CBSE MATH

Made Simple

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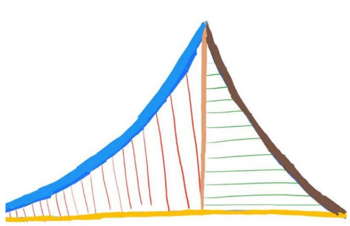

Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. 2023

1.1.1. 10

1.1.1 In what ratio, does x-axis divide the line segment joinin the points $\mathbf{A}(3,6)$ and $\mathbf{B}(-12,-3)$?

(a) $1:2$

(b) $1:4$

(c) $4:1$

(d) $2:1$

1.1.2 The distance between the point $(0,2\sqrt{5})$ and $(-2\sqrt{5},0)$ is

(a) $2\sqrt{10}$ units

(b) $4\sqrt{10}$ units

(c) $2\sqrt{20}$ units

(d) 0 units

1.1.3 If $(-5, 3)$ and $(5, 3)$ are two vetices of an equilateral triangle,then coordinates of the third vertex,given that origin lies inside the triangle $(take\sqrt{3} = 1.7)$

1.1.4 Show that the points $(-2, 3)$, $(8, 3)$ and $(6, 7)$ are the verices of right-angled triangle

1.1.5 If $\mathbf{Q} = (0, 1)$ is equidistant from $\mathbf{P} = (5, -3)$ and $\mathbf{R} = (x, 6)$, find the value of x.

1.1.6 The distance of the point $(-6, 8)$ from origin is :

(a) 6

(b) -6

(c) 8

(d) 10

1.1.7 The points $(-4, 0)$ $(4, 0)$ and $(0, 3)$ are the vertices of a :

(a) right triangle

(b) isosceles triangle

(c) equilateral triangle

(d) scalene triangle

1.1.2. 10

1. The area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is :

(a) ab

(b) $\frac{1}{2}ab$

(c) $\frac{1}{4}ab$

(d) $2ab$

2. Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and remaining for growing vegetables as shown in figure. 2.1 . In the field , there is a pole marked as O .

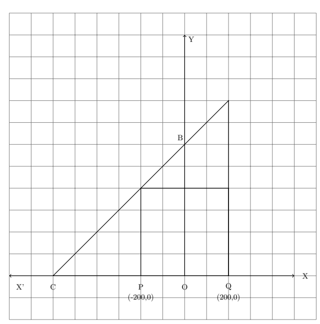

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Figure 2.1: Image

Based on the above information,answer the following equations:

(a) Taking O as origin , coordinates of P are (-200,0) and of Q are (200,0). PQRS being a square, what are the coordinates of R and S?

(b) i. What is the area of square PQRS?

  ii. What is the length of diagonal PR in PQRS?

(c) If S divides CA in the ratio K:1,what is the value of K,where point A is (200,800)?

1.1.3. 12

1.1.1. Unit vector along **PQ**, where coordinates of **P** and **Q** respectively are (2,1,-1)and(4,4,-7), is

(a) $2\hat{i} + 3\hat{j} - 6\hat{k}$

(b) $-2\hat{i} - 3\hat{j} + 6\hat{k}$

(c) $-\frac{2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$

(d) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

1.1.2. If in $\triangle$ABC, $\overrightarrow{BA}$=2$\overrightarrow{a}$ and $\overrightarrow{BC}$=3$\overrightarrow{b}$, then $\overrightarrow{AC}$ is

(a) $2\overrightarrow{a} + 3\overrightarrow{b}$

(b) 2$\overrightarrow{a}$ - 3$\overrightarrow{b}$

(c) 3$\overrightarrow{b}$ - 2$\overrightarrow{a}$

(d) $-2\overrightarrow{a}$ -3$\overrightarrow{b}$

1.1.3. Equation of line passing through origin and making 30°, 60° and 90°with x, y, z axes respectively is

(a) $\frac{2x}{\sqrt{3}}=\frac{y}{2}=\frac{z}{0}$

(b) $\frac{2x}{\sqrt{3}}=\frac{2y}{1}=\frac{z}{0}$

(c) 2x=$\frac{2y}{\sqrt{3}}=\frac{z}{1}$

(d) $\frac{2x}{\sqrt{3}}=\frac{2y}{1}=\frac{z}{1}$

1.1.4. If $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ are three non-zero unequal vectors such that $\overrightarrow{a}.\overrightarrow{b} = \overrightarrow{a}.\overrightarrow{c}$, then find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ - $\overrightarrow{c}$.

1.1.5. If the equation of a line is

$$x = ay + b, z = cy + d, \tag{1.1.5.1}$$

then find the direction ratios of the line and a point on the line.

1.1.6. Using Integration, find the area of triangle whose vertices are (-1, 1), (0, 5) and (3, 2).

1.2. 2022

1.2.1. 10

1.2.1. The distance between the points $(0, 0)$ and $(a - b, a + b)$ is

(a) $2\sqrt{ab}$

(b) $\sqrt{2a^2 + ab}$

(c) $2\sqrt{a^2 + b^2}$

(d) $\sqrt{2a^2 + 2b^2}$

1.2.2. The value of m which makes the point $(0, 0)$, $(2m, -4)$ and $(3, 6)$ collinear, is _____

1.2.3. A circle has its center at $(4, 4)$. If one end of a diameter is $(4, 0)$, then find the coordinates of other end.

1.2.4. Find the area of the quadrilateral ABCD whose vertices are $\mathbf{A}(-4, -3)$, $\mathbf{B}(3, -1)$, $\mathbf{C}(0, 5)$ and $\mathbf{D}(-4, 2)$

1.2.5. If the points $\mathbf{A}(2, 0)$, $\mathbf{B}(6, 1)$, and $\mathbf{C}(p, q)$ form a triangle of area 12sq. units (positive only) and

$$2p + q = 10 \tag{1.2.5.1}$$

, then find the values of p and q.

1.2.2. 12

1.2.1. $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that

$$\left|2\overrightarrow{a} + 3\overrightarrow{b}\right| = \left|3\overrightarrow{a} - 2\overrightarrow{b}\right|. \tag{1.2.1.1}$$

Find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

1.2.2. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\overrightarrow{a} = \hat{i} - \hat{j} + \hat{k} \tag{1.2.2.1}$$

and

$$\overrightarrow{b} = 2\hat{i} - \hat{j} - 3\hat{k} \tag{1.2.2.2}$$

then find the vector $\overrightarrow{c}$, given that

$$\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b} \tag{1.2.2.3}$$

and

$$\overrightarrow{a} . \overrightarrow{c} = 4. \tag{1.2.2.4}$$

1.2.3. If

$$\left|\overrightarrow{a} \times \overrightarrow{b}\right|^2 + \left|\overrightarrow{a} . \overrightarrow{b}\right|^2 = 400 \tag{1.2.3.1}$$

and

$$\left|\overrightarrow{b}\right| = 5 \tag{1.2.3.2}$$

find the value of $|\overrightarrow{a}|$.

1.2.4. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a} . \overrightarrow{b} = 1 \tag{1.2.4.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k} \tag{1.2.4.2}$$

, then find $\left|\overrightarrow{b}\right|$

1.2.5. If

$$\left|\overrightarrow{a}\right| = 3, \left|\overrightarrow{b}\right| = 2\sqrt{3} \tag{1.2.5.1}$$

and

$$\overrightarrow{a} . \overrightarrow{b} = 6, \tag{1.2.5.2}$$

then find the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$.

1.2.6. $\left|\overrightarrow{a}\right| = 8, \left|\overrightarrow{b}\right| = 3$ and $\overrightarrow{a} . \overrightarrow{b} = 12\sqrt{3}$, then the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$ is

(a) 24

(b) 144

(c) 2

(d) 12

1.2.7. If

$$\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.2.7.1}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.2.7.2}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

1.2.8. $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ and $\overrightarrow{d}$ are four non-zeros vectors such that

$$\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{d} \tag{1.2.8.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{c} = 4\overrightarrow{b} \times \overrightarrow{d} \tag{1.2.8.2}$$

, then show that $(\overrightarrow{a} - 2\overrightarrow{d})$ is parallel to $(2\overrightarrow{b} - \overrightarrow{c})$ where

$$\overrightarrow{a} \neq 2\overrightarrow{d}, \overrightarrow{c} \neq 2\overrightarrow{b} \tag{1.2.8.3}$$

1.2.9. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{1.2.9.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k}, \tag{1.2.9.2}$$

then find $\left|\overrightarrow{b}\right|$

1.2.10. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{b}\right|, \tag{1.2.10.1}$$

then prove that $(\overrightarrow{a} + 2\overrightarrow{b})$ is perpendicular to $\overrightarrow{a}$.

1.2.11. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vectors and θ is the angle between them , then prove that sin

$$\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.2.11.1}$$

1.2.12. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that and θ is the angle between them, then prove that

$$sin\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.2.12.1}$$

1.2.13. If

$$\overrightarrow{a} = 2\hat{i} + y\hat{j} + \hat{k} \tag{1.2.13.1}$$

and

$$\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k} \tag{1.2.13.2}$$

are two vectors for which the vector $(\overrightarrow{a} + \overrightarrow{b})$ is perpendicular to the vector $(\overrightarrow{a} - \overrightarrow{b})$ then find all the possible values of y.

1.2.14. Write the projection of the vector $(\overrightarrow{b} + \overrightarrow{c})$ on the vector $\overrightarrow{a}$, where

$$\overrightarrow{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + 2\hat{j} - 2\hat{k} \tag{1.2.14.1}$$

and

$$\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}. \tag{1.2.14.2}$$

1.2.15. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + \hat{j} - 2\hat{k} \tag{1.2.15.1}$$

and

$$\overrightarrow{c} = \hat{i} + 3\hat{j} - \hat{k} \tag{1.2.15.2}$$

and the projection of vector $\overrightarrow{c} + \lambda\overrightarrow{b}$ on vector $\overrightarrow{a}$ is $2\sqrt{6}$, find the value of λ.

1.2.16. If

$$\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.2.16.1}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.2.16.2}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

1.2.17. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k} \tag{1.2.17.1}$$

and

$$\overrightarrow{b} = 5\hat{i} - 3\hat{j} - 4\hat{k} \tag{1.2.17.2}$$

, then find the ratio $\dfrac{projectionofvector \overrightarrow{a} onvector \overrightarrow{b}}{projectionofvector \overrightarrow{b} onvector \overrightarrow{a}}$

1.2.18. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right-angled triangle. If

$$\overrightarrow{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \overrightarrow{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.2.18.1}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} \tag{1.2.18.2}$$

are such that the vector $(\overrightarrow{a} + \lambda \overrightarrow{b})$ is perpendicular to vector $\overrightarrow{c}$, then find the value of λ.

1.2.19. If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are the position vectors of the points $\mathbf{A}(2, 3, -4)$, $\mathbf{B}(3, -4, -5)$ and $\mathbf{C}(3, 2, -3)$ and respectively, then $\left|\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right|$ is equal to

(a) $\sqrt{113}$

(b) $\sqrt{185}$

(c) $\sqrt{203}$

(d) $\sqrt{209}$

1.2.20. Find the values λ, for which the distance of point $(2, 1, \lambda)$ from plane

$$3x + 5y + 4z = 11 \tag{1.2.20.1}$$

is $2\sqrt{2}$ units.

1.2.21. Find the coordinates of the point where the line through $(3, 4, 1)$ crosses the ZX-plane

1.2.22. Using vectors, find the area of the triangle withvertices $\mathbf{A}(-1, 0, -2)$, $\mathbf{B}(0, 2, 1)$ and $\mathbf{C}(-1, 4, 1)$

1.2.23. Using integration, find the area of triangle region whose vertices are $(2,0)$, $(4,5)$ and $(1,4)$.

1.2.24. If a line makes $60°$ and $45°$ angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line.

1.2.25. The Cartesian equation of a line AB is :

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3} \tag{1.2.25.1}$$

.

1.2.26. Find the directions cosines of a line parallel to line AB.

1.2.27. Find the direction cosines of a line whose cartesian equation is given as

$$3x+1 = 6y-2 = 1-z. \tag{1.2.27.1}$$

1.2.28. A vector of magnitude 9 units in the direction of the vector $-2\hat{\imath}-\hat{\jmath}+2\hat{k}$ is ______

1.2.29. The two adajacent sides of a parallelogram are represented by $2\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

1.2.30. The two adjacent sides of a parallelogram are represented by vectors

$2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also,find the area of the parallelogram.

1.2.31. If

$$\overrightarrow{a} = \overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k} \tag{1.2.31.1}$$

and

$$\overrightarrow{b} = 2\hat{i} + 4\hat{j} - 5\hat{k} \tag{1.2.31.2}$$

represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram.

1.3. 2021

1.3.1. 10

1.3.1.1. Find the distance between the points $\mathbf{A}(-\frac{7}{3}, 5)$ and $\mathbf{B}(\frac{2}{3}, 5)$.

1.3.1.2. Check whether 13cm, 12cm, 5cm can be the sides of a right triangle.

1.3.1.3. (a) If PL and PM are two tangents to a circle with centre $\mathbf{O}$ from an external point $\mathbf{P}$ and $PL = 4$ cm, find the length of OP, where radius of the circle is 3 cm.

(b) Find the distance between two parallel tangents of a cicle of radius 2.5 cm.

1.3.1.4. Find the coordinates of the points which divides the line segment joining the points $\mathbf{A}(7,-1)$ and $\mathbf{B}(-3,-4)$ in the ratio $2:3$.

1.3.1.5. To divide a line segment QP internally in the ratio $2:3$, we draw a ray QY such that $\angle$ PQY is acute. What will be the minimum number of points to be located at equal distances on the ray QY ?

1.3.1.6. Answer any four of the following questions :

(i) The point which divides the line segment joining the points $(7,-6)$ and $(3,4)$ in the ratio $1:2$ lies in

(A) I quadrant

(B) II quadrant

(C) III quadrant

(D) IV quadrant

(ii) If the $\mathbf{A}(1,2)$, $\mathbf{O}(0,0)$ and $\mathbf{C}(a,6)$ are collinear, then the value of a is

(A) 6

(B) $\frac{3}{2}$

(C) 3

(D) 12

(iii) The distance between the points $\mathbf{A}(0,6)$ and $\mathbf{B}(0,-2)$ is

(A) 6 units

(B) 8 units

(C) 4 units

(D) 2 units

(iv) If $(\frac{a}{3}, 4)$ is the mid-point of the line segment joining the points $(-6, 5)$ and $(-2, 3)$, then the value of 'a ' is

(A) -4

(B) 4

(C) -12

(D) 12

(v) What kind of triangle is formed with vertices $\mathbf{A}(0, 2)$, $\mathbf{B}(-3, 0)$ and $\mathbf{C}(3, 0)$?

(A) A right triangle

(B) An equilateral triangle

(C) An isosceles triangle

(D) A scalene triangle

1.3.1.7. (a) If the distance between the points $(k, -2)$ and $(3, -6)$ is 10 units, find the positive value of k.

(b) Find the length of the segment joining $\mathbf{A}(-6, 7)$ and $\mathbf{B}(-1, -5)$.Also, find the mid-point of AB.

1.3.1.8. A man goes 5 metres due to West and then 12 metres due North. How far is he from the starting point ?

1.3.1.9. Students of a school are standing in rows and columns in their school playground to celebrate their annual sports day. **A**, **B**, **C** and **D** are the positions of four students as shown in the figure.

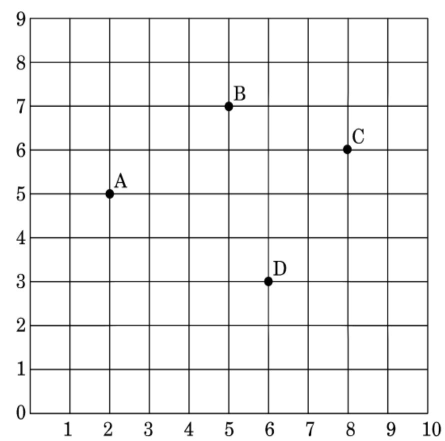


Figure 1.3.1.9.1: Based on the above, answer the following question :

(i) The figure formed by the points **A**, **B**, **C** and **D** is a

(A) sqaure

(B) parallelogram

(C) rhombus

(D) quadrilateral

(ii) If the sports teacher is sitting at the origin, then which of the four students is closest to him ?

(A) **A**

(B) **B**

(C) **C**

(D) **D**

(iii) The distance between **A** and **C** is

(A) $\sqrt{37}$ units

(B) $\sqrt{35}$ units

(C) 6 units

(D) 5 units

(iv) The coordinates of the mid-point of line segment AC are

(v) If a point $\mathbf{P}$ divides the line segment AD in the ratio $1:2$, then coordinates of $\mathbf{P}$ are

(A) $(\frac{8}{3}, \frac{8}{3})$

(B) $(\frac{10}{3}, \frac{13}{3})$

(C) $(\frac{13}{3}, \frac{10}{3})$

(D) $(\frac{16}{3}, \frac{11}{3})$

1.3.1.10. (a) Check whether the points $\mathbf{P}(5,-2)$, $\mathbf{Q}(6,4)$ and $\mathbf{R}(7,-2)$ are the vertices of an isosceles triangle PQR.

(b) Find the ratio in which $\mathbf{P}(4,5)$ divides the join of $\mathbf{A}(2,3)$ and $\mathbf{B}(7,8)$.

1.3.1.11. The coordinate of the three consecutive vertices of a parallelogram ABCD are $\mathbf{A}(1,3)$, $\mathbf{B}(-1,2)$, and $\mathbf{C}(2,5)$. Find the cordinates of the fourth vertex $\mathbf{D}$.

1.3.1.12. (a) If $\mathbf{P}(2,2)$, $\mathbf{Q}(-4,-4)$ and $\mathbf{R}(5,-8)$ are the vertices of a $\triangle$PQR, then find the length of the median through $\mathbf{R}$.

(b) Find the ratio in which y-axis divides the line segment joining the points $\mathbf{A}(5,-6)$ and $\mathbf{B}(-1,-4)$.Also, find the coordinates of the point of intersection.

1.3.1.13. (a) Find the ratio in which the line segment joining the points $\mathbf{A}(1,-5)$

and $\mathbf{B}(-4,5)$ is divided by the ax-axis. Also, find coordinates of the point of division.

(b) The points $\mathbf{A}(0,3)$, $\mathbf{B}(-2,a)$ and $\mathbf{C}(-1,4)$ are the vertices of a rigth triangle, right-angled at $\mathbf{A}$. Find the value of a.

1.3.2. 12

1. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are position vectors of the points A(2,3,-4), B(3,-4,-5) and C(3,2,-3) respectively, then $|\mathbf{a}+\mathbf{b}+\mathbf{c}|$ is equal to

   (a) $\sqrt{113}$

   (b) $\sqrt{185}$

   (c) $\sqrt{203}$

   (d) $\sqrt{209}$

2. Find the distance of the point (a,b,c) from the x-axis

3. If $\mathbf{a} = 2\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $\mathbf{b} = 5\hat{\imath}-3\hat{\jmath}-4\hat{k}$, then find the ratio $\frac{\text{projection of vector } \mathbf{a} \text{ on } \mathbf{b}}{\text{projection of vector } \mathbf{b} \text{ on vector } \mathbf{a}}$

4. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\mathbf{c} = \hat{a} + 2\hat{b}$ and $\mathbf{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

5. Show that $|\mathbf{a}|\,\mathbf{b}$ + $|\mathbf{b}|\,\mathbf{a}$ is perpendicular to $|\mathbf{ab}|$- $|\mathbf{b}|\,\mathbf{a}$, for any two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$.

6. Prove that three points A,B and C with position vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ respectively are collinear if and only if $(\mathbf{b}\times\mathbf{c})+(\mathbf{c}\times\mathbf{a})+(\mathbf{a}\times\mathbf{b}) = \mathbf{0}$.

Chapter 2

Linear Forms

2.1. 2023

2.1.1. 10

2.1.1. **Assertion (A):** Point $\mathbf{P}(0,2)$ is the point of intersection of $y - axis$ with the line $3x + 2y = 4$.

Reason (R): The distance of point $\mathbf{P}(0,2)$ from $x - axis$ is 2 units.

2.1.2. If the pair of equations $3x - y + 8 = 0$ and $6x - ry + 16 = 0$ represent coincident lines, then the value of 'r' is:

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) -2

(d) 2

2.1.3. The of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are:

(a) intersecting

(b) parallel

(c) coincident

(d) either intersecting or parallel

2.1.4. Find the equations of the diagonals of the parallelogram **PQRS** whose vertices are **P**(4,2,-6), **Q**(5,-3,1), **R**(12,4,5) and **S**(11,9,-2). Use these equations to find the point of intersection of diagonals.

2.1.5. A line l passes through point (-1,3,-2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the ctor equation of the line l. Hence, obtain its distance from origin.

2.1.2. 12

1. Equation of line passing through origin and making $30°, 60°$ and $90°$ with x, y, z axes respectively is

   (a) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$

   (b) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$

   (c) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$

   (d) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$

2. If the equation of a line is $x = ay + b, z = cy + d$,then find the direction ratios of the line and a point on the line.

3. (a) Find the equations of the diagonals of the parallelogram $PQRS$ whose vertices are $P(4, 2, -6), Q(5, -3, 1), R(12, 4, 5), S(11, 9, -2)$.Use these equations to find the point of intersection of diagonals.

   (b) A line l passes through point$(-1, 3, -2)$ and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.Find the vector equation of the line l.Hence,obtain its distance from origin.

2.2. 2022

2.2.1. Solve the equations $x + 2y = 6$ and $2x - 5y = 12$ graphically.

2.2.2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \tag{2.2.2.1}$$

$$(bx + ay) + (b - 4a) = 0 \tag{2.2.2.2}$$

2.2.3. Find the co-ordinates of the point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2}, 0, 0\right), (0, 7, 0), (0, 0, 7)$.

2.2.4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

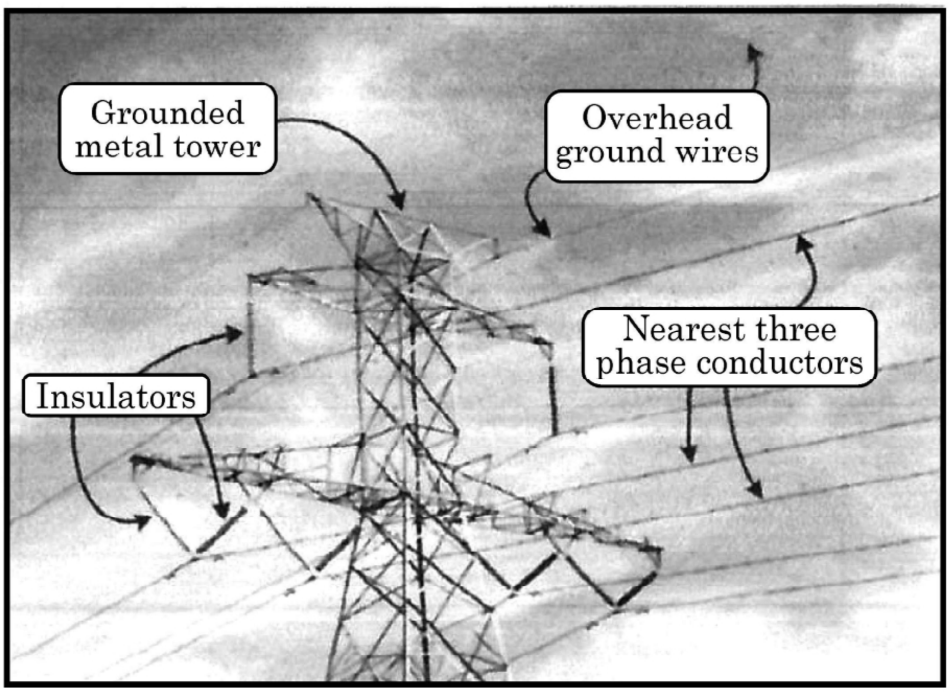


Figure 2.2.4.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.2.4.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \tag{2.2.4.1}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \tag{2.2.4.2}$$

Based on the given information, answer the following questions:

(a) Are the l_1 and l_2 coplanar? Justify your answer.

(b) Find the point of intersection of lines l_1 and l_2.

2.2.5. Write the cartesian equation of the line PQ passing through points

P$(2,2,1)$ and Q$(5,1,-2)$. Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2.

2.2.6. Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

2.2.7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k}) \tag{2.2.7.1}$$

$$\mathbf{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k}) \tag{2.2.7.2}$$

2.2.8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.2.8.1) represented by the following lines

$$\mathbf{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) \tag{2.2.8.1}$$

$$\mathbf{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k}) \tag{2.2.8.2}$$

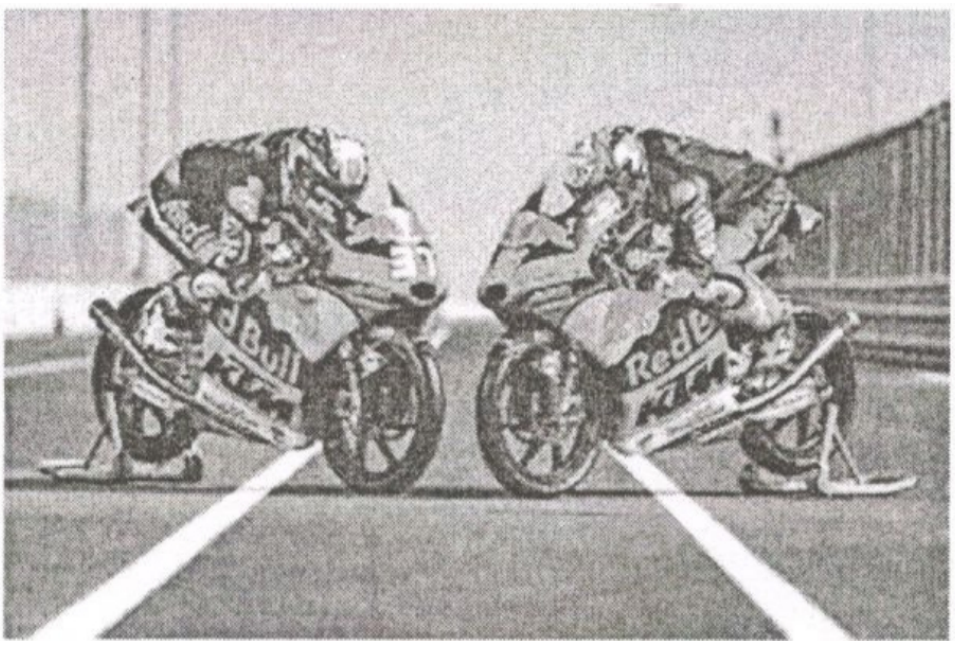


Figure 2.2.8.1: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

(a) Find the shortest distance between the given lines.

(b) Find a point at which the motorcycles may collide.

2.2.9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{\imath} + (\lambda + 4)\hat{\jmath} - (\lambda - 3)\hat{k} \tag{2.2.9.1}$$

$$\mathbf{r} = (3 - \mu)\hat{\imath} + (2\mu + 2)\hat{\jmath} + (\mu + 6)\hat{k} \tag{2.2.9.2}$$

2.2.10. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{2.2.10.1}$$

2.2.11. Find the equation of the plane passing through the points $(2,1,0), (3,-2,-2)$ and $(1,1,7)$. Also, obtain its distance from the origin.

2.2.12. The foot of a perpendicular drawn from the point $(-2,-1,-3)$ on a plane is $(1,-3,3)$. Find the equation of the plane.

2.2.13. Find the cartesian and the vector equation of a plane which passes through the point $(3,2,0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

2.2.14. The distance between the planes $4x-4y+2z+5 = 0$ and $2x-2y+z+6 = 0$ is

(a) $\frac{1}{6}$
(b) $\frac{7}{6}$
(c) $\frac{11}{6}$
(d) $\frac{16}{6}$

2.2.15. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{\imath} + 3\hat{\jmath}) + 6 = 0 \tag{2.2.15.1}$$

$$\mathbf{r} \cdot (3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0 \tag{2.2.15.2}$$

which is at a unit distance from the origin.

2.2.16. If the distance of the point $(1, 1, 1)$ from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ.

2.2.17. Find the distance of the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$.

2.2.18. Find the distance of the point $P(4, 3, 2)$ from the plane determined by the points $A(-1, 6, -5), B(-5, -2, 3)$ and $C(2, 4, -5)$.

2.2.19. The distance of the line

$$\mathbf{r} = (\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 5\hat{\jmath} + \hat{k}) \tag{2.2.19.1}$$

from the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + 4\hat{k}) = 5 \tag{2.2.19.2}$$

is

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{3\sqrt{2}}$

(d) $\frac{-2}{3\sqrt{2}}$

2.2.20. Find a unit vector perpendicular to each of the vectors $(\mathbf{a} + \mathbf{b})$ and

$(\mathbf{a} - \mathbf{b})$ where

$$\mathbf{a} = \hat{\imath} + \hat{\jmath} + \hat{k} \tag{2.2.20.1}$$

$$\mathbf{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} \tag{2.2.20.2}$$

2.2.21. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}) \tag{2.2.21.1}$$

and the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 10. \tag{2.2.21.2}$$

2.2.22. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

2.2.23. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

2.2.24. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.2.24.1:

Table 2.2.24.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40

2.2.1. 12

1. Find the values of λ,for which the distance of point (2,1,λ) from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units.

2. If the distance of the point (1,1,1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$,find the value(s) of λ.

2.3. 2021

2.3.1. 10

2.3.1. If the graph of a pair of lines $x - 2y + 3 = 0$ and $2x - 4y = 5$ be drawn, that what type of lines are drawn ?

2.3.2. 12

1. If the two lines

$$L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2} \tag{1.1}$$

$$L_1 : x = 2, \frac{y}{-1} = \frac{z}{z-\alpha} \tag{1.2}$$

are perpendicular,then the value of α

(a) $\frac{2}{3}$

(b) 3

(c) 4

(d) $\frac{7}{3}$

2. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z \tag{2.1}$$

$$\frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{2.2}$$

3. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r}.(i + 3j) + 6 = 0 \tag{3.1}$$

$$\mathbf{r}.(3i - j - 4k) = 0 \tag{3.2}$$

which is at a unit distance from the origin.

4. If segment of the line intercepted between the co-ordinate-axes is bisected at the point $M\,(2, 3)$, then the equation of this line is

$$2x + 3y = 13 \tag{4.1}$$

$$x + y = 5 \tag{4.2}$$

$$2x + y = 7 \tag{4.3}$$

$$3x + 2y = 12 \tag{4.4}$$

5. The equation of a line through $(2, -4)$ and parallel to x-axis is ___________.

6. Find the equation of the median through vertex A of the triangle ABC, having vertices $A\,(2, 5)$, $B\,(-4, 9)$ and $C\,(-2, -1)$.

7. Solve the system of linear equations, using matrix method :

$$7x + 2y = 11 \tag{7.1}$$

$$4x - y = 2 \tag{7.2}$$

Chapter 3

Circles

3.1. 2022

3.1.1. 10

3.1.1. In tangents **PA** and **PB** from an external point P to a circle with centre O , are inclined to each other at an angle of 80°, then $\angle AOB$ is equal to

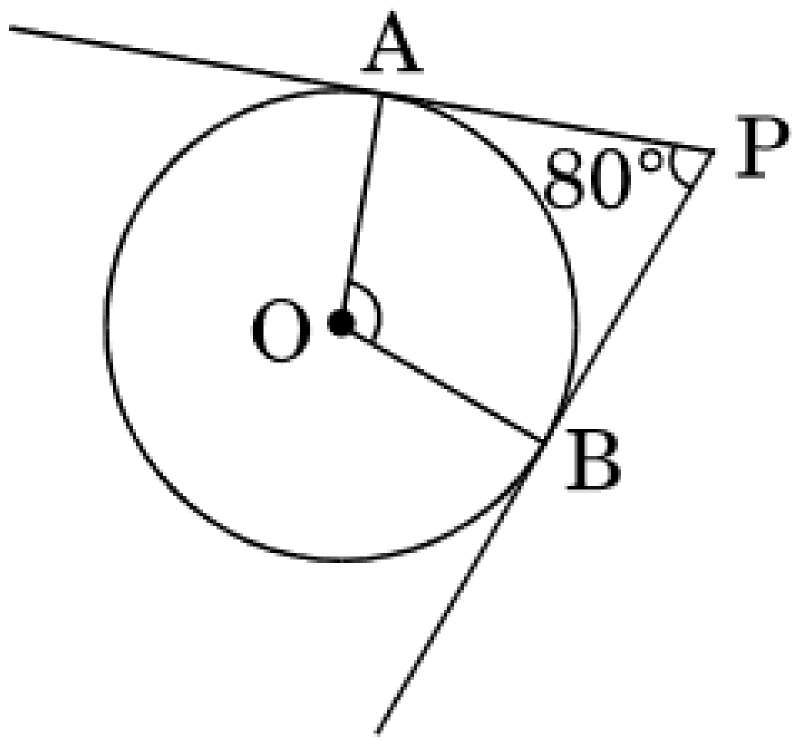


Figure 1.1: Tangents PA and PB

(a) 100°

(b) 60°

(c) 80°

(d) 50°

3.1.2. Two concentric circles are of radii $4cm$ and $3cm$. Find the length of the chord of the larger circle which touches the smaller circle.

3.1.3. In a triangle ABC with $\angle AOB$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point P. Prove that the tangent drawn at point P bisects BC.

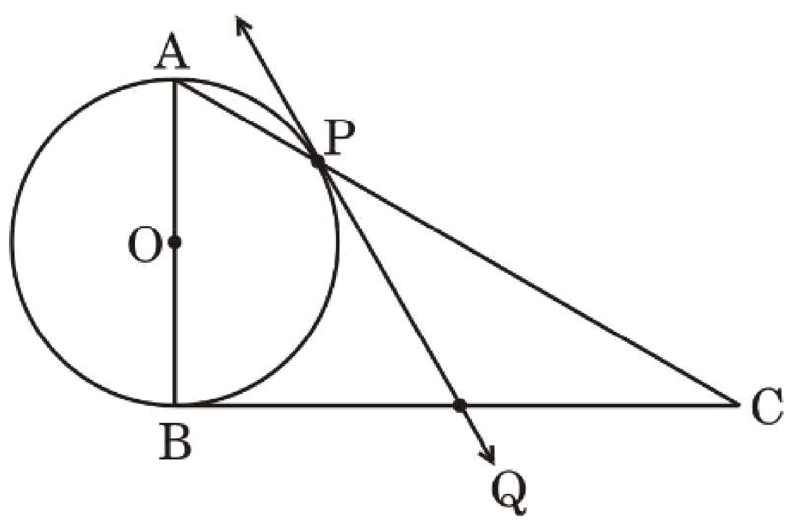


Figure 3.1: Concentric circles

3.1.4. Prove that a Parallelogram circumscribing a circle is a rhombus.

3.1.5. In two circles with centres at O and O of radii $2r$ and r respectively, touch each other internally at A. A chord AB of the bigger circle meets the smaller circle at C. Show that C bisects AB.

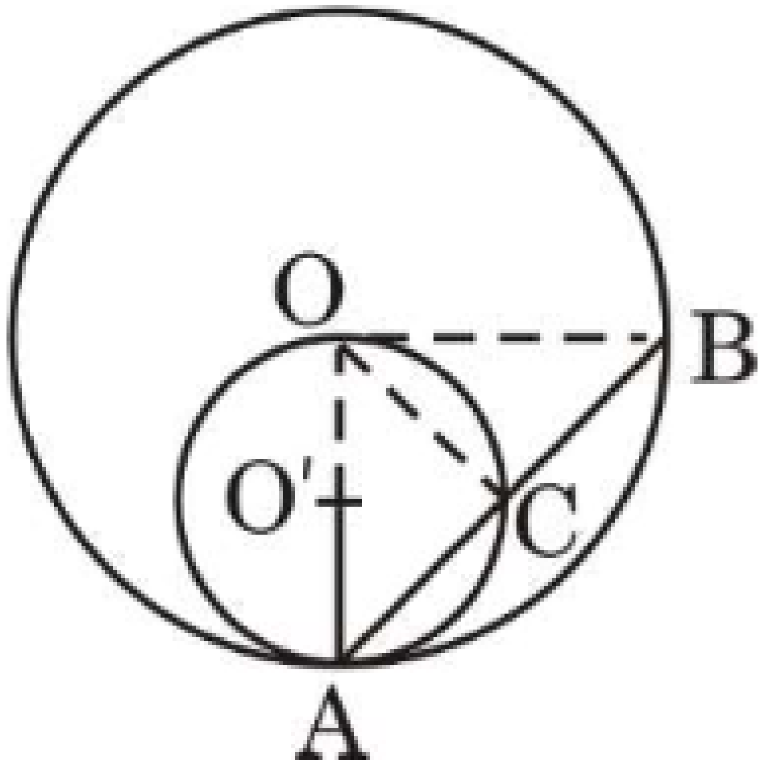


Figure 5.1: Two circles with center

3.1.6. In O is centre of a circle of radius $5cm$. PA and BC are tangents to the circle at A and B respectively. If $OP = 13cm$, then find the length of tangents PA and BC.

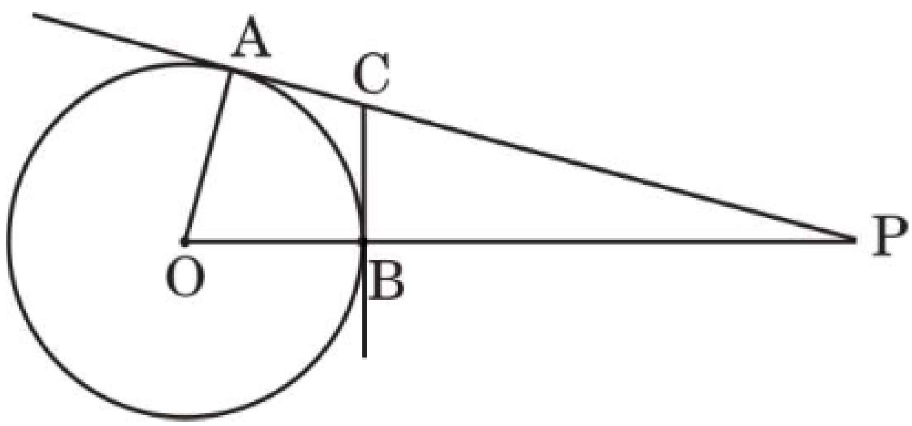


Figure 6.1: The center of circle radius is $5m$

3.1.7. In two concentric circles, a chord of length $48cm$ of the larger circle is a tangent to the smaller circle, whose radius is $7cm$. Find the radius of the larger circle.

3.1.8. At a point on the level ground, the angle of elevation of the top of a vertical tower is found to be α, such that $\tan \alpha = \frac{5}{12}$. On walking $192m$ towards the tower, the angle of elevation β is such that $\tan \beta = \frac{3}{4}$. Find the height of the tower.

3.2. 2023

3.2.1. 10

3.2.1. In the given figure Fig. 3.2.1.1, PQ is tangent to the circle centred at **O**. If $\angle AOB = 95°$, then measure of $\angle ABQ$ will be

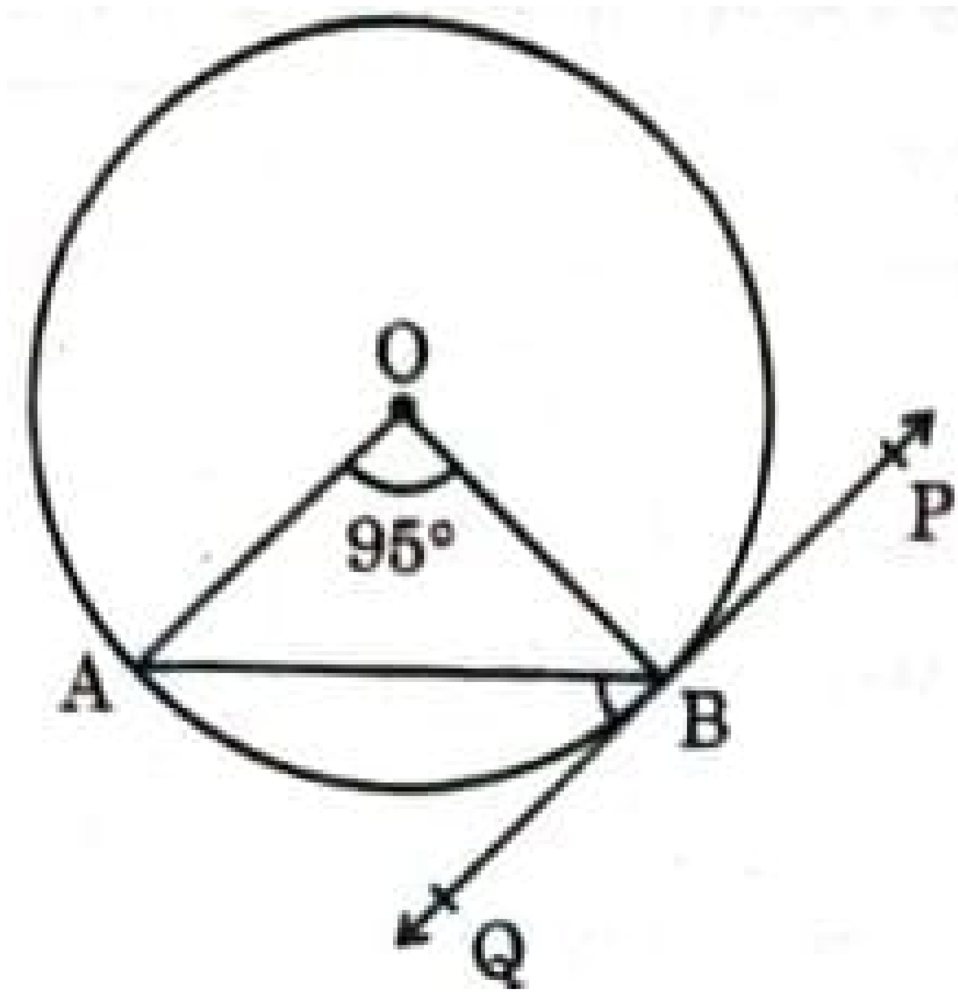


Figure 3.2.1.1:

(a) 47.5°

(b) 42.5°

(c) 85°

(d) 95°

3.2.2. (a) In the given figure Fig. 3.2.2.1, two tangents TP and TQ are drawn to be a circle with centre **O** from an external point **T**. Prove that $\angle PTQ = 2\angle OPQ$.

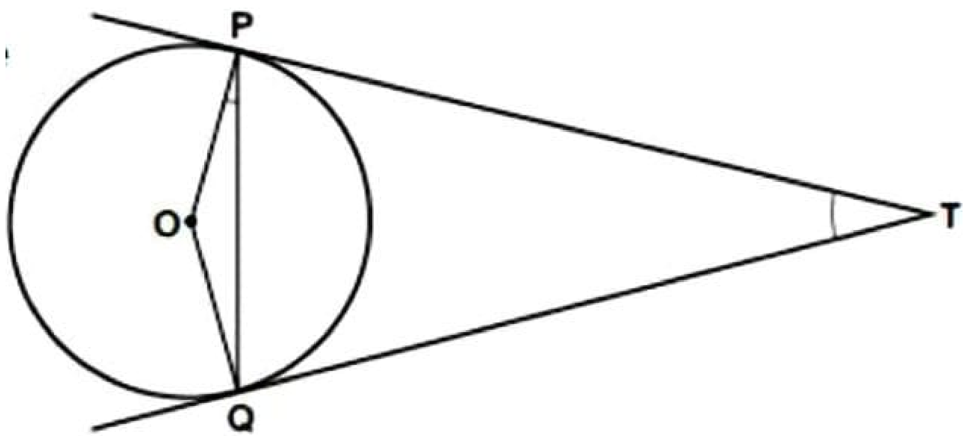


Figure 3.2.2.1:

(b) In the given figure Fig. 3.2.2.2, a circle is inscribed in a quadrilateral $ABCD$ in which $\angle B = 90°$. If $AD = 17cm, AB = 20cm$ and $DS = 3cm$, then find the radius of the circle.

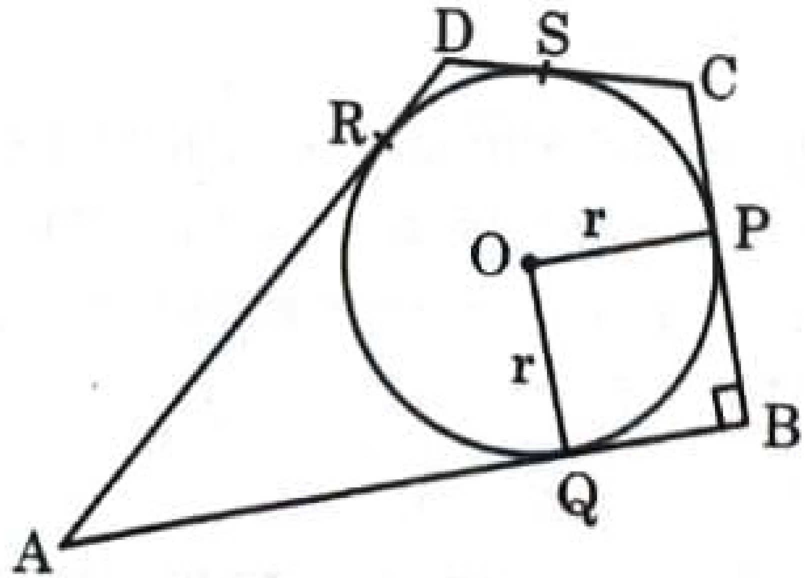


Figure 3.2.2.2:

3.2.3. The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a cicrle as shown in Fig. 3.2.3.1 below, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.

Figure 3.2.3.1:

In the given figure Fig. 3.2.3.2, AB is one such tangent to a circle of radius 75 cm. Point **O** is centre of the circle and $\angle ABO = 30^\circ . PQ$ is parallel to OA.

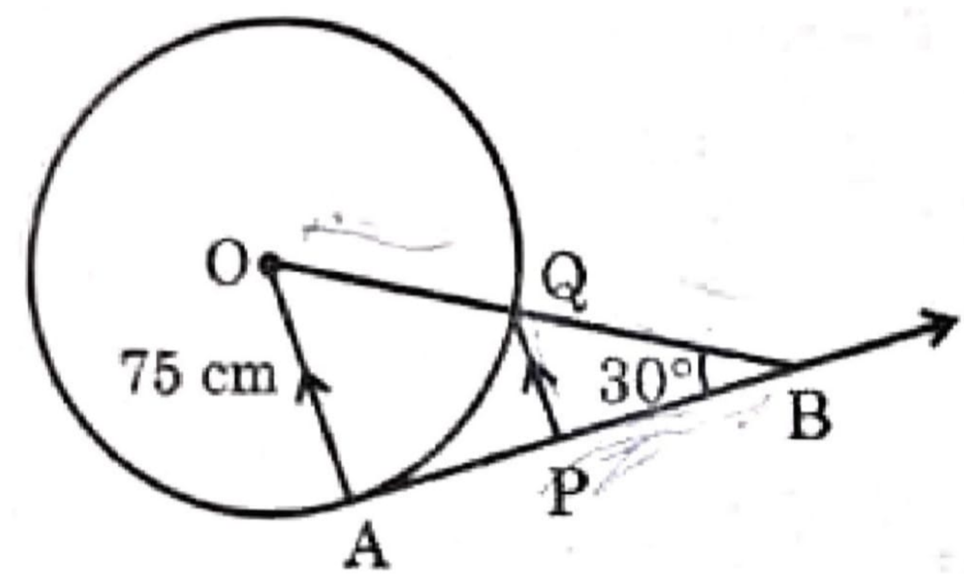


Figure 3.2.3.2:

Based on above information :

(a) find the length of AB.

(b) find the length of OB.

(c) find the length of AP.

(d) find the length of PQ.

3.2.4. In the given figure Fig. 3.2.4.1, the quadrilateral $PQRS$ circumscribes a circle. Here $PA + CS$ is equal to :

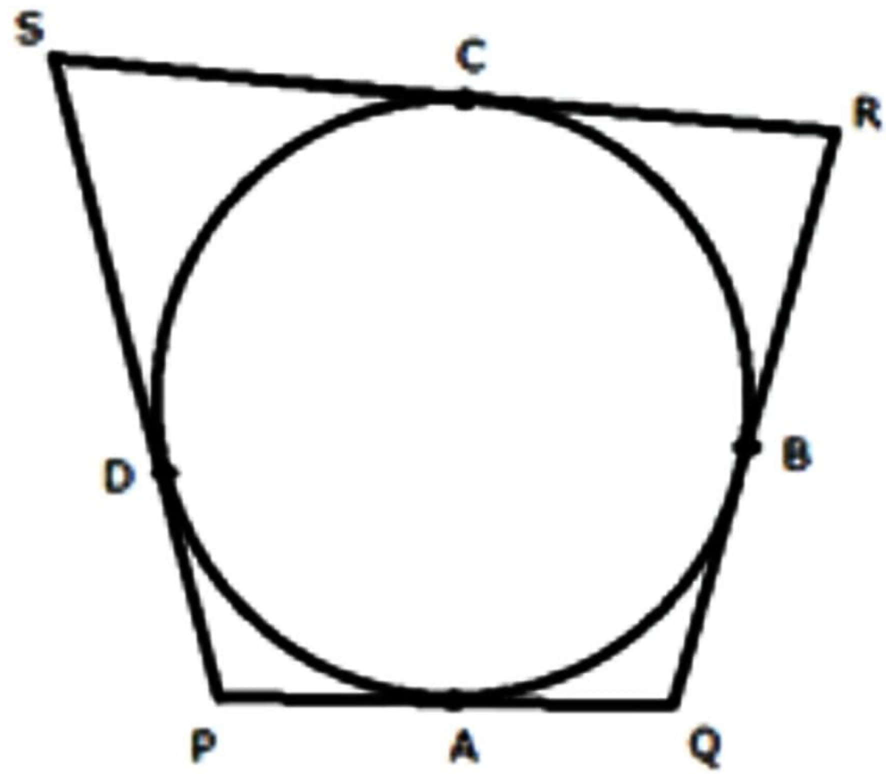


Figure 3.2.4.1:

(a) QR

(b) PR

(c) PS

(d) PQ

3.2.5. In the given figure Fig. 3.2.5.1, **O** is the centre of the circle. AB and AC are tangents drawn to the circle from point **A**. If $\angle BAC = 65°$, then find the measure of $\angle BOC$.

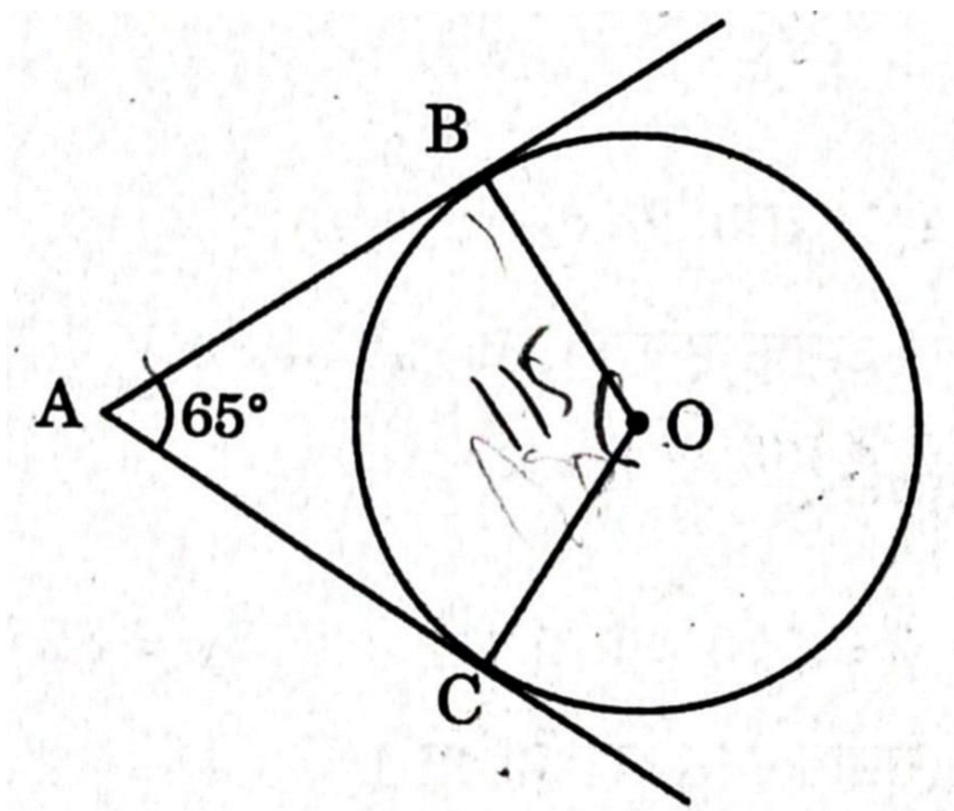


Figure 3.2.5.1:

3.2.6. In the given figure Fig. 3.2.6.1, **O** is the centre of the circle and QPR is the tangent to it at **P**. Prove that $\angle QAP + \angle APR = 90°$.

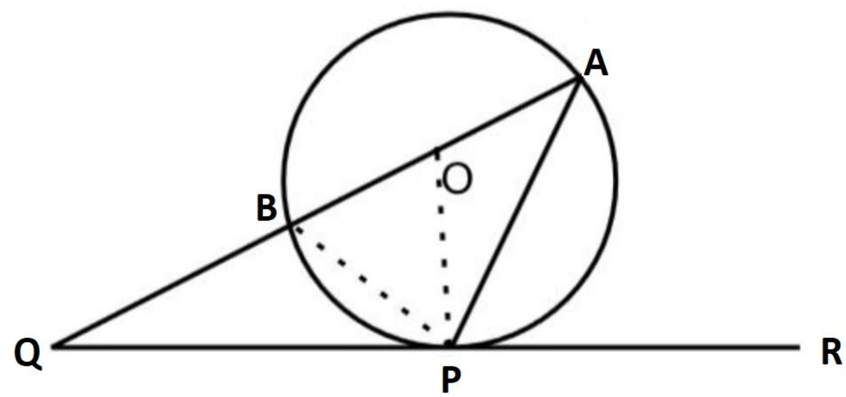


Figure 3.2.6.1:

3.2.7. In the given figure Fig. 3.2.7.1, TA is a tangent to the circle with centre **O** such that $OT = 4cm, \angle OTA = 30°$, then length of TA is :

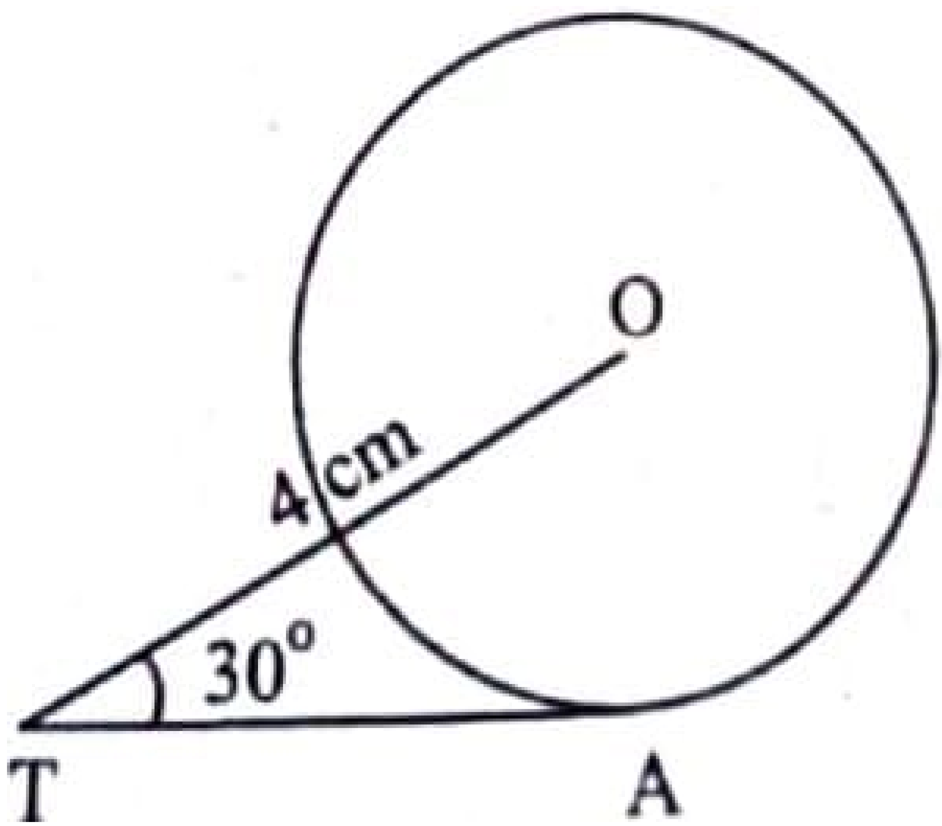


Figure 3.2.7.1:

(a) $2\sqrt{3}cm$

(b) $2cm$

(c) $2\sqrt{2}cm$

(d) $\sqrt{3}cm$

3.2.8. In the given figure Fig. 3.2.8.1, PT is a tangent at **T** to the circle with centre **O**. If $\angle TPO = 25°$, then x is equal to :

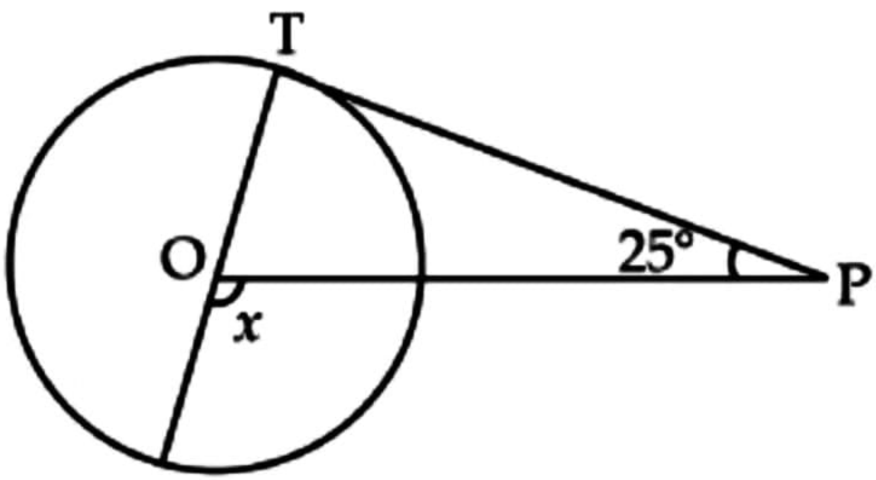


Figure 3.2.8.1:

(a) 25°

(b) 65°

(c) 90°

(d) 115°

3.2.9. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

3.3. 2022

3.3.1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.

3.3.2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.

3.3.3. In Figure 3.3.3.1, there are two concentric circles with centre **O**. If ARC and AQB are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of AC, if $AQ = 5$ cm.

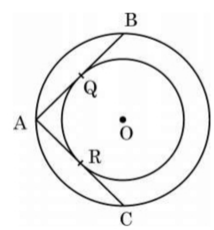


Figure 3.3.3.1: Two concentric circles with **O** as centre

3.3.4. In Figure 3.3.4.1, if a circle touches the side QR of ΔPQR at **S** and extended sides PQ and PR at **M** and **N**, respectively,

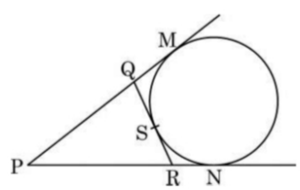


Figure 3.3.4.1: Two tangents are drawn from point **P** to the circle

prove that $PM = \frac{1}{2}(PQ + QR + PR)$

3.3.5. In Figure 3.3.5.1, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact **D** are of lengths 6 cm and 8 cm respectively. If the area of ΔABC is 84 cm^2, find the lengths of sides AB and AC.

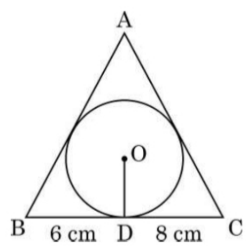


Figure 3.3.5.1: Circle with **O** as center circumscribed in triangle ABC

3.3.6. In Figure 3.3.6.1, PQ and PR are tangents to the circle centered at **O**. If $\angle OPR = 45°$, then prove that $ORPQ$ is a square.

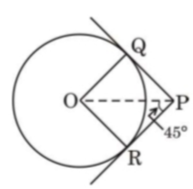


Figure 3.3.6.1: Two tangents drawn from point **P** to a circle whose centre is **O**

3.3.7. In Figure 3.3.7.1, **O** is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at **A** and **B** respectively. If OP is 13 cm, then find the length of tangents PA and BC.

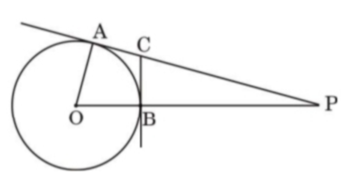


Figure 3.3.7.1: Two tangents drawn from point **C** to a circle whose centre is **O**

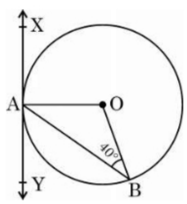


Figure 3.3.9.1: The line XAY is tangent to the circle centered at **O**

3.3.8. In Figure 3.3.8.1, AB is diameter of a circle centered at **O**. BC is tangent to the circle at **B**. If OP bisects the chord AD and $\angle AOP = 60°$, then find $m\angle C$.

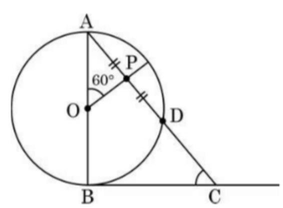


Figure 3.3.8.1: Tangent BC is drawn from point **C** to a circle whose centre is **O**

3.3.9. In Figure 3.3.9.1, XAY is a tangent to the circle centered at **O**. If $\angle ABO = 60°$,then find $m\angle BAY$ and $m\angle AOB$.

3.3.10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

3.3.11. In Figure 3.3.11.1, a triangle ABC with $\angle B = 90°$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point **P**. Prove that the tangent drawn at point **P** bisects BC.

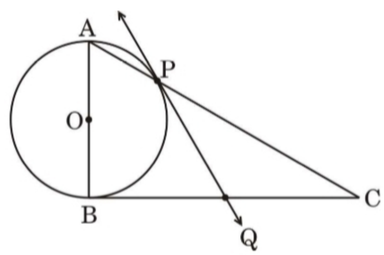


Figure 3.3.11.1: PQ is tangent to the circle centered at **O**. AB is the diameter and $\angle B = 90°$

3.3.12. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

3.4. 2021

3.4.1. 10

1. A quadrilateral $ABCD$ is drawn to circumscribe a circle (see Figure-1). Prove that $AB + CD = AD + BC$.

2. Draw a pair of tangents to a circle of radius $4cm$ which are inclined to each other at an angle of $45°$.

3. A point **T** is $13cm$ away from the centre of a circle. The length of the tangent drawn from **T** to the circle is $12cm$. Find the radius of the circle.

4. Two tangents TP and PQ are drawn to a circle with centre **O** from an external point **T**. Prove that $\angle PTQ = 2\angle OPQ$.

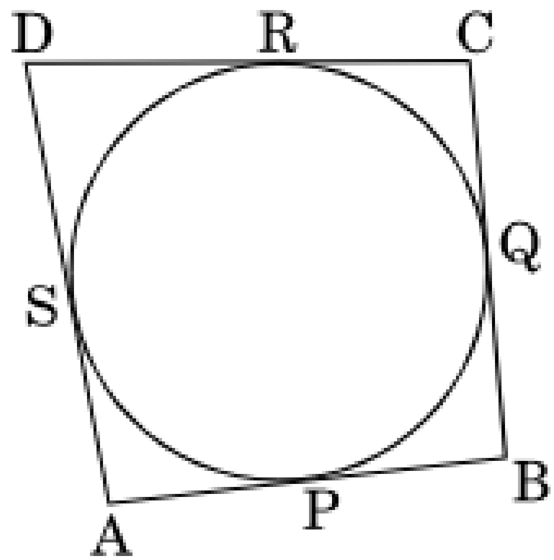


Figure 1.1:

5. PQ is a tangent to a circle with centre **O** at the point **P** on the circle. If $\triangle OPQ$ is an isosceles triangle, then find $\angle OQP$.

6. Two concentric circles have radii $10cm$ and $6cm$. Find the length of the chord of the larger circle which touches the smaller circle.

7. If tangents PA and PB from an external point **P** to a circle with centre **O** are inclined to each other at an angle of $70°$, then find $\angle POA$.

8. ABC is right triangle, right-angled at **B** with $BC = 6cm$ and $AB = 8cm$. A circle with centre **O** and radius r cm has been inscribed in $\triangle ABC$ as shown in the figure. Find the value of r.

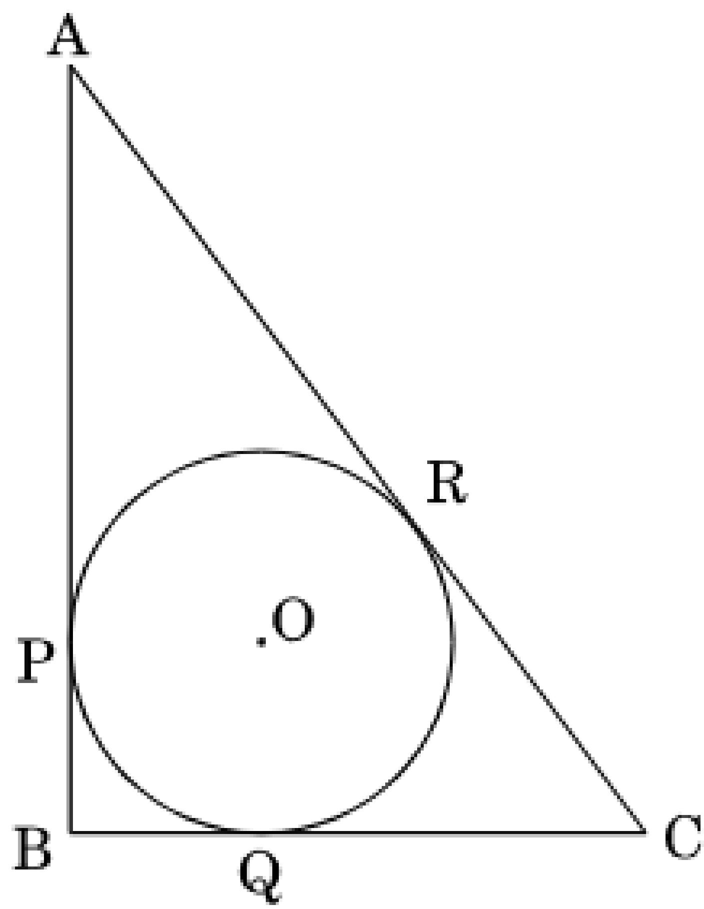


Figure 8.1:

9. Draw a circle of radius $5cm$. From a point $8cm$ away from its centre, construct a pair of tangents to the circle.

10. In the given figure, PT and PS are tangents to a circle with centre **O**, from a point **P**, such that $PT = 4cm$ and $\angle TPS = 60°$. Find the length of the chord TS. Also, find the radius of the circle.

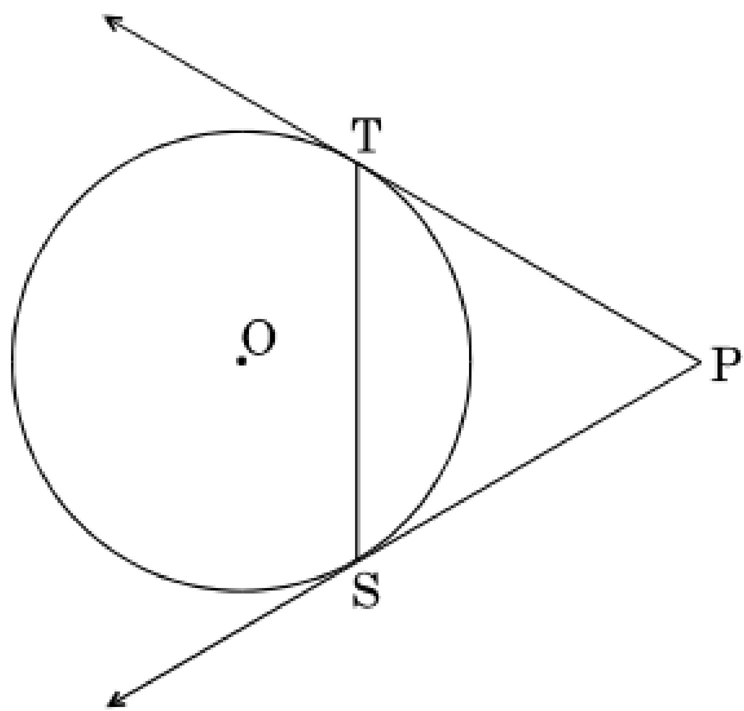


Figure 10.1:

11. (a) In a right triangle ABC, right-angled at **B**, $BC = 6cm$ and $AB = 8cm$. A circle is inscribed in the $\triangle ABC$. Find the radius of the incircle.

    (b) Two circles touch externally at **P** and AB is a common tangent, touching one circle at **A** and the other at **B**. Find the measure of $\angle APB$.

12. From an external point **P**, tangents PQ and PR are drawn to a circle with centre **O**, touching the circle at **Q** and **R**. If $\angle QOR = 140°$, find the measure of $\angle QPR$.

13. A circle touches all the sides of a quadrilateral $ABCD$. Prove that $AB + CD = DA + BC$.

14. Write the steps of construction of a circle of diameter $6cm$ and drawing of a pair of tangents to the circle from a point $5cm$ away from the centre.

Chapter 4

Intersection of Conics

4.1. 2022

4.1.1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

4.1.2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

4.1.3. Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

4.1.4. If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

4.1.5. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

4.1.6. If the area of the region bounded bythe line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

4.1.7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

4.1.8. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

4.1.9. Find the area of the region $\{(x, y) : x^2 \le y \le x\}$, using integration

4.2. 2021

4.2.1. 12

1. The point at which the normal to the curve

$$y = x + \frac{1}{x}, x > 0 \tag{1.1}$$

is perpendicular to the line

$$3x - 4y - 7 = 0 \tag{1.2}$$

(a) $\left(2, \frac{5}{2}\right)$

(b) $\left(\pm 2, \frac{5}{2}\right)$

(c) $\left(-\frac{1}{2}, \frac{5}{2}\right)$

(d) $\left(\frac{1}{2}, \frac{5}{2}\right)$

2. The points on the curve

$$\frac{x^2}{9} + \frac{y^2}{16} = 1 \tag{2.1}$$

at which the tangents are parallel to y-axis are:

(a) $(0, \pm 4)$

(b) $(\pm 4, 0)$

(c) $(\pm 3, 0)$

(d) $(0, \pm 3)$

3. For which value of m is the line

$$y = mx + 1 \tag{3.1}$$

a tangent to the curve

$$y^2 = 4x \tag{3.2}$$

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 3

Chapter 5

Probability

5.1. 2021

5.1.1. 10

5.1.1. During the lockdown period,many families got bored of watching TV all the time.Out of these families, one family of 6 members decided to play a card game.17 cards numbered 1,2,3,4,..,17 are put in a box and mixed thoroughly.One card is drawn by one member at random and other family members bet for the chances of drawing the number either prime,odd or even etc.

Figure 5.1.1.1: Family of six

Based on the above,answer the following questions:

(i) The first member of the family draws a card at random and another member bets that it is an even prime number.What is the probability of his winning the bet?

(A) $\frac{2}{17}$

(B) $\frac{3}{17}$

(C) $\frac{1}{17}$

(D) $\frac{4}{17}$

(ii) The second member of the family draws a card at random and some other member bets that it is an even number. What is the probability of his winning the bet ?

(A) $\frac{7}{17}$

(B) $\frac{8}{17}$

(C) $\frac{9}{17}$

(D) $\frac{10}{17}$

(iii) What is the probability that the number on the card drawn at random is divisible by 5 ?

(A) $\frac{5}{17}$

(B) $\frac{4}{17}$

(C) $\frac{3}{17}$

(D) $\frac{2}{17}$

(iv) What is the probability that the number on the card drawn at random is a multiple of 3 ?

(A) $\frac{5}{17}$

(B) $\frac{6}{17}$

(C) $\frac{7}{17}$

(D) $\frac{8}{17}$

5.1.2. (a) Two different coins are tossed simultaneously. Write all the possible outcomes.

(b) A die is thrown once. Write the probability of getting a number less than 7.

5.1.3. If the probability of occurrence of event E,$\Pr(E)$=0.99, what is the probability of non-occurrence of the event E,$\Pr(notE)$?

5.1.4. (a) A bag contains 5 white balls and 7 red balls.A ball is drawn at random from the bag.What is the probability that it is either a white or a red ball?

(b) Two coins are tossed together once.What is the probability of getting at least one head?

5.1.5. Cards marked with numbers 1,2,3,4, ..., 100 are placed in a bag and mixed together throughly. A card is randomly drawn from the bag.Find the probability that the numbers on the card is

(i) an even number,

(ii) a 2-digit number,

(iii) a perfect square.

5.1.6. (a) How many outcomes are possible when three dice are thrown together?

(b) if $\Pr(E)$=0.015, then find $\Pr(notE)$.

5.1.7. During summer break, Harish wanted to play with his friends but it was too hot outside, so he decided to play some indoor game with his friends. He collects 20 identical Icards and writes the numbers 1 to 20 on them (one number on one card). He puts them in a box. He and his friends make a bet for the chances of drawing various cards out of the box. Ench was given a chance to tell the probability of picking one card out of the box.

Based on the above,answer the following questions:

(i) The probability that the number on the card drawn is an odd prime number,is

(A) $\frac{3}{5}$

(B) $\frac{2}{5}$

(C) $\frac{9}{20}$

(D) $\frac{7}{20}$

(ii) The probability that the number on the card drawn is a composite number is

(A) $\frac{11}{20}$

(B) $\frac{3}{5}$

(C) $\frac{4}{5}$

(D) $\frac{1}{2}$

(iii) The probability that the number on the card drawn is a multiple of 3,6 and 9 is

(A) $\frac{1}{20}$

(B) $\frac{1}{20}$

(C) $\frac{3}{20}$

(D) 0

(iv) The probability that the number on the card drawn is a multiple of 3 and 7is

(A) $\frac{3}{10}$

(B) $\frac{1}{10}$

(C) 0

(D) $\frac{2}{5}$

(v) If all cards having odd numbers written on them are removed from the box and then one card is drawn from the remaining cards, the probability of getting a card having a prime number is

(A) $\frac{1}{20}$

(B) $\frac{1}{10}$

(C) 0

(D) $\frac{1}{5}$

5.1.8. (a) In a single throw of a pair of dice,find the probability that both dice have the same number.

(b) A card is drawn from a well-shuffled pack of 52 cards.Find the probability that it is not an ace.

5.1.2. 12

1. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively.If both try to solve the question independently, the probability that the question is solved is

(a) $\frac{7}{15}$

(b) $\frac{8}{15}$

(c) $\frac{2}{15}$

(d) $\frac{14}{15}$

2. From a pack of 52 cards, 3 cards are drawn at random (without replacement).The probability that they are two red cards and one black card is__________.

3. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

4. Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

5. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement), both of which are found to be red. Find the probability that these two balls are drawn from the second bag.

6. An unbiased die is thrown. What is the probability of getting an odd number or a multiple of 3 ?

   (a) $\frac{3}{4}$

   (b) $\frac{1}{2}$

   (c) $\frac{2}{3}$

   (d) $\frac{1}{3}$

7. A card is drawn from an ordinary pack of 52 cards and a gambler bets that it is a heart or a king card. What are the odds against his winning

this bet ?

(a) 4 : 9

(b) 1 : 4

(c) 4 : 1

(d) 9 : 4

8. In a lottery of 25 tickets, numbered 1 to 25, two tickets are drawn simultaneously. Find the probability that none of the tickets has prime number.

9. If E_1 and E_2 are two events, where E_1 is a subset of E_2, then evaluate $P(E_2 \mid E_1)$.

10. Two dice are thrown simultaneously. Find the probability of getting a multiple of 3 on one dice and a multiple of 2 on the other dice.

11. An urn contains 4 white, 7 green and 9 blue balls. If two balls are drawn at random, find the probability that the drawn balls are of the same colour.

5.2. 2023

5.2.1. 10

5.2.1. Probability of happining of an event is denoted by p and probability of non-happening of the event is denoted by q. Relation between p and q is

(a) $p+q=1$

(b) $p=1$, $q=1$

(c) $p=q-1$

(d) $p+q+1=0$

5.2.2. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought ?

(a) 40

(b) 240

(c) 480

(d) 750

5.2.3. In a group of 20 people, 5 can't swim. If one person is selected at random, then the probability that he/sh can swim, is

(a) $\frac{3}{4}$

(b) $\frac{1}{3}$

(c) 1

(d) $\frac{1}{4}$

5.2.4. A bag contain 4 red, 3 blue and 2 yellow balls. One ball is drawn at random from the bag. Find the probability that drawn ball is

(a) red

(b) yellow

5.2.5. A bag contain 100 cards numbered 1 to 100.Acard is drawn at random from the b. What is the probability that the number on the card is a perfect cube ?

(a) $\frac{1}{20}$

(b) $\frac{3}{50}$

(c) $\frac{1}{25}$

(d) $\frac{7}{100}$

5.2.6. If three coins are tossed simultaneously, what is the probability of getting a most one trail ?

(a) $\frac{3}{8}$

(b) $\frac{4}{8}$

(c) $\frac{5}{8}$

(d) $\frac{7}{8}$

5.2.7. Two dics are thrown together. The probability of getting the difference of numbers on their upper faces equals to 3 is :

(a) $\frac{1}{9}$

(b) $\frac{2}{9}$

(c) $\frac{1}{6}$

(d) $\frac{1}{12}$

5.2.8. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that the card drawn is not an ace is :

(a) $\frac{1}{13}$

(b) $\frac{9}{13}$

(c) $\frac{4}{13}$

(d) $\frac{12}{13}$

5.2.9. **Assertion (A) :** The probability that a leap year has 53 Students is $\frac{2}{7}$.

Reason (R) : The probability that a non-leap year has 53 Sundays is $\frac{5}{7}$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

5.2.2. 12

5.2.1. If A and B are two events such that

$$\Pr(A|B) = 2 \times \Pr(B|A)\Pr(A) + \Pr(B) = \frac{2}{3} \tag{5.2.1.1}$$

then $\Pr(B)$ is equal to

(a) $\frac{2}{9}$

(b) $\frac{7}{9}$

(c) $\frac{4}{9}$

(d) $\frac{5}{9}$

5.2.2. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.

(b) A and B throw a die alternately till one of them gets '6' and wins the game. Find their respective probabilities of winning, if A starts the game first.

5.2.3. Recent studies suggest that roughly 12% of the world population is left handed.

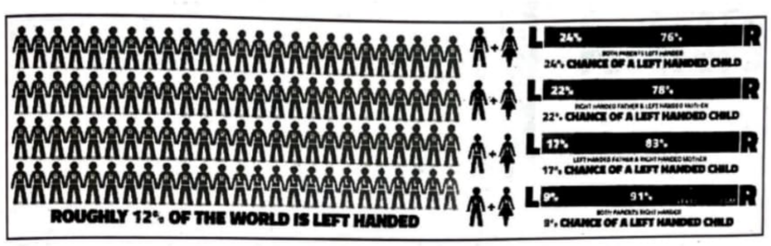


Figure 1: chance of left hand, depending upon parents

Figure 5.2.3.1: chance of left hand, depending upon parents

Depending upon the parents, the chances of having a left handed child are as follows :

(a) When both father and mother are left handed : Chances of left handed child is 24%.

(b) When father is right handed and mother is left handed : Chances of left handed child is 22%.

(c) when father is left handed and mother is right handed : Chances of left handed child is 17%.

(d) When both father nd mother are right handed : Chances of left handed child is 9%.

Assuming that $\Pr(A) = \Pr(B) = \Pr(C) = \Pr(D) = \frac{1}{4}$ and L denotes the event that child is left handed. Based on the above information, answer the following questions :

(a) Find $\Pr(L|C)$

(b) Find $\Pr\left(\overline{L}|A\right)$

(c) Find $\Pr(A|L)$

(d) Find the probability that a randomly selected child is left handed given that exactly one of the parent is left handed.

5.3. 2022

5.3.1. 10

5.3.1. Two dice are thrown simultaneously. The probability that the sum of two numbers appearing on the top of the dice is less than 12, is

(a) $\frac{1}{36}$

(b) $\frac{35}{36}$

(c) 0

(d) 1

5.3.2. A jar contains 18 marbles. Some are red and others are yellow. If a marble is drawn at random from the jar, the probability that it is red is $\frac{2}{3}$. Find the number of yellow marbles in the jar.

5.4. 2022

5.4.1. 12

5.4.1. Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A|B) = \frac{3}{4}$. Find the value of $P(B|A)$.

5.4.2. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variables X denotes the number of red balls. Find the probabillity distribution of X.

5.4.3. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.

5.4.4. Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved.

5.4.5. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number apprearing on at least one die is 3.

5.4.6. At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. such a coin is unbaised with equal probabilities of getting head and tail Fig. 5.4.6.1 .

Figure 5.4.6.1: Toss before the match

Based on the above information, answer the following question:

(a) If such a coin is tossed 2 times, then find the probability distibution of numbers of tails.

(b) Find the probability of getting at least one head in three tosses of such a coin.

5.4.7. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards.

5.4.8. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on at least one die.

5.4.9. The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probabillity that the target is hit.

5.4.10. A shopkeeper sells three types of flower seeds A_1 , A_1 , A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germinaton rates of the three types of seeds are 45%, 60% and 35% respectively Fig. 5.4.10.1.

Figure 5.4.10.1: Three types of flowers

Based on the above information :

(a) Calculate the probability that a randomly chosen seed will germinate.

(b) Calculate the probability that the seed is of type A_2, given that a randomly choosen seed germinates.

5.4.11. Three friends A, B and C got their photograph clicked. Find the probability that B is standing at the central position, given that A is standing at the left corner.

5.4.12. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are alloted points 9 to 1 in sequential order moving outwards.Archer A is likely to earn 10 points with a probability of 0.8 and Archer B is likely the earn 10 points with a probability of 0.9 Fig. 5.4.12.1.

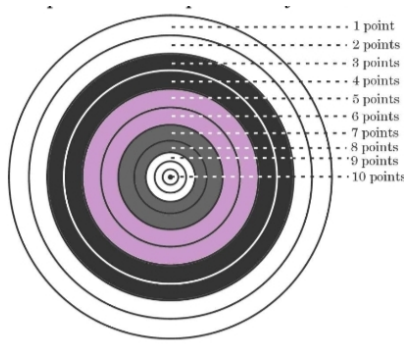


Figure 5.4.12.1: centermost ring

Based on the above innformation, answer the following questions :

(a) exactly one of them earns 10 points .

(b) both of them earn 10 point.

5.4.13. Event A and B are such that

$$P(A) = \frac{1}{2}, P(B) = \frac{7}{12} \tag{5.4.13.1}$$

and

$$P(\bar{A} \cup \bar{B}) = \frac{1}{4} \tag{5.4.13.2}$$

Find whether the events A and B are independent or not.

5.4.14. A box B_1 contain 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn are of the same colour.

5.4.15. Let X be random variable which assumes values x_1, x_2, x_3, x_4 such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4). \tag{5.4.15.1}$$

Find the probability distribution of X.

5.4.16. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes , is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.

5.4.17. In a toss of three different coins, find the probability of comming up of three heads, if it is known that at least one head comes up.

5.4.18. A laboratory blood text is 98% effective in detecting a certain disease when it is fact, present. However, the text also yeilds a false positive result for 0.4% of the healthy person tested. From a large population, it is given that 0.2% of the population actually has the diseases. Based on the above, answer the following questtion :

(a) one person, from the population, is taken at random and given the test. Find the probabiliy of his getting a positive test result.

(b) what is the probability that the person actually has the disease, given that his test result is positive ?

5.4.19. Two cards are drawn from a well-shuffled pack of playing cards one-by-one with replacement. The probability that the first card is a king and the second card is a queen is

(a) $\frac{1}{13} + \frac{1}{13}$

(b) $\frac{1}{13} \times \frac{4}{51}$

(c) $\frac{4}{52} \times \frac{3}{51}$

(d) $\frac{1}{13} \times \frac{1}{13}$

5.4.20. For two events A and B if P(A) $= \frac{4}{10}, PB = \frac{8}{10}$ and $P(B|A) = \frac{6}{10}$ then find $P(A \cup B)$.

5.4.21. Bag I contain 4 red and 3 black balls. Bag II contains 3 red and 5 black balls. One of two bags is selected at random and a ball is drawn from the bag, which if found to be red. Find the probability that the ball is drawn from bag II.

5.4.22. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces and hence find its mean.

5.4.23. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively . If both try to solve the question independently, the probability that the question is solved is

(a) $\frac{7}{15}$

(b) $\frac{8}{15}$

(c) $\frac{2}{15}$

(d) $\frac{14}{15}$

5.4.24. A card is picked at random from a pack of 52 playing cards. Given that the picked up card is a queen, the probability of it being a queen of spades is ______.

5.4.25. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

5.4.26. Find the probability distribution of the numbers of successes in two tosses of a die, when a success is defined as number greater than 5.

5.4.27. Ten cartoons are taken at random from an automatic packing machine. The mean net weight of the ten carton is 11.8 kg and standard deviation is 0.15 kg. Does the sample mean differ significantly from the intended mean of 12 kg ? [Given that for d.f. = 9, $t_{0.05} = 2.26$]

5.4.28. A Coin is tossed twice. The following table5.4.28.2 shows the probability distribution of numbers of tails:

X	0	1	2
P(X)	K	6K	9K

Table 5.4.28.2: Table shows the probability distribution of numbers of tails

(a) Find the value of K.

(b) Is the coin tossed biased or unbaised? Justify your answer.

5.4.29. If X is a random variable with probability distribution as given below 5.4.29.2:

X	0	1	2
P(X)	K	4K	K

Table 5.4.29.2: table shows the proability distribution

The value of K and the mean of the distribution respectively are

(a) $\frac{1}{7}, 1$

(b) $\frac{1}{6}, 2$

(c) $\frac{1}{6}, 1$

(d) $1, \frac{1}{6}$

5.4.30. The random variable X has a probability function P(x) as defined below, where K is some number :

$$P(X) = \begin{cases} K, & \text{if } x = 0 \\ 2K, & \text{if } x = 1 \\ 3K, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases} \tag{5.4.30.1}$$

Find:

(a) The value of K.

(b) $P(X < 2), P(X \leq 2), P(X \geq 2)$.

5.4.31. Two rotten apples are mixed with 8 fresh apples. Find the probability distribution of number of rotten apples, if two apples are drawn at random, one-by-one without replacement.

5.4.32. A die is thrown twice. What is the probability that

(i) 5 will come up at least once, and

(ii) 5 will not come up either time ?

5.4.33. Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A/B) = \frac{3}{4}$. Find the value of $P(B/A)$.

5.4.34. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability distribution of X.

5.4.35. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.

5.4.36. Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved.

5.4.37. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3.

5.4.38. In Fig. 5.4.38.1,At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.

Figure 5.4.38.1: Tossing a coin

Based on the above information, answer the following questions :

(a) If such a coin is tossed 2 times, then find the probability distribution of number of tails.

(b) Find the probability of getting at least one head in three tosses of such a coin.

5.4.39. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards.

5.4.40. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die.

5.4.41. In Fig. 5.4.41.1, A shopkeeper sells three types of flower seeds $A1$, $A2$, $A3$. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Figure 5.4.41.1: Three Types of Flower Seeds

Based on the above information:

(a) Calculate the probability that a randomly chosen seed will germinate;

(b) Calculate the probability that the seed is of type $A2$, given that a randomly chosen seed germinates.

5.4.42. Three friends A, B and C got their photograph clicked. Find the probability that B is standing at the central position, given that A is standing at the left corner.

5.4.43. In Fig. 5.4.43.1 A coin is tossed twice. The following table shows the probability distribution of number of tails :

X	0	1	2
P(X)	K	6K	9K

Figure 5.4.43.1: Probability Distribution of number of tails

(a) Find the value of K.

(b) Is the coin tossed biased or unbiased ? Justify your answer.

5.4.44. In Fig. 5.4.44.1 In a game of Archery, each ring of the Archery target is valued. The centre most ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.

Archer A is likely to earn 10 points with a probability of 08 and Archer B is likely the earn 10 points with a probability of 09.

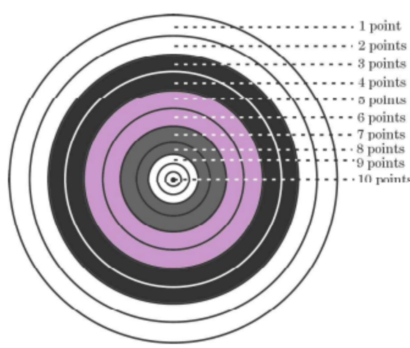


Figure 5.4.44.1: Ring of the Archery Target

Based on the above information, answer the following questions : If both of them hit the Archery target, then find the probability that

(a) exactly one of them earns 10 points.

(b) both of them earn 10 points.

5.4.45. (a) Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\overline{A} \cup \overline{B}) = \frac{1}{4}$ Find whether the events A and B are independent or not.

(b) A box B_1 contains 1 white ball and 3 red balls.Another box B_2 contains 2 white balls and 3 red balls.If one ball is drawn at random from each of the boxes B_1 and B_2 then find the probability that the two balls drawn are of the same colour.

5.4.46. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes, is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.

5.4.47. In a toss of three different coins, find the probability of coming up of three heads, if it is known that at least one head comes up.

5.4.48. Two rotten apples are mixed with 8 fresh apples. Find the probability distribution of number of rotten apples, if two apples are drawn at random, one-by-one without replacement.

5.4.49. A laboratory blood test is 98% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 04% of the healthy person tested. From a large population, it is given that 0·2% of the population actually has the disease. Based on the above, answer the following questions :

(a) One person, from the population, is taken at random and given the test. Find the probability of his getting a positive test result.

(b) What is the probability that the person actually has the disease, given that his test result is positive ?

5.4.50. Two cards are drawn from a well-shuffled pack of playing cards one-by-one with replacement. The probability that the first card is a king and the second card is a queen is

(a) $\frac{1}{13} + \frac{1}{13}$

(b) $\frac{1}{13} \times \frac{4}{51}$

(c) $\frac{4}{52} \times \frac{3}{51}$

(d) $\frac{1}{13} \times \frac{1}{13}$

5.4.51. In Fig. 5.4.51.1 If X is a random variable with probability distribution as given below :

X	0	1	2
P(X)	k	4k	k

Figure 5.4.51.1: Probability Distribution

The value of k and the mean of the distribution respectively are

(a) $\frac{1}{7}$,1

(b) $\frac{1}{6}$,2

(c) $\frac{1}{6}$,1

(d) $\frac{1}{6}$

5.4.52. For two events A and B if P(A) = $\frac{4}{10}$, P(B) = $\frac{8}{10}$ and $P(B \mid A)$=$\frac{6}{10}$, then find $P(A \cup B)$.

5.4.53. Bag I contains 4 red and 3 black balls. Bag II contains 3 red and 5 black balls. One of the two bags is selected at random and a ball is drawn from the bag, which is found to be red. Find the probability that the ball is drawn from Bag II.

5.4.54. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces and hence find its mean.

5.4.55. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the question independently, the probability that the question is solved is

(a) $\frac{7}{15}$

(b) $\frac{8}{15}$

(c) $\frac{2}{15}$

(d) $\frac{14}{15}$

5.4.56. A card is picked at random from a pack of 52 playing cards. Given that the picked up card is a queen, the probability of it being a queen of spades is ?

5.4.57. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

5.4.58. Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

5.4.59. The random variable X has a probability function $P(x)$ as defined below, where k is some number :

$$p(x) = \begin{cases} k, & \text{if } x = 0, \\ 2k, & \text{if } x = 1, \\ 3k, & \text{if } x = 2, \\ 0, & \text{otherwise.} \end{cases} \quad (5.4.59.1)$$

Find :

(i) The value of k

(ii) $P(X < 2)$, $P(X \leq 2)$, $P(X \geq 2)$

5.4.60. Consider the following hypothesis :

$$H0 : \mu = 35 \quad (5.4.60.1)$$

$$H1 : \mu \neq 35 \quad (5.4.60.2)$$

A sample of 81 items is taken whose mean is 375 and the standard deviation is 5. Test the hypothesis at 5% level of significance.

[Given : Critical value of Z for a two-tailed test at 5% level of significance is 1.96]

5.4.61. In Fig. 5.4.61.1 Fit a straight line trend by the method of least squares and find the trend value for the year 2008 for the following data :

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40
2007	36

Figure 5.4.61.1: Years and Production

Chapter 6

Construction

6.0.1. In the given figure, XZ is parallel to BC. $AZ = 3$cm, $ZC = 2$cm, BM $= 3$cm and $MC = 5$cm. Find the length of XY.

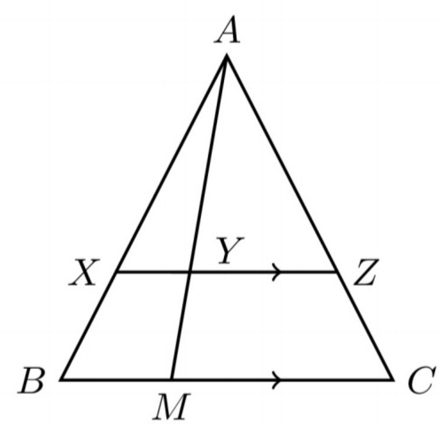


Figure 1: Isosceles Triangle

Figure 6.0.1.1: Isosceles Triangle

6.0.2. In the given figure, $DE \parallel BC$. If $AD = 2$units, $DB = AE = 3$units and $EC = x$units, then find the value of x is:

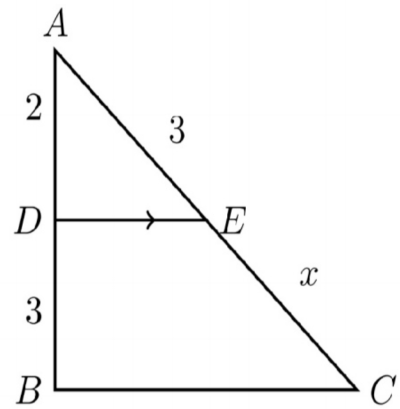


Figure 2: Right Angle Triangle

Figure 6.0.2.1: Right Angle Triangle

(a) 2

(b) 3

(c) 5

(d) $\frac{9}{2}$

6.0.3. In the given figure, ΔABC and ΔDBC are on te same base BC. If AD intersects BC at **O**, prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.

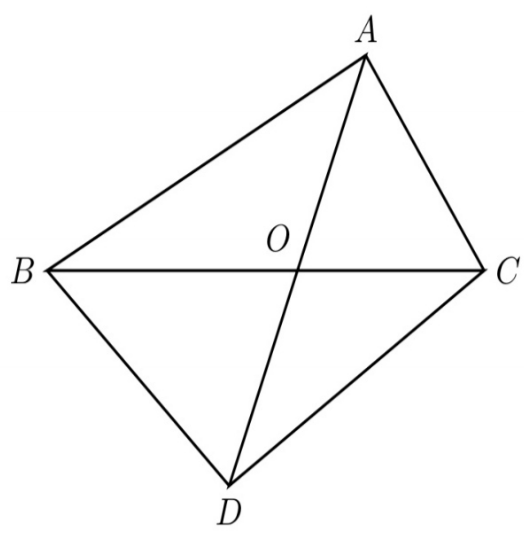


Figure 3: Triangles with same base

Figure 6.0.3.1: Triangles with same base

6.1. 2022

6.1.1. 10

6.1.1. In figure,Fig. 6.1.1.1 BN and CM are medians of a △ ABC right-angled at A. Prove that

$$4(BN^2 + CM^2) = 5BC^2 \tag{6.1.1.1}$$

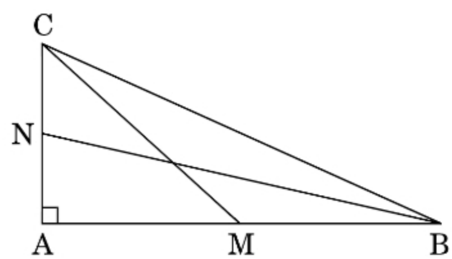


Figure 6.1.1.1: Right-angled triangle

6.1.2. **CaseStudy − 1** :

KiteFestival

Kite festival is celebrated in many countries at different times of the year. in India, every year 14th January is celebrated as international kite Day. on his day many people visit India and participate in the festival by flying various kinds of kites.

The picture given below Fig. 6.1.2.1 , three kites flying together.

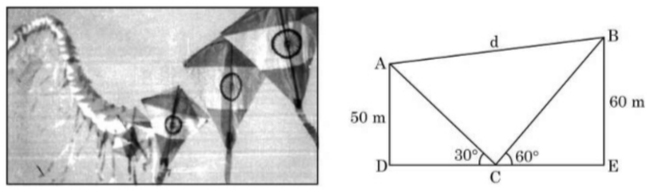


Figure 6.1.2.1: kites flying to gether

In Fig. 6.1.2.1, the angles of elevation of two kites (point C) are found to be °30 and °60 respectively. Taking

$$AD = 50m \tag{6.1.2.1}$$

and

$$BE = 60m \tag{6.1.2.2}$$

find

(a) The length of string used (take them straight) for kites A and B as shown in the figure.

(b) The distance 'd' between these two kites

6.1.3. In Fig. 6.1.3.1, $PQ \parallel BC$, $PQ = 3cm$, $BC = 9cm$ and $AC = 7.5cm$. Find the length of AQ.

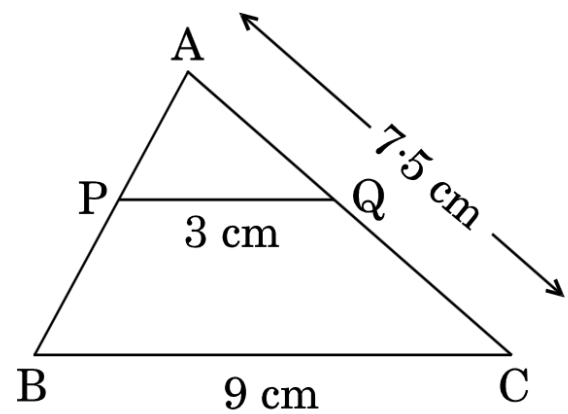


Figure 6.1.3.1: $PQ \,||\, BC$

6.1.4. Draw a circle of radius $2.5cm$. Take a point P outside the circle at a distance of $7cm$ from the centre. Then construct a pair of tangents to the circle from point P.

6.1.5. Sides AB and AC and median AD of $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.

6.1.6. In Fig. 6.1.6.1 BN and CM are medians of a $\triangle ABC$ right-angled at A. Prove that $4(BN^2 + CM^2) = 5BC^2$.

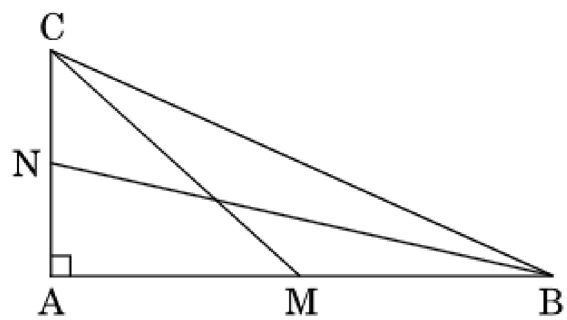


Figure 6.1.6.1: BN and CM are medians

6.1.7. Construct a pair of tangents to a circle of radius $4cm$ from a point P lying outside the circle at a distance of $6cm$ from the centre.

6.1.8. (a) Draw a line segment AB of length $8cm$ and locate a point P on AB such that $AP : PB = 1 : 5$.

(b) Draw a circle of radius $3cm$. From a point P lying outside the circle at a distance of $6cm$ from its centre, construct two tangents PA and PB to the circle.

6.1.9. Construct a pair of tangents to a circle of radius $5cm$ which are inclined each other at an angle of 60^o.

6.1.10. Write the steps of construction for constructing a pair of tangents to a circle of radius $4cm$ from a point P, at a distance of $7cm$ from its centre O.

6.2. 2021

6.2.1. 10

6.2.1. (a) **D** and **E** are points on the sides CA and CB respectively of a triangle ABC, right-angled at **C**.

Prove that $AE^2 + BD^2 = AB^2 + DE^2$

(b) Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point **O**. If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD.

6.2.2. Write the steps of construction of drawing a line segment $AB = 4.8$ cm and finding a point **P** on it suchthat $AP = \frac{1}{4}AB$.

6.2.3. Answer any $four$ of the following questions :

(a) Given $\triangle ABC$ $\triangle PQR$. If $\frac{AB}{PQ} = \frac{1}{3}$, than $\frac{ar(\triangle ABC)}{ar(\triangle PQR)}$ is

i. $\frac{1}{3}$

ii. 3

iii. $\frac{2}{3}$

iv. $\frac{1}{9}$

(b) The length of an altitude of n equilateral triangle of side 8 cm is

i. 4 cm

ii. $4\sqrt{3}$ cm

iii. $\frac{8}{3}$ cm

iv. 12 cm

(c) In $\triangle PQR, PQ = 6\sqrt{3}$ cm, $PR = 12cm$ and $QR = 6cm$. The measure of angle **Q** is

i. $120°$

ii. $60°$

iii. $90°$

iv. $45°$

(d) If $\triangle ABC$ $\triangle PQR$ and $\angle B = 46°$ and $\angle R = 69°$, then the measure of $\angle A$ is

i. $65°$

ii. $111°$

iii. $44°$

iv. $115°$

(e) **P** and **Q** are the points on the sides AB and AC respectively of a $\triangle ABC$ such that $PQ \parallel BC$. If $AP \parallel PB = 2:3$ and $AQ = 4cm$ then AC is equal to

i. 6 cm

ii. 8 cm

iii. 10 cm

iv. 12 cm

6.2.4. Answer any $four$ of the following questions :

(a) ABC and BDE are two equilateral triangles such that **D** is the mid-point of BC. The ratio of the areas of the triangles ABC and BDE is

i. 2 : 1

ii. 1 : 2

iii. 4 : 1

iv. 1 : 4

(b) In $\triangle ABC, AB = 4\sqrt{3}$ cm, $AC = 8cm$ and $BC = 4cm$. The angle **B** is

i. 120°

ii. 90°

iii. 60°

iv. 45°

(c) The perimeters of two similar triangles are 35 cm and 21 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of the second triangle is

i. 5.4 cm

ii. 4.5 cm

iii. 5.6 cm

iv. 15 cm

(d) In a $\triangle ABC$, **D** and **E** are points on the sides AB and Ac respectively such that $DE \parallel BC$ and $AD : DB = 3 : 1$. If $AE = 3.3cm$, then AC is equal to

i. 4 cm

ii. 1.1 cm

iii. 4.4 cm

iv. 5.5 cm

(e) In the isosceles triangle ABC, if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C$ is equal to

i. $30°$

ii. $45°$

iii. $60°$

iv. $90°$

Chapter 7

Optimization

7.1. 2023

1. The objective function $Z = ax + by$ of an LLP has maximum value 42 at (4,6) and minimum value 19 at (3,2).Which of the following is true?

   (a) $a = 9, b = 1$

   (b) $a = 5, b = 2$

   (c) $a = 3, b = 5$

   (d) $a = 5, b = 3$

2. The corner point of the feasible region of a linear programming problem are (0,4), (8,0)and $(\frac{20}{3}, \frac{4}{3})$.if $Z = 30x + 24y$ is the objective fnction, then (maximum value of Z-minimum value of Z) is equal to

   (a) 40

   (b) 96

   (c) 120

   (d) 136

3. Solve the following linear programming problem graphically :

$$Maximum : Z = x + 2y$$
$$subjecttoconstraints : x + 2y \geq 100,$$
$$2x - y \leq 0,$$
$$2x + y \leq 200,$$
$$x \geq 0, y \geq 0.$$

4. Engine displacement is the measure of the cylinder volume swept by all the pistons engine.The piston move inside the cylinder bore

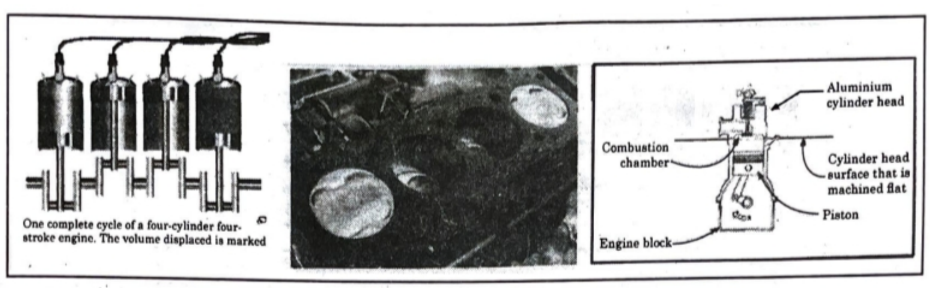


Figure 4.1: Engine

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi cm^2$

Based on the above information,answer the following questions:

(a) if the radius of cylinder is r cm and height is h cm,then write the volme V of cylinder in terms of radius r.

(b) Find $\frac{dV}{dr}$.

(c) i. Find the radius of cylinder when its volume is maximum.

ii. For maximum volume,$h > r$.State true or false and justify.

7.2. 2021

7.2.1. 12

7.2.1. If the corner points $(3,4)$ and $(5,0)$ of the feasible region in an LPP, give the same maximum value for the objective function $z = ax + by$, where a,b > 0, then we have

(a) $a = 2b$

(b) $2a = b$

(c) $2a = 3b$

(d) $3b = 2a$

7.2.2. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs 1 ₹50 per kg to purchase Food I and

₹70 per kg to purchase Food II. Formulate this problem as a Linear Programming Problem for minimizing the cost of such a mixture.

7.2.3. Show that of all the rectangles inscribed in a given fixed circle, the square has maximum area.

Find the intervals in which the function f given by $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

7.2.4. A company produces two types of goods, A and B that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold, while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹120 and that of type B ₹150 , then find the number of units of each type that the company should produce to maximize profit.

Formulate the above LPP and solve it graphically. Also, find the maximum profit.

7.2.5. Find the intervals in which the function f defined as $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is strictly increasing or decreasing.

Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

7.2.6. Maximize $z = 3x + 4y$, if possible, subject to the constraints :

$$x - y \leq -1 \tag{7.2.6.1}$$

$$-x + y \leq 0 \tag{7.2.6.2}$$

$$x, y \geq 0 \tag{7.2.6.3}$$

7.2.7. A dietician wishes to mix two types of foods F1 and F2 in such a way that the vitamin content of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food F1 contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C, while Food F2 contains 1 unit/kg of vitamin A and 2units/kg of vitamin C. It costs ₹5 per kg to purchase Food F1 and ₹7 per kg to purchase Food F2

Based on the above information, answer the following questions:

(a) To find out the minimum cost of such a mixture, formulate the above problem as a LPP.

(b) Determine the minimum cost of the mixture.

7.2.8. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

7.3. 2022

7.3.1. 12

1. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires $3g$ of silver and $1g$ of gold, while that of type B requires $1g$ of silver and $2g$ of gold. The company can use at the most $9g$ of silver and $8g$ of gold. If each unit of type A brings a profit of ₹ 120 and that of type B ₹ 150, then find the number of units of each type that the company should produce to maximise profit. Formulate the above LPP and solve it graphically. Also, find the maximum profit.

2. Find the maximum value of $7x + 6y$ subject to the constrains:

$$x + y \geq 2 \tag{2.1}$$

$$2x + 3y \leq 6 \tag{2.2}$$

$$x \geq 0 \text{and} y \geq 0 \tag{2.3}$$

3. A window is in the form of a rectangular mounted by a semi-circular opening.The total perimeter of the window to admit maximum light through the whole opening.

4. Divide the number 8 into two positive numbers such that the sum of the cube of one and the square of the other is maximum.

5. Find the maximum and the minimum values of

$$z = 5x + 2y \tag{5.1}$$

subject to the constrains:

$$-2x - 3y \leq -6 \tag{5.2}$$

$$x - 2y \leq 2 \tag{5.3}$$

$$6x + 4y \leq 24 \tag{5.4}$$

$$-3x + 2y \leq 3 \tag{5.5}$$

$$x \geq 0, y \geq 0 \tag{5.6}$$

6. A furniture dealer deals in only two items : chairs and tables. He has ₹ 5,000 to invest and a space to store at most 60 pieces. A table costs him ₹ 250 and a chair ₹ 50. He sells a table at a profit of ₹ 50 and a chair at a profit of ₹ 15. Assuming that he can sell all the items he buys, how should he invest his money in order that he may maximize his profit ? Formulate the above as a linear programming problem.

7. The least value of the function

$$f(x) = 2\cos(x) + x \tag{7.1}$$

in the closed interval $\left[0, \frac{\pi}{2}\right]$ is:

(a) 2

(b) $\frac{\pi}{6} + \sqrt{3}$

(c) $\frac{\pi}{2}$

(d) The least value does not exist.

8. A linear programming problem is as follows: Minimize

$$Z = 30x + 50y \tag{8.1}$$

subject to the constrains,

$$3x + 5y \geq 15 \tag{8.2}$$

$$2x + 3y \leq 18 \tag{8.3}$$

$$x \geq 0, y \geq 0 \tag{8.4}$$

In the feasible region, the minimum value of Z occurs at

(a) a unique point

(b) no point

(c) infinitely many points

(d) two points only

9. The area of a trapezium is defined by function f and given by

$$f(x) = (10 + x)\sqrt{100 - x^2} \tag{9.1}$$

, then the area when it is maximised is:

(a) $75cm^2$

(b) $7\sqrt{3}cm^2$

(c) $75\sqrt{3}cm^2$

(d) $5cm^2$

10. For an objective function

$$Z = ax + by \tag{10.1}$$

,where $a, b > 0$;the corner points of the feasible region determined by a set of constrains (linear inequalities) are $(0, 20)$, $(10, 10)$, $(30, 30)$, and $(0, 40)$.The condition on a and b such that the maximum Z occurs at the points $(30, 30)$ and $(0, 40)$ is:

(a) $b - 3a = 0$

(b) $a = 3b$

(c) $a + 2b = 0$

(d) $2a - b = 0$

11. In a linear programming problem, the constrains on the decision variables x and y are $x - 3y \geq 0, y \geq 0, 0 \leq x \leq 3$.The feasible region

(a) is not in the first quadrant

(b) is bounded in the first quadrant

(c) is unbounded in the first quadrant

(d) does not exist

12. Based on the given shaded region in figure 12.1 as the feasible region in the graph, at which point(S) is the objective function

$$Z = 3x + 9y \tag{12.1}$$

maximum?

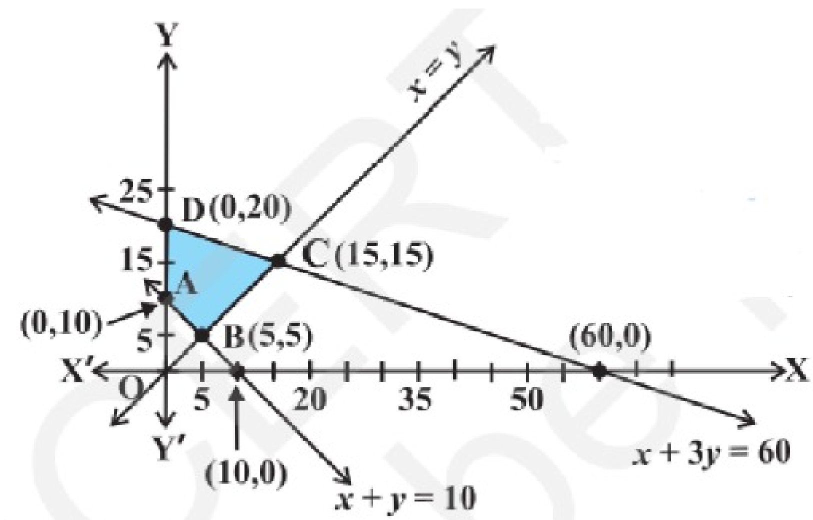


Figure 12.1: Optimization graph

(a) point B

(b) point C

(c) point D

(d) every point on the line segment CD

13. In figure 13.1, the feasible region for a LPP is shaded.The objective

function

$$Z = 2x - 3y \tag{13.1}$$

,will be minimum at:

(a) $(4, 10)$

(b) $(6, 8)$

(c) $(0, 8)$

(d) $(6, 5)$

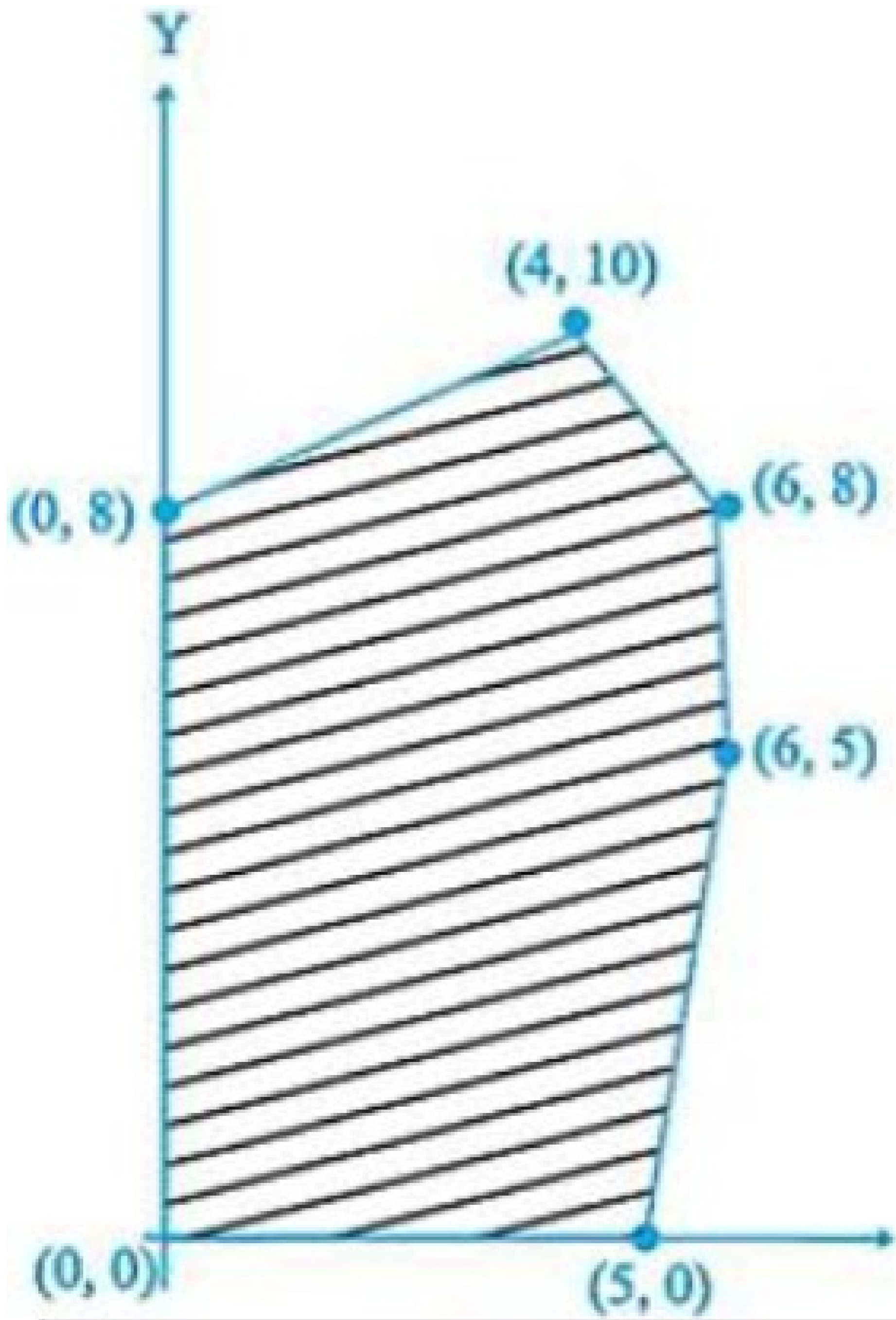


Figure 13.1: Optimization graph

Chapter 8

Algebra

8.1. 2023

8.1.1. 10

1. If one zero of the polynomial

$$p(x) = 6x^2 + 37x - (k - 2) \tag{1.1}$$

is reciprocal of the other, then find the value of k?

2. Find the value of $'p'$ for which one root of the quadratic equation

$$px^2 - 14x + 18 = 0 \tag{2.1}$$

is 6 times the other?

3. (a) prove that

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A \tag{3.1}$$

(b)

$$\sec A(1-\sin A)(\sec A+\tan A)=1 \tag{3.2}$$

4. Which of the following quadratic equations has sum of its roots as 4?

   (a) $2x^2-4x+8=0$

   (b) $-x^2+4x+4=0$

   (c) $\sqrt{2x^2}-\frac{4}{\sqrt{2}}x+1=0$

   (d) $4x^2-4x+4=0$

5. if one zero of the polynomial

$$6x^2+37x-(k-2) \tag{5.1}$$

   is reciprocal of the other,then what is the value of k?

   (a) -4

   (b) -6

   (c) 6

   (d) 4

6. The zeroes of the polynomial

$$p(x)=x^2+4x+3 \tag{6.1}$$

   are given by:

(a) 1,3

(b) -1,3

(c) 1,-3

(d) -1,-3

7. If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - ax - b$, then the value of $\alpha^2 + \beta^2$ is:

(a) $a^2 - 2b$

(b) $a^2 + 2b$

(c) $b^2 - 2a$

(d) $b^2 + 2a$

8. The below is the Assertion and Reason based question. Two statements are given, one labelled as Assertion(A) and the other is labelled as Reason(R). Select the correct answer to these questions from the codes (a),(b),(c) and (d) as given below.

(a) Both Assertion(A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A).

(b) Both Assertion(A) and Reason(R) are true, but Reason(R) is not the correct explanation of the Assertion(A).

(c) Assertion(A) is true, but Reason(R) is false.

(d) Assertion(A) is false, but Reason(R) is true.

Assertion(A): The polynomial $p(x) = x^2 + 3x + 3$ has two real zeroes.

Reason(R): A quadratic polynomial can have at most two real zeroes.

9. (a) If

$$4\cot^2 45° - \sec^2 60° + \sin^2 60° + p = \frac{3}{4}, \tag{9.1}$$

then find the value of p.

(b) If

$$\cos A + \cos^2 A = 1, \tag{9.2}$$

then find the value of

$$\sin^2 A + \sin^4 A. \tag{9.3}$$

10. Prove that:

$$\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right) = \frac{1}{\tan\theta + \cot\theta} \tag{10.1}$$

11. The value of k for which the pair of equations $kx = y+2$ and $6x = 2y+3$ has infinitely many solutions,

(a) is $k = 3$

(b) does not exist

(c) is $k = -3$

(d) is $k = 4$

12. If $2\tan A = 3$, then the value of $\frac{4 sin A + 3\cos A}{4\sin A - 3\cos A}$ is

    (a) $\frac{7}{\sqrt{13}}$

    (b) $\frac{1}{\sqrt{13}}$

    (c) 3

    (d) does not exist

13. If α, β are the zeroes of a polynomial $p(x) = x^2 + x - 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ equals to

    (a) 1

    (b) 2

    (c) -1

    (d) $\frac{-1}{2}$

14. $\left(\sec^2\theta - 1\right)\left(\csc^2\theta - 1\right)$ is equal to:

    (a) -1

    (b) 1

    (c) 0

    (d) 2

15. The roots of equation

$$x^2 + 3x - 10 = 0 \tag{15.1}$$

are:

(a) $(2, -5)$

(b) $(-2, 5)$

(c) $(2, 5)$

(d) $(-2, -5)$

16. If α , β are zeroes of the polynomial $x^2 - 1$,then value of $(\alpha + \beta)$ is:

(a) 2

(b) 1

(c) 1

(d) 0

17. If α, β are the zeroes of the polynomial

$$p(x) = 4x^2 - 3x - 7 \tag{17.1}$$

,then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ is equal to:

(a) $\frac{7}{3}$

(b) $\frac{-7}{3}$

(c) $\frac{3}{7}$

(d) $\frac{-3}{7}$

18. Find the sum and product of the roots of the quadratic equation

$$2x^2 - 9x + 4 = 0 \tag{18.1}$$

19. Find the discriminant of the quadratic equation

$$4x^2 - 5 = 0 \tag{19.1}$$

and hence comment on the nature of roots of the equation.

20. Evaluate $2\sec^2\theta + 3\csc^2\theta - 2\sin\theta\cos\theta$ if

$$\theta = 45° \tag{20.1}$$

21. If

$$\sin\theta - \cos\theta = 0 \tag{21.1}$$

,then find the value of $\sin^4\theta + \cos^4\theta$.

8.2. 2022

8.2.1. 10

1. If $\sin\theta = 0$, then the value of $\tan^2\theta + \cot^2\theta$ is

   (a) 2

   (b) 4

   (c) 1

   (d) $\frac{10}{9}$

2. The value(s) of k for which the quadratic equation

$$3x^2 - kx + 3 = 0 \tag{2.1}$$

   has equal roots, is (are)

   (a) 6

   (b) -6

   (c) ± 6

(d) 9

3. $5\tan^2\theta - 5\sec^2\theta =$ __________

4. If α, β are zeroes of the polynomial $2x^2 - 5x - 4$, then $\frac{1}{\alpha} + \frac{1}{\beta}$.

5. In Fig. 5.1, a tower stands vertically on the ground. From a point on the ground, which is $80m$ away from the foot of the tower, the angle of elevation of the tower is found to be $30°$. Find the height of the tower.

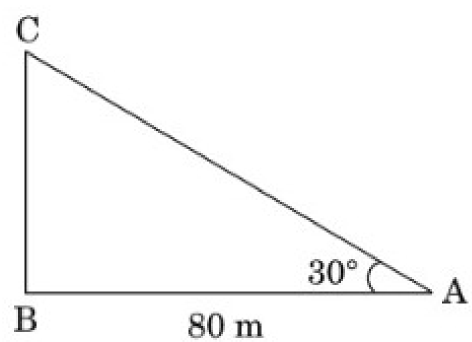


Figure 5.1: as.jpeg

6. Solve

$$9x^2 - 6a^2x + a^4 - b^4 = 0 \tag{6.1}$$

using the quadratic formula.

7. Show that

$$\cos(38°)\cos(52°) - \sin(38°)\sin(52°) = \cos(90°). \tag{7.1}$$

8. Prove that

$$\frac{\sin\theta}{\cot\theta+\csc\theta} = 2 + \frac{\sin\theta}{\cot\theta-\csc\theta}. \tag{8.1}$$

9. Given

$$15\cot(A) = 8, \tag{9.1}$$

find the values of $\sin(A)$ and $\sec(A)$.

10. The angles of depression of the top and bottom of a tower as seen from the top of a $60\sqrt{3}m$ high cliff are 45° and 60° respectively. Find the height of the tower. (Use $\sqrt{3} = 1.73$)

11. A and B jointly finish a piece of work in 15 days. When they work separately, A takes 16 days less than the number of days taken by B to finish the same piece of work. Find the number of days taken by B to finish the work.

12. If the polynomial

$$f(x) = 3x^4 - 9x^3 + x^2 + 15x + k \tag{12.1}$$

is completely divisible by $3x^2 - 5$, then find the value of k. Using the quotient obtained, find two zeroes of the polynomial.

13. Find all the zeroes of the polynomial

$$f(x)x^4 - 8x^3 + 23x^2 - 28x + 12 \tag{13.1}$$

if two of its zeroes are 2 and 3.

14. Find the value of m for which the quadratic equation

$$(m-1)x^2 + 2(m-1)x + 1 = 0 \tag{14.1}$$

has two real and equal roots.

15. Solve the following quadratic equation for x

$$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0 \tag{15.1}$$

16. The product of Rehan's age (in years) 5 years ago and his age 7 years from now is one more than twice his age. Find his present age.

17. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 meters high, then find the height of the building.

18. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 60° respectively. If the bridge is at a height of 3 meters from the banks, then find the width of the river.

19. In Fig. 19.1, Gadisar Lake is located in the Jaisalmer district of Rajasthan. It was built by the King of Jaisalmer and rebuilt by Gadsi Singh in the 14th century. The lake has many Chhatris. One of them is shown below:

Figure 19.1: ak.jpg

Observe the picture. From a point A h meters above the water level, the angle of elevation of the top of Chhatri (point B) is $45°$ and the angle of depression of its reflection in the water (point C) is $60°$. If the height of Chhatri above water level is (approximately) 10 meters, then

(a) Draw a well-labeled figure based on the above information.

(b) Find the height (h) of the point A above water level. (Use $\sqrt{3} = 1.73$)

20. Solve the quadratic equation

$$x^2 + \sqrt{2}x - 6 = 0 \tag{20.1}$$

for x.

21. In Fig. 21.1, from a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are $30°$ and $45°$. If the bridge is at a height of 8 meters from the banks, then find the width of the river.

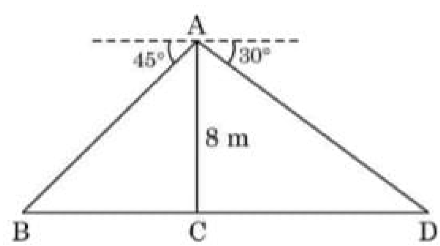


Figure 21.1: su.jpg

22. A 2-digit number is such that the product of its digits is 24. If 18 is subtracted from the number, the digits interchange their places. Find the numbers.

23. The difference of the squares of two numbers is 180. The square of the smaller number is 8 times the greater number. Find the two numbers.

24. Case Study-1:

    In Fig. 24.1, Kite Festival is celebrated in many countries at different times of the year. In India, every year on 14^{th} January is celebrated as International Kite Day. On this day, many people visit India and participate in the festival by flying various kinds of kites.

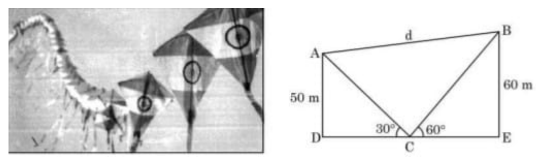


Figure 24.1: kites

In Fig. 5, the angles of elevation of two kites (Point A and B) from the hands of a man (Point C) are found to be $30°$ and $60°$ respectively. Taking $AD = 50$ meters and $BE = 60$ meters, find:

(a) The lengths of strings used (take them straight) for kites A and B as shown in the figure.

(b) The distance d between these two kites.

25. Solve the quadratic equation for x:

$$x^2 - 2ax - (4b^2 - a^2) = 0 \tag{25.1}$$

26. If the quadratic equation

$$(1 + a^2)x^2 + 2abx + (b^2 - c^2) = 0 \tag{26.1}$$

has equal and real roots, then prove that:

$$b^2 = c^2(1 + a^2) \tag{26.2}$$

27. Two boats are sailing in the sea 80 meters apart from each other towards a cliff AB. The angles of depression of the boats from the top

of the cliff are 30° and 45° respectively, as shown in Fig. 27.1

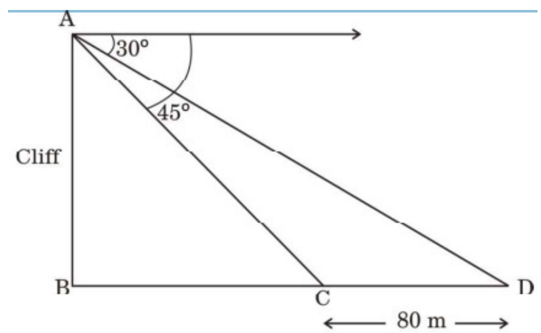


Figure 27.1: boat

Find the height of the cliff.

28. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 meters vertically above X, the angle of elevation of the top Q of tower PQ is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3} = 1.73$)

29. Find the value of k for which the quadratic equation

$$2kx^2 - 40 + 25 = 0 \tag{29.1}$$

has real and equal roots.

30. Solve for x:

$$\frac{5}{2}x^2 + \frac{2}{5} = 1 - 2x \tag{30.1}$$

31. An Aeroplane at an altitude of 200 meters observes the angles of de-

pression of opposite points on the two banks of a river to be $45°$ and $60°$. Find the width of the river. (Use $\sqrt{3} = 1.732$)

32. Find the value(s) of $'p'$ for which the quadratic equation $(px-4)(x-2)$ has real and equal roots.

33. Had Aarush scored 8 more marks in a Mathematics test, out of 35 marks, 7 times these marks would have been 4 less than the square of his actual marks. How many marks did he get in the test?

34. From the top of an 8 meter high building, the angle of elevation of the top of a cable tower is $60°$ and the angle of depression of its foot is $45°$. Determine the height of the tower. (Take $\sqrt{3} = 1.732$).

35. Find the roots of the quadratic equation

$$9x^2 - 6\sqrt{2}x + 2 = 0 \tag{35.1}$$

36. The product of two consecutive odd positive integers is 255. Find the integers, by formulating a quadratic equation.

37. Find the value(s) of k for the quadratic equation,

$$(k+3)x^2 + kx + 1 = 0 \tag{37.1}$$

to have two real and equal roots.

38. As observed from the top of a lighthouse 60 meters high from the sea level, the angles of depression of two ships are $45°$ and $60°$. If one ship

is exactly behind the other on the same side of the lighthouse, then find the distance between the two ships. (Use $\sqrt{3} = 1.732$)

39. At a point on the level ground, the angle of elevation of the top of a vertical tower is found to be α, such that $\tan\alpha = \frac{5}{12}$. On walking 192 meters towards the tower, the angle of elevation β is such that $\tan\beta = \frac{3}{4}$. Find the height of the tower.

40. $\tan^{-1}\frac{1}{\sqrt{3}} - \cot^{-1}\frac{-1}{\sqrt{3}}$

41. Show that the relation R in the set of all real numbers, defined as $R = \{(a, b) : a \leq b^2\}$.

42. Two angles of a triangle are $\cot^{-1} 2$ and $\cot^{-1} 3$. The third angle of the triangle is?

43. Solve for x:

$$\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2} \tag{43.1}$$

44. Find the present value of a perpetuity of ₹ 18,000 at the end of 6 months if it is worth 8% p.a. compounded semi-annually.

[Given that: $1.00833^{12} = 1.1047$]

45. Find the effective rate which is equivalent to a nominal rate of 10% p.a. compounded monthly.

[Given that: $1.00833^{12} = 1.1047$]

46. Abhay bought a mobile phone for ₹ 30,000. The mobile phone is estimated to have a scrap value of ₹ 3,000 after a span of 3 years. Using the linear depreciation method, find the book value of the mobile phone at the end of 2 years.

47. Madhu exchanged her old car valued at ₹ 1,50,000 with a new one priced at ₹ 65,000. She paid ₹ x as a down payment, and the balance in 20 monthly equal installments of ₹ 21,000. The rate of interest offered to her is 9% p.a. Find the value of x.

[Given that: $1.0075^{-20} = 0.86118985$]

48. Calculate the EMI under the 'Flat Rate System' for a loan of | $5, 00, 000$ with 10% annual interest rate for 5 years.

49. A machine costing ₹ 2,00,000 has an effective life of 7 years, and its scrap value is ₹ 30,000. What amount should the company put into a sinking fund earning 5% p.a., so that it can replace the machine after its usual life? Assume that a new machine will cost ₹ 3,00,000 after 7 years.

[Given that: $(1.05)^7 = 1.407$]

50. A start-up company invested ₹ 3,00,000 in shares for 5 years. The value of this investment was ₹ 3,50,000 at the end of the second year, ₹ 3,80,000 at the end of the third year, and on maturity, the final value stood at ₹ 4,50,000.Calculate the Compound Annual Growth Rate (CAGR) on the investment.

[Given that: $(1.5)^{\frac{1}{5}} = 1.084$]

8.3. 2021

8.3.1. 10

1. Find the sum and product of zeroes of the polynomial $p(x) = x^2+5x+6$

2. If $2\cos\theta = \sqrt{3}$, then find the value of θ

3. Find the discriminant of the quadratic equation $2x^2 - 5x - 6 = 0$.

4. In $\triangle ABC$, right-angled at A, if $AB = 7cm$ and $AC = 24cm$, then find $\sin B$ and $\tan C$.

5. (a) If $\sin(A+B) = \sqrt{3}/2, \sin(A-B) = 1/2$, Where $0° < A+B < 90°; A > B$, then find the values of A and B.

   (b) Simplify :

$$\frac{\sin 30° + \tan 45° - \cos 60°}{\sec 30° + \cos 60° + \cot 45°} \tag{5.1}$$

6. The greater of two supplementary angles exceeds the smaller by $18°$. Find the two angles.

7. Prove that $7\sqrt{2}$ is an irrational number ,given that $\sqrt{2}$ is an irrational number.

8. (a) Prove that :

$$\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta) = 1 \tag{8.1}$$

(b) Prove that :

$$\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A} \tag{8.2}$$

9. If α, β are the zeroes of the quadratic polynomial $x^2 + 9x + 20$, from a quadratic polynomial whose zeroes are $(\alpha + 1)$ and $(\beta + 1)$.

10. (a) The diagonal of a rectangular field is 60 meters more than the shorter side, find the sides of the field.

    (b) The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present ages

11. Write a quadratic polynomial sum of Whose zeroes is -5 and product is 6.

12. If the sum of the zeroes of the polynomial $2x^2 - 3ax + 4$ is 6, then the value of a

    (a) 4

    (b) -4

    (c) 2

    (d) -2

13. The common zero of the polynomials $x^3 + 1, x^2 - 1$ and $x^2 + 2x + 1$ is

    (a) -2

    (b) -1

(c) 1

(d) 2

14. If α, β are the zeroes of the polynomial $x^2 - 4x + 6$, then the value of $\alpha\beta$ is

(a) 4

(b) -4

(c) 6

(d) -6

15. The zeroes of the polynomial $3x^2 - 5x - 2$ are

(a) $\frac{1}{3}$,2

(b) $-\frac{1}{3}$,2

(c) $\frac{1}{3}$,-2

(d) $-\frac{1}{3}$,-2

16. If is a zero of the polynomial $p(x) = ax^2 - 3(a-1)x - 1$ then the value of a is

(a) $\frac{1}{3}$,2

(b) $-\frac{1}{3}$,2

(c) $\frac{1}{3}$,-2

(d) $-\frac{1}{3}$,-2

17. If $\tan\theta = 4/3$, find the value $\frac{2\sin\theta - 3\cos\theta}{2\sin\theta + 3\cos\theta}$

18. If x= $a\cos\theta$ and y=$b\sin\theta$, then find the value of $b^2x^2 + a^2y^2$

19. A number consists of two digits whose sum is 9. if 27 is added to the number, the digits are reversed. Find the number

20. Prove that :

$$\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \tan^2\theta - \cot^2\theta \tag{20.1}$$

21. Prove that:

$$(\sec\theta - \tan\theta)^2 = \frac{1+\sin\theta}{1-\sin\theta} \tag{21.1}$$

22. The sum of the squares of three consecutive positive integers is 110. Find the positive integers.

23. Ram can row a boat at the rate of 4 km/hour in still water. If he takes 8 hours in going 12 km upstream and 12 km downstream, find the speed of the stream.

24. Write the quadratic equation in x whose roots are 2 and -5.

25. If α and β are zeros of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

26. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then find the value of k.

27. If $3\sin A = 1$, then find the value of $\sec A$.

28. Show that: $\frac{1+\cot^2\theta}{1+\tan^2\theta} = \cot^2\theta$.

29. Simplify :

$$\csc^2 60^\circ \sin^2 30^\circ - \sec^2 60^\circ$$

30. If $\tan\theta + \cot\theta = \frac{4\sqrt{3}}{3}$, then find the value of $\tan^2\theta + \cot^2\theta$.

31. Divide the polynomial $f(x) = 5x^3 + 10x^2 - 30x - 15$ by the polynomial $g(x) = x^2 + 1 + x$ and hence, find the quotient and the remainder.

32. Prove:

$$\frac{1}{(\cot A)(\sec A) - \cot A} - \csc A = \csc A - \frac{1}{(\cot A)(\sec A) + \cot A}$$

33. Prove:

$$\sin^6 A + 3\sin^2 A\cos^2 A = 1 - \cos^6 A$$

34. One of the root of the quadratic equation $2x^2 - 8x - k = 0$ is $\frac{5}{2}$. Find the value of k, Also find the root.

35. Using quadratic formula, solve the following equation for x:

$$abx^2 + (b^2 - ac)x - bc = 0$$

.

36. With vertices A,B and C of a triangle ABC as centers, arcs are drawn with radii 2 cm each as/ shown in the figure. If AB = 6 cm, BC = 8 cm and AC = 10 cm, find the area of the shaded region.

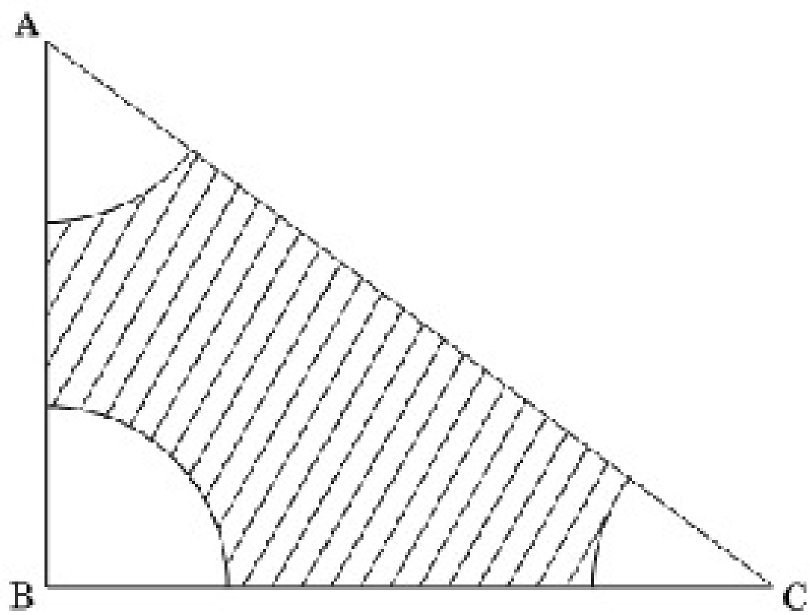


Figure 36.1:

37. Water is being pumped out through a circular pipe whose internal diameter is 8 cm. If the rate of flow of water is 80 cm/s, then how many liters of water is being pumped out through this pipe in one hour ?

38. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 18 minutes for the angle of depression to change from 30° to 60°, how soon after this will the car reach the tower ?

39. A girl on a ship standing on a wooden platform, which is 50 m above water level, observes the angle of elevation of a top of a hill as 30° and

the angle of depression of the base of the hill as $60°$. Calculate the distance of the hill from the platform and the height of the hill.

40. If one zero of the polynomial $p(x) = (a^2+4)X^2+20X+4a$ is reciprocal of the other, find the value of a.

41. Find the roots of the quadratic equation

$$X^2 + X - (a+1)(a+2) = 0 \tag{41.1}$$

42. Solve for x :

$$10X - \frac{1}{X} = 3, X \neq 0 \tag{42.1}$$

43. In $\triangle ABC$, $\angle B = 90°$ and $\tan A = \frac{1}{\sqrt{3}}$.Then find the value of $\sin A \cos C + \cos A \sin C$

44. If $X = a\sin\theta + b\cos\theta$ and $y = a\cos\theta - b\sin\theta$,then find the value of $(X^2 + Y^2)$.

45. Answer any **four** of the following questions:

    (a) The sum and the product of the zeroes of a quadratic polynomial are -1 and -12 respectively. The polynomial is

        i. $X^2 - X - 12$

        ii. $X^2 + X - 12$

        iii. $X^2 - X + 12$

        iv. $X^2 + X + 12$

(b) The zeroes of the quadratic polynomial $x^2 + 20x + 91$ are

   i. both positive.

   ii. both equal.

   iii. both negative.

   iv. one positive and one negative.

(c) If the zeroes of the polynomial $5x^2 - 26x + k$ are reciprocal of each other, then the value of k is

   i. 5

   ii. -5

   iii. $\frac{1}{5}$

   iv. -$\frac{1}{5}$

(d) If α, β are the zeroes of the polynomial $x^2 - 5x - 14$, then the value of $\alpha\beta - \alpha - \beta$ is

   i. -9

   ii. 19

   iii. 9

   iv. -19

(e) What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of the resulting polynomial ?

   i. 5

   ii. 4

   iii. 2

   iv. 1

46. If $2\sin 2A = \sqrt{3}$, then find the value of A.

47. If $7\sin^2\theta + 3\cos^2\theta = 4$, then show that $\tan\theta = \frac{1}{\sqrt{3}}$, $0^\circ < \theta < 90^\circ$

48. Find the quadratic polynomial whose zeroes are $(\sqrt{5}-4)$and$(\sqrt{5}+4)$.

49. If the sum of LCM and HCF of two numbers is 1260 and the LCM is 900 more than their HCF, find their LCM.

50. Find the values of m and n for which x=2 and x=3 are the roots of the quadratic equation $3x^2 - 2mx + 2n = 0$.

51. Divide 19 into two parts such that sum of their squares is 193.

52. The angles of depression of the top and bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multi-storeyed building.

53. From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed on the top of a 20m high building are 45° and 60° respectively. Find the height of the tower.

54. As observed from the top of 75m high lighthouse from the sea-level,the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

55. It takes 12 hours to fill a swimming pool using two pipes together. If the larger pipe is used for 4 hours and smaller pipe is used for 9 hours, only half of the pool is filled. How long will it take for each pipe alone to fill the pool?

8.4. 2021

8.4.1. 12

1. Two angles of a triangle are $\cot^{-1} 2$ and $\cot^{-1} 3$.The third angle of the triangle is ______

2. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

3. Akbar invested ₹6060 in the shares of face value ₹100 each of a company.At the end of the year,the company declared dividend of 15 % which gave him an income of ₹600.At what price was the share quoted if the brokerage was 1% ?

4. $\sin\ [[]\ \frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})]$ is equal to:

   (a) $\frac{1}{2}$

   (b) $\frac{1}{3}$

   (c) -1

   (d) 1

5. $\sin(\tan^{-1} x)$,where $|x| \leq 1$,is equal to:

   (a) $\frac{x}{\sqrt{1-x^2}}$

   (b) $\frac{1}{\sqrt{1-x^2}}$

   (c) $\frac{1}{\sqrt{1+x^2}}$

   (d) $\frac{x}{\sqrt{1+x^2}}$

6. Simplest form of $\tan^{-1}(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}), \pi < x < \frac{3\pi}{2}$ is:

(a) $\frac{\pi}{4} - \frac{x}{2}$

(b) $\frac{3\pi}{2} - \frac{x}{2}$

(c) $-\frac{x}{2}$

(d) $\pi - \frac{x}{2}$

Chapter 9

Geometry

9.1. 2023

9.1.1. 10

1. The hour-hand of a clock is 6 cm long.The angle swept by it between $7:20$ a.m. and $7:55$ a.m. is:

   (a) $\left(\frac{35}{4}\right)^\circ$

   (b) $\left(\frac{35}{2}\right)^\circ$

   (c) 35°

   (d) 70°

2. In the given Fig. 2.1, $AB \parallel PQ$.If $AB = 6$ cm,$PQ = 2$ cm and $OB = 3$ cm,then the length of OP is:

   (a) 9cm

   (b) 3cm

   (c) 4cm

(d) 1cm

3. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower.Find the angle of elevation of the sun.

4. The angle of elevation of the top of a tower from a point on the ground which is 30 m away from the foot of the tower,is 30° .Find the height of the tower.

5. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle of 120°. Find the total area cleaned at each sweep of the two blades.

6. As observed from the top of a 75 m high lighthouse from the sea-level,the angles of depression of two ships are 30° and 60°.If one ship is exactly behind the other on the same side of the lighthouse,find the distance between two ships.$\left(Use\sqrt{3} = 1.73\right)$

7. From a point on the ground,the angle of elevation of the bottom and top of a transmission tower fixed at the top of 30 m high building are 30° and 60°, respectively.Find the height of the transmission tower.$\left(Use\sqrt{3} = 1.73\right)$

8. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.

9. Through the mid-point M of the side CD of a parallelogram $ABCD$,the line BM is drawn intersecting AC in L and AD(produced) in E.Prove

that

$$EL = 2BL. \tag{9.1}$$

10. In an annual day function of a school,the organizers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in the Fig. 10.1 and its base $ABCD$ is shown from the front side. The rate of silver plating is ₹ 20 $per\ cm^2$.

    Based on the above, answer the following questions:

    (i) What is the area of the quadrant $ODCO$?

    (ii) Find the area of $\triangle AOB$.

    (iii) What is the total cost of silver plating the shaded part $ABCD$?

    (iv) what is the length of arc CD?

11. What is the length of the arc of the sector of a circle with radius 14 cm and of central angle 90°.

    (a) 22 cm

    (b) 44 cm

    (c) 88 cm

    (d) 11 cm

12. if$\triangle ABC \sim \triangle PQR$ with $\angle A = 32^\circ$ and $\angle R = 65^\circ$,then the measure of $\angle B$ is:

(a) 32°

(b) 65°

(c) 83°

(d) 97°

13. What is the total surface area of a solid hemisphere of diameter $'d'$?

    (a) $3\pi d^2$

    (b) $2\pi d^2$

    (c) $\frac{1}{2}\pi d^2$

    (d) $\frac{3}{4}\pi d^2$

14. In $\triangle ABC$,$DE \parallel BC$.if $AD = 2$ units,$DB = AE = 3$ units and $EC = x$ units,then the value of x is :

    (a) 2

    (b) 3

    (c) 5

    (d) $\frac{9}{2}$

15. A straight highway leads to the foot of a tower.A man standing on the top of the 75 m high tower observes two cars at angles of depression of 30° and 60°,Which are approaching the foot of the tower.If one car is exactly behind the other on the same side of the tower,find the distance between the two cars.

16. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is $60°$ and the angle of depression of its foot is $30°$.Determine the height of the tower.(take $\sqrt{3} = 1.73$)

17. Governing council of local public development authority of Dehradun decided to build and adventurous playground on the top of a hill,Which will have adequate space for parking. After survey,it was decided to build rectangular playground,with a semi-circular area allocated for parking at one end of the playground.The length and breadth of the rectangular playground are 14 units and 7 units,respectively.There are two quadrants of radius 2 units on one side for special seats:

    (a) What is the total perimeter of the parking area?

    (b) What is the total area of parking and the two quadrants?

    (c) What is the ratio of area of playground to the area of parking area?

    (d) Find the cost of fencing the playground and parking area at the rate of ₹2 per unit.

18. What is the total surface area of a solid hemisphere of diameter $'d'$?

    (a) $3\pi d^2$

    (b) $2\pi d^2$

    (c) $\frac{1}{2}\pi d^2$

    (d) $\frac{3}{4}\pi d^2$

19. In the given Fig. 19.1, $DE \parallel BC$. If AD=2 units, $DB = AE = 3$ units and $EC = x$ units, then the value of x is:

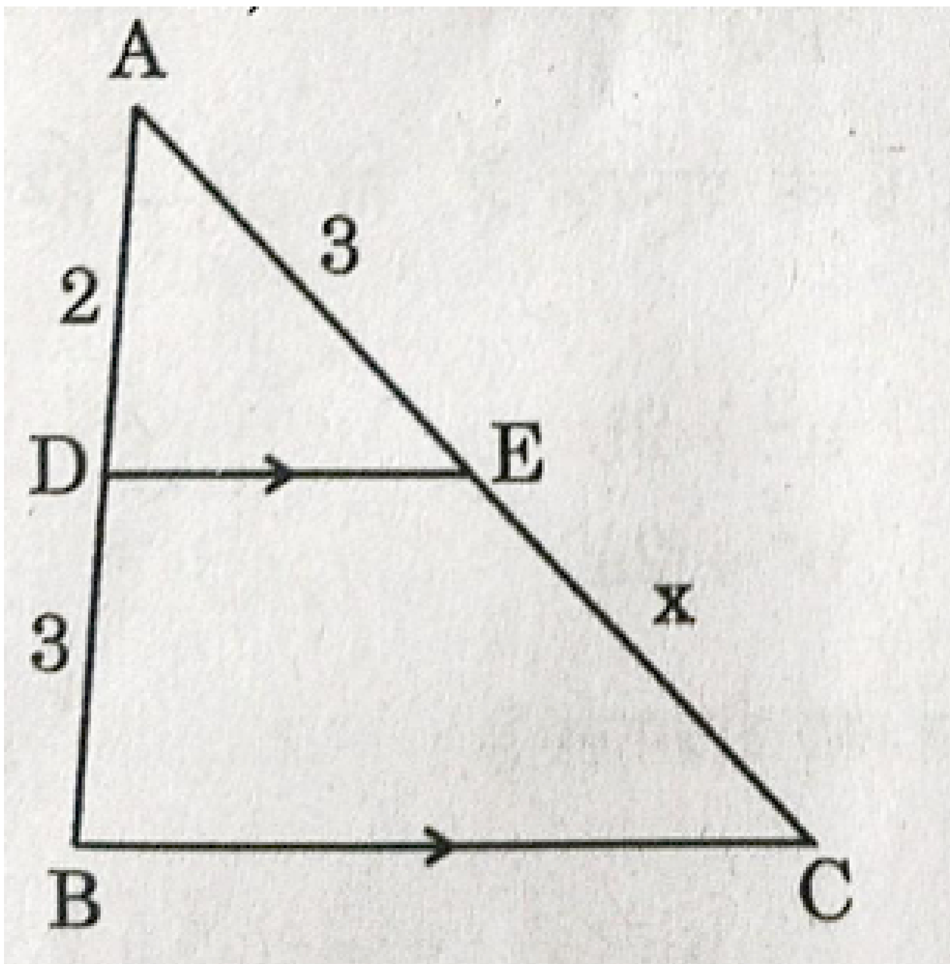


Figure 19.1:

(a) 2

(b) 3

(c) 5

(d) $\frac{9}{2}$

20. In the given Fig. 20.1, XZ is parallel to BC. $AZ = 3$ cm, $ZC = 2$ cm, $BM = 3$ cm, and $MC = 5$ cm. Find the length of XY.

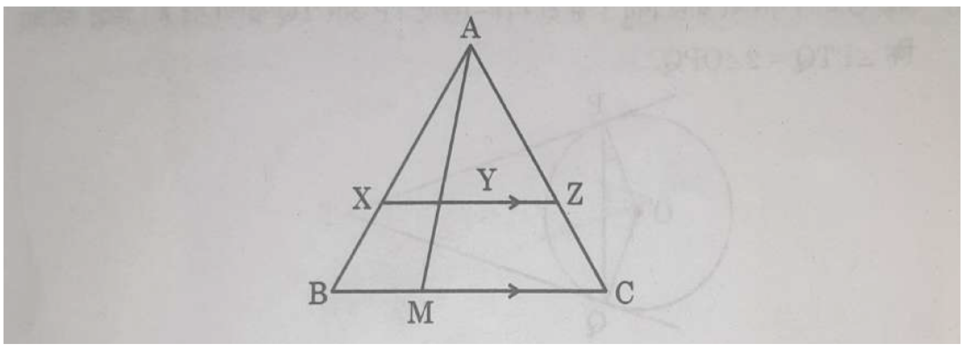


Figure 20.1:

21. A room is in the form of a cylinder surmounted by a hemi-spherical dome. The base radius of hemisphere is one-half the height of cylindrical part. Find total height of the room if it contains $\left(\frac{1408}{21}\right)$ m^3 of air.Take $\left(\pi = \frac{22}{7}\right)$

22. In the given Fig. 22.1, An empty cone is of radius 3 cm and height 12 cm. Ice-cream is filled so that lower part of the cone which is $\left(\frac{1}{6}\right)$th of the volume of the cone is unfilled but hemisphere is formed on the top. Find volume of the ice-cream.Take$(\pi = 3.14)$

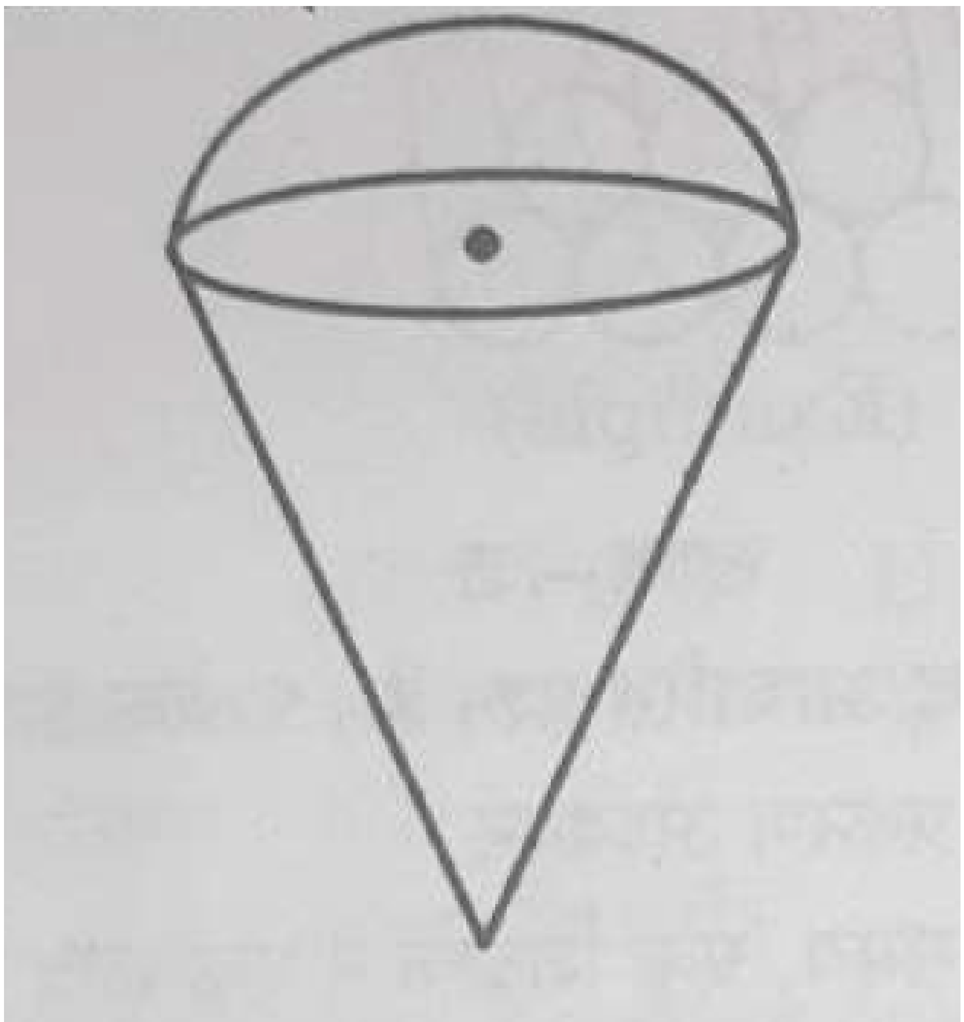


Figure 22.1:

23. If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio.

24. The angle of elevation of the top of a tower 24 m high from the foot of

another tower in the same plane is 60°. The angle of elevation of the top of second tower from the foot of the first tower is 30°. Find the distance between two towers and the height of the other tower. Also, find the length of the wire attached to the tops of both the towers.

25. A spherical balloon of radius r subtends an angle of $60°$ at the eye of an observer. If the angle of elevation of its centre is $45°$ from the same point, then prove that height of the centre of the balloon is $\sqrt{2}$ times its radius.

26. A chord of a circle of radius 14 cm subtends an angle of $60°$ at the centre. Find the area of the corresponding minor segment of the circle. Also find the area of the major segment of the circle.

9.2. 2022

9.2.1. 10

9.2.1. A solid spherical ball fits exactly inside the cubical box of side $2a$. The volume of the ball is

(a) $\frac{16}{3}\pi a^3$

(b) $\frac{1}{6}\pi a^3$

(c) $\frac{32}{3}\pi a^3$

(d) $\frac{4}{3}\pi a^3$

9.2.2. A frustum of a right circular cone which is of height $8cm$ with radii of its circular ends as $10cm$ and $4cm$, has its slant height equal to

(a) $14cm$

(b) $28cm$

(c) $10cm$

(d) $\sqrt{260}cm$

9.2.3. The capacity of a cylindrical glass tumbler is $125.6cm^3$. If the radius of the glass tumbler is $2cm$, then find its height. (Use $\pi = 3.14$)

9.2.4. A mint moulds four types of copper coins A,B,C and D whose diameters vary from $0.5cm$ to $5cm$. The first coin A has a diameter of $0.7cm$. The second coin B has double the diameter of coin A and from then onwards the diameters increase by 50%. Thickness of each coin is $0.25cm$. After reading the above, answer the following questions :

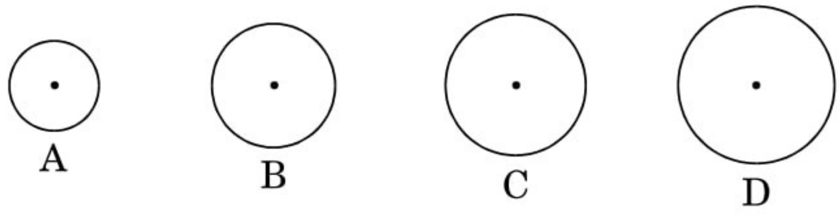


Figure 4.1: Cirls

- Fill in the diameters of the coins required in the following table :

Type of Coin	Diameter (in cm)
A	0·7
B	---

Figure 4.2: table

- Complete the following table :

Type of Coin	Area (in cm^2) of one face	Volume (in cm^3)
A	0·385	0·09625
B	---	---

Figure 4.3: table2

9.2.5. A well of diameter $3m$ is dug $14m$ deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width $4m$ to form a platform. Find the height of the platform. (Take $\pi = \frac{22}{7}$)

9.2.6. A solid metallic sphere of radius $10.5cm$ is melted and recast into a number of smaller cones, each of radius $3.5cm$ and height $3cm$. Find the number of cones so formed.

9.2.7. In Figure 2, from a solid cube of side $7cm$, a cylinder of radius $2.1cm$

and height $7cm$ is scooped out. Find the total surface area of the remaining solid.

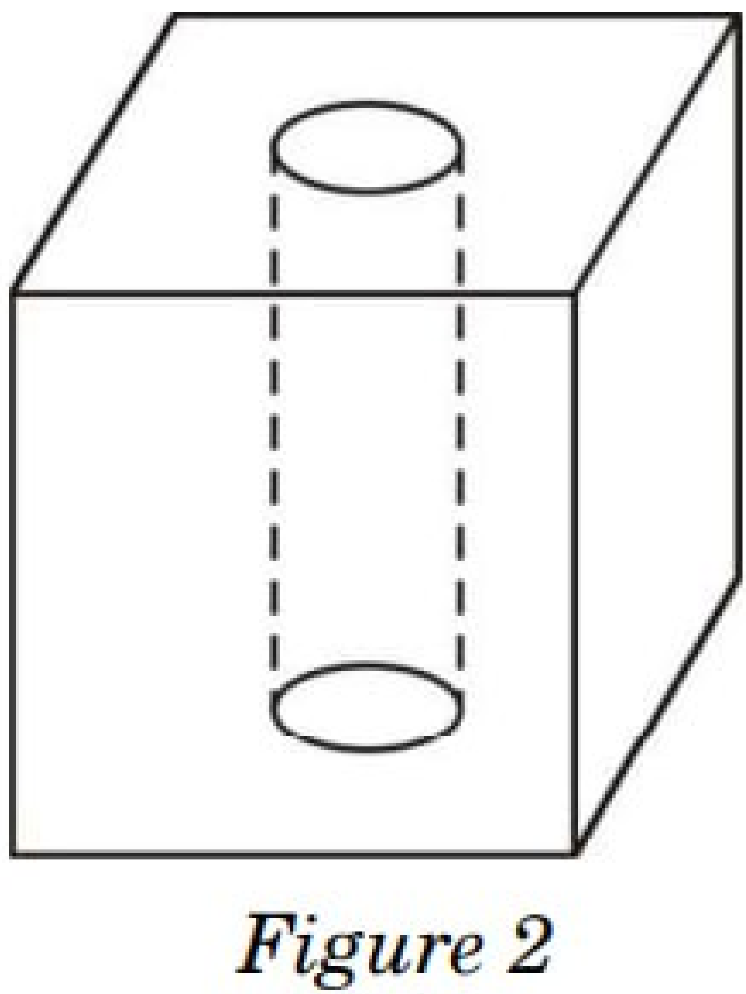


Figure 7.1: soildcube

9.2.8. A well of diameter $5m$ is dug $24m$ deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width

$3m$ to form an embankment. Find the height of the embankment.

9.2.9. A solid piece of metal in the form of a cuboid of dimensions $11cm \times 7cm \times 7cm$ is melted to form n number of solid spheres of radii $\frac{7}{2}cm$ each. Find the value of n.

9.2.10. A 'circus' is a company of performers who put on shows of acrobats, clowns etc. to entertain people started around 250 years back, in open fields, now generally performed in tents

One such 'Circus Tent' is shown below.

Figure 10.1: circustent

The tent is in the shape of a cylinder surmounted by a conical top. if the height and diameter of cylinder part are $9m$ and $30m$ respectively and height of conical part is 8 cm with same diameter as that of the cylindrical part, then find.

(a) The area of the canvas used in making the tent

(b) The cost of the canvas bought for the tent at the rate ₹ 200 per sq. m, if 30 sq m canvas was wasted during stitching.

9.2.11. (a) 150 spherical marbles, each of diameter $1.4cm$ are dropped in a cylinder vessel of diameter $7cm$ containing some water, and are completely immersed in water.Find the rise in the level of water in the cylindrical vessel

(b) Three cubes of side $6cm$ each,are joined as shown in Figure 2

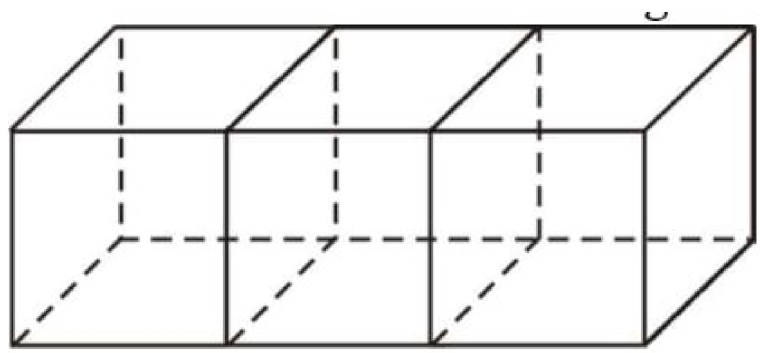


Figure 2

Figure 11.1: Cuboid

9.2.12. In the picture given below, one can see a rectangular in-ground swimming pool installed by a family in their backyard. There is a concrete sidewalk around the pool of width xm. The outside edges of the sidewalk measure $7m$ and $12m$. The area of the pool is $36sq.m$.

Figure 12.1: swimmingpool

(a) Based on the information given above, form a quadratic equation in terms of x

(b) Find the width of the sidewalk around the pool.

9.2.13. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions:

Cake : Cylindrical shape with diameter $24cm$ and height $14cm$.

Cap : Conical shape with base circumference $44cm$ and height $24cm$.

Figure 13.1: birthdaycake

Based on the above information, answer the following questions :

(a) How many square cm paper would be used to make 4 such caps ?

(b) The bakery shop sells cakes by weight ($0.5kg, 1kg, 1.5kg, etc.$). To have the required dimensions, how much cake should they order, if $650cm^3$ equals $100g$ of cake ?

9.2.14. (a) The curved surface area of a right circular cylinder is $176sqcm$ and its volume is$1232cucm$ what is the height of the cylinder?

(b) The largest sphere is carved out of a solid cube of side $21cm$ Find the volume of sphere

9.2.15. Khurja is a city in the Indian state of Uttar Pradesh famous for the pottery.Khurja pottery is traditional Indian pottery work which has attracted Indians as well as foreighners with a variety of tea sets,crockery and ceramic tile works. A huge portion of the ceramics used in the country is supplied by Khurja and is also referred as 'The Ceramic Town' One of the private schools of Bulandshahr organised an Educational Tour for class 10 students to Khurja. Students were very excited about the trip. Following are the few pottery objects of Khurja

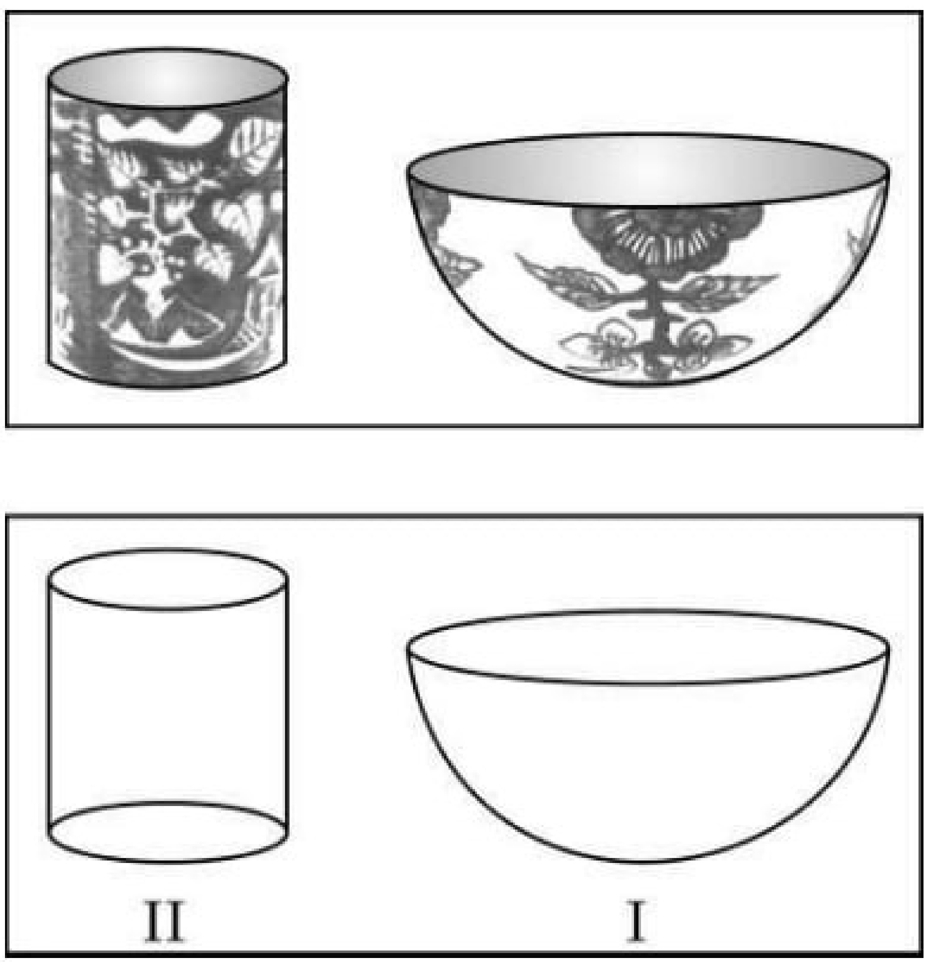


Figure 15.1: pottery

Students found the shapes of the objects very interesting and they could easily relate them with mathematical shapes viz sphere, hemisphere, cylinder etc. Maths teacher who was accompanying the students asked following question:

(a) The internal radius of hemispherical bowl(filled completely with water)in I is $9cm$ and radius and height of cylindrical jar in II is $1.5cm$ and $4cm$ respectively. If the hemisherical bowl is to be emptied in cylindrical jars, then how many cylindrical jars are required?

(b) If in the cylindrical jar full of water, a conical funnel of same height and same diameter is immersed, then how much water will flow out of the jar?

9.2.16. How many spherical shots each having diameter $3cm$ can be made by melting a cuboidal solid of dimensions $18cm \times 22cm \times 6cm$?

9.2.17. Conical bottom tanks in which an inverted cone at the bottom is surmounted by a cylinder of same diameter, are very advantageous in industry, specially where getting every last drop from the tank is important.

Vikas designed a conical bottom tank where the height of the conical part is equal to its radius and the height of the cylindrical part is two times of its radius. The tank is closed from the top.

(a) If the radius of the cylindrical part is $3m$, then find the volume of the tank.

(b) Find the ratio of the volume of the cylindrical part to the volume of the conical part.

9.3. 2021

9.3.1. 10

1. (a) Write the expression for the volume of the cone of radius 'r' and height three times the radius 'r'.

   (b) Write the expression for total surface area of a solid hemisphere of radius 'r'.

2. A vertical pole is 100 metres high. Find the angle subtended by the pole at a point on the ground $100\sqrt{3}$ meters from the base of the pole.

3. (a) Find the area of that sector of a circle of radius 3.5cm whose central angle is $90°$.

   (b) The length of the minute hand of a clock is 14 cm.Find the area swept by the minute hand in 5 minutes.
   (Take $\pi = \frac{22}{7}$)

4. A semicircular ground of radius 1.5 m is to be fenced with wire. Find the cost of wiring at the rate of ₹30 per metre.

5. (a) The angle of elevation of the top of a tower from a point is found to be $60°$.At a point 40 m above the first point, the angle of

elevation of the top of the tower is 45°.Find the height of the tower.

(b) A statue 1.6m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°.Find the height of the pedestal.

6. The areas of two similar triangles are $121cm^2$ and $64cm^2$ respectively. If one median of the first triangle is 12.1 cm long, then find the length of the corresponding median of the other triangle.

7. (a) In a triangle ABC, a line is drawn parallel to base BC meeting AB in D and AC at E. If $\frac{AB}{BD} = 4$ and $CE = 2cm$, BD then find the value of AE.

   (b) Two poles, 6m and 11 m high, stand vertically on the ground. If the distance between their feet is 12 m, find the distance between their tops.

8. Answer any **four** of the following questions :

   (i) If the sum of the areas of two circles with radii $r1$ and $r2$ is equal to the area of a circle of radius r, then

   (A) $r_1 + r_2 = r$

   (B) ${r_1}^2 + {r_2}^2 = r^2$

   (C) $r_1 + r_2 < r$

   (D) ${r_1}^2 + {r_2}^2 < r^2$

(ii) The area of a circle that can be inscribed in a square of side 8 cm is

(A) $64\pi cm^2$

(B) $24\pi cm^2$

(C) $16\pi cm^2$

(D) $8\pi cm^2$

(iii) The area of a square that can be inscribed in a circle of radius 6 cm is

(A) $36\ cm^2$

(B) $72\ cm^2$

(C) $18\ cm^2$

(D) $32\sqrt{2}\ cm^2$

(iv) The radius of a circle whose circumference is equal to the sum of the circumferences of two circles of diameters 36 cm and 20cm is

(A) $56cm$

(B) $42cm$

(C) $28cm$

(D) $16cm$

(v) If the circumference of a circle is equal to the perimeter of a square, then the ratio of their areas is

(A) $22 : 7$

(B) $14 : 11$

(C) $7 : 22$

(D) 11 : 24

9. A solid right circular cone is 4.1cm high and the radius of its base is 2.1 cm. Another solid right circular cone is 4.3 cm high and radius of its base is 2.1 cm. Both the cones are melted and recast into a sphere. Find the diameter of the sphere.

10. Answer any **four** of the following questions :

    (i) The radius of a solid hemisphere is 'r' cm.It is divided into two equal hemispherical parts. The whole surface area of one part is

    (A) $2\pi r^2$sq.cm

    (B) $3\pi r^2$sq.cm

    (C) $\frac{2}{3}\pi r^3$sq.cm

    (D) $\frac{1}{3}\pi r^3$sq.cm

    (ii) The diameter of the largest sphere that can be carved out of a cube of side 21 cm is

    (A) 42cm

    (B) 7cm

    (C) 21cm

    (D) $\frac{21}{2}$cm

    (iii) The total surface area of a solid right circular cylinder having the radius of the base as 7 cm and the height as 10 cm is

    (A) 154sq.cm

    (B) 440sq.cm

(C) 308sq.cm

(D) 748sq.cm

(iv) A cone and a cylinder are of the same height. If the radii of their bases are in the ratio 3 : 1, then the ratio of their volumes is

(A) 1 : 1

(B) 1 : 3

(C) 3 : 1

(D) 2 : 3

(v) The slant height of a cone of radius 5 cm and height 12 cm(in cm) is

(A) 12

(B) 13

(C) 5

(D) 17

11. The angle of elevation of the top of a tower from a point on the ground,which is 30 m away from the foot of the tower is 45° .What is the height of the tower ?

12. Find the sun's altitude if the shadow of a 15 m high tower is $15\sqrt{3}$ m.

13. A circular piece of land is 40 m in diameter. A well of diameter 16 m has been dug to a depth of 28 m and the earth taken out has been spread evenly over the remaining area. How much has the level of ground been raised ?

14. From a point on the ground, 20 m away from the foot of vertical tower, the angle of elevation of the top of the tower is 60°. Find the height of the tower.

15. (a) In a right triangle ABC, right-angled at B, $BC = 6cm$ and $AB = 8cm$. A circle is inscribed in the $\triangle ABC$. Find the radius of the incircle.

    (b) Two cricles touch externally at P and AB is a common tangent, touching one circle at A and the other at B. Find the measure of $\angle APB$.

16. A solid sphere of radius r is melted and cast into the shape of a solid cone of height r. What is the radius of the base of the cone in terms of r ?

17. Answer any four of the following questions :

    (i) ABC and BDE are two equilateral triangles such that D is the mid-point of BC. The ratio of the areas of the triangles ABC and BDE is

    (A) $2:1$

    (B) $1:2$

    (C) $4:1$

    (D) $1:4$

    (ii) In $\triangle$ ABC, $AB = 4\sqrt{3}$ cm, $AC = 8cm$ and $BC = 4cm$. The angle B is

(A) 120°

(B) 90°

(C) 60°

(D) 45°

(iii) The perimeters of two similar triangles are $35cm$ and $21cm$ respectively. If one side of the first triangle is $9cm$,then the corresponding side of the second triangle is

(A) $5.4cm$

(B) $4.5cm$

(C) $5.6cm$

(D) $15cm$

(iv) In a $\triangle\ ABC, D$ and E are points on the sides AB and AC respectively such that $DE||BC$ and $AD : DB = 3 : 1$. If $AE = 3.3cm$,then AC is equal to

(A) $4cm$

(B) $1.1cm$

(C) $4.4cm$

(D) $5.5cm$

(v) In an isosceles triangle ABC, if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C$ is equal to

(A) 30°

(B) 45°

(C) 60°

(D) 90°

18. To explain how trignometry can be used measure the height of an inaccessible object, a teacher gave the following example to students :

A TV tower stands vertically n the bank of a canal. From a point on the other bank direct opposite the tower, the angle of the elevation of the top of the tower is 60°.From another point 20 m away from this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (as shown in Figure 1).

Based on the above, answer the following questions :

(i) The width of the canal is

(A) $10\sqrt{3}m$

(B) $20\sqrt{3}m$

(C) $10m$

(D) $20m$

(ii) Height of the tower is

(A) $10\sqrt{3}m$

(B) $10m$

(C) $20\sqrt{3}m$

(D) $20m$

(iii) Distance of the foot of the tower from the point D is

(A) $20m$

(B) $30m$

(C) $10m$

(D) $20\sqrt{3}m$

(iv) The angle formed by the line of sight with the horizontal when it is above the horizontal line is known as

(A) angle of depression

(B) line of sight

(C) angle of elevation

(D) obtuse angle

(v) In above figure, measure of angle XAC is

(A) $30°$

(B) $60°$

(C) $90°$

(D) $45°$

19. A children's park is in the triangular shape as shown in the below figure.In the middle of the park, there is a circular region for younger children to play. It is fenced with three layers of wire. The radius of the circular region is $3m$. Based on the above,answer the following questions:

(i) The perimeter(or circumference) of the circular region is

(A) $3\pi m$

(B) $18\pi m$

(C) $6\pi m$

(D) $9\pi m$

(ii) The Total length of wire used is

(A) $9\pi m$

(B) $18\pi m$

(C) $54\pi m$

(D) $27\pi m$

(iii) The area of the circular region is

(A) $54\pi m^2$

(B) $3\pi m^2$

(C) $18\pi m^2$

(D) $9\pi m^2$

(iv) If $BD = 6m$, $DC = 9m$ and ar $(\triangle ABC = 54\ m^2$,then the length of sides AB and AC, respectively, are)

(A) $9m, 12m$

(B) $12m, 9m$

(C) $10m, 12m$

(D) $12m, 10m$

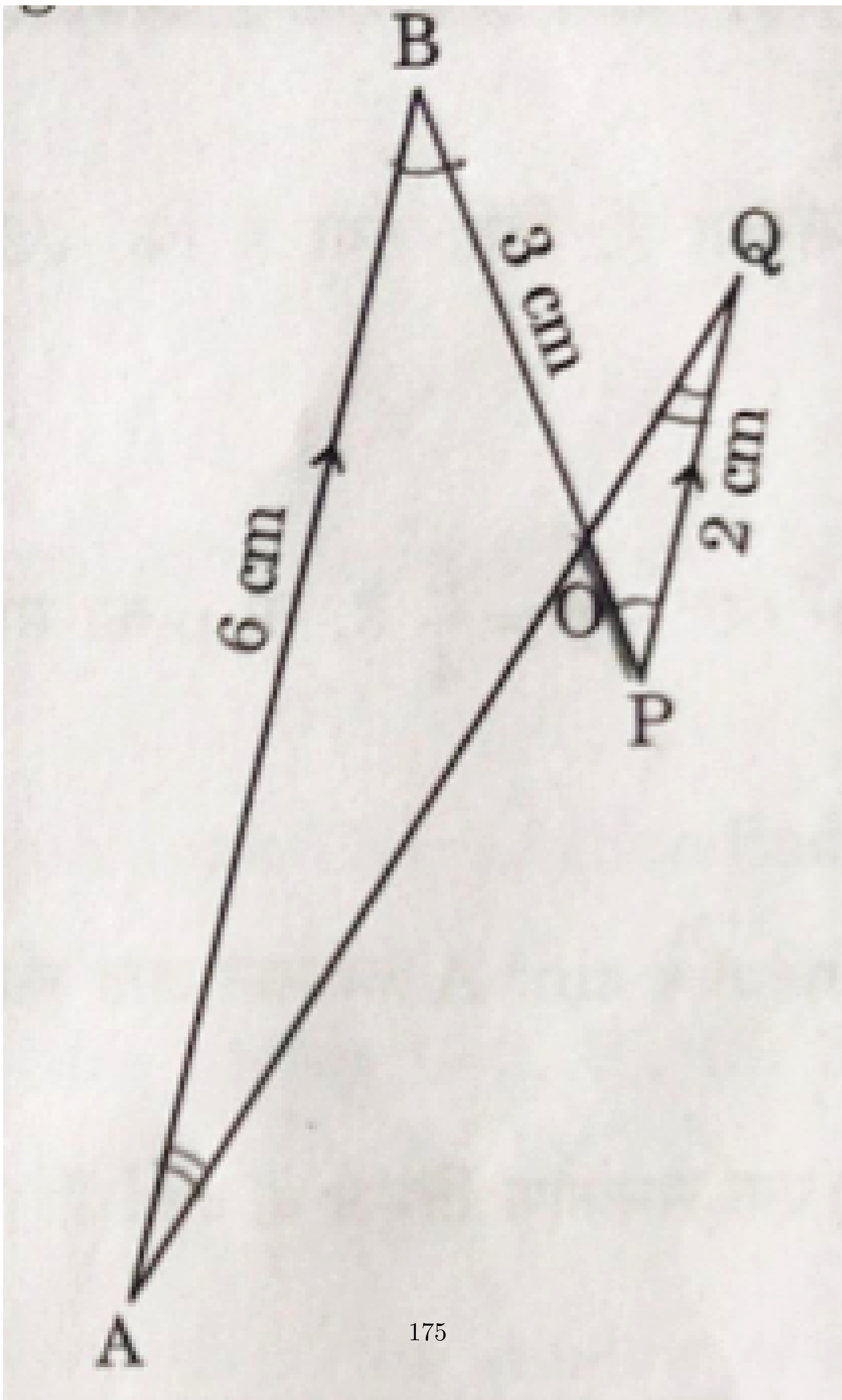
(v) The perimter of $\triangle ABC$ is

(A) $28m$

(B) $37m$

(C) $36m$

(D) $38m$


B
Q
3 cm
6 cm
2 cm
O
P
A

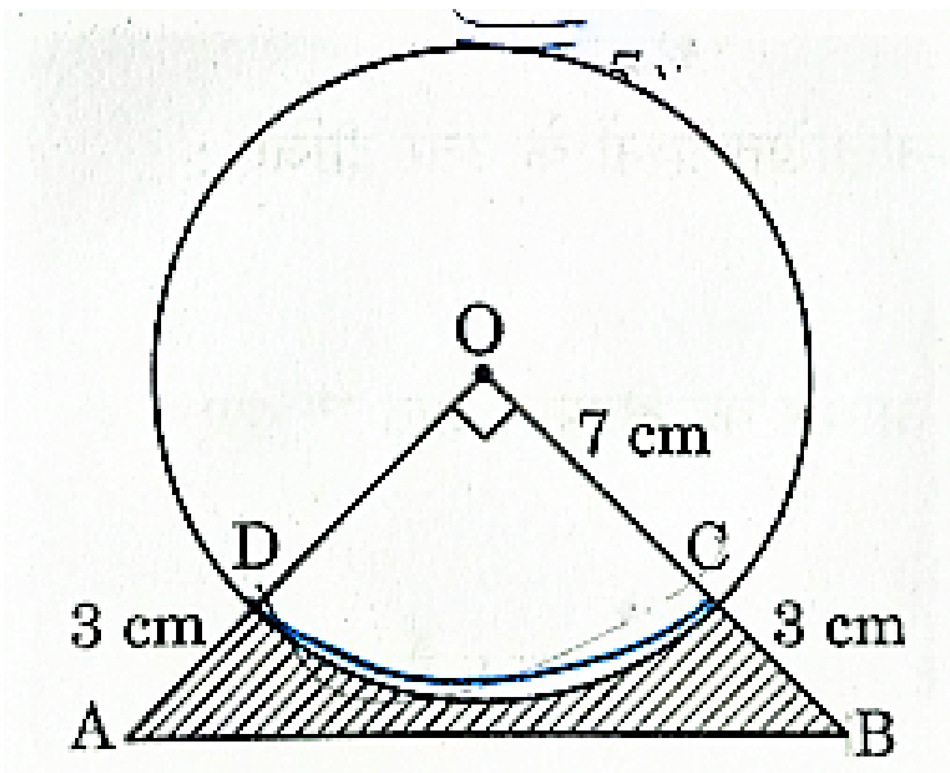


Figure 10.1: memento

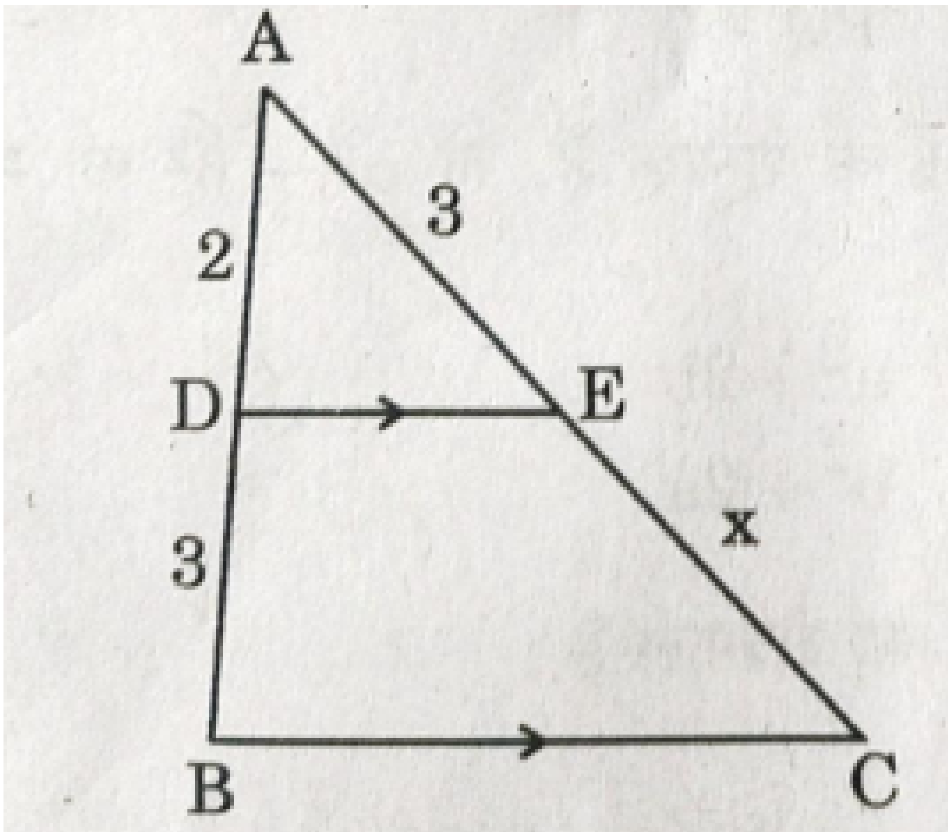


Figure 14.1: $\triangle ABC$

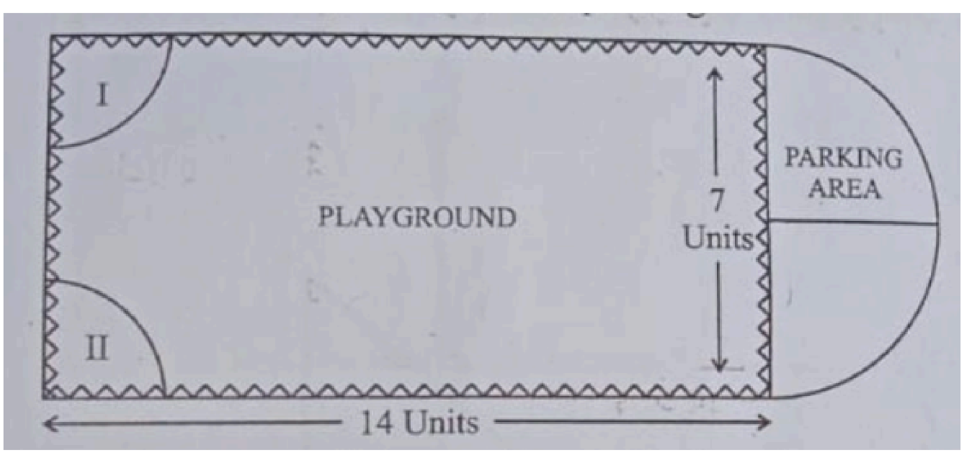


Figure 17.1: Playground

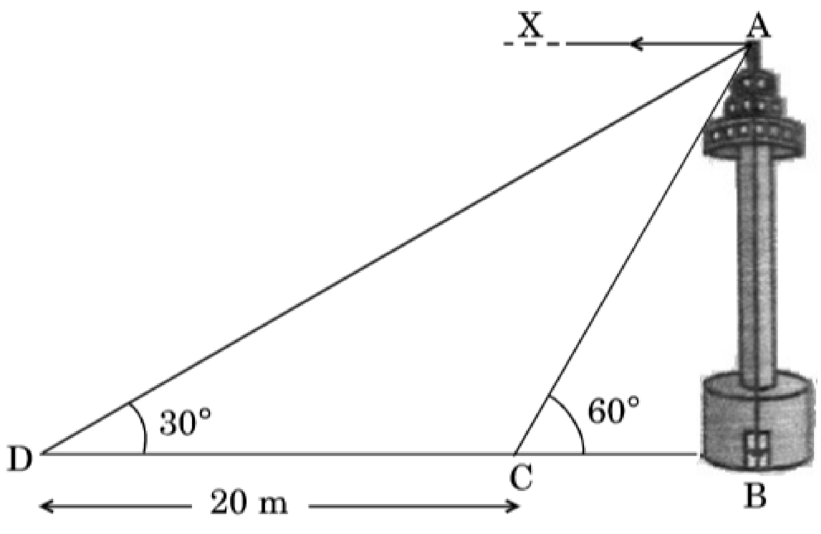


Figure 18.1: Projection of Tower

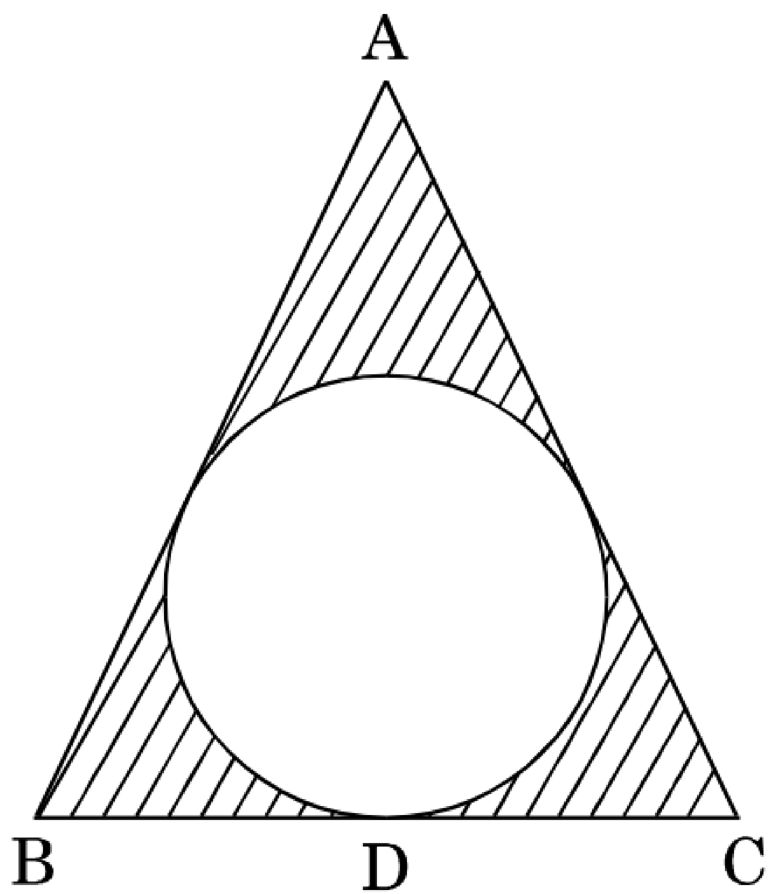


Figure 19.1: Children's Park in triangular shape

Chapter 10

Discrete

10.1. 2022

10.1.1. 10

1. If $-\frac{5}{7}$, a, 2 are consecutive terms in an Arthimetic Progression, then the value of a is

   (a) $\frac{9}{7}$

   (b) $\frac{9}{14}$

   (c) $\frac{19}{7}$

   (d) $\frac{19}{14}$

2. If two positive integers p and q can be expressed as $p = ab^3$ and $q = a^2b$; a and b being prime numbers, then find LCM of (p, q) .

3. Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$ for some integer q.

4. Prove that $\sqrt{5}$ is an irrational number.

5. (a) Find the sum of first 16 terms of an Arithmetic Progression whose 4^{th} and 9^{th} terms are -15 and -30 respectively.

   (b) If the sum of first 14 terms of an Arithmetic Progression is 1050 and its fourth term is 40, find its 20^{th} term.

6. (a) Find the sum of the first twelve 2-digit numbers which are multiples of 6.

   (b) In an AP, if $a_2 = 26$ and $a_{15} = -26$, then write the AP.

7. In Mathematics, relations can be expressed in various ways. The matchstick patterns are based on linear relations. Different strategies can be used to calculate the number of matchsticks used in different Fig. 7.1 One such pattern is shown below. Observe the pattern and answer the following questions using Arithmetic Progression :

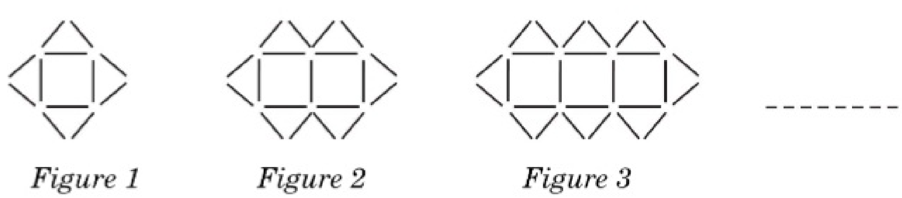


Figure 7.1: patterns of Figure1, figure2 ,figure3

   (a) Write the AP for the number of triangles used in the Fig. 7.1. Also, write the nth term of this AP.

   (b) Which figure has 61 matchsticks ?

8. (a) In an A.P. if the sum of third and seventh term is zero, find its 5^{th} term.

(b) Determine the AP whose third term is 5 and seventh term is 9.

9. Find the sum of the first 20 terms of an A.P. whose n^{th} term is given as $a_n = 5 - 2n$

10. Find the common difference 'd' of an AP whose first term is 10 and the sum of the first 14 terms is 1505.

11. For what value of 'n', are the n^{th} terms of the APs: $9, 7, 5, \ldots$ and $15, 12, 9, \ldots$ the same?

12. (a) The curved surface area of a right circular cylinder is $176 sq.cm$ and its volume is $1232 cu.cm$. What is the height of the cylinder?

    (b) The largest sphere is carved out of a soild cube of side $21 cm$. Find the volume of the sphere.

13. The sum of the first three terms of an A.P is 33. If the product of first and third term exceeds the second term by 29, find the A.P.

14. (a) Find the number of terms in the following A.P:

$$5, 11, 17, \ldots, 203 \tag{14.1}$$

    (b) Find the sum of the first 20 terms of an AP whose n^{th} term is given as $a_n = 5 - 3n$

15. While buying an expensive item like a house or a car, it becomes easier for a middle-class person to take a loan from a bank and then repay the loan along with interest in easy instalments. Aman buys a car by

taking a loan of ₹2,36,000 from the bank and starts repaying the loan in monthly instalments. He pays ₹2,000 as the first instalment and then increases the instalment by ₹500 every month.

(a) Find the amount he pays in the 25^{th} installment.

(b) Find the total amount paid by him in the first 25 installments.

10.2. 2023

10.2.1. 10

1. The ratio of HCF to LCM of the least composite number and the least prime number is :

   (a) 1 : 2

   (b) 2 : 1

   (c) 1 : 1

   (d) 1 : 3

2. The next term of the A.P.: $\sqrt{7}$, $\sqrt{28}$, $\sqrt{63}$ is :

   (a) $\sqrt{70}$

   (b) $\sqrt{80}$

   (c) $\sqrt{97}$

   (d) $\sqrt{112}$

3. Two numbers are in the ratio 2 : 3 and their LCM is 180. what is the HCF of these numbers ?

4. How many terms are there in A.P whose first and fifth term are - 14 and 2, respectively and the last term is 62.

5. Which term of the A.P.:65, 61, 57, 53, ... is the first negative term ?

6. Prove that $\sqrt{5}$ is an irrational number.

7. If p and q are natural numbers and p is the multiple of q, then what is the HCF of p and q ?

   (a) pq

   (b) p

   (c) q

   (d) $p + q$

8. Prove that $2+\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

9. Find by prime factorisation the LCM of the numbers 18180 and 7575. Also, find the HCF of the two numbers

10. Three bells ring at intervals of 6, 12 and 18 minutes. If all the three bells rang at 6 a.m., when will they ring together again ?

11. How many terms of the arithmetic progression 45, 39, 33, ... must be taken so that their sum is 180 ? Explain the double answer.

12. If $p-1$, $p+1$ and $2p+3$ are in A.P., then the value of p is

    (a) -2

    (b) 4

    (c) 0

    (d) 2

13. Assertion (A): The perimeter of $\triangle ABC$ is a rational number.

    Reason (R): The sum of the squares of two rational numbers is always rational.

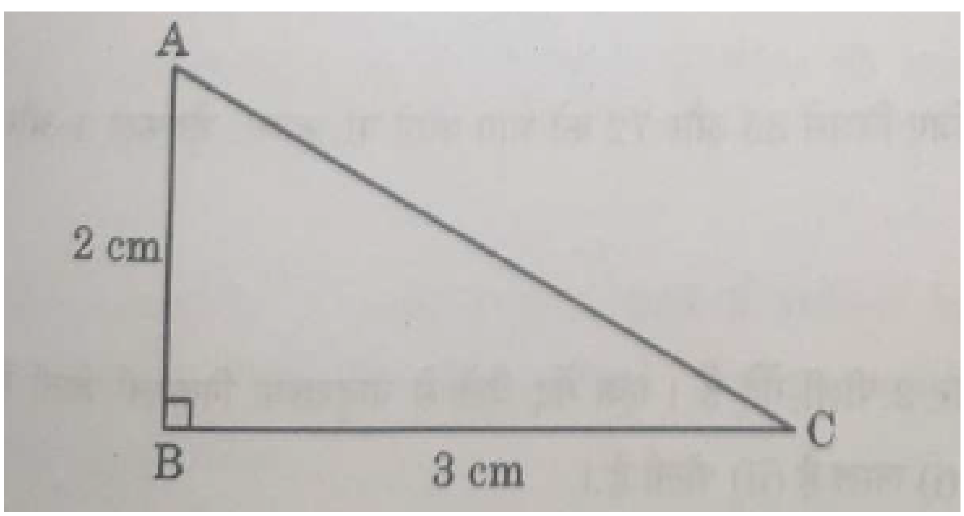


Figure 13.1: $\triangle ABC$

14. Find the greatest number which divides 85 and 72 leaving remainders 1 and 2 respectively.

15. Prove that $\sqrt{5}$ is an irrational number.

16. The ratio of the 11^{th} term to 17^{th} term of an A.P. is 3 : 4. Find the ratio of 5^{th} to 21^{th} of the same A.P. Also, find the ratio of the sum of first 5 terms to that of first 21 terms

17. 250 logs are stacked in the following manner:

    22 logs in the bottom row , 21 in the next row, 20 in the row next to it and so on(as shown by an example). In how many, are the 250 logs placed and how many logs are there in top row ?

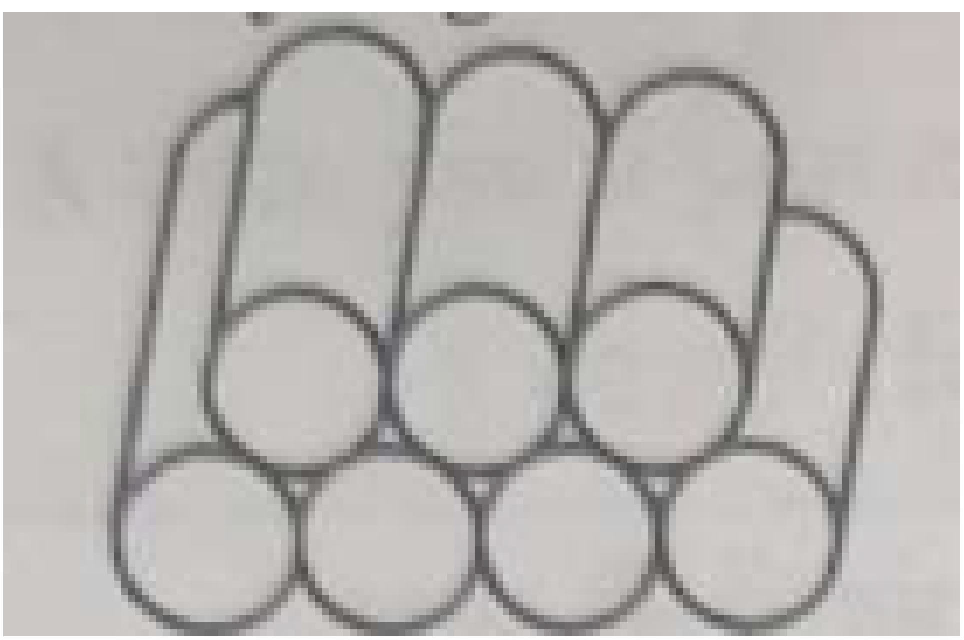


Figure 17.1: Pile of logs

10.3. 2021

10.3.1. 10

1. Write the common difference of the A.P. :$\frac{1}{5}, \frac{4}{5}, \frac{7}{5}, \frac{10}{5}, \cdots$

2. Find the 8^{th} term of the A.P. whose first term is -2 and common difference is 3.

3. Roshini being a plant lover decides to start a nursery. She bought few plants with pots.She placed the pots in such a way that the number of pots in the first row is 2, in the second is 5, in the third row is 8 and so on. Based on the above, answer the following questions :

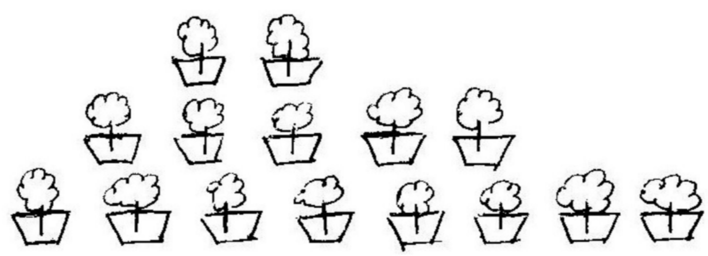


Figure 3.1: Plants

   (i) How many pots were placed in the 7^{th} row ?

      A 20

      B 23

      C 77

      D 29

   (ii) If Roshini wants to place 100 pots in total, then total number of rows formed in the arrangement will be ?

A 8

B 9

C 10

D 12

(iii) How many pots are placed in the last row ?

A 20

B 23

C 26

D 29

(iv) If Roshini ha sufficient space for 12 rows, then how many total number of pots are placed by her wih the same arrangement ?

A 222

B 155

C 187

D 313

4. Find the LCM and HCF of two numbers 26 and 91 by the method of prime factorization.

5. For two numbers x and y, if $xy = 1344$ and HCF $(x, y) = 8$, then find LCM(x, y).

6. Find the HCF of 96 and 404 by prime factorisation.

7. Express 792 as the product of its prime factors.

8. The sum of the first 4 terms of an A.P. is zero and its 4^{th} term is 2. Find the A.P.

9. If the sum of the first n terms of an A.P. is given by $S_n = 4n - n^2$, then find its n^{th} term. Hence, find the 25^{th} term and the sum if the first 25 terms of this A.P.

10. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

11. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then find the value of k.

12. Find the mean of first 10 composite numbers.

13. If S_n denotes the sum of first n terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$.

14. After how many decimal places will the decimal expansion of the rational number $\frac{14587}{1250}$ terminate ?

15. State giving reason whether $5 \times 7 \times 11 + 11$ is a composite number or a prime number.

16. If the 6^{th} and 14^{th} terms of an A.P. are 29 and 69 respectively, then find the 10^{th} term of the A.P.

17. If the first three consecutive terms of an A.P. are $3y - 1, 3y + 5$ and $5y + 1$ find the value of y.

Chapter 11

Differentiation

11.1. 2023

11.1.1. 12

1. If $\tan\left(\frac{x+y}{x-y}\right) = k$,then $\dfrac{dy}{dx}$ is equal to

2. $\frac{-y}{x}$

3. $\frac{y}{x}$

4. $\sec^2\left(\frac{y}{x}\right)$

5. $-\sec^2\left(\frac{y}{x}\right)$

6. **Assertion(A) :**Maximum value of $\left(\cos^{-1}\right)^2$ is π^2.

   Reason(R): Range of the principle value branch of $\cos^{-1} x$ is $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$.

7. If $y = \sqrt{ax+b}$, prove that $y\left(\dfrac{d^2y}{dx^2}\right) + \left(\dfrac{dy}{dx}\right)^2 = 0$

8. If the circumference of circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius.

9. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine.The piston moves inside the cylinder bore

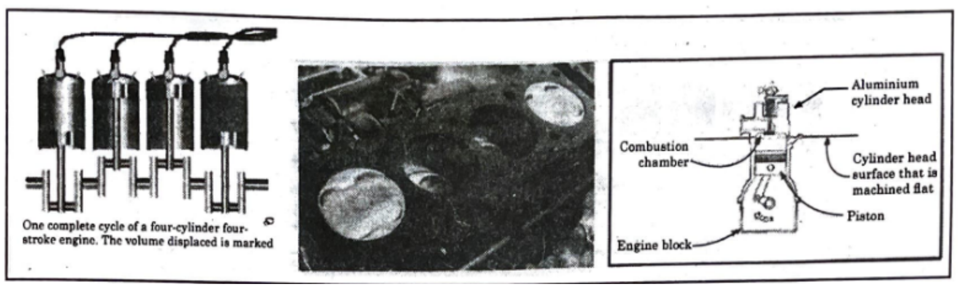


Figure 9.1:

10. The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi\ cm^2$.

    Based on the above information , answer the following questions:

    (i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

    (ii) Find $\dfrac{dv}{dr}$

    (iii) (a) Find the radius of cylinder when its volume is maximum.

    (b) For maximum volume, $h > r$.State true or false and justify.

11. The use of electric vehicles will curb air pollution in the long run.

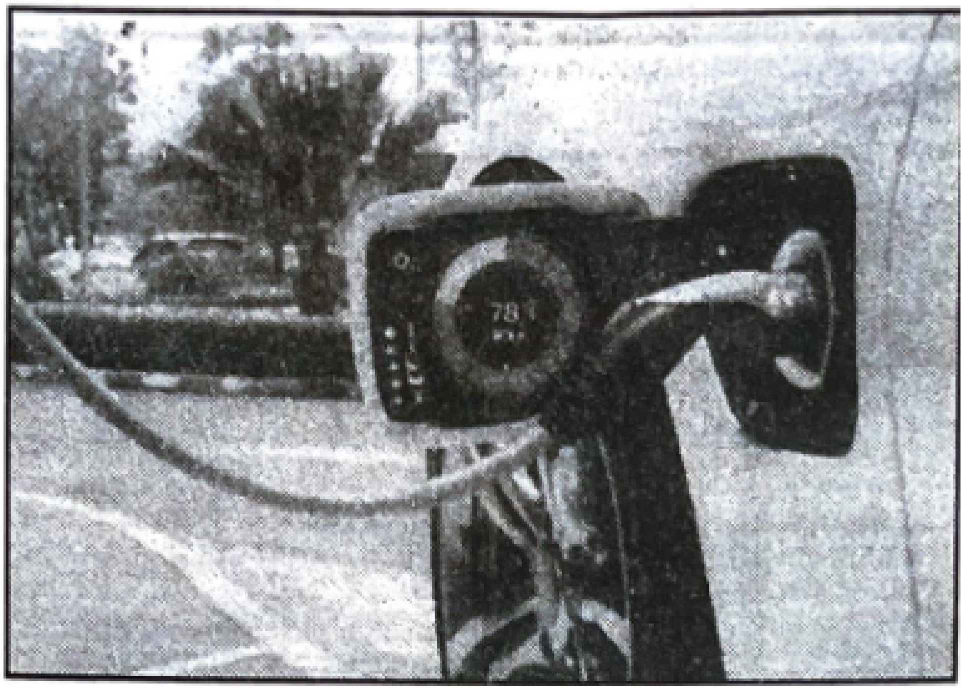


Figure 11.1:

12. The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2 \qquad (12.1)$$

Where t represents the time and t=1,2,3... corresponds to year 2001,2002,2003... respectively. Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.

(ii) Prove that the function V(t) is an increasing function.

13. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is ______.

14. The total revenue (in ₹) received from sale of x units of a product is $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 12$ is ______.

15. If $\sin y = x\sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin x^2(a+y)}{\sin a}$.

16. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point(3,22).

17. If $Y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$,$-\frac{1}{\sqrt{3}}$ $<$x $<\frac{1}{\sqrt{3}}$ then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

18. If $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$, then $\frac{dy}{dx}$ is equal to

19. The oder and degree of the differential equation of the family of parabolas having at organ and axis along positive x-axis is

20. If $y = \log x$, then $\frac{d^2y}{dx^2}$=

21. Find the intervals in which the function f defined as $f(x) = \sin(x) + \cos(x)$, $0 \leq x \leq 2\pi$ is strictly increasing or decreasing.

22. Prove that the radius of the right circular cylindar of greatest curved surface area which can be inscribed in a given cone is half of thatof the cone.

23. If $y = x^{\sin x} + \sin^{-1}(\sqrt{x})$, the find $\frac{dy}{dx}$.

24. The supply function of a commodity is $100p = (x + 20)^2$. Find the Producer's Surplus(PS),whne the market price is ₹25.

25. Find: $\int \frac{2x^2+1}{x^2-3x+2}$dx

11.2. 2022

11.2.1. 12

11.2.1. The order and degree of the differential equation of the family of parabolas having vertex at origin and axis along positive x-axis is:

(a) 1,1

(b) 1,2

(c) 2,1

(d) 2,2

11.2.2. If $y = \log x$, then $\frac{d^2y}{dx^2} =$ ______

11.2.3. If $y = \sqrt{a + \sqrt{a + x}}$:

11.2.4. Find the intervals in which the function f defined as $f(x) = \sin x + \cos x, 0 \leq x \leq 2$ is strictly increasing or decreasing.

11.2.5. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

11.2.6. If $y = x^{\sin x} + \sin^{-1} x$,then find $\frac{dy}{dx}$.

11.2.7. If $\sin y = x \sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

11.2.8. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point (3,22).

11.2.9. If $y = \tan^{-1}(\frac{3x-x^3}{1-3x^2}) - \frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

11.2.10. If $y = (\tan^{-1} x)^2$, then show that $(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$.

11.2.11. Show that of all the rectangles inscribed in a given fixed circle, the square has maximum area.

11.2.12. Find the intervals in which the function f given by $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

11.2.13. If $\sec^{-1}(\frac{1+x}{1-y}) = a$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{x-1}{y-1}$

(b) $\frac{x-1}{y+1}$

(c) $\frac{y-1}{x+1}$

(d) $\frac{y+1}{x-1}$

11.3. 2021

11.3.1. 10

1. The order and degree of the differential equation of the family of parabolas having vertex at origin and axis along positive x-axis is

(a) $1, 1$

(b) $1, 2$

(c) $2, 1$

(d) $2, 2$

2. If $y = \log x$, then $\frac{d^2y}{dx^2} =$ ______.

3. If $y = e^x + e^{-x}$, then show that $\frac{dy}{dx} = \sqrt{y^2 - 4}$.

4. If $y = x^{\sin x} + \sin^{-1}(\sqrt{x})$, the find $\frac{dy}{dx}$.

5. Find the intervals in which the function f defined as $f(x) = \sin(x) + \cos(x)$, $0 \leq x \leq 2\pi$ is strictly increasing or decreasing.

6. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

7. $\lim_{x \to 0} \frac{e^{-x} - e^x}{x}$ is equal to

(a) 2

(b) 1

(c) -1

(d) 2

11.4. 2021

11.4.1. 12

1. The point at which the normal to the curve $y = x + \frac{1}{x}, X > 0$ is perpendicular to the line $3x - 4y7 = 0$ is:

   (a) $(2, \frac{5}{2})$

   (b) $(\pm 2, \frac{5}{2})$

   (c) $(-\frac{1}{2}, \frac{5}{2})$

   (d) $(\frac{1}{2}, \frac{5}{2})$

2. If $y = log(\cos e^x)$, then $\frac{dx}{dy}$ is:

   (a) $\cos e^{x-1}$

   (b) $e^{-x} \cos e^x$

   (c) $e^x \sin e^x$

   (d) $-e^x \tan e^x$

3. The least value of the function $f(x) = 2\cos x + x$ in the closed interval $[0, \frac{\pi}{2}]$ is:

   (a) 2

   (b) $\frac{\pi}{6} + \sqrt{3}$

   (c) $\frac{\pi}{2}$

   (d) The least value does not exist.

4. If $x = a\sec\theta, y = b\tan\theta$, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ is:

   (a) $\frac{-3\sqrt{3}b}{a^2}$

   (b) $\frac{-2\sqrt{3}b}{a}$

   (c) $\frac{-3\sqrt{3}b}{a}$

   (d) $\frac{-b}{3\sqrt{3}a^2}$

5. The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t $\sin^{-1}x, -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$, is:

   (a) 2

   (b) $\frac{\pi}{2} - 2$

   (c) $\frac{\pi}{2}$

   (d) -2

6. The point(s) on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$ is/are:

   (a) $(-2, 19)$

   (b) $(2, -9)$

   (c) $(\pm 2, 19)$

   (d) $(-2, 19) and (2, -9)$

7. For which value of m is the line $y = mx + 1$ a tangent to the curve $y^2 = 4x$?

   (a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 3

8. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$ is:

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) $\sqrt{3}\frac{1}{3}$

11.5. 2021

11.5.1. 12

1. A firm knows that the demand function for one of its product is linear. It also knows that it can sell 1400 units when the price is ₹4 per unit and it can sell 1800 units at a price ₹2 per unit.Find the marginal revenue function of this product.

2. Find the intervals in which the function $f(x) = x^4–4x^3 + 6x^2–4x + 1$ is increasing or decreasing.

3. If $\sqrt{1-x^2} + \sqrt{1-y^2} = 4(x-y)$,then show that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

4. If $f'(x) = 3x^2 - 4x - \frac{2}{x^3}$ and $f(1) = 0$, then find $f(2)$.

5. A window is in the form of a rectangle mounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the rectangular part of the window to admit maximum light through the whole opening.

6. Divide the number 8 into two positive numbers such that the sum of the cube of one and the square of the other is minimum.

7. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ is:

   (a) e^{y-x}

   (b) e^{x+y}

   (c) $-e^{y-x}$

   (d) $2e^{x-y}$

8. $Ify = 5\cos x - 3\sin x$, then $\frac{d^2y}{dx^2}$ is equal to

   (a) -y

   (b) y

   (c) 25y

   (d) 9y

9. The points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to y-axis are:

   (a) $(0, \pm 4)$

   (b) $(\pm 4, 0)$

   (c) $(\pm 3, 0)$

(d) $(0, \pm 3)$

(Differentiation 2021)

Chapter 12

Integration

12.1. 2023

12.1.1. 12

1. If

$$\frac{d}{dx}(f(x)) = 2x + \frac{3}{x} \tag{1.1}$$

   and $f(1) = 1$,then $f(x)$is

   (a) $x^2 + 3\log|x| + 1$

   (b) $x^2 + 3\log|x|$

   (c) $2 - \frac{3}{x^2}$

   (d) $x^2 + 3\log|x| - 4$

2. The integral factor of the differential equation

$$\left(1 - y^2\right)\frac{dx}{dy} + yx = ay, (-1 < y < 1) \tag{2.1}$$

is

(a) $\frac{1}{y^2-1}$

(b) $\frac{1}{\sqrt{y^2-1}}$

(c) $\frac{1}{1-y^2}$

(d) $\frac{1}{\sqrt{1-y^2}}$

3. Anti derivative of $\frac{\tan(x)-1}{\tan(x)+1}$ with respect to x is:

   (a) $\sec^2(\frac{\pi}{4}-x)+c$

   (b) $-\sec^2(\frac{\pi}{4}-x)+c$

   (c) $\log\mid\sec(\frac{\pi}{4}-x)\mid+c$

   (d) $-\log\mid\sec(\frac{\pi}{4}-x)\mid+c$

4. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}}\left(\frac{1}{(e^x+e^{-x})(e^x-e^{-x})}\right)dx$

5. (a) Find the general solution of the differential equation:

$$\left(xy-x^2\right)dy=y^2dx \tag{5.1}$$

   (b) Find the general solution of the differential equation:

$$\left(x^2+1\right)\frac{dy}{dx}+2xy=\sqrt{x^2+4} \tag{5.2}$$

6. (a) Evaluate $\int_{-1}^{1}\left|x^4-x\right|dx$

   (b) Find $\int e^x\left(\frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}\right)dx$

7. Find $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

12.2. 2022

12.2.1. 12

12.2.1. Integration:

$$\int_0^1 x^2 e^x \, dx$$

12.2.2. Find the general solution for this differential equation:

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

12.2.3. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m using integration.

12.2.4. Find:

$$\int \frac{1}{e^x + 1} \, dx$$

12.2.5. Evaluate:

$$\int_1^4 \{|x| + |3 - x|\} \, dx$$

12.2.6. Evaluate:

$$\int_{-3}^3 \frac{x^4}{1 + e^x} \, dx$$

12.2.7. Find the particular solution of the differential equation:

$$x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$$

given that $y(1) = 0$.

12.2.8. Find the general solution of the differential equation:

$$x(y^3 + x^3)\,dy = (2y^4 + 5x^3y)\,dx$$

12.2.9. Find:

$$\int \frac{dx}{\sqrt{4x - x^2}}$$

12.2.10. Find the general solution of the following equation:

$$\frac{dy}{dx} = e^x - yx^2e^{-y}$$

12.2.11. Find:

$$\int e^x \sin(2x)\,dx$$

12.2.12. Find:

$$\int \frac{2x}{(x^2+1)(x^2+2)}\,dx$$

12.2.13. Evaluate:

$$\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\,dx$$

12.2.14. Solve the following differential equations:

$$(y-\sin^2 x)\,dx+\tan(x)\,dy=0$$

12.2.15. Find the general solution of the differential equation:

$$(x^3+y^3)\,dy=x^2y\,dx$$

12.2.16. Find:

$$\int \frac{1}{\sqrt{12+4x-x^2}}\,dx$$

12.2.17. Find:

$$\int \frac{xe^x}{(x+4)^5}\,dx$$

12.2.18. Find the general solution of the following differential equation:

$$(4+y^2)(3+\log x)\,dx+x\,dy=0$$

12.2.19. Evaluate:

$$\int_0^{\frac{\pi}{3}} |\cos(3x)| \, , dx$$

12.2.20. Find the general solution of the following differential equation:

$$2xe^{\frac{y}{x}}\, dy + (x - 2ye^{\frac{y}{x}})\, dx = 0$$

12.2.21. Find the particular solution of the differential equation:

$$(2x^2 + y)\frac{dx}{dy} = x$$

given that $y = 2$ when $x = 1$.

12.2.22. Find:

$$\int \frac{x^2 + x + 1}{(x+1)(x^2+4)}\, dx$$

12.2.23. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

12.2.24. Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration.

12.2.25. Find:

$$\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\cot x}}\, dx$$

is equal to:

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

12.2.26. Find:

$$\int \frac{(x+2)(x+2\log x)^3}{x}\,dx$$

12.2.27. Find:

$$\int_0^{\frac{\pi}{2}} \log(\tan x)\,dx$$

12.2.28. Find:

$$\int_{-1}^{2} |x|\ dx$$

12.2.29. Find:

$$\int x^2 \log x.dx$$

12.2.30. Find the general solution of the following differential equation :

$$\frac{dy}{dx} = (1+x)(1+y)$$

12.2.31. Find the integrating factor for the following differential equation:

$$\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x (x \neq 0)$$

12.2.32. Find:

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

12.2.33. Find the following differential equation :

$$x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$$

12.2.34. Find the sum of the order and the degree of the differential equation : $\left(x + \frac{dx}{dy}\right)^2 = \left(\frac{dy}{dx}\right)^2 + 1$

12.2.35. If $\frac{d}{dx}(x) = \frac{\sec^4 x}{\csc^4 x}$ and $F\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$, then find $F(x)$

12.2.36. Find : $\int \frac{\log x - 3}{(\log x)^4} dx$.

12.2.37. Find : $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$.

12.2.38. Evaluate : $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(4+\sin x)} dx$

12.2.39. Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x} dx$.

12.2.40. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

12.2.41. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$, semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

12.2.42. Find the product of the order and the degree of the differential equation $\frac{d}{dx}\left(xy^2\right) \cdot \frac{dy}{dx} + y = 0$.

12.2.43. Find : $\int \frac{\sqrt{\cot x}}{\sin x \cos x} dx$

12.2.44. Find : $\int \frac{1}{x(x^2+4)} dx$

12.2.45. Evaluate : $\int_0^1 \tan^{-1} x dx$

12.2.46. Find : $\int \frac{2x}{x^2+3x+2} dx$

12.2.47. Solve the following differential equation : $\left(1 + e^{y/x}\right) dy + e^{y/x}(1 - \frac{y}{x})dx = 0$

12.2.48. Evaluate : $\int_0^1 x(1-x)^n dx$

12.2.49. Using integration, find the area of the smaller region enclosed the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$

12.2.50. If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq.units, then using integration, find the value of a, where $a > 0$.

12.2.51. Find the general solution of the differential equation : $\frac{dy}{dx} = \dfrac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$

12.2.52. Find : $\int \frac{dx}{x^2-6x+13}$

12.2.53. Find the particular solution of the differential equation $x\frac{dy}{dx} - y = x^2 \cdot e^x$, given $y(1) = 0$.

12.2.54. Find the general solution of the differential equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$

12.2.55. Evaluate : $\int_{-\pi/2}^{\pi/2} (\sin|x| + \cos|x|)\, dx$

12.2.56. Find : $\int \frac{x^2}{(x^2+1)(3x^2+4)} dx$

12.2.57. Evaluate : $\int_{-2}^{1} \sqrt{5-4x-x^2} dx$

12.2.58. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

12.2.59. $\int \frac{\cos 8x+1}{\tan 2x-\cot 2x} dx = \lambda \cos 8x + c$, then the value of λ is

(a) $\frac{1}{16}$

(b) $\frac{1}{8}$

(c) $-\frac{1}{16}$

(d) $-\frac{1}{8}$

12.2.60. $\int_0^1 \tan(\sin^{-1} x) dx$ equals

(a) 2

(b) 0

(c) -1

(d) 1

12.2.61. The integrating factor of the differential equation $x\left(\frac{dy}{dx}\right) - y = \log x$ is ___________.

12.2.62. Find the solution of the differential equation $\log\left(\frac{dy}{dx}\right) = ax + by$

12.2.63. Solve the following homogeneous differential equation : $x\left(\frac{dy}{dx}\right) = x+y$

12.2.64. Evaluate $\int_1^3 (x^2 + 1 + e^x) dx$ as the limit of sums.

12.2.65. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a using integration.

12.2.66. Find : $\int \frac{x}{(x+1)^2(x+2)} dx$

12.2.67. Evaluate : $\int_0^1 \frac{xe^x}{(x+1)^2} dx$

12.2.68. Solve the following differential equation : $\left(\frac{dy}{dx}\right) = e^{x+y} + x^2 e^y$

12.2.69. The supply function of a commodity is $100p = (x + 20)^2$. Find producer's surplus (PS), when the market price is ₹25.

12.2.70. Find : $\int \frac{2x^2+1}{x^2-3x+2} dx$

12.2.71. In a certain culture of bacteria, the rate of increase of bacteria is proportional to the number present. It is found that there are $10,000$ bacteria at the end of 3 hours and $40,000$ bacteria at the end of 5 hours . determine the number of bacteria present int the beginning

12.3. 2021

12.3.1. 12

1. If $f(x) = \frac{1-x}{1+x}$,then find $f \circ f(x)$.

2. Let W denote the set of words in the English dictionary. Define the relation R by

   $R = (x, y) \in W \times W$such that x and y have at least one letter in common.

Show that this relation R is reflexive and symmetric, but not transitive.

3. Find the inverse of the function $f(x) = \frac{4x}{3x+4}$.

4. $\int x\sqrt{x+2}$ dx is equal to

   (a) $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{2}{3}(x+2)^{\frac{3}{2}} + C$

   (b) $\frac{5}{2}(x+2)^{\frac{5}{2}} + \frac{3}{2}(x+2)^{\frac{3}{2}} + C$

   (c) $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

   (d) $\frac{2}{5}(x+2)^{\frac{5}{2}} + \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

   where C is the constant of integration.

5. $\int_0^1 \tan(\sin^{-1} x)$ dx equals

   (a) 2

   (b) 0

   (c) -1

   (d) 1

6. $\int \frac{e^x}{x+1}|1 + (x+1)\log(x+1)|dx$ equals

   (a) $\frac{e^x}{x+1} + c$

   (b) $e^x \frac{e^x}{x+1} + c$

   (c) $e^x \log(x+1) + e^x + c$

   (d) $e^x \log(x+1) + c$

7. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}$dx

8. Find: $\int \frac{x}{(x-1)^2(x+2)}$dx

9. $\int_0^{\frac{\pi}{2}} (\sin^{100} x - \cos^{100} x)$ dx equals

   (a) $\frac{\pi}{100}$

   (b) 0

   (c) $\frac{1}{100}$

   (d) $\frac{\lfloor 100}{(100)^{100}}$

10. $\int \frac{\cos 8x+1}{\tan 2x - \cot 2x} dx = \lambda \cos 8x + c$,then the value of λ is

   (a) $\frac{1}{16}$

   (b) $\frac{1}{8}$

   (c) $\frac{-1}{16}$

   (d) $\frac{-1}{8}$

Chapter 13

Functions

13.1. 2023

13.1.1. 12

1. The function $f(x) = x\,|x|$ is

   (a) continuous and differentiable at $x = 0$.

   (b) continuous but not differentiable at $x = 0$.

   (c) differentiable but not continuous at $x = 0$.

   (d) neither differentiable nor continuous at $x = 0$.

2. If

$$f(x) = \begin{cases} ax + b & 0 < x \leq 1 \\ 2x^2 - x & 1 < x < 2 \end{cases} \tag{2.1}$$

   is a differentiable function is (0,2), then find the values of a and b.

3. A function $f : [-4, 4] \to [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function.Further,find all possible values of 'a' for which $f(a) = \sqrt{7}$.

13.2. 2022

13.2.1. 12

1. Let R be the relation defined in N, as $R = \{(x, y) : 2x + 3y = 15, x, y \in N\}$, then R= {_____,_____}.

2. If the function $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if} x \neq \frac{\pi}{2} \\ 2, & \text{if} x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of k is _____.

3. Show that the relation R in the set $\mathbb{R}$ of all real numbers,defined as $\mathbb{R} = \left\{(a, b) : a \leq b^2\right\}$ is neither reflexive nor symmetric.

4. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$

5. Let a function $f : \mathbb{R} - \left\{\frac{-4}{3}\right\} \to \mathbb{R}$ be defined as $f(x) = \frac{4x}{3x+4}$.To show that f is one-one function. Hence, find the inverse of the function $f : \mathbb{R} - \left\{\frac{-4}{3}\right\} \to \text{Range of } f$.

6. If $f : R \to R$ be given by $f(x) = \left(3 - x^3\right)^{1/3}$, then find $(fof)(x)$.

7. Let W denote the set of words in the English dictionary. Define the relation R by $R = (x, y) \in W \times W$ such x and y have at least one letter in common. Show that this relation R is reflexive and symmetric, but not transitive.

8. Find the inverse of the function $f(x) = \left(\frac{4x}{3x+4}\right)$

13.3. 2021

13.3.1. 10

i

1. The graph of $y = p(x)$ is shown in Figure 1 for some polynomial $p(x)$.

   Find the number of zeroes of $p(x)$.

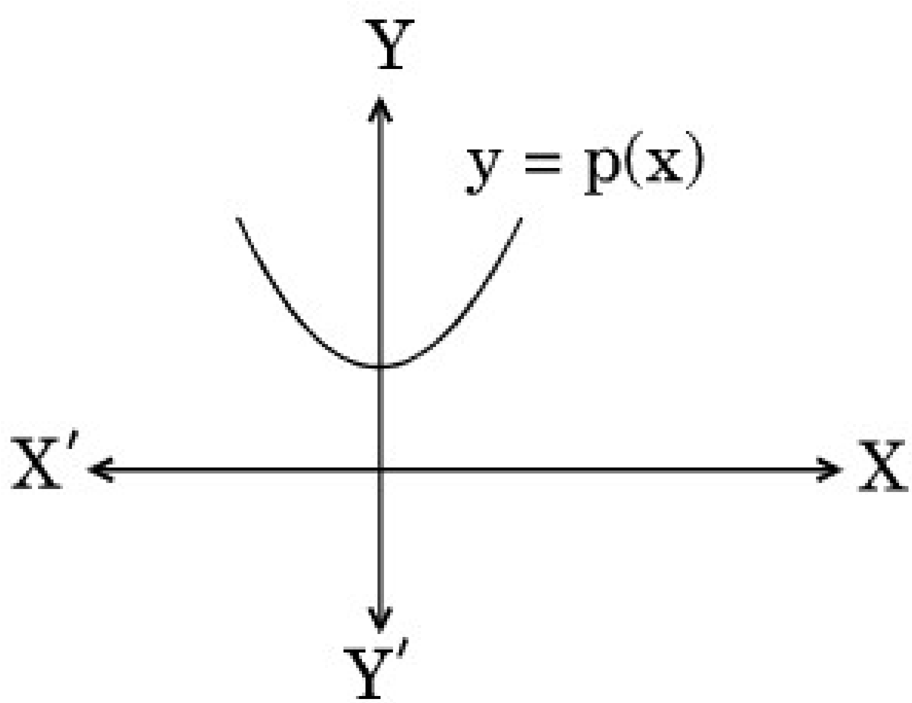


Figure 1.1:

13.3.2. 12

1. If $f(x) = \frac{1-x}{1+x}$, then find $(f \circ f)(x)$.

2. Let W denote the set of words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W \mid\}$ x and y have at least one letter in common. Show that this relation R is reflexive and symmetric, but not transitive.

3. Find the inverse of the function $f(x) = (\frac{4x}{3x+4})$.

4. The value of $k(k < 0)$ for which the function f defined as

$$f(x) = \begin{cases} x^2, & \text{if } x < 0 \\ \sin(x), & \text{if } x \geq 0 \end{cases}$$

is continuous at $x = 0$ is:

(a) ± 1

(b) ± 1

(c) $\pm \frac{1}{2}$

(d) $\frac{1}{2}$

5. Find the intervals in which the function f given by $f(x) = x^2 – 4x + 6$ is strictly increasing:

(a) $(-\infty, 2) \bigcup (2, \infty)$

(b) $(2, \infty)$

(c) $(-\infty, 2)$

(d) $(-\infty, 2) \bigcup (2, \infty)$

6. The real function $f(x) = 2x3 – 3x2 – 36x + 7$ is:

(a) Strictly increasing in $(-\infty,-2)$ and strictly decreasing in $(-2,\infty)$

(b) Strictly decreasing in $(-2,3)$

(c) Strictly decreasing in $(-\infty,3)$ and strictly increasing in $(3,\infty)$

(d) Strictly decreasing in $(-\infty,2)\bigcup(3,\infty)$

7. The value of b for which the function $f(x) = x + cosx + b$ is strictly decreasing over $\mathbf{R}$ is:

(a) $b < 1$

(b) No value of b exists

(c) $b \leq 1$

(d) $b \geq 1$

8. The point(s), at which the function f given by

$$f(x) = \begin{cases} \frac{x}{|x|}, x < 0 \\ -1, x \geq 0 \end{cases}$$

is continuous, is/are:

(a) $x\varepsilon R$

(b) $x = 0$

(c) $x\varepsilon R - \{0\}$

(d) $x = -1$ and 1

9. The area of a trapezium is defined by function f and given by $f(x) = (10+x)\sqrt{100-x^2}$, then the area when it is maximised is:

   (a) $75cm^2$

   (b) $7\sqrt{3}cm^2$

   (c) $75\sqrt{3}cm^2$

   (d) $5cm^2$

10. If $tan^{-1}x = y$, then:

   (a) $-1 < y < 1$

   (b) $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$

   (c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

   (d) $y\varepsilon\{\frac{-\pi}{2}, \frac{\pi}{2}\}$

13.4. 2020

13.4.1. 12

1. The interval in which the function f given by $f(x) = x^2e^{-x}$ is strictly increasing, is

   (a) $(-\infty, \infty)$

   (b) $(-\infty, 0)$

   (c) $(2, \infty)$

   (d) $(0, 2)$

2. The function $f(x) = \frac{x-1}{x(x^2-1)}$ is discontinuous at

   (a) exactly one point

   (b) exactly two points

   (c) exactly three points

   (d) no points

3. The function $f : \mathbb{R} \to [-1, 1]$ defined by $f(x) = \cos x$ is

   (a) both one-one and onto

   (b) not one-one, but onto

   (c) one-one, but onto

   (d) neither one-one, nor onto

4. The range of the principal value branch of the function $y = \sec^{-1} x$ is

5. The principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is

6. Find the value of k, so that the function $f(x) = \begin{cases} kx^2 + 5 & \text{if } x \leq 1, \\ 2 & \text{if } x > 1 \end{cases}$

   is continuous at $x = 1$.

7. Check whether the relation $\mathbb{R}$ in the set $\mathbb{N}$ of natural numbers given by

$$\mathbb{R} = \{(a, b) : \text{a is divisor of b}\} \tag{7.1}$$

   is reflexive, symmetric or transitive.Also determine whether $\mathbb{R}$ is an equivalence relation.

8. Prove that:

$$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right) \tag{8.1}$$

Chapter 14

Matrices

14.1. 2022

14.1.1. 10

1. Solve the equation $x + 2y = 6$ and $2x - 5y = 12$ graphically.

2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \tag{2.1}$$

$$(bx + ay) + (b - 4a) = 0 \tag{2.2}$$

14.1.2. 12

1. If $\begin{vmatrix} 3x & 3 \\ 13 & x \end{vmatrix} = \begin{vmatrix} 4 & -2 \\ 8 & 5 \end{vmatrix}$, then the value of x is :

   (a) 3

   (b) ± 5

(c) 25

(d) ± 1

2. For $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, if $A + A' = O$, then the value of α is:

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$

(d) π

3. For the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \\ 2 & -1 & 3 \end{pmatrix}$,
show that $A^3 - 6A^2 + 5A + 11I = 0$. Hence, find A^{-1}.

4. Using the properties of determinants, solve the following for x:

$$\begin{vmatrix} x+3 & x+7 & x-1 \\ x+7 & x-1 & x+3 \\ x-1 & x+3 & x+7 \end{vmatrix} = 0 \tag{4.1}$$

5. Find the value of x, if $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix}$=0.

6. If $A = \begin{pmatrix} 4x & 0 \\ 2x & 2x \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$, then $x =$ __________.

7. If $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, then A^2 equals

(a) $\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & -2 \\ -2 & -2 \end{pmatrix}$

(c) $\begin{pmatrix} -2 & -2 \\ -2 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$

8. The roots of the equation $\begin{vmatrix} x & 0 & 8 \\ 4 & 1 & 3 \\ 2 & 0 & x \end{vmatrix} = 0$ are

(a) $-4, 4$

(b) $2, -4$

(c) $2, 4$

(d) $2, 8$

9. A square matrix A is said to be singular if __________.

10. If $A = \begin{pmatrix} 3 & -5 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 17 \\ 0 & -10 \end{pmatrix}$, then $\left|AB\right| =$ __________.

11. if $\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ is symmetric matrix, then find the value of x.

12. If A is a square matrix such that $A^2 = A$, then find $(2+A)^3 - 19A$.

13. For the matrix $A = \begin{pmatrix} 2 & 3 \\ -4 & -6 \end{pmatrix}$, verify the following:

$$A(adjA) = (adjA)A = \left|A\right| I \tag{13.1}$$

14. Using properties of determinants show that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2) \tag{14.1}$$

15. Find the equation of the line join $A\,(1,3)$ and $B\,(0,0)$, using determinants. Also find k if $D\,(k,0)$ is a point such that the area of $\triangle ABD$ is 3 square units.

14.2. 2023

14.2.1. 10

1. The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are :

(a) intersecting

(b) parallel

(c) coincident

(d) either intersecting or parallel

2. Two schools P and Q decided to award prizes to their students for two games of Hockey ₹x per students and cricket ₹y per student. School P decided to award a total of ₹$9,500$ for the two games to 5 and 4 students respectively; while school Q decided to award ₹$7,370$ for the two games to 4 and 3 students respectively.

Figure 2.1: trophies

Based on the given information, answer the following questions :

(i) Represent the following information algebraically(in terms of x and y).

(ii) (a) what is the prize amount for hockey ?

(b) Prize amount on which game is more and by how much ?

(iii) what will be the total prize amount if there are 2 students each from two games ?

3. If the pair of equations $3x - y + 8 = 0$ and $6x - ry + 16 = 0$ represents coincident lines,then the values of r is :

   (a) $-\frac{1}{2}$

   (b) $\frac{1}{2}$

   (c) 2

   (d) -2

4. The pair of equations $x = a$ and $y = b$ graphically represents lines which are :

   (a) parallel

   (b) intersecting at (b, a)

   (c) coincident

   (d) intersecting at (a, b)

5. (a) If the system of linear equations $2x+3y = 7$ and $2ax+(a + b)\,y = 28$ have infinite number of solutions, then find the values of a and b.

   (b) If $217x + 131y = 913$ and $131x + 217y = 827$, then solve the equations for the values of x and y.

6. Half of the difference between two numbers is 2. The sum of the greater number and twice the smaller number is 3.Find the numbers.

14.2.2. 12

1. If (a,b),(c,d) and (e,f) are the vertices of $\triangle ABC$ and Δ denotes the area of $\triangle ABC$, then

$$\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2 \tag{1.1}$$

is equal to

   (a) $2\Delta^2$
   (b) $4\Delta^2$
   (c) 2Δ
   (d) 2Δ

2. If $\begin{pmatrix} 2 & 0 \\ 5 & 4 \end{pmatrix} = P+Q$ is a symmetric and Q is a skew symmetric matrix, then Q is equal to

   (a) $\begin{pmatrix} 2 & \frac{5}{2} \\ \frac{5}{2} & 4 \end{pmatrix}$
   (b) $\begin{pmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{pmatrix}$
   (c) $\begin{pmatrix} 0 & \frac{5}{2} \\ -\frac{5}{2} & 0 \end{pmatrix}$
   (d) $\begin{pmatrix} 2 & -\frac{5}{2} \\ \frac{5}{2} & 4 \end{pmatrix}$

3. If $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{pmatrix}$ is non-singular matrix and $a \in A$, then the set A is

(a) $\mathbb{R}$

(b) $\{0\}$

(c) $\{4\}$

(d) $\mathbb{R} - \{4\}$

4. If $\left|A\right| = \left|kA\right|$, where A is a square matrix of order 2,then sum of all possible values of k is

(a) 1

(b) -1

(c) 2

(d) 0

5. (a) If $A = \begin{pmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{pmatrix}$,then find AB and use it to solve the following system of equations :

$$x - 2y = 3 \tag{5.1}$$

$$2x - y - z = 2 \tag{5.2}$$

$$-2y + z = 3 \tag{5.3}$$

(b) If $f(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$,then prove that

$$f(\alpha) . f(-\beta) = f((\alpha - \beta)) . \tag{5.4}$$

14.3. 2021

14.3.1. 12

1. If $\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ is a symmetric, find the value of x.

2. If A is a square matrix such that $A^2 = A$, find $(2+A)^3 - 19A$.

3. For the matrix $A = \begin{pmatrix} 2 & 3 \\ -4 & -6 \end{pmatrix}$, verify the fallowing $A(adjA) = (adjA)A = \left|A\right| I$.

4. Using properties of determinants shows that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

5. Find the equation of the line joining $A(1,3)$ and $B(0,0)$ using determinants. Also, find k if $D(k,0)$ is a point such that the area of ΔABD is 3 square units.

6. Solve the system of linear equations using the matrix method:

$$7x + 2y = 11$$
$$4x - 7y = 2$$

7. Find the value of x, if $\begin{pmatrix} x & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} x \\ 3 \end{pmatrix} = 0$

8. If $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then $A^4 =$ __________.

9. Given $A = \begin{pmatrix} 1 & -1 & 1 \\ 3 & -2 & 1 \\ -2 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 1 & -2 \end{pmatrix}$, the order of the matrix AB is __________.

10. if $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $(i^2 = -1)$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, then AB is equal to

    (a) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

    (b) $\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

    (c) $\begin{pmatrix} i & -i \\ 0 & 1 \end{pmatrix}$

    (d) $\begin{pmatrix} 0 & 0 \\ i & 0 \end{pmatrix}$

11. If A is a $5 \times p$ matrix, B is a $2 \times q$ matrix, then the order of the matrix AB is 5×4. What are the values of p and q?

(a) $p = 2, q = 4$

(b) $p = 4, q = 2$

(c) $p = 2, q = 2$

(d) $p = 4, q = 4$

12. Value of k, for which $A = \begin{pmatrix} k & 8 \\ 1 & 2k \end{pmatrix}$ is a singular matrix is:

(a) 4

(b) -4

(c) ± 4

(d) 0

13. If $A = [a_i j]$ is a square matrix of order 2 such that $a_i = \begin{cases} 1, & i+j \\ 0, & i-j \end{cases}$, then A^2 is:

(a) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

14.4. 2021

14.4.1. 12

1. If $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, then A^2 equals

   (a) $\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$

   (b) $\begin{pmatrix} 2 & -2 \\ -2 & -2 \end{pmatrix}$

   (c) $\begin{pmatrix} -2 & -2 \\ -2 & 2 \end{pmatrix}$

   (d) $\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$

2. $\begin{vmatrix} 43 & 44 & 45 \\ 44 & 45 & 46 \\ 45 & 46 & 47 \end{vmatrix}$

   (a) 0

   (b) -1

   (c) 1

(d) 2

3. A square matrix A is said to be singular if ___________.

4. If $A = \begin{pmatrix} 3 & -5 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 17 \\ 0 & -10 \end{pmatrix}$, then $\left|AB\right| =$ ___________.

14.4.2. 10

1. Find whether the following pair of linear equations are consistent or inconsistent:

$$5x - 3y = 11, -10x + 6y = 22.$$

2. Solve for x and y:

$$x + y = 6, 2x - 3y = 4.$$

3. Find out whether the pair of equations $2x + 3y = 0$ and $2x - 3y = 26$ is consistent or inconsistent.

4. For what values of k, does the pair of linear equations $kx - 2y = 3$ and $3x + y = 5$ have a unique solution?

5. What type of lines will you get by drawing the graph of the pair of equations $x - 2y + 3 = 0$ and $2x - 4y = 5$?

6. The sum of the numerator and the denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to $\frac{1}{3}$. Find the

fraction.

7. Find the value of k for which the system of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has no solution.

8. If 2 tables and 2 chairs cost ₹700 and 4 tables and 3 chairs cost ₹1, 250, then find the cost of one table.

9. If the graph of a pair of lines $x - 2y + 3 = 0$ and $2x - 4y = 5$ be drawn, then what type of lines are drawn?

14.5. 2021

14.5.1. 12

1. Given that A is a square matrix of order 3 and $|A| = -4$, then $|adj A|$ is equal to:

   (a) -4

   (b) 4

   (c) -16

   (d) 16

2. If $\begin{pmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix}$, then the value of $a + b - c + 2d$ is:

   (a) 8

(b) 10

(c) 4

(d) -8

3. Given that matrices A and B are of order $3 \times n$ and $m \times 5$ respectively, then the order of matrix $C = 5A + 3B$ is:

   (a) 3×5

   (b) 5×3

   (c) 3×3

   (d) 5×5

4. For matrix $A = \begin{pmatrix} 2 & 5 \\ -11 & 7 \end{pmatrix}$, $(\text{adj}A)'$ is equal to:

   (a) $\begin{pmatrix} -2 & -5 \\ 11 & -7 \end{pmatrix}$

   (b) $\begin{pmatrix} 7 & 5 \\ 11 & 2 \end{pmatrix}$

   (c) $\begin{pmatrix} 7 & 11 \\ -5 & 2 \end{pmatrix}$

   (d) $\begin{pmatrix} 7 & -5 \\ 11 & 2 \end{pmatrix}$

5. Given that $A = [a_{ij}]$ is a square matrix of order 3×3 and $|A| = -7$, then the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$,where A_{ij} denotes the cofactor of element a_{ij} is:

(a) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

6. If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, then

(a) $A^{-1} = B$

(b) $A^{-1} = 6B$

(c) $B^{-1} = B$

(d) $B^{-1} = \frac{1}{6}A$

7. Given that A is a non-singular matrix of order 3 such that $A^2 = 2A$, then the value of $|2A|$ is:

(a) 4

(b) 8

(c) 64

(d) 16

8. If $A = \begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$ and $kA = \begin{pmatrix} 0 & 3a \\ 2b & 24 \end{pmatrix}$, then the values of k, a and b respectively are:

   (a) $-6, -12, -18$

   (b) $-6, -4, -9$

   (c) $-6, 4, 9$

   (d) $-6, 12, 18$

9. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

   (a) A

   (b) $I + A$

   (c) $I - A$

   (d) I

10. For $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, then $14A^{-1}$ is given by:

   (a) $14 \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$

   (b) $\begin{pmatrix} 4 & -2 \\ 2 & 6 \end{pmatrix}$

   (c) $2 \begin{pmatrix} 2 & -1 \\ 1 & -3 \end{pmatrix}$

(d) $2\begin{pmatrix} -3 & -1 \\ 1 & -2 \end{pmatrix}$

11. Given that $A = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$ and $A^2 = 3I$, then:

(a) $1 + \alpha^2 + \beta\gamma = 0$

(b) $1 - \alpha^2 - \beta\gamma = 0$

(c) $3 - \alpha^2 - \beta\gamma = 0$

(d) $3 + \alpha^2 + \beta\gamma = 0$

12. Let $A = \begin{pmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{pmatrix}$, where $0 \leq \alpha \leq 2\pi$, then:

(a) $\left|A\right| = 0$

(b) $\left|A\right| \in (2, \infty)$

(c) $\left|A\right| \in (2, 4)$

(d) $\left|A\right| \in \left|2, 4\right|$