Physical Layer Design for a Narrow Band Communication System

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***Abstract*—This a simple document explaining a question about the concept of similar triangles.**

Download all python codes from

```
svn co https://github.com/SiddharthPh/
    Summer2020/trunk/geometry/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Specifications

1.0.1. Constellation diagram of BPSK

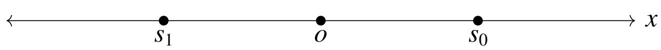

Fig. 1.0.1.1: Constellation diagram

1.0.2. Encoding

We will encode bits as symbols s_0 and s_1.Here, we will transmit s_0 if bit is 0 and we transmit s_1 if bit is 1.

$$s = \begin{cases} s_0, & bit = 0 \\ s_1, & bit = 1 \end{cases}$$

1.0.3. Decision rule for BPSK.

Given symbols s_0 and s_1 are equiprobable and assume symbol carries $\sqrt{E_b}$ per bit and consider a additive white gaussian noise(AWGN) with mean 0 and variance $\frac{N_o}{2}$ and take symbols as equiprobable. The received symbols can be:

$$y|s_0 = \sqrt{(E_b)} + n \tag{1.0.3.1}$$

$$y|s_1 = -\sqrt{(E_b)} + n \tag{1.0.3.2}$$

According to MAP detction rule, we will decode the received signal as symbol s for which p($s|y$) is more.

$$\hat{s} = max_{s\in\{s_0,s_1\}} p\left(s|y\right) \tag{1.0.3.3}$$

$$\implies p\left(s_0|y\right) \underset{s_1}{\overset{s_0}{\gtrless}} p\left(s_1|y\right) \tag{1.0.3.4}$$

Using Bayes rule,

$$p\left(s_0|y\right) = \frac{p\left(y|s_0\right)p\left(s_0\right)}{p\left(y\right)} \tag{1.0.3.5}$$

$$p\left(s_1|y\right) = \frac{p\left(y|s_1\right)p\left(s_1\right)}{p\left(y\right)} \tag{1.0.3.6}$$

Since symbols are equi probable. p(s_0) & p(s_1) are equal.

$$\frac{p\left(y|s_0\right)p\left(s_0\right)}{p\left(y\right)} \underset{s_1}{\overset{s_0}{\gtrless}} \frac{p\left(y|s_1\right)p\left(s_1\right)}{p\left(y\right)} \tag{1.0.3.7}$$

$$\implies p\left(y|s_0\right) \underset{s_1}{\overset{s_0}{\gtrless}} p\left(y|s_1\right) \tag{1.0.3.8}$$

$$\implies \frac{1}{\sqrt{2\pi}}\exp - \frac{(y-\sqrt{E_b})^2}{\frac{N_o}{2}} \underset{s_1}{\overset{s_0}{\gtrless}} \tag{1.0.3.9}$$

$$\frac{1}{\sqrt{2\pi}}\exp - \frac{(y+\sqrt{E_b})^2}{\frac{N_o}{2}} \tag{1.0.3.10}$$

$$\implies (y+\sqrt{E_b})^2 \underset{s_1}{\overset{s_0}{\gtrless}} (y-\sqrt{E_b})^2 \tag{1.0.3.11}$$

$$\implies y \underset{s_1}{\overset{s_0}{\gtrless}} 0 \tag{1.0.3.12}$$

The decision region of BPSK is:

$$y \underset{s_1}{\overset{s_0}{\gtrless}} 0 \tag{1.0.3.13}$$

1.0.4. Decoding

Consider, y is the received symbol .Then, we need to decode this symbol into bits.Here,we will use decision region to decode into bits.

$$bit = \begin{cases} y > 0, & bit = 0 \\ y < 0, & bit = 1 \end{cases}$$

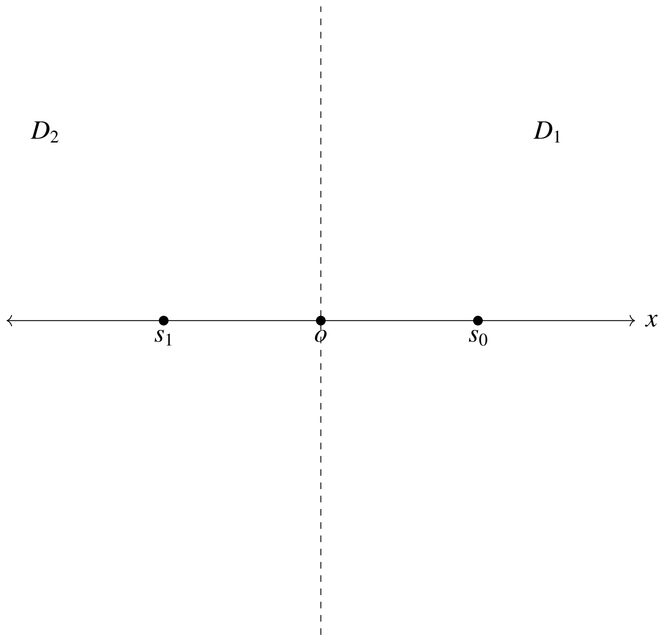

Fig. 1.0.3.1: Decision region for BPSK

So, if received symbol y >0 ,we decode that symbol into 0 and if y<0 we decode it as 1.

1.0.5. The following code has simulation of ber of BPSK.

```
codes/bpsk_ber.py
```