Optimization

simplex method

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Simplex problem

Q)Solve

$$z_{max} = \begin{pmatrix} 4 & 1 \end{pmatrix} X$$

such that

$$\begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix} * X \leqslant \begin{pmatrix} 50 \\ 90 \end{pmatrix}$$

$$X > 0$$

$Z_{max} = 4x_1 + x_2$
s.t.

$$x_1 + x_2 + s_1 = 50$$

$$3x_1 + x_2 + s_2 = 90$$

$$x_1, x_2, s_1, s_2 > 0$$

- Iteration_1 :

$\Rightarrow$ step 0: Starting basic feasible solution at O(0,0)

$$\Omega = (S_1, S_2)\ ,\ c_N = (4, 1)\ , c_B = (0, 0)$$

$$x_B = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}\ ,\ x_N = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\ , \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\ ,$$

$$N = \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}\ ,\ b = \begin{pmatrix} 50 \\ 90 \end{pmatrix}$$

$\Rightarrow$ Step-1 :

$$B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\Rightarrow$ Step-2 :

$$z_{x_1} - c_{x_1} = c_B B^{-1} N_{x_1} - c_{x_1} = (0,0) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} - 4 = -4$$

$$z_{x_2} - c_{x_2} = c_B B^{-1} N_{x_2} - c_{x_2} = (0,0) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} - 1 = -1$$

The entering variable is x_1.

$\Rightarrow$ Step -3 :

$$B^{-1}N_{x_1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$B^{-1}b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 50 \\ 90 \end{pmatrix} = \begin{pmatrix} 50 \\ 90 \end{pmatrix}$$

Then , $\frac{(B^{-1}b)_{s_1}}{(B^{-1}N_{x_1})_{s_1}} = \frac{50}{1} = 50$

Then , $\frac{(B^{-1}b)_{s_2}}{(B^{-1}N_{x_1})_{s_2}} = \frac{90}{3} = 30$

The leaving variable is S_2

$\Rightarrow$ Step - 4 :

The new basis is $\Omega = \{S_1.S_2\} \cup \{x_1\} - \{S_2\} = \{S_1, x_1\}$

- Iteration$_2$:

$\Rightarrow$ step 0: Starting basic feasible solution at O(0,0)

$$\Omega = (S_1, x_1) \ , \ c_N = (1, 0) \ , c_B = (0, 4)$$

$$x_B = \begin{pmatrix} S_1 \\ x_1 \end{pmatrix} , \ x_N = \begin{pmatrix} x_2 \\ S_2 \end{pmatrix} , \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix} ,$$

$$N = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix},\ b = \begin{pmatrix} 50 \\ 90 \end{pmatrix}$$

$\Rightarrow$ Step-1 :

$$B^{-1} = \begin{pmatrix} 1 & -1/3 \\ 0 & 1/3 \end{pmatrix}$$

$\Rightarrow$ Step-2 :

$$z_{x_2} - c_{x_2} = c_B B^{-1} N_{x_2} - c_{x_2} = (0,4) \begin{pmatrix} 1 & -1/3 \\ 0 & 1/3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} - 1 = 1/3$$

$$z_{S_2} - c_{S_2} = c_B B^{-1} N_{S_2} - c_{S_2} = (0,4) \begin{pmatrix} 1 & -1/3 \\ 0 & 1/3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} - 0 = 0$$

As $z_j - c_j > 0$, for all j nb.The optimal solution is reached.
The Optimal solution is

$$x_B* = \begin{pmatrix} S_1* \\ x_2* \end{pmatrix} = B^{-1}b = \begin{pmatrix} 1 & -1/3 \\ 0 & 1/3 \end{pmatrix} \begin{pmatrix} 50 \\ 90 \end{pmatrix} = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$$

$$Z* = c_B x_B* = \begin{pmatrix} 0 & 4 \end{pmatrix} \begin{pmatrix} 20 \\ 30 \end{pmatrix} = 120$$

The optimal solution is $\mathrm{x}_1* = 30, x_2* = 0, z_{max} = 120$