Modern digital communication in DVB S2

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Contents

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1. Modulation and Demodulation

The basedband signal representaion for PSK in [1] can be expressed as

$$Y_k = X_k + V_k, \quad k = 1, \dots, N \tag{1.1}$$

where X_k is the transmitted symbol from the constellation symbols of specified mapping $\{X\}$ in the kth time slot and $V_k \sim \mathcal{N}\left(0, \sigma^2\right)$.

A. QPSK

Constellation Mapping symbol set $\{X\}$ is generated by

$$X_k \in \left\{e^{j\frac{2\pi n}{4}}\right\} \quad n = 0, 1, 2, 3 \tag{1.2}$$

Demapping can be done by using,

$$\frac{2\pi}{4} i < \angle Y_k < \frac{2\pi}{4}(i+1) \implies \hat{X}_k = X_i \quad i = 0, \dots, 3 \tag{1.3}$$

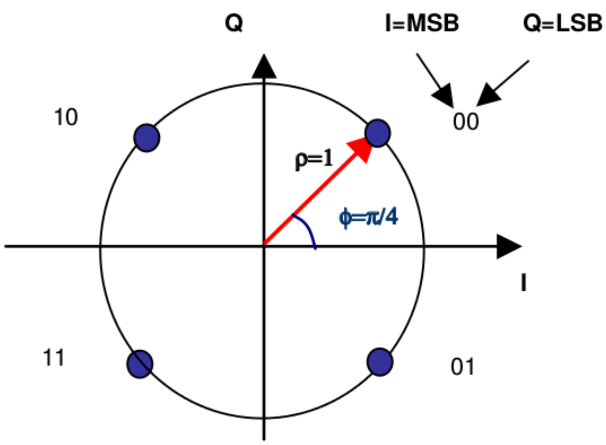

Fig. 1: Constellation diagram of QPSK

Fig. 1 Shows the Constellation mapping for QPSK scheme and similarly Fig. 2 Shows the Simulation diagram.

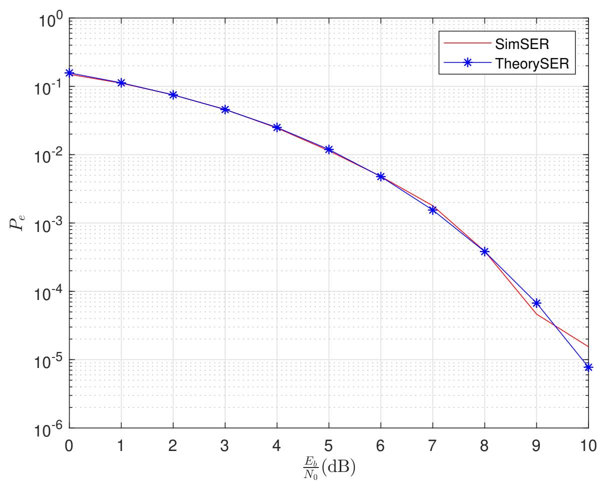

Fig. 2: SNR vs BER for QPSK

B. 8PSK

Constellation Mapping symbol set $\{X\}$ is generated by

$$X_k \in \left\{e^{j\frac{2\pi n}{8}}\right\} \quad n = 0, 1, \dots, 7 \tag{1.4}$$

Demapping can be done by using,

$$\frac{2\pi}{8} i < \angle Y_k < \frac{2\pi}{8}(i+1) \implies \hat{X}_k = X_i \quad i = 0, \dots, 7 \tag{1.5}$$

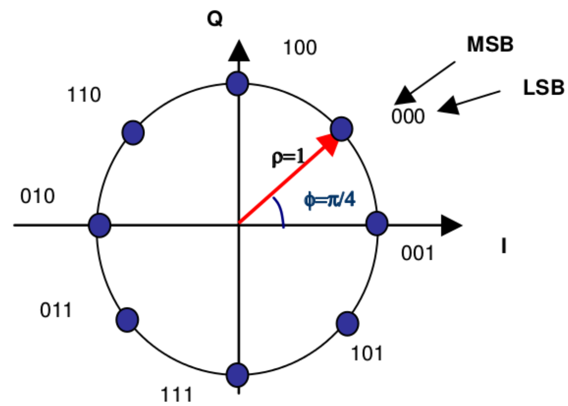

Fig. 3: Constellation diagram of 8-PSK

Fig. 3 Shows the Constellation mapping for 8-PSK symbols and Fig. 4 Shows the Simulation diagram.

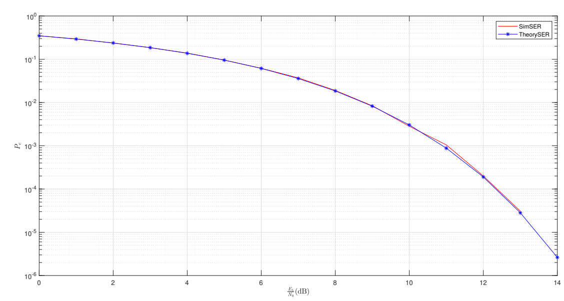

Fig. 4: SNR vs BER for 8-PSK

2. Amplitude Phase Shift Keying (APSK)

A. 16-APSK

Constellation Mapping symbol set $\{X\}$ is generated by

$$X_k \in \{X\} = \begin{cases} r_1 e^{j\left(\phi_1 + \frac{2\pi}{4} n\right)} & n = 0, \dots, 3 \\ r_2 e^{j\left(\phi_2 + \frac{2\pi}{12} n\right)} & n = 0, 1, \dots, 11 \end{cases} \tag{2.1}$$

Demapping can be done by using,

$$|Y_k| < \frac{r_1 + r_2}{2} \&\& \frac{2\pi}{4} i < \angle Y_k < \frac{2\pi}{4}(i+1) \tag{2.2}$$

$$\implies \hat{X}_k = X_i \quad i = 0, \dots, 3 \tag{2.3}$$

$$|Y_k| > \frac{r_1 + r_2}{2} \&\& \frac{2\pi}{12} i < \angle Y_k < \frac{2\pi}{12}(i+1) \tag{2.4}$$

$$\implies \hat{X}_k = X_i \quad i = 4, \dots, 15 \tag{2.5}$$

Where $\frac{r_2}{r_1} = 2.6, \phi_1 = 45, \phi_2 = 15$ Fig. 5 Shows the Constellation mapping for 16-APSK symbols and Fig. 6 Shows the Simulation diagram.

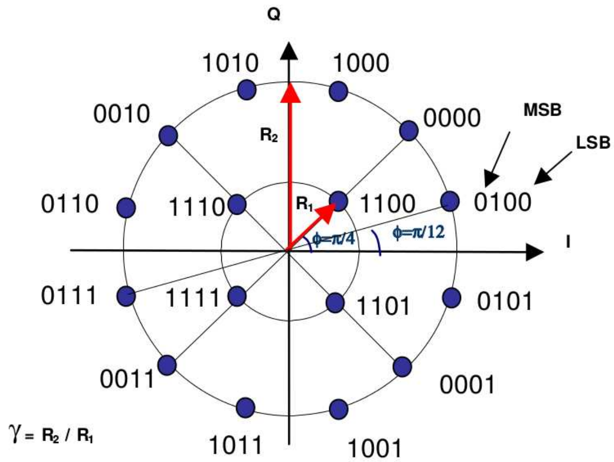

Fig. 5: Constellation diagram of 16APSK

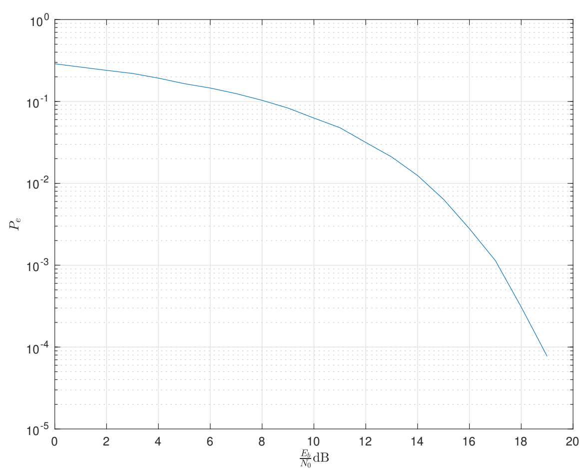

Fig. 6: SNR vs BER for 16-APSK

B. 32-APSK

Constellation Mapping symbol set $\{X\}$ is generated by

$$X_k \in \{X\} = \begin{cases} r_1 e^{j\left(\phi_1 + \frac{2\pi}{4} n\right)} & n = 0, \dots, 3 \\ r_2 e^{j\left(\phi_2 + \frac{2\pi}{12} n\right)} & n = 0, 1, \dots, 11 \\ r_3 e^{j\left(\phi_3 + \frac{2\pi}{16} n\right)} & n = 0, 1, \dots, 16 \end{cases} \quad (2.6)$$

Where $\frac{r_2}{r_1} = 2.54, \frac{r_3}{r_2} = 4.33,, \phi_1 = 45, \phi_2 = 15, \phi_3 = 0$.

Demapping can be done by using,

$$|Y_k| < \frac{r_1 + r_2}{2} \&\& \frac{2\pi}{4} i < \angle Y_k < \frac{2\pi}{4}(i+1) \quad (2.7)$$

$$\implies \hat{X}_k = X_i \quad i = 0, \dots, 3 \quad (2.8)$$

$$\frac{r_1 + r_2}{2} < |Y_k| < \frac{r_2 + r_3}{2} \&\& \frac{2\pi}{12} i < \angle Y_k < \frac{2\pi}{12}(i+1) \quad (2.9)$$

$$\implies \hat{X}_k = X_i \quad i = 4, \dots, 15 \quad (2.10)$$

$$|Y_k| > \frac{r_2 + r_3}{2} \&\& \frac{2\pi}{12} i < \angle Y_k < \frac{2\pi}{12}(i+1) \quad (2.11)$$

$$\implies \hat{X}_k = X_i \quad i = 16, \dots, 31 \quad (2.12)$$

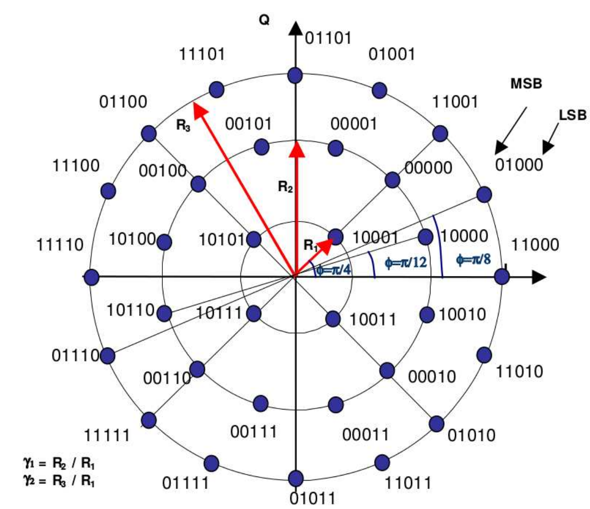

Fig. 7: Constellation diagram of 32APSK

Fig. 7 shows the Constellation mapping for 32-APSK symbols and Fig. 8 shows the Simulation diagram.

3. BCH: Generator Polynomial

For a BCH code, the minimal polynomials are given by

$$g_1(x) = 1 + x + x^3 + x^5 + x^{14} \quad (3.1)$$

$$g_2(x) = 1 + x^6 + x^8 + x^{11} + x^{14} \quad (3.2)$$

$$g_3(x) = 1 + x + x^2 + x^6 + x^9 + x^{10} + x^{14} \quad (3.3)$$

$$g_4(x) = 1 + x^4 + x^7 + x^8 + x^{1}0 + x^{12} + x^{14} \quad (3.4)$$

$$g_5(x) = 1 + x^2 + x^4 + x^6 + x^8 + x^9 + x^{11}$$

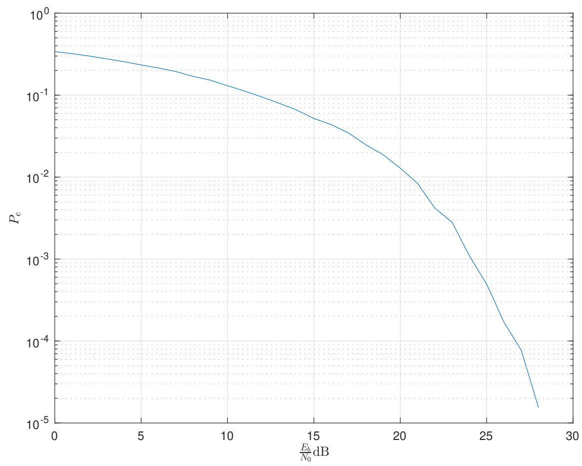

Fig. 8: SNR vs BER for 32-APSK

$$+ x^{13} + x^{14} \tag{3.5}$$

$$g_6(x) = 1 + x^3 + x^7 + x^8 + x^9 + x^{13} + x^{14} \tag{3.6}$$

$$g_7(x) = 1 + x^2 + x^5 + x^6 + x^7 + x^{10} + x^{11} + x^{13} + x^{14} \tag{3.7}$$

$$g_8(x) = 1 + x^5 + x^8 + x^9 + x^{10} + x^{11} + x^{14} \tag{3.8}$$

$$g_9(x) = 1 + x + x^2 + x^3 + x^9 + x^{10} + x^{14} \tag{3.9}$$

$$g_{10}(x) = 1 + x^3 + x^6 + x^9 + x^{11} + x^{12} + x^{14} \tag{3.10}$$

$$g_{11}(x) = 1 + x^4 + x^{11} + x^{12} + x^{14} \tag{3.11}$$

$$g_{12}(x) = 1 + x + x^2 + x^3 + x^5 + x^6 + x^7 + x^8 + x^{10} + x^{13} + x^{14} \tag{3.12}$$

The generator polynomial is obtained as

$$g(x) = \prod_{i=1}^{m} g_i(x) \tag{3.13}$$

4. BCH: Encoding

Let $\vec{m}$ be a $k \times 1$ message vector and

$$m(x) = m_{k-1}x^{k-1} + m_{k-2}x^{k-2} + \cdots + m_1 x + m_0 \tag{4.1}$$

be the corresponding Message polynomial. Let

$$m(x)x^{n-k} = q(x)g(x) + d(x) \tag{4.2}$$

and

$$c(x) = m(x)x^{n-k} + d(x) \tag{4.3}$$

5. Berlekamp's Decoding Algorithm

Let $\vec{\alpha}_i, 2 \le i \le 2m+1$ be the ith row of $\vec{A}$ and $\alpha_i(x)$ be the corresponding polynomial. Let $\vec{r}$ be the received codeword (noisy). $r(x)$ is then defined to be the received polynomial.the correspondig syndromes.

$$S_i(x) = r(\alpha_i(x)) \tag{5.1}$$

Intialization : $k = 0, \Lambda^{(0)}(x) = 1, T^{(0)} = 1$ Let $\Delta^{(2k)}$ be the coefficient of x^{2k+1} in $\Lambda^{(2k)}[1+S(x)]$ Compute

$$\Delta^{(2k+2)}(x) = \Lambda^{(2k)}(x) + \Delta^{(2k)}[x.T^{(2k)}(x)] \tag{5.2}$$

Compute

$$T^{(2k+2)}(x) = \begin{cases} x^2 T^{(2k)}(x) & \text{if } \Delta^{(2k)} = 0 \text{ or deg}\left[\Lambda^{(2k)}(x)\right] > k \\ \frac{x\Lambda^{(2k)}(x)}{\Delta^{(2k)}} & \text{if } \Delta^{(2k)} \neq 0 \text{ or deg}\left[\Lambda^{(2k)}(x)\right] \le k \end{cases} \tag{5.3}$$

Set $k = k + 1$.If $k < t$ then go to step3. Return the Error Locator polynomial $\Lambda(x) = \Lambda^{(2k)}(x)$

6. BCH Decoding: The Chien's Search Algorithm

6.1 Take α^j as test root . $0 \le j \le n-1$.

6.2 if Λ_i test every root and if its equals to zero. Then that is root.

6.3 Flip the bit values at root positions.

6.4 Let the Received polynomial be $r(x)$ i.e which contains both transmitted codeword polynomial $c(x)$ and the error polynomial $e(x)$

$$e(x) = e_0 + e_1 x^1 + ... + e_{n-1}x^{n-1} \tag{6.1}$$

Where e_i represents the value of the error at the location. For binary BCH codes e_i is either 0 or 1.

$$r(x) = c(x)+e(x) = r_{n-1}x^{n-1}+r_{n-2}x^{n-2}+\cdots+r_1 x+r_0 \tag{6.2}$$

Definie, Syndrome

$$S_i = r(\alpha^i) = c(\alpha^i) + e(\alpha^i) = e(\alpha^i) \tag{6.3}$$

Where α^i is a root of the codeword. Suppose that v errors occurred, and $0 \le v \le t$. Let the error occurs at $i_1, i_2, \ldots, i_v$. The Decoding Process, for a t-error correcting code will follows the basic steps,

7. LDPC: Introduction

Let the Channel model be,

$$Y_k = X_k + V_k, \quad k = 0, \ldots, 6 \tag{7.1}$$

where X_k is the transmitted symbol in the kth time slot using the BPSK modulation and $V_k \sim \mathcal{N}\left(0, \sigma^2\right)$.

8. LDPC: Encoding

LDPC codes are popular linear block codes with closest Shannon limit channel capacity [2]. As an example, lets take (7,4) Hamming parity check matrix.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{8.1}$$

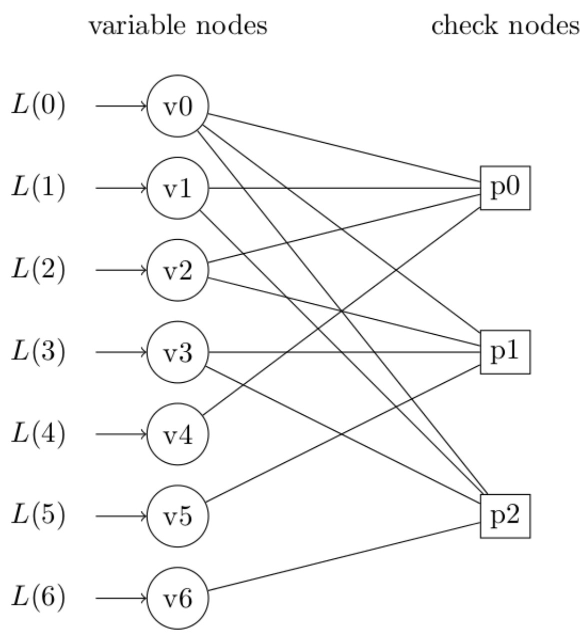

Fig. 9: Tanner Graph Representation for (7,4) Hamming parity check matrix

Encoding can be carried out by using

$$H \times c^T = 0 \tag{8.2}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = 0 \tag{8.3}$$

solving we get

$$p_0 = m_0 \oplus m_1 \oplus m_2 \tag{8.4}$$

$$p_1 = m_0 \oplus m_2 \oplus m_3 \tag{8.5}$$

$$p_2 = m_0 \oplus m_1 \oplus m_3 \tag{8.6}$$

This is called Systematic Encoding.i.e Encoder will ensures information bits followed by parity bits.

9. LDPC: Decoding

A. Useful Calculations for proceeding LDPC Decoding

1) Calculation of Input Channel Log Likelihood Ratio LLR

$$L(x_j) = \log\left(\frac{Pr(x_j = 1|y)}{Pr(x_j = -1|y)}\right) \quad X = 1 - 2c \tag{9.1}$$

$$= \log\left(\frac{f(y|x_j = 1)Pr(x_j = 1)}{f(y|x_j = -1)Pr(x_j = -1)}\right) \tag{9.2}$$

$$= \log\left(\frac{\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_j-1)^2}{2\sigma^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_j+1)^2}{2\sigma^2}}}\right) \tag{9.3}$$

$$= \log\left(e^{\frac{2y_j}{\sigma^2}}\right) \tag{9.4}$$

$$L(x_j) = \frac{2y_j}{\sigma^2} \tag{9.5}$$

2) Check Node Operation :
Lets assume that we have initilized all LLR values to variable nodes and we sent to check nodes. V_j represents all the variable nodes which are connected to j^{th} check node. Using the min-sum approximation [3], the message from j^{th} check node to i^{th} variable node given by, since parity node equation for the first check node is $p_0 = m_0 + m_1 + m_2 + m_4$. we need to calculate

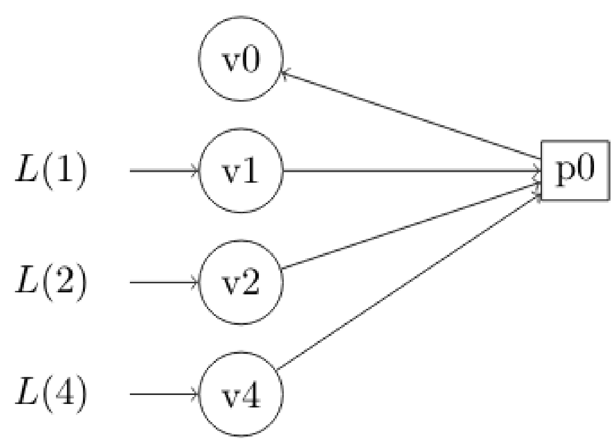

Fig. 10: Check node operation

$$L_{ext0,0} = \log\left(\frac{Pr(x_0 = 0|y_1, y_2, y_4)}{Pr(x_0 = 1|y_1, y_2, y_4)}\right) \tag{9.6}$$

Defining,

$$L_1 = \log\left(\frac{Pr(x_1 = 0|y_1)}{Pr(x_1 = 1|y_1)}\right) = \log\left(\frac{p_1}{1-p_1}\right) \quad (9.7)$$

$$L_2 = \log\left(\frac{Pr(x_2 = 0|y_2)}{Pr(x_2 = 1|y_2)}\right) = \log\left(\frac{p_2}{1-p_2}\right) \quad (9.8)$$

$$L_4 = \log\left(\frac{Pr(x_4 = 0|y_4)}{Pr(x_4 = 1|y_4)}\right) = \log\left(\frac{p_4}{1-p_4}\right) \quad (9.9)$$

$$(9.10)$$

Using Table. I we can find the

c_0	c_1	c_2	c_4
0	0	0	0
1	0	0	1
1	0	1	0
0	0	1	1
1	1	0	0
0	1	0	1
0	1	1	0
1	1	1	1

TABLE I: Probability of a varibale node from other check nodes

$$p_0 = Pr(c_0 = 0|c_1, c_2, c_4) \quad (9.11)$$

$$\begin{aligned} p_0 &= p_1p_2p_4 + p_1(1-p_2)(1-p_4) \\ &\quad + (1-p_1)p_2(1-p_4) + (1-p_1)(1-p_2)p_4 \\ 1 - p_0 &= p_1p_2(1-p_4) + p_1(1-p_2)p_4 \\ &\quad + (1-p_1)p_2p_4 + (1-p_1)(1-p_2)(1-p_4) \end{aligned}$$

by rearranging above equations,

$$p_0-(1-p_0) = p_1-(1-p_1)+p_2-(1-p_2)+p_4-(1-p_4) \quad (9.12)$$

Where p_i is the probability. getting message from check to variable node by taking all variable node informations.

$$\frac{1-\frac{1-p_0}{p_0}}{1+\frac{1-p_0}{p_0}} = \frac{1-\frac{1-p_1}{p_1}}{1+\frac{1-p_1}{p_1}} \times \frac{1-\frac{1-p_2}{p_2}}{1+\frac{1-p_2}{p_2}} \times \frac{1-\frac{1-p_4}{p_4}}{1+\frac{1-p_4}{p_4}} \quad (9.13)$$

$$\frac{1-e^{-L_{ext0,0}}}{1+e^{-L_{ext0,0}}} = \frac{1-e^{-L_1}}{1+e^{-L_1}} \times \frac{1-e^{-L_2}}{1+e^{-L_2}} \times \frac{1-e^{-L_4}}{1+e^{-L_4}} \quad (9.14)$$

$$-\tanh\left(\frac{L_{ext0,0}}{2}\right) = \left(-\tanh\left(\frac{L_1}{2}\right)\right)\left(-\tanh\left(\frac{L_2}{2}\right)\right)\left(-\tanh\left(\frac{L_4}{2}\right)\right) \quad (9.15)$$

$$L_{ext0,0} = \left(\prod_{k\in V_j\backslash i} \alpha_{k,0}\right) f\left(\sum_{k\in V_j\backslash i} f(\beta_{k,0})\right) \quad (9.16)$$

Where,

$$\alpha_{k,j} = sign(L_{k,j}) \quad (9.17)$$

$$\beta_{k,j} = |L_{k,j}| \quad (9.18)$$

$$f(x) = -\log\left(\tanh\frac{x}{2}\right) \quad (9.19)$$

Using the Fig 11 and using its 45° symmetricity, we can approximate the above equation as, given by minimum sum approximation [3]

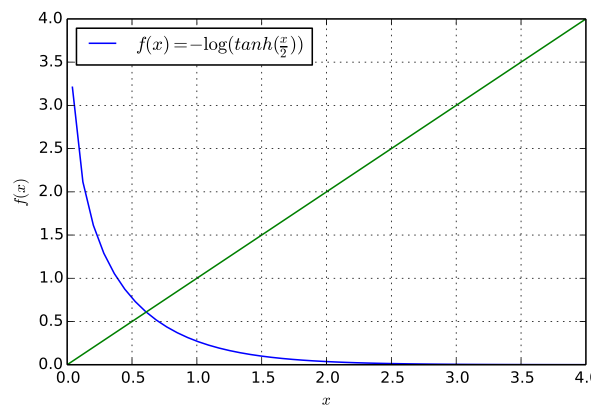

Fig. 11: Plot of function $f(x)$

$$f\left(\sum_{k\in V_j\backslash i} f(\beta_{k,0})\right) \approx f\left(f\left(\min_{k\in V_j\backslash i}(\beta_{k,0})\right)\right) \quad (9.20)$$

$$= \min_{k\in V_j\backslash i}(\beta_{k,0}) \quad (9.21)$$

Combining (9.21) in (9.16),

$$L(r_{j=0,i=0}) = \left(\prod_{k\in V_j\backslash i} \alpha_{k,0}\right)\left(\min_{k\in V_j\backslash i}(\beta_{k,0})\right) \quad (9.22)$$

3) Variable Node Operation :
Let C_i denotes all the check nodes connected to i^{th} variable node. The message from i^{th} variable node to j^{th} check node given by,

$$L(q_{i=0,j=0}) = \log\left(\frac{Pr(x_j = 1|y_0, y_1, y_2)}{Pr(x_j = -1|y_0, y_1, y_2)}\right) \quad X = 1-2c \quad (9.23)$$

$$= \log\left(\frac{f(y_0, y1, y_2|x_j = 1)Pr(x_j = 1)}{f(y_0, y_1, y_2|x_j = -1)Pr(x_j = -1)}\right) \quad (9.24)$$

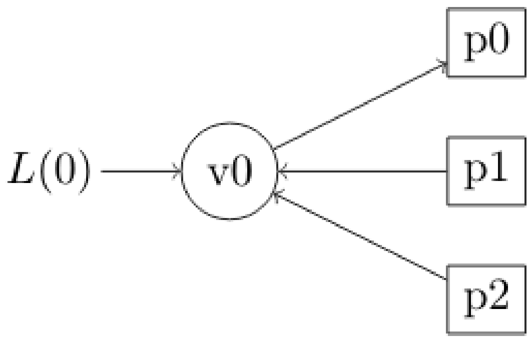

Fig. 12: Variable node operation

$$= \log\left(\frac{\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^3 e^{\frac{-(y_0-1)^2}{2\sigma^2}} e^{\frac{-(y_1-1)^2}{2\sigma^2}} e^{\frac{-(y_2-1)^2}{2\sigma^2}}}{\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^3 e^{\frac{-(y_0+1)^2}{2\sigma^2}} e^{\frac{-(y_1+1)^2}{2\sigma^2}} e^{\frac{-(y_2+1)^2}{2\sigma^2}}}\right) \tag{9.25}$$

$$= \log\left(e^{\frac{2(y_0+y_1+y_2)}{\sigma^2}}\right) \tag{9.26}$$

$$L(q_{i=0,j=0}) = \frac{2(y_0+y_1+y_2)}{\sigma^2} = L(x_i) + \sum_{k\in C_i \backslash j} L(r_{ki}) \tag{9.27}$$

B. Message Passing Algorithm using min-sum Approximation

Transmitted frames = N, Total number of bits = N × 7 and Total number of information bits = N × 4. For Each Frame,

1) Initialize $L(q_{ij})$ using (9.5) for all i, j for which $h_{ij} = 1$ with channel LLR's.
2) Update $\left\{L(r_{ji})\right\}$ using (9.22)
3) Update $\left\{L(q_{ji})\right\}$ using (9.27).
4) Update $\{L(V_i)\}$ using,

$$L(V_i) = L(x_i) + \sum_{k\in C_i} L(r_{ki}) \quad i = 0, \ldots, 6. \tag{9.28}$$

5) Proceed to step 2.
   After maximum specified iterations,

Decoding can be done using,

$$\hat{c}_i = \begin{cases} 1 & L(V_i) < 0 \\ 0 & else \end{cases} \tag{9.29}$$

C. Simulation Results

For frames N=10000. Fig 13 Shows the Comparison of Probability error with channel coding and without channel coding. Since the parity check matrix taken was not much sparse, we are not getting near shannon limit performance. (Good sparse matrix i.e number of entries in H << $m \times n$)

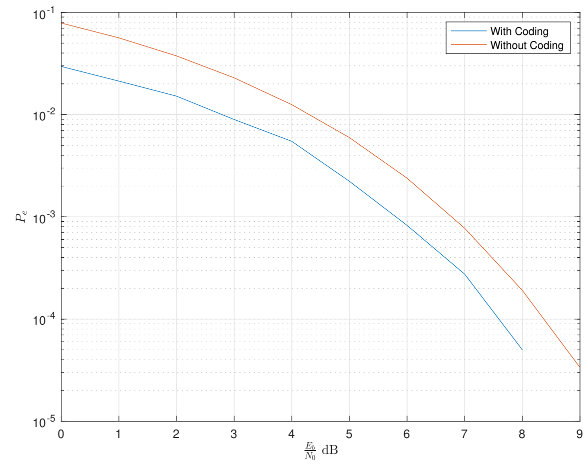

Fig. 13: SNR vs BER curves using LDPC channel coding and no channel coding

10. LDPC Decoding for Higher Order Mapping Schemes

In the case of higher order mapping schemes, calculation of Log Likelihood Ratio's are crutial. A generalized and approximated Log Likelihood Ratio given by [4]

A. General Expression for Calculation of Log Likelihood Ratio(LLR)

$$LLR(b_j) = \log\left(\frac{Pr(b_j = 0|y)}{Pr(b_j = 1|y)}\right) \tag{10.1}$$

$$LLR(b_j) = \log\left(\frac{\sum_{i\in\left\{K_{b_j=0}\right\}} \frac{1}{2\sigma^2} e^{\frac{-|y-x_i|^2}{2\sigma^2}}}{\sum_{i\in\left\{K_{b_j=1}\right\}} \frac{1}{2\sigma^2} e^{\frac{-|y-x_i|^2}{2\sigma^2}}}\right) \tag{10.2}$$

Where the alphabet $\left\{K_{b_j=b}\right\}$ is the set of all symbols representing the j^{th} bit equals to $b = 0$ or $b = 1$.

$$\log\left(e^a + e^b\right) \approx \max(a, b) \tag{10.3}$$

Shows the Jacobian-Logarithmic approximation using [5]. By Using the Above equation, The LLR expression can be written as,

$$LLR(b_j) \approx \max_{i \in \left\{K_{b_j=0}\right\}} \left(e^{\frac{-|y-x_i|^2}{2\sigma^2}}\right) - \max_{i \in \left\{K_{b_j=1}\right\}} \left(e^{\frac{-|y-x_i|^2}{2\sigma^2}}\right) \tag{10.4}$$

B. Approximated LLR's for QPSK Mapping Scheme

For the QPSK mapping scheme showed in Fig. 14 is grey code constellation and b_1, b_0 are the MSB and LSB of the mapped symbols.

$$LLR(b_1) \approx \max(L_0, L_1) - \max(L_3, L_2) \tag{10.5}$$

$$LLR(b_0) \approx \max(L_0, L_2) - \max(L_1, L_3) \tag{10.6}$$

Where,

$$L_i = e^{-\frac{|y-x_i|^2}{2\sigma^2}} \quad i = 0, 1, 2, 3 \tag{10.7}$$

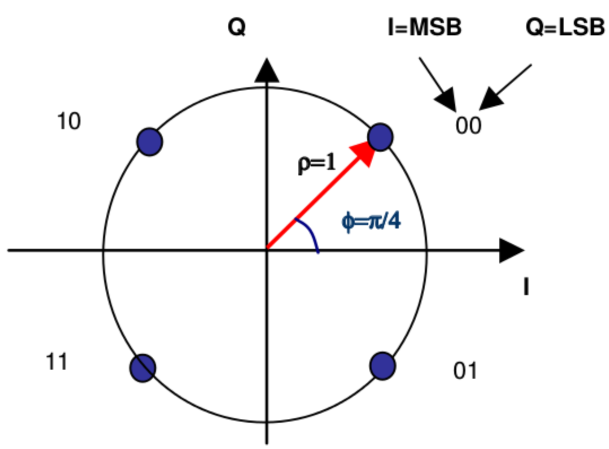

Fig. 14: Constellation Diagram of QPSK

C. Approximated LLR's for 8-PSK Mapping Scheme

For the 8-PSK mapping scheme showed in Fig. 15 is grey code constellation and b_2, b_0 are the MSB and LSB of the mapped symbols.

$$LLR(b_2) \approx \max(L_0, L_1, L_3, L_2) - \max(L_6, L_7, L_5, L_4) \tag{10.8}$$

$$LLR(b_1) \approx \max(L_0, L_1, L_5, L_4) - \max(L_3, L_2, L_6, L_7) \tag{10.9}$$

$$LLR(b_0) \approx \max(L_0, L_2, L_6, L_4) - \max(L_1, L_3, L_7, L_5) \tag{10.10}$$

Where,

$$L_i = e^{-\frac{|y-x_i|^2}{2\sigma^2}} \quad i = 0, 1, \dots, 7 \tag{10.11}$$

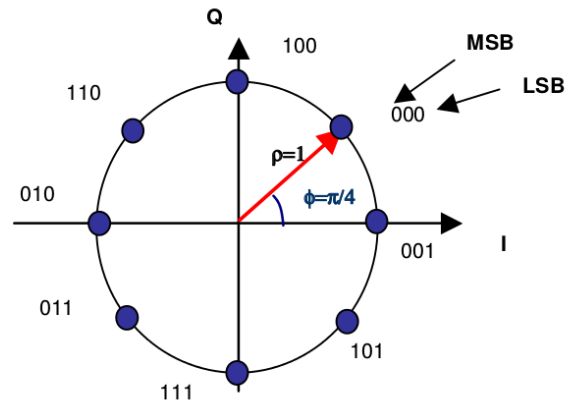

Fig. 15: Constellation Diagram of 8-PSK

11. Time Offset: Gardner TED

Let the mth sample in the rth received symbol time slot be

$$Y_k(m) = X_k + V_k(m), \quad k = 1, \dots, N, m = 1, \dots, M. \tag{11.1}$$

where X_k is the transmitted symbol in the kth time slot and $V_k(m) \sim \mathcal{N}\left(0, \sigma^2\right)$. The decision variable for the kth symbol is [6]

$$U_k = \frac{1}{N} \sum_{i=1}^{N} Y_{k-i}\left(\frac{M}{2}\right)\left[Y_{k-i+1}(M) - Y_{k-i}(M)\right] \tag{11.2}$$

A. Plots

Fig. 16 is generated by the following code

```
https://github.com/gadepall/EE5837/raw/master/
    synctech/codes/time_sync_offsets.py
```

and shows the variation of the BER with respect to the SNR with different timing offsets τ for $N = 6$.

12. Frequency Offset: LR Technique

Let the frequency offset be Δf [7] . Then

$$Y_k = X_k e^{j2\pi \Delta f k M} + V_k, \quad k = 1, \dots, N \tag{12.1}$$

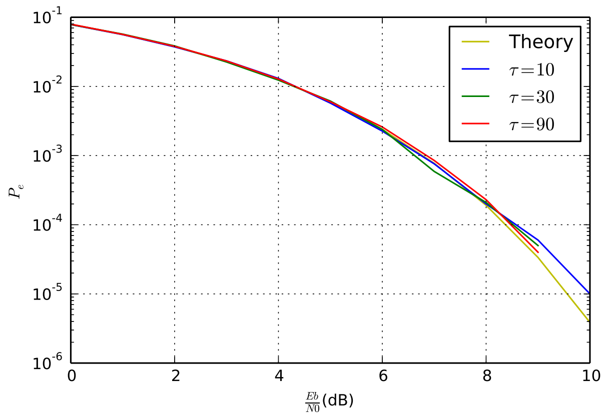

Fig. 16: SNR vs BER for varying τ.

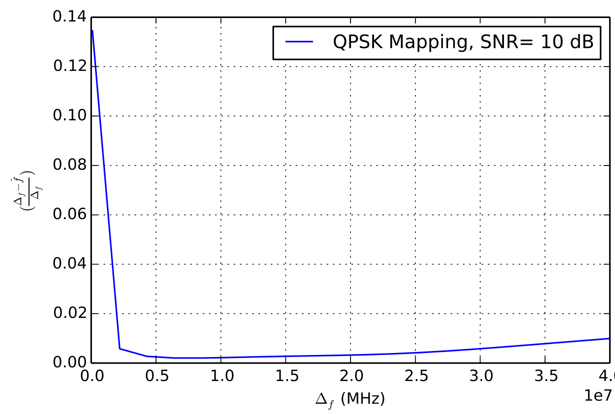

Fig. 17: Error variation with respect to frequency offset.

From (12.1),

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta fkM} + X_k^* V_k \tag{12.2}$$

$$\implies r_k = e^{j2\pi\Delta fkM} + \bar{V}_k \tag{12.3}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{12.4}$$

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^{N} r_i r_{i-k}^*, 1 \leq k \leq N-1 \tag{12.5}$$

Where N is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta\hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^{P} \text{Im}(R(k))}{\sum_{k=1}^{P} k\text{Re}(R(k))}, \quad P\Delta fM << 1 \tag{12.6}$$

where P is the number of pilot symbols.

A. Plots

The number of pilot symbols is $P = 18$. The codes for generating the plots are available at

Fig. 17 shows the variation of the error in the offset estimate with respect to the offset Δf when the SNR = 10 dB. Similarly Fig. 18 shows the variation of the error with respect to the SNR for $\Delta f = 5$MHz.

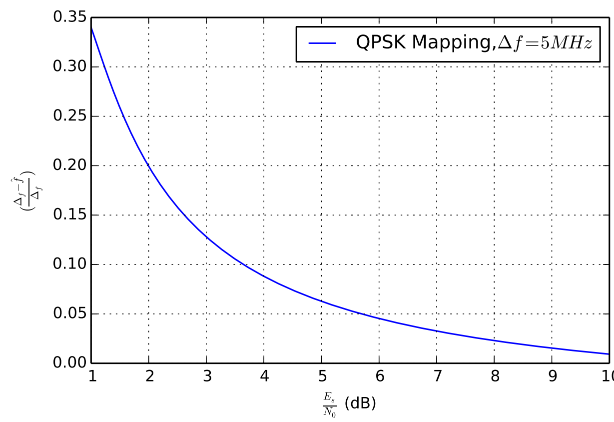

Fig. 18: Error variation with respect to the SNR. $\Delta f = 5$ MHz, Center frequency $f_c = 25$ GHz

13. Phase Offset: Feed Forward Maximum Likelihood (FF-ML)technique

Let the phase offset be $\Delta\phi$ [8] . Then for the kth pilot,

$$Y_k = X_k e^{j\Delta\phi_k} + V_k, \quad k = 1, \ldots, P \tag{13.1}$$

From (13.1),

$$Y_k X_k^* = |X_k|^2 e^{j\Delta\phi_k} + X_k^* V_k \tag{13.2}$$

$$\implies r_k = e^{j\Delta\phi_k} + \bar{V}_k \tag{13.3}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{13.4}$$

From (13.3), the estimate for the kth pilot is obtained as

$$\Delta\hat{\phi}_k = \arg(r_k) \tag{13.5}$$

The phase estimate is then obtained using $\Delta\hat{\phi}_k$ in the following update equation as

$$\Delta\theta_k = \Delta\theta_{k-1} + \alpha SAW\left[\Delta\hat{\phi}_k - \Delta\theta_{k-1}\right] \tag{13.6}$$

Where SAW is sawtooth non-linearity

$$SAW[\phi] = [\phi]_{-\pi}^{\pi} \tag{13.7}$$

and $\alpha \leq 1$. The estimate is then obtained as $\Delta\theta_P$.

A. Plots

Fig. 19 is generated using

```
https://github.com/gadepall/EE5837/raw/master/
    synctech/codes/Error_vs_lp.py
```

and shows the variation of the phase error in the offset estimate with respect to the pilot symbols when the SNR = 10 dB and $\alpha = 0.5$.

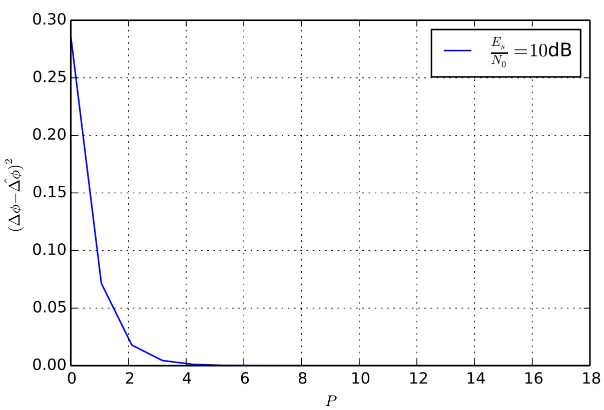

Fig. 19: Phase error variation with respect to pilot symbols

Similarly Fig. 20 generated by

```
https://github.com/gadepall/EE5837/blob/master/
    synctech/codes/Error_vs_snr.py
```

shows the variation of the error with respect to the SNR for pilot symbols $P = 18$ and $\alpha = 1$.

14. Automatic Gain Controller (AGC): Data-Aided Vector-Tracker (DA-VT)

Let the random AGC offset α, then the received symbol equation with amplitude offset as,

$$Y_k = \alpha X_k + V_k \quad k = 1, \ldots, P \tag{14.1}$$

where $\alpha = \alpha_I + j\alpha_Q$ is the gain parameter. According to [9], the $\hat{\alpha}_k$ estimate for the kth pilot is

$$\alpha_{k+1} = \alpha_k - \gamma\left[\alpha_k Y_k^p - X_k^p\right]\left[X_k^p\right]^*, \tag{14.2}$$

where γ is the AGC step size.

A. Plots

The following code plots the real and imaginary parts of the gain parameter α with respect to the number of pilot symbols P. in Fig. 21. $\gamma = 10^{-3}, SNR = 10dB.$

```
https://github.com/gadepall/EE5837/raw/master/
    synctech/codes/
    Digital_AGC_with_fixed_SNR.py
```

15. Frame Synchronization : Global Summation of SOF/PLSC Detectors

Let the frequency offset be Δf and phase offset be $\Delta\phi$.Then,

$$Y_k = X_k e^{j(2\pi\Delta f kM + \phi_k)} + V_k, \quad k = 1, \ldots, N \tag{15.1}$$

assuming that no pilot symbols are trasmitted. Let the phase information be θ_k, and defined as

$$\theta(k) = \frac{Y_k}{|Y_k|} \tag{15.2}$$

At the receiver, the header information is available in the form of

$$g_i(l) = x_s(l)x_s(l-i), l = 0, \ldots, SOF - 1 \tag{15.3}$$
$$h_i(l) = x_p(l)x_p(l-i), l = 0, \ldots, PLSC - 1 \tag{15.4}$$

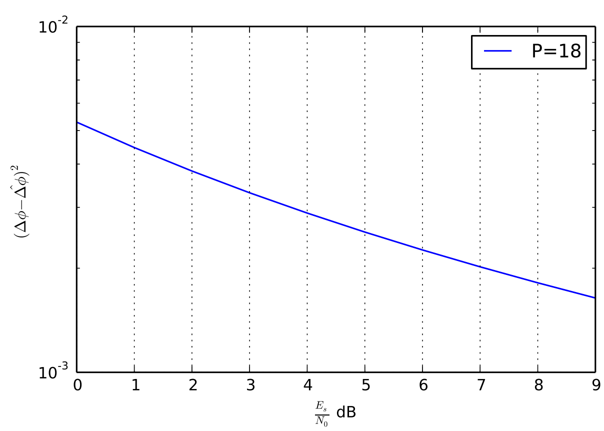

Fig. 20: $\Delta f = 5$ MHz

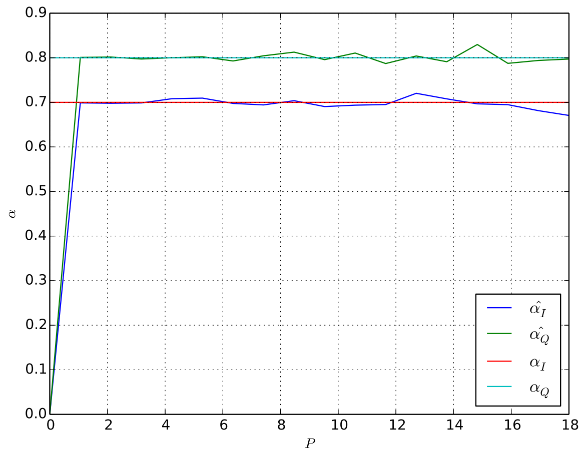

Fig. 21: Convergence of Digital AGC with resepct to P.

where x_s are the mapped SOF symbols, x_p are the scrambled PLSC symbols, both modulated using $\pi/2$ BPSK for $i = 1, 2, 4, 8, 16, 32$. A special kind of correlation is performed to obtain

$$m_i(k) = \sum_{l=0}^{PLSC-1} e^{j(\theta(k-l)-\theta(k-l-i))} h_i(l), \tag{15.5}$$

$$n_i(k) = \sum_{l=0}^{SOF-1} e^{j(\theta(k-l)-\theta(k-l-i))} g_i(l), \tag{15.6}$$

$$k = 1, \ldots, N \tag{15.7}$$

Compute

$$p_i(k) = \begin{cases} \max\left(|n_i(k - PLSC) + m_i(k)|,\right. & \\ \left.|n_i(k - PLSC) - m_i(k)|\right) & k > PLSC \\ \max |m_i(k)| & k < 64 \end{cases} \tag{15.8}$$

GLOBAL variable $G_{R,T}(k)$ [10] defined as,

$$G_{R,T}(k) = \sum_{i \geq 1} p_i(k), \quad i = 1, 2, 4, 8, 16, 32 \tag{15.9}$$

At the receiver, let us consider we have sent two types of transmission. One is PLHEADER+DATA (Y_{k1}) and another is only DATA (Y_{k2}) and the GLOBAL variables for (Y_{k1}) and (Y_{k2}) from (15.9) are $G1_{R,T}(k)$, $G2_{R,T}(k)$ respectively.

A. *Global Threshold Calculation*

The Global Threshold variable is defined as

$$T = \max\left(\max\left(G1_{R,T}(k)\right), \max\left(G2_{R,T}(k)\right)\right) \tag{15.10}$$

The probability of false detection of plheader when only DATA frame (Y_{k2}) has been sent is defined as

$$P_{FA} = \frac{\sum \frac{sign(|Y_{k2} - T|) + 1}{2}}{N} \tag{15.11}$$

The probability of missed detection of plheader when PLHEADER+DATA (Y_{k1}) has been sent is defined as

$$P_{MD} = \frac{\sum \frac{sign(T - |Y_{k1}|) + 1}{2}}{N + PLSC + SOF} \tag{15.12}$$

B. *Plots*

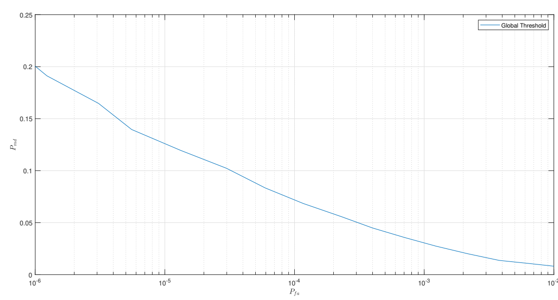

Fig. 22: Frame Synchronization Receiver Operating Characteristcs (ROC)

Fig.22 shows the ROC curve ($P_{FA} vs P_{MD}$) at the receiver for frame synchronization at $\frac{E_b}{N_0} = -2$ dB.

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