

# Physical Layer Design for a Narrow Band Communication System

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**Abstract**—This a simple document explaining a question about the concept of similar triangles.

Download all python codes from

svn co <https://github.com/SiddharthPh/Summer2020/trunk/geometry/codes>

and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

## 1 SPECIFICATIONS

### 1.1 Modulation

1.1.1. See Fig.1.1.1.1 for the constellation diagram.  
The transmitted symbol set is given by

$$\mathbf{s}_m = \begin{pmatrix} \cos \frac{2m\pi}{8} \\ \sin \frac{2m\pi}{8} \end{pmatrix}, \quad m \in \{0, 1, \dots, 3\}. \quad (1.1.1.1)$$

The numerical values for  $\mathbf{s}_m$  are listed in Table 1.1.2

1.1.2. See Table 1.1.2 for the encoding scheme.

Symbol	Grey Code	Co-ordinates
$s_0$	00	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$s_1$	01	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$s_2$	11	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
$s_3$	10	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

TABLE 1.1.2

### 1.2 Demodulation

1.2.1. The received symbol is then obtained as

$$\mathbf{y} = \sqrt{E_s} \mathbf{s} + \mathbf{n} \quad (1.2.1.1)$$

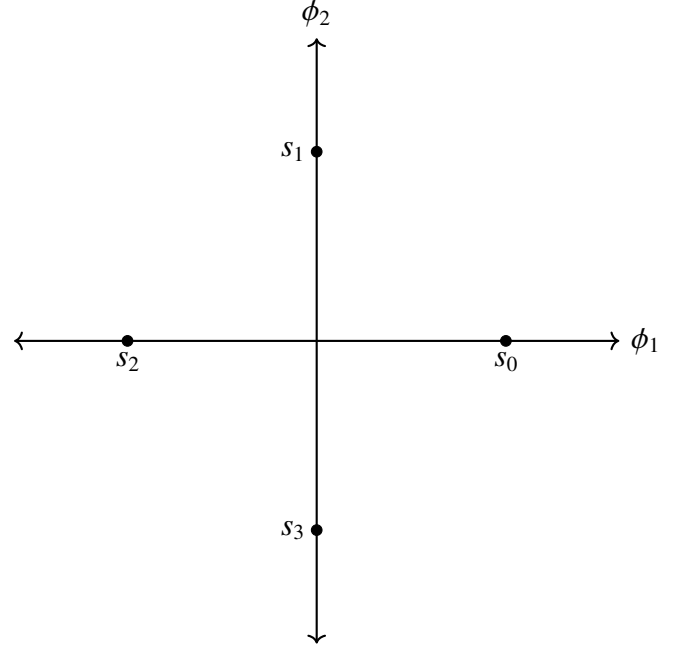


Fig. 1.1.1.1: constellation diagram

where  $E_s$  is the symbol energy and

$$\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}, \frac{N_0}{2} \mathbf{I}\right) \quad (1.2.1.2)$$

$$\mathbf{s} \in \{\mathbf{s}_m\}_{m=0}^3 \quad (1.2.1.3)$$

1.2.2. The decision rule is given by Fig.1.2.2.1 and can be expressed as

**Minimum distance Criterion:**

$$\hat{s} = \min \|\mathbf{y} - \mathbf{s}\| \quad (1.2.2.1)$$

From eq.1.2.2.1,  $s_0$  is chosen if

$$\|\mathbf{y} - \mathbf{s}_0\|^2 < \|\mathbf{y} - \mathbf{s}_1\|^2 \quad (1.2.2.2)$$

$$\|\mathbf{y} - \mathbf{s}_0\|^2 < \|\mathbf{y} - \mathbf{s}_2\|^2 \quad (1.2.2.3)$$

$$\|\mathbf{y} - \mathbf{s}_0\|^2 < \|\mathbf{y} - \mathbf{s}_3\|^2 \quad (1.2.2.4)$$

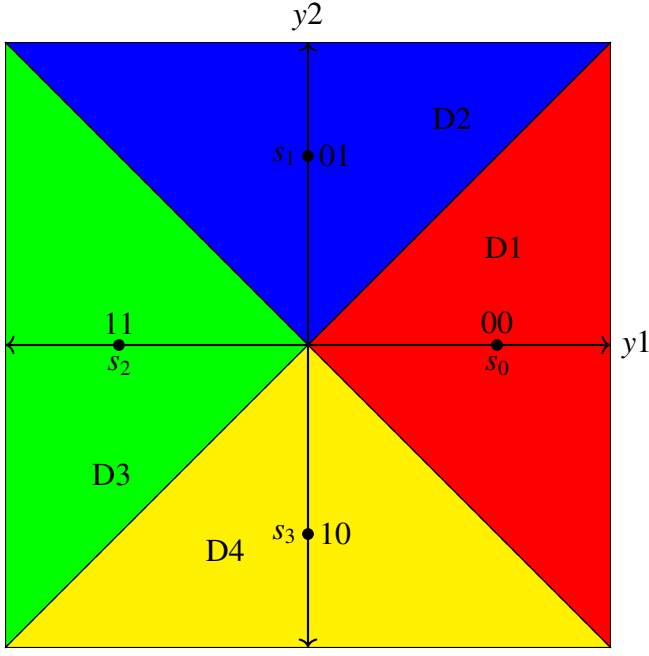


Fig. 1.2.2.1: decision regions

The above conditions can be simplified to obtain the region

$$(s_0 - s_1)^T \mathbf{y} > 0 \quad (1.2.2.5)$$

$$(s_0 - s_2)^T \mathbf{y} > 0 \quad (1.2.2.6)$$

$$(s_0 - s_3)^T \mathbf{y} > 0 \quad (1.2.2.7)$$

Substituting the values of  $s_0, s_1, \dots, s_7$  in the above

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}^T \mathbf{y} > 0 \quad (1.2.2.8)$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \mathbf{y} > 0 \quad (1.2.2.9)$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}^T \mathbf{y} > 0 \quad (1.2.2.10)$$

yielding  $|y_2| < y_1$  i.e D1 region (red) is detected at the receiver.

Similarly, from eq.1.2.2.1

For detecting  $s_1$ ,  $y_1 > -y_2$  and  $y_1 < y_2$  i.e D2 region (blue) is detected at the receiver.

For detecting  $s_2$ ,  $y_1 < -y_2$  and  $y_1 < y_2$  i.e D3 region (green) is detected at the receiver.

For detecting  $s_3$ ,  $y_1 < -y_2$  and  $y_1 > y_2$  i.e D4 region (yellow) is detected at the receiver.

Symbol	Decision region	Decision Rule
$s_0$	D1	$y_1 > y_2, y_1 > -y_2$
$s_1$	D2	$y_1 < y_2, y_1 > -y_2$
$s_2$	D3	$y_1 < y_2, y_1 < -y_2$
$s_3$	D4	$y_1 > y_2, y_1 < -y_2$

TABLE 1.2.2

1.2.3. The following code has simulation of QPSK.

```
codes/qpsk.py
```

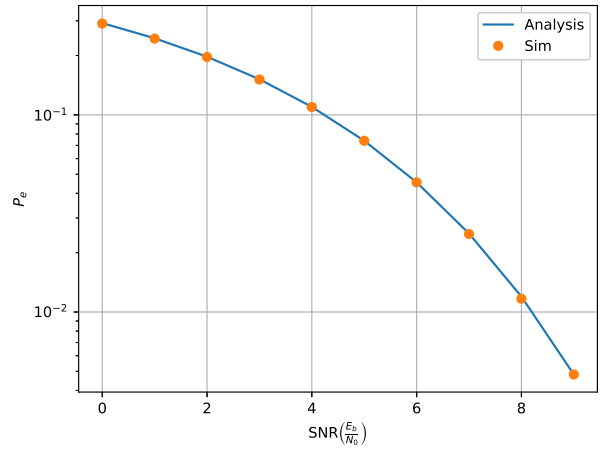


Fig. 1.2.3.1: Result from Simulation