# Linear Forms

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Contents



***Abstract*—This manual provides solved problems in linear algebra from CBSE Class 10 and 12 board exam papers.**

## 1 Vectors

1.1. Find the value of $k$, if the points

$$\mathbf{A} = \begin{pmatrix} k \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \tag{1.1.1}$$

are collinear.
**Solution:**
Let

$$\mathbf{M} = \begin{pmatrix} \mathbf{A}-\mathbf{B} & \mathbf{B}-\mathbf{C} \end{pmatrix}^\top = \begin{pmatrix} k-6 & 3-(-2) \\ 6-(-3) & -2-4 \end{pmatrix} \tag{1.1.2}$$

$$= \begin{pmatrix} k-6 & 5 \\ 9 & -6 \end{pmatrix} \tag{1.1.3}$$

Upon row reduction,

$$\begin{pmatrix} k-6 & 5 \\ 9 & -6 \end{pmatrix} \xleftrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 9 & -6 \\ k-6 & 5 \end{pmatrix} \tag{1.1.4}$$

$$\leftrightarrow \begin{pmatrix} 9 & -6 \\ 0 & 9+6k \end{pmatrix} \tag{1.1.5}$$

$$\xleftrightarrow{R_2 \to \frac{R_2}{6}} \begin{pmatrix} 9 & -6 \\ 0 & \frac{3}{2}+k \end{pmatrix} \tag{1.1.6}$$

$$\xleftrightarrow{R_1 \to \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{3}{2}+k \end{pmatrix} \tag{1.1.7}$$

$\because$ the points are collinear, rank$(\mathbf{M}) = 1$. Hence,

$$\frac{3}{2} + k = 0 \tag{1.1.8}$$

$$\implies k = \frac{-3}{2} \tag{1.1.9}$$

This is verified in Fig. 1.1.1.

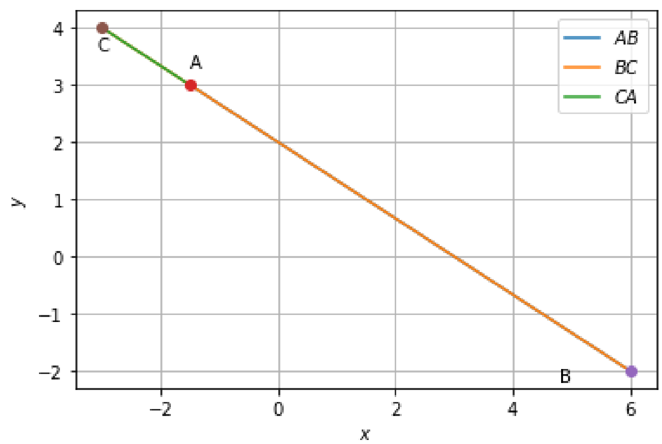


Fig. 1.1.1: Graphical solution

## 2 Linear Forms

2.1. Draw the graphs of the following equations:

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = -6 \tag{2.1.1}$$

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 9 \tag{2.1.2}$$

Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis.
**Solution:**

a) The intersection of the lines is given by

$$\begin{pmatrix} 3 & -4 \\ 3 & 9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -6 \\ 9 \end{pmatrix} \tag{2.1.3}$$

for which, the augmented matrix is

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix} \tag{2.1.4}$$

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which can be reduced as

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix} \xleftrightarrow[R_1 \leftarrow R_2]{R_2 \leftarrow R_1} \begin{pmatrix} 3 & 1 & 9 \\ 3 & -4 & -6 \end{pmatrix} \tag{2.1.5}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 3 & -4 & -6 \end{pmatrix} \tag{2.1.6}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & -5 & -15 \end{pmatrix} \tag{2.1.7}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{5} R_2} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.1.8}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{1}{3} R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \tag{2.1.9}$$

$$\therefore \mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{2.1.10}$$

is the point of intersection of the lines and the vertex of the triangle formed by the two lines with x-axis as base.

b) The equation of the x axis is

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2.1.11}$$

Thus, the intersection of (2.1.1) with the x axis is given by the set

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = -6 \tag{2.1.12}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2.1.13}$$

The augmented matrix for above is

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.1.14}$$

which can be reduced as

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{1}{3} R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.1.15}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{4}{3} R_2} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.1.16}$$

$$\tag{2.1.17}$$

$$\therefore \mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2.1.18}$$

is the point of intersection of the line (2.1.1) with the x axis.

c) Similarly, the intersection of (2.1.2) with the x axis is given by the set

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 9 \tag{2.1.19}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2.1.20}$$

with augmented matrix

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.1.21}$$

Twhich can be reduced as

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{1}{3} R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.1.22}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{1}{3} R_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.1.23}$$

$$\tag{2.1.24}$$

resulting in

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{2.1.25}$$

as the point of intersection of the line (2.1.2) with the x axis.

These points are then plotted in Fig. 2.1.1 for verification.

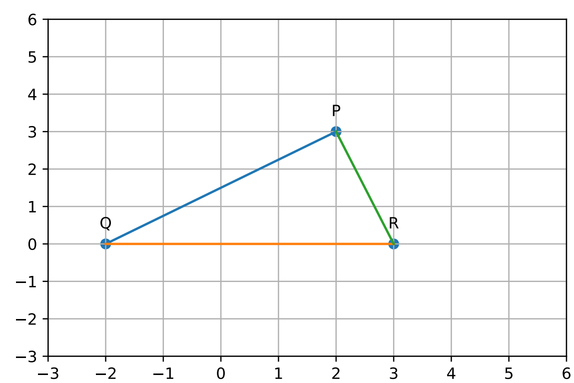


Fig. 2.1.1: Two lines representing given equations meet at point $\begin{pmatrix} 2 & 3 \end{pmatrix}$

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