CBSE MATH

Made Simple

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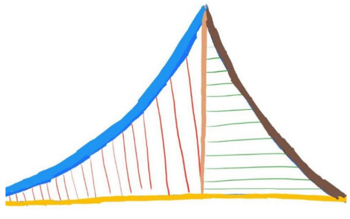

Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

Chapter 2

Linear Forms

2.0.1. Solve the equations $x + 2y = 6$ and $2x - 5y = 12$ graphically.

2.0.2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \tag{2.0.2.1}$$

$$(bx + ay) + (b - 4a) = 0 \tag{2.0.2.2}$$

2.0.3. Find the co-ordinates of the point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2}, 0, 0\right)$, $(0, 7, 0)$, $(0, 0, 7)$.

2.0.4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

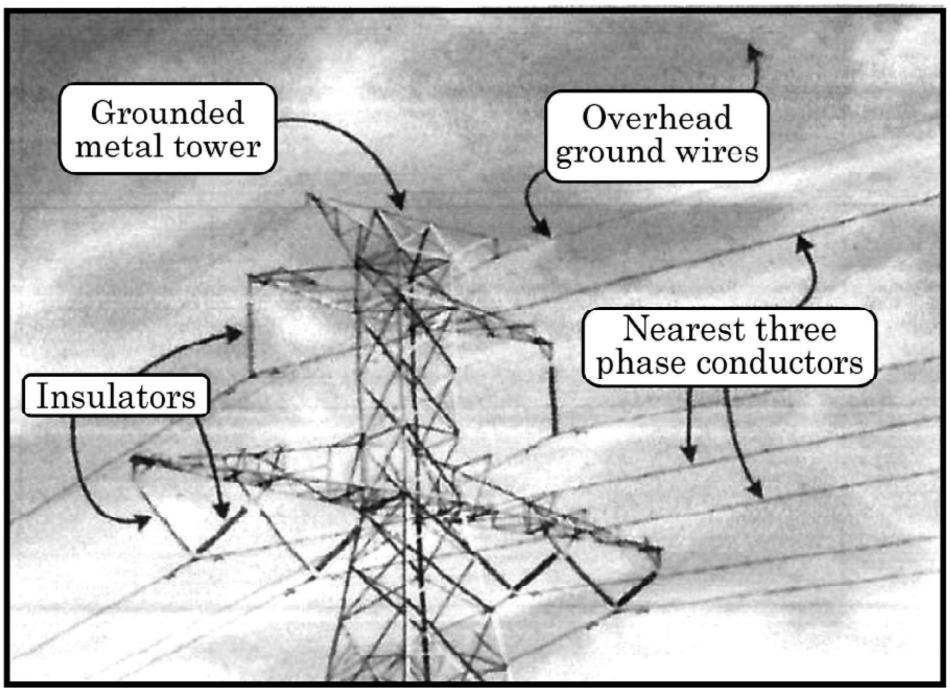

Figure 2.0.4.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.0.0.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \tag{2.0.4.1}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \tag{2.0.4.2}$$

Based on the given information, answer the following questions:

(a) Are the l_1 and l_2 coplanar? Justify your answer.

(b) Find the point of intersection of lines l_1 and l_2.

2.0.5. Write the cartesian equation of the line PQ passing through points

P$(2,2,1)$ and Q$(5,1,-2)$. Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2.

2.0.6. Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

2.0.7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + \hat{k}) \tag{2.0.7.1}$$

$$\mathbf{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(7\hat{\imath} - 6\hat{\jmath} + \hat{k}) \tag{2.0.7.2}$$

2.0.8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.0.0.2) represented by the following lines

$$\mathbf{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k}) \tag{2.0.8.1}$$

$$\mathbf{r} = (3\hat{\imath} + 3\hat{\jmath}) + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k}) \tag{2.0.8.2}$$

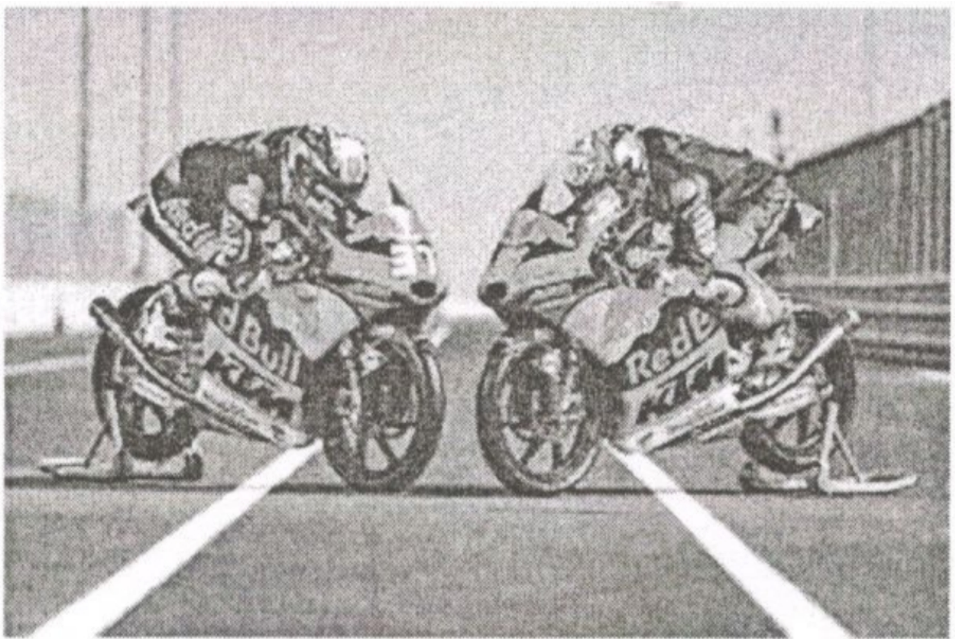

Figure 2.0.8.1: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

(a) Find the shortest distance between the given lines.

(b) Find a point at which the motorcycles may collide.

2.0.9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{\imath} + (\lambda + 4)\hat{\jmath} - (\lambda - 3)\hat{k} \tag{2.0.9.1}$$

$$\mathbf{r} = (3 - \mu)\hat{\imath} + (2\mu + 2)\hat{\jmath} + (\mu + 6)\hat{k} \tag{2.0.9.2}$$

2.0.10. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{2.0.10.1}$$

2.0.11. Find the equation of the plane passing through the points $(2,1,0), (3,-2,-2)$ and $(1,1,7)$. Also, obtain its distance from the origin.

2.0.12. The foot of a perpendicular drawn from the point $(-2,-1,-3)$ on a plane is $(1,-3,3)$. Find the equation of the plane.

2.0.13. Find the cartesian and the vector equation of a plane which passes through the point $(3,2,0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

2.0.14. The distance between the planes $4x-4y+2z+5 = 0$ and $2x-2y+z+6 = 0$ is

(a) $\frac{1}{6}$
(b) $\frac{7}{6}$
(c) $\frac{11}{6}$
(d) $\frac{16}{6}$

2.0.15. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{\imath} + 3\hat{\jmath}) + 6 = 0 \tag{2.0.15.1}$$

$$\mathbf{r} \cdot (3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0 \tag{2.0.15.2}$$

which is at a unit distance from the origin.

2.0.16. If the distance of the point $(1,1,1)$ from the plane $x-y+z+\lambda=0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ.

2.0.17. Find the distance of the point $(2,3,4)$ measured along the line $\frac{x-4}{3}=\frac{y+5}{6}=\frac{z+1}{2}$ from the plane $3x+2y+2z+5=0$.

2.0.18. Find the distance of the point $P(4,3,2)$ from the plane determined by the points $A(-1,6,-5), B(-5,-2,3)$ and $C(2,4,-5)$.

2.0.19. The distance of the line

$$\mathbf{r}=(\hat{\imath}-\hat{\jmath})+\lambda(\hat{\imath}+5\hat{\jmath}+\hat{k}) \tag{2.0.19.1}$$

from the plane

$$\mathbf{r}\cdot(\hat{\imath}-\hat{\jmath}+4\hat{k})=5 \tag{2.0.19.2}$$

is

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{3\sqrt{2}}$

(d) $\frac{-2}{3\sqrt{2}}$

2.0.20. Find a unit vector perpendicular to each of the vectors $(\mathbf{a}+\mathbf{b})$ and

$(\mathbf{a} - \mathbf{b})$ where

$$\mathbf{a} = \hat{\imath} + \hat{\jmath} + \hat{k} \tag{2.0.20.1}$$

$$\mathbf{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} \tag{2.0.20.2}$$

2.0.21. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}) \tag{2.0.21.1}$$

and the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 10. \tag{2.0.21.2}$$

2.0.22. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

2.0.23. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

2.0.24. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.0.0.1:

Table 2.0.24.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40

Chapter 3

Intersection of Conics

1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

OR

2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

3. (a) Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

OR

(b) If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

4. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

5. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

6. (a) Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

OR

(b) Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration.

7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

Chapter 4

Tangent And Normal

4.1. Construction

4.1.1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.

4.1.2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.

4.1.3. In Figure 4.1.0.1, there are two concentric circles with centre **O**. If ARC and AQB are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of AC, if $AQ = 5$ cm.

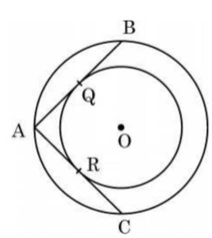

Figure 4.1.3.1: Two concentric circles with **O** as centre

4.1.4. In Figure 4.1.0.2, if a circle touches the side QR of ΔPQR at **S** and extended sides PQ and PR at **M** and **N**, respectively,

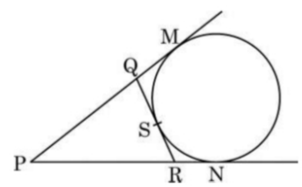

Figure 4.1.4.1: Two tangents are drawn from point **P** to the circle

prove that $PM = \frac{1}{2}(PQ + QR + PR)$

4.1.5. In Figure 4.1.0.3, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact **D** are of lengths 6 cm and 8 cm respectively. If the area of ΔABC is 84 cm^2, find the lengths of sides AB and AC.

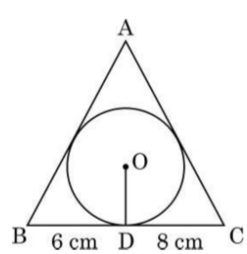

Figure 4.1.5.1: Circle with **O** as center circumscribed in triangle ABC

4.1.6. In Figure 4.1.0.4, PQ and PR are tangents to the circle centered at **O**. If $\angle OPR = 45°$, then prove that $ORPQ$ is a square.

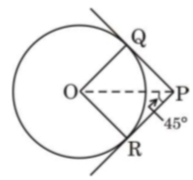

Figure 4.1.6.1: Two tangents drawn from point **P** to a circle whose centre is **O**

4.1.7. In Figure 4.1.0.5, **O** is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at **A** and **B** respectively. If OP is 13 cm, then find the length of tangents PA and BC.

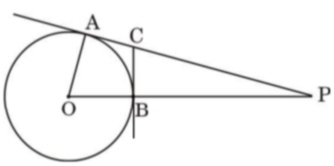

Figure 4.1.7.1: Two tangents drawn from point **C** to a circle whose centre is **O**

4.1.8. In Figure 4.1.0.6, AB is diameter of a circle centered at **O**. BC is tangent to the circle at **B**. If OP bisects the chord AD and $\angle AOP = 60°$, then find $m\angle C$.

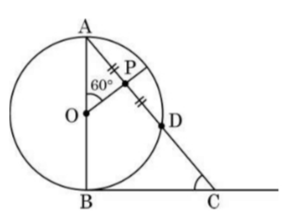

Figure 4.1.8.1: Tangent BC is drawn from point **C** to a circle whose centre is **O**

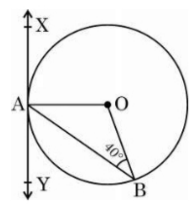

Figure 4.1.9.1: The line XAY is tangent to the circle centered at **O**

4.1.9. In Figure 4.1.0.7, XAY is a tangent to the circle centered at **O**. If $\angle ABO = 60°$,then find $m\angle BAY$ and $m\angle AOB$.

4.1.10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

4.1.11. In Figure 4.1.0.8, a triangle ABC with $\angle B = 90°$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point **P**. Prove that the tangent drawn at point **P** bisects BC.

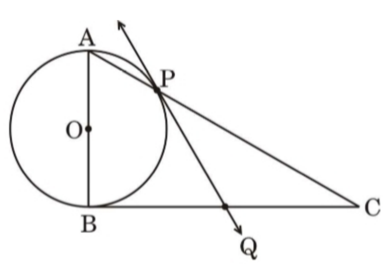

Figure 4.1.11.1: PQ is tangent to the circle centered at **O**. AB is the diameter and $\angle B = 90°$

4.2. Properties

4.2.1. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.