Digital Synchronization Techniques for Reliable Communication

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***Abstract*—This manual provides a brief description about the design and implementation of digital synchronization techniques for reliable communication.**

1. Time Offset: Gardner TED

Let the mth sample in the rth received symbol time slot be

$$Y_k(m) = X_k + V_k(m), \quad k = 1, \ldots, N, m = 1, \ldots, M. \tag{1.1}$$

where X_k is the transmitted symbol in the kth time slot and $V_k(m) \sim \mathcal{N}\left(0, \sigma^2\right)$. The decision variable for the kth symbol is [1]

$$U_k = \frac{1}{N} \sum_{i=1}^{N} Y_{k-i}\left(\frac{M}{2}\right)\left[Y_{k-i+1}(M) - Y_{k-i}(M)\right] \tag{1.2}$$

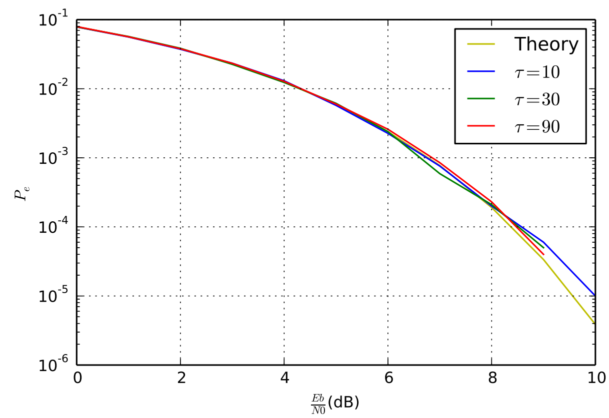

Fig. 1: SNR vs BER for varying τ.

A. Plots

Fig. 1 is generated by the following code

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https://github.com/gadepall/EE5837/raw/master/
    synctech/codes/time_sync_offsets.py
```

and shows the variation of the BER with respect to the SNR with different timing offsets τ for $N = 6$.

2. Frequency Offset: LR Technique

Let the frequency offset be Δf [2] . Then

$$Y_k = X_k e^{j2\pi\Delta f kM} + V_k, \quad k = 1, \ldots, N \tag{2.1}$$

From (5.1),

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta f kM} + X_k^* V_k \tag{2.2}$$
$$\implies r_k = e^{j2\pi\Delta f kM} + \bar{V}_k \tag{2.3}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{2.4}$$

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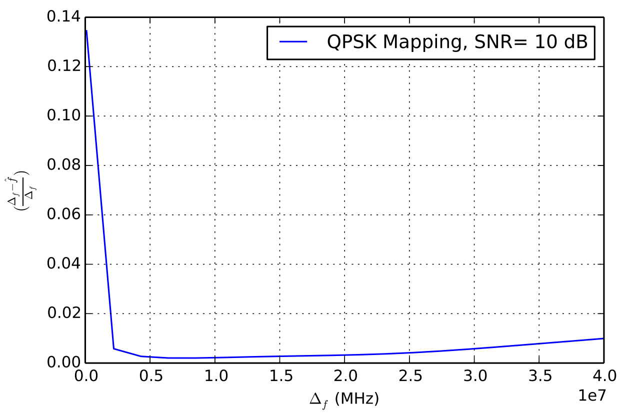

Fig. 2: Error variation with respect to frequency offset.

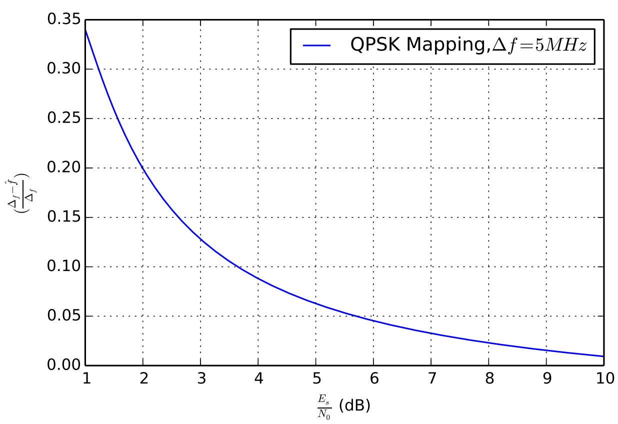

Fig. 3: Error variation with respect to the SNR. $\Delta f = 5$ MHz, Center frequency $f_c = 25$ GHz

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^{N} r_i r_{i-k}^*, 1 \le k \le N-1 \qquad (2.5)$$

Where N is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta \hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^{P} \operatorname{Im}(R(k))}{\sum_{k=1}^{P} k \operatorname{Re}(R(k))}, \quad P \Delta f M << 1 \qquad (2.6)$$

where P is the number of pilot symbols.

A. Plots

The number of pilot symbols is $P = 18$. The codes for generating the plots are available at

Fig. 2 shows the variation of the error in the offset estimate with respect to the offset Δf when the SNR = 10 dB. Similarly Fig. 3 shows the variation of the error with respect to the SNR for Δf = 5MHz.

3. Phase Offset: Feed Forward Maximum Likelihood (FF-ML)technique

Let the phase offset be $\Delta\phi$ [3] . Then for the kth pilot,

$$Y_k = X_k e^{j\Delta\phi_k} + V_k, \quad k = 1, \dots, P \qquad (3.1)$$

From (3.1),

$$Y_k X_k^* = |X_k|^2 e^{j\Delta\phi_k} + X_k^* V_k \qquad (3.2)$$
$$\implies r_k = e^{j\Delta\phi_k} + \bar{V}_k \qquad (3.3)$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \qquad (3.4)$$

From (3.3), the estimate for the kth pilot is obtained as

$$\Delta\hat{\phi}_k = \arg(r_k) \qquad (3.5)$$

The phase estimate is then obtained using $\Delta\hat{\phi}_k$ in the following update equation as

$$\Delta\theta_k = \Delta\theta_{k-1} + \alpha SAW\left[\Delta\hat{\phi}_k - \Delta\theta_{k-1}\right] \qquad (3.6)$$

Where SAW is sawtooth non-linearity

$$SAW[\phi] = [\phi]_{-\pi}^{\pi} \qquad (3.7)$$

and $\alpha \le 1$. The estimate is then obtained as $\Delta\theta_P$.

A. Plots

Fig. 4 is generated using

```
https://github.com/gadepall/EE5837/raw/master/
    synctech/codes/Error_vs_lp.py
```

and shows the variation of the phase error in the offset estimate with respect to the pilot symbols when the SNR = 10 dB and $\alpha = 0.5$.

Similarly Fig. 5 generated by

```
https://github.com/gadepall/EE5837/blob/master/
    synctech/codes/Error_vs_snr.py
```

shows the variation of the error with respect to the SNR for pilot symbols $P = 18$ and $\alpha = 1$.

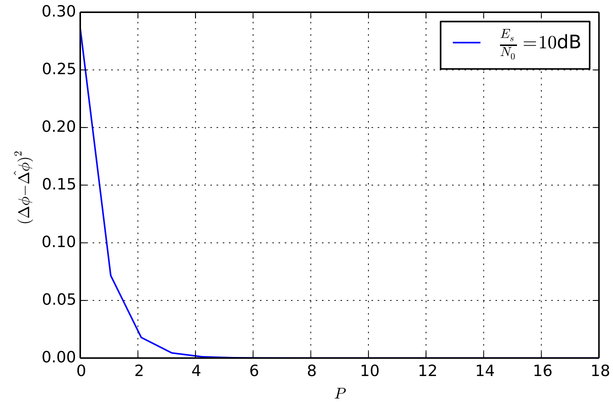

Fig. 4: Phase error variation with respect to pilot symbols

4. Automatic Gain Controller (AGC): Data-Aided Vector-Tracker (DA-VT)

Let the random AGC offset α, then the received symbol equation with amplitude offset as,

$$Y_k = \alpha X_k + V_k \quad k = 1, \ldots, P \tag{4.1}$$

where $\alpha = \alpha_I + j\alpha_Q$ is the gain parameter. According to [4], the $\hat{\alpha}_k$ estimate for the kth pilot is

$$\alpha_{k+1} = \alpha_k - \gamma \left[\alpha_k Y_k^p - X_k^p\right] \left[X_k^p\right]^*, \tag{4.2}$$

where γ is the AGC step size.

A. Plots

The following code plots the real and imaginary parts of the gain parameter α with respect

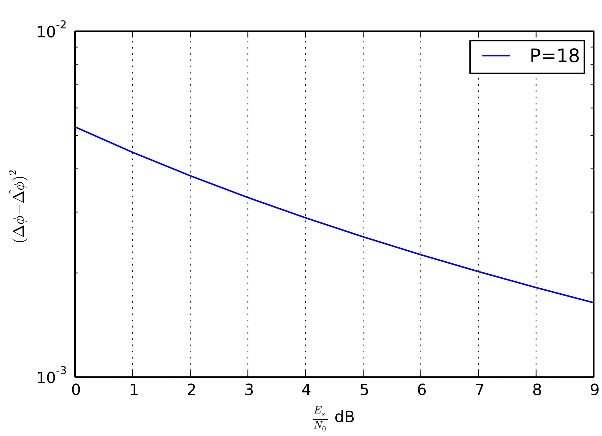

Fig. 5: $\Delta f = 5$ MHz

to the number of pilot symbols P. in Fig. 6. $\gamma = 10^{-3}, SNR = 10dB$.

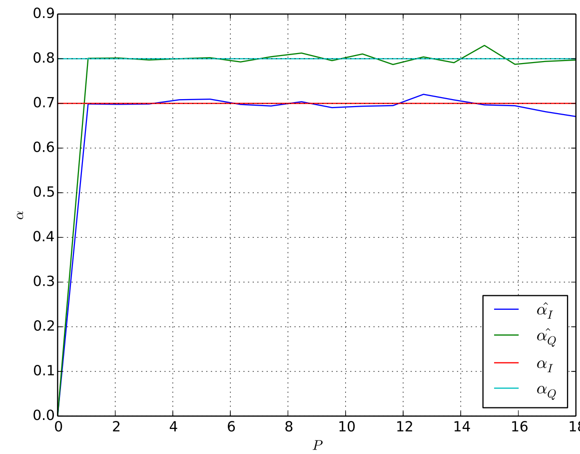

Fig. 6: Convergence of Digital AGC with resepct to P.

https://github.com/gadepall/EE5837/raw/master/synctech/codes/Digital_AGC_with_fixed_SNR.py

5. Frame Synchronization : Global Summation of SOF/PLSC Detectors

Let the frequency offset be Δf and phase offset be $\Delta\phi$.Then,

$$Y_k = X_k e^{j(2\pi\Delta f k M + \phi_k)} + V_k, \quad k = 1, \ldots, N \tag{5.1}$$

assuming that no pilot symbols are trasmitted. Let the phase information be θ_k, and defined as

$$\theta(k) = \frac{Y_k}{|Y_k|} \tag{5.2}$$

At the receiver, the header information is available in the form of

$$g_i(l) = x_s(l)x_s(l-i), l = 0, \ldots, SOF - 1 \tag{5.3}$$
$$h_i(l) = x_p(l)x_p(l-i), l = 0, \ldots, PLSC - 1 \tag{5.4}$$

where x_s are the mapped SOF symbols, x_p are the scrambled PLSC symbols, both modulated using

$\pi/2$ BPSK for $i = 1, 2, 4, 8, 16, 32$. A special kind of correlation is performed to obtain

$$m_i(k) = \sum_{l=0}^{PLSC-1} e^{j(\theta(k-l)-\theta(k-l-i))} h_i(l), \tag{5.5}$$

$$n_i(k) = \sum_{l=0}^{SOF-1} e^{j(\theta(k-l)-\theta(k-l-i))} g_i(l), \tag{5.6}$$

$$k = 1, \ldots, N \tag{5.7}$$

Compute

$$p_i(k) = \begin{cases} \max\left(|n_i(k-PLSC)+m_i(k)|\right., & \\ \left.|n_i(k-PLSC)-m_i(k)|\right) & k > PLSC \\ \max|m_i(k)| & k < 64 \end{cases} \tag{5.8}$$

GLOBAL variable $G_{R,T}(k)$ [5] defined as,

$$G_{R,T}(k) = \sum_{i\geq 1} p_i(k), \quad i = 1, 2, 4, 8, 16, 32 \tag{5.9}$$

Where,

$$\tag{5.10}$$

$$\tag{5.11}$$

Where $xsof$,$xscr$ are the mapped symbols of sof and scr using $\frac{\pi}{2}BPSK$.

References

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