MUSIC

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Contents

***Abstract*—This manual explains the MUSIC algorithm through examples.**

1 System Model

The system parameters are listed in Table 1.1.

1.1 See Table 1.1. The signal $s(t)$ transmitted by each of the M nodes can be expressed as

$$s(t) = Ae^{\jmath 2\pi f_c t}. \tag{1.1}$$

The received signal vector is

$$\mathbf{x}(t) = \mathbf{a}(\theta)s(t) + \mathbf{n}(t), \tag{1.2}$$

where

$$\mathbf{a}(\theta) = \begin{pmatrix} 1 & e^{-\frac{\jmath 2\pi f_c d\cos\theta}{c}} & \cdots & e^{-\frac{\jmath 2\pi f_c (N-1) d\cos\theta}{c}} \end{pmatrix}^T \tag{1.3}$$

2 Estimating DOA using MUSIC

2.1 Let

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}(0) & \mathbf{x}(1) & \mathbf{x}(2) & \cdots \mathbf{x}(P-1) \end{pmatrix}^T \tag{2.1}$$

be an $M \times P$ matrix with samples of the received vector array and

$$\mathbf{S} = \frac{\mathbf{X}\mathbf{X}^H}{P} \tag{2.2}$$

be the corresponding empirical covariance.

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Parameter	**Symbol**	**Value**
Number of transmitters	M	5
Number of receivers	N	10
Number of Samples	P	100
Sigal frequency	f_c	10 MHz
Sampling frequency	f_s	100 MHz
Speed of light	c	$3 \times 10^8 ms^{-1}$
Transmitted signal	$s(t)$	
Received Signal	$x(t)$	
AWGN	$n(t)$	$\mathcal{N}(0,1)$
Direction of arrival	θ	

TABLE 1.1

2.2 Let

$$\mathbf{W} = \begin{pmatrix} \lambda_1 & \lambda_2 & \ldots & \lambda_M \end{pmatrix}$$
$$\mathbf{U} = \begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_M \end{pmatrix}, \tag{2.3}$$

such that

$$\mathbf{S}\mathbf{v}_i = \lambda_i \mathbf{v}_i \tag{2.4}$$

and

$$\lambda_1 > \lambda_2 > \cdots > \lambda_M \tag{2.5}$$

2.3 For $M > N$, partition the eigenspace into source and noise subspaces.

$$\mathbf{U}_S = \begin{pmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_N \end{pmatrix},$$
$$\mathbf{U}_N = \begin{pmatrix} \mathbf{v}_{N+1} & \mathbf{v}_{N+2} & \ldots & \mathbf{v}_M \end{pmatrix}, \tag{2.6}$$

2.4 Define

$$P(\theta) = \frac{1}{\mathbf{a}(\theta)\,\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta)} \tag{2.7}$$

Plot $P(\theta)$ for $0 < \theta < \pi$. The peaks of the graph are the estimated DOA.

3 Simulation Results

3.1 Generate the source signal using the following code

```
https://github.com/gadepall/EE5837/raw/
    master/MUSIC/codes/1
    _simulate_source_signal.py
```

3.2 Generate the received signal using the followin code

```
https://github.com/gadepall/EE5837/raw/
    master/MUSIC/codes/2
    _simulate_recieved_signal.py
```

3.3 Use the following code to plot the DOA in Fig. 3.1

```
https://github.com/gadepall/EE5837/raw/
    master/MUSIC/codes/3
    _DOA_estimation_MUSIC.py
```

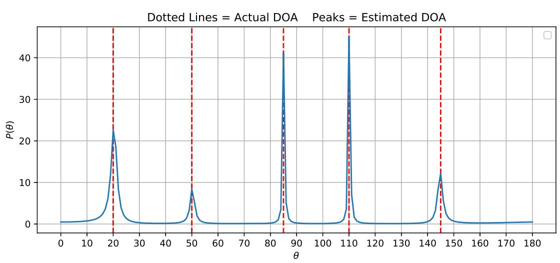

Fig. 3.1