Convex-Optimization

1) There are three food items available, corn, milk, and bread. The table contains, the cost per serving, the amount of Vitamin A per serving, and the number of calories per serving for each food item. Also, there are restrictions on the total number of calories (between 2000 and 2250) and the total amount of Vitamin A (between 5000 and 50,000) intake in the diet.The goal of the diet problem is to select a set of food items that will satisfy a set of daily nutritional requirement at minimum cost. The maximum number of servings is 10 per food item. Table 1 shows all the content per serving.

Food	Cost (**c**)	Vitamin A (**v**)	calories (**u**)
Corn	0.18 USD	107	72
Milk	0.23 USD	500	121
Wheat Bread	0.05 USD	0	65

TABLE I

Solution:
Consider the vector **c** as cost vector, vector **v** representing the vitamin A and vector **u** representing calories.
Let **x** be the variable vector.

$$\mathbf{c} = \begin{pmatrix} 0.18 \\ 0.23 \\ 0.05 \end{pmatrix} \tag{1}$$

$$\mathbf{v} = \begin{pmatrix} 107 \\ 500 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{u} = \begin{pmatrix} 72 \\ 121 \\ 65 \end{pmatrix} \tag{3}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{4}$$

Objective function

$$z = \min_{\mathbf{x}} \mathbf{c}^T \mathbf{x} \tag{5}$$

Constraints

$$5000 \leq 107x + 500y + 0z \leq 50000 \tag{6}$$
$$2000 \leq 72x + 121y + 65z \leq 2250 \tag{7}$$
$$0 \leq x \leq 10 \tag{8}$$
$$0 \leq y \leq 10 \tag{9}$$
$$0 \leq z \leq 10 \tag{10}$$

Writing all the constraints in the matrix form

$$\mathbf{px} = \mathbf{q} \tag{11}$$

$$\begin{pmatrix} 107 & 500 & 0 \\ -107 & -500 & 0 \\ 72 & 121 & 65 \\ -72 & -121 & -65 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 50000 \\ -5000 \\ 2250 \\ -2000 \\ 10 \\ 10 \\ 10 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{12}$$

By providing the objective function and constraints to cvxpy, we get the optimal cost (z) and optimal no.of servings (**x**).
cvxpy code,

```
https://github.com/gadepall/EE5606-
    optimization/codes/opt_1.py
```

From cvxpy, we get

$$z = 3.16\text{USD} \tag{13}$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ 10 \\ 10 \end{pmatrix} \tag{14}$$

2) Funding an expense stream. Your task is to fund an expense stream over n time periods. We consider an expense stream $\mathbf{e} \in R^n$, so that e_t is our expenditure at time t. One possibility for funding the expense stream is through our bank account. At time period t, the account has balance b_t and we withdraw an amount w_t. The value of our bank account accumulates with an

interest rate ρ per time period, less withdrawals

$$b_{t+1} = (1+\rho)b_t - w_t$$

We assume the account value must be nonnegative, so that $b_t \geq 0$ for all t. We can also use other investments to fund our expense stream, which we purchase at the initial time period t = 1, and which pay out over the n time periods. The amount each investment type pays out over the n time periods is given by the payout matrix **P**, defined so that P_{tj} is the amount investment type j pays out at time period t per dollar invested. There are m investment types, and we purchase $x_j \geq 0$ dollars of investment type j. In time period t, the total payout of all investments purchased is therefore given by $(Px)_t$. In each time period, the sum of the withdrawals and the investment payouts must cover the expense stream, so that

$$w_t + (Px)_t \geq e_t$$

for all t = 1, . . . , n. The total amount we invest to fund the expense stream is the sum of the initial account balance, and the sum total of the investments purchased: $b_1 + 1^T\mathbf{x}$.
Using the data in **expense stream data.*****,**Show that the minimum initial investment that funds the expense stream can be found by solving a convex optimization problem.
Solution:
Given to minimize the inital investment.
Inputs to the problem, given in the data set **expense stream data.py** are $\mathbf{P}, \mathbf{e}, \rho$, Time period = n and No. of investments = m.
Objective Function,

$$Z = \min_{b1,\mathbf{x}} b_1 + 1^T\mathbf{x} \tag{15}$$

Optimization variables,

$$\mathbf{w}_{nx1}$$
$$\mathbf{b}_{nx1}$$
$$\mathbf{x}_{mx1}$$

Constraints,

$$\mathbf{b} \geq 0$$
$$\mathbf{x} \geq 0$$
$$w_t + (Px)_t \geq e_t$$
$$b_{t+1} = (1+\rho)b_t - w_t.$$

Solution of the above objective function with the constraints is obtained by CVXPY.
CVXPY code,

```
https://github.com/gadepall/EE5606−
    optimization/codes/opt_2.py
```

```
https://github.com/gadepall/EE5606−
    optimization/codes/expense_stream _data
    .py
```

The optimal initial investment Z = 177.51