IIT HYDERABAD
DEPARTMENT OF MATHEMATICS
MA 101 - CALCULUS I

9, September 2009 90 mins **25** marks

(1) Give examples of sequences / series with the following properties:
   (a) A sequence (x_n) such that $(|x_n|)$ converges but the original sequence (x_n) does not.
   (b) Two divergent sequences $(x_n), (y_n)$ such that the sequence $(x_n \cdot y_n)$ converges.
   (c) An unbounded sequence that has a convergent subsequence.
   (d) Two divergent series $\sum x_n, \sum y_n$ such that the series $\sum x_n \cdot y_n$ converges.
   (e) Two convergent series $\sum x_n, \sum y_n$ such that the series $\sum x_n \cdot y_n$ diverges.
   (f) A convergent series $\sum x_n$ such that the series $\sum x_n^2$ diverges.
   (g) A convergent series $\sum x_n$ such that the series $\sum \sqrt{x_n}$ diverges.

(**7** marks)

(2) Show that the sequence $x_n = \dfrac{4 - 7n^6}{n^6 + 3}$ converges using the (ϵ, K)-definition.

(**3** marks)

(3) Comment on the convergence of the sequence $x_n = \sin n$.

(**4** marks)

(4) Does the recursively defined sequence $s_1 = 1;\ s_{n+1} = \dfrac{s_n + 1}{5}$ converge ? If so, find its limit.

(**3** marks)

(5) Comment on the convergence of the following series:

$(i)\ \displaystyle\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{(2n)!}$ $(ii)\ \displaystyle\sum_{n=1}^{\infty} \frac{e^{n\pi}}{\pi^{ne}}$ $(iii)\ \displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)^n$

(**6** marks)

(6) Is the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 \ln n}$ convergent ? Is it also absolutely convergent ?

(**2** marks)

ALL THE BEST