Least Mean Square Algorithm

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Contents

***Abstract*—This manual provides an introduction to the LMS algorithm.**

1 Source Files

1) Get the git source and enter the local directory

```
git clone https://github.com/
    gadepall/adsp.git
cd adsp/audio_source
```

2) Play the **signal_noise.wav** and **noise.wav** file.

2 Problem Formulation

The **signal_noise.wav** $d(n)$ contains a human voice along with an instrument sound in the background. This sound is captured in **noise.wav** $X(n)$. The goal is to suppress $X(n)$ in d_n. Let

$$d(n) = e(n) + y(n) \tag{2.1}$$

where $e(n)$ is the desired signal. We want an estimate of $I(n)$ from $X(n)$. This can be done by considering

$$y(n) = W^T(n)X(n) \tag{2.2}$$

where

$$X(n) = \begin{bmatrix} X(n) \\ X(n-1) \\ X(n-2) \\ .. \\ .. \\ X(n-M+1) \end{bmatrix}_{MX1} \tag{2.3}$$

$$W(n) = \begin{bmatrix} w_1(n) \\ w_2(n) \\ w_3(n) \\ .. \\ .. \\ w_{n-M+1}(n) \end{bmatrix}_{MX1} \tag{2.4}$$

and estimating $W(n)$. The human voice can be characterized as

$$e(n) = d(n) - W^T(n)X(n) \tag{2.5}$$

The goal is to find $W(n)$ that will allow $W^T(n)X(n)$ to mimic the instrument sound in $d(n)$. This is possible if $e(n)$ is minimum. This problem can be expressed as

$$\min_{W(n)} e^2(n) \tag{2.6}$$

3 Gradient Descent Method

Consider the problem of finding the square root of a number c. This can be expressed as the equation

$$x^2 - c = 0 \tag{3.1}$$

Problem 3.1. Show that (3.1) results from

$$\min_x f(x) = x^3 - 3xc \tag{3.2}$$

Problem 3.2. Find a numerical solution for (3.1).

Solution: A numerical solution for (3.1) is obtained

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as

$$x_{n+1} = x_n - \mu f'(x) \tag{3.3}$$
$$= x_n - \mu\left(3x_n^2 - 3c\right) \tag{3.4}$$

where x_0 is an inital guess.

Problem 3.3. Write a program to implement (3.4).

Solution: Execute **square_root.py** in the **lms** directory.

4 LMS Algorithm

Problem 4.1. Show using (2.5) that

$$\nabla_{W(n)} e^2(n) = \frac{\partial e^2(n)}{\partial W(n)} \tag{4.1}$$
$$= -2X(n)d(n) + 2X(n)X^T(n)W(n) \tag{4.2}$$

Problem 4.2. Use the gradient descent method to obtain an algorithm for solving

$$\min_{W(n)} e^2(n) \tag{4.3}$$

Solution: The desired algorithm can be expressed as

$$W(n+1) = W(n) - \bar{\mu}[\nabla_{W(n)} e^2(n)] \tag{4.4}$$
$$W(n+1) = W(n) + \mu X(n)e(n) \tag{4.5}$$

where $\mu = \bar{\mu}$.

Problem 4.3. Write a program to suppress $X(n)$ in $d(n)$.

Solution: Execute **LMS_NC_SPEECH.py** in the **lms** directory.