Problem Set: Differentiation

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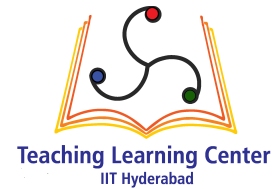IIT Hyderabad

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1) Find the derivatives of the following functions from the definition.

(i) $\frac{3+x}{3-x}, x \neq 3$

(v) $\frac{1}{\sqrt{10+x}}$

(ix) $x^2 \sin\left(\frac{1}{x}\right), x \neq 0; f(0) = 0$

(ii) $\sqrt{2x-1}$

(vi) $\sin \frac{1}{(x-1)^2}$

(x) $\sqrt{x + \sqrt{x}}$

(iii) $\ln(1 + \sin x)$

(vii) $\sin\left(\frac{1}{\cos x}\right)$

(xi) $\cos(\sqrt{1 + x^2})$

(iv) $\frac{1+\cos x}{3+\sin x}$

(viii) $\sqrt{(x-3)(6-x)}$, $3 \leqslant x \leqslant 6$

(xii) $\frac{\sqrt{1+|\sin x|}}{x}$

2) Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$

(a). Is f differentiable at $x = 0$?

(b). Is f' is continuous at $x = 0$?

3) Use L'Hospital's rule to evaluate the following limits.

(i) $\lim\limits_{x \to 0} \frac{e^{2x}-1}{x}$

(iv) $\lim\limits_{x \to \infty} x^2 e^{-x}$

(vii) $\lim\limits_{x \to \infty} \frac{5x^2-3x}{7x^2+1}$

(ii) $\lim\limits_{x \to 0} \frac{1+\cos \pi x}{x^2-2x+1}$

(v) $\lim\limits_{x \to 0} (\cos x)^{\frac{1}{x^2}}$

(viii) $\lim\limits_{x \to \infty} \left(x - \sqrt{x + x^2}\right)$

(iii) $\lim\limits_{x \to \infty} \frac{3x^2-x+5}{5x^2+6x-3}$

(vi) $\lim\limits_{x \to 1} \frac{(2x-x^4)^{\frac{1}{2}}-x^{\frac{1}{3}}}{1-x^{\frac{3}{4}}}$

(ix) $\lim\limits_{x \to \infty} \frac{\sqrt{x+2}}{\sqrt{x+1}}$

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