Digital Modulation Techniques

G V V Sharma*

Contents

***Abstract*—The manual frames the problems of receiver design and performance analysis in digital communication as applications of probability theory.**

Download all codes in this manual from

```
svn co https://github.com/gadepall/comm/trunk/
    modulation/manual/codes
```

1 BPSK

1.1. The *signal constellation diagram* for BPSK is given by Fig. 1.1. The symbols s_0 and s_1 are equiprobable. $\sqrt{E_b}$ is the energy transmitted per bit. Assuming a zero mean additive white gaussian noise (AWGN) with variance $\frac{N_0}{2}$, obtain the symbols that are received.
Solution: The possible received symbols are

$$y|s_0 = \sqrt{E_b} + n \tag{1.1.1}$$

$$y|s_1 = -\sqrt{E_b} + n \tag{1.1.2}$$

where the AWGN $n \sim \mathcal{N}\left(0, \frac{N_0}{2}\right)$.

1.2. From Fig. 1.1 obtain a decision rule for BPSK.
Solution: The decision rule is

$$y \underset{s_1}{\overset{s_0}{\gtrless}} 0 \tag{1.2.1}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

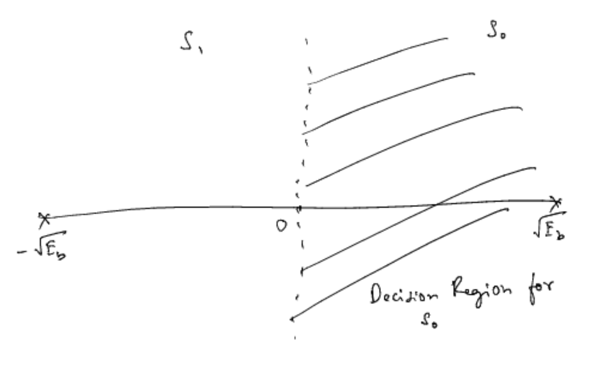

Fig. 1.1

1.3. Find the PDFs of $y|s_0$ and $y|s_0$
Solution: $\because$ $y|s_0$ is Gaussian with

$$E\left[y|s_0\right] = \sqrt{E_b} \tag{1.3.1}$$

$$\text{var}\left(y|s_0\right) = \frac{N_0}{2}, \tag{1.3.2}$$

$$\implies p\left(y|s_0\right) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{\left(y - \sqrt{E_b}\right)^2}{\frac{N_0}{2}}\right\} \tag{1.3.3}$$

Similarly,

$$p\left(y|s_1\right) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{\left(y + \sqrt{E_b}\right)^2}{\frac{N_0}{2}}\right\} \tag{1.3.4}$$

1.4. Obtain (1.2.1) using the MAP criterion.
Solution: According to the MAP rule,

$$\hat{s} = \max_{s \in \{s_0, s_1\}} p\left(s|y\right) \tag{1.4.1}$$

$$\implies p\left(s_0|y\right) \underset{s_1}{\overset{s_0}{\gtrless}} p\left(s_1|y\right) \tag{1.4.2}$$

Using Bayes' rule,

$$p\left(s_0|y\right) = \frac{p\left(y|s_0\right) p(s_0)}{p\left(y\right)} \tag{1.4.3}$$

$$p\left(s_1|y\right) = \frac{p\left(y|s_1\right) p(s_1)}{p\left(y\right)} \tag{1.4.4}$$

which, upon substituting in (1.4.2), yields

$$\frac{p(y|s_0)\,p(s_0)}{p(y)} \underset{s_1}{\overset{s_0}{\gtrless}} \frac{p(y|s_1)\,p(s_1)}{p(y)} \tag{1.4.5}$$

$$p(y|s_0) \underset{s_1}{\overset{s_0}{\gtrless}} p(y|s_1) \tag{1.4.6}$$

$\because$ the symbols are equiprobable. Substituting from and in

$$\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\left(y-\sqrt{E_b}\right)^2}{\frac{N_0}{2}}\right\} \underset{s_1}{\overset{s_0}{\gtrless}} \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\left(y+\sqrt{E_b}\right)^2}{\frac{N_0}{2}}\right\} \tag{1.4.7}$$

$$\implies \left(y+\sqrt{E_b}\right)^2 \underset{s_1}{\overset{s_0}{\gtrless}} \left(y-\sqrt{E_b}\right)^2 \tag{1.4.8}$$

$$\text{or, } y \underset{s_1}{\overset{s_0}{\gtrless}} 0 \tag{1.4.9}$$

after some algebra.

1.5. Using the decision rule in Problem 1.2, obtain an expression for the probability of error for BPSK.
Solution: Since the symbols are equiprobable, it is sufficient if the error is calculated assuming that a 0 was sent. This results in

$$P_e = \Pr(y < 0|s_0) = \Pr\left(\sqrt{E_b} + n < 0\right) \tag{1.5.1}$$

$$= \Pr\left(-n > \sqrt{E_b}\right) = \Pr\left(n > \sqrt{E_b}\right) \tag{1.5.2}$$

since n has a symmetric pdf. Let $w \sim \mathcal{N}(0,1)$. Then $n = \sqrt{\frac{N_0}{2}}w$. Substituting this in (1.5.2),

$$P_e = \Pr\left(\sqrt{\frac{N_0}{2}}w > \sqrt{E_b}\right) = \Pr\left(w > \sqrt{\frac{2E_b}{N_0}}\right) \tag{1.5.3}$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{1.5.4}$$

where

$$Q(x) \triangleq \Pr(w > x), x \geq 0. \tag{1.5.5}$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{x^2}{2\sin^2\theta}}\,d\theta \tag{1.5.6}$$

The PDF of $w \sim \mathcal{N}(0,1)$ is given by

$$p_w(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty \tag{1.5.7}$$

and the complementary error function is defined as

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}\,dt. \tag{1.5.8}$$

1.6. Show that

$$Q(x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right) \tag{1.6.1}$$

Solution: From (1.5.5)

$$Q(x) = \Pr(w > x), x \geq 0 \tag{1.6.2}$$

$$= \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{t^2}{2}}\,dt. \tag{1.6.3}$$

$$= \frac{1}{\sqrt{\pi}}\int_{\frac{x}{\sqrt{2}}}^{\infty} e^{-y^2}\,dy. \quad \left(t = \sqrt{2}y\right) \tag{1.6.4}$$

resulting in

1.7. Verify the bit error rate (BER) plots for BPSK through simulation and analysis for 0 to 10 dB.
Solution: The following code

```
codes/bpsk_ber.py
```

yields Fig. 1.7

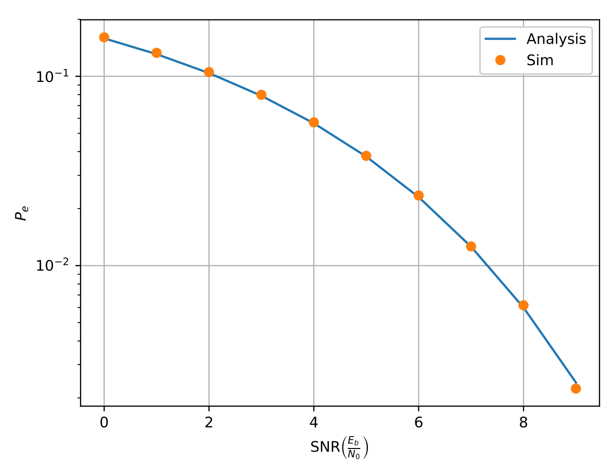

Fig. 1.7

2 Coherent BFSK

2.1. The signal constellation for binary frequency shift keying (BFSK) is given in Fig. 2.1. Obtain

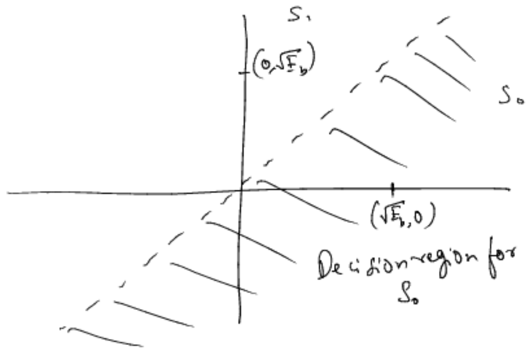

Fig. 2.1

the equations for the received symbols.
Solution: The received symbols are given by

$$\mathbf{y}|s_0 = \begin{pmatrix} \sqrt{E_b} \\ 0 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}, \tag{2.1.1}$$

and

$$\mathbf{y}|s_1 = \begin{pmatrix} 0 \\ \sqrt{E_b} \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}, \tag{2.1.2}$$

where $n_1, n_2 \sim \mathcal{N}\left(0, \frac{N_0}{2}\right)$. and $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$.

2.2. Obtain a decision rule for BFSK from Fig. 2.1.
Solution: The decision rule is

$$y_1 \underset{s_1}{\overset{s_0}{\gtrless}} y_2 \tag{2.2.1}$$

2.3. The multivariate Gaussian distribution is defined as

$$p_{\mathbf{x}}(x_1, \dots, x_k) = \frac{1}{\sqrt{(2\pi)^k |\mathbf{\Sigma}|}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \mathbf{\Sigma}^{-1} (\mathbf{x}-\boldsymbol{\mu})\right\} \tag{2.3.1}$$

where $\boldsymbol{\mu}$ is the mean vector, $\mathbf{\Sigma} = E\left[(\mathbf{x}-\boldsymbol{\mu})(\mathbf{x}-\boldsymbol{\mu})^T\right]$ is the covariance matrix and $|\mathbf{\Sigma}|$ is the determinant of $\mathbf{\Sigma}$. Show that the PDF of the *bivariate* Gaussian is

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho^2)} \times \left\{\frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y}\right\}\right] \tag{2.3.2}$$

where

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}, \mathbf{\Sigma} = \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix} \tag{2.3.3}$$

For equiprobable symbols, the MAP criterion is defined as

$$p(\mathbf{y}|s_0) \underset{s_1}{\overset{s_0}{\gtrless}} p(\mathbf{y}|s_1) \tag{2.3.4}$$

2.4. Use (2.3.2) in (2.3.4) to obtain (2.2.1).
Solution: According to the MAP criterion, assuming equiprobably symbols,

$$p(\mathbf{y}|s_0) \underset{s_1}{\overset{s_0}{\gtrless}} p(\mathbf{y}|s_1) \tag{2.4.1}$$

2.5. Derive and plot the probability of error. Verify through simulation.
Solution: Given that s_0 was transmitted, the received symbols are

$$\mathbf{y}|s_0 = \begin{pmatrix} \sqrt{E_b} \\ 0 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}, \tag{2.5.1}$$

From (2.2.1), the probability of error is given by

$$P_e = \Pr(y_1 < y_2 | s_0) = \Pr\left(\sqrt{E_b} + n_1 < n_2\right) \tag{2.5.2}$$

$$= \Pr\left(n_2 - n_1 > \sqrt{E_b}\right) \tag{2.5.3}$$

Note that $n_2 - n_1 \sim \mathcal{N}(0, N_0)$. Thus,

$$P_e = \Pr\left(\sqrt{N_0}w > \sqrt{E_b}\right) \tag{2.5.4}$$

$$= \Pr\left(w > \sqrt{\frac{E_b}{N_0}}\right) \tag{2.5.5}$$

$$\implies P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{2.5.6}$$

where $w \sim \mathcal{N}(0,1)$. The following code plots the BER curves in Fig. 2.5

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codes/fsk_ber.py
```

3 Maximum Likelihood: Higher Order Modulation

3.1. In higher order modulation, the number of possible symbols can be $M > 2$. Suggest a model for the received vector.
Solution: The model can be expressed as

$$\mathbf{y} = \mathbf{s} + \mathbf{n} \tag{3.1.1}$$

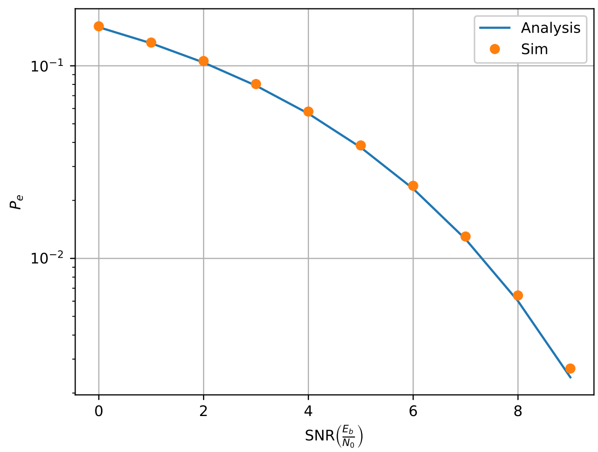

Fig. 2.5

where

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_N \end{pmatrix}, \tag{3.1.2}$$

$$\mathbf{s} \in \{\mathbf{s}_0, \mathbf{s}_1 \dots \mathbf{s}_M\}, \tag{3.1.3}$$

$$\mathbf{n} \sim \mathcal{N}\left(0, \frac{N_0}{2}\mathbf{I}\right) \tag{3.1.4}$$

3.2. Find $p(\mathbf{y}|\mathbf{s})$.
Solution: For (3.1.1), the parameters in (2.3.1) are

$$\boldsymbol{\mu} = \mathbf{s} \tag{3.2.1}$$

$$\boldsymbol{\Sigma} = \frac{N_0}{2}\mathbf{I} \tag{3.2.2}$$

$$\implies p(\mathbf{y}|\mathbf{s}) = \frac{1}{(\pi N_0)^{\frac{1}{2}}} e^{-\frac{\|\mathbf{y}-\mathbf{s}\|^2}{N_0}} \tag{3.2.3}$$

3.3. Extending (1.4.2) and (1.4) for arbitrary M

$$\hat{\mathbf{s}} = \max_{\mathbf{s} \in \{\mathbf{s}_0, \mathbf{s}_1\} \dots \mathbf{s}_M} p(\mathbf{s}|\mathbf{y}) \tag{3.3.1}$$

$$\implies \hat{\mathbf{s}} = \max_{\mathbf{s} \in \{\mathbf{s}_0, \mathbf{s}_1 \dots \mathbf{s}_M\}} p(\mathbf{y}|\mathbf{s}) \tag{3.3.2}$$

which is known as the *maximum likelihood* decision. Using (3.2.3), show that (3.3.2) reduces to the *minimum distance* criterion

$$\hat{\mathbf{s}} = \min_{\mathbf{s} \in \{\mathbf{s}_0, \mathbf{s}_1 \dots \mathbf{s}_M\}} \|\mathbf{y} - \mathbf{s}\| \tag{3.3.3}$$

Solution: Using (3.2.3), (3.3.2) can be expressed as

$$\hat{\mathbf{s}} = \max_{\mathbf{s} \in \{\mathbf{s}_0, \mathbf{s}_1 \dots \mathbf{s}_M\}} \frac{1}{(\pi N_0)^{\frac{1}{2}}} e^{-\frac{\|\mathbf{y}-\mathbf{s}\|^2}{N_0}} \tag{3.3.4}$$

$$= \min_{\mathbf{s} \in \{\mathbf{s}_0, \mathbf{s}_1 \dots \mathbf{s}_M\}} \frac{\|\mathbf{y}-\mathbf{s}\|^2}{N_0} \tag{3.3.5}$$

$$= \min_{\mathbf{s} \in \{\mathbf{s}_0, \mathbf{s}_1 \dots \mathbf{s}_M\}} \|\mathbf{y}-\mathbf{s}\|^2 \tag{3.3.6}$$

yielding (3.3.3).

4 QPSK

4.1. Let

$$\mathbf{y} = \mathbf{s} + \mathbf{n} \tag{4.1.1}$$

where $\mathbf{s} \in \{\mathbf{s}_0, \mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3\}$ and

$$\mathbf{s}_0 = \begin{pmatrix} \sqrt{E_s} \\ 0 \end{pmatrix}, \mathbf{s}_1 = \begin{pmatrix} 0 \\ \sqrt{E_s} \end{pmatrix}, \tag{4.1.2}$$

$$\mathbf{s}_2 = \begin{pmatrix} -\sqrt{E_s} \\ 0 \end{pmatrix}, \mathbf{s}_3 = \begin{pmatrix} 0 \\ -\sqrt{E_s} \end{pmatrix}, \tag{4.1.3}$$

$$E[\mathbf{n}] = \mathbf{0}, E\left[\mathbf{n}\mathbf{n}^T\right] = \frac{N_0}{2}\mathbf{I} \tag{4.1.4}$$

4.2. Using (3.3.3), show that the MAP decision for detecting $\mathbf{s}_0$ results in

$$|y_2| < y_1 \tag{4.2.1}$$

Sketch this region.
Solution: From (3.3.3), $\mathbf{s}_0$ is chosen if

$$\|\mathbf{y} - \mathbf{s}_0\|^2 < \|\mathbf{y} - \mathbf{s}_1\|^2 \tag{4.2.2}$$

$$\|\mathbf{y} - \mathbf{s}_0\|^2 < \|\mathbf{y} - \mathbf{s}_2\|^2 \tag{4.2.3}$$

$$\|\mathbf{y} - \mathbf{s}_0\|^2 < \|\mathbf{y} - \mathbf{s}_3\|^2 \tag{4.2.4}$$

$\because \|\mathbf{s}_i\|^2 = \sqrt{E_s}$, the above conditions can be simplified to obtain the region

$$(\mathbf{s}_0 - \mathbf{s}_1)^T \mathbf{y} > 0 \tag{4.2.5}$$

$$(\mathbf{s}_0 - \mathbf{s}_2)^T \mathbf{y} > 0 \tag{4.2.6}$$

$$(\mathbf{s}_0 - \mathbf{s}_3)^T \mathbf{y} > 0 \tag{4.2.7}$$

Substituting from (4.1.2), in the above and eliminating E_s, the desired region is

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{y} > 0 \tag{4.2.8}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{y} > 0 \tag{4.2.9}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{y} > 0 \tag{4.2.10}$$

yielding (4.2.1).

4.3. Show that (4.2.1) is convex.

4.4. Express $\Pr(\hat{\mathbf{s}} = \mathbf{s}_0|\mathbf{s} = \mathbf{s}_0)$ in terms of y_1, y_2.
Solution: From (4.2.1) and (4.1.2) ,

$$\Pr(\hat{\mathbf{s}} = \mathbf{s}_0|\mathbf{s} = \mathbf{s}_0) = \Pr\left(|y_2| < y_1 | y_1 = \sqrt{E_s}, y_2 = 0\right) \quad (4.4.1)$$

4.5. Let

$$X = n_2 - n_1, \quad (4.5.1)$$
$$Y = -n_2 - n_1, \quad (4.5.2)$$

where $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$.
Show that $X, Y \sim \mathcal{N}(0, N_0)$.

4.6. The correlation coefficient of X, Y is defined as

$$\rho = \frac{E\left[(X - \mu_x)\left(Y - \mu_y\right)\right]}{\sigma_x \sigma_y} \quad (4.6.1)$$

X and Y are said to be uncorrelated if $\rho = 0$ Show that X and Y are uncorrelated. Verify this numerically.
Solution: From (4.1.4),

$$\mu_x = E[X] = 0 \quad (4.6.2)$$
$$\mu_y = E[Y] = 0 \quad (4.6.3)$$
$$\implies \rho = E[XY] = E[(n_2 - n_1)(-n_2 - n_1)]$$
$$= 0 \quad (4.6.4)$$

upon substituting from (4.5.1) and (4.5.2).

4.7. Show that X and Y are independent, i.e.

$$p_{XY}(x, y) = p_X(x) p_Y(y). \quad (4.7.1)$$

Solution: Use (4.6.4) in (2.3.2) to get (4.7.1). Uncorrelated Gaussians are independent.

4.8. Show that

$$\Pr(\hat{\mathbf{s}} = \mathbf{s}_0|\mathbf{s} = \mathbf{s}_0) = \Pr\left(X < \sqrt{E_s}\right)\Pr\left(Y < \sqrt{E_s}\right). \quad (4.8.1)$$

Solution: From (4.4.1) and (4.1.1)

$$\Pr(\hat{\mathbf{s}} = \mathbf{s}_0|\mathbf{s} = \mathbf{s}_0) = \Pr\left(|n_2| < \sqrt{E_s} + n_1\right) \quad (4.8.2)$$

which can be expressed as

$$\Pr\left(n_2 < \sqrt{E_s} + n_1, -n_2 > \sqrt{E_s} + n_1\right) \quad (4.8.3)$$
$$= \Pr\left(X < \sqrt{E_s}, Y < \sqrt{E_s}\right) \quad (4.8.4)$$
$$= \Pr\left(X < \sqrt{E_s}\right)\Pr\left(Y < \sqrt{E_s}\right) \quad (4.8.5)$$

after some algebra, using the fact that X, Y are independent.

4.9. Show that

$$\Pr(\hat{\mathbf{s}} = \mathbf{s}_0|\mathbf{s} = \mathbf{s}_0) = \left(1 - Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right)^2 \quad (4.9.1)$$

Solution: From ,

$$\Pr\left(X > \sqrt{E_s}\right) = \Pr\left(Y > \sqrt{E_s}\right) = Q\left(\sqrt{\frac{E_s}{N_0}}\right) \quad (4.9.2)$$

yielding (4.9.1).

4.10. Verify the above through simulation.
Solution: This is shown in Fig. 4.10 through the following code.

```
codes/qpsk.py
```

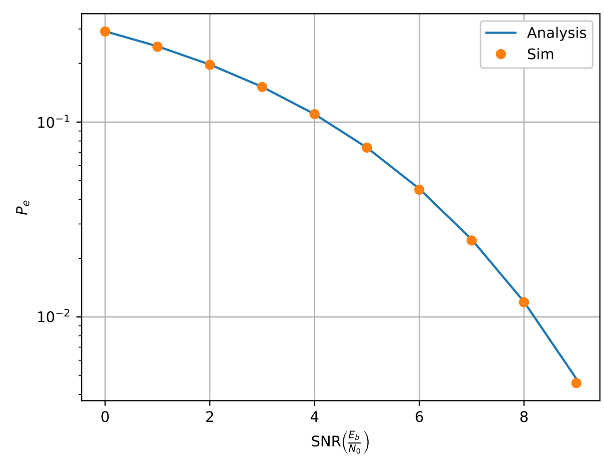

Fig. 4.10

4.11. Modify the above script to obtain the probability of symbol error.

5 M-PSK

5.1. Consider a system where $\mathbf{s}_i = \begin{pmatrix} \cos\left(\frac{2\pi i}{M}\right) \\ \sin\left(\frac{2\pi i}{M}\right) \end{pmatrix}, i = 0, 1, \dots M - 1$. Let

$$\mathbf{y}|s_0 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \sqrt{E_s} + n_1 \\ n_2 \end{pmatrix} \quad (5.1.1)$$

where $n_1, n_2 \sim \mathcal{N}\left(0, \frac{N_0}{2}\right)$.
Substituting

$$y_1 = R\cos\theta \tag{5.1.2}$$

$$y_2 = R\sin\theta \tag{5.1.3}$$

show that the joint pdf of R, θ is

$$p(R,\theta) = \frac{R}{\pi N_0}\exp\left(-\frac{R^2 - 2R\sqrt{E_s}\cos\theta + E_s}{N_0}\right) \tag{5.1.4}$$

5.2. Show that

$$\lim_{\alpha\to\infty}\int_0^\infty (V-\alpha)\,e^{-(V-\alpha)^2}\,dV = 0 \tag{5.2.1}$$

$$\lim_{\alpha\to\infty}\int_0^\infty e^{-(V-\alpha)^2}\,dV = \sqrt{\pi} \tag{5.2.2}$$

5.3. Using the above, show that

$$\int_0^\infty V\exp\left\{-\left(V^2 - 2V\sqrt{\gamma}\cos\theta + \gamma\right)\right\}dV$$
$$= e^{-\gamma\sin^2\theta}\sqrt{\gamma\pi}\cos\theta \tag{5.3.1}$$

for large values of γ.
Solution: The integrand in (5.3.1) can be expressed as

$$Ve^{-\left(V^2-2V\sqrt{\gamma}\cos\theta+\gamma\right)}$$
$$= \{(V - \sqrt{\gamma}\cos\theta) + (\sqrt{\gamma}\cos\theta)\}$$
$$\times e^{-\left(V-\sqrt{\gamma}\cos\theta\right)^2} e^{-\sqrt{\gamma}\sin^2\theta}$$
$$\implies \int_0^\infty Ve^{-\left(V^2-2V\sqrt{\gamma}\cos\theta+\gamma\right)}\,dV$$
$$= e^{-\sqrt{\gamma}\sin^2\theta}$$
$$\times\left\{\int_0^\infty (V - \sqrt{\gamma}\cos\theta)\,e^{-\left(V-\sqrt{\gamma}\cos\theta\right)^2}\,d\theta\right.$$
$$\left.+\int_0^\infty (\sqrt{\gamma}\cos\theta)\,e^{-\left(V-\sqrt{\gamma}\cos\theta\right)^2}\,d\theta\right\} \tag{5.3.2}$$

yielding (5.3.1) from (5.2.1) and (5.2.2)

5.4. Find a compact expression for

$$I = 1 - \sqrt{\frac{\gamma}{\pi}}\int_{-\frac{\pi}{M}}^{\frac{\pi}{M}} e^{-\gamma\sin^2\theta}\cos\theta\,d\theta \tag{5.4.1}$$

Solution: The above integral can be expressed as

$$I = 1 - 2\sqrt{\frac{\gamma}{\pi}}\int_0^{\frac{\pi}{M}} e^{-\gamma\sin^2\theta}\cos\theta\,d\theta \tag{5.4.2}$$

$$= 1 - 2\left\{Q(0) - Q\left(\sqrt{2\gamma}\sin\frac{\pi}{M}\right)\right\} \tag{5.4.3}$$

$$= 2Q\left(\sqrt{2\gamma}\sin\frac{\pi}{M}\right) \tag{5.4.4}$$

$\because Q(0) = \frac{1}{2}$.

5.5. Find the decision region for the symbol $\mathbf{s}_0$.

5.6. Show that

$$P_{e|\mathbf{s}_0} = 2Q\left(\sqrt{2\left(\frac{E_s}{N_o}\right)}\sin\frac{\pi}{M}\right) \tag{5.6.1}$$

5.7. Verify the SER through simulation.