BCH Codes

G V V Sharma*

Contents

***Abstract*—This manual provides an introduction to BCH codes.**

1 Introduction

1.1 For a BCH code, the minimal polynomials are given by

$$g_1(x) = 1 + x + x^3 + x^5 + x^{14} \tag{1.1}$$
$$g_2(x) = 1 + x^6 + x^8 + x^{11} + x^{14} \tag{1.2}$$
$$g_3(x) = 1 + x + x^2 + x^6 + x^9 + x^{10} + x^{14} \tag{1.3}$$
$$g_4(x) = 1 + x^4 + x^7 + x^8 + x^10 + x^{12} + x^{14} \tag{1.4}$$
$$g_5(x) = 1 + x^2 + x^4 + x^6 + x^8 + x^9 + x^{11} + x^{13} + x^{14} \tag{1.5}$$
$$g_6(x) = 1 + x^3 + x^7 + x^8 + x^9 + x^{13} + x^{14} \tag{1.6}$$
$$g_7(x) = 1 + x^2 + x^5 + x^6 + x^7 + x^{10} + x^{11} + x^{13} + x^{14} \tag{1.7}$$
$$g_8(x) = 1 + x^5 + x^8 + x^9 + x^{10} + x^{11} + x^{14} \tag{1.8}$$
$$g_9(x) = 1 + x + x^2 + x^3 + x^9 + x^{10} + x^{14} \tag{1.9}$$
$$g_{10}(x) = 1 + x^3 + x^6 + x^9 + x^{11} + x^{12} + x^{14} \tag{1.10}$$
$$g_{11}(x) = 1 + x^4 + x^{11} + x^{12} + x^{14} \tag{1.11}$$
$$g_{12}(x) = 1 + x + x^2 + x^3 + x^5 + x^6 + x^7 + x^8 + x^{10} + x^{13} + x^{14} \tag{1.12}$$

Obtain the minimal polynomial matrix.

1.2 Obtain the generator polynomial. **Solution:** The generator polynomial is obtained as

$$g(x) = \prod_{i=1}^{m} g_i(x) \tag{1.13}$$

The following code computes g.

1.3 where $\mathbf{w}$ is $p \times 1$ and $\mathbf{X}$ is $N \times p$. Show that

$$E\left(\hat{\mathbf{w}}\right) = \mathbf{w} \tag{1.14}$$

1.4 If the covariance matrix of $\mathbf{y}$ is

$$\mathbf{C_y} = \sigma^2 \mathbf{I} \tag{1.15}$$

show that

$$\mathbf{C_w} = \sigma^2 \left(\mathbf{X}^T\mathbf{X}\right)^{-1} \tag{1.16}$$

1.5 Let

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\mathbf{w}} \tag{1.17}$$
$$\hat{\sigma}^2 = \frac{1}{N-p}\|\mathbf{y} - \hat{\mathbf{y}}\|^2 \tag{1.18}$$
$$\mathbf{y} - \hat{\mathbf{y}} \sim \mathcal{N}\left(\mathbf{0}, \sigma^2\mathbf{I}\right) \tag{1.19}$$

Show that

$$(N-p)\,\hat{\sigma}^2 \sim \sigma^2\chi^2_{N-p} \tag{1.20}$$

1.6 Let

$$\hat{z} = \frac{w_j}{\hat{\sigma}\sqrt{v_j}} \tag{1.21}$$

where v_j is the diagonal element of $\left(\mathbf{X}^T\mathbf{X}\right)^{-1}$. If $w_j = 0$, show that z_j has a t_{N-p} distribution.

1.7 Plot $\Pr\left(|Z| > z\right)$ for t_{30}, t_{100} and the standard normal distribution.

2 Applications

2.1 Explain how (**??**) can be used to obtain the Nearest Neighbour approximation.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

2.2 Repeat the exercise for the least squares method.