Modern Digital Synchronization Techniques for Reliable Communication

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Contents

***Abstract*—A brief description about the design and implementation of Modern Digital Synchronization Techniques for reliable communication.**

1. Time Offset: Gardner TED

Let the mth sample in the rth received symbol time slot be

$$Y_k(m) = X_k + V_k(m), \quad k = 1, \dots, N, m = 1, \dots, M. \tag{1.1}$$

where X_k is the transmitted symbol in the kth time slot and $V_k(m) \sim \mathcal{N}\left(0, \sigma^2\right)$. The decision variable for the kth symbol is [2]

$$U_k = \frac{1}{N} \sum_{i=1}^{N} Y_{k-i} \left(\frac{M}{2}\right) \left[Y_{k-i+1}(M) - Y_{k-i}(M)\right] \tag{1.2}$$

A. Plots

Fig. 1 is generated by the following code

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https://github.com/gadepall/EE5837/raw/master/
    synctech/codes/time_sync_offsets.py
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and shows the variation of the BER with respect to the SNR with different timing offsets τ for $N = 6$.

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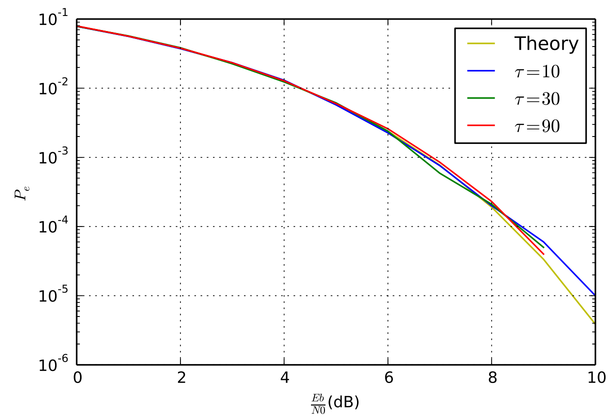

Fig. 1: SNR vs BER for varying τ.

2. Frequency Offset: LR Technique

Let the frequency offset be Δf [3]. Then

$$Y_k = X_k e^{j2\pi\Delta f kM} + V_k, \quad k = 1, \dots, N \tag{2.1}$$

From (2.1),

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta f kM} + X_k^* V_k \tag{2.2}$$
$$\implies r_k = e^{j2\pi\Delta f kM} + \bar{V}_k \tag{2.3}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{2.4}$$

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^{N} r_i r_{i-k}^*, 1 \le k \le N-1 \tag{2.5}$$

Where N is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta\hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^{P} \mathrm{Im}(R(k))}{\sum_{k=1}^{P} k\mathrm{Re}(R(k))}, \quad P\Delta f M << 1 \tag{2.6}$$

where P is the number of pilot symbols.

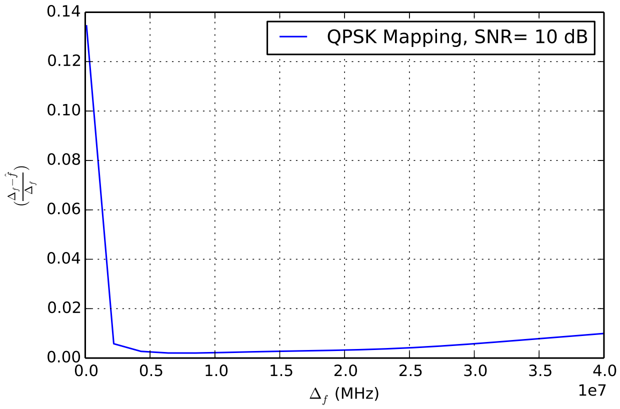

Fig. 2: Error variation with respect to frequency offset.

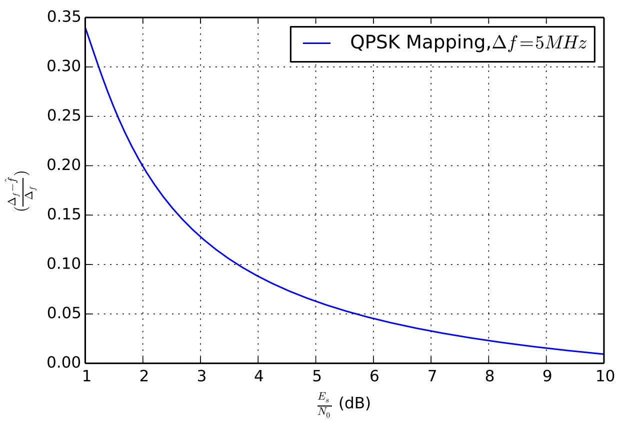

Fig. 3: $\Delta f = 5$ MHz, Center frequency $f_c = 25$ GHz

A. *Plots*

The number of pilot symbols is $P = 18$. The codes for generating the plots are available at

Fig. 2 shows the variation of the error in the offset estimate with respect to the offset Δf when the SNR = 10 dB. Similarly Fig. 3 shows the variation of the error with respect to the SNR for Δf = 5MHz.

3. Phase Offset: Feed Forward Maximum Likelihood (FF-ML)technique

Let the phase offset be $\Delta\phi$ [4]. Then for the kth pilot,

$$Y_k = X_k e^{j\Delta\phi_k} + V_k, \quad k = 1, \dots, P \tag{3.1}$$

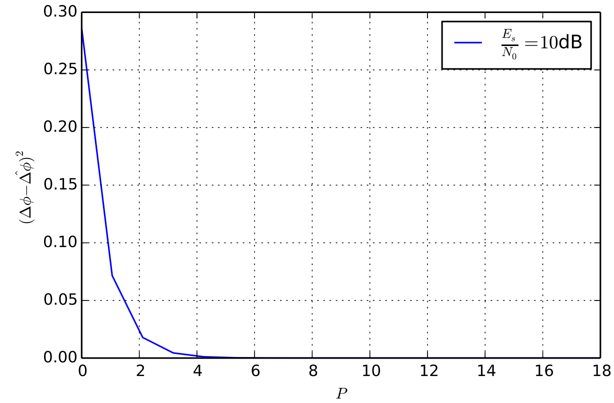

Fig. 4: Phase error variation with respect to pilot symbols

From (3.1),

$$Y_k X_k^* = |X_k|^2 e^{j\Delta\phi_k} + X_k^* V_k \tag{3.2}$$

$$\implies r_k = e^{j\Delta\phi_k} + \bar{V}_k \tag{3.3}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{3.4}$$

From (3.3), the estimate for the kth pilot is obtained as

$$\Delta\hat{\phi}_k = \arg(r_k) \tag{3.5}$$

The phase estimate is then obtained using $\Delta\hat{\phi}_k$ in the following update equation as

$$\Delta\theta_k = \Delta\theta_{k-1} + \alpha SAW\left[\Delta\hat{\phi}_k - \Delta\theta_{k-1}\right] \tag{3.6}$$

Where SAW is sawtooth non-linearity

$$SAW[\phi] = [\phi]_{-\pi}^{\pi} \tag{3.7}$$

and $\alpha \leq 1$. The estimate is then obtained as $\Delta\theta_P$.

A. *Plots*

Fig. 4 is generated using

and shows the variation of the phase error in the offset estimate with respect to the pilot symbols. The codes for generating the plots are available at

Δf when the SNR = 10 dB. Similarly Fig. 5 shows the variation of the error with respect to the SNR for pilot symbols $P = 18$.

References

[1] U. Mengali and A. N. D'Andrea:'synchronization Techniques for Digital Receivers,' New York: Plenum, 1997.

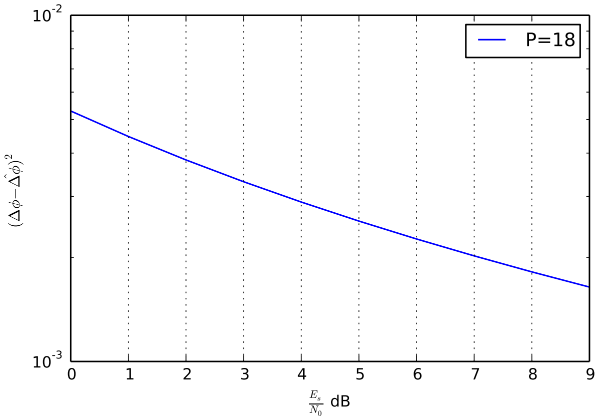

Fig. 5: $\Delta f = 5$ MHz

[2] F. M. Gardner:'A BPSK/QPSK timing-error detector for sampled receivers,' IEEE TRANSACTIONS ON COMMUNICATIONS, VOL.COM-34,NO.5,MAY 1986

[3] M. Luise and R. Reggiannini:'Carrier frequency recovery in all-digital modems for burst mode transmissions,' IEEE Trans. Commun., vol. 43, no. 2/3/4, pp. 1169-1178, Feb/Mar/Apr 1995.

[4] E. Casini, R. De Gaudenzi, and A. Ginesi:'DVB-S2 modem algorithms design and performance over typical satellite channels,' International Journal of Satellite Communications and Networking 2004, vol. 22(3), pp. 281-318, 2004.