

BCH Codes



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Abstract—This manual provides an introduction to BCH codes.

1 Introduction

1.1 For a BCH code, the minimal polynomials are given by

$$g_1(x) = 1 + x + x^3 + x^5 + x^{14}$$
 (1.1)

$$g_2(x) = 1 + x^6 + x^8 + x^{11} + x^{14}$$
 (1.2)

$$g_3(x) = 1 + x + x^2 + x^6 + x^9 + x^{10} + x^{14}$$
 (1.3)

$$g_4(x) = 1 + x^4 + x^7 + x^8 + x^10 + x^{12} + x^{14}$$
 (1.4)

$$g_5(x) = 1 + x^2 + x^4 + x^6 + x^8 + x^9 + x^{11} + x^{13} + x^{14}$$
 (1.5)

$$g_6(x) = 1 + x^3 + x^7 + x^8 + x^9 + x^{13} + x^{14}$$
(1.6)

$$g_7(x) = 1 + x^2 + x^5 + x^6 + x^7 + x^{10} + x^{11} + x^{13} + x^{14}$$

$$(1.7)$$

$$g_8(x) = 1 + x^5 + x^8 + x^9 + x^{10} + x^{11} + x^{14}$$
 (1.8)

$$g_9(x) = 1 + x + x^2 + x^3 + x^9 + x^{10} + x^{14}$$
 (1.9)

$$g_{10}(x) = 1 + x^3 + x^6 + x^9 + x^{11} + x^{12} + x^{14}$$
(1.10)

$$g_{11}(x) = 1 + x^4 + x^{11} + x^{12} + x^{14}$$
 (1.11)

$$g_{12}(x) = 1 + x + x^2 + x^3 + x^5 + x^6 + x^7 + x^8 + x^{10} + x^{13} + x^{14}$$
 (1.12)

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Obtain the minimal polynomial matrix. **Solution:**

wget https://github.com/gadepall/EE6317/blob/master/BCH/codes/min_poly_mat.py

1.2 Obtain the generator polynomial. **Solution:** The generator polynomial is obtained as

$$g(x) = \prod_{i=1}^{m} g_i(x)$$
 (1.13)

The following code computes g.

1.3 where w is $p \times 1$ and X is $N \times p$. Show that

$$E\left(\hat{\mathbf{w}}\right) = \mathbf{w} \tag{1.14}$$

1.4 If the covariance matrix of y is

$$\mathbf{C_y} = \sigma^2 \mathbf{I} \tag{1.15}$$

show that

$$\mathbf{C}_{\mathbf{w}} = \sigma^2 \left(\mathbf{X}^T \mathbf{X} \right)^{-1} \tag{1.16}$$

1.5 Let

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\mathbf{w}} \tag{1.17}$$

$$\hat{\sigma}^2 = \frac{1}{N - n} ||\mathbf{y} - \hat{\mathbf{y}}||^2 \tag{1.18}$$

$$\mathbf{y} - \hat{\mathbf{y}} \sim \mathcal{N}\left(\mathbf{0}, \sigma^2 \mathbf{I}\right)$$
 (1.19)

Show that

$$(N-p)\,\hat{\sigma}^2 \sim \sigma^2 \chi_{N-n}^2$$
 (1.20)

1.6 Let

$$\hat{z} = \frac{w_j}{\hat{\sigma} \sqrt{v_j}} \tag{1.21}$$

where v_j is the diagonal element of $(\mathbf{X}^T\mathbf{X})^{-1}$. If $w_j = 0$, show that z_j has a t_{N-p} distribution.

1.7 Plot Pr (|Z| > z) for t_{30} , t_{100} and the standard normal distribution.

2 Applications

- 2.1 Explain how (??) can be used to obtain the Nearest Neighbour approximation.

 2.2 Repeat the exercise for the least squares
- method.