 + (1-\theta) 1^T P_2$$

$$= 1.$$

* **Polyhedron** :-

$$\{x : Ax \leq b\}$$

$$\{x : Ax \leq b, \ Cx = d\} = \{x : Ax \leq b, \ Cx \leq d, \ -Cx \leq -d\}$$

$$= \left\{x : \begin{bmatrix} A \\ C \\ -C \end{bmatrix} \leq \begin{bmatrix} b \\ d \\ -d \end{bmatrix}\right\}$$

P is called probability simplex.

* Given :-

$v_1, v_2, \ldots\ldots, v_k \in R^n$

$V = \text{Con}\{\theta_1, \theta_2, \ldots\ldots, \theta_k\}$ be the convex hull.

then V is polyhedron.

Q.) Prove this.