MUSIC

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Contents

***Abstract*—This manual explains the MUSIC algorithm through examples.**

1 System Model

The system parameters are listed in Table 1.1.

1.1 See Table 1.1. The signal $s(t)$ transmitted by each of the M nodes can be expressed as

$$s(t) = Ae^{\jmath 2\pi f_c t}. \tag{1.1}$$

The received signal vector is

$$\mathbf{x}(t) = \mathbf{a}(\theta)s(t) + \mathbf{n}(t), \tag{1.2}$$

where

$$\mathbf{a}(\theta) = \begin{pmatrix} 1 & e^{-\frac{\jmath 2\pi f_c d\cos\theta}{c}} & \dots & e^{-\frac{\jmath 2\pi f_c (N-1) d\cos\theta}{c}} \end{pmatrix}^T \tag{1.3}$$

Parameter	Symbol	Value
Number of transmitters	M	5
Number of receivers	N	10
Number of Samples	P	100
Sigal frequency	f_c	10 MHz
Sampling frequency	f_s	100 MHz
Speed of light	c	$3 \times 10^8 ms^{-1}$
Transmitted signal	$s(t)$	
Received Signal	$x(t)$	
AWGN	$n(t)$	$\mathcal{N}(0,1)$
Direction of arrival	θ	

TABLE 1.1

2 Estimating DOA using MUSIC

2.1 Let

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}(0) & \mathbf{x}(1) & \mathbf{x}(2) & \cdots \mathbf{x}(P-1) \end{pmatrix}^T \tag{2.1}$$

be an $M \times P$ matrix with samples of the received vector array and

$$\mathbf{S} = \frac{\mathbf{X}\mathbf{X}^H}{P} \tag{2.2}$$

be the corresponding empirical covariance.

2.2 Let

$$\mathbf{W} = \begin{pmatrix} \lambda_1 & \lambda_2 & \dots & \lambda_M \end{pmatrix}$$
$$\mathbf{U} = \begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \dots & \mathbf{v}_M \end{pmatrix}, \tag{2.3}$$

such that

$$\mathbf{S}\mathbf{v}_i = \lambda_i \mathbf{v}_i \tag{2.4}$$

and

$$\lambda_1 > \lambda_2 > \cdots > \lambda_M \tag{2.5}$$

2.3 For $M > N$, partition the eigenspace into source and noise subspaces.

$$\mathbf{U}_S = \begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \dots & \mathbf{v}_N \end{pmatrix},$$
$$\mathbf{U}_N = \begin{pmatrix} \mathbf{v}_{N+1} & \mathbf{v}_{N+2} & \dots & \mathbf{v}_M \end{pmatrix}, \tag{2.6}$$

2.4 Define

$$P(\theta) = \frac{1}{\mathbf{a}(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta)} \tag{2.7}$$

Plot $P(\theta)$ for $0 < \theta < \pi$. The peaks of the graph are the estimated DOA.

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3 Simulation Results

3.1 Generate the source signal using the following code

3.2 Generate the received signal using the followin code

3.3 Use the following code to plot the DOA in Fig. 3.1

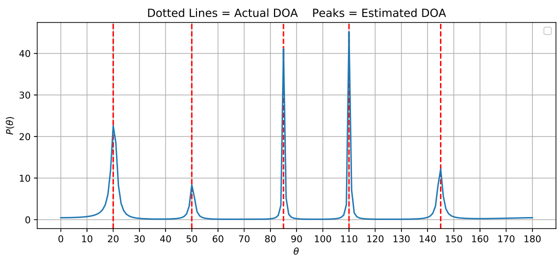

Fig. 3.1