Digital Synchronization Techniques for Reliable Communication

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***Abstract*—This manual provides a brief description about the design and implementation of digital synchronization techniques for reliable communication.**

1. Time Offset: Gardner TED

Let the mth sample in the rth received symbol time slot be

$$Y_k(m) = X_k + V_k(m), \quad k = 1, \dots, N, m = 1, \dots, M. \tag{1.1}$$

where X_k is the transmitted symbol in the kth time slot and $V_k(m) \sim \mathcal{N}\left(0, \sigma^2\right)$. The decision variable for the kth symbol is [1]

$$U_k = \frac{1}{N}\sum_{i=1}^{N} Y_{k-i}\left(\frac{M}{2}\right)\left[Y_{k-i+1}(M) - Y_{k-i}(M)\right] \tag{1.2}$$

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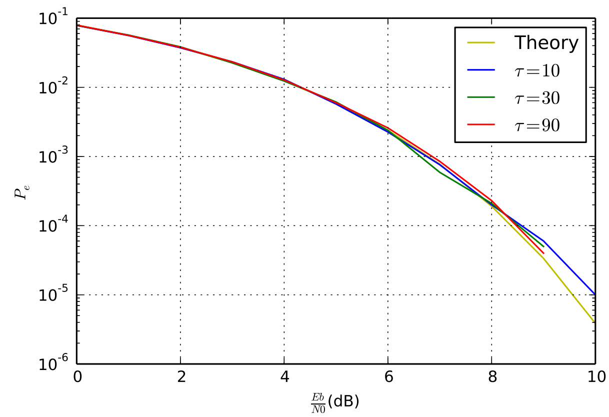

Fig. 1: SNR vs BER for varying τ.

A. Plots

Fig. 1 is generated by the following code

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https://github.com/gadepall/EE5837/raw/master/
    synctech/codes/time_sync_offsets.py
```

and shows the variation of the BER with respect to the SNR with different timing offsets τ for $N = 6$.

2. Frequency Offset: LR Technique

Let the frequency offset be Δf [2] . Then

$$Y_k = X_k e^{j2\pi\Delta f kM} + V_k, \quad k = 1, \dots, N \tag{2.1}$$

From (2.1),

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta f kM} + X_k^* V_k \tag{2.2}$$

$$\implies r_k = e^{j2\pi\Delta f kM} + \bar{V}_k \tag{2.3}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{2.4}$$

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k}\sum_{i=k+1}^{N} r_i r_{i-k}^*, 1 \le k \le N-1 \tag{2.5}$$

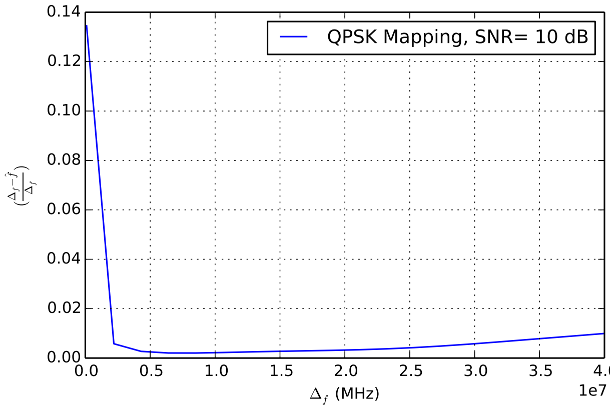

Fig. 2: Error variation with respect to frequency offset.

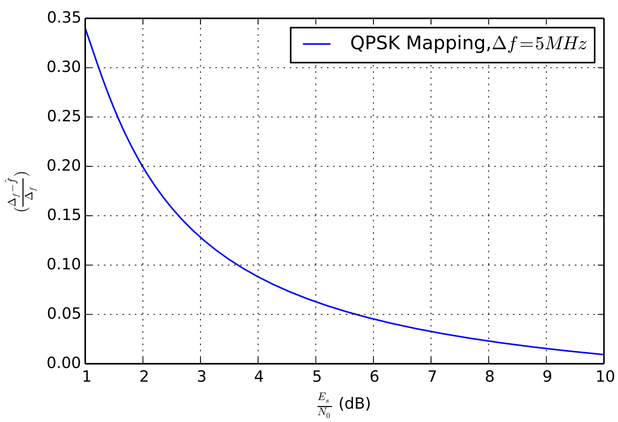

Fig. 3: Error variation with respect to the SNR. $\Delta f = 5$ MHz, Center frequency $f_c = 25$ GHz

Where N is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta\hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^{P} \mathrm{Im}(R(k))}{\sum_{k=1}^{P} k\mathrm{Re}(R(k))}, \quad P\Delta f M << 1 \tag{2.6}$$

where P is the number of pilot symbols.

A. Plots

The number of pilot symbols is $P = 18$. The codes for generating the plots are available at

Fig. 2 shows the variation of the error in the offset estimate with respect to the offset Δf when the SNR = 10 dB. Similarly Fig. 3 shows the variation of the error with respect to the SNR for Δf = 5MHz.

3. Phase Offset: Feed Forward Maximum Likelihood (FF-ML)technique

Let the phase offset be $\Delta\phi$ [3] . Then for the kth pilot,

$$Y_k = X_k e^{j\Delta\phi_k} + V_k, \quad k = 1, \ldots, P \tag{3.1}$$

From (3.1),

$$Y_k X_k^* = |X_k|^2 e^{j\Delta\phi_k} + X_k^* V_k \tag{3.2}$$

$$\implies r_k = e^{j\Delta\phi_k} + \bar{V}_k \tag{3.3}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{3.4}$$

From (3.3), the estimate for the kth pilot is obtained as

$$\Delta\hat{\phi}_k = \arg(r_k) \tag{3.5}$$

The phase estimate is then obtained using $\Delta\hat{\phi}_k$ in the following update equation as

$$\Delta\theta_k = \Delta\theta_{k-1} + \alpha SAW\left[\Delta\hat{\phi}_k - \Delta\theta_{k-1}\right] \tag{3.6}$$

Where SAW is sawtooth non-linearity

$$SAW[\phi] = [\phi]_{-\pi}^{\pi} \tag{3.7}$$

and $\alpha \leq 1$. The estimate is then obtained as $\Delta\theta_P$.

A. Plots

Fig. 4 is generated using

https://github.com/gadepall/EE5837/raw/master/synctech/codes/Error_vs_lp.py

and shows the variation of the phase error in the offset estimate with respect to the pilot symbols when the SNR = 10 dB and $\alpha = 0.5$.

Similarly Fig. 5 generated by

https://github.com/gadepall/EE5837/blob/master/synctech/codes/Error_vs_snr.py

shows the variation of the error with respect to the SNR for pilot symbols $P = 18$ and $\alpha = 1$.

4. Automatic Gain Controller (AGC): Data-Aided Vector-Tracker (DA-VT)

Let the random AGC offset α, then the received symbol equation with amplitude offset as,

$$Y_k = \alpha X_k + V_k \quad k = 1, \ldots, P \tag{4.1}$$

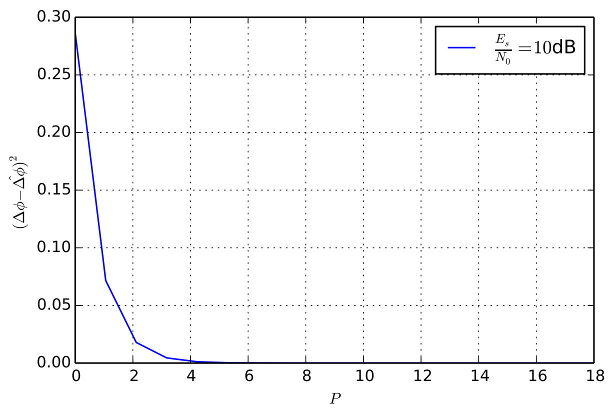

Fig. 4: Phase error variation with respect to pilot symbols

where $\alpha = \alpha_I + j\alpha_Q$ is the gain parameter. According to [4], the $\hat{\alpha}_k$ estimate for the kth pilot is

$$\alpha_{k+1} = \alpha_k - \gamma \left[\alpha_k Y_k^p - X_k^p\right] \left[X_k^p\right]^*, \tag{4.2}$$

where γ is the AGC step size.

A. Plots

The following code plots the real and imaginary parts of the gain parameter α with respect to the number of pilot symbols P. in Fig. 6. $\gamma = 10^{-3}, SNR = 10dB$.

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https://github.com/gadepall/EE5837/raw/master/
    synctech/codes/
    Digital_AGC_with_fixed_SNR.py
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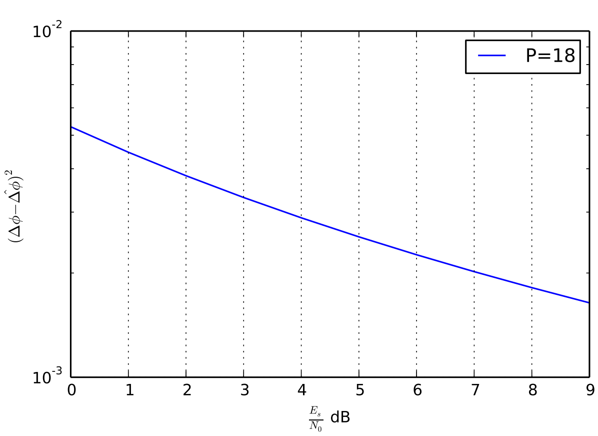

Fig. 5: $\Delta f = 5$ MHz

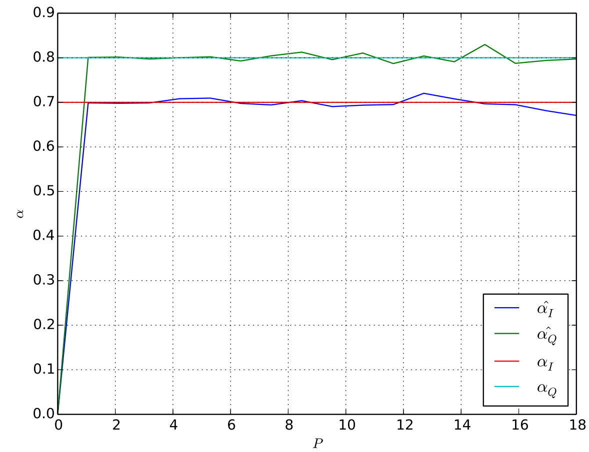

Fig. 6: Convergence of Digital AGC with resepct to P.

5. Frame Synchronization : Global Summation of SOF/PLSC Detectors

Let the frequency offset be Δf and phase offset be $\Delta\phi$.Then,

$$Y_k = X_k e^{j(2\pi\Delta f k M + \phi_k)} + V_k, \quad k = 1, \dots, N \tag{5.1}$$

assuming that no pilot symbols are trasmitted. Let the phase information be θ_k, and defined as

$$\theta(k) = \frac{Y_k}{|Y_k|} \tag{5.2}$$

At the receiver, the header information is available in the form of

$$g_i(l) = x_s(l)x_s(l-i), l = 0, \dots, SOF - 1 \tag{5.3}$$

$$h_i(l) = x_p(l)x_p(l-i), l = 0, \dots, PLSC - 1 \tag{5.4}$$

where x_s are the mapped SOF symbols, x_p are the scrambled PLSC symbols, both modulated using $\pi/2$ BPSK for $i = 1, 2, 4, 8, 16, 32$. A special kind of correlation is performed to obtain

$$m_i(k) = \sum_{l=0}^{PLSC-1} e^{j(\theta(k-l) - \theta(k-l-i))} h_i(l), \tag{5.5}$$

$$n_i(k) = \sum_{l=0}^{SOF-1} e^{j(\theta(k-l) - \theta(k-l-i))} g_i(l), \tag{5.6}$$

$$k = 1, \dots, N \tag{5.7}$$

Compute

$$p_i(k) = \begin{cases} \max\left(\left|n_i(k-PLSC)+m_i(k)\right|,\right. \\ \left.\left|n_i(k-PLSC)-m_i(k)\right|\right) & k > PLSC \\ \max\left|m_i(k)\right| & k < 64 \end{cases} \tag{5.8}$$

GLOBAL variable $G_{R,T}(k)$ [5] defined as,

$$G_{R,T}(k) = \sum_{i\geq 1} p_i(k), \quad i = 1,2,4,8,16,32 \tag{5.9}$$

At the receiver, let us consider we have sent two types of transmission. One is PLHEADER+DATA (Y_{k1}) and another is only DATA (Y_{k2}) and the GLOBAL variables for (Y_{k1}) and (Y_{k2}) from (5.9) are $G1_{R,T}(k)$, $G2_{R,T}(k)$ respectively.

A. Global Threshold Calculation

The Global Threshold variable is defined as

$$T = \max\left(\max\left(G1_{R,T}(k)\right), \max\left(G2_{R,T}(k)\right)\right) \tag{5.10}$$

The probability of false detection of plheader when only DATA frame (Y_{k2}) has been sent is defined as

$$P_{FA} = \frac{\sum \frac{sign(|Y_{k2}-T|)+1}{2}}{N} \tag{5.11}$$

The probability of missed detection of plheader when PLHEADER+DATA (Y_{k1}) has been sent is defined as

$$P_{MD} = \frac{\sum \frac{sign(T-|Y_{k1}|)+1}{2}}{N+PLSC+SOF} \tag{5.12}$$

B. Plots

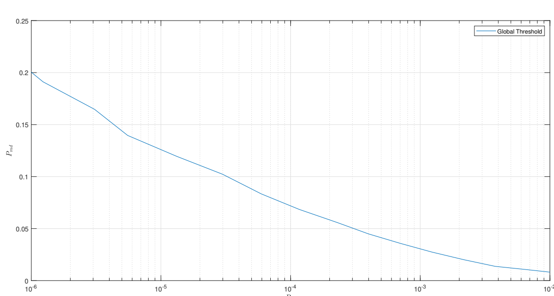

Fig. 7: Frame Synchronization Receiver Operating Characteristcs (ROC)

Fig.7 shows the ROC curve ($P_{FA} vs P_{MD}$) at the receiver for frame synchronization at $\frac{E_b}{N_0} = -2$ dB.

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