MA 101 - Calculus I
Problem Sheet : Series

August 24, 2009

1. Discuss the convergence / divergence of the following series:

(i) $u_n = \left(\frac{1}{\sqrt{2}}\right)^n$ (ii)$u_n = (-1)^{n+1}\frac{3}{2^n}$ (iii) $u_n = \sqrt{n+1} - \sqrt{n}$

(iv) $u_n = e^{-2n}$ (v) $u_n = \frac{2}{10^n}$ (vi)$u_n = \frac{2^n - 1}{3^n}$

(vii) $u_n = \frac{\cos n\pi}{5^n}$ (viii) $u_n = \frac{n!}{1000^n}$ (ix) $u_n = \ln\frac{n}{n+1}$

(x) $u_n = \left(\frac{e}{\pi}\right)^n$ (xi) $u_n = \frac{n!}{(2^n)^3}$ (xii) $u_n = \frac{n^3}{2^n}$

(xiii) $u_n = \frac{1}{n\sqrt{n+1}}$ (xiv) $u_n = \frac{\sin^2 n}{n^2}$ (xv) $u_n = \frac{e^{n\pi}}{\pi^{ne}}$

2. Show that the following series diverge:

(i) $(-1)^n$ (ii) $\frac{n}{n+1}$ (iii) $\frac{n}{\sqrt{n^2+1}}$ (iv) $\cos\frac{n\pi}{2}$

3. Determine the conditional / absolute convergence of the following series:

(i) $u_n = \frac{(-1)^n}{\ln n^2}$ (ii) $u_n = \frac{(-2)^n}{n!}$ (iii) $u_n = \frac{(-1)^n}{n\ln n^2}$

(iv) $u_n = \frac{1}{\sqrt{n}} - \frac{1}{n}$ (v) $u_n = \frac{(-3)^n}{n!}$ (iv) $u_n = \frac{\cos n\pi}{\sqrt{n}}$

4. Let $p \geq 0$ and consider the series $\sum \frac{(-1)^{n-1}}{n^p}$. Determine for what values of p is the series conditionally / absolutely convergent.