MA 101 - CALCULUS I
PROBLEM SHEET : SEQUENCES

(1) Show that the following sequences converge by $\epsilon - K$ definition:

$$\text{(i)} \lim_{n\to\infty} \frac{2n}{n+4\sqrt{n}} = 2 \qquad \text{(ii)} \lim_{n\to\infty} \frac{10^7}{n} = 0$$
$$\text{(iii)} \lim_{n\to\infty} \frac{n^2-1}{2n^2+3} = \frac{1}{2} \qquad \text{(iv)} \lim_{n\to\infty} \frac{3n+1}{2n+3} = \frac{3}{2}$$

(2) Show that the sequence $x_n = \dfrac{1}{\ln(n+1)}$ converges to 0 using the (ϵ, K)-definition. Also find the constant $K(\epsilon)$ when $\epsilon = \frac{1}{2}$ and $\epsilon = \frac{1}{10}$.

(3) Discuss the convergence / divergence of the following sequences ($0 < a < 1$ and $b > 1$).

$$\text{(i) } x_n = \frac{n^2}{n+5} \qquad \text{(ii)} x_n = \frac{n}{10^7} \qquad \text{(iii) } x_n = \sqrt{n+1} - \sqrt{n}$$

$$\text{(iv) } x_n = \frac{(-1)^n}{n+1} \qquad \text{(v) } x_n = \frac{1-2n}{1+2n} \qquad \text{(vi)} x_n = \frac{1-5n^4}{n^4+8n^3}$$

$$\text{(vii) } x_n = \frac{\cos n}{n} \qquad \text{(viii) } x_n = \frac{1}{3^n} \qquad \text{(ix) } x_n = \frac{n^2}{e^n}$$

$$\text{(x) } x_n = a^n \qquad \text{(xi) } x_n = \frac{n!}{n^n} \qquad \text{(xii) } x_n = \frac{2^{3n}}{3^{2n}}$$

$$\text{(xiii) } x_n = \frac{n}{b^n} \qquad \text{(xiv) } x_n = \frac{b^n}{n^2} \qquad \text{(xv) } x_n = \frac{5^n}{n!}$$

(4) Show that each sequence is monotone and bounded. Then find the limit.

$$\text{(i) } x_1 = 1;\ x_{n+1} = \frac{x_n+1}{3} \qquad \text{(ii) } x_1 = 2;\ x_{n+1} = \sqrt{2x_n+1}$$

(5) Discuss whether the following sequences are Cauchy or not:

$$\text{(i) } x_n = \frac{1}{n^2} \qquad \text{(ii) } x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$$

$$\text{(iii) } x_n = \ln n^2 \qquad \text{(iv) } x_n = \sqrt{n}$$