# Digital Synchronization Techniques for Reliable Communication

Theresh Babu Benguluri, Raktim Goswami, Abhishek Bairagi, Siddharth Maurya, Pappu Manasa and G V V Sharma*

Contents


**1 Time Offset: Gardner TED** 1
1.1 Plots . . . . . . . . . . . . . . 1

**2 Frequency Offset: LR Technique** 1
2.1 Plots . . . . . . . . . . . . . . 2

**3 Phase Offset: Feed Forward Maximum Likelihood (FF-ML)technique** 2
3.1 Plots . . . . . . . . . . . . . . 2

**References** 3


***Abstract*—This manual provides a brief description about the design and implementation of digital synchronization techniques for reliable communication.**

## 1. Time Offset: Gardner TED

Let the $m$th sample in the $r$th received symbol time slot be

$$Y_k(m) = X_k + V_k(m), \quad k = 1, \dots, N, m = 1, \dots, M. \tag{1.1}$$

where $X_k$ is the transmitted symbol in the $k$th time slot and $V_k(m) \sim \mathcal{N}\left(0, \sigma^2\right)$. The decision variable for the $k$th symbol is [1]

$$U_k = \frac{1}{N} \sum_{i=1}^{N} Y_{k-i}\left(\frac{M}{2}\right)\left[Y_{k-i+1}(M) - Y_{k-i}(M)\right] \tag{1.2}$$

### A. Plots

Fig. 1 is generated by the following code

```
https://github.com/gadepall/EE5837/raw/master/
    synctech/codes/time_sync_offsets.py
```

and shows the variation of the BER with respect to the SNR with different timing offsets $\tau$ for $N = 6$.

*The authors are with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

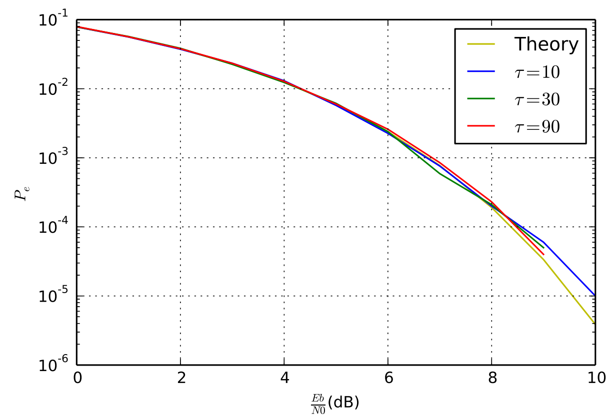


Fig. 1: SNR vs BER for varying $\tau$.

## 2. Frequency Offset: LR Technique

Let the frequency offset be $\Delta f$ [2] . Then

$$Y_k = X_k e^{j2\pi\Delta f k M} + V_k, \quad k = 1, \dots, N \tag{2.1}$$

From (2.1),

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta f k M} + X_k^* V_k \tag{2.2}$$
$$\implies r_k = e^{j2\pi\Delta f k M} + \bar{V}_k \tag{2.3}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{2.4}$$

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^{N} r_i r_{i-k}^*, 1 \le k \le N-1 \tag{2.5}$$

Where N is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta\hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^{P} \mathrm{Im}(R(k))}{\sum_{k=1}^{P} k\mathrm{Re}(R(k))}, \quad P\Delta f M << 1 \tag{2.6}$$

where $P$ is the number of pilot symbols.

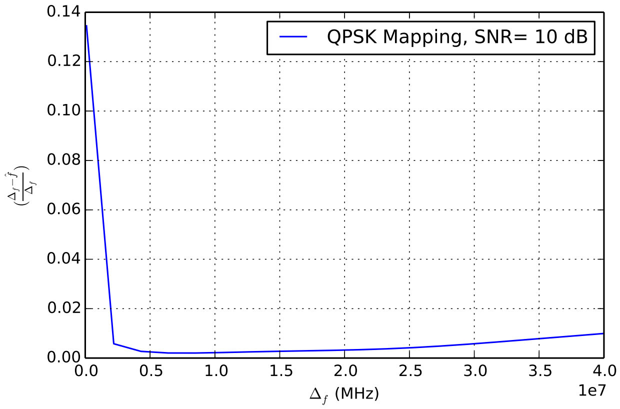


Fig. 2: Error variation with respect to frequency offset.

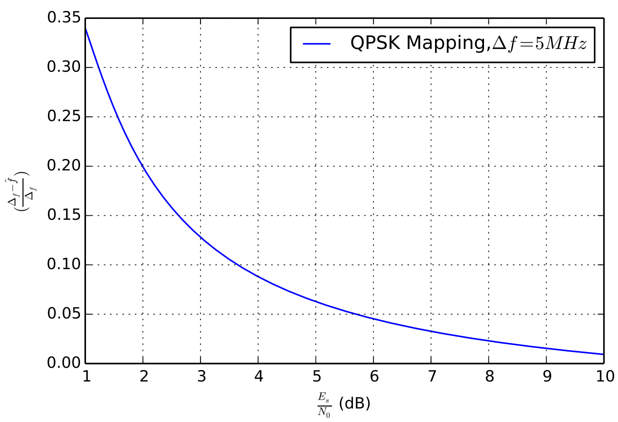


Fig. 3: Error variation with respect to the SNR. $\Delta f$ = 5 MHz, Center frequency $f_c$ = 25 GHz

## A. Plots

The number of pilot symbols is $P = 18$. The codes for generating the plots are available at

Fig. 2 shows the variation of the error in the offset estimate with respect to the offset $\Delta f$ when the SNR = 10 dB. Similarly Fig. 3 shows the variation of the error with respect to the SNR for $\Delta f$ = 5MHz.

# 3. Phase Offset: Feed Forward Maximum Likelihood (FF-ML)technique

Let the phase offset be $\Delta\phi$ [3] . Then for the $k$th pilot,

$$Y_k = X_k e^{j\Delta\phi_k} + V_k, \quad k = 1, \dots, P \tag{3.1}$$

From (3.1),

$$Y_k X_k^* = |X_k|^2 e^{j\Delta\phi_k} + X_k^* V_k \tag{3.2}$$

$$\implies r_k = e^{j\Delta\phi_k} + \bar{V}_k \tag{3.3}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{3.4}$$

From (3.3), the estimate for the $k$th pilot is obtained as

$$\Delta\hat{\phi}_k = \arg(r_k) \tag{3.5}$$

The phase estimate is then obtained using $\Delta\hat{\phi}_k$ in the following update equation as

$$\Delta\theta_k = \Delta\theta_{k-1} + \alpha SAW\left[\Delta\hat{\phi}_k - \Delta\theta_{k-1}\right] \tag{3.6}$$

Where $SAW$ is sawtooth non-linearity

$$SAW[\phi] = [\phi]_{-\pi}^{\pi} \tag{3.7}$$

and $\alpha \leq 1$. The estimate is then obtained as $\Delta\theta_P$.

## A. Plots

Fig. 4 is generated using

https://github.com/gadepall/EE5837/raw/master/synctech/codes/Error_vs_lp.py

and shows the variation of the phase error in the offset estimate with respect to the pilot symbols when the SNR = 10 dB and $\alpha$ = 0.5.

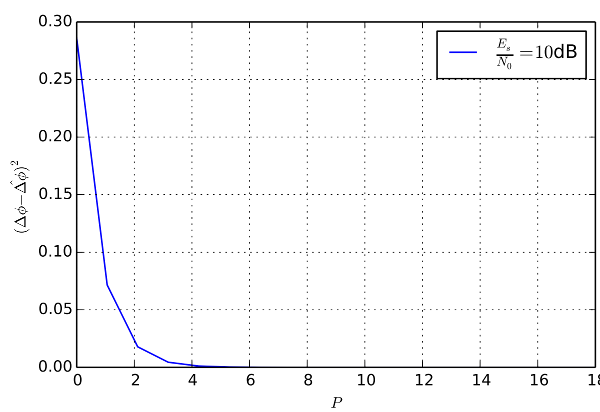


Fig. 4: Phase error variation with respect to pilot symbols

Similarly Fig. 5 generated by

https://github.com/gadepall/EE5837/blob/master/synctech/codes/Error_vs_snr.py shows the variation of the error with respect to the SNR for pilot symbols $P = 18$ and $\alpha = 1$.

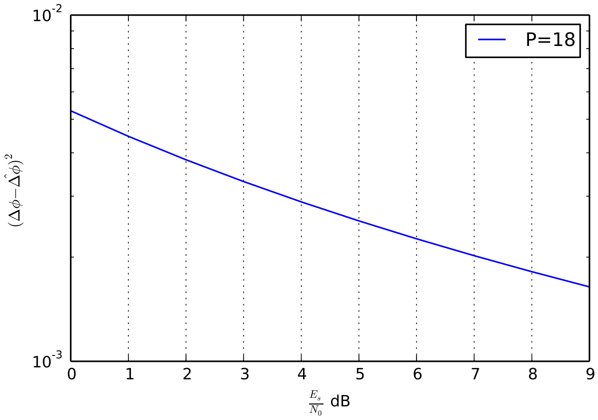


Fig. 5: $\Delta f = 5$ MHz

## References


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