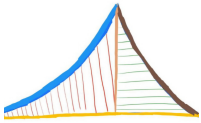
Algebra



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About this Book

This book introduces quadratic equations, complex numbers and other concepts in algebra. All problems in the book are from NCERT mathematics textbooks from Class 9-12. Exercises are from CBSE and JEE exam papers.

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Github: https://github.com/gadepall/algebra

Contents

1 Identities

1.1 NCERT

Verify

1.1.1. $x^3 + y^3 = (x + y)\left(x^2 - xy + y^2\right)$

1.1.2. $x^3 - y^3 = (x - y)\left(x^2 + xy + y^2\right)$

1.1.3. $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]$

Factorize each of the following

1.1.1. $8a^3 + b^3 + 12a^2b + 6ab^2$

1.1.2. $8a^3 - b^3 - 12a^2b + 6ab^2$

1.1.3. $27 - 125a^3 - 135a + 225a^2$

1.1.4. $64a^3 - 27b^3 - 144a^2b + 108ab^2$

1.1.5. $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{p}{4}$

1.1.6. $27y^3 + 125z^3$

1.1.7. $64m^3 - 343n^3$

1.1.8. $27x^3 + y^3 + z^3 - 9xyz$

Find the value of each of the following

1.1.9. $(-12)^3 + (7)^3 + (5)^3$

1.1.10. $(28)^3 + (-15)^3 + (-13)^3$

Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given

1.1.11. $25a^2 - 35a + 12$

1.1.12. $35a^2 + 13y - 12$

What are the possible expressions for the dimensions of the cuboids whose volumes are given below

1.1.13. $3x^2 - 12x$

1.1.14. $12ky^2 + 8ky - 20k$

2 Polynomials

2.1 NCERT

2.1.15 Divide $p(x)$ by $g(x)$, where $p(x) = x + 3x^2$–1 and $g(x) = 1 + x$.

2.1.16 Divide the polynomial $p(x) = 3x^4 - 4x^3 - 3x - 1$ by $x - 1$.

2.1.17 Find the remainder obtained upon dividing $p(x) = x^3 + 1$ by $x + 1$.

2.1.18 Find the remainder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$.

2.1.19 Check whether the polynomial $q(t) = 4t^3 + 4t^2 - t - 1$ is a multiple of $2t + 1$.

2.1.20 Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

a) $x + 1$
b) $x - \frac{1}{2}$
c) x
d) $x + \pi$
e) $5 + 2x$

2.1.21 Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

2.1.22 Find the value of k, if x–1 is a factor of $p(x) = 4x^3 + 3x^2 - 4x + k$.

2.1.23 Determine which of the following polynomials has $x + 1$ as a factor

a) $x^3 + x^2 + x + 1$
b) $x^4 + x^3 + x^2 + x + 1$
c) $x^4 + 3x^3 + 3x^2 + x + 1$
d) $x^3 - x^2 - \left(2 + \sqrt{2}\right)x + \sqrt{2}$

2.1.24 Factorize $x^3 - 23x^2 + 142x - 120$.

2.1.25 Divide $2x^2 + 3x + 1$ by $x + 2$.

2.1.26 Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.

2.1.27 Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

2.1.28 Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

2.1.29 Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

a) $p(x) = x^2 + x + k$
b) $p(x) = kx^2 - \sqrt{2}x + 1$
c) $p(x) = 2x^2 + kx + \sqrt{2}$
d) $p(x) = kx^2 - 3x + k$

2.1.30 Divide the polnyomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

a) $p(x) = x^3 - 3x^2 + 5x - 3, g(x) = x^2 - 2$.
b) $p(x) = x^4 - 3x^2 + 4x + 5, g(x) = x^2 + 1 - x$.
c) $p(x) = x^4 - 5x + 6, g(x) = 2 - x^2$.

2.1.31 Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

a) $t^2 - x, 2t^4 + 3t^3 - 2t^2 - 9t - 12$.
b) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$.
c) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$.

2.1.32 Obtain all the other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$and $-\sqrt{\frac{5}{3}}$.

2.1.33 On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$respectively. Find $g(x)$.

2.1.34 Verify that the numbers given alongside the cubic polynomials below are their zeroes. Also verify if the relationship between the zeroes and the coefficients in each case:

a) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$
b) $x^3 - 4x^2 + 5x - 2; 2, 1, 1$

2.1.35 Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -4 respectively.

2.1.36 If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find the other zeroes.

2.1.37 If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

2.1.38 Use the factor theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases.

a) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$
b) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$
c) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

2.1.39 Factorise

a) $12x^2 - 7x + 1$
b) $2x^2 + 7x + 3$
c) $6x^2 + 5x - 6$
d) $3x^2 - x - 4$
e) $x^3 - 2x^2 - x + 2$
f) $x^3 - 3x^2 - 9x - 5$
g) $x^3 + 13x^2 + 32x + 20$
h) $2y^3 + y^2 - 2y + 1$

2.1.40 Factorise $y^2 - 5y + 6$ using the factor theorem.

3 Roots

3.1 NCERT

Find the roots of the following equations graphically or otherwise.

3.1.1 $x^2 - 2x = 0$
3.1.2 $x^3 - x^2 + 2 = 0$
3.1.3 $x^5 - x^4 + 3 = 0$
3.1.4 $2 - y^2 - y^3 + 2y^8 = 0$
3.1.5 $x - x^3 = 1$
3.1.6 $5x^3 + 4x^2 + 7x = 0$
3.1.7 $4 - y^2 = 0$
3.1.8 $x^2 + x + 2 = 0$
3.1.9 $4x^2 - 3x + 7 = 0$
3.1.10 $y^2 + \sqrt{2} = 0$
3.1.11 $3\sqrt{t} + t\sqrt{2} = 1$
3.1.12 $y + \frac{2}{y} = 1$
3.1.13 $x^2 + x = 1$
3.1.14 $y + y^2 + 4 = 0$
3.1.15 $y(x) = 5x^2 - 3x + 7 = 0$. Find $y(1)$.
3.1.16 $3y^3 - 4y + \sqrt{11} = 0$. Find $y(2)$.
3.1.17 $4t^4 + 5t^3 - t^2 + 6 = 0$
3.1.18 $5x^3 - 2x^2 + 3x - 2 = 0$. Find $y(1), y(0)$ and $y(-1)$.
3.1.19 $5x - 4x^2 + 3 = 0$. Find $y(2), y(0)$ and $y(-1)$.
3.1.20 $(x - 2)^2 + 1 = 2x - 3$
3.1.21 $x(2x + 3) = x^2 + 1$
3.1.22 $(x + 2)^3 = x^3 - 4$
3.1.23 $(x + 1)^2 = 2(x - 3)$
3.1.24 $x^2 - 2x = -2(3 - x)$
3.1.25 $(2x - 1)(x - 3) = (x + 5)(x - 1)$
3.1.26 $(x + 2)^3 = 2x\left(x^2 - 1\right)$
3.1.27 $x^3 - 4x^2 - x + 1 = (x - 2)^3$
3.1.28 $2x^2 - 5x + 3 = 0$
3.1.29 $6x^2 - x - 2 = 0$
3.1.30 $3x^2 - 2\sqrt{6}x + 2 = 0$
3.1.31 $x^2 - 3x - 10 = 0$
3.1.32 $2x^2 + x - 6 = 0$
3.1.33 $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
3.1.34 $2x^2 - x + \frac{1}{8} = 0$
3.1.35 $100x^2 - 20x + 1 = 0$
3.1.36 $5x^2 - 6x - 2 = 0$
3.1.37 $4x^2 + 3x + 5 = 0$
3.1.38 $3x^2 - 5x + 2 = 0$
3.1.39 $x^2 + 4x + 5 = 0$
3.1.40 $2x^2 - 2\sqrt{2}x + 1 = 0$
3.1.41 $x + \frac{1}{x} = 3, x \neq 0$
3.1.42 $\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$
3.1.43 $3x^2 - 2x + \frac{1}{3} = 0$.
3.1.44 $x^2 - 4x + 3 = 0$.
3.1.45 $2x^2 - 4x + 3 = 0$.
3.1.46 $x - \frac{1}{x} = 3, x \neq 0$
3.1.47 $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$
3.1.48 $2x^2 - 3x + 5 = 0$
3.1.49 $3x^2 - 4\sqrt{3}x + 4 = 0$
3.1.50 $2x^2 - 6x + 3 = 0$

Find $p(0), p(1)$ and $p(2)$ for each of the following polynomials.

3.1.51 $p(y) = y^2 - y + 1$
3.1.52 $p(t) = 2 + t + 2t^2 - t^3$
3.1.53 $p(x) = x^3$
3.1.54 $p(x) = (y - 1)(y + 1)$

Find the values of k for each of the following quadratic equations, so that they have two equal roots

3.1.55 $2x^2 + kx + 3 = 0$
3.1.56 $kx(x - 2) + 6 = 0$

Verify whether the following are zeroes of the polynomial, indicated against them.

3.1.57 $p(x) = x^2 - 1, \quad x = -1, 1.$
3.1.58 $p(x) = (x - 2)(x + 1), \quad x = -1, 2.$
3.1.59 $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}.$

Solve

3.1.60 $x^2 + 2 = 0$
3.1.61 $x^2 + x + 1 = 0$
3.1.62 $\sqrt{5}x^2 + x + 5 = 0$
3.1.63 $x^2 + 3 = 0$
3.1.64 $2x^2 + x + 1 = 0$
3.1.65 $x^2 + 3x + 9 = 0$
3.1.66 $-x^2 + x - 2 = 0$
3.1.67 $x^2 + 3x + 5 = 0$
3.1.68 $x^2 - x + 2 = 0$
3.1.69 $\sqrt{2}x^2 + x + \sqrt{2} = 0$
3.1.70 $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$
3.1.71 $x^2 + x + \frac{1}{\sqrt{2}} = 0$
3.1.72 $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$
3.1.73 $3x^2 - 4x + \frac{20}{3} = 0$
3.1.74 $x^2 - 2x + \frac{3}{2} = 0$
3.1.75 $27x^2 - 10x + 1 = 0$
3.1.76 $21x^2 - 28x + 10 = 0$

4 Quadratic Equations

4.1 NCERT

4.1.1 Janak and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

4.1.2 A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹750. We would like to find out the number of toys produced on that day.

4.1.3 The product of Sunita's age (in years) two years ago and her age four years from now is one more than twice her present age. What is her present age?

4.1.4 Find two consecutive odd positive integers, sum of whose squares is 290.

4.1.5 A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

4.1.6 The product of two consecutive positive integers is 306. We need to find the integers.

4.1.7 Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

4.1.8 A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

4.1.9 Find two numbers whose sum is 27 and product is 182.

4.1.10 Find two consecutive positive integers, sum of whose squares is 365.

4.1.11 A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹90, find the number of articles produced and the cost of each article.

4.1.12 The sum of the reciprocals of Raman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

4.1.13 In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

4.1.14 The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

4.1.15 A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

4.1.16 Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

4.1.17 An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

4.1.18 Sum of the areas of two squares is 468 $m^2$2 find the sides of the two squares.

4.1.19 Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

4.1.20 The area of a rectangular plot is $528m^2$. The length of the plot is one more than twice the breadth. We need to find the length and breadth of the plot.

4.1.21 A temple courtyard has a carpet area of $300m^2$ with its length one metre more than twice its breadth. What should be the length and breadth of the hall.

4.1.22 The altitude of a right triangle is $7cm$ less than its base. If the hypotenuse is $13cm$, find the other two sides.

4.1.23 A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres than the area of a park that has already been made in the shape of a isoceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find its length and breadth.

4.1.24 The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

4.1.25 The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

4.1.26 A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it

would have taken 1 hour less for the same journey. Find the speed of the train.

4.1.27 Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank seperately. Find the time in which each tap can seperately fill the tank.

4.1.28 A pole has to be erected at a point on the boundary of a circular park of diameter 1.3 metres in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gatees should the pole be erected?

4.1.29 Sum of the areas of two squares is $468m^2$. If the difference of their perimeter is 24m, find the sides of the two squares.

4.1.30 Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is $800m^2$? If so, find its length and breadth.

4.1.31 Is the following situation possible? If so, determine their present ages.
The sum of the ages of the two friends is 20 years. Four years ago, the product of their ages in years was 48.

4.1.32 Is it possible to design a rectangular park of perimeter 80m and area of $400m^2$. If so, find its length and breadth.

5 Complex Numbers

5.1 NCERT

5.1.1 If $4x + 1(3x - y) = 3 + 1(-6)$, where x and y are real numbers, then find the values of x and y.

5.1.2 If $z_1 = 2 - 1, z_2 = 1 + 1$, find $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$.

5.1.3 Find the real numbers x and y if $(x - 1y)(3 + 51)$ is the conjugate of $-6 - 241$.

5.1.4 If $(a + 1b)(c + 1d)(e + 1f)(g + 1h) = A + 1B$, then show that

$$\left(a^2 + b^2\right)\left(c^2 + d^2\right)\left(d^2 + f^2\right)\left(f^2 + h^2\right)$$

.

Express the following in the form of $a + b1$

5.1.5 $(-51)\left(\frac{1}{8}\right)$

5.1.6 $(-1)(-21)\left(\frac{1}{8}\right)^3$

5.1.7 $(51)\left(-\frac{31}{5}\right)$

5.1.8 $(5 - 31)^3$

5.1.9 $\frac{5-\sqrt{2}1}{1-\sqrt{2}1}$

5.1.10 1^{-35}

5.1.11 $1^9 + 1^{19}$

5.1.12 1^{-39}

5.1.13 $3(7 + 17) + 1(7 + 17)$

5.1.14 $(1–i)–(-1 + i6)$

5.1.15 $\left(\frac{1}{5} + \frac{21}{5}\right) - \left(4 + \frac{51}{2}\right)$

5.1.16 $\left[\left(\frac{1}{3} + \frac{71}{3}\right) + \left(4 + \frac{1}{3}\right)\right] - \left(1 - \frac{4}{3}\right)$

5.1.17 $\frac{(3+1\sqrt{5})(3-1\sqrt{5})}{(\sqrt{3}+1\sqrt{2})-(\sqrt{3}-1\sqrt{2})}$

5.1.18 $\left[1^18 + \left(\frac{1}{1}\right)^{25}\right]$

5.1.19 $\left(\frac{1}{1-41} - \frac{2}{1+1}\right)\left(\frac{3-41}{5+1}\right)$

Find the multiplicative inverse of the following

5.1.20 $2 - 3\imath$.

5.1.21 $4 - 3\imath$.

5.1.22 $\sqrt{5} + 3\imath$.

5.1.23 $-\imath$.

Find the conjugate of

5.1.24 $\frac{(3-2\imath)(2+3\imath)}{(1+2\imath)(2-\imath)}$