1. Simulating Source Signal :

Let M = no. of sources

N = no. of sensors

p = no. of time snapshots of signal

f_c = freq of signal source

f_s = sampling freq of sensor.

(∴ Sampling times are $\frac{1}{f_s}, \frac{2}{f_s}, \ldots, \frac{p}{f_s}$)

$s(t)$ = source signal

$c = 3 \times 10^8 \ ms^{-1}$

Take $M = 5$, $N = 10$, $p = 100$,

$f_c = 10^6$ Hz, $f_s = 10^7$ Hz,

$$s(t) = \rho \exp(j \, 2\pi f_c \, t)$$

where $\rho \sim \mathcal{N}(0,1)$

Assume M sources are independent.

② Simulating Recieved Signal

Assume N sensors are placed linearly at distance d apart.

$x(t)$ = recieved signal

$$x(t) = \begin{bmatrix} 1 \\ \exp(-j\, 2\pi f_c\, d\cos(\theta^*)/c) \\ \exp(-j\, 2\pi f_c\, 2d\cos(\theta^*)/c) \\ \vdots \\ \exp(-j\, 2\pi f_c\, (N-1)d\cos(\theta^*)/c) \end{bmatrix} s(t) + n(t)$$

$$x(t) = a(\theta^*)\, s(t) + n(t)$$

where $n(t)$ = White Gaussian Noise

θ^* = direcⁿ of arrival of signal $s(t)$.

③ Estimating DOA using MUSIC.

X = recieved signal $(X_{M \times p})$.

S = empirical covariance of X

$$S_{M \times M} = \frac{X X^H}{p}$$

Let λ_i, v_i be eigen values and corresponding eigenvectors of S.

Arrange as :

$$W = [\lambda_1 \ \lambda_2 \ \cdots \ \lambda_M]$$

$$U = [v_1 \ v_2 \ \cdots \ v_M]$$

$$\lambda_1 > \lambda_2 > \cdots > \lambda_M$$

$$S v_i = \lambda_i v_i$$

Partition eigenspace into source and noise subspace.

$$U_s = [v_1 \ v_2 \ \cdots \ v_N] \quad U_n = [v_{N+1} \ \cdots \ v_M]$$

Define $$P(\theta) = \frac{1}{a(\theta)^H U_n U_n^H a(\theta)}$$

Plot θ v/s $P(\theta)$ for $\theta = 0°$ to $180°$.

The peaks of the graph are estimated DOA.