

# Equalization

B Swaroop reddy, Raja Pradyumna and G V V Sharma\*

**Abstract**—This manual provides a brief description about the design and implementation of different Equalization techniques for mitigating the effects of Inter Symbol Interference in wireless communication systems.

## 1. MAXIMUM LIKELIHOOD SEQUENCE ESTIMATION (MLSE)

Let  $\mathbf{a} = (a_0, \dots, a_{m-1})$  be the sequence to be estimated. Where  $m$  is the length of the sequence. The received symbol with the channel filter coefficients  $\mathbf{h} = (h_1, \dots, h_{L-1})$  is

$$\mathbf{r} = \mathbf{s} + \mathbf{n} \quad (1.1)$$

where,

$$\mathbf{s} = \mathbf{a} * \mathbf{h} \quad (1.2)$$

The MLSE or Minimum distance receiver estimation is given by

$$\hat{\mathbf{a}} = \underset{(a_0, \dots, a_{m-1})}{\operatorname{argmin}} d \quad (1.3)$$

where,

$$d = \sum_k |r_k - s_k|^2 \quad s_k = \sum_{l=0}^{L-1} a_l h_{k-l} \quad (1.4)$$

For M-ary modulation formats,  $\mathbf{a}$  takes on  $M^m$  values. So, Viterbi Algorithm does this in an efficient way.

### A. Viterbi Algorithm

Let

$$r_k = a_k + 0.5a + k - 1 + n_k \quad (1.5)$$

Where,

$$a_k \in \{0, 1\} \quad n_k \sim N(0, \sigma^2) \quad (1.6)$$

TABLE I: Truth Table for Trellis

$a_k$	$\Phi_k = a_{k-1}$	$s_k$	$\Phi_{k+1} = a_k$
0	$\Phi_0 = 0$	0	$\Phi_0 = 0$
1	$\Phi_0 = 0$	1	$\Phi_1 = 1$
0	$\Phi_1 = 1$	0.5	$\Phi_0 = 0$
1	$\Phi_1 = 1$	1.5	$\Phi_1 = 1$

Where,

$$a_k = \text{current\_symbol} \quad \Phi_0 = \text{Current\_state} \quad (1.7)$$

$$s_k = a_k + a_{k-1} \quad \Phi_{k+1} = \text{Next\_state} \quad (1.8)$$

The finite state machine for the above truth table is shown in Fig. 1 The Trellis diagram for the FSM shown in Fig. 1 is given in Fig. 2

The multi-stage trellis diagram is shown in Fig. 3.

Suppose, we receive  $r_k = [0.2, 0.6, 0.9, 0.1]$  symbols at time instants  $k = 0, 1, 2, 3$

- With Symbol-by-Symbol detection:
  - Detection threshold = 0.5.
  - Detected Symbols are  $[0, 1, 1, 0]$
- ML detection/Minimum distance metric to be minimized

$$\zeta_i = \sum_k |r_k - s_{k,i}|^2 \quad (1.9)$$

Where,  $i$  is over different transmit symbol vectors

$$\zeta_i = (r_0 - s_{0,i})^2 + (r_2 - s_{2,i})^2 + (r_2 - s_{2,i})^2 + (r_3 - s_{3,i})^2 \quad (1.10)$$

So, the branch metric at  $k^{th}$  symbol period is

$$\zeta_{k,i} = (r_k - s_{k,i})^2 \quad (1.11)$$

Sum of these branch metrics to be minimized in MLSE.

The Trellis Diagram with branch metrics is shown in Fig. 4. The shortest path using Viterbi Algorithm (VA) is shown in Fig. 5. The symbol-by-symbol detection gives  $[0, 1, 1, 0]$  and Maximum Likelihood

\*The authors are with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

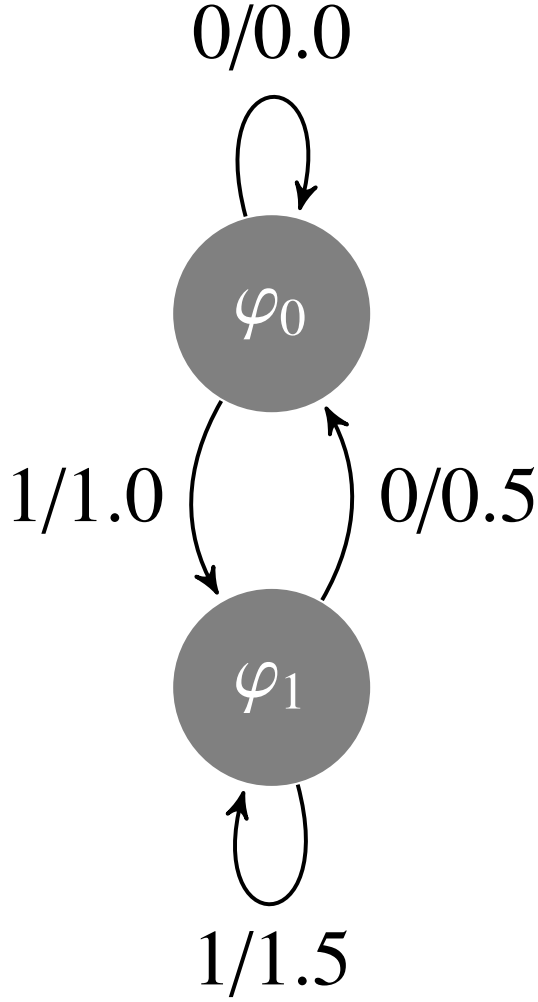


Fig. 1: Finite state machine.

sequence estimation (Via VA) gives  $[0, 1, 0, 0]$ .

1) *Trellis structure in Code:* There are total  $M^{L-1}$  states in the trellis. The complexity of viterbi algorithm increases with the length of the channel filter. Suppose  $L = 3$  and number of states in trellis 64. The total number of outputs from the trellis is

$$trellis\_out\_size = 64 * M \quad (1.12)$$

*Trellis\_in\_struct* and *Trellis\_out\_struct*

The ML sequence estimator structure using viterbi algorithm [?] is shown in [?] and [?]. To estimate a sequence of length  $m$ , the receiver complexity is  $M^m$  symbols for finding the most likely sequence among  $M^m$  sequences. However, this complexity can be reduced from  $M^m$  to  $M^{L-1}$ , where  $L$  is the number of states in trellis used for viterbi Algorithm ( $L < m$ ).

The number of the states in the trellis increases

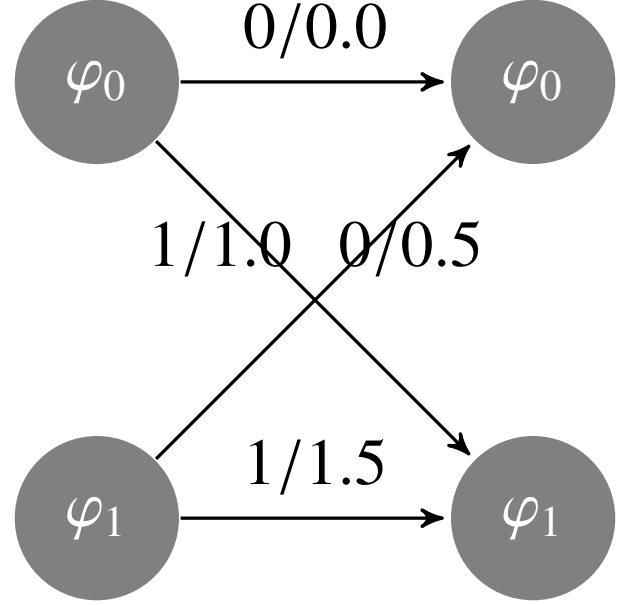


Fig. 2: Trellis Diagram.

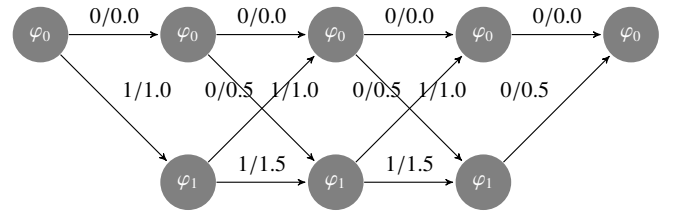


Fig. 3: Trellis Diagram.

exponentially ( $M^{L-1}$ ) with length of the channel impulse response ( $L$ ). Since the bandwidth requirement is 250 KHz, the length of the impulse response is sufficient to assume as 3 and implemented the viterbi algorithm with 64 states.

## 2. CHANNEL ESTIMATION

The MLSE via Viterbi Algorithm needs the channel state information for finding the shortest path. The channel can be estimated using Fast Fourier transforms for a sequence of pilot symbols.

We have used 10 pilot symbols in the code for the estimation of channel. The observed symbols at the receiver for the pilot symbols is given by

$$\mathbf{r}_p = \mathbf{h} * \mathbf{x}_p + \mathbf{n}_p \quad (2.1)$$

The steps for channel estimation is given below

- Make  $\mathbf{r}_p$  into a circular convolution of  $\mathbf{h}$  and  $\mathbf{x}_p$  by removing last  $L - 1$  symbols from  $\mathbf{r}_p$  to add them to the first  $L - 1$  symbols .

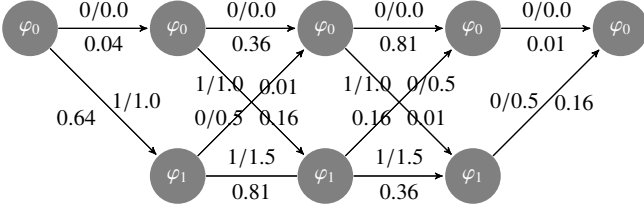


Fig. 4: Trellis Diagram with branch metrics.

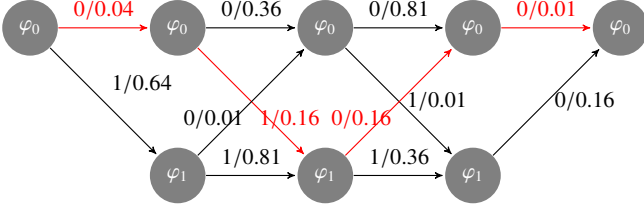


Fig. 5: Trellis Diagram with branch metrics.

- Find

$$Y = fft(\mathbf{r}_p) \quad X = fft(\mathbf{x}_p) \quad (2.2)$$

- Find first three taps of  $h_{est}$  from

$$h_{est} = ifft\left(\frac{Y}{X}\right) \quad (2.3)$$

The second method for channel estimation based on Toeplitz matrix method with pilot symbols is given below

- Construct a toeplitz matrix  $X$  with pilot symbols  $x_p$ .

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$$h_{est} = (X^* X)^{-1} X^* y_p \quad (2.4)$$

### 3. ZERO FORCING EQUALIZER AND MMSE EQUALIZER

The Zero-Forcing Equalizer and MMSE are implemented using Toeplitz matrices [?].

#### A. Zero Forcing Equalizer

- Construct a toeplitz matrix  $H$  with estimated channel  $h_{est}$ .

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$$x_{hat} = (H^* H)^{-1} H^* y \quad (3.1)$$

#### B. MMSE Equalizer

- Construct a toeplitz matrix  $H$  with estimated channel  $h_{est}$ .

$$x_{hat} = \left( H^* H + \frac{I}{SNR} \right)^{-1} H^* y \quad (3.2)$$

Where,  $I$  is an identity matrix and  $SNR$  is the signal-to-noise ratio.