# CBSE MATH

## Made Simple

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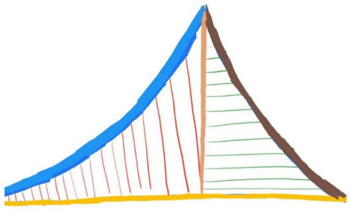


# Contents

# Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

# Chapter 1

# Vectors

## 1.1. 2020

### 1.1.1. 10

1. The distance between the points $(m, -n)$ and $(-m, n)$ is

   (a) $\sqrt{m^2 + n^2}$

   (b) $m + n$

   (c) $2\sqrt{m^2 + n^2}$

   (d) $\sqrt{2m^2 + 2n^2}$

2. The point on the x-axis which is equidistant from $(-4, 0)$ and $(10, 0)$ is

   (a) $(7, 0)$

   (b) $(5, 0)$

   (c) $(0, 0)$

   (d) $(3, 0)$

3. The centre of a circle whose end points of a diameter are $(-6, 3)$ and $(6, 4)$ is

   (a) $(8, -1)$

   (b) $(4, 7)$

   (c) $\left(0, \frac{7}{2}\right)$

   (d) $\left(4, \frac{7}{2}\right)$

4. $AOBC$ is a rectangle whose three vertices are $\mathbf{A}\,(0, -3)$, $\mathbf{O}\,(0, 0)$ and $\mathbf{B}\,(4, 0)$. The length of its diagonal is ________________.

5. Find the ratio in which the $y - axis$ divides the line segment joining the points $(6, -4)$ and $(-2, -7)$. Also find the point of intersection.

6. Show that the points $(7, 10)$, $(-2, 5)$ and $(3, 4)$ are vertices of an isosceles right triangle.

### 1.1.2. 12

1. The area of a triangle formed by vertices $\mathbf{O}$, $\mathbf{A}$ and $\mathbf{B}$, where $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is

   (a) $3\sqrt{5}$ sq. units

   (b) $5\sqrt{5}$ sq. units

   (c) $6\sqrt{5}$ sq. units

   (d) 4 sq. units

2. The coordinates of the foot of the perpendicular drawn from the point $(2, -3, 4)$ on the $y - axis$ is

   (a) $(2, 3, 4)$

   (b) $(-2, -3, -4)$

   (c) $(0, -3, 0)$

   (d) $(2, 0, 4)$

3. The angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$ is

   (a) $\frac{-\pi}{3}$

   (b) $0$

   (c) $\frac{\pi}{3}$

   (d) $\frac{2\pi}{3}$

4. If $\left|\overrightarrow{a}\right| = 4$ and $-3 \leq \lambda \leq 2$, then $\left|\lambda\overrightarrow{a}\right|$ lies in

   (a) $[0, 12]$

   (b) $[2, 3]$

   (c) $[8, 12]$

   (d) $[-12, 8]$

5. The distance between parallel planes $2x + y - 2z - 6 = 0$ and $4x + 2y - 4z = 0$ is ______________ units.

6. If $\mathbf{P}\,(1, 0, -3)$ is the foot of the perpendicular from the origin to the plane, then the cartesian equation of the plane is ______________.

7. Find the coordinates of the point where the line $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$ cuts the $xy - plane$.

8. Find a vector $\overrightarrow{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.

9. Find the angle between unit vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ so that $\sqrt{3}\overrightarrow{a}$ - $\overrightarrow{b}$ is also a unit vector.

10. Show that the plane $x - 5y - 2z = 1$ contains the line $\frac{x-5}{3} = \text{y} = 2 - z$.

11. Find the equation of the plane passing through the points $(1, 0, -2)$, $(3, -1, 0)$ and perpendicular to the plane $2x - y + z = 8$. Also find the distance of the plane thus obtained from the origin.

## 1.2. 2023

### 1.2.1. 10

1. In what ratio, does $x$-axis divide the line segment joinin the points $\mathbf{A}(3, 6)$ and $\mathbf{B}(-12, -3)$ ?

   (a) $1 : 2$

   (b) $1 : 4$

   (c) $4 : 1$

   (d) $2 : 1$

2. The distance between the point $(0, 2\sqrt{5})$ and $(-2\sqrt{5}, 0)$ is

(a) $2\sqrt{10}$ units

(b) $4\sqrt{10}$ units

(c) $2\sqrt{20}$ units

(d) 0 units

3. If $(-5, 3)$ and $(5, 3)$ are two vetices of an equilateral triangle,then co-ordinates of the third vertex,given that origin lies inside the triangle $(take \sqrt{3} = 1.7)$

4. Show that the points $(-2, 3)$ , $(8, 3)$ and $(6, 7)$ are the verices of right-angled triangle

5. If $\mathbf{Q} = (0, 1)$ is equidistant from $\mathbf{P} = (5, -3)$ and $\mathbf{R} = (x, 6)$, find the value of $x$.

6. The distance of the point $(-6, 8)$ from origin is :

(a) 6

(b) $-6$

(c) 8

(d) 10

7. The points $(-4, 0)$ $(4, 0)$ and $(0, 3)$ are the vertices of $a$ :

(a) right triangle

(b) isosceles triangle

(c) equilateral triangle

(d) scalene triangle

8. The area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is :

(a) $ab$

(b) $\frac{1}{2}ab$

(c) $\frac{1}{4}ab$

(d) $2ab$

9. Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and remaining for growing vegetables as shown in figure. 1.1 . In the field , there is a pole marked as O .

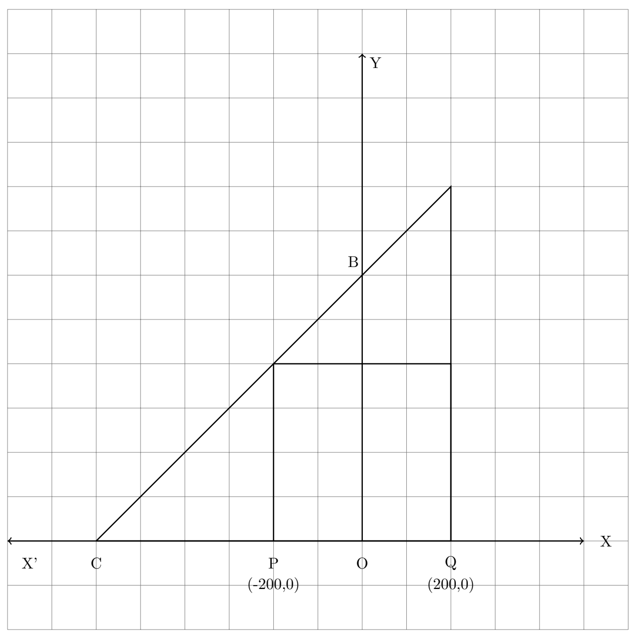



Figure 1.1: Image

Based on the above information,answer the following equations:

(a) Taking O as origin , coordinates of P are (-200,0) and of Q are (200,0). PQRS being a square, what are the coordinates of R and S?

(b) i. What is the area of square PQRS?

ii. What is the length of diagonal PR in PQRS?

(c) If S divides CA in the ratio K:1,what is the value of K,where point A is (200,800)?

### 1.2.2. 12

1. Unit vector along **PQ**, where coordinates of **P** and **Q** respectively are (2,1,-1)and(4,4,-7), is

   (a) $2\hat{i}+3\hat{j}-6\hat{k}$

   (b) $-2\hat{i}-3\hat{j}+6\hat{k}$

   (c) $-\frac{2\hat{i}}{7}-\frac{3\hat{j}}{7}+\frac{6\hat{k}}{7}$

   (d) $\frac{2\hat{i}}{7}+\frac{3\hat{j}}{7}-\frac{6\hat{k}}{7}$

2. If in $\triangle$ABC, $\overrightarrow{BA}$=2$\overrightarrow{a}$ and $\overrightarrow{BC}$=3$\overrightarrow{b}$, then $\overrightarrow{AC}$ is

   (a) $2\overrightarrow{a}$ + $3\overrightarrow{b}$

   (b) $2\overrightarrow{a}$ - $3\overrightarrow{b}$

   (c) $3\overrightarrow{b}$ - $2\overrightarrow{a}$

   (d) $-2\overrightarrow{a}$ -$3\overrightarrow{b}$

3. Equation of line passing through origin and making 30°, 60° and 90°with x, y, z axes respectively is

(a) $\frac{2x}{\sqrt{3}}=\frac{y}{2}=\frac{z}{0}$

(b) $\frac{2x}{\sqrt{3}}=\frac{2y}{1}=\frac{z}{0}$

(c) 2x=$\frac{2y}{\sqrt{3}}=\frac{z}{1}$

(d) $\frac{2x}{\sqrt{3}}=\frac{2y}{1}=\frac{z}{1}$

4. If $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ are three non-zero unequal vectors such that $\overrightarrow{a}.\overrightarrow{b} = \overrightarrow{a}.\overrightarrow{c}$, then find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ - $\overrightarrow{c}$.

5. If the equation of a line is

$$x = ay + b, z = cy + d, \tag{1.1}$$

then find the direction ratios of the line and a point on the line.

6. Using Integration, find the area of triangle whose vertices are (-1, 1), (0, 5) and (3, 2).

# 1.3. 2022

## 1.3.1. 10

1. The distance between the points $(0, 0)$ and $(a - b, a + b)$ is

(a) $2\sqrt{ab}$

(b) $\sqrt{2a^2 + ab}$

(c) $2\sqrt{a^2 + b^2}$

(d) $\sqrt{2a^2 + 2b^2}$

2. The value of m which makes the point $(0,0)$ , $(2m,-4)$ and $(3,6)$ collinear, is _____

3. A circle has its center at $(4,4)$. If one end of a diameter is $(4,0)$, then find the coordinates of other end.

4. Find the area of the quadrilateral ABCD whose vertices are $\mathbf{A}(-4,-3)$ , $\mathbf{B}(3,-1)$, $\mathbf{C}(0,5)$ and $\mathbf{D}(-4,2)$

5. If the points $\mathbf{A}(2,0)$, $\mathbf{B}(6,1)$, and $\mathbf{C}(p,q)$ form a triangle of area 12sq. units (positive only) and

$$2p+q=10 \tag{1.2}$$

, then find the values of p and q.

### 1.3.2. 12

1. $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that

$$\left|2\overrightarrow{a}+3\overrightarrow{b}\right|=\left|3\overrightarrow{a}-2\overrightarrow{b}\right|. \tag{1.3}$$

Find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

2. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\overrightarrow{a}=\hat{i}-\hat{j}+\hat{k} \tag{1.4}$$

and

$$\overrightarrow{b} = 2\hat{i} - \hat{j} - 3\hat{k} \tag{1.5}$$

then find the vector $\overrightarrow{c}$, given that

$$\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b} \tag{1.6}$$

and

$$\overrightarrow{a}.\overrightarrow{c} = 4. \tag{1.7}$$

3. If

$$\left|\overrightarrow{a} \times \overrightarrow{b}\right|^2 + \left|\overrightarrow{a}.\overrightarrow{b}\right|^2 = 400 \tag{1.8}$$

and

$$\left|\overrightarrow{b}\right| = 5 \tag{1.9}$$

find the value of $|\overrightarrow{a}|$.

4. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{1.10}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k} \tag{1.11}$$

, then find $\left|\overrightarrow{b}\right|$

5. If

$$|\overrightarrow{a}| = 3, \left|\overrightarrow{b}\right| = 2\sqrt{3} \tag{1.12}$$

and

$$\overrightarrow{a}.\overrightarrow{b} = 6, \tag{1.13}$$

then find the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$.

6. $|\overrightarrow{a}| = 8, \left|\overrightarrow{b}\right| = 3$ and $\overrightarrow{a}.\overrightarrow{b} = 12\sqrt{3}$, then the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$ is

   (a) 24

   (b) 144

   (c) 2

   (d) 12

7. If

$$\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.14}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.15}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

8. $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ and $\overrightarrow{d}$ are four non-zeros vectors such that

$$\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{d} \tag{1.16}$$

and

$$\overrightarrow{a} \times \overrightarrow{c} = 4\overrightarrow{b} \times \overrightarrow{d} \tag{1.17}$$

, then show that $(\overrightarrow{a} - 2\overrightarrow{d})$ is parallel to $(2\overrightarrow{b} - \overrightarrow{c})$ where

$$\overrightarrow{a} \neq 2\overrightarrow{d}, \overrightarrow{c} \neq 2\overrightarrow{b} \tag{1.18}$$

9. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{1.19}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k}, \tag{1.20}$$

then find $\left|\overrightarrow{b}\right|$

10. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{b}\right|, \tag{1.21}$$

then prove that $(\overrightarrow{a} + 2\overrightarrow{b})$ is perpendicular to $\overrightarrow{a}$.

11. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vectors and $\theta$ is the angle between them , then prove that sin

$$\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.22}$$

12. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that and $\theta$ is the angle between them, then prove that

$$sin\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.23}$$

13. If

$$\overrightarrow{a} = 2\hat{i} + y\hat{j} + \hat{k} \tag{1.24}$$

and

$$\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k} \tag{1.25}$$

are two vectors for which the vector $(\overrightarrow{a} + \overrightarrow{b})$ is perpendicular to the vector $(\overrightarrow{a} - \overrightarrow{b})$ then find all the possible values of y.

14. Write the projection of the vector $(\overrightarrow{b} + \overrightarrow{c})$ on the vector $\overrightarrow{a}$ , where

$$\overrightarrow{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + 2\hat{j} - 2\hat{k} \tag{1.26}$$

and

$$\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}. \tag{1.27}$$

15. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + \hat{j} - 2\hat{k} \tag{1.28}$$

and

$$\overrightarrow{c} = \hat{i} + 3\hat{j} - \hat{k} \tag{1.29}$$

and the projection of vector $\overrightarrow{c} + \lambda\overrightarrow{b}$ on vector $\overrightarrow{a}$ is $2\sqrt{6}$, find the value of $\lambda$.

16. If

$$\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.30}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.31}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

17. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k} \tag{1.32}$$

and

$$\overrightarrow{b} = 5\hat{i} - 3\hat{j} - 4\hat{k} \tag{1.33}$$

, then find the ratio $\frac{projection of vector \overrightarrow{a} on vector \overrightarrow{b}}{projection of vector \overrightarrow{b} on vector \overrightarrow{a}}$

18. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ , and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right-angled triangle. If

$$\overrightarrow{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \overrightarrow{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.34}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} \tag{1.35}$$

are such that the vector $(\overrightarrow{a} + \lambda \overrightarrow{b})$ is perpendicular to vector $\overrightarrow{c}$, then find the value of $\lambda$.

19. If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are the position vectors of the points $\mathbf{A}(2, 3, -4)$, $\mathbf{B}(3, -4, -5)$ and $\mathbf{C}(3, 2, -3)$ and respectively, then $\left|\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right|$ is equal to

   (a) $\sqrt{113}$

(b) $\sqrt{185}$

(c) $\sqrt{203}$

(d) $\sqrt{209}$

20. Find the values $\lambda$, for which the distance of point $(2, 1, \lambda)$ from plane

$$3x + 5y + 4z = 11 \tag{1.36}$$

is $2\sqrt{2}$ units.

21. Find the coordinates of the point where the line through $(3, 4, 1)$ crosses the ZX-plane

22. Using vectors, find the area of the triangle withvertices $\mathbf{A}(-1, 0, -2)$, $\mathbf{B}(0, 2, 1)$ and $\mathbf{C}(-1, 4, 1)$

23. Using integration, find the area of triangle region whose vertices are $(2, 0)$ , $(4, 5)$ and $(1, 4)$.

24. If a line makes $60°$ and $45°$ angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line.

25. The Cartesian equation of a line $AB$ is :

$$\frac{2x - 1}{12} = \frac{y + 2}{2} = \frac{z - 3}{3} \tag{1.37}$$

.

26. Find the directions cosines of a line parallel to line $AB$.

27. Find the direction cosines of a line whose cartesian equation is given as

$$3x + 1 = 6y - 2 = 1 - z. \tag{1.38}$$

28. A vector of magnitude 9 units in the direction of the vector $-2\hat{i} - \hat{j} + 2\hat{k}$ is ______

29. The two adajacent sides of a parallelogram are represented by $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

30. The two adjacent sides of a parallelogram are represented by vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also,find the area of the parallelogram.

31. If

$$\overrightarrow{a} = \overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k} \tag{1.39}$$

and

$$\overrightarrow{b} = 2\hat{i} + 4\hat{j} - 5\hat{k} \tag{1.40}$$

represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram.

## 1.4. 2021

### 1.4.1. 10

1. Find the distance between the points $\mathbf{A}(-\frac{7}{3}, 5)$ and $\mathbf{B}(\frac{2}{3}, 5)$.

2. Check whether 13cm, 12cm, 5cm can be the sides of a right triangle.

3. (a) If $PL$ and $PM$ are two tangents to a circle with centre **O** from an external point **P** and $PL = 4$ cm, find the length of $OP$, where radius of the circle is 3 cm.

   (b) Find the distance between two parallel tangents of a cicle of radius 2.5 cm.

4. Find the coordinates of the points which divides the line segment joining the points $\mathbf{A}(7, -1)$ and $\mathbf{B}(-3, -4)$ in the ratio $2 : 3$.

5. To divide a line segment $QP$ internally in the ratio $2 : 3$, we draw a ray $QY$ such that $\angle$ PQY is acute. What will be the minimum number of points to be located at equal distances on the ray $QY$ ?

6. Answer any four of the following questions :

   (i) The point which divides the line segment joining the points $(7, -6)$ and $(3, 4)$ in the ratio $1 : 2$ lies in

      i. I quadrant

      ii. II quadrant

      iii. III quadrant

      iv. IV quadrant

(ii) If the $\mathbf{A}(1,2)$, $\mathbf{O}(0,0)$ and $\mathbf{C}(a,6)$ are collinear, then the value of a is

(A) 6

(B) $\frac{3}{2}$

(C) 3

(D) 12

(iii) The distance between the points $\mathbf{A}(0,6)$ and $\mathbf{B}(0,-2)$ is

(A) 6 units

(B) 8 units

(C) 4 units

(D) 2 units

(iv) If $(\frac{a}{3},4)$ is the mid-point of the line segment joining the points $(-6,5)$ and $(-2,3)$, then the value of 'a ' is

(A) $-4$

(B) 4

(C) $-12$

(D) 12

(v) What kind of triangle is formed with vertices $\mathbf{A}(0,2)$, $\mathbf{B}(-3,0)$ and $\mathbf{C}(3,0)$ ?

(A) A right triangle

(B) An equilateral triangle

(C) An isosceles triangle

(D) A scalene triangle

7. (a) If the distance between the points $(k, -2)$ and $(3, -6)$ is 10 units, find the positive value of k.

   (b) Find the length of the segment joining $\mathbf{A}(-6, 7)$ and $\mathbf{B}(-1, -5)$.Also, find the mid-point of $AB$.

8. A man goes 5 metres due to West and then 12 metres due North. How far is he from the starting point ?

9. Students of a school are standing in rows and columns in their school playground to celebrate their annual sports day. **A**, **B**, **C** and **D** are the positions of four students as shown in the figure.

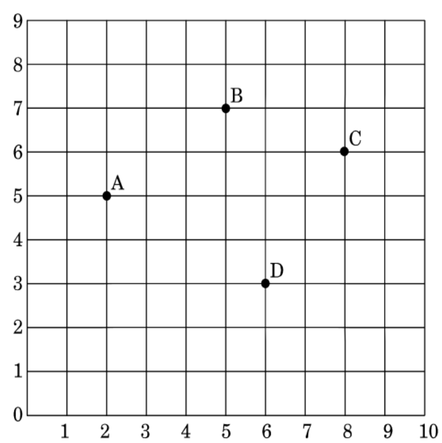



Figure 1.2: Based on the above, answer the following question :

   (i) The figure formed by the points **A**, **B**, **C** and **D** is a

      (A) sqaure

      (B) parallelogram

      (C) rhombus

(D) quadrilateral

(ii) If the sports teacher is sitting at the origin, then which of the four students is closest to him ?

(A) **A**

(B) **B**

(C) **C**

(D) **D**

(iii) The distance between **A** and **C** is

(A) $\sqrt{37}$ units

(B) $\sqrt{35}$ units

(C) 6 units

(D) 5 units

(iv) The coordinates of the mid-point of line segment $AC$ are

(v) If a point **P** divides the line segment $AD$ in the ratio $1 : 2$, then coordinates of **P** are

(A) $(\frac{8}{3}, \frac{8}{3})$

(B) $(\frac{10}{3}, \frac{13}{3})$

(C) $(\frac{13}{3}, \frac{10}{3})$

(D) $(\frac{16}{3}, \frac{11}{3})$

10. (a) Check whether the points $\mathbf{P}(5, -2)$, $\mathbf{Q}(6, 4)$ and $\mathbf{R}(7, -2)$ are the vertices of an isosceles triangle PQR.

(b) Find the ratio in which $\mathbf{P}(4, 5)$ divides the join of $\mathbf{A}(2, 3)$ and $\mathbf{B}(7, 8)$.

11. The coordinate of the three consecutive vertices of a parallelogram ABCD are $\mathbf{A}(1, 3)$, $\mathbf{B}(-1, 2)$, and $\mathbf{C}(2, 5)$. Find the cordinates of the fourth vertex $\mathbf{D}$.

12. (a) If $\mathbf{P}(2, 2)$, $\mathbf{Q}(-4, -4)$ and $\mathbf{R}(5, -8)$ are the vertices of a $\triangle$PQR, then find the length of the median through $\mathbf{R}$.

    (b) Find the ratio in which y-axis divides the line segment joining the points $\mathbf{A}(5, -6)$ and $\mathbf{B}(-1, -4)$.Also, find the coordinates of the point of intersection.

13. (a) Find the ratio in which the line segment joining the points $\mathbf{A}(1, -5)$ and $\mathbf{B}(-4, 5)$ is divided by the ax-axis. Also, find coordinates of the point of division.

    (b) The points $\mathbf{A}(0, 3)$, $\mathbf{B}(-2, a)$ and $\mathbf{C}(-1, 4)$ are the vertices of a rigth triangle, right-angled at $\mathbf{A}$. Find the value of a.

### 1.4.2. 12

1. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are position vectors of the points A(2,3,-4), B(3,-4,-5) and C(3,2,-3) respectively, then $|\mathbf{a} + \mathbf{b} + \mathbf{c}|$ is equal to

    (a) $\sqrt{113}$

    (b) $\sqrt{185}$

    (c) $\sqrt{203}$

    (d) $\sqrt{209}$

2. Find the distance of the point (a,b,c) from the x-axis

3. If $\mathbf{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\mathbf{b} = 5\hat{i} - 3\hat{j} - 4\hat{k}$, then find the ratio $\frac{\text{projection of vector } \mathbf{a} \text{ on } \mathbf{b}}{\text{projection of vector } \mathbf{b} \text{ on vector } \mathbf{a}}$

4. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\mathbf{c} = \hat{a} + 2\hat{b}$ and $\mathbf{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

5. Show that $|\mathbf{a}|\,\mathbf{b} + |\mathbf{b}|\,\mathbf{a}$ is perpendicular to $|\mathbf{ab}|$- $|\mathbf{b}|\,\mathbf{a}$, for any two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$.

6. Prove that three points A,B and C with position vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ respectively are collinear if and only if $(\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) + (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$.

## 1.5. 2019

### 1.5.1. 12

1. A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

2. If $\left|\overrightarrow{a}\right| = 2, \left|\overrightarrow{b}\right| = 7$ and $\overrightarrow{a} \times \overrightarrow{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

3. Find the volume of a cuboid whose edges are given by $-3\hat{i} + 7\hat{j} + 5\hat{k}$,$-5\hat{i} + 7\hat{j} - 3\hat{k}$ and $7\hat{i} - 5\hat{j} - 3\hat{k}$.

4. The scalar product of the vector $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\overrightarrow{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\overrightarrow{c} = \hat{\lambda} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of $\lambda$ and hence find the unit vector along $\overrightarrow{b} + \overrightarrow{c}$.

5. Find the vector and cartesian equations of the plane passing through the points having position vectors $\hat{i}+\hat{j}-2\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$. Write the equation of a plane passing through a point $\left(2,3,7\right)$ and parallel to the plane obtained above. Hence, find the distance between the two parallel planes.

6. Find the cartesian and vector equations of the plane passing through the points $A(2,5,-3)$, $B(-2,-3,5)$ and $C(5,3,-3)$.

7. Show that the points $A\left(-2\hat{i}+3\hat{j}+5\hat{k}\right)$, $B\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $C\left(7\hat{i}-\hat{k}\right)$ are collinear.

8. Find $\left|\vec{a}\times\vec{b}\right|$, if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$.

9. Using integration, find the area of the triangular region whose sides have the equations $y=2x+1$, $y=3x+1$ and $x=4$.

10. Using method of integration, find the area of the triangle whose vertices are $\left(1,0\right)$,$\left(2,2\right)$ and $\left(3,1\right)$.

11. Find the direction cosines of a line which makes equal angles with the coordinate axes.

12. If the lines $\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3\lambda}=\frac{y-1}{2}=\frac{z-6}{-5}$ are perpendicular, find the value of $\lambda$. Hence find weather the lines are intersecting or not.

13. Find the equation of the line passing through $(2,-1,2)$ and $(5,3,4)$ and of the plane passing through $(2,0,3)$, $(1,1,5)$ and $(3,2,4)$. Also, find their point of intersection.

14. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$ , $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$.

15. Find the vector equation of a line passing through the point $(2,3,2)$ and parallel to the line $\vec{r}=\left(-2\hat{i}+3\hat{j}\right)\lambda\left(2\hat{i}-3\hat{j}+6\hat{k}\right)$. Also, find the distance between these two lines.

16. Find the value of **x** such that the four points with position vectors, $\mathbf{A}\left(3\hat{i}+2\hat{j}+\hat{k}\right), \mathbf{B}\left(4\hat{i}+x\hat{j}+5\hat{k}\right), \mathbf{C}\left(4\hat{i}+2\hat{j}-2\hat{k}\right), \mathbf{D}\left(6\hat{i}+5\hat{j}-\hat{k}\right)$ are coplanar.

17. Find the vector equation of the plane determined by the points $\mathbf{A}(3,-1,2), \mathbf{B}(5,2,4)$ and $\mathbf{C}(-1,-1,6)$. Hence, find the distance of the plane, thus obtained,from the origin.

18. Find the coordinates of the foot of the perpendicular **Q** drawn from $\mathbf{P}(3,2,1)$ to the plane $2x-y+z+1=0$. Also, find the distance **PQ** and the image of the point **P** treating this plane as a mirror.

19. Find the value of $\lambda$ for which the following lines are perpendicular to each other :

$$\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1};\frac{x}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3}.$$

Hence, find whether the lines intersect or not.

20. Find the value of $p$ for which the following lines are perpendicular :

$$\frac{1-x}{3} = \frac{2y-14}{2p} = \frac{z-3}{2}; \frac{1-x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}.$$

21. Find the vector and Cartesian equations of the plane passing through the points $(2,5,-3)$, $(-2,-3,5)$, and $(5,3,-3)$.Also, find the point of intersection of this plane with the line passing through points $(3,1,5)$ and $(-1,-3,-1)$.

22. If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines ?

23. Find the equation of the plane passing through the intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right) = 1$ and $\vec{r}.2\hat{i}+3\hat{j}-\hat{k}+4=0$ and parallel to $x-axis$. Hence, find the distance of the plane from $x-axis$.

24. Find the Cartesian equation of the line which passes through the point $(-2,4,-5)$ and is parallel to the line

$$\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}.$$

25. Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

26. Show that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}, -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.

27. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 2$ and

$|\overrightarrow{c}| = 3$ .If the projection of $\overrightarrow{b}$ along $\overrightarrow{a}$ is equal to the projection of $\overrightarrow{c}$ along $\overrightarrow{a}$; and $\overrightarrow{b}, \overrightarrow{c}$ are perpendicular to each other, then find $\left|3\overrightarrow{a} - 2\overrightarrow{b} + 2\overrightarrow{c}\right|$.

28. Find the direction cosines of the line joining the points $P\,(4, 3, -5)$ and $Q\,(-2, 1, -8)$.

29. Find the area of the triangle whose vertices are $(-1, 1)\,, (0, 5)$ and $(3, 2)$,using integration.

30. Find the equation of planes passing through the intersection of planes $\overrightarrow{r}.\left(3\hat{i} + 6\hat{j}\right) + 12 = 0$ and $\overrightarrow{r}.\left(3\hat{i} - \hat{j} + 4\hat{k}\right) = 0$ and are at a unit distance from origin.

31. Find the angle between the line $\overrightarrow{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and the plane $\overrightarrow{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.

32. Find the co-ordinates of the point where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the $yz$-plane.

33. Using vectors, prove that the points $(2, -1, 3)$, $(3, -5, 1)$ and $(-1, 11, 9)$ are co-linear.

34. For any two vectors $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$ prove that

$$\left(\overrightarrow{\mathbf{a}} \times \overrightarrow{\mathbf{b}}\right)^2 = \overrightarrow{\mathbf{a}}^2\overrightarrow{\mathbf{b}}^2 - \left(\overrightarrow{\mathbf{a}}.\overrightarrow{\mathbf{b}}\right)^2$$

35. Using integration, find the area of $\triangle\, ABC$ bounded by the lines

$$4x - y + 5 = 0,$$
$$x + y - 5 = 0,$$
$$x - 4y + 5 = 0.$$

36. Let $\overrightarrow{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\overrightarrow{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vector. Show that vector $\left(\overrightarrow{a} + \overrightarrow{b}\right)$ and $\left(\overrightarrow{a} - \overrightarrow{b}\right)$ are perpendicular to each other.

37. $X$ and $Y$ are two points with position vectors $3\overrightarrow{a} + \overrightarrow{b}$ and $\overrightarrow{a} - 3\overrightarrow{b}$ respectively. Write the position vector of a point $Z$ which divides the line segment $XY$ in the ratio $2 : 1$ externally.

38. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

39. Find the vector equation of the line passing through $(2, 1, -1)$ and parallel to the line $\overrightarrow{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$. Also find the distance between these two lines.

40. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

41. Find the coordinates of the foot $Q$ of the perpendicular drawn from the point $P\,(1, 3, 4)$ to the plane $2x - y + z + 3 = 0$. Find the distance $PQ$ and the image of $P$ treating the plane as a mirror.

42. Using vectors find the value of **x** such that the four points $A\,(x, 5, -1)$, $B\,(3, 2, 1)$ ,$C\,(4, 5, 5)$ and $D\,(4, 2, -2)$ are co-planar.

## 1.6. 2019

### 1.6.1. 10

1. The point $R$ divides the line segment $AB$, where $A(-4, 0)$ and $B(0, 6)$ such that $AR = \frac{3}{4}AB$. Find the coordinates of $R$.

2. In what ratio does the point $P(-4, y)$ divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$ ? Hence find the value of $y$.

3. Find the distance between the points $(a, b)$ and $(-a, -b)$.

4. Find the value of $p$ for which the points $(-5, 1)$, $(1, p)$ and $(4, -2)$ are collinear.

5. Find the area of a triangle whose vertices are given as $(1, -1), (-4, 6)$ and $(-3, -5)$.

6. Write the coordinates of a point $P$ on x-axis which is equidistant from the points $A(-2, 0)$ and $B(6, 0)$.

7. Find a relation between $x$ and $y$ if the points $A(x, y)$, $B(-4, 6)$ and $C(-2, 3)$ are collinear.

8. Point **A** lies on the line segment **XY** joining $X(6, -6)$ and $Y(-4, -1)$ in such a way that $\frac{XA}{XY} = \frac{2}{5}$. If point $A$ also lies on the line $3x + k(y + 1) = 0$, find the value of $k$.

9. Find the ratio in which the y-axis divides the line segment joining the points $(-1, -4)$ and $(5, -6)$. Also find the coordinates of the point of intersection.

10. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Find the coordinates of the point of intersection.

11. Find the value $(s)$ of $x$, if the distance between the points $A(0, 0)$ and $B(x, -4)$ is 5 units.

12. Points $A(3, 1), B(5, 1), C(a, b)$ and $D(4, 3)$ are vertices of a parallelogram $ABCD$. Find the values of $a$ and $b$.

13. Points $P$ and $Q$ trisect the line segment joining the points $A(-2, 0)$ and $B(0, 8)$ such that $P$ is near to $A$. Find the coordinates of points $P$ and $Q$.

14. Find the area of the triangle formed by joining the mid-points of the sides of the triangle ABC, whose vertices are $A(0, -1)$, $B(2, 1)$ and $C(0, 3)$.

15. Find the value of '$a$' so that the point $(3, a)$ lies on the line represented by $2x - 3y = 5$.

16. The mid-point of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(1, 2a + 1)$. Find the values of $a$ and $b$.

17. Point P divides the line segment joining the points $A(2, 1)$ and $B(5, -8)$ such that $\frac{AP}{AB} = \frac{1}{3}$.If $P$ lies on the line $2x - y + k = 0$, find the value of $k$.

18. For what value of $p$, are the points $(2, 1)$, $(p, -1)$ and $(-1, 3)$ collinear ?

19. Find the coordinates of a point $A$, where $AB$ is a diameter of the circle with centre $(3, -1)$ and the point $B$ is $(2, 6)$.

20. Find the values of $x$ for which the distance between the points $A\,(x, 2)$ and $B\,(9, 8)$ is 10 units.

21. The mid-point of the line segment joining $A\,(2a, 4)$ and $B\,(-2, 3b)$ is $(1, 2a + 1)$. Find the values of $a$ and $b$.

22. Find the coordinates of a point **A**, where **AB** is diameter of a circle whose center is $(2, -3)$ and **B** is the point $(1, 4)$.

23. Find the ratio in which the segment joining the points $(1, 3)$ and $(4, 5)$ is divided by $x-axis$? Also find the coordinates of this point on $x$-axis.

24. Find the point on $y-axis$ which is equidistant from the points $(5, -2)$ and $(-3, 2)$.

25. The line segment joining the points $\mathbf{A}\,(2, 1)$ and $\mathbf{B}\,(5, -8)$ is trisected at the points **P** and **Q** such that **P** is nearer to **A**. If **P** also lies on the line given by $2x - y + k = 0$, find the value of $k$.

26. Find the coordinates of a point $A$, where $AB$ is a diameter of the circle with centre $(-2, 2)$ and $B$ is the point with coordinates $(3, 4)$.

## 1.7. 2018

### 1.7.1. 10

1. $\mathbf{A}(-2,1), \mathbf{B}(a,0), \mathbf{C}(4,b)$ and $\mathbf{D}(1,2)$ are the vertices of a parallelogram ABCD, find the values of $a$ and $b$. Hence find the lengths if its sides.

2. $\mathbf{A}(-5,7), \mathbf{B}(-4,-5), \mathbf{C}(-1,-6)$ and $\mathbf{D}(4,5)$ are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

3. Find the distance of a point $\mathbf{P}(x,y)$ from the origin.

4. Find the ratio in which $\mathbf{P}(4,m)$ divides the line segment joining the points $\mathbf{A}(2,3)$ and $\mathbf{B}(6,-3)$. Hence find m.

## 1.8. Construction

5. In an equilateral $\triangle$ ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9(AD)^2 = 7(AB)^2$.

6. Prove that, in a right triangle, the square on the hypotenuse is equal to sum of the squares on the other two sides.

7. Prove that the area of an equilateral triangle described on one side of the square is equal to half of the area of the equilateral triangle described on one of its diagonal.

8. If the area of two similar triangles are equal, prove that they are congruent.

9. Draw a triangle ABC with $BC = 6cm, AB = 5cm$ and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the $\triangle ABC$.

10. Given $\triangle ABC \sim \triangle PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{ar\triangle ABC}{ar\triangle PQR}$.

### 1.8.1. 12

1. Find the magnitude of each of the vectors $\overrightarrow{\mathbf{a}}$ and $\overrightarrow{\mathbf{b}}$, having the same magnitude such that the angle between them is $60°$ and their scalar product is $\frac{9}{2}$.

2. Find the shortest distance between the lines $\overrightarrow{\mathbf{r}} = \left(4\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and $\overrightarrow{\mathbf{r}} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \mu\left(2\hat{i} + 4\hat{j} - 5\hat{k} -\right)$.

3. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{x}{x^2+1}, \forall \mathbf{x} \in \mathbb{R}$ is neither one-one nor onto. Also, if $g : \mathbb{R} \to \mathbb{R}$ is defined as $g(x) = 2x - 1$, find $fog(x)$.

4. If $\theta$ is the angle between the two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.

5. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\overrightarrow{\mathbf{r}} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\overrightarrow{\mathbf{r}} \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 5$.

6. Let $\overrightarrow{\mathbf{a}} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}, \overrightarrow{\mathbf{b}} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\overrightarrow{\mathbf{c}} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$. find a vector $\overrightarrow{\mathbf{d}}$ which is perpendicular to both $\overrightarrow{\mathbf{c}}$ and $\overrightarrow{\mathbf{b}}$ and $\overrightarrow{\mathbf{d}} \cdot \overrightarrow{\mathbf{a}} = 21$

7. If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines ?

8. Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

9. Find a unit vector perpendicular to both the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ , where $\overrightarrow{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}$ and $\overrightarrow{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ .

10. Show that the vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and $\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ are coplanar.

11. Let $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ be three vectors such that $|\overrightarrow{a}| = 1$, $\left|\overrightarrow{b}\right| = 2$ and $|\overrightarrow{c}| = 3$. If the projection of $\overrightarrow{b}$ along $\overrightarrow{a}$ is equal to the projection of $\overrightarrow{c}$ along $\overrightarrow{a}$ ; and $\overrightarrow{b}$, $\overrightarrow{c}$ are perpendicular to each other, then find $\left|3\overrightarrow{a} - 2\overrightarrow{b} + 2\overrightarrow{c}\right|$.

12. Find the vector and cartesian equations of the plain passing through the points $(2, 5, -3), (-2, -3, 5)$ and $(5, 3, -3)$. also find the point of intersection of this plane with the line passing through points $(3, 1, 5)$ and $(-1, -3, -1)$.

13. Find the equation of the plane passing through the intersection of the planes $\overrightarrow{r} \cdot \left(\hat{\imath} + \hat{\jmath} + \hat{k}\right) = 1$ and $\overrightarrow{r} \cdot \left(2\hat{\imath} + 3\hat{\jmath} - \hat{k}\right) + 4 = 0$ and parallel to $x - axis$. Hence, find the distance of plane from $x - axis$.

14. Find the direction cosines of the line joining points $P$ $(4, 3, -5)$ and $Q$ $(-2, 1, 8)$.

15. Find the value of $P$ for which the following lines are perpendicular:

$$\frac{1-x}{3} = \frac{2y-14}{2p} = \frac{z-3}{2}; \frac{1-x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

16. Find the value of $\lambda$ for which the following lines are perpendicular to each other:

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$

Hence, find whether the lines insert or not.

17. Fnd the value of $x$ such that the four points with position vectors, $\mathbf{A}\left(3\hat{i}+2\hat{j}+\hat{k}\right), \mathbf{B}\left(4\hat{i}+x\hat{j}+5\hat{k}\right), \mathbf{C}\left(4\hat{i}+2\hat{j}-2\hat{k}\right), \mathbf{D}\left(6\hat{i}+5\hat{j}-\hat{k}\right)$ are coplanar.

18. Find the vector equation of the plane which contains the line of intersection of the planes $\overrightarrow{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$ , $\overrightarrow{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ and which is perpendicular to the plane $\overrightarrow{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0.$

19. Find the value of $x$, for which the four points $\mathbf{A}\,(x, 1, -1)$, $\mathbf{B}\,(4, 5, 1)$, $\mathbf{C}\,(3, 9, 4)$ and $\mathbf{D}\,(-4, 4, 4)$ are coplanar.

20. Find the acute angle between the planes $\mathbf{r}(\hat{i}-\hat{2j}-\hat{2k})=1$ and $\mathbf{r}(\hat{3i}-\hat{6j}+\hat{2k})=0$

21. Let $\mathbf{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\mathbf{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vectors. Show that the vectors $(\mathbf{a} + \mathbf{b})$ and $(\mathbf{a} - \mathbf{b})$ are perpendicular to each other.

22. Find the coordinates of the foot of the perpendicular $\mathbf{Q}$ drawn from $\mathbf{P}(3, 2, 1)$ to the plane $2x - y + z + 1 = 0$. Also, find distance $\mathbf{PQ}$ and the image of the point $\mathbf{P}$ treating this plane as a mirror.

23. Find the vector equation of the plane determined by the points $\mathbf{A}\,(3, -1, 2)$, $\mathbf{B}\,(5, 2, 4)$, $\mathbf{C}\,(-1, -1, 6)$. Hence, find the distance of the plane, thus obtained, from the origin.

24. $\mathbf{X}$ and $\mathbf{Y}$ are two points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively. Write the position vector of a point $\mathbf{V}$ which divides the line segment $\mathbf{XY}$ in the ratio $2 : 1$ externally.

25. Find the vector equation of a line passing through the point $(2, 3, 2)$ and parallel to the line $\vec{r} = \left(-2\hat{i} + 3\hat{j}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$. Also, find the distance these two lines.

26. Find the vector equation of the line passing through $(2, 1, -1)$ and parallel to the line $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$. Also, find the distance between these two lines.

27. For any two vectors $\vec{a}$ and $\vec{b}$, prove that

$$\left(\vec{a} \times \vec{b}\right)^2 = \vec{a}^2 \;\; \vec{b}^2 - \left(\vec{a} \cdot \vec{b}\right)^2$$

28. Find the equation of planes passing through the intersection of the planes $\vec{r} \cdot \left(2\hat{i} + 6\hat{j}\right) + 12 = 0$ and $\vec{r} \cdot \left(3\hat{i} - \hat{j} + 4\hat{k}\right) = 0$ and are at a

unit distance from origin.

29. Using vectors, find the value of $x$ such that the four points **A** $(x, 5, -1)$,**B** $(3, 2, 1)$,**C** $(4, 5, 5)$ and **D** $(4, 2, -2)$ are coplanar.

30. Find the angle between the line $\overrightarrow{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and the plane $\overrightarrow{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.

31. Using vectors, prove that the points $(2, -1, 3)$, $(3, -5, 1)$ and $(-1, 11, 9)$ are collinear.

32. Find the direction cosine of the line which makes equal angles with the coordinate axes.

33. Find the vector and cartesian equations of the plane passing through the points having position vectors $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. Write the equation of the plane passing through a point $(2, 3, 7)$ and parllel to the plane obtained above. Hence, find the distance between the two parallel planes.

34. If $|\overrightarrow{a}| = 2$, $\left|\overrightarrow{b}\right| = 7$ and $\overrightarrow{a}\ X\ \overrightarrow{b} = \hat{3i} + \hat{2j} + \hat{6k}$, find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

35. Find the volume of a cuboid whose edges are given by $-3\hat{i} + 7\hat{j} + 5\hat{k}$,$-5\hat{i} + 7\hat{j} - 3\hat{k}$ and $7\hat{i} - 5\hat{j} - 3\hat{k}$.

36. The scalar product of the vector $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vector $\overrightarrow{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\overrightarrow{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of $\lambda$ and hence find the unit vector along $\overrightarrow{b} + \overrightarrow{c}$.

37. A line passes through the point with position vector $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and is in the direction of the vector $\hat{\imath} + \hat{\jmath} - 2\hat{k}$. Find the equation of the line in cartesian form.

38. Show that the points $A\left(-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\right)$, $B\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right)$ and $C\left(7\hat{\imath} - \hat{k}\right)$ are collinear.

39. Find $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$, if $\overrightarrow{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\overrightarrow{b} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$.

40. Find the cartesian and vector equations of the plane passing through the points $A\ (2, 5, -3)$, $B\ (-2, -3, 5)$, and $C\ (5, 3, -3)$.

41. If the sum of two unit vectors is a unit vector,prove that the magnitude of their difference is $\sqrt{3}$.

42. If $\overrightarrow{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$, $\overrightarrow{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\overrightarrow{c} = -3\hat{\imath} + \hat{\jmath} + 2\hat{k}$,Find $\left(\overrightarrow{a}\ \ \overrightarrow{b}\ \ \overrightarrow{c}\right)$

43. If $\hat{\imath} + \hat{\jmath} + k, 2\hat{\imath} + 5\hat{\jmath}, 3\hat{\imath} + 2\hat{\jmath} - 3k, \hat{\imath} - 6\hat{\jmath} - k$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not.

44. Find the vector equation of the line which passes through the point $(3, 4, 5)$ and is parallel to the vector $2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

45. Find the vector cartesian equations of the plane passing through the points $(2, 2, -1), (3, 4, 2)$ and $(7, 0, 6)$.Also find the vector equations of plane passing through $(4, 3, 1)$ and parallel to the plane obtained above.

46. Find the vector equation of the plane that contains the lines $r = \left(\hat{\imath}+\hat{\jmath}\right)+\lambda\left(\hat{\imath}+2\hat{\jmath}-\hat{\jmath}\right)$ and the point $(-1,3,-4)$. Also,find the length of the perpendicular drawn from the point $(2,1,4)$ to the plane, thus obtained.

47. Find the value of $\lambda$, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

48. If a line makes angles $90°, 135°, 45°$ with the x,y and z axes respectively,find its direction cosines.

## 1.9. 2017

### 1.9.1. 10

### 1.9.2. 12

1. Show that the points $A, B, C$ with position vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}$, $\hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

2. Find the value of $\lambda$, if four points with position vectors $3\hat{\imath}+6\hat{\jmath}+9\hat{k}$ , $\hat{\imath}+2\hat{\jmath}+3\hat{k}$ , $2\hat{\imath}+3\hat{\jmath}+\hat{k}$ and $4\hat{\imath}+6\hat{\jmath}+\lambda\hat{k}$ are coplanar.

3. If $\mathbf{a}=2\hat{\imath}-\hat{\jmath}-2\hat{k}$ and $\mathbf{b}=7\hat{\imath}+2\hat{\jmath}-3\hat{k}$, then express $\mathbf{b}$ in the form of $\mathbf{b}=\mathbf{b_1}+\mathbf{b_2}$, where $\mathbf{b_1}$ is parallel to $\mathbf{a}$ and $\mathbf{b_2}$ is perpendicular to $\mathbf{a}$.

# 1.10. 2016

## 1.10.1. 10

1. Prove that the points $\left(3,0\right)$, $\left(6,4\right)$ and $\left(-1,3\right)$ are the vertices of a right angled isosceles triangle.

2. If the point $P\left(x,y\right)$ is equidistant from the points $A\left(a+b,b-a\right)$ and $B\left(a-b,a+b\right)$. Prove that $bx = ay$.

3. In fig 1.3, the vertices of $\triangle ABC$ are $A\left(4,6\right)$, $B\left(1,5\right)$ and $C\left(7,2\right)$. A line-segment $DE$ is drawn to intersect the sides $AB$ and $AC$ at $D$ and $E$ respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of $\triangle ADE$ and compare it with area of $\triangle ABC$.

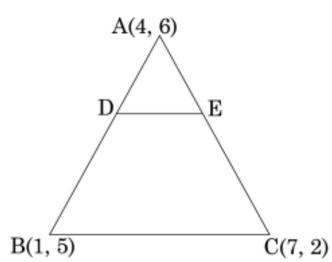



Figure 1.3:

4. Let $P$ and $Q$ be the points of trisection of the line segment joining the points $A\left(2,-2\right)$ and $B\left(-7,4\right)$ such that $P$ is nearer to $A$. Find the coordinates of $P$ and $Q$.

## 1.10.2. 12

1. If vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ are such that $|\overrightarrow{a}| = \frac{1}{2}$, $\left|\overrightarrow{b}\right| = \frac{4}{\sqrt{3}}$ and $\left|\overrightarrow{a} \times \overrightarrow{b}\right| = \frac{1}{\sqrt{3}}$, then find $\left|\overrightarrow{a}.\overrightarrow{b}\right|$.

2. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vectors, then what is the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ for $\overrightarrow{a} - \sqrt{2}\overrightarrow{b}$ to be an unit vector ?

3. Find the distance between the planes

$$\overrightarrow{r}.\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) - 4 = 0$$

and

$$\overrightarrow{r}.\left(6\hat{i} - 9\hat{j} + 18\hat{k}\right) + 30 = 0$$

4. Given that vectors $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ form a triangle such that $\overrightarrow{a} = \overrightarrow{b} + \overrightarrow{c}$. Find $p$, $q$, $r$, $s$ such that area of triangle is $5\sqrt{6}$ where $\overrightarrow{a} = p\hat{i}+q\hat{j}+r\hat{k}$, $\overrightarrow{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$ and $\overrightarrow{c} = 3\hat{i} + \hat{j} - 2\hat{k}$.

5. Find the co-ordinates of the point where the line $\overrightarrow{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ meets the plane which is perpendicular to the vector $\overrightarrow{n} = \hat{i} + \hat{j} + 3\hat{k}$ and at a distance of $\frac{4}{\sqrt{11}}$ from origin.

6. Write the sum of intercepts cut off by the plane $\overrightarrow{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) - 5 = 0$ on the three axes.

7. Find $\lambda$ and $\mu$ if

$$\left(\hat{i}+3\hat{j}+9\hat{k}\right)\times\left(3\hat{i}-\lambda\hat{j}+\mu\hat{k}\right)=\overrightarrow{0}.$$

8. If $\overrightarrow{a}=4\hat{i}-\hat{j}+\hat{k}$ and $\overrightarrow{b}=2\hat{i}-2\hat{j}+\hat{k}$, then find a unit vector parallel to the vector $\overrightarrow{a}+\overrightarrow{b}$.

9. Find the equation of the plane which contains the line of intersection of the planes

$$\overrightarrow{r}.\left(\hat{i}-2\hat{j}+3\hat{k}\right)-4=0 \text{ and}$$
$$\overrightarrow{r}.\left(-2\hat{i}+\hat{j}+\hat{k}\right)+5=0$$

and whose intercept on $x$-axis is equal to that of on $y$-axis.

10. Find the coordinates of the foot of perpendicular and perpendicular distance from the point $P\,(4,3,2)$ to the plane

$$x+2y+3z=2$$

Also find the image of $P$ in the plane.

11. Find the angle between the vectors $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$ if

$$\mathbf{a}=2\hat{i}-\hat{j}+3\hat{k} \quad \text{and}$$
$$\mathbf{b}=3\hat{i}+\hat{j}-2\hat{k}$$

and hence find a vector perpendicular to both $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$.

12. If $|\mathbf{a}|=4, |\mathbf{b}|=3$ and $\mathbf{a}.\mathbf{b}=6\sqrt{3}$, then find the value of $|\mathbf{a}\times\mathbf{b}|$.

13. Find the position vector of the point which divides the join of points with position vectors $\mathbf{a}+3\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$ internally in the ratio $1:3$.

14. Write the position vector of the point which divides the join of the point s with position vectors $3\mathbf{a}-2\mathbf{b}$ and $2\mathbf{a}+3\mathbf{b}$ in the ratio $2:1$.

15. Write the number of vectors of unit lenght perpendicular to both the vector

$$\mathbf{a}=2\hat{\imath}+\hat{\jmath}+2\hat{k} \quad \text{and}$$
$$\mathbf{b}=\hat{\jmath}+\hat{k}.$$

16. Find the vector equationof the plane with intercepts $3,-4$ and $2$ on $x, y$ and $z$-axis respectively.

17. Find the coordinates of the point where the line through the points $A(3,4,1)$ and $B(5,1,6)$ crosses the $XZ$ plane. Also find the angle which this line makes with the $XZ$ plane.

18. The two adjecent sides of a parallelogram are $2\hat{\imath}-4\hat{\jmath}-5\hat{k}$ and $2\hat{\imath}+2\hat{\jmath}+3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

19. Find the position vector of the foot of perpendicular and the perpendicular distance from the point $P$ with position vector $2\hat{\imath}+3\hat{\jmath}+\hat{k}$ to

the plane

$$\mathbf{r} \cdot \left(2\hat{i} + \hat{j} + 3\hat{k}\right) - 26 = 0$$

Also find image of $P$ in the plane.

20. Write the number of vectors of unit length perpendicular to both the vectors $\overrightarrow{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{b} = \hat{j} + \hat{k}$

21. Write the position vector of the point which divides the join of points with position vectors $3\overrightarrow{a} - 2\overrightarrow{b}$ and $2\overrightarrow{a} + 3\overrightarrow{b}$ in the ratio $2:1$.

22. Find the vector equation of the plane with intercepts $3, -4$ and $2$ on $x, y$ and $z$ axis respectively.

23. Find the co-ordinates of the point where the line through the points A $(3, 4, 1)$ and B $(5, 1, 6)$ crosses the XZ plane. Also find the angle which this line makes with the XZ plane.

24. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vectors parallel to its diagnols. Using the diagnol vectors, find the area of the parallelogram.

25. Find the position vector of the foot of perpendicular and the perpendicular distance from the point $P$ with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ to the plane $\overrightarrow{r}.\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) - 26 = 0$. Also find the image of $P$ in the plane.

26. If $|\overrightarrow{a}| = 4, \left|\overrightarrow{b}\right| = 3$ and $\overrightarrow{a}.\overrightarrow{b} = 6\sqrt{3}$, then find the the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$.

27. Write the coordinates of the point which is the reflection of the point $(\alpha, \beta, \gamma)$ in the $XZ$-plane.

28. Find the position vector of the point which divides the join of points with position vectors $\vec{a} + 3\vec{b}$ and $\vec{a} - \vec{b}$ internally in the ratio $1:3$.

29. Show that the lines $\dfrac{x-1}{3} = \dfrac{y-1}{-1} = \dfrac{z+1}{0}$ and $\dfrac{x-4}{2} = \dfrac{y}{0} = \dfrac{z+1}{3}$ intersect. Find their point of intersection.

30. Find the angle between the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ if $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$, and hence find a vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

31. Find the coordinates of the foot of perpendicular and perpendicular distance from the point $P(4, 3, 2)$ to the plane $x + 2y + 3z = 2$. Also find the image of $P$ in the plane.

32. Show that the relation $R$ defined by $(a, b)\mathrm{R}(c, d) \Rightarrow a + d = b + c$ on the $A \times A$, where $A = \{1, 2, 3, \ldots, 10\}$ is an equivalence relation. Hence write the equivalence class $[(3, 4)]\,; a, b, c, d \in A$.

33. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$.

34. Find $\lambda$ and $\mu$ if
$$\left(\hat{i} + 3\hat{j} + 9\hat{k}\right) \times \left(3\hat{i} - \lambda\hat{j} + \mu\hat{k}\right) = \vec{0}$$

35. Write the sum of intercepts cut by the plane $\vec{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) - 5 = 0$ on the three axes.

36. Find the equations of the plane which contains the line of intersection of the planes

$$\overrightarrow{r} \cdot \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) - 4 = 0$$
$$\overrightarrow{r} \cdot \left(-2\hat{i} + \hat{j} + \hat{k}\right) + 5 = 0$$

and whose intercept on x-axis is equal to that of y-axis.

# Chapter 2

# Linear Forms

## 2.1. 2023

### 2.1.1. 10

1. **Assertion (A):** Point $\mathbf{P}(0,2)$ is the point of intersection of $y-axis$ with the line $3x+2y=4$.

   **Reason (R):** The distance of point $\mathbf{P}(0,2)$ from $x-axis$ is 2 units.

2. If the pair of equations $3x-y+8=0$ and $6x-ry+16=0$ represent coincident lines, then the value of '$r$' is:

   (a) $-\frac{1}{2}$

   (b) $\frac{1}{2}$

   (c) -2

   (d) 2

3. The of linear equations $2x=5y+6$ and $15y=6x-18$ represents two lines which are:

(a) intersecting

(b) parallel

(c) coincident

(d) either intersecting or parallel

4. Find the equations of the diagonals of the parallelogram **PQRS** whose vertices are **P**(4,2,-6), **Q**(5,-3,1), **R**(12,4,5) and **S**(11,9,-2). Use these equations to find the point of intersection of diagonals.

5. A line $l$ passes through point (-1,3,-2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the ctor equation of the line $l$. Hence, obtain its distance from origin.

### 2.1.2. 12

1. Equation of line passing through origin and making $30°, 60°$ and $90°$ with $x, y, z$ axes respectively is

   (a) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$

   (b) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$

   (c) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$

   (d) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$

2. If the equation of a line is $x = ay+b, z = cy+d$,then find the direction ratios of the line and a point on the line.

3. (a) Find the equations of the diagonals of the parallelogram $PQRS$ whose vertices are $P(4,2,-6), Q(5,-3,1), R(12,4,5), S(11,9,-2)$. Use these equations to find the point of intersection of diagonals.

   (b) A line $l$ passes through point $(-1,3,-2)$ and is perpendicular to both the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$. Find the vector equation of the line $l$. Hence, obtain its distance from origin.

## 2.2. 2022

### 2.2.1. 10

1. Solve the equations $x+2y=6$ and $2x-5y=12$ graphically.

2. Solve the following equations for $x$ and $y$ using cross-multiplication method:

$$(ax-by)+(a+4b)=0 \tag{2.1}$$

$$(bx+ay)+(b-4a)=0 \tag{2.2}$$

3. Find the co-ordinates of the point where the line $\dfrac{x-3}{-1}=\dfrac{y+4}{1}=\dfrac{z+5}{6}$ crosses the plane passing through the points $\left(\dfrac{7}{2},0,0\right),(0,7,0),(0,0,7)$.

4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

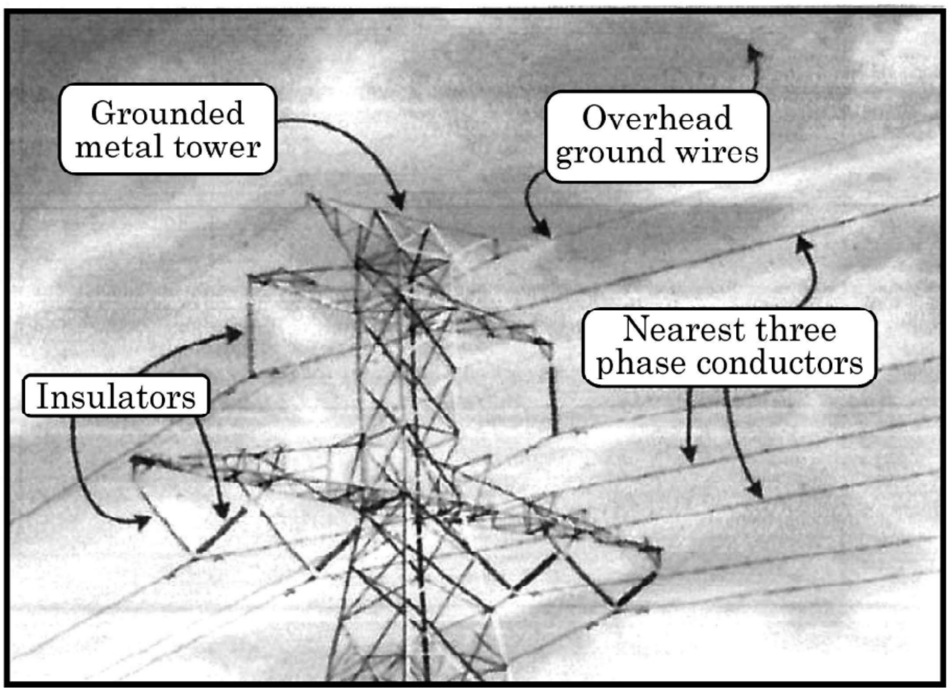



Figure 2.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \tag{2.3}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \tag{2.4}$$

Based on the given information, answer the following questions:

(a) Are the $l_1$ and $l_2$ coplanar? Justify your answer.

(b) Find the point of intersection of lines $l_1$ and $l_2$.

5. Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, −2). Hence, find the y-coordinate of the point on

the line PQ whose z-coordinate is -2.

6. Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k}) \tag{2.5}$$

$$\mathbf{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k}) \tag{2.6}$$

8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.2) represented by the following lines

$$\mathbf{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) \tag{2.7}$$

$$\mathbf{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k}) \tag{2.8}$$

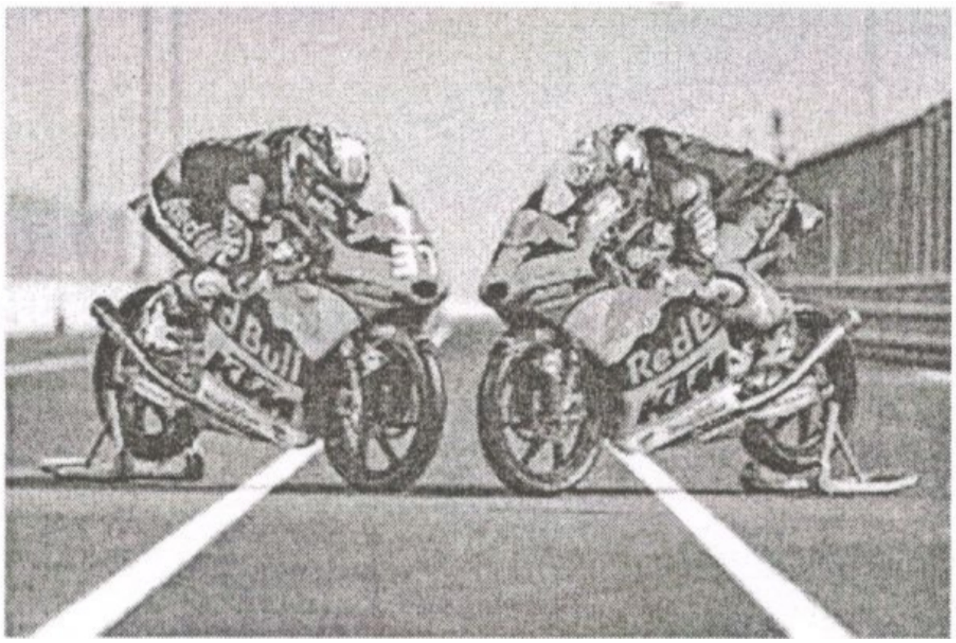


Figure 2.2: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

(a) Find the shortest distance between the given lines.

(b) Find a point at which the motorcycles may collide.

9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{i} + (\lambda + 4)\hat{j} - (\lambda - 3)\hat{k} \tag{2.9}$$

$$\mathbf{r} = (3 - \mu)\hat{i} + (2\mu + 2)\hat{j} + (\mu + 6)\hat{k} \tag{2.10}$$

10. Find the shortest distance between the following lines and hence write

whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{2.11}$$

11. Find the equation of the plane passing through the points $(2,1,0), (3,-2,-2)$ and $(1,1,7)$. Also, obtain its distance from the origin.

12. The foot of a perpendicular drawn from the point $(-2,-1,-3)$ on a plane is $(1,-3,3)$. Find the equation of the plane.

13. Find the cartesian and the vector equation of a plane which passes through the point $(3,2,0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

14. The distance between the planes $4x-4y+2z+5=0$ and $2x-2y+z+6=0$ is

    (a) $\frac{1}{6}$
    (b) $\frac{7}{6}$
    (c) $\frac{11}{6}$
    (d) $\frac{16}{6}$

15. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{\imath} + 3\hat{\jmath}) + 6 = 0 \tag{2.12}$$

$$\mathbf{r} \cdot (3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0 \tag{2.13}$$

which is at a unit distance from the origin.

16. If the distance of the point $(1, 1, 1)$ from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of $\lambda$.

17. Find the distance of the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$.

18. Find the distance of the point $P(4, 3, 2)$ from the plane determined by the points $A(-1, 6, -5), B(-5, -2, 3)$ and $C(2, 4, -5)$.

19. The distance of the line

$$\mathbf{r} = (\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 5\hat{\jmath} + \hat{k}) \tag{2.14}$$

from the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + 4\hat{k}) = 5 \tag{2.15}$$

is

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{3\sqrt{2}}$

(d) $\frac{-2}{3\sqrt{2}}$

20. Find a unit vector perpendicular to each of the vectors $(\mathbf{a} + \mathbf{b})$ and

$(\mathbf{a}-\mathbf{b})$ where

$$\mathbf{a} = \hat{\imath} + \hat{\jmath} + \hat{k} \tag{2.16}$$

$$\mathbf{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} \tag{2.17}$$

21. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}) \tag{2.18}$$

and the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 10. \tag{2.19}$$

22. Find the area bounded by the curves $y = |x-1|$ and $y = 1$, using integration.

23. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

24. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.1:

Table 2.1: Table showing yearly trend of production of goods in lakh tonnes

| Year | Production (in lakh tonnes) |
|---|---|
| 2001 | 30 |
| 2002 | 35 |
| 2003 | 36 |
| 2004 | 32 |
| 2005 | 37 |
| 2006 | 40 |

### 2.2.2. 12

1. Find the values of $\lambda$,for which the distance of point (2,1,$\lambda$) from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units.

2. If the distance of the point (1,1,1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$,find the value(s) of $\lambda$.

## 2.3. 2021

### 2.3.1. 10

1. If the graph of a pair of lines $x - 2y + 3 = 0$ and $2x - 4y = 5$ be drawn, that what type of lines are drawn ?

### 2.3.2. 12

1. If the two lines

$$L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2} \tag{2.20}$$

$$L_1 : x = 2, \frac{y}{-1} = \frac{z}{z-\alpha} \tag{2.21}$$

are perpendicular,then the value of $\alpha$

(a) $\frac{2}{3}$

(b) 3

(c) 4

(d) $\frac{7}{3}$

2. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z \tag{2.22}$$

$$\frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{2.23}$$

3. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r}.\,(i + 3j) + 6 = 0 \tag{2.24}$$

$$\mathbf{r}.\,(3i - j - 4k) = 0 \tag{2.25}$$

which is at a unit distance from the origin.

4. If segment of the line intercepted between the co-ordinate-axes is bisected at the point $M(2,3)$, then the equation of this line is

$$2x + 3y = 13 \tag{2.26}$$

$$x + y = 5 \tag{2.27}$$

$$2x + y = 7 \tag{2.28}$$

$$3x + 2y = 12 \tag{2.29}$$

5. The equation of a line through $(2,-4)$ and parallel to x-axis is ____________.

6. Find the equation of the median through vertex $A$ of the triangle $ABC$, having vertices $A(2,5)$, $B(-4,9)$ and $C(-2,-1)$.

7. Solve the system of linear equations, using matrix method :

$$7x + 2y = 11 \tag{2.30}$$

$$4x - y = 2 \tag{2.31}$$

# 2.4. 2019

## 2.4.1. 12

1. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

2. Using the method of integration, find the area of the region bounded

by the lines $3x - 2y + 1 = 0, 2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

3. Using integration, find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

4. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

5. Find the acute angle between the planes

$$\overrightarrow{r}.\ \left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 1$$

and

$$\overrightarrow{r}.\ \left(3\hat{i} - 6\hat{j} + 2\hat{k}\right) = 0.$$

6. Find the length of the intercept, cut off by the plane $2x + y - z = 5$ on the x-axis.

### 2.4.2. 10

1. Draw the graph of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Using this graph, find the values of $x$ and $y$ which satisfy both the equations.

## 2.5. 2018

### 2.5.1. 12

1. If $a * b$ denotes the larger of '$a$' and '$b$' and if $aob = (a * b) + 3$, then write the value of $(5)\, o\, (10)$, where $*$ and $o$ are binary operations.

2. Find the equations of the tangent and the normal, to the curve $16x^2 + 9y^2 = 145$ at the point $(x_1, y_1)$, where $x_1 = 2$ and $y_1 > 0$.

3. Using the integration, find the area of the region in the first quadrant enclosed by the X-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

4. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

5. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

6. Find the length of the intercept, cut off by the plane $2x + y - z = 5$ on the x-axis.

7. Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1$=0, $2x + 3y - 21 = 0$ , and $x - 5y + 9 = 0$

8. Using integration, find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

9. Using integration, find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates and the ordinates $x = -2$ and $x = 1$.

10. Using integration, find the area of the following region:

$$\left\{(x,y) : x^2 + y^2 \leq 16a^2 \text{ and } y^2 \leq 6ax\right\}$$

11. If the line $\dfrac{x-1}{-3} = \dfrac{y-2}{2\lambda} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{3\lambda} = \dfrac{y-1}{2} = \dfrac{z-6}{-5}$ are perpendicular, find the value of $\lambda$. Hence find whether the lines are intersecting or not.

12. Find the equation of the line passing through $(2,-1,2)$ and $(5,3,4)$ and of the plane passing through $(2,0,3)$, $(1,1,5)$ and $(3,2,4)$. Also, find their point of intersection.

# 2.6. 2017

## 2.6.1. 10

## 2.6.2. 12

1. Find the value of $x$ such that the points $A\,(3,2,1)\,, B\,(4,x,5)\,, C\,(4,2,-2)$ and $D\,(6,5,-1)$ are coplanar.

2. The x-coordinate of a point on the line joining the points $P\,(2,2,1)$and $Q\,(5,1,-2)$ is 4. Find its z-coordinate.

3. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2.5y + 5z = 20$ .

4. Find the coordinates of the point where the line through the points $(3,-4,-5)$ and $(2,-3,1)$, crosses the plane determined by the points $(1,2,3)\,,(4,2,-3)$ and $(0,4,3)$ .

# 2.7. 2016

## 2.7.1. 12

1. Find the equation of plane passing through the points $A\left(3,2,1\right)$, $B\left(4,2,-2\right)$ and $C\left(6,5,-1\right)$ and hence find the value of$\lambda$ for which $A\left(3,2,1\right)$, $B\left(4,2,-2\right)$, $C\left(6,5,-1\right)$ and $D\left(\lambda,5,5\right)$ are coplanar.

2. Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$ Also, find if the plane thus obtained contains the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ or not.

3. For what value of $k$, the system of linear equations

$$x + y + z = 2$$
$$2x + y - z = 3$$
$$3x + 2y + kz = 4$$

has a unique solution?

4. Show that the four points $A(4,5,1)$, $B(0,-1,-1)$, $C(3,9,4)$ and $D(-4,4,4)$ are coplanar.

5. Find the coordinates of the foot of perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$. Hence find the image of the point $A$ in the line $BC$.

# Chapter 3

# Circles

## 3.1. 2023

### 3.1.1. 10

1. In the given figure Fig. 3.1, $PQ$ is tangent to the circle centred at **O**. If $\angle AOB = 95°$, then measure of $\angle ABQ$ will be

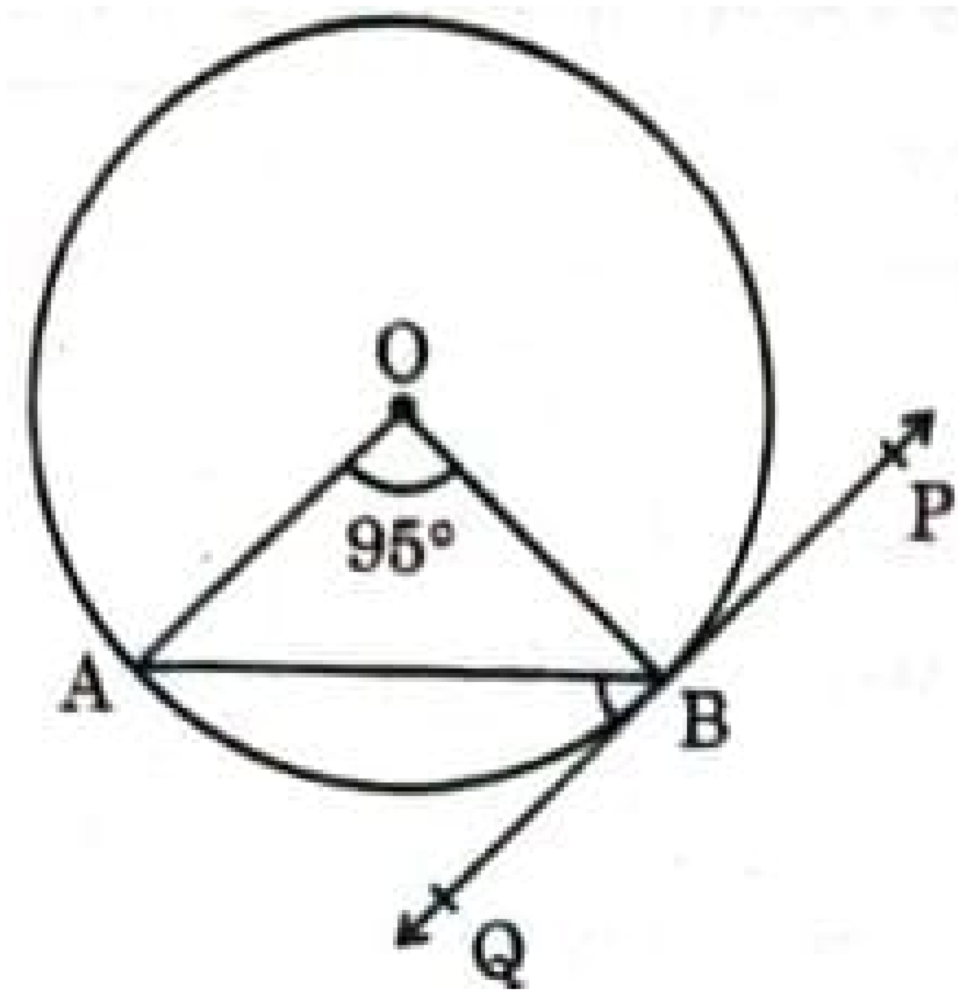



Figure 3.1:

(a) 47.5°

(b) 42.5°

(c) 85°

(d) 95°

2. (a) In the given figure Fig. 3.2, two tangents $TP$ and $TQ$ are drawn

to be a circle with centre **O** from an external point **T**. Prove that $\angle PTQ = 2\angle OPQ$.

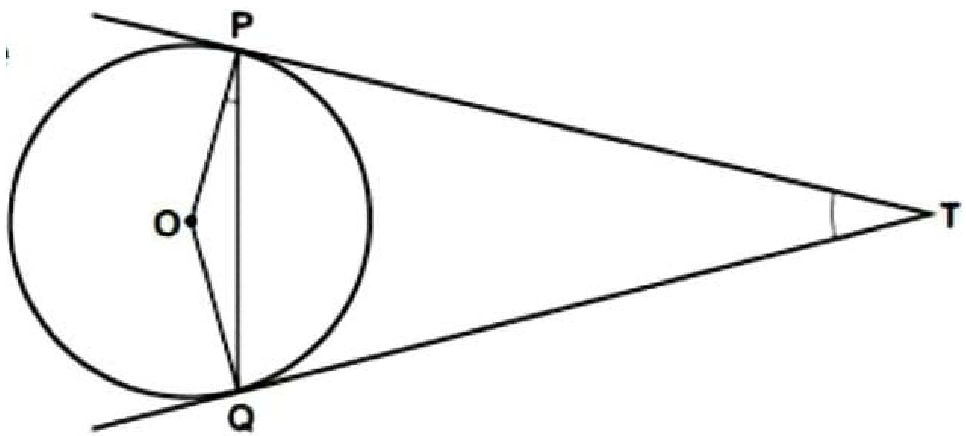



Figure 3.2:

(b) In the given figure Fig. 3.3, a circle is inscribed in a quadrilateral $ABCD$ in which $\angle B = 90°$. If $AD = 17cm, AB = 20cm$ and $DS = 3cm$, then find the radius of the circle.

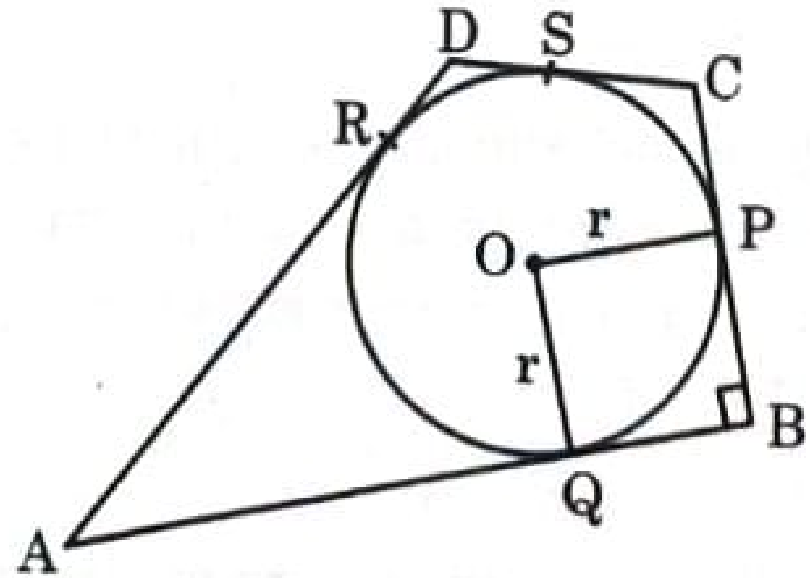



Figure 3.3:

3. The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a cicrle as shown in Fig. 3.4 below, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.


Figure 3.4:

In the given figure Fig. 3.5, $AB$ is one such tangent to a circle of radius 75 cm. Point **O** is centre of the circle and $\angle ABO = 30°.PQ$ is parallel to $OA$.

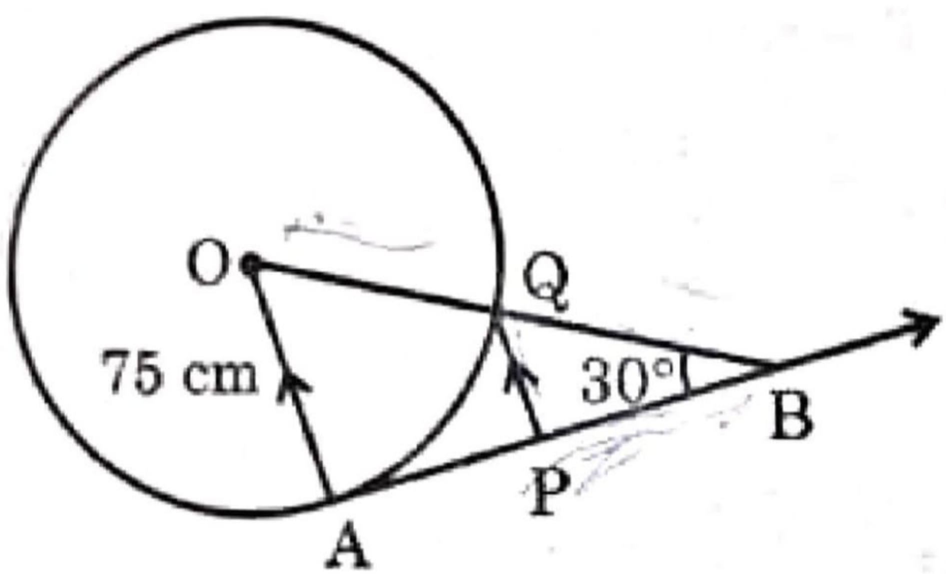



Figure 3.5:

Based on above information :

(a) find the length of $AB$.

(b) find the length of $OB$.

(c) find the length of $AP$.

(d) find the length of $PQ$.

4. In the given figure Fig. 3.6, the quadrilateral $PQRS$ circumscribes a circle. Here $PA + CS$ is equal to :

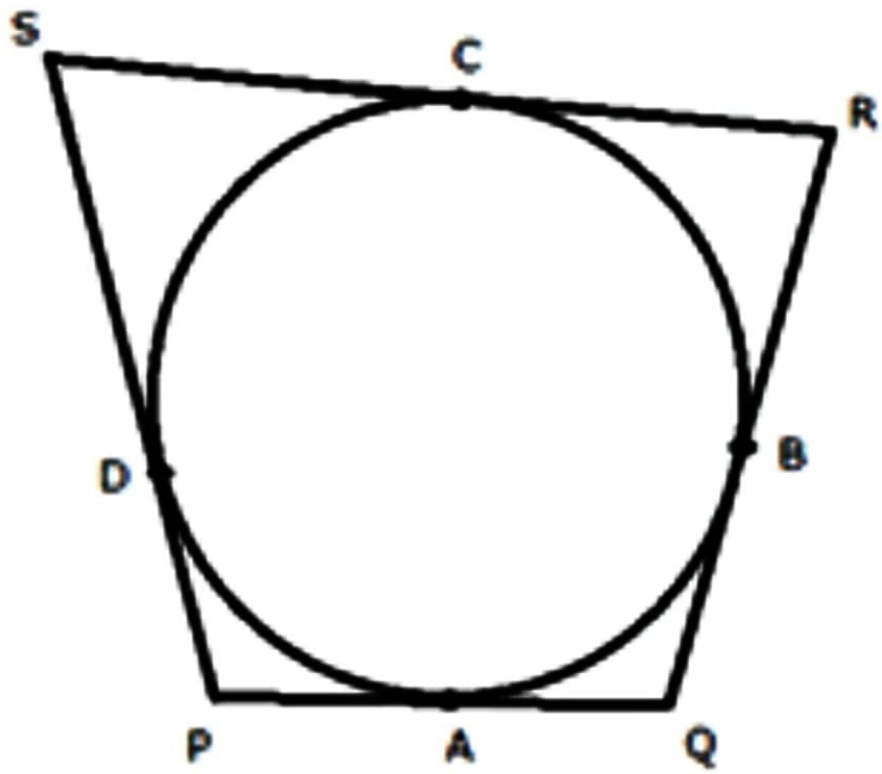



Figure 3.6:

(a) $QR$

(b) $PR$

(c) $PS$

(d) $PQ$

5. In the given figure Fig. 3.7, **O** is the centre of the circle. $AB$ and $AC$ are tangents drawn to the circle from point **A**. If $\angle BAC = 65°$, then find the measure of $\angle BOC$.

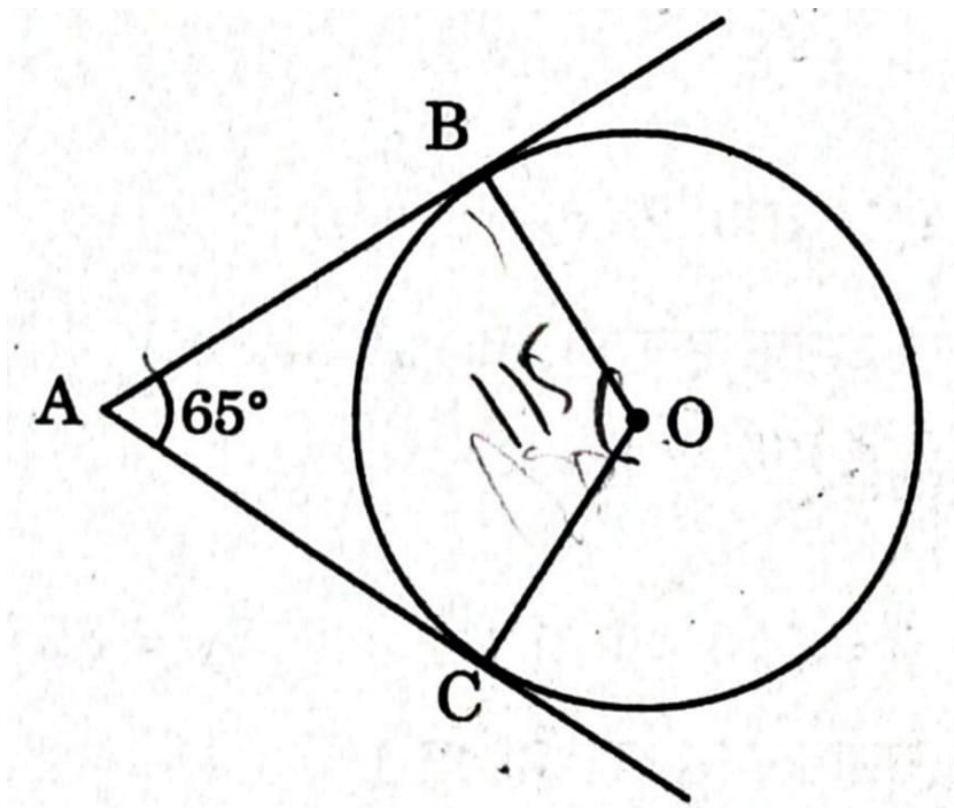



Figure 3.7:

6. In the given figure Fig. 3.8, **O** is the centre of the circle and $QPR$ is the tangent to it at **P**. Prove that $\angle QAP + \angle APR = 90°$.

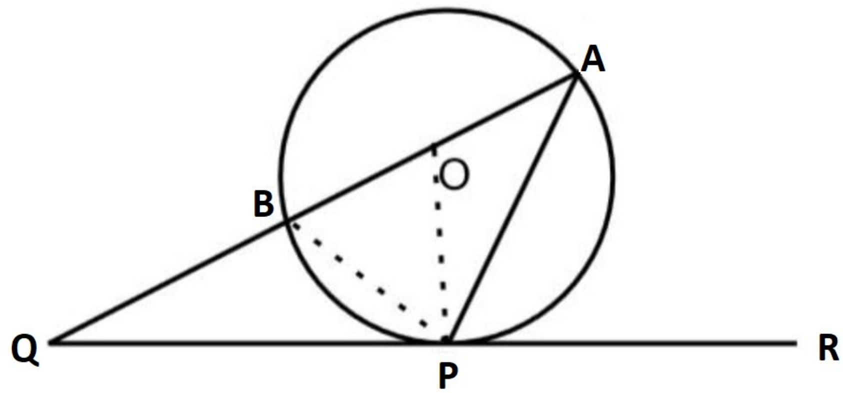



Figure 3.8:

7. In the given figure Fig. 3.9, $TA$ is a tangent to the circle with centre **O** such that $OT = 4cm, \angle OTA = 30°$, then length of $TA$ is :

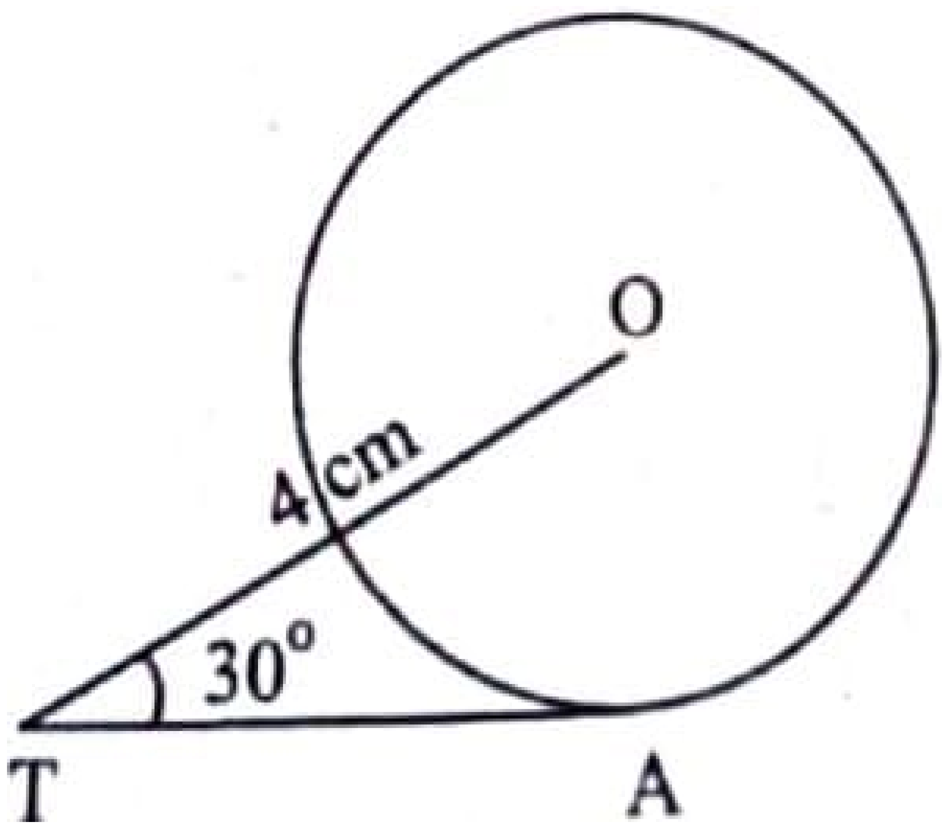



Figure 3.9:

(a) $2\sqrt{3}cm$

(b) $2cm$

(c) $2\sqrt{2}cm$

(d) $\sqrt{3}cm$

8. In the given figure Fig. 3.10, $PT$ is a tangent at **T** to the circle with centre **O**. If $\angle TPO = 25°$, then $x$ is equal to :

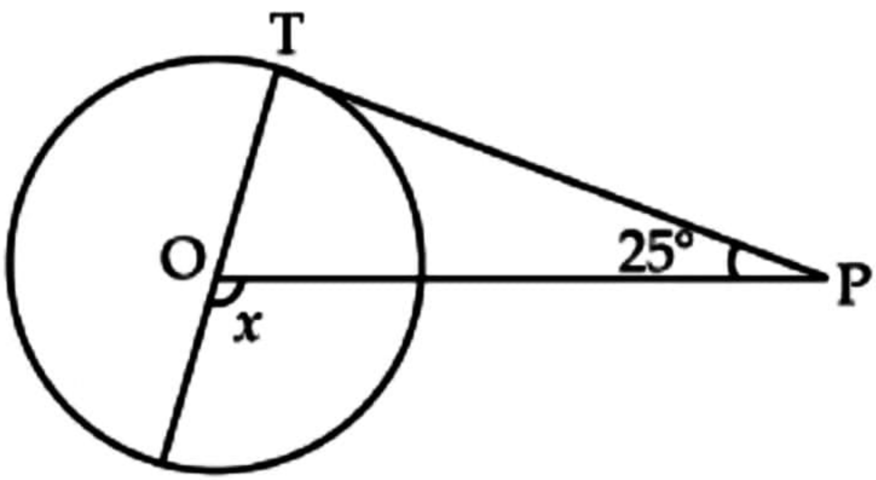



Figure 3.10:

(a) 25°

(b) 65°

(c) 90°

(d) 115°

9. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

## 3.2. 2022

### 3.2.1. 12

1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to

the circle from point **P**.

2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.

3. In Figure 3.11, there are two concentric circles with centre **O**. If $ARC$ and $AQB$ are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of $AC$, if $AQ = 5$ cm.

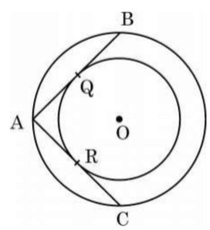



Figure 3.11: Two concentric circles with **O** as centre

4. In Figure 3.12, if a circle touches the side $QR$ of $\Delta PQR$ at **S** and extended sides $PQ$ and $PR$ at **M** and **N**, respectively,

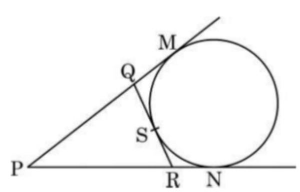



Figure 3.12: Two tangents are drawn from point **P** to the circle

prove that $PM = \frac{1}{2}(PQ + QR + PR)$

5. In Figure 3.13, a triangle $ABC$ is drawn to circumscribe a circle of radius 4 cm such that the segments $BD$ and $DC$ into which $BC$ is

divided by the point of contact **D** are of lengths 6 cm and 8 cm respectively. If the area of $\Delta ABC$ is 84 $cm^2$, find the lengths of sides $AB$ and $AC$.

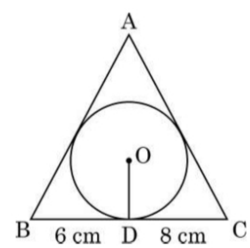



Figure 3.13: Circle with **O** as center circumscribed in triangle $ABC$

6. In Figure 3.14, $PQ$ and $PR$ are tangents to the circle centered at **O**. If $\angle OPR = 45°$, then prove that $ORPQ$ is a square.

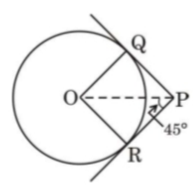



Figure 3.14: Two tangents drawn from point **P** to a circle whose centre is **O**

7. In Figure 3.15, **O** is the centre of a circle of radius 5 cm. $PA$ and $BC$ are tangents to the circle at **A** and **B** respectively. If $OP$ is 13 cm, then find the length of tangents $PA$ and $BC$.

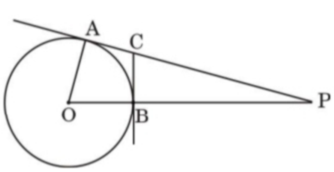



Figure 3.15: Two tangents drawn from point **C** to a circle whose centre is **O**

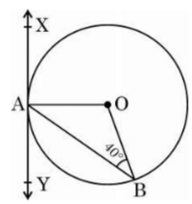



Figure 3.17: The line $XAY$ is tangent to the circle centered at **O**

8. In Figure 3.16, $AB$ is diameter of a circle centered at **O**. $BC$ is tangent to the circle at **B**. If $OP$ bisects the chord $AD$ and $\angle AOP = 60°$, then find $m\angle C$.

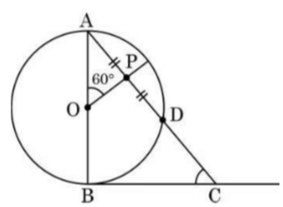



Figure 3.16: Tangent $BC$ is drawn from point **C** to a circle whose centre is **O**

9. In Figure 3.17, $XAY$ is a tangent to the circle centered at **O**. If $\angle ABO = 60°$,then find $m\angle BAY$ and $m\angle AOB$.

10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

11. In Figure 3.18, a triangle $ABC$ with $\angle B = 90°$ is shown. Taking $AB$ as diameter, a circle has been drawn intersecting $AC$ at point **P**. Prove that the tangent drawn at point **P** bisects $BC$.

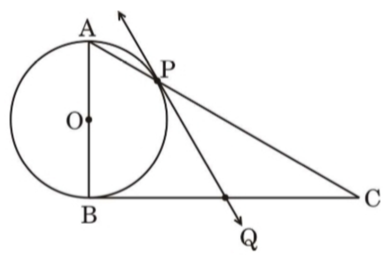



Figure 3.18: $PQ$ is tangent to the circle centered at **O**. $AB$ is the diameter and $\angle B = 90°$

12. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

## 3.2.2. 10

1. In tangents **PA** and **PB** from an external point $P$ to a circle with centre $O$ , are inclined to each other at an angle of $80^{\circ}$, then $\angle AOB$ is equal to

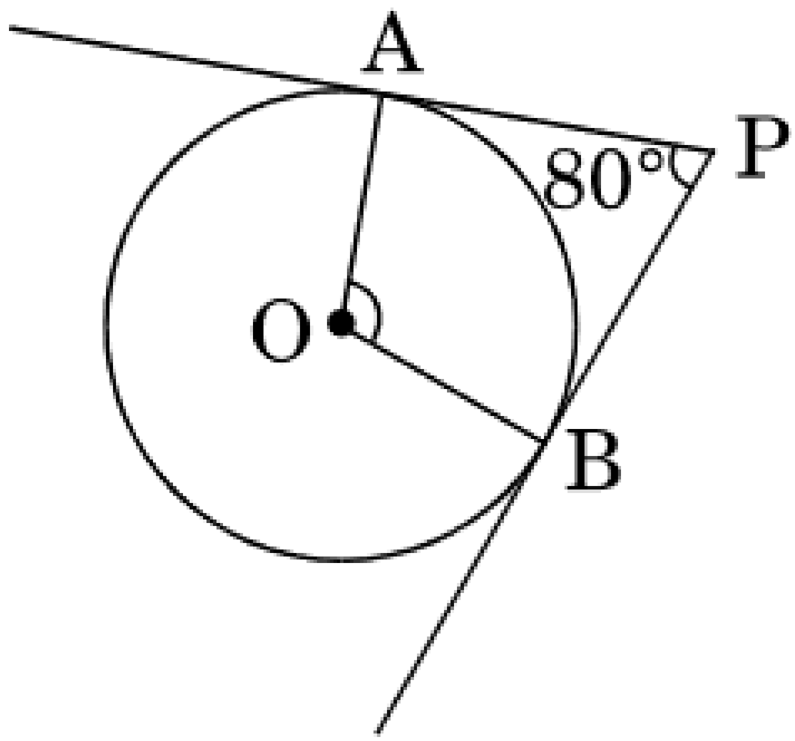



Figure 3.19: Tangents $PA$ and $PB$

(a) $100^{\circ}$

(b) $60^{\circ}$

(c) $80^{\circ}$

(d) $50^{\circ}$

2. Two concentric circles are of radii $4cm$ and $3cm$. Find the length of the chord of the larger circle which touches the smaller circle.

3. In a triangle $ABC$ with $\angle AOB$ is shown. Taking $AB$ as diameter,

a circle has been drawn intersecting $AC$ at point $P$. Prove that the tangent drawn at point $P$ bisects $BC$.

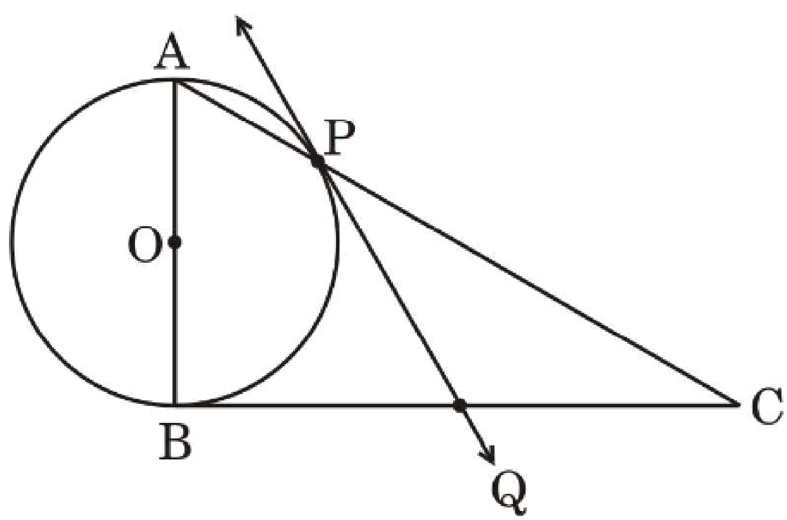



Figure 3.20: Concentric circles

4. Prove that a Parallelogram circumscribing a circle is a rhombus.

5. In two circles with centres at $O$ and $O$ of radii $2r$ and $r$ respectively, touch each other internally at $A$. A chord $AB$ of the bigger circle meets the smaller circle at $C$. Show that $C$ bisects $AB$.

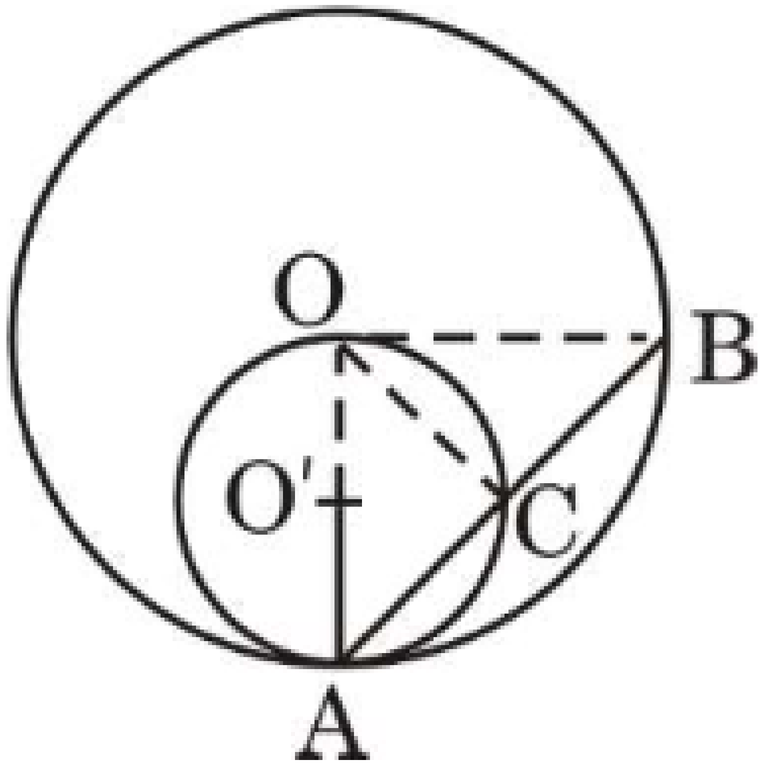



Figure 3.21: Two circles with center

6. In $O$ is centre of a circle of radius $5cm$. $PA$ and $BC$ are tangents to the circle at $A$ and $B$ respectively. If $OP = 13cm$, then find the length of tangents $PA$ and $BC$.

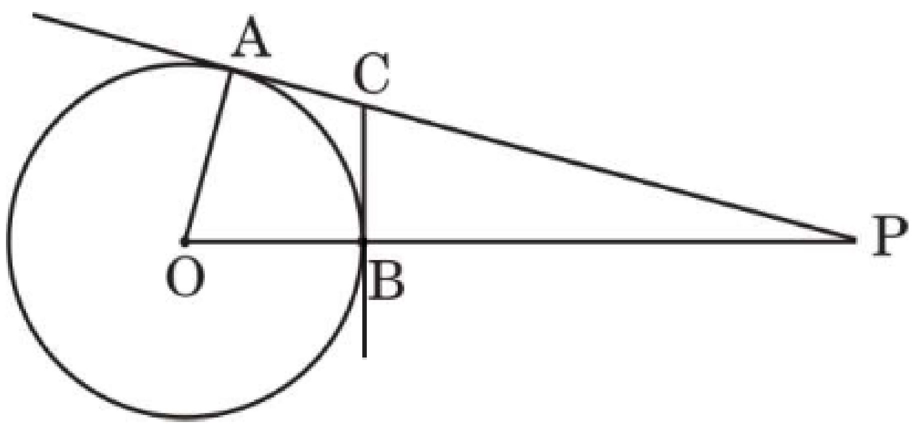



Figure 3.22: The center of circle radius is $5m$

7. In two concentric circles, a chord of length $48cm$ of the larger circle is a tangent to the smaller circle, whose radius is $7cm$. Find the radius of the larger circle.

8. At a point on the level ground, the angle of elevation of the top of a vertical tower is found to be $\alpha$, such that $\tan\alpha = \frac{5}{12}$. On walking $192m$ towards the tower, the angle of elevation $\beta$ is such that $\tan\beta = \frac{3}{4}$. Find the height of the tower.

## 3.3. 2021

### 3.3.1. 10

1. A quadrilateral $ABCD$ is drawn to circumscribe a circle (see Figure-1). Prove that $AB + CD = AD + BC$.

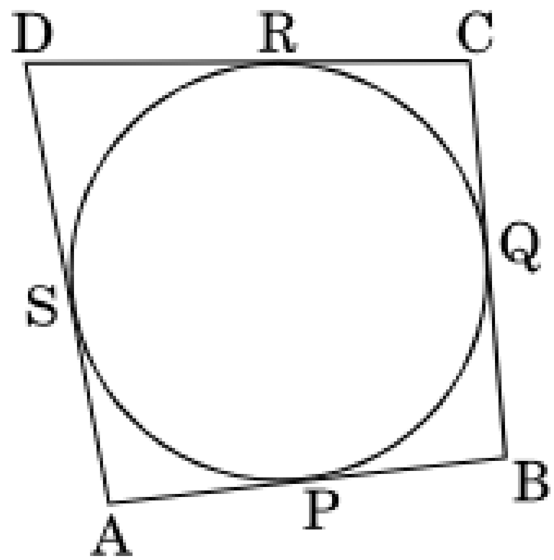



Figure 3.23:

2. Draw a pair of tangents to a circle of radius $4cm$ which are inclined to each other at an angle of $45°$.

3. A point **T** is $13cm$ away from the centre of a circle. The length of the tangent drawn from **T** to the circle is $12cm$. Find the radius of the circle.

4. Two tangents $TP$ and $PQ$ are drawn to a circle with centre **O** from an external point **T**. Prove that $\angle PTQ = 2\angle OPQ$.

5. $PQ$ is a tangent to a circle with centre **O** at the point **P** on the circle. If $\triangle OPQ$ is an isosceles triangle, then find $\angle OQP$.

6. Two concentric circles have radii $10cm$ and $6cm$. Find the length of the chord of the larger circle which touches the smaller circle.

7. If tangents $PA$ and $PB$ from an external point **P** to a circle with centre **O** are inclined to each other at an angle of $70°$, then find $\angle POA$.

8. $ABC$ is right triangle, right-angled at **B** with $BC = 6cm$ and $AB = 8cm$. A circle with centre **O** and radius $r$ cm has been inscribed in $\triangle ABC$ as shown in the figure. Find the value of $r$.

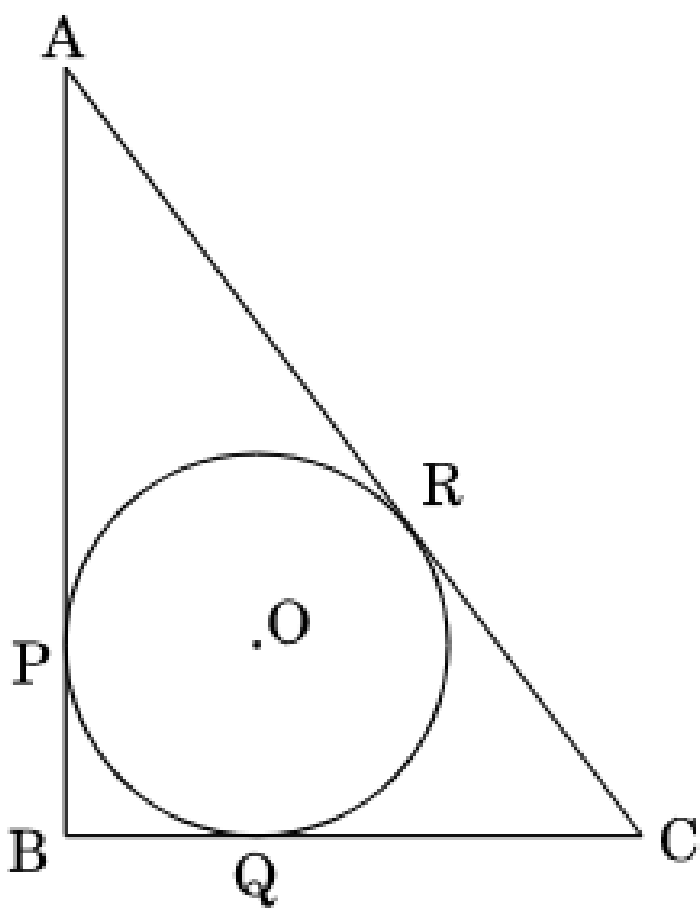



Figure 3.24:

9. Draw a circle of radius $5cm$. From a point $8cm$ away from its centre, construct a pair of tangents to the circle.

10. In the given figure, $PT$ and $PS$ are tangents to a circle with centre **O**, from a point **P**, such that $PT = 4cm$ and $\angle TPS = 60°$. Find the length of the chord $TS$. Also, find the radius of the circle.

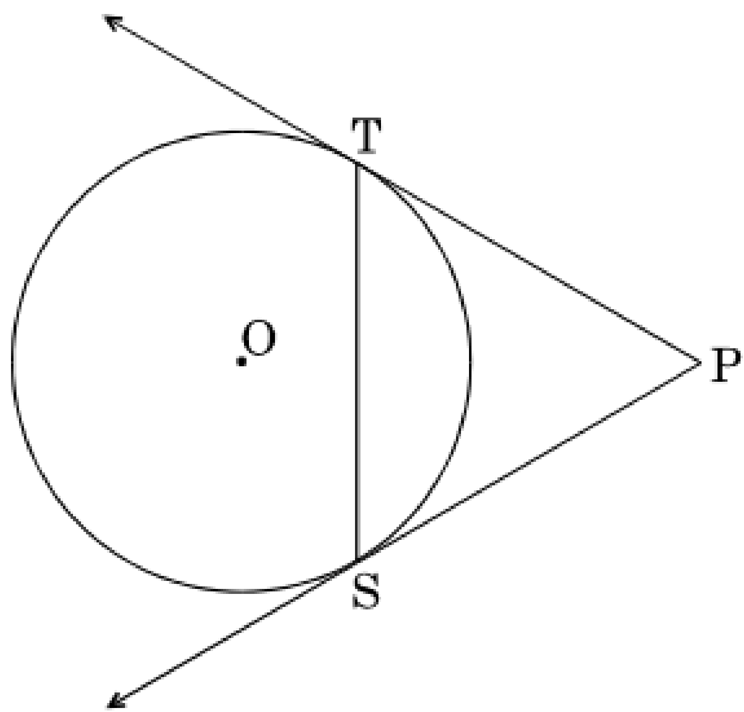



Figure 3.25:

11. (a) In a right triangle $ABC$, right-angled at **B**, $BC = 6cm$ and $AB = 8cm$. A circle is inscribed in the $\triangle ABC$. Find the radius of the incircle.

    (b) Two circles touch externally at **P** and $AB$ is a common tangent, touching one circle at **A** and the other at **B**. Find the measure of $\angle APB$.

12. From an external point **P**, tangents $PQ$ and $PR$ are drawn to a circle with centre **O**, touching the circle at **Q** and **R**. If $\angle QOR = 140°$, find the measure of $\angle QPR$.

13. A circle touches all the sides of a quadrilateral $ABCD$. Prove that $AB + CD = DA + BC$.

14. Write the steps of construction of a circle of diameter $6cm$ and drawing of a pair of tangents to the circle from a point $5cm$ away from the centre.

## 3.4. 2020

### 3.4.1. 10

1. In Fig. 3.26, from an external point $P$, two tangent $PQ$ and $PR$ are drawn to a circle of radius $4cm$ with center $O$. If$\angle PQR = 90°$, then length of $PQ$ is ____________________.

   (a) $3cm$

   (b) $4cm$

   (c) $2cm$

   (d) $2\sqrt{2}cm$

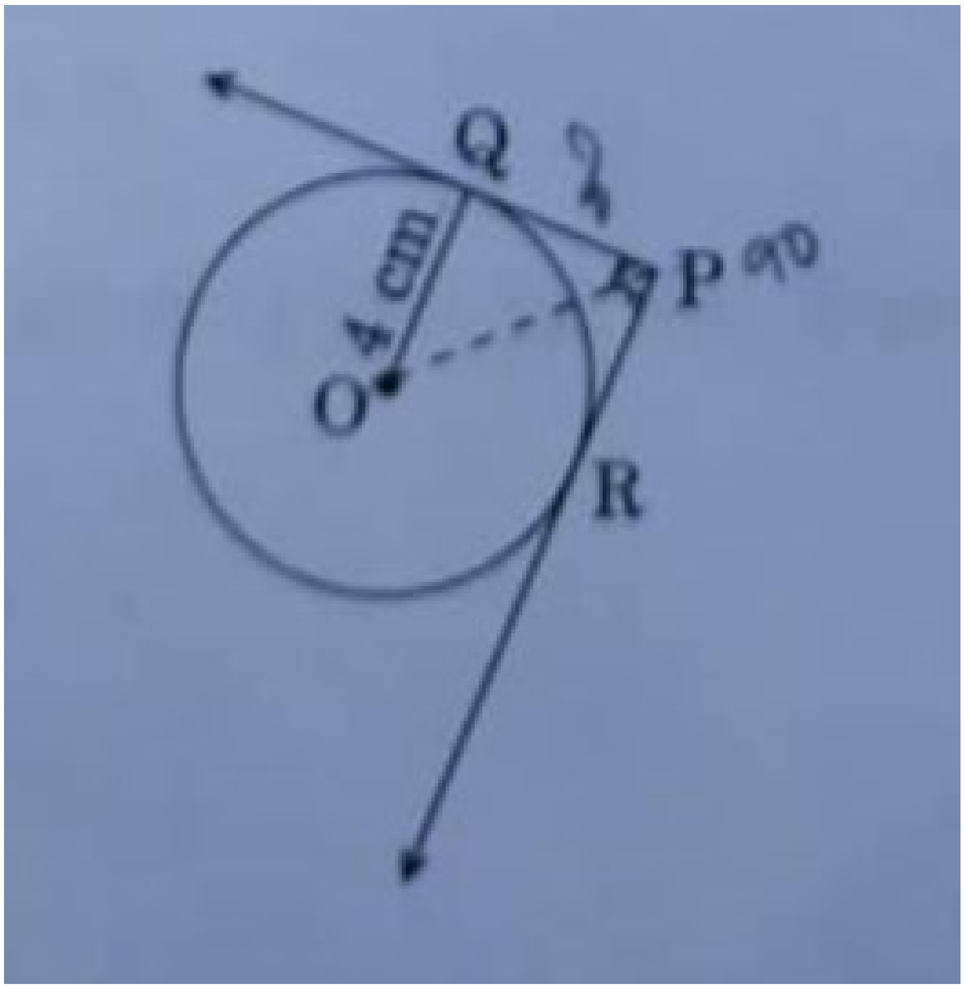



Figure 3.26:

2. In Fig. 3.27, $PQ$ is tangent to the circe with center at $O$, at the point $B$. If$\angle AOB = 100°$, then$\angle ABP$ is equal to

   (a) $50°$

   (b) $40°$

(c) 60°

(d) 80°

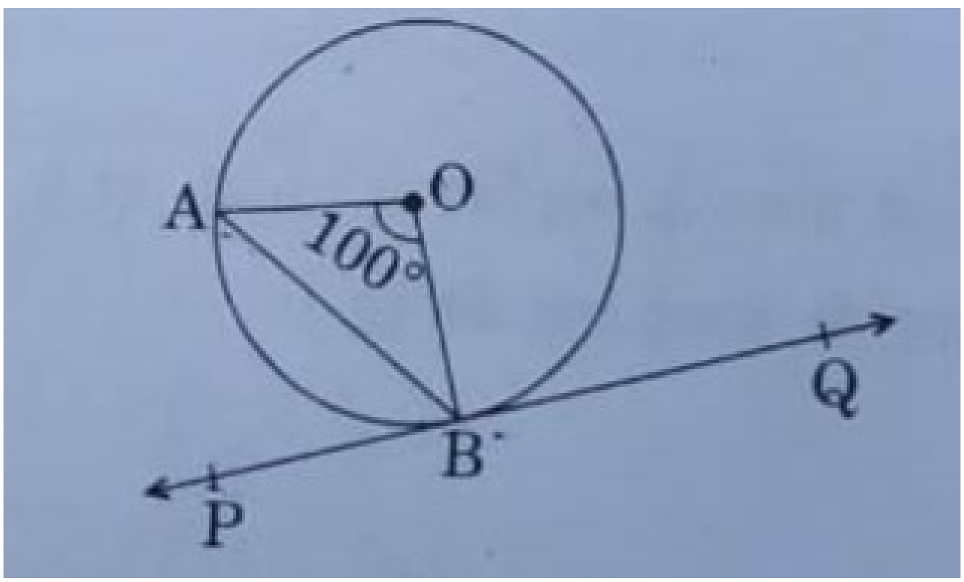



Figure 3.27:

3. In Fig. 3.28, quadrilateral $ABCD$ is drawn to circumscribe a circle. Prove that

$AB + CD = BC + AD$

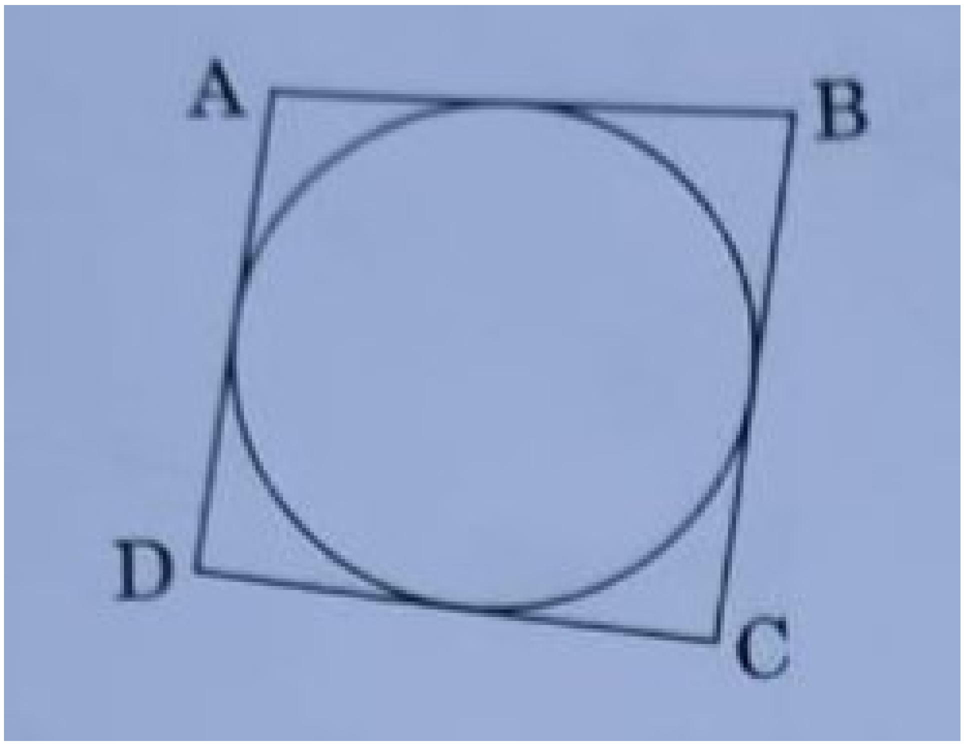



Figure 3.28:

4. In Fig. 3.29, find the perimeter of $\triangle ABC$, if $AP = 12cm$

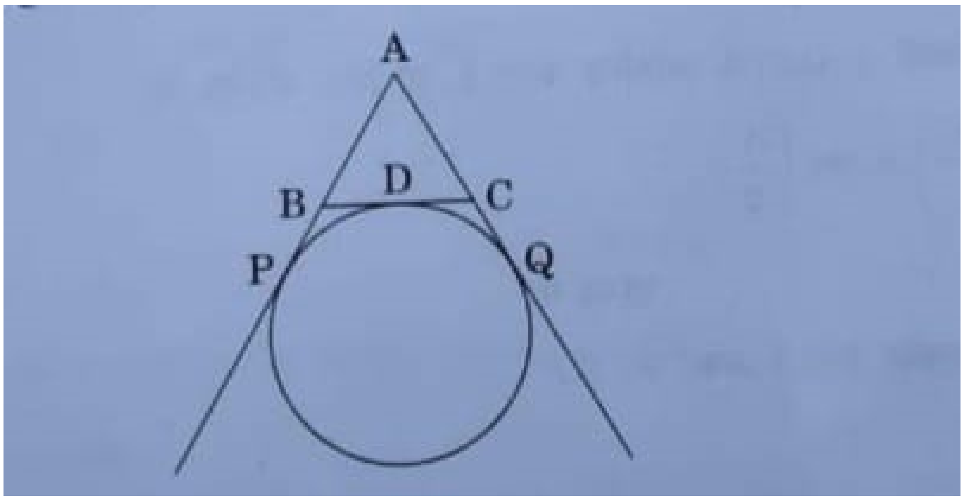



Figure 3.29:

# 3.5. 2019

## 3.5.1. 10

1. $ABC$ is a right triangle in which $\angle B = 90°$. If $AB = 8cm$ and $BC = 6cm$, find the diameter of the circle inscribed in the triangle.

2. Draw two concentric circles of radii $2cm$ and $5cm$. Take a point $P$ on the outer circle and construct a pair of tangents $PA$ and $PB$ to the smaller circle. Measure $PA$.

3. In Fig. 3.30, $PQ$ and $RS$ are two parallel tangents to a circle with centre $O$ and another tangent $AB$ with point of contact $C$ intersecting $PQ$ at $A$ and $RS$ at $B$. Prove that $\angle AOB = 90°$

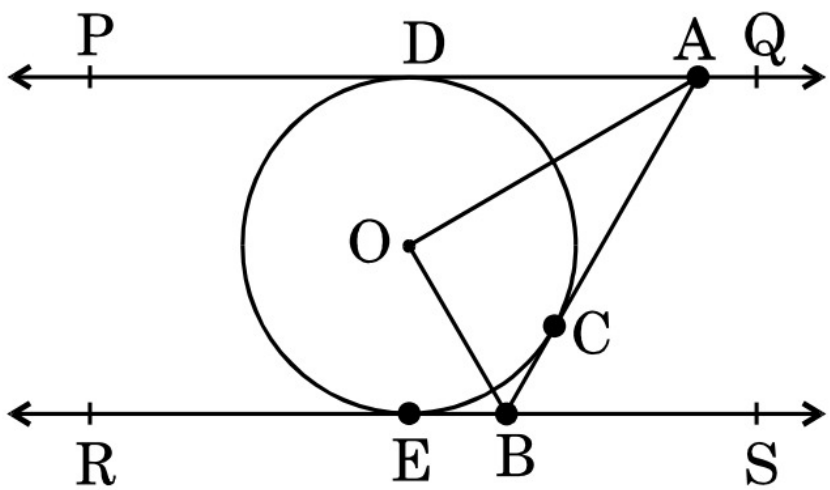


Figure 3.30: Tangent and Circle

4. In Fig. 3.31, PQ is a chord of length $8cm$ of a circle of radius $5cm$. The tangents at $P$ and $Q$ intersect at a point $T$. Find the length $TP$.

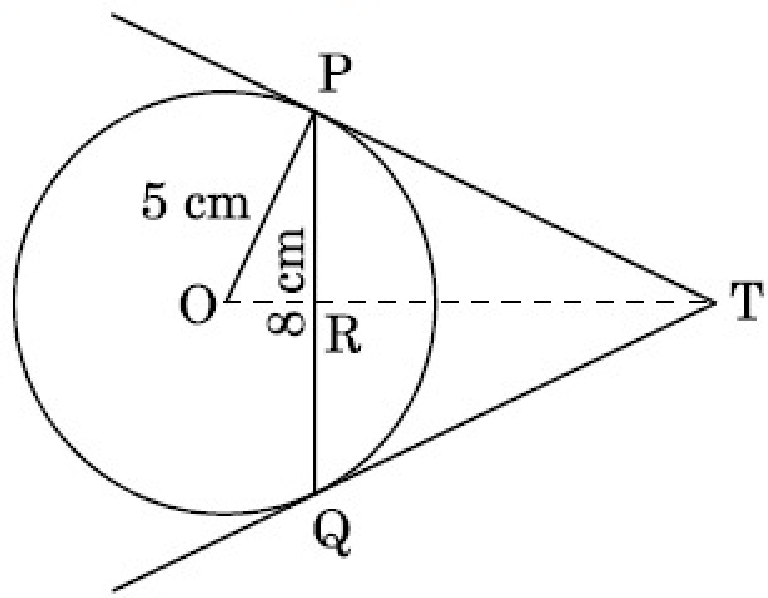



Figure 3.31: CIRCLE

5. A chord of a circle of radius $14cm$ subtends an angle of $60°$ at the centre. Find the area of the corresponding minor segment of the circle. (Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.732$)

6. In Fig. 3.32, $PQ$ is a chord of length $8cm$ of a circle of radius $5cm$ and centre **O**. The tangents at $P$ and $Q$ intersect at point $T$. Find the length of $TP$.

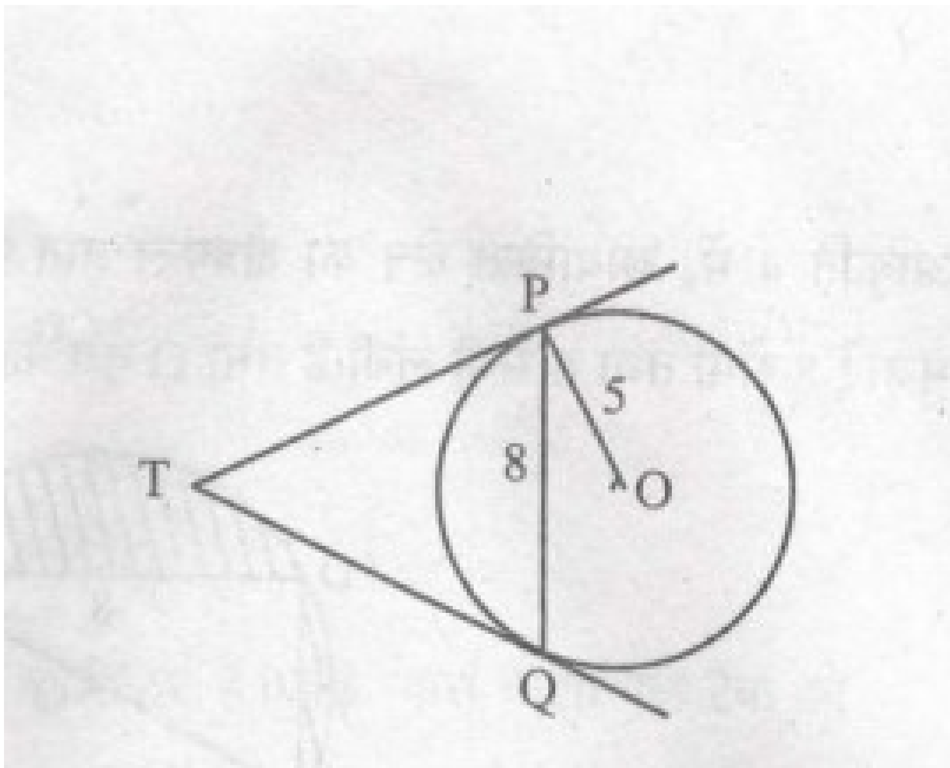



Figure 3.32: triangle45

7. Find the area of the segment shown in Fig. 3.33, if radius of the circle is $21cm$ and $\angle AOB = 120°$ Use $\left(n = \frac{22}{7}\right)$

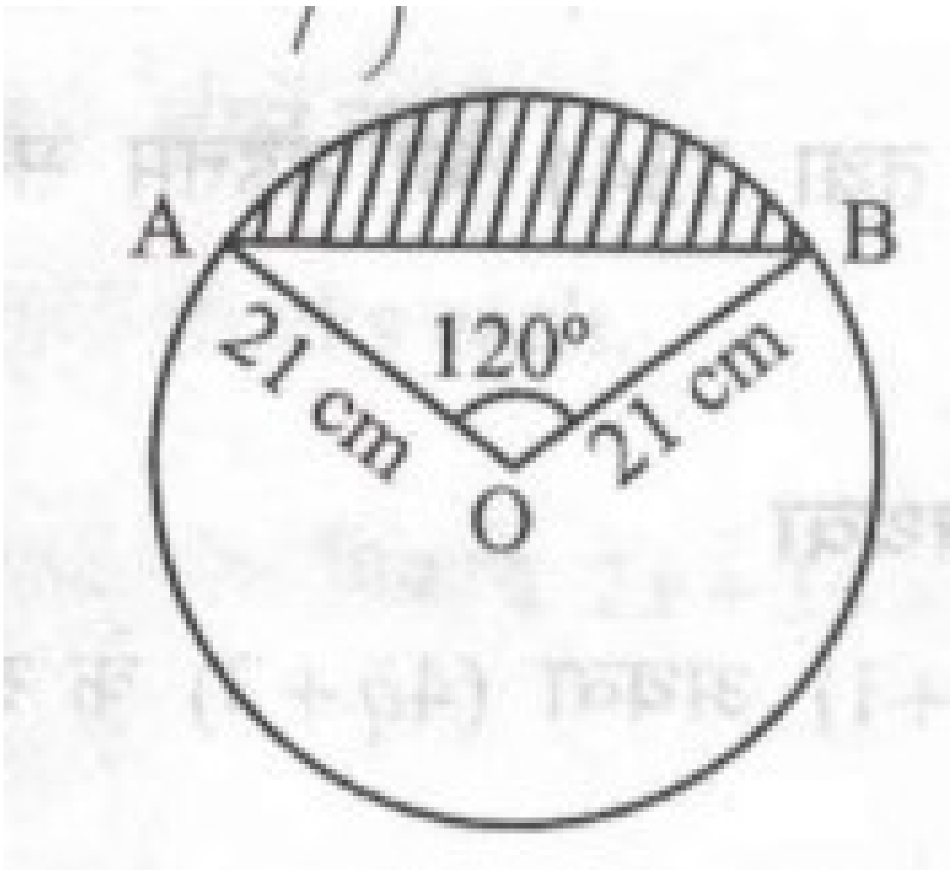



Figure 3.33: circles45

8. In Fig. 3.34, a circle is inscribed in a $\triangle ABC$ having sides $BC = 8cm$, $AB = 10cm$ and $AC = 12cm$. Find the lengths $BL$, $CM$ and $AN$.

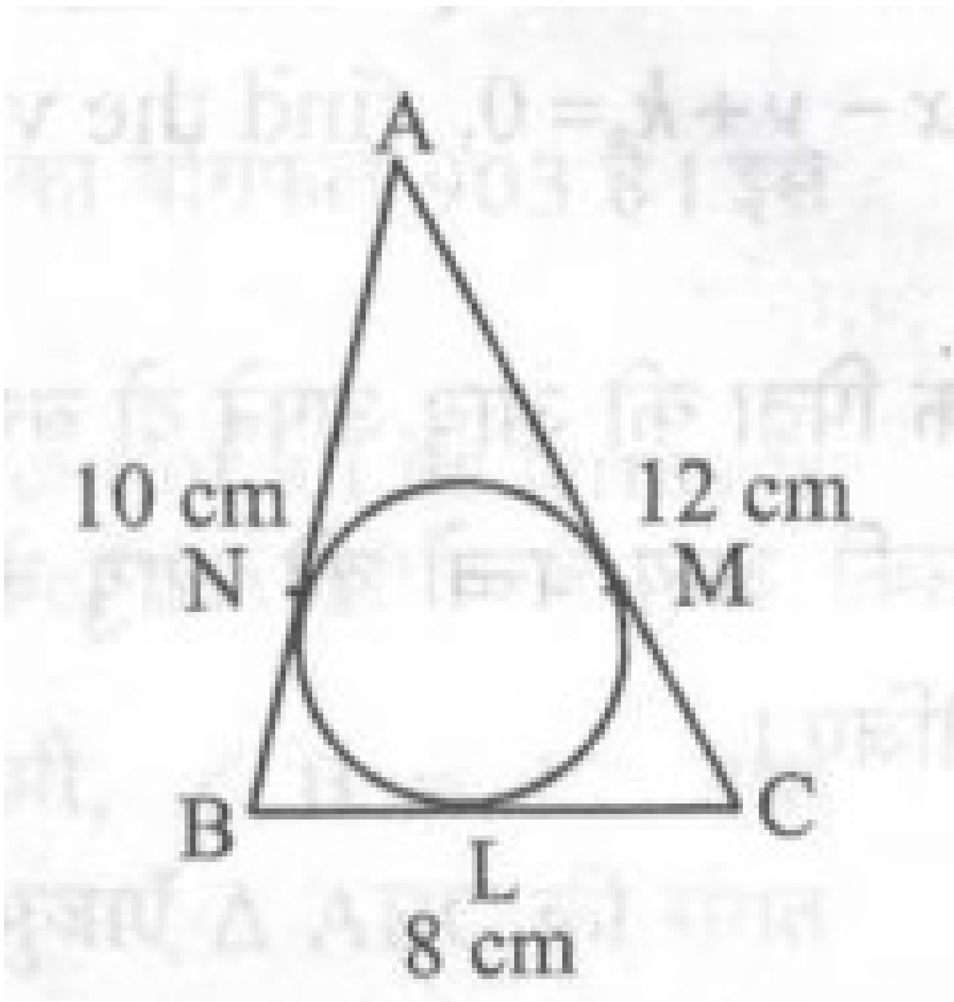



Figure 3.34: circles45

## 3.6. 2018

### 3.6.1. 10

1. Prove that the lengths of tangents drawn from an external point to a circle are equal.

### 3.6.2. 12

1. The sum of the perimeters of circle and a square is K, where K is some constant. Prove that the sum of their areas is least when the side of the square is twice the radius of the circle.

2. Find the equations of the tangent and normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

## 3.7. 2016

### 3.7.1. 10

1. In Fig 3.35, $PQ$ is a tangent at point $C$ to a circle with centre $O$. If $AB$ is a diameter and $\angle CAB = 30°$, Find $\angle PCA$.

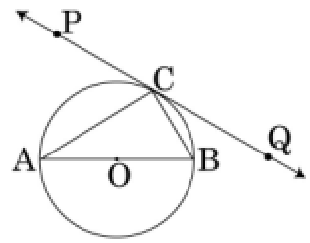



Figure 3.35:

2. In Fig 3.36, a quadrilateral $ABCD$ is drawn to circumscribe a circle, with centre $O$, in such a way that the sides $AB, BC, CD$ and $DA$ touch the circle at the points $P, Q, R$ and $S$ respectively. Prove that $AB + CD = BC + DA$.

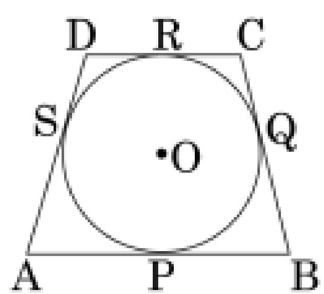



Figure 3.36:

3. In Fig 3.37, from an external point $P$, two tangents $PT$ and $PS$ are drawn to a circle with centre $O$ and radius $r$. If $OP = 2r$, show that $\angle OTS = \angle OST = 30°$.

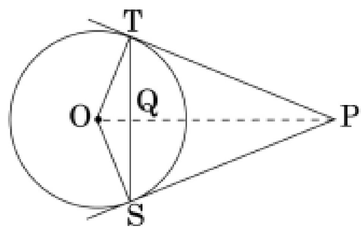



Figure 3.37:

4. In fig 3.38, $O$ is the centre of a circle such that diameter $AB = 13$ cm and $AC = 12$ cm. $BC$ is joined. Find the area of the shaded region. (Take $\pi = 3.14$)

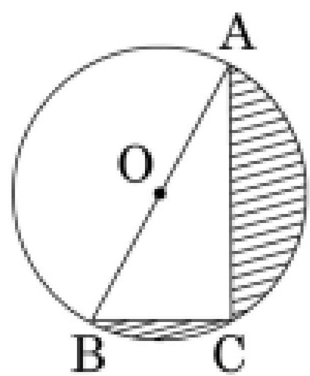



Figure 3.38:

5. In Fig 3.39 , two equal circles, with centres $O$ and $O'$, touch each other at $X.OO\prime$ produced meets the circle with centre $O'$ at $A$. $AC$ is tangent to the circle with centre $O$, at the point $C$. $O'D$ is perpendicular to $AC$. Find the value of $\frac{DO'}{CO'}$.

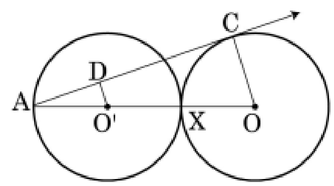



Figure 3.39:

6. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

# Chapter 4

# Intersection of Conics

## 4.1. 2022

1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

3. Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

4. If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

5. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

6. If the area of the region bounded bythe line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

8. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

9. Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration

## 4.2. 2021

### 4.2.1. 12

1. The point at which the normal to the curve

$$y = x + \frac{1}{x}, x > 0 \tag{4.1}$$

is perpendicular to the line

$$3x - 4y - 7 = 0 \tag{4.2}$$

(a) $\left(2, \frac{5}{2}\right)$

(b) $\left(\pm 2, \frac{5}{2}\right)$

(c) $\left(-\frac{1}{2}, \frac{5}{2}\right)$

(d) $\left(\frac{1}{2}, \frac{5}{2}\right)$

2. The points on the curve

$$\frac{x^2}{9} + \frac{y^2}{16} = 1 \tag{4.3}$$

at which the tangents are parallel to $y$-axis are:

(a) $(0, \pm 4)$

(b) $(\pm 4, 0)$

(c) $(\pm 3, 0)$

(d) $(0, \pm 3)$

3. For which value of m is the line

$$y = mx + 1 \tag{4.4}$$

a tangent to the curve

$$y^2 = 4x \tag{4.5}$$

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 3

## 4.3. 2019

### 4.3.1. 12

1. Using method of integration, find the area of the region enclosed between two circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

2. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $\left(-1, 4\right)$.

3. Find the point on the curve $y^2 = 4x$, which is nearest to the point $\left(2, -8\right)$.

4. Find the area of the region bounded by the curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$ using integration.

5. Find the area of the region

$$\left\{(x, y) : 0 \leq y \leq x^2, 0 \leq y \leq x + 2, -1 \leq x \leq 3\right\}.$$

6. Find the area of the region bounded by the curves

$$\begin{aligned}(x-1)^2 + y^2 &= 1\\ x^2 + y^2 &= 1\end{aligned}$$

using integration.

7. Find the equations of the tangent and the normal to the curve

$$y = \frac{x-7}{(x-2)(x-3)}$$

at the point where it cuts the x-axis.

## 4.4. 2018

### 4.4.1. 12

1. Find the point on the curve $y^2$=$4x$,which is nearest to the point $(2, -8)$.

2. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $(-1, 4)$.

3. Using integration, find the area of triangle ABC, whose vertices are A$(2, 5)$, B$(4, 7)$ and C$(6, 2)$.

4. Find the area of the region lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$

5. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by sides $x = 0, x = 4, y = 4$, and $y = 0$ into three equal parts.

6. Using integration, find the area of the triangle whose vertices are $(2, 3), (3, 5)$ and $(4, 4)$

7. Find the equation of tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$.and Also write the equation of the normal to the curve at the contact.

# Chapter 5

# Probability

## 5.1. 2021

### 5.1.1. 10

1. During the lockdown period,many families got bored of watching TV all the time.Out of these families, one family of 6 members decided to play a card game.17 cards numbered 1,2,3,4,..,17 are put in a box and mixed thoroughly.One card is drawn by one member at random and other family members bet for the chances of drawing the number either prime,odd or even etc.

Figure 5.1: Family of six

Based on the above,answer the following questions:

(a) The first member of the family draws a card at random and another member bets that it is an even prime number.What is the probability of his winning the bet?

   i. $\frac{2}{17}$

   ii. $\frac{3}{17}$

   iii. $\frac{1}{17}$

   iv. $\frac{4}{17}$

(b) The second member of the family draws a card at random and some other member bets that it is an even number. What is the probability of his winning the bet ?

   i. $\frac{7}{17}$

   ii. $\frac{8}{17}$

   iii. $\frac{9}{17}$

   iv. $\frac{10}{17}$

(c) What is the probability that the number on the card drawn at random is divisible by 5 ?

   i. $\frac{5}{17}$

   ii. $\frac{4}{17}$

   iii. $\frac{3}{17}$

   iv. $\frac{2}{17}$

(d) What is the probability that the number on the card drawn at random is a multiple of 3 ?

   i. $\frac{5}{17}$

   ii. $\frac{6}{17}$

   iii. $\frac{7}{17}$

   iv. $\frac{8}{17}$

2.

3. Two different coins are tossed simultaneously. Write all the possible outcomes.

4. A die is thrown once. Write the probability of getting a number less than 7.

5. If the probability of occurrence of event E,Pr $(E)$=0.99, what is the probability of non-occurrence of the event E,Pr $(notE)$?

6. (a) A bag contains 5 white balls and 7 red balls.A ball is drawn at random from the bag.What is the probability that it is either a white or a red ball?

   (b) Two coins are tossed together once.What is the probability of getting at least one head?

7. Cards marked with numbers 1,2,3,4, ..., 100 are placed in a bag and mixed together throughly. A card is randomly drawn from the bag.Find the probability that the numbers on the card is

   (a) an even number,

   (b) a 2-digit number,

   (c) a perfect square.

8. (a) How many outcomes are possible when three dice are thrown together?

   (b) if Pr $(E)$=0.015, then find Pr $(notE)$.

9. During summer break, Harish wanted to play with his friends but it was too hot outside, so he decided to play some indoor game with his friends. He collects 20 identical Icards and writes the numbers 1 to 20 on them (one number on one card). He puts them in a box. He and

his friends make a bet for the chances of drawing various cards out of the box. Ench was given a chance to tell the probability of picking one card out of the box.

Based on the above,answer the following questions:

(a) The probability that the number on the card drawn is an odd prime number,is

   i. $\frac{3}{5}$

   ii. $\frac{2}{5}$

   iii. $\frac{9}{20}$

   iv. $\frac{7}{20}$

(b) The probability that the number on the card drawn is a composite number is

   i. $\frac{11}{20}$

   ii. $\frac{3}{5}$

   iii. $\frac{4}{5}$

   iv. $\frac{1}{2}$

(c) The probability that the number on the card drawn is a multiple of 3,6 and 9 is

   i. $\frac{1}{20}$

   ii. $\frac{1}{20}$

   iii. $\frac{3}{20}$

   iv. 0

(d) The probability that the number on the card drawn is a multiple of 3 and 7is

  i. $\frac{3}{10}$

  ii. $\frac{1}{10}$

  iii. 0

  iv. $\frac{2}{5}$

(e) If all cards having odd numbers written on them are removed from the box and then one card is drawn from the remaining cards, the probability of getting a card having a prime number is

  i. $\frac{1}{20}$

  ii. $\frac{1}{10}$

  iii. 0

  iv. $\frac{1}{5}$

10. (a) In a single throw of a pair of dice,find the probability that both dice have the same number.

    (b) A card is drawn from a well-shuffled pack of 52 cards.Find the probability that it is not an ace.

### 5.1.2. 12

1. The probability of solving a specific question independently by $A$ and $B$ are $\frac{1}{3}$ and $\frac{1}{5}$ respectively.If both try to solve the question independently, the probability that the question is solved is

(a) $\frac{7}{15}$

(b) $\frac{8}{15}$

(c) $\frac{2}{15}$

(d) $\frac{14}{15}$

2. From a pack of 52 cards, 3 cards are drawn at random (without replacement).The probability that they are two red cards and one black card is__________.

3. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

4. Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

5. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement), both of which are found to be red. Find the probability that these two balls are drawn from the second bag.

6. An unbiased die is thrown. What is the probability of getting an odd number or a multiple of 3 ?

   (a) $\frac{3}{4}$

   (b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{1}{3}$

7. A card is drawn from an ordinary pack of 52 cards and a gambler bets that it is a heart or a king card. What are the odds against his winning this bet ?

   (a) 4 : 9

   (b) 1 : 4

   (c) 4 : 1

   (d) 9 : 4

8. In a lottery of 25 tickets, numbered 1 to 25, two tickets are drawn simultaneously. Find the probability that none of the tickets has prime number.

9. If $E_1$ and $E_2$ are two events, where $E_1$ is a subset of $E_2$, then evaluate $P(E_2 \mid E_1)$.

10. Two dice are thrown simultaneously. Find the probability of getting a multiple of 3 on one dice and a multiple of 2 on the other dice.

11. An urn contains 4 white, 7 green and 9 blue balls. If two balls are drawn at random, find the probability that the drawn balls are of the same colour.

## 5.2. 2023

### 5.2.1. 10

1. Probability of happining of an event is denoted by $p$ and probability of non-happening of the event is denoted by $q$. Relation between $p$ and $q$ is

   (a) $p$+$q$=1

   (b) $p$=1, $q$=1

   (c) $p$=$q$-1

   (d) $p$+$q$+1=0

2. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought ?

   (a) 40

   (b) 240

   (c) 480

   (d) 750

3. In a group of 20 people, 5 can't swim. If one person is selected at random, then the probability that he/sh can swim, is

   (a) $\frac{3}{4}$

   (b) $\frac{1}{3}$

(c) 1

(d) $\frac{1}{4}$

4. A bag contain 4 red, 3 blue and 2 yellow balls. One ball is drawn at random from the bag. Find the probability that drawn ball is

   (a) red

   (b) yellow

5. A bag contain 100 cards numbered 1 to 100.Acard is drawn at random from the b. What is the probability that the number on the card is a perfect cube ?

   (a) $\frac{1}{20}$

   (b) $\frac{3}{50}$

   (c) $\frac{1}{25}$

   (d) $\frac{7}{100}$

6. If three coins are tossed simultaneously, what is the probability of getting a most one trail ?

   (a) $\frac{3}{8}$

   (b) $\frac{4}{8}$

   (c) $\frac{5}{8}$

   (d) $\frac{7}{8}$

7. Two dics are thrown together. The probability of getting the difference of numbers on their upper faces equals to 3 is :

(a) $\frac{1}{9}$

(b) $\frac{2}{9}$

(c) $\frac{1}{6}$

(d) $\frac{1}{12}$

8. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that the card drawn is not an ace is :

(a) $\frac{1}{13}$

(b) $\frac{9}{13}$

(c) $\frac{4}{13}$

(d) $\frac{12}{13}$

9. **Assertion (A) :** The probability that a leap year has 53 Students is $\frac{2}{7}$.

**Reason (R) :** The probability that a non-leap year has 53 Sundays is $\frac{5}{7}$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

### 5.2.2. 12

1. If A and B are two events such that

$$\Pr(A|B) = 2 \times \Pr(B|A)\Pr(A) + \Pr(B) = \frac{2}{3} \tag{5.1}$$

   then $\Pr(B)$ is equal to

   (a) $\frac{2}{9}$

   (b) $\frac{7}{9}$

   (c) $\frac{4}{9}$

   (d) $\frac{5}{9}$

2. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.

   (b) A and B throw a die alternately till one of them gets '6' and wins the game. Find their respective probabilities of winning, if A starts the game first.

3. Recent studies suggest that roughly 12% of the world population is left handed.

   Depending upon the parents, the chances of having a left handed child are as follows :

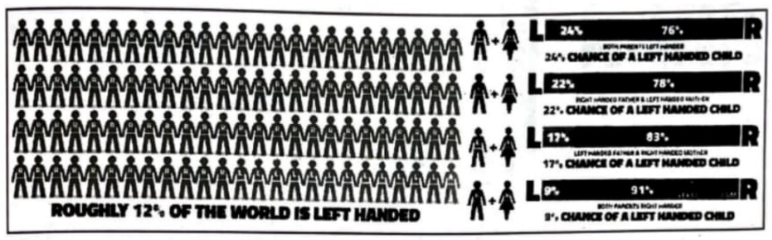



Figure 1: chance of left hand, depending upon parents

Figure 5.2: chance of left hand, depending upon parents

(a) When both father and mother are left handed : Chances of left handed child is 24%.

(b) When father is right handed and mother is left handed : Chances of left handed child is 22%.

(c) when father is left handed and mother is right handed : Chances of left handed child is 17%.

(d) When both father nd mother are right handed : Chances of left handed child is 9%.

Assuming that $\Pr(A) = \Pr(B) = \Pr(C) = \Pr(D) = \frac{1}{4}$ and L denotes the event that child is left handed. Based on the above information, answer the following questions :

(a) Find $\Pr(L|C)$

(b) Find $\Pr\left(\overline{L}|A\right)$

(c) Find $\Pr(A|L)$

(d) Find the probability that a randomly selected child is left handed given that exactly one of the parent is left handed.

# 5.3. 2022

## 5.3.1. 10

1. Two dice are thrown simultaneously. The probability that the sum of two numbers appearing on the top of the dice is less than 12, is

   (a) $\frac{1}{36}$

   (b) $\frac{35}{36}$

   (c) 0

   (d) 1

2. A jar contains 18 marbles. Some are red and others are yellow. If a marble is drawn at random from the jar, the probability that it is red is $\frac{2}{3}$. Find the number of yellow marbles in the jar.

# 5.4. 2022

## 5.4.1. 12

1. Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A|B) = \frac{3}{4}$. Find the value of $P(B|A)$.

2. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variables X denotes the number of red balls. Find the probabillity distribution of X.

3. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.

4. Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved.

5. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number apprearing on at least one die is 3.

6. At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. such a coin is unbaised with equal probabilities of getting head and tail Fig. 5.3 .

Figure 5.3: Toss before the match

Based on the above information, answer the following question:

(a) If such a coin is tossed 2 times, then find the probability distibution of numbers of tails.

(b) Find the probability of getting at least one head in three tosses of such a coin.

7. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards.

8. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on at least one die.

9. The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probabillity that the target is hit.

10. A shopkeeper sells three types of flower seeds $A_1$ , $A_1$ , $A_3$. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germinaton rates of the three types of seeds are 45%, 60% and 35% respectively Fig. 5.4.
Based on the above information :

    (a) Calculate the probability that a randomly chosen seed will germinate.

    (b) Calculate the probability that the seed is of type $A_2$, given that a randomly choosen seed germinates.

Figure 5.4: Three types of flowers

11. Three friends A, B and C got their photograph clicked. Find the probability that B is standing at the central position, given that A is standing at the left corner.

12. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are alloted points 9 to 1 in sequential order moving outwards.Archer A is likely to earn 10 points with a probability of 0.8 and Archer B is likely the earn 10 points with a probability of 0.9 Fig. 5.5.

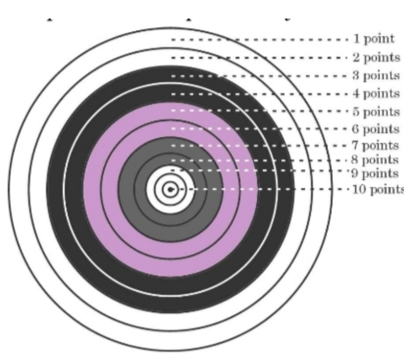



Figure 5.5: centermost ring

Based on the above innformation, answer the following questions :

(a) exactly one of them earns 10 points .

(b) both of them earn 10 point.

13. Event A and B are such that

$$P(A) = \frac{1}{2}, P(B) = \frac{7}{12} \tag{5.2}$$

and

$$P(\bar{A} \cup \bar{B}) = \frac{1}{4} \tag{5.3}$$

Find whether the events A and B are independent or not.

14. A box $B_1$ contain 1 white ball and 3 red balls. Another box $B_2$ contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes $B_1$ and $B_2$, then find the probability that the two balls drawn are of the same colour.

15. Let X be random variable which assumes values $x_1$, $x_2$, $x_3$, $x_4$ such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4). \tag{5.4}$$

Find the probability distribution of X.

16. There are two boxes, namely box-I and box-II. Box-I contains 3 red

and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes , is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.

17. In a toss of three different coins, find the probability of comming up of three heads, if it is known that at least one head comes up.

18. A laboratory blood text is 98% effective in detecting a certain disease when it is fact, present. However, the text also yeilds a false positive result for 0.4% of the healthy person tested. From a large population, it is given that 0.2% of the population actually has the diseases.
Based on the above, answer the following questtion :

    (a) one person, from the population, is taken at random and given the test. Find the probabiliy of his getting a positive test result.

    (b) what is the probability that the person actually has the disease, given that his test result is positive ?

19. Two cards are drawn from a well-shuffled pack of playing cards one-by-one with replacement. The probability that the first card is a king and the second card is a queen is

    (a) $\frac{1}{13} + \frac{1}{13}$

    (b) $\frac{1}{13} \times \frac{4}{51}$

    (c) $\frac{4}{52} \times \frac{3}{51}$

    (d) $\frac{1}{13} \times \frac{1}{13}$

20. For two events A and B if P(A) $= \frac{4}{10}, PB = \frac{8}{10}$ and $P(B|A) = \frac{6}{10}$ then find $P(A \cup B)$.

21. Bag I contain 4 red and 3 black balls. Bag II contains 3 red and 5 black balls. One of two bags is selected at random and a ball is drawn from the bag, which if found to be red. Find the probability that the ball is drawn from bag II.

22. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces and hence find its mean.

23. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively . If both try to solve the question independently, the probability that the question is solved is

    (a) $\frac{7}{15}$

    (b) $\frac{8}{15}$

    (c) $\frac{2}{15}$

    (d) $\frac{14}{15}$

24. A card is picked at random from a pack of 52 playing cards. Given that the picked up card is a queen, the probability of it being a queen of spades is _____.

25. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first

one in the bag. Find the probability distribution of the number of even numbers on the ticket.

26. Find the probability distribution of the numbers of successes in two tosses of a die, when a success is defined as number greater than 5.

27. Ten cartoons are taken at random from an automatic packing machine. The mean net weight of the ten carton is 11.8 kg and standard deviation is 0.15 kg. Does the sample mean differ significantly from the intended mean of 12 kg ? [Given that for d.f. = 9, $t_{0.05} = 2.26$]

28. A Coin is tossed twice. The following table5.2 shows the probability distribution of numbers of tails:

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | K | 6K | 9K |

Table 5.2: Table shows the probability distribution of numbers of tails

(a) Find the value of $K$.

(b) Is the coin tossed biased or unbaised? Justify your answer.

29. If X is a random variable with probability distribution as given below 5.4:

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | K | 4K | K |

Table 5.4: table shows the proability distribution

The value of K and the mean of the distribution respectively are

(a) $\frac{1}{7}, 1$

(b) $\frac{1}{6}, 2$

(c) $\frac{1}{6}, 1$

(d) $1, \frac{1}{6}$

30. The random variable X has a probability function P($x$) as defined below, where K is some number :

$$P(X) = \begin{cases} K, & \text{if } x = 0 \\ 2K, & \text{if } x = 1 \\ 3K, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases} \tag{5.5}$$

Find:

(a) The value of $K$.

(b) $P(X < 2), P(X \leq 2), P(X \geq 2)$.

31. Two rotten apples are mixed with 8 fresh apples. Find the probability distribution of number of rotten apples, if two apples are drawn at random, one-by-one without replacement.

32. A die is thrown twice. What is the probability that

(i) 5 will come up at least once, and

(ii) 5 will not come up either time ?

33. Let $A$ and $B$ be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A/B) = \frac{3}{4}$. Find the value of $P(B/A)$.

34. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable $X$ denotes the number of red balls. Find the probability distribution of $X$.

35. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.

36. Probabilities of $A$ and $B$ solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved.

37. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3.

38. In Fig. 5.6,At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.

Figure 5.6: Tossing a coin

Based on the above information, answer the following questions :

(a) If such a coin is tossed 2 times, then find the probability distribution of number of tails.

(b) Find the probability of getting at least one head in three tosses of such a coin.

39. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards.

40. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die.

41. In Fig. 5.7, A shopkeeper sells three types of flower seeds $A1$, $A2$, $A3$. They are sold in the form of a mixture, where the proportions of these

seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Figure 5.7: Three Types of Flower Seeds

Based on the above information:

(a) Calculate the probability that a randomly chosen seed will germinate;

(b) Calculate the probability that the seed is of type $A2$, given that a randomly chosen seed germinates.

42. Three friends $A$, $B$ and $C$ got their photograph clicked. Find the probability that $B$ is standing at the central position, given that $A$ is standing at the left corner.

43. In Fig. 5.8 A coin is tossed twice. The following table shows the probability distribution of number of tails :

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | K | 6K | 9K |

Figure 5.8: Probability Distribution of number of tails

(a) Find the value of $K$.

(b) Is the coin tossed biased or unbiased ? Justify your answer.

44. In Fig. 5.9 In a game of Archery, each ring of the Archery target is valued. The centre most ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.

Archer A is likely to earn 10 points with a probability of 08 and Archer $B$ is likely the earn 10 points with a probability of 09.

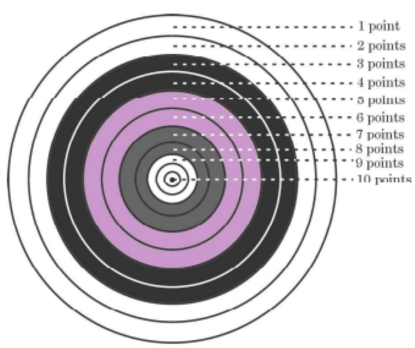



Figure 5.9: Ring of the Archery Target

Based on the above information, answer the following questions : If both of them hit the Archery target, then find the probability that

(a) exactly one of them earns 10 points.

(b) both of them earn 10 points.

45. (a) Events $A$ and $B$ are such that P(A) $= \frac{1}{2}$ , P(B) $= \frac{7}{12}$ and $P(\overline{A} \cup \overline{B}) = \frac{1}{4}$ Find whether the events $A$ and $B$ are independent or not.

(b) A box $B_1$ contains 1 white ball and 3 red balls.Another box $B_2$ contains 2 white balls and 3 red balls.If one ball is drawn at random from each of the boxes $B_1$ and $B_2$ then find the probability that the two balls drawn are of the same colour.

46. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes, is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.

47. In a toss of three different coins, find the probability of coming up of three heads, if it is known that at least one head comes up.

48. Two rotten apples are mixed with 8 fresh apples. Find the probability distribution of number of rotten apples, if two apples are drawn at random, one-by-one without replacement.

49. A laboratory blood test is 98% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 04% of the healthy person tested. From a large population, it is given that 0·2% of the population actually has the disease. Based on the above, answer the following questions :

(a) One person, from the population, is taken at random and given the test. Find the probability of his getting a positive test result.

(b) What is the probability that the person actually has the disease, given that his test result is positive ?

50. Two cards are drawn from a well-shuffled pack of playing cards one-by-one with replacement. The probability that the first card is a king and the second card is a queen is

(a) $\frac{1}{13} + \frac{1}{13}$

(b) $\frac{1}{13} \times \frac{4}{51}$

(c) $\frac{4}{52} \times \frac{3}{51}$

(d) $\frac{1}{13} \times \frac{1}{13}$

51. In Fig. 5.10 If $X$ is a random variable with probability distribution as given below :

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | k | 4k | k |

Figure 5.10: Probability Distribution

The value of $k$ and the mean of the distribution respectively are

(a) $\frac{1}{7}$,1

(b) $\frac{1}{6}$,2

(c) $\frac{1}{6}$,1

(d) $\frac{1}{6}$

52. For two events $A$ and $B$ if P(A) = $\frac{4}{10}$, P(B) = $\frac{8}{10}$ and $P(B \mid A)$=$\frac{6}{10}$, then find $P(A \cup B)$.

53. Bag I contains 4 red and 3 black balls. Bag II contains 3 red and 5 black balls. One of the two bags is selected at random and a ball is drawn from the bag, which is found to be red. Find the probability that the ball is drawn from Bag II.

54. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces and hence find its mean.

55. The probability of solving a specific question independently by $A$ and $B$ are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the question independently, the probability that the question is solved is

(a) $\frac{7}{15}$

(b) $\frac{8}{15}$

(c) $\frac{2}{15}$

(d) $\frac{14}{15}$

56. A card is picked at random from a pack of 52 playing cards. Given that the picked up card is a queen, the probability of it being a queen of spades is ?

57. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

58. Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

59. The random variable $X$ has a probability function $P(x)$ as defined below, where $k$ is some number :

$$p(x) = \begin{cases} k, & \text{if } x = 0, \\ 2k, & \text{if } x = 1, \\ 3k, & \text{if } x = 2, \\ 0, & \text{otherwise.} \end{cases} \tag{5.6}$$

Find :

(a) The value of $k$

(b) $P(X < 2)$, $P(X \leq 2)$, $P(X \geq 2)$

60. Consider the following hypothesis :

$$H0 : \mu = 35 \tag{5.7}$$

$$H1 : \mu \neq 35 \tag{5.8}$$

A sample of 81 items is taken whose mean is 375 and the standard deviation is 5. Test the hypothesis at 5% level of significance.

[Given : Critical value of $Z$ for a two-tailed test at 5% level of significance is 1.96]

61. In Fig. 5.11 Fit a straight line trend by the method of least squares and find the trend value for the year 2008 for the following data :

| Year | Production (in lakh tonnes) |
|---|---|
| 2001 | 30 |
| 2002 | 35 |
| 2003 | 36 |
| 2004 | 32 |
| 2005 | 37 |
| 2006 | 40 |
| 2007 | 36 |

Figure 5.11: Years and Production

## 5.5. 2020

### 5.5.1. 10

1. If the probability of an event $E$ happening is 0.023,then $P(\overline{E}) =$ __________.

2. Read the following passage and answer the questions given at the end:

   **Diwali Fair**

   A game in booth Diwali Fair involves using a spinner first.Then,if the spinner stops on an even member,the player is allowed to pick a marble from a bag.The spinner and the marbles in the bag are represented in Fig. 5.12.

Prizes are given,when a block marble is picked.Shooter plays the game once.

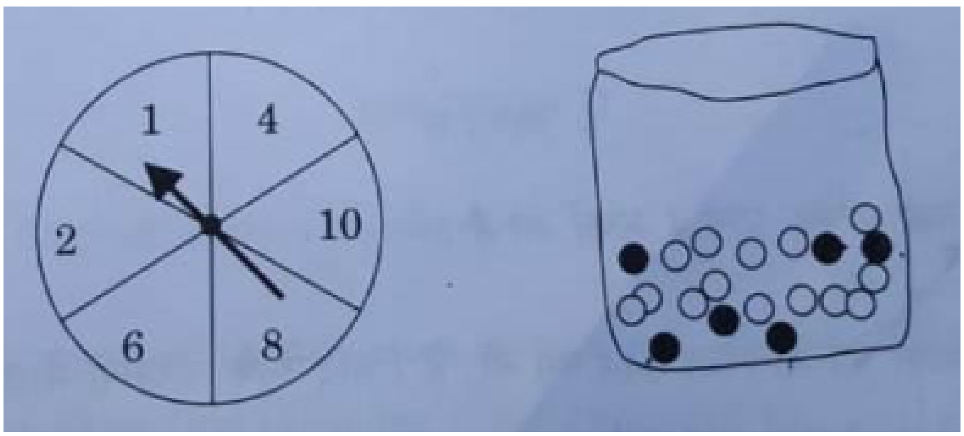



Figure 5.12: A bag contains of marbles

3. what is the probability that she will be allowed to pick a mobile from the bag?

4. suppose she is allowed to pick a marble from the bag,what if the probability of getting a prize,when it is given that the bag contains 20 balls out of which 6 came black?

## 5.5.2. 12

1. A fair dice is thrown two times.Find the probability distribution of the number of sixes. Also determine the mean of the number of sixes.

2. A card from a pack of 52 cards is lost.From the remaining cards of the pack ,two cards are drawn randomly one-by-one without replacement

and are found to be both kings.Find the probability of the last card being a king.

# 5.6. 2019

## 5.6.1. 12

1. If $P(notA) = 0.7$, $P(B) = 0.7$ and $P(B \mid A) = 0.5$, then find $P(A \mid B)$.

2. A coin is tossed 5 times. What is the probability of getting

   (i) 3 heads

   (ii) at most 3 heads

3. Find the probability distribution of $X$, the number of heads in a simultaneous toss of two coins.

4. There are three coins. One is a two-headed coin, another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows heads, what is the probability that it is the two-headed coin ?

5. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement) both of which are found to be red. Find the probability that the balls are drawn from the second bag.

6. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

7. The probabilities of solving a specific problem independently by $A$ and $B$ are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.

8. There are three coins. One is a coin having tails on both faces, another is a biased coin that comes up tails 70% of the time and the third is an unbiased coin. One of the coins is chosen at random and tossed, it shows tail. Find the probability that it was a coin with tail on both the faces.

9. Let $X$ be a random variable which assumes values $x_1, x_2, x_3, x_4$ such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4).$$

Find the probability distribution of $X$.

10. Mother, father and son line up at random for a family photo. If $A$ and $B$ are two events given by $A$ = Son on one end, $B$ = Father in the middle, find $P(B \mid A)$.

11. A coin is tossed 5 times. Find the probability of getting

    i. at least 4 heads

    ii. at most 4 heads.

12. There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and $'n'$ black balls. One of the two boxes, box I and box I is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box I is $\frac{3}{5}$,find the value of $'n'$.

13. Find the mean and variance of the random variable $X$ which denotes the number of doublets in four throws of a pair of dice.

14. A bag contains 5 red and 3 black balls and another bag contain 2 red and 6 black balls. Two balls are drawn at random (without replacement) from one of the bags an both are found to be red. Find the probability that balls are drawn from the first bag.

15. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, two cards are drawn at random (without replacement) and both are found to be spades. Find the probability of the lost card being a spade.

16. In answering a question on a multiple choice questions with four choices in each question out of which only one is correct a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability that he copies is also $\frac{1}{4}$. The probability that the answer is correct given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to the question given that he correctly answered it.

17. 12 cards numbered 1 to 12 (one number on one card), are placed in a

box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is greater than 5, find the probability that the card bears an odd number.

18. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

19. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

20. An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are 0.3, 0.05 and 0.02 respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist ?

21. If $P(A) = 0.6, P(B) = 0.5$ and $P\left(\frac{B}{A}\right) = 0.4$ find $P(A \cup B)$ and $P\left(\frac{A}{B}\right)$.

22. Four cards are drawn one by one with replacement from a well-shuffled deck of playing cards. Find the probability that at least three cards are diamonds.

23. The Probability of two students $A$ and $B$ coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$ respectively, Assuming that the events '$A$ coming on time' and '$B$ coming on time' are independent find the probability of only

one of them coming to school on time.

24. If $A$ and $B$ are independent events with $P(A) = \dfrac{3}{7}$ and $P(B) = \dfrac{2}{5}$, then find $P(A' \cap B')$

# 5.7. 2019

## 5.7.1. 10

1. A bag contains 15 balls, out of which some are white and the others are black. If the probability of drawing a black ball at random from the bag is $\frac{2}{3}$, then find how many white balls are there in the bag.

2. A card is drawn at random from a pack of 52 playing cards. Find the probability of drawing a card which is neither a spade nor a king.

3. A die is thrown once. Find the probability of getting

    (a) a prime number

    (b) an odd number.

4. Three different coins are tossed simultaneously. Find the probability of getting exactly one head.

5. A die is thrown twice. Find the probability that

    (a) 5 will come up at least once .

    (b) 5 will not come up either time .

6. The probability of selecting a blue marble at random from a jar that contains only blue, black and green marbles is $\frac{1}{5}$. The probability of selecting a black marble at random from the same jar is $\frac{1}{4}$. If the jar contains 11 green marbles, find the total number of marbles in the jar.

7. A die is thrown once. Find the probability of getting

   (a) a composite number,

   (b) a prime number.

8. Cards numbered 7 to 40 were put in a box. Poonam selects a card at random. What is the probability that Poonam selects a card which is a multiple of 7 ?

9. A child has a die whose 6 faces show the letters given below :

$$\boxed{\text{A}} \quad \boxed{\text{B}} \quad \boxed{\text{C}} \quad \boxed{\text{A}} \quad \boxed{\text{A}} \quad \boxed{\text{B}}$$

10. Cards marked with numbers 5 to 50 (one number on one card) are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that the number on the card taken out is

    (a) a prime number less than 10,

    (b) a number which is a perfect square.

11. Find the probability that a number selected at random from the numbers $3, 4, 4, 4, 5, 5, 6, 6, 6, 7$ will be their mean.

12. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

13. A die is thrown once. Find the probability of getting a number.

    (a) which is a prime number

    (b) lies between 2 and 6.

## 5.8. 2018

### 5.8.1. 10

1. Two different dice are tossed together. Find the probability:

    (i) of getting a doublet

    (ii) of getting a sum 10, of the numbers on the two dice.

2. An integer is choosen at random between 1 and 100. Find the probability that it is:

    (i) divisible by 8.

    (ii) not divisible by 8.

### 5.8.2. 12

1. The total cost $C(x)$ associated with the production of $x$ units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the

material cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

2. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

3. Two numbers are selected at random (*without replacement*) from the first five positive integers. Let $X$ denote the larger of the two numbers obtained. Find the mean and variance of $X$.

4. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ?

5. Mother, father and son line up at random for a family photo. If $A$ and $B$ are two events given by $A$ = Son on one end, $B$ = Father in the middle, find $P\left(\frac{B}{A}\right)$.

6. Let $X$ be a random variable which assumes values $x_1, x_2, x_3, x_4$such that

$$2P\left(X = x_1\right) = 3P\left(X = x_2\right) = P\left(X = x_3\right) = 5P\left(X = x_4\right).$$

Find the probability distribution of $X$.

7. A coin is tossed 5 times. Find the probability of getting $(i)$ at least 4 heads, and $(ii)$ at most 4 heads.

8. There are two boxes $I$ and $II$. Box $I$ contains 3 red and 6 black balls. Box $II$ contains 5 red and $'n'$ black balls. One of the two boxes, box $I$ and box $II$ is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box $II$ is $\frac{3}{5}$, find the value of $'n'$.

9. Find the mean and variance of the random variable $X$ which denotes the number of doublets in four throws of a pair of dice.

10. The probabilities of solving a specific problem independently by $A$ and $B$ are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.

11. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

12. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for quiz competition. Find the probability that 2 boys and 2 girls are selected .

13. An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver are 0.3, 0.5 and 0.02 respectively. One of the insured person meets with an accident. what is the probability that he is a cyclist ?

14. 12 cards numbered 1 to 12 (one number on one card), are placed in a box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that number on the drawn card is greater than 5, find the probability that the card bears an odd number.

15. There are three coins. One is a coin having tails on both faces, another is a biased coin that comes up tail 70% of the time and the third is an unbiased coin. One of the coins is chosen at random and tossed, it shows tail. Find the probability that it was a coin with tail on both the faces.

16. If $A$ and $B$ are independent events with $P(A) = \frac{3}{7}$ and $P(B) = \frac{2}{5}$, then find $P(A' \bigcap B')$.

17. If $P(A) = 0.6$, $P(B) = 0.5$ and $P(B \mid A) = 0.4$, find $P(A \cup B)$ and $P(A \mid B)$.

18. The probability of two students $A$ and $B$ coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively. Assuming that the events '$A$ coming on time' and '$B$ coming on time' are independent, find the probability of only one of them coming to school on time.

19. A bag contains 5 red and 3 black balls and another bag contains 2 red and 6 black balls. Two balls are drawn at random (without replacement) from one of the bags and both are found to be red. Find the probability that balls are drawn from the first bag.

20. In answering a question on a multiple choice questions test with four choices in each questions, out of which only one is correct, a student

either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability the he copies is also $\frac{1}{4}$. The probability that the answer is correct, given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to the question, given that he correctly answered it.

21. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, two cards are drawn at random (without replacement) and both are found to be spades. Find the probability of the lost card being a spade.

22. Four cards are drawn one by one with replacement from a well-shuffled deck of playing cards. Find the probability that at least three cards are of diamonds.

23. If $P\,(notA) = 0.7, P\,(B) = 0.7$ and $P\,(B \mid A) = 0.5$, then find $P\,(A \mid B)$.

24. A coin is tossed 5 times. What is the probability of getting

    (i) 3 heads,

    (ii) at most 3 heads ?

25. Find the probability distribution of $X$, the number of heads in a simultaneous toss of two coins.

26. There are three coins. One is a two-headed coin, another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows heads, what is the probability that it is the two-headed coin ?

27. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A ?

28. The random variable $X$ has a probability distribution $P(X)$ of the following form, where $k$ is some number:

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of $k$.

29. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of kings.

30. A die marked $1, 2, 3$ in red and $4, 5, 6$ in green is tossed. Let $A$ be the event "number is even" and $B$ be the event "number is marked red." Find whether the events $A$ and $B$ are independent or not.

31. A die is thrown 6 times. If "getting an odd number" is a "success",

what is the probability of: $(i)$ 5 successes? $(ii)$ at most 5 successes?

## 5.9. 2017

### 5.9.1. 10

### 5.9.2. 12

1. A die,whose faces are marked $1, 2, 3$ in red and $4, 5, 6$ in green, is tossed . Let A be the event "number obtained is even" and B be the event "number obtained is red" . Find if A and B are independent events.

2. There are 4 cards numbered $1, 3, 5$ and 7, one number on one card . Two cards are drawn at random without replacement . Let $X$ denote the sum of the numbers on the two drawn cards . Find the mean and variance of $X$ .

3. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular . Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination . At the end of the year, one student is chosen at random from the school and he was found to have an A grade . What is the probability that the student has 100% attendance ? Is regularity required only in school ? Justify your answer .

## 5.10. 2016

### 5.10.1. 10

1. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen.

2. There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card

   (i) is divisible by 9 and is a perfect square

   (ii) is a prime number greater than 80

3. A number $x$ is selected at random from the numbers $1, 2, 3$ and 4. Another number $y$ is selected at random from the numbers $1, 4, 9$ and 16. Find the probability that product of $x$ and $y$ is less than 16.

4. From a pack of 52 playing cards, Jacks, Queens and Kings of red colour are removed. From the remaining, a card is drawn at random. Find the probability that drawn card is :

   (i) a black King

   (ii) a card of red colour

   (iii) a card of black colour

5. Three different coins are tossed together. Find the probability of getting

   (i) exactly two heads

   (ii) at least two heads

   (iii) at least two tails.

### 5.10.2. 12

1. There are two bags $A$ and $B$. Bag $A$ contains 3 white and 4 red balls whereas bag $B$ contains 4 white and 3 red balls. Three balls are drawn at random (without replacement) from one of the bags and are found to be two white and one red. Find the probability that these were drawn from bag $B$.

2. Three numbers are selected at random (without replacement) from first six positive integers. If $X$ denotes the smallest of the three numbers obtained, find the probability distribution of $X$. Also find the mean and variance of the distribution.

3. A bag $X$ contains 4 white balls and 2 black balls, while another bag $Y$ contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag $Y$.

4. $A$ and $B$ throw a pair of dice alternately, till one of them gets a total of

10 and wins the game. Find their respective probabilities of winning, if $A$ starts first.

5. Three numbers are selected at random (without replacement) from first six positive integers. Let $X$ denote the largest of the three numbers obtained. Find the probability distribution of $X$. Also, find the mean and variance of the distribution.

6. $A, B$ and $C$ throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if $A$ starts first.

7. A random variable $X$ has the following probability distribution : Find

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $\Pr(X)$ | $C$ | $2C$ | $2C$ | $3C$ | $C^2$ | $2C^2$ | $7C^2 + C$ |

the value of $C$ and also calculate mean of the distribution.

8. A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

9. Five bad oranges are accidently mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean and variance of the distribution.

10. In a game, a man wins ₹5 for getting a number greater than 4 and loses ₹1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amound he wins/loses.

11. A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white ?

12. A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

13. A random variable $X$ has the following probability distribution:

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P(X)$ | $C$ | $2C$ | $2C$ | $3C$ | $C^2$ | $2C^2$ | $7C^2 + C$ |

Find the value of $C$ and also calculate mean of the distribution.

14. $A$, $B$ and $C$ throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if $A$ starts first.

15. A bag $X$ contains 4 white balls and 2 black balls, while another bag $Y$ contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag $Y$.

16. $A$ and $B$ throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if $A$ starts first.

17. Three numbers are selected at random (without replacement) from first six positive integers. Let $X$ denote the largest of the three numbers obtained. Find the probability distribution of $X$. Also, find the mean and variance of the distribution.

## 5.11. 2015

### 5.11.1. 10

1. Two different dice are tossed together. Find the probability that the product of the two numbers on the top of the dice is 6.

2. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is

   (a) a card of spade or an ace.

   (b) a black king.

   (c) neither a jack nor a king.

   (d) either a king or a queen

3. A bag contains, white, black and red balls only. A ball is drawn at random from the bag. If the probability of getting a white balls is $\frac{3}{10}$ and that of black ball is $\frac{2}{5}$, then find the probability of getting a red

ball. If the bag contains 20 black balls, then find the total number of balls in the bag.

# Chapter 6

# Construction

## 6.1. 2023

### 6.1.1. 10

1. In the given figure, $XZ$ is parallel to $BC$. $AZ = 3$cm, $ZC = 2$cm, $BM = 3$cm and $MC = 5$cm. Find the length of $XY$.

2. In the given figure, $DE \parallel BC$. If $AD = 2$units, $DB = AE = 3$units and $EC = x$units, then find the value of $x$ is:

   (a) 2

   (b) 3

   (c) 5

   (d) $\frac{9}{2}$

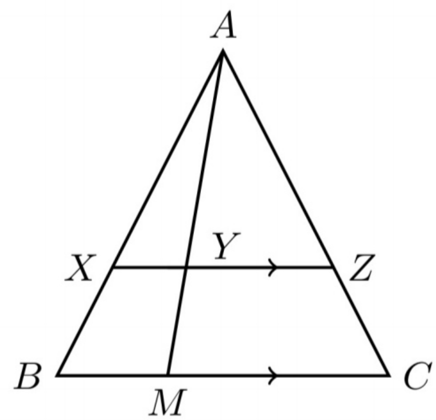



Figure 1: Isosceles Triangle

Figure 6.1: Isosceles Triangle

3. In the given figure, $\Delta ABC$ and $\Delta DBC$ are on te same base $BC$. If $AD$ intersects $BC$ at **O**, prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.

## 6.2. 2022

### 6.2.1. 10

1. In figure,Fig. 6.4 BN and CM are medians of a $\triangle$ ABC right-angled at A. Prove that

$$4(BN^2 + CM^2) = 5BC^2 \tag{6.1}$$

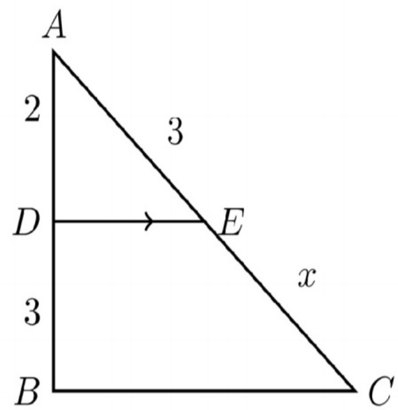



Figure 2: Right Angle Triangle

Figure 6.2: Right Angle Triangle

2. **CaseStudy – 1** :

**KiteFestival**

Kite festival is celebrated in many countries at different times of the year. in India, every year 14th January is celebrated as international kite Day. on his day many people visit India and participate in the festival by flying various kinds of kites.

The picture given below Fig. 6.5 , three kites flying together.

In Fig. 6.5, the angles of elevation of two kites (point C) are found to

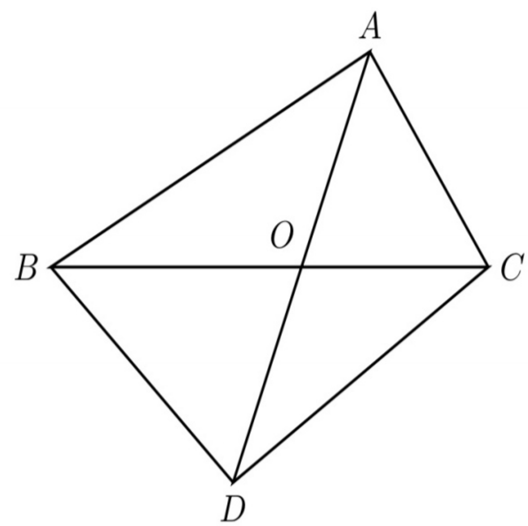



Figure 3: Triangles with same base

Figure 6.3: Triangles with same base

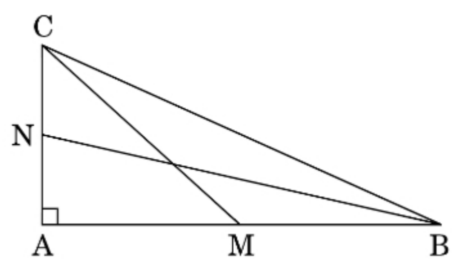
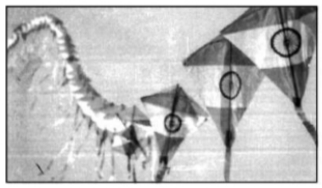



Figure 6.4: Right-angled triangle

be °30 and °60 respectively. Taking

$$AD = 50m \tag{6.2}$$



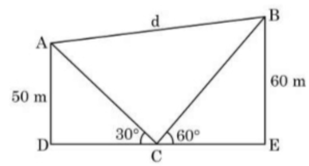



Figure 6.5: kites flying to gether

and

$$BE = 60m \tag{6.3}$$

find

(a) The length of string used (take them straight) for kites A and B as shown in the figure.

(b) The distance 'd' between these two kites

3. In Fig. 6.6, $PQ \parallel BC$, $PQ = 3cm$, $BC = 9cm$ and $AC = 7.5cm$. Find the length of $AQ$.

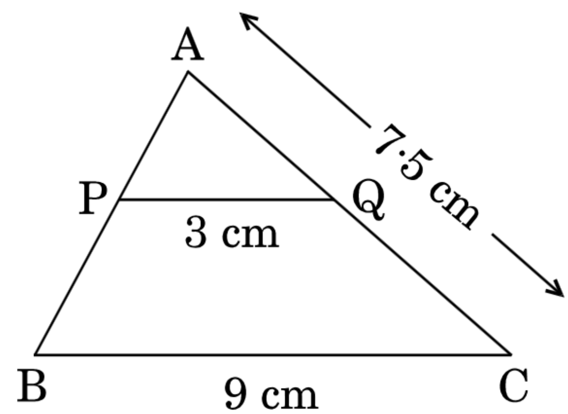



Figure 6.6: $PQ \,||\, BC$

4. Draw a circle of radius $2.5cm$. Take a point $P$ outside the circle at a distance of $7cm$ from the centre. Then construct a pair of tangents to the circle from point $P$.

5. Sides $AB$ and $AC$ and median $AD$ of $\triangle ABC$ are respectively proportional to sides $PQ$ and $PR$ and median $PM$ of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.

6. In Fig. 6.7 $BN$ and $CM$ are medians of a $\triangle ABC$ right-angled at $A$. Prove that $4(BN^2 + CM^2) = 5BC^2$.

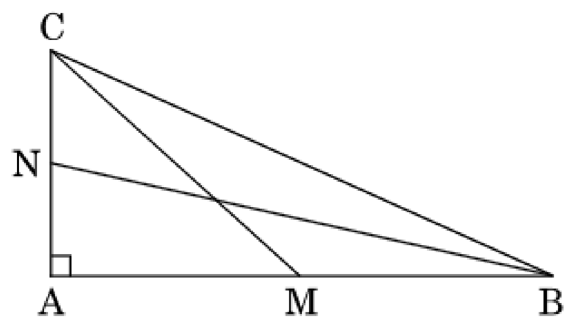



Figure 6.7: $BN$ and $CM$ are medians

7. Construct a pair of tangents to a circle of radius $4cm$ from a point $P$ lying outside the circle at a distance of $6cm$ from the centre.

8. (a) Draw a line segment $AB$ of length $8cm$ and locate a point $P$ on $AB$ such that $AP : PB = 1 : 5$.

   (b) Draw a circle of radius $3cm$. From a point $P$ lying outside the circle at a distance of $6cm$ from its centre, construct two tangents $PA$ and $PB$ to the circle.

9. Construct a pair of tangents to a circle of radius $5cm$ which are inclined each other at an angle of $60^o$.

10. Write the steps of construction for constructing a pair of tangents to a circle of radius $4cm$ from a point $P$, at a distance of $7cm$ from its centre $O$.

## 6.3. 2021

### 6.3.1. 10

1. (a) **D** and **E** are points on the sides $CA$ and $CB$ respectively of a triangle $ABC$, right-angled at **C**.

   Prove that $AE^2 + BD^2 = AB^2 + DE^2$

   (b) Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point **O**. If $AB = 2CD$, find the ratio of the areas of triangles $AOB$ and $COD$.

2. Write the steps of construction of drawing a line segment $AB = 4.8$ cm and finding a point **P** on it suchthat $AP = \frac{1}{4}AB$.

3. Answer any $four$ of the following questions :

   (a) Given $\triangle ABC \ \triangle PQR$. If $\frac{AB}{PQ} = \frac{1}{3}$, than $\frac{ar(\triangle ABC)}{ar(\triangle PQR)}$ is

   i. $\frac{1}{3}$

   ii. 3

   iii. $\frac{2}{3}$

   iv. $\frac{1}{9}$

   (b) The length of an altitude of n equilateral triangle of side 8 cm is

   i. 4 cm

   ii. $4\sqrt{3}$ cm

   iii. $\frac{8}{3}$ cm

   iv. 12 cm

(c) In $\triangle PQR, PQ = 6\sqrt{3}$ cm, $PR = 12cm$ and $QR = 6cm$. The measure of angle **Q** is

i. $120°$

ii. $60°$

iii. $90°$

iv. $45°$

(d) If $\triangle ABC\ \triangle PQR$ and $\angle B = 46°$ and $\angle R = 69°$, then the measure of $\angle A$ is

i. $65°$

ii. $111°$

iii. $44°$

iv. $115°$

(e) **P** and **Q** are the points on the sides $AB$ and $AC$ respectively of a $\triangle ABC$ such that $PQ \parallel BC$. If $AP \parallel PB = 2 : 3$ and $AQ = 4cm$ then $AC$ is equal to

i. 6 cm

ii. 8 cm

iii. 10 cm

iv. 12 cm

4. Answer any $four$ of the following questions :

(a) $ABC$ and $BDE$ are two equilateral triangles such that **D** is the mid-point of $BC$. The ratio of the areas of the triangles $ABC$ and $BDE$ is

i. 2 : 1

ii. 1 : 2

iii. 4 : 1

iv. 1 : 4

(b) In $\triangle ABC, AB = 4\sqrt{3}$ cm, $AC = 8cm$ and $BC = 4cm$. The angle **B** is

i. 120°

ii. 90°

iii. 60°

iv. 45°

(c) The perimeters of two similar triangles are 35 cm and 21 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of the second triangle is

i. 5.4 cm

ii. 4.5 cm

iii. 5.6 cm

iv. 15 cm

(d) In a $\triangle ABC$, **D** and **E** are points on the sides $AB$ and $Ac$ respectively such that $DE \parallel BC$ and $AD : DB = 3 : 1$. If $AE = 3.3cm$, then $AC$ is equal to

i. 4 cm

ii. 1.1 cm

iii. 4.4 cm

iv. 5.5 cm

(e) In the isosceles triangle $ABC$, if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C$ is equal to

i. 30°

ii. 45°

iii. 60°

iv. 90°

## 6.4. 2020

### 6.4.1. 10

1. Draw a circle of radius $3.5cm$. Take a point $P$ outside the circle at a distance of $7cm$ from the centre of the circle and construct a pair of tangents to the circle from that point.

2. Contruct a $\triangle ABC$ with sides $BC = 6cm$, $AB = 5cm$ and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of $\triangle ABC$.

3. In Fig. 6.8, $DE \parallel BC$. If $\frac{AD}{DB} = \frac{3}{2}$ and $AE = 2.7cm$, then $EC$ is equal to

(a) $2.0cm$

(b) $1.8cm$

(c) $4.0cm$

(d) $2.7cm$

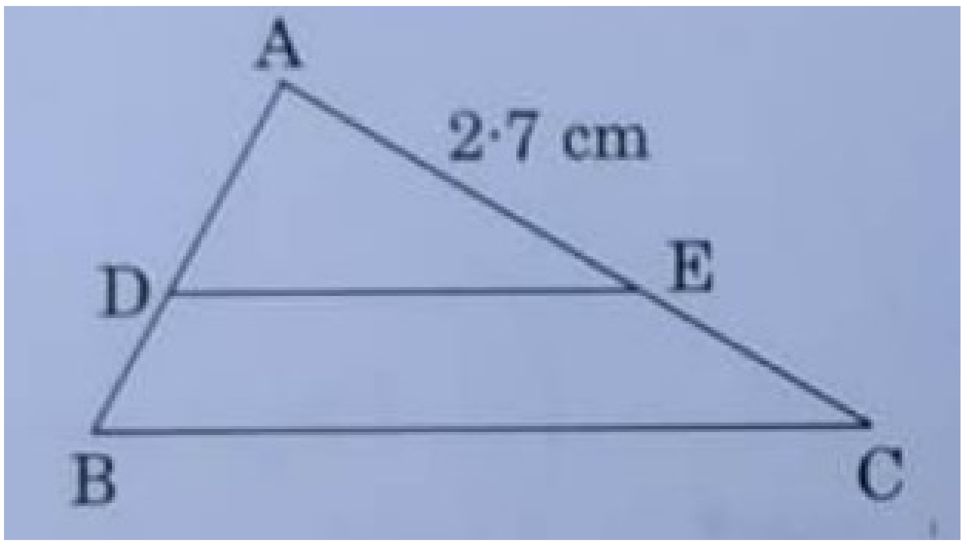



Figure 6.8:

4. In Fig. 6.9, if $PQ \parallel BC$ and $PR \parallel CD$ that $\frac{QB}{AQ} = \frac{DR}{AR}$.

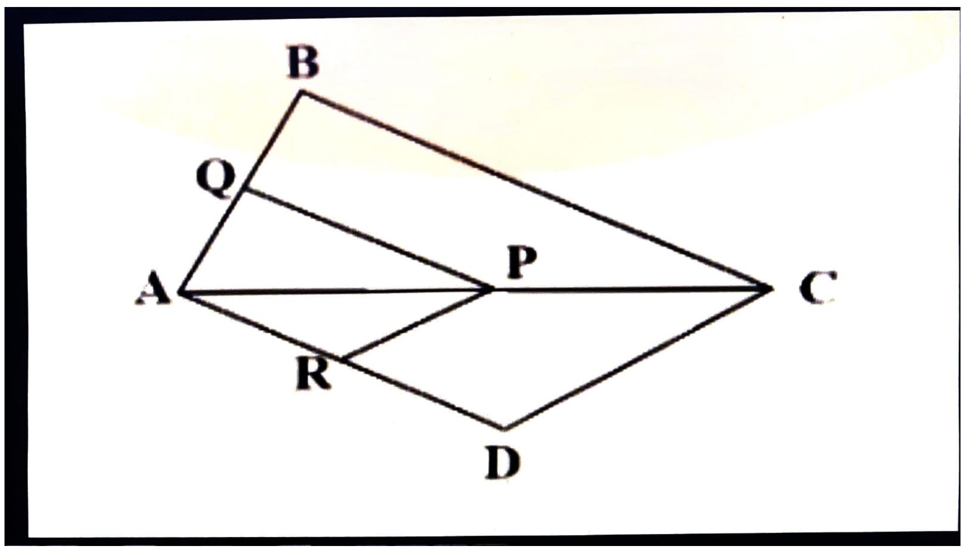



Figure 6.9:

# 6.5. 2019

## 6.5.1. 10

1. In $\triangle ABC$ Fig. 6.10, $AD \perp BC$. Prove that

   $AC^2 = AB^2 + BC^2 - 2BC \times BD$

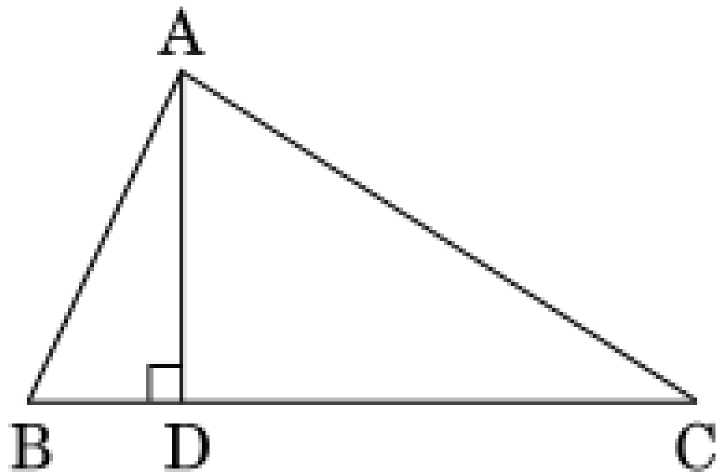



Figure 6.10:

2. Draw a circle of radius 4 cm. From a point 6 cm away from its centre, construct a pair of tangents to the circle and measure their lengths.

3. Construct a triangle with sides $5cm$, $6cm$ and $7cm$ and then another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the first triangle.

4. Let $\triangle ABC \sim \triangle DEF$ and their areas be respectively, $64cm^2$ and $121cm^2$. If $EF = 15.4cm$, find $BC$.

5. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

6. In Fig. 6.11, $BL$ and $CM$ are medians of a $\triangle ABC$ right-angled at $A$. Prove that $4(BL^2 + CM^2) = 5BC^2$.

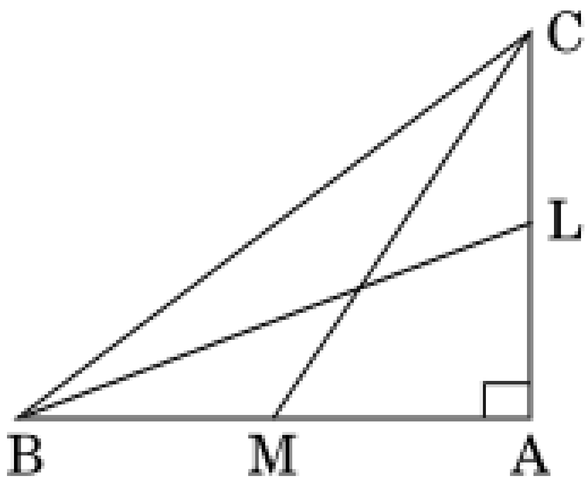



Figure 6.11: Triangle ABC

7. In Fig. 6.12, $ABC$ is an isosceles triangle right angled at $C$ with $AC = 4cm$. Find the length of $AB$.

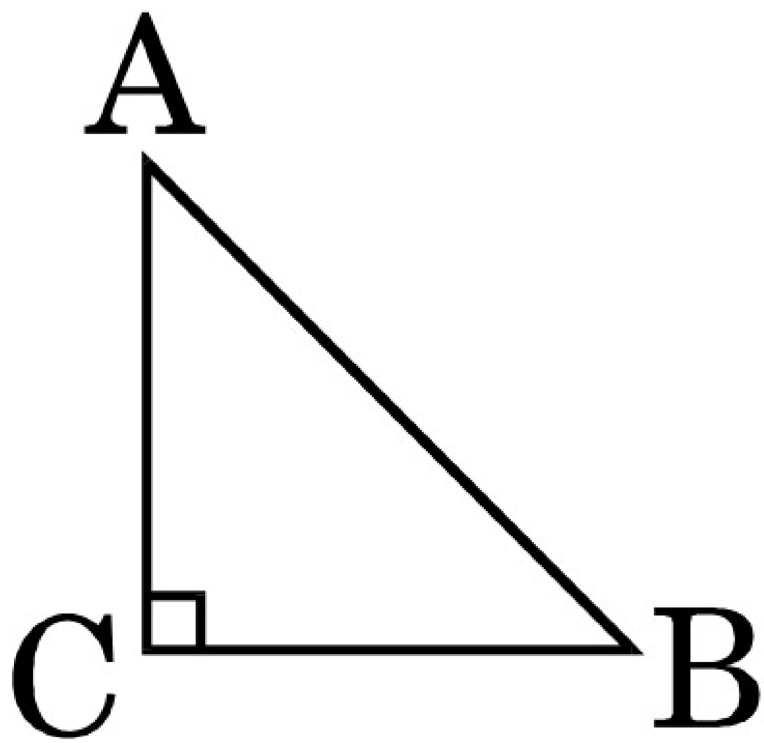



Figure 6.12: Triangle $ABC$

8. In Fig. 6.13, $DE \parallel BC$. Find the length of side $AD$, given that $AE = 1.8cm$, $BD = 7.2cm$ and $CE = 5.4cm$.

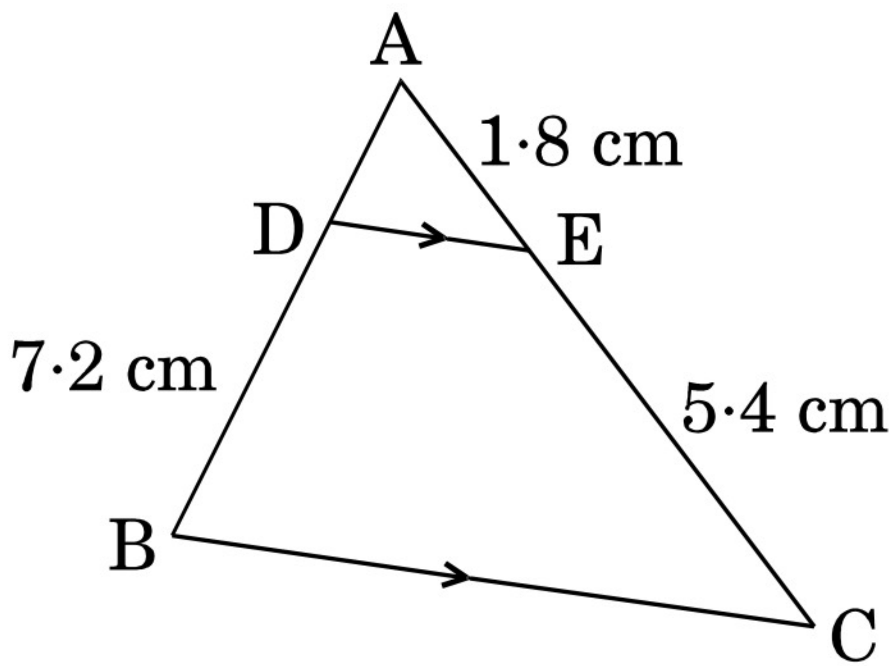



Figure 6.13: Triangle $ABC$

9. Two right triangles $ABC$ and $DBC$ are drawn on the same hypotenuse $BC$ and on the same side of $BC$. If $AC$ and $BD$ intersect at $P$, prove that $AP \times PC = BP \times DP$.

10. Construct an equilateral $\triangle ABC$ with each side $5cm$. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of $\triangle ABC$.

11. Diagonals of a trapezium $PQRS$ intersect each other at the point $O$, $PQ \parallel RS$ and $PQ = 3RS$. Find the ratio of the areas of triangles $POQ$ and $ROS$.

12. In Fig. 6.14, $PS = 3cm$, $QS = 4cm$, $\angle\ PRQ = \theta$, $\angle\ PSQ = 90°$, $PQ \perp RQ$ and $RQ = 9cm$. Evaluate tan $\theta$.

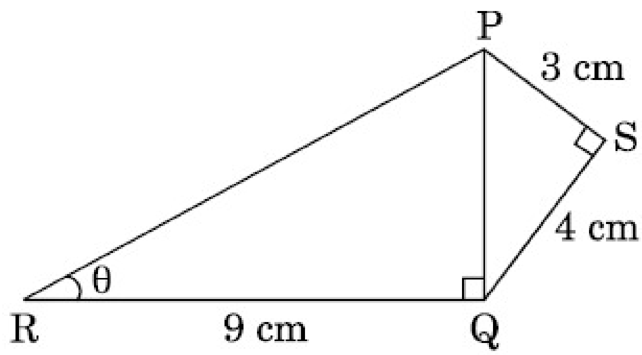



Figure 6.14: Triangle PSQR

13. Two concentric circles of radii $a$ and $b$ $(a > b)$ are given. Find the length of the chord of the larger circle which touches the smaller circle.

14. Construct a triangle, the lengths of whose sides are $5cm$, $6cm$ and $7cm$. Now construct another triangle whose sides are $\frac{5}{7}$ times the corresponding sides of the first triangle.

15. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

16. Construct a triangle $ABC$ with side $BC = 6cm$, $AB = 5cm$ and $\angle ABC = 60°$. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle $ABC$

17. The perpendicular from $A$ on side $BC$ of a $\triangle ABC$ meets $BC$ at $D$

such that $DB = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$.

18. $AD$ and $PM$ are medians of triangles $ABC$ and $PQR$ respectively where $\triangle ABC \sim \triangle PQR$. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

19. Prove that tangents drawn at the ends of a diameter of a circle are parallel.

20. The area of two similar triangles are $25 sq.cm$ and $121 sq.cm$. Find the ratio of their corresponding sides.

21. In Fig. 6.15, $E$ is a point on $CB$ produced of an isosceles $\triangle ABC$, with side $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.

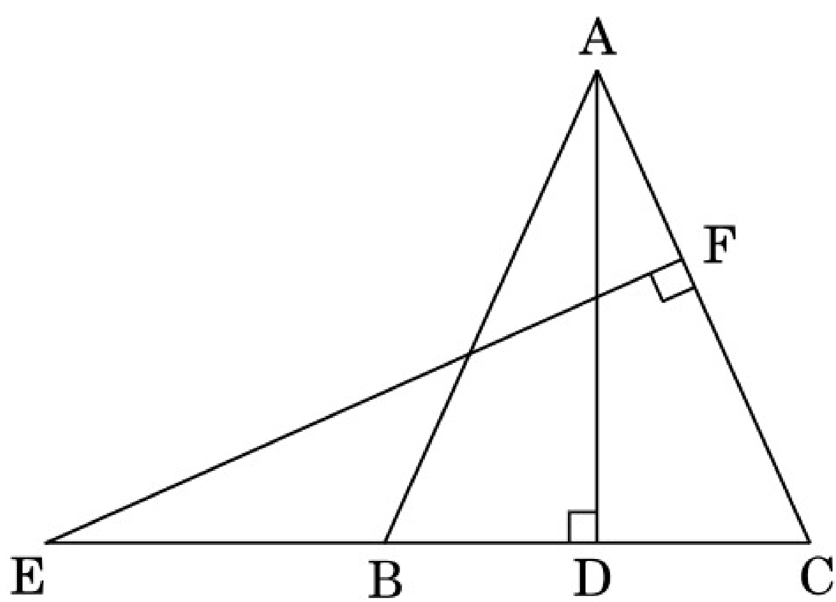



Figure 6.15: Triangle

22. Prove that the parallelogram circumscribing a circle is a rhombus.

23. In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.

24. In two concentric circles, prove that all chords of the outer circle which touch the inner circle, are of equal length.

25. Construct an isosceles triangle whose base is $8cm$ and altitude $4cm$ and then another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the isosceles triangle.

26. Construct a right triangle in which sides (other than the hypotenuse) are $8cm$ and $6cm$. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the right triangle.

27. Construct a $\triangle ABC$ in which $CA = 6cm$ , $AB = 5cm$ and $BAC = 45°$. Then construct a triangle whose sides are $\frac{3}{5}$ of the corresponding sides of $\triangle ABC$.

28. Prove that in a right angle triangle, the square of the hypotenuse is equal the sum of squares of the other two sides.

29. In Fig. 6.16, $DE \parallel BC$, $AD = 1cm, BD = 2cm$. What is the ratio of the area of $\triangle ABC$ to the area of $\triangle ADE$?

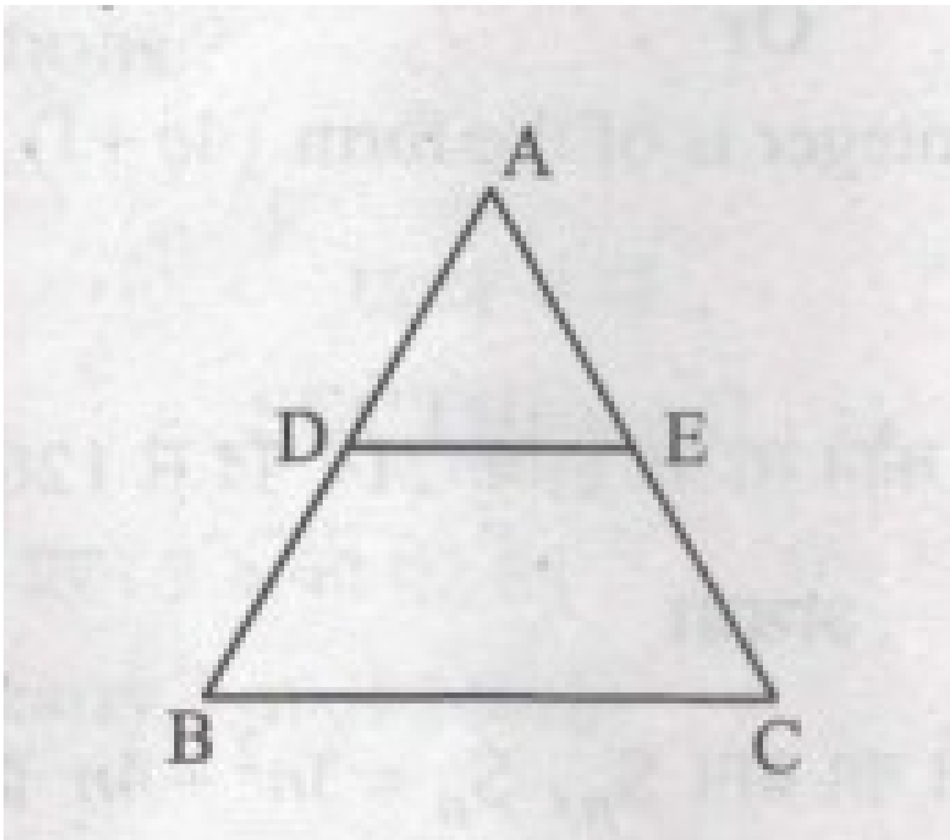



Figure 6.16: triangle

30. In Fig. 6.17, angle $ACB = 90^\circ$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.

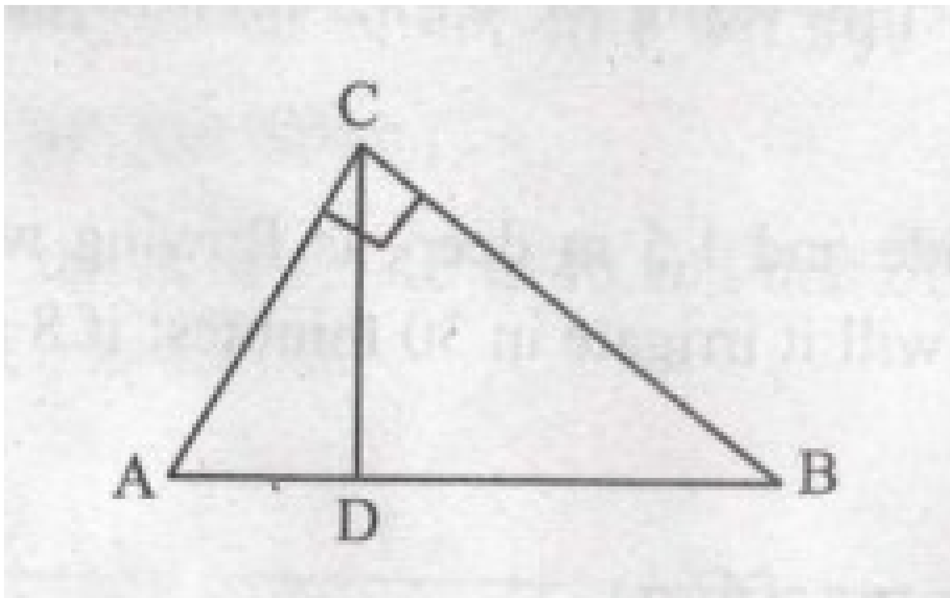



Figure 6.17: circles

31. Construct a triangle $ABC$ with side $BC = 6cm$, $\angle B = 45°, \angle A = 105°$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the $\triangle ABC$

32. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

# 6.6. 2018

## 6.6.1. 10

1. In an equilateral $\triangle$ ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9(AD)^2 = 7(AB)^2$.

2. Prove that, in a right triangle, the square on the hypotenuse is equal to sum of the squares on the other two sides.

3. Prove that the area of an equilateral triangle described on one side of the square is equal to half of the area of the equilateral triangle described on one of its diagonal.

4. If the area of two similar triangles are equal, prove that they are congruent.

5. Draw a triangle ABC with $BC = 6cm, AB = 5cm$ and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the $\triangle ABC$.

6. Given $\triangle ABC \sim \triangle PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{ar\triangle ABC}{ar\triangle PQR}$.

## 6.7. 2016

### 6.7.1. 10

1. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of $60°$ to each other.

2. Draw a triangle with sides 5 cm, 6 cm and 7 cm. Then draw another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of first triangle.

3. Draw an isosceles $\triangle ABC$ in which $BC = 5.5cm$ and altitude $AL = 5.3cm$. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of $\triangle ABC$.

# 6.8. 2015

## 6.8.1. 10

1. Construct a triangle $ABC$ in which $AB = 5$ cm, $BC = 6$ cm and $\angle ABC = 60°$. Now construct another triangle whose sides are $\frac{5}{7}$ times the corresponding sides of $\triangle ABC$.

# Chapter 7

# Optimization

## 7.1. 2023

1. The objective function $Z = ax + by$ of an LLP has maximum value 42 at (4,6) and minimum value 19 at (3,2).Which of the following is true?

   (a) $a = 9, b = 1$

   (b) $a = 5, b = 2$

   (c) $a = 3, b = 5$

   (d) $a = 5, b = 3$

2. The corner point of the feasible region of a linear programming problem are (0,4), (8,0)and $(\frac{20}{3}, \frac{4}{3})$.if $Z = 30x + 24y$ is the objective fnction, then ( maximum value of Z-minimum value of Z) is equal to

   (a) 40

   (b) 96

   (c) 120

   (d) 136

3. Solve the following linear programming problem graphically :

$$Maximum : Z = x + 2y$$
$$subjecttoconstraints : x + 2y \geq 100,$$
$$2x - y \leq 0,$$
$$2x + y \leq 200,$$
$$x \geq 0, y \geq 0.$$

4. Engine displacement is the measure of the cylinder volume swept by all the pistons engine.The piston move inside the cylinder bore

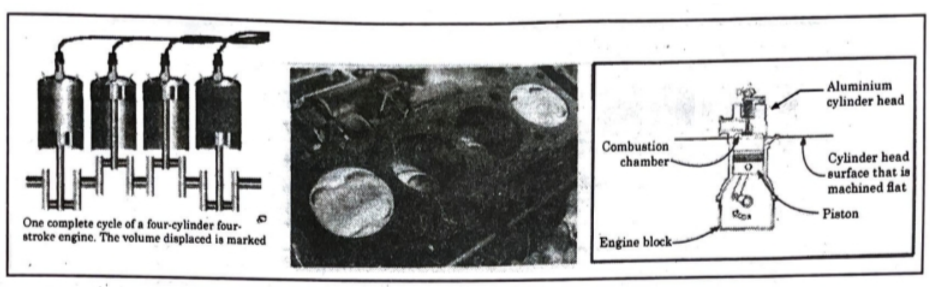



Figure 7.1: Engine

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi cm^2$

Based on the above information,answer the following questions:

(a) if the radius of cylinder is r cm and height is h cm,then write the volme V of cylinder in terms of radius r.

(b) Find $\frac{dV}{dr}$.

(c) i. Find the radius of cylinder when its volume is maximum.

ii. For maximum volume,$h > r$.State true or false and justify.

## 7.2. 2021

### 7.2.1. 12

1. If the corner points $(3, 4)$ and $(5, 0)$ of the feasible region in an LPP, give the same maximum value for the objective function $z = ax + by$, where a,b $> 0$, then we have

   (a) $a = 2b$

   (b) $2a = b$

   (c) $2a = 3b$

   (d) $3b = 2a$

2. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs 1 ₹50 per kg to purchase Food I and

₹70 per kg to purchase Food II. Formulate this problem as a Linear Programming Problem for minimizing the cost of such a mixture.

3. Show that of all the rectangles inscribed in a given fixed circle, the square has maximum area.

   Find the intervals in which the function f given by $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

4. A company produces two types of goods, A and B that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold, while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹120 and that of type B ₹150 , then find the number of units of each type that the company should produce to maximize profit.

   Formulate the above LPP and solve it graphically. Also, find the maximum profit.

5. Find the intervals in which the function f defined as $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is strictly increasing or decreasing.

   Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

6. Maximize $z = 3x + 4y$, if possible,

subject to the constraints :

$$x - y \leq -1 \tag{7.1}$$

$$-x + y \leq 0 \tag{7.2}$$

$$x, y \geq 0 \tag{7.3}$$

7. A dietician wishes to mix two types of foods F1 and F2 in such a way that the vitamin content of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food F1 contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C, while Food F2 contains 1 unit/kg of vitamin A and 2units/kg of vitamin C. It costs ₹5 per kg to purchase Food F1 and ₹7 per kg to purchase Food F2

   Based on the above information, answer the following questions:

   (a) To find out the minimum cost of such a mixture, formulate the above problem as a LPP.

   (b) Determine the minimum cost of the mixture.

8. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

## 7.3. 2022

### 7.3.1. 12

1. A company produces two types of goods, $A$ and $B$, that require gold and silver. Each unit of type $A$ requires $3g$ of silver and $1g$ of gold, while that of type $B$ requires $1g$ of silver and $2g$ of gold. The company can use at the most $9g$ of silver and $8g$ of gold. If each unit of type $A$ brings a profit of ₹ 120 and that of type $B$ ₹ 150, then find the number of units of each type that the company should produce to maximise profit. Formulate the above LPP and solve it graphically. Also, find the maximum profit.

2. Find the maximum value of $7x + 6y$ subject to the constrains:

$$x + y \geq 2 \tag{7.4}$$

$$2x + 3y \leq 6 \tag{7.5}$$

$$x \geq 0 \text{and} y \geq 0 \tag{7.6}$$

3. A window is in the form of a rectangular mounted by a semi-circular opening.The total perimeter of the window to admit maximum light through the whole opening.

4. Divide the number 8 into two positive numbers such that the sum of the cube of one and the square of the other is maximum.

5. Find the maximum and the minimum values of

$$z = 5x + 2y \tag{7.7}$$

subject to the constrains:

$$-2x - 3y \leq -6 \tag{7.8}$$

$$x - 2y \leq 2 \tag{7.9}$$

$$6x + 4y \leq 24 \tag{7.10}$$

$$-3x + 2y \leq 3 \tag{7.11}$$

$$x \geq 0, y \geq 0 \tag{7.12}$$

6. A furniture dealer deals in only two items : chairs and tables. He has ₹ 5,000 to invest and a space to store at most 60 pieces. A table costs him ₹ 250 and a chair ₹ 50. He sells a table at a profit of ₹ 50 and a chair at a profit of ₹ 15. Assuming that he can sell all the items he buys, how should he invest his money in order that he may maximize his profit ? Formulate the above as a linear programming problem.

7. The least value of the function

$$f(x) = 2\cos(x) + x \tag{7.13}$$

in the closed interval $\left[0, \frac{\pi}{2}\right]$ is:

(a) 2

(b) $\frac{\pi}{6} + \sqrt{3}$

(c) $\frac{\pi}{2}$

(d) The least value does not exist.

8. A linear programming problem is as follows: Minimize

$$Z = 30x + 50y \tag{7.14}$$

subject to the constrains,

$$3x + 5y \geq 15 \tag{7.15}$$

$$2x + 3y \leq 18 \tag{7.16}$$

$$x \geq 0, y \geq 0 \tag{7.17}$$

In the feasible region, the minimum value of $Z$ occurs at

(a) a unique point

(b) no point

(c) infinitely many points

(d) two points only

9. The area of a trapezium is defined by function $f$ and given by

$$f(x) = (10 + x)\sqrt{100 - x^2} \tag{7.18}$$

, then the area when it is maximised is:

(a) $75cm^2$

(b) $7\sqrt{3}cm^2$

(c) $75\sqrt{3}cm^2$

(d) $5cm^2$

10. For an objective function

$$Z = ax + by \tag{7.19}$$

,where $a, b > 0$;the corner points of the feasible region determined by a set of constrains (linear inequalities) are $(0, 20)$, $(10, 10)$, $(30, 30)$, and $(0, 40)$.The condition on $a$ and $b$ such that the maximum $Z$ occurs at the points $(30, 30)$ and $(0, 40)$ is:

(a) $b - 3a = 0$

(b) $a = 3b$

(c) $a + 2b = 0$

(d) $2a - b = 0$

11. In a linear programming problem, the constrains on the decision variables $x$ and $y$ are $x - 3y \geq 0, y \geq 0, 0 \leq x \leq 3$.The feasible region

(a) is not in the first quadrant

(b) is bounded in the first quadrant

(c) is unbounded in the first quadrant

(d) does not exist

12. Based on the given shaded region in figure 7.2 as the feasible region in the graph, at which point(S) is the objective function

$$Z = 3x + 9y \tag{7.20}$$

maximum?

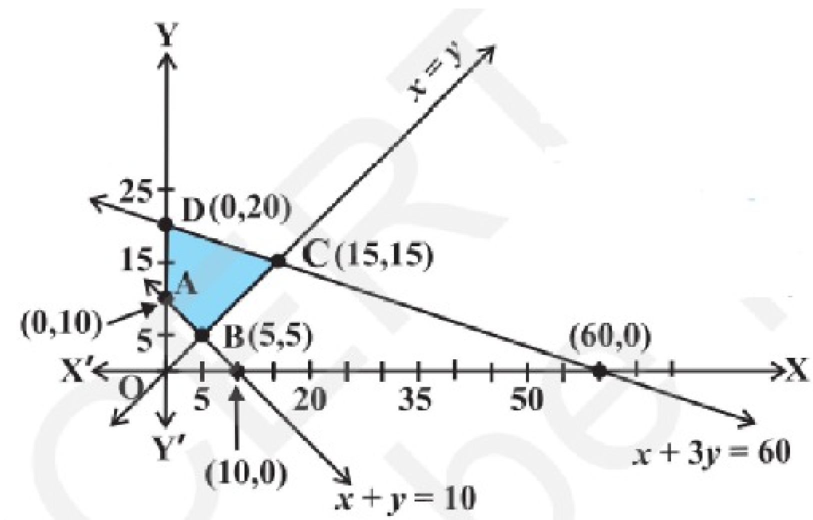



Figure 7.2: Optimization graph

(a) point $B$

(b) point $C$

(c) point $D$

(d) every point on the line segment $CD$

13. In figure 7.3, the feasible region for a LPP is shaded.The objective

function

$$Z = 2x - 3y \tag{7.21}$$

,will be minimum at:

(a) $(4, 10)$

(b) $(6, 8)$

(c) $(0, 8)$

(d) $(6, 5)$

# 7.4. 2020

## 7.4.1. 12

1. The corner points of the feasible region of an LLP are $(0, 0)$,$(0, 8)$,$(2, 7)$,$(5, 4)$ and $(6, 0)$. The maximum profit $P = 3x + 2y$ occure at the point ____________________.

# 7.5. 2019

## 7.5.1. 12

1. A company produces two types of goods, $A$ and $B$, that require gold and silver. Each unit of type $A$ requires 3 g of silver and 1 g of gold

while that of type $B$ requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type $A$ brings a profit of ₹40 and that of type $B$ ₹50, find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it graphically and also find the maximum profit.

2. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius $R$ is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

3. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of its edge is $12cm$ ?

4. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin $A$ and 10 units of vitamin $C$. Food $I$ contains 2 units/kg of vitamin $A$ and 1 unit/kg of vitamin $C$. It costs ₹50 per kg to produce food $I$. Food $II$ contains 1 unit/kg of vitamin $A$ and 2 units/kg of vitamin $C$ and it costs ₹70 per kg to produce food $II$. Formulate this problem as a LPP to minimise the cost of a mixture that will produce the required diet. Also find the minimum cost.

5. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine $A$ and 3 hours on machine $B$ to produce a package of nuts. It takes 3 hours on machine $A$ and 1 hour on machine $B$ to produce a package of bolts. He earns a profit of ₹ 35 per package of nuts and ₹ 14

per package of bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates each machine for atmost 12 hours a day ? Convert it into an LPP and solve graphically.

6. $A$ company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type $A$ require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type $B$ require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹50 each for type $A$ and ₹60 each for type $B$ souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit ? Formulate the above LPP and solve it graphically and also find the maximum profit .

7. Find the local maxima and local minima, if any, of the following function. Also find the local maximum and the local minimum values, as the case may be :

$$f(x) = \sin x + \frac{1}{2}\cos 2x, 0 \leq x \leq \frac{\pi}{2}.$$

8. Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.

9. The sum of the perimeters of circle and a square is K is some constant. Prove that the sum of their area is least when the side of the square is twice the radius of the radius of the circle.

10. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

11. A company manufactures two types of novelty souvenirs made of plywood. souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type $B$ require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4hours for assembling. The profit for type $A$ souvenirs is ₹100 each and for type $B$ souvenirs, profit is ₹120 each. How many souvenirs of each type should the company manufacture in order to maximum the profit ? Formulate the problem as a LPP and then solve it graphically.

12. An isosceles triangle of vertical angle $2\theta$ is inscribed in a circle of radius $a$. Show that the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

## 7.6. 2018

### 7.6.1. 12

1. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this

question ?

2. A factory manufactures two types of screws $A$ and $B$, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws '$A$' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws '$B$'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws '$A$' at a profit of 70 paise and screws '$B$' at a profit of ₹1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit ? Formulate the above LPP and solve it graphically and find the maximum profit.

3. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type $A$ require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type $B$ require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹ 50 each for type $A$ and ₹60 each for type $B$ souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit ? Formulate the above LPP and solve it graphically and also find the maximum profit.

4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine $A$ and 3 hours on machine $B$ to produce a package of nuts.

It takes 3 hours on machine $A$ and 1 hour on machine $B$ to produce a package of bolts. He earns a profit of ₹35 per package of nuts and ₹14 per package of bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates each machine for at most 12 hours a day ? Convert it into an LPP and solve graphically.

5. A dietitian wishes to mix two types of food in such a way the vitamin contents of the mixtures contain at least 8 units of vitamin A and 10 units of vitamin C. It costs ₹50 per Kg to produce food I. Food II contain 1 unit/kg of vitamin A and 2 units/kg of Vitamin C and it costs ₹70 per kg to produce food II. Formulate this problem as a LPP to minimise the cost of the mixture that will produce the required diet. Also find the minimum cost.

6. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type $A$ require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type $B$ require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit for type $A$ souvenirs is ₹100 each and for type $B$ souvenirs, profit is ₹120 each. How many souvenirs of each type should the company manufacture in order to maximise the profit ? Formulate the problem as a LPP and then solve it graphically.

7. A company produces two types of goods, $A$ and $B$, that require gold and silver. Each unit of type $A$ requires 3g of silver and 1g of gold, while that of type $B$ requires 1g of silver and 2g of gold. The company

can use at most 9g of silver and 8g of gold. If each unit of type $A$ brings a profit of ₹40 and that of type $B$ ₹50, find the number of units of each type that the company should produce to maximize profit. Formulate the above Linear Programming Problem ($LPP$) and solve it graphically. Also, find the maximum profit.

8. A manufacturer has employed 5 skilled men and 10 semi-skilled men and make two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours of work by a semi-skilled man.one item of model B requires 1 hour by skilled man and 3 hours by a semi-skilled man.No man is expected to work more than 8 hours per day.The manufacturers profit on an item of a model A is ₹15 and on an item of modal B is ₹10. How many of items of each modal should be made per day in order to maximize daily profit?Formulate the above LPP and solve it graphically and find the maximum profit.

9. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is $8m^3$ . If building of tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides, what is the cost of least expensive tank ?

## 7.7. 2017

### 7.7.1. 10

### 7.7.2. 12

1. Two tailors, $A$ and $B$, earn ₹300 and ₹400 per day respectively . $A$ can stitch 6 shirts and 4 pairs of trousers while $B$ can stitch 10 shirts and 4 pairs of trousers per day . To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

2. Maximise $Z = x + 2y$ subject to the constraints

$$\begin{aligned} x + 2y &\geq 100 \\ 2x - y &\leq 0 \\ 2x + y &\leq 200 \\ x, y &\geq 0 \end{aligned}$$

   Solve the above LPP graphically.

3. Solve the following linear programming problem graphically :

   Maximise $Z = 7x + 10y$

subject to te constraints

$$4x + 6y \leq 240$$
$$6x + 3y \leq 240$$
$$x \geq 10$$
$$x \geq 0, y \geq 0$$

4. Solve the following linear programming problem graphically :

   Maximise $Z = 34x + 45y$

   under the following constraints

$$x + y \leq 300$$
$$2x + 3y \leq 70$$
$$x \geq 0, y \geq 0$$

## 7.8. 2016

### 7.8.1. 12

1. A diet is to contain at least 80 units of Vitamin $A$ and 100 units of minerals. Two foods $\mathrm{F}_1$ and $\mathrm{F}_2$ are available costing 5 rupees per unit and 6 rupees per unit respectively. One unit of food $\mathrm{F}_1$ contains 4 units of vitamin $A$ and 3 units of minerals whereas one unit of food $\mathrm{F}_2$ contains 3 units of vitamin $A$ and 6 units of minerals. Formulate

this as a linear programming problem. Find the minimum cost of diet that consists of mixture of these two foods and also meets minimum nutritional requirement.

2. A retired person wants to invest an amount of $50,000$ rupees. His broker recommends investing in two type of bonds $A$ and $B$ yielding 10% and 9% return respectively on the invested amount. He decides to invest at least $20,000$ rupees in bond $A$ and at least $10,000$ rupees in bond $B$. He also wants to invest at least as much in bond $A$ as in bond $B$. Solve this linear programming problem graphically to maximise his returns.

3. A typist charges 145 rupees for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are 180 rupees. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only 2 rupees per page from a poor student Shayam for 5 Hindi pages. How mcuh less was charged from ths poor boy? which values are reflected in this problem?

4. There are two types of fertilisers $A$ and $B$. $A$ consists of 12% nitrogen and 5% phosphoric acid whereas $B$ consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If $A$ costs ₹10 per kg and $B$ cost ₹8 per kg, then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost.

5. A company manufactures two types of cardigans : type $A$ and type $B$. It costs ₹360 to make a type $A$ cardigan and ₹120 to make a type $B$ cardigan. The company can make at most 300 cardigans and spend at most ₹72, 000 a day. The number of cardigans of type $B$ cannot exceed the number of cardigans of type $A$ by more than 200. The company makes a profit of ₹100 for each cardigan of type $A$ and ₹50 for every cardigan of type $B$. Formulate this problem as a linear programming problem to maximise the profit to the company. Solve it graphically and find maximum profit.

6. A typist charges ₹145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are ₹180. Using matrices, find the charges of typing one English and one English page separately. However typist charged only ₹2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy? Which values are reflected in this problem?

7. A retired person wants to invest an amount of ₹50, 000. His broker recommends investing in the type of bonds '$A$' and '$B$' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least ₹20, 000 in bond '$A$' and at least ₹10, 000 in bond '$B$'. He also wants to invest at least as much in bond '$A$' as in bond '$B$'. Solve this linear programming problem graphically to maximise his returns.

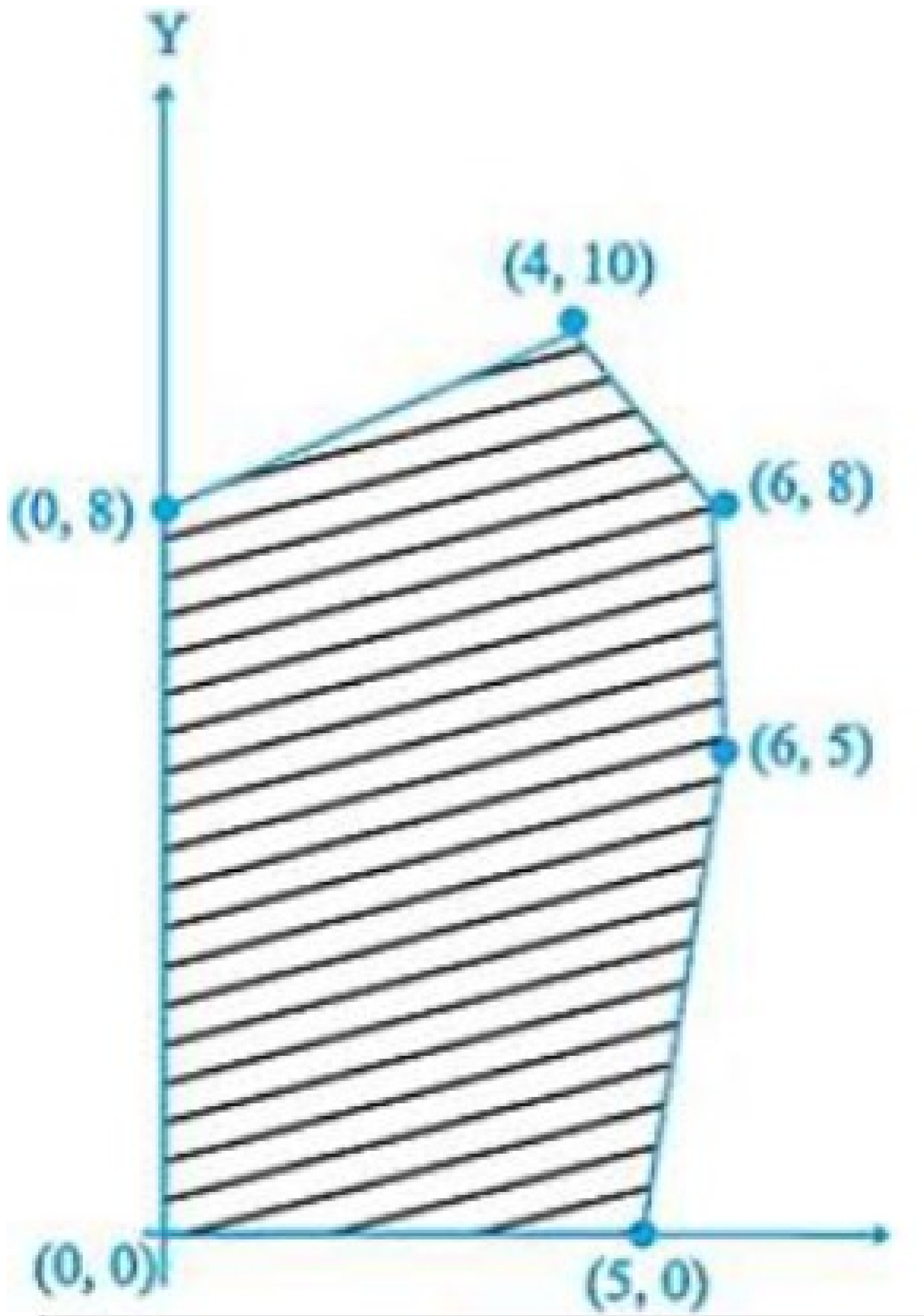



Figure 7.3: Optimization graph

# Chapter 8

# Algebra

## 8.1. 2020

### 8.1.1. 10

1. The value(s) of $k$ for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is

   (A) 4

   (B) $\pm 4$

   (C) $-4$

   (D) 0

2. on dividing a polynomial $p(x)$ by $x^2 - 4$,quotient and remainder are found to be $x$ and 3 respectively. The polynomial $p(x)$ is

   (A) $3x^2 + x - 12$

   (B) $x^3 - 4x + 3$

   (C) $x^2 + 3x - 4$

(D) $x^3 - 4x - 3$

3. Simplest form of

$$\frac{1+\tan^2 A}{1+\cot^2 A} \tag{8.1}$$

is .

4. Write the value of

$$\sin^2 30^\circ + \cos^2 60^\circ \tag{8.2}$$

.

5. From the quadratic polynomial, the sum and product of whose zeroes are $(-3)$ and 2 respectively.

6. Can $\left(x^2 - 1\right)$ be a reminder while dividing $x^4 - 3x^2 + 5x - 9$ by $\left(x^2 + 3\right)$ ? Justify your answer with reasons.

7. If $A$, $B$ and $C$ are interior angles of $\triangle ABC$, then show that

$$\cos\left(\frac{B+C}{2}\right) = \sin\left(\frac{A}{2}\right) \tag{8.3}$$

8. Prove that :

$$(\sin^4\theta - \cos^4\theta + 1)\csc^2\theta = 2 \tag{8.4}$$

9. Sum of the areas of two squares is $544m^2$. If the difference of their

perimeters is $32m$, find the sides of the two squares.

10. A motor boat whose speed is 18km/h in still water takes 1 hour more to go 24km upstream than to return down stream to the same spot. Find the speed of the stream.

11. Obtain the zeroes of the polynomial $p(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$ if two zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

12. What minimum is added to $2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial will be divisible by $x^2 - 4x + 8$ ?

## 8.2. 2020

### 8.2.1. 12

1. If

$$\cos\left(\sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1} x\right) = 0 \tag{8.5}$$

then $x$ is equal to

(A) $\frac{1}{\sqrt{5}}$

(B) $-\frac{2}{\sqrt{5}}$

(C) $\frac{2}{\sqrt{5}}$

(D) 1

## 8.3. 2023

### 8.3.1. 10

1. If one zero of the polynomial

$$p(x) = 6x^2 + 37x - (k-2) \tag{8.6}$$

is reciprocal of the other, then find the value of $k$?

2. Find the value of $'p'$ for which one root of the quadratic equation

$$px^2 - 14x + 18 = 0 \tag{8.7}$$

is 6 times the other?

3. (a) prove that

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A \tag{8.8}$$

(b)

$$\sec A(1 - \sin A)(\sec A + \tan A) = 1 \tag{8.9}$$

4. Which of the following quadratic equations has sum of its roots as 4?

(a) $2x^2 - 4x + 8 = 0$

(b) $-x^2 + 4x + 4 = 0$

(c) $\sqrt{2x^2} - \frac{4}{\sqrt{2}}x + 1 = 0$

(d) $4x^2 - 4x + 4 = 0$

5. if one zero of the polynomial

$$6x^2 + 37x - (k - 2) \tag{8.10}$$

is reciprocal of the other,then what is the value of $k$?

(a) -4

(b) -6

(c) 6

(d) 4

6. The zeroes of the polynomial

$$p(x) = x^2 + 4x + 3 \tag{8.11}$$

are given by:

(a) 1,3

(b) -1,3

(c) 1,-3

(d) -1,-3

7. If $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $p(x) = x^2 - ax - b$, then the value of $\alpha^2 + \beta^2$ is:

(a) $a^2 - 2b$

(b) $a^2 + 2b$

(c) $b^2 - 2a$

(d) $b^2 + 2a$

8. The below is the Assertion and Reason based question. Two statements are given, one labelled as Assertion(A) and the other is labelled as Reason(R). Select the correct answer to these questions from the codes (a),(b),(c) and (d) as given below.

   (a) Both Assertion(A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A).

   (b) Both Assertion(A) and Reason(R) are true, but Reason(R) is not the correct explanation of the Assertion(A).

   (c) Assertion(A) is true, but Reason(R) is false.

   (d) Assertion(A) is false, but Reason(R) is true.

   **Assertion(A):** The polynomial $p(x) = x^2 + 3x + 3$ has two real zeroes.

   **Reason(R):** A quadratic polynomial can have at most two real zeroes.

9. (a) If

$$4\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + p = \frac{3}{4}, \tag{8.12}$$

then find the value of $p$.

(b) If

$$\cos A + \cos^2 A = 1, \tag{8.13}$$

then find the value of

$$\sin^2 A + \sin^4 A. \tag{8.14}$$

10. Prove that:

$$\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right) = \frac{1}{\tan\theta + \cot\theta} \tag{8.15}$$

11. The value of k for which the pair of equations $kx = y+2$ and $6x = 2y+3$ has infinitely many solutions,

    (a) is $k = 3$

    (b) does not exist

    (c) is $k = -3$

    (d) is $k = 4$

12. If $2\tan A = 3$, then the value of $\frac{4sinA+3\cos A}{4\sin A-3\cos A}$ is

    (a) $\frac{7}{\sqrt{13}}$

    (b) $\frac{1}{\sqrt{13}}$

    (c) 3

    (d) does not exist

13. If $\alpha$, $\beta$ are the zeroes of a polynomial $p(x) = x^2 + x - 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ equals to

    (a) 1

    (b) 2

    (c) $-1$

    (d) $\frac{-1}{2}$

14. $\left(\sec^2\theta - 1\right)\left(\csc^2\theta - 1\right)$ is equal to:

    (a) $-1$

    (b) 1

    (c) 0

    (d) 2

15. The roots of equation

$$x^2 + 3x - 10 = 0 \tag{8.16}$$

    are:

    (a) $(2, -5)$

    (b) $(-2, 5)$

    (c) $(2, 5)$

    (d) $(-2, -5)$

16. If $\alpha$ , $\beta$ are zeroes of the polynomial $x^2 - 1$,then value of $(\alpha + \beta)$ is:

(a) 2

(b) 1

(c) 1

(d) 0

17. If $\alpha$, $\beta$ are the zeroes of the polynomial

$$p(x) = 4x^2 - 3x - 7 \tag{8.17}$$

,then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ is equal to:

(a) $\frac{7}{3}$

(b) $\frac{-7}{3}$

(c) $\frac{3}{7}$

(d) $\frac{-3}{7}$

18. Find the sum and product of the roots of the quadratic equation

$$2x^2 - 9x + 4 = 0 \tag{8.18}$$

19. Find the discriminant of the quadratic equation

$$4x^2 - 5 = 0 \tag{8.19}$$

and hence comment on the nature of roots of the equation.

20. Evaluate $2\sec^2\theta + 3\csc^2\theta - 2\sin\theta\cos\theta$ if

$$\theta = 45° \tag{8.20}$$

21. If

$$\sin\theta - \cos\theta = 0 \tag{8.21}$$

,then find the value of $\sin^4\theta + \cos^4\theta$.

# 8.4. 2022

## 8.4.1. 10

1. If $\sin\theta = 0$, then the value of $\tan^2\theta + \cot^2\theta$ is
   (a) 2
   (b) 4
   (c) 1
   (d) $\frac{10}{9}$
2. The value(s) of $k$ for which the quadratic equation

$$3x^2 - kx + 3 = 0 \tag{8.22}$$

   has equal roots, is (are)
   (a) 6
   (b) $-6$
   (c) $\pm 6$

(d) 9

3. $5\tan^2\theta - 5\sec^2\theta =$ __________

4. If $\alpha$, $\beta$ are zeroes of the polynomial $2x^2 - 5x - 4$, then $\frac{1}{\alpha} + \frac{1}{\beta}$.

5. In Fig. 8.1, a tower stands vertically on the ground. From a point on the ground, which is $80m$ away from the foot of the tower, the angle of elevation of the tower is found to be $30°$. Find the height of the tower.

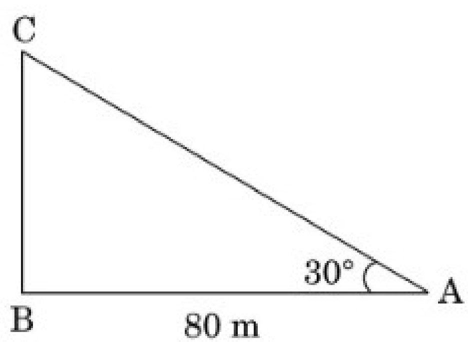



Figure 8.1: as.jpeg

6. Solve

$$9x^2 - 6a^2x + a^4 - b^4 = 0 \tag{8.23}$$

using the quadratic formula.

7. Show that

$$\cos(38°)\cos(52°) - \sin(38°)\sin(52°) = \cos(90°). \tag{8.24}$$

8. Prove that

$$\frac{\sin\theta}{\cot\theta+\csc\theta} = 2 + \frac{\sin\theta}{\cot\theta-\csc\theta}. \tag{8.25}$$

9. Given

$$15\cot(A) = 8, \tag{8.26}$$

find the values of $\sin(A)$ and $\sec(A)$.

10. The angles of depression of the top and bottom of a tower as seen from the top of a $60\sqrt{3}m$ high cliff are $45°$ and $60°$ respectively. Find the height of the tower. (Use $\sqrt{3} = 1.73$)

11. $A$ and $B$ jointly finish a piece of work in 15 days. When they work separately, $A$ takes 16 days less than the number of days taken by $B$ to finish the same piece of work. Find the number of days taken by $B$ to finish the work.

12. If the polynomial

$$f(x) = 3x^4 - 9x^3 + x^2 + 15x + k \tag{8.27}$$

is completely divisible by $3x^2 - 5$, then find the value of $k$. Using the quotient obtained, find two zeroes of the polynomial.

13. Find all the zeroes of the polynomial

$$f(x)x^4 - 8x^3 + 23x^2 - 28x + 12 \tag{8.28}$$

if two of its zeroes are 2 and 3.

14. Find the value of $m$ for which the quadratic equation

$$(m-1)x^2 + 2(m-1)x + 1 = 0 \tag{8.29}$$

has two real and equal roots.

15. Solve the following quadratic equation for $x$

$$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0 \tag{8.30}$$

16. The product of Rehan's age (in years) 5 years ago and his age 7 years from now is one more than twice his age. Find his present age.

17. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 meters high, then find the height of the building.

18. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 60° respectively. If the bridge is at a height of 3 meters from the banks, then find the width of the river.

19. In Fig. 8.2, Gadisar Lake is located in the Jaisalmer district of Rajasthan. It was built by the King of Jaisalmer and rebuilt by Gadsi Singh in the 14th century. The lake has many Chhatris. One of them is shown below:


Figure 8.2: ak.jpg

Observe the picture. From a point $A$ $h$ meters above the water level, the angle of elevation of the top of Chhatri (point $B$) is $45°$ and the angle of depression of its reflection in the water (point $C$) is $60°$ . If the height of Chhatri above water level is (approximately) 10 meters, then

(a) Draw a well-labeled figure based on the above information.

(b) Find the height ($h$) of the point $A$ above water level. (Use $\sqrt{3} = 1.73$)

20. Solve the quadratic equation

$$x^2 + \sqrt{2}x - 6 = 0 \tag{8.31}$$

for $x$.

21. In Fig. 8.3, from a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are $30°$ and $45°$. If the bridge is at a height of 8 meters from the banks, then find the width of the river.

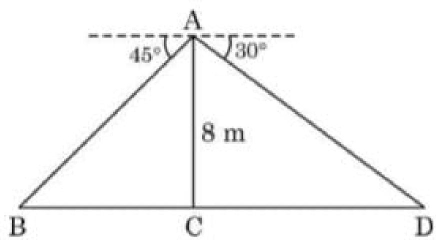



Figure 8.3: su.jpg

22. A 2-digit number is such that the product of its digits is 24. If 18 is subtracted from the number, the digits interchange their places. Find the numbers.

23. The difference of the squares of two numbers is 180. The square of the smaller number is 8 times the greater number. Find the two numbers.

24. Case Study-1:

    In Fig. 8.4, Kite Festival is celebrated in many countries at different times of the year. In India, every year on $14^{th}$ January is celebrated as International Kite Day. On this day, many people visit India and participate in the festival by flying various kinds of kites.

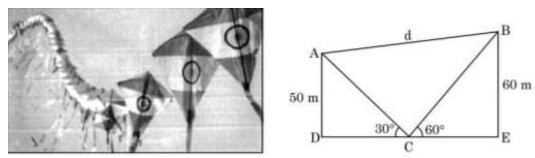



Figure 8.4: kites

In Fig. 5, the angles of elevation of two kites (Point $A$ and $B$) from the hands of a man (Point $C$) are found to be $30°$ and $60°$ respectively. Taking $AD = 50$ meters and $BE = 60$ meters, find:

(a) The lengths of strings used (take them straight) for kites $A$ and $B$ as shown in the figure.

(b) The distance $d$ between these two kites.

25. Solve the quadratic equation for $x$:

$$x^2 - 2ax - (4b^2 - a^2) = 0 \tag{8.32}$$

26. If the quadratic equation

$$(1 + a^2)x^2 + 2abx + (b^2 - c^2) = 0 \tag{8.33}$$

has equal and real roots, then prove that:

$$b^2 = c^2(1 + a^2) \tag{8.34}$$

27. Two boats are sailing in the sea 80 meters apart from each other towards a cliff $AB$. The angles of depression of the boats from the top

of the cliff are 30° and 45° respectively, as shown in Fig. 8.5

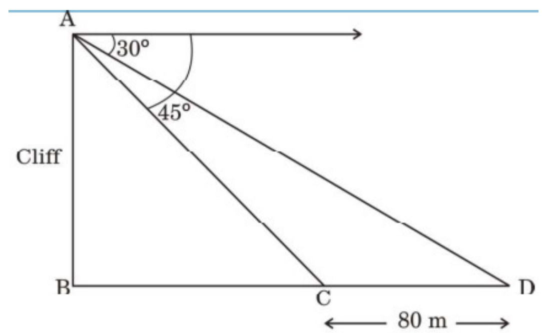



Figure 8.5: boat

Find the height of the cliff.

28. The angle of elevation of the top $Q$ of a vertical tower $PQ$ from a point $X$ on the ground is 60°. From a point $Y$, 40 meters vertically above $X$, the angle of elevation of the top $Q$ of tower $PQ$ is 45°. Find the height of the tower $PQ$ and the distance $PX$. (Use $\sqrt{3} = 1.73$)

29. Find the value of $k$ for which the quadratic equation

$$2kx^2 - 40 + 25 = 0 \tag{8.35}$$

has real and equal roots.

30. Solve for $x$:

$$\frac{5}{2}x^2 + \frac{2}{5} = 1 - 2x \tag{8.36}$$

31. An Aeroplane at an altitude of 200 meters observes the angles of de-

pression of opposite points on the two banks of a river to be 45° and 60°. Find the width of the river. (Use $\sqrt{3} = 1.732$)

32. Find the value(s) of $'p'$ for which the quadratic equation $(px-4)(x-2)$ has real and equal roots.

33. Had Aarush scored 8 more marks in a Mathematics test, out of 35 marks, 7 times these marks would have been 4 less than the square of his actual marks. How many marks did he get in the test?

34. From the top of an 8 meter high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower. (Take $\sqrt{3} = 1.732$).

35. Find the roots of the quadratic equation

$$9x^2 - 6\sqrt{2}x + 2 = 0 \tag{8.37}$$

36. The product of two consecutive odd positive integers is 255. Find the integers, by formulating a quadratic equation.

37. Find the value(s) of $k$ for the quadratic equation,

$$(k+3)x^2 + kx + 1 = 0 \tag{8.38}$$

to have two real and equal roots.

38. As observed from the top of a lighthouse 60 meters high from the sea level, the angles of depression of two ships are 45° and 60°. If one ship

is exactly behind the other on the same side of the lighthouse, then find the distance between the two ships. (Use $\sqrt{3} = 1.732$)

39. At a point on the level ground, the angle of elevation of the top of a vertical tower is found to be $\alpha$, such that $\tan\alpha = \frac{5}{12}$. On walking 192 meters towards the tower, the angle of elevation $\beta$ is such that $\tan\beta = \frac{3}{4}$. Find the height of the tower.

40. $\tan^{-1}\frac{1}{\sqrt{3}} - \cot^{-1}\frac{-1}{\sqrt{3}}$

41. Show that the relation $R$ in the set of all real numbers, defined as $R = \{(a, b) : a \leq b^2\}$.

42. Two angles of a triangle are $\cot^{-1} 2$ and $\cot^{-1} 3$. The third angle of the triangle is?

43. Solve for $x$:

$$\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2} \tag{8.39}$$

44. Find the present value of a perpetuity of ₹ 18,000 at the end of 6 months if it is worth 8% p.a. compounded semi-annually.

[Given that: $1.00833^{12} = 1.1047$]

45. Find the effective rate which is equivalent to a nominal rate of 10% p.a. compounded monthly.

[Given that: $1.00833^{12} = 1.1047$]

46. Abhay bought a mobile phone for ₹ 30,000. The mobile phone is estimated to have a scrap value of ₹ 3,000 after a span of 3 years. Using the linear depreciation method, find the book value of the mobile phone at the end of 2 years.

47. Madhu exchanged her old car valued at ₹ 1,50,000 with a new one priced at ₹ 65,000. She paid ₹ x as a down payment, and the balance in 20 monthly equal installments of ₹ 21,000. The rate of interest offered to her is 9% p.a. Find the value of $x$.

    [Given that: $1.0075^{-20} = 0.86118985$]

48. Calculate the EMI under the 'Flat Rate System' for a loan of | $5,00,000$ with 10% annual interest rate for 5 years.

49. A machine costing ₹ 2,00,000 has an effective life of 7 years, and its scrap value is ₹ 30,000. What amount should the company put into a sinking fund earning 5% p.a., so that it can replace the machine after its usual life? Assume that a new machine will cost ₹ 3,00,000 after 7 years.

    [Given that: $(1.05)^7 = 1.407$]

50. A start-up company invested ₹ 3,00,000 in shares for 5 years. The value of this investment was ₹ 3,50,000 at the end of the second year, ₹ 3,80,000 at the end of the third year, and on maturity, the final value stood at ₹ 4,50,000.Calculate the Compound Annual Growth Rate (CAGR) on the investment.

    [Given that: $(1.5)^{\frac{1}{5}} = 1.084$]

## 8.5. 2021

### 8.5.1. 10

1. Find the sum and product of zeroes of the polynomial $p(x) = x^2+5x+6$

2. If $2\cos\theta = \sqrt{3}$ , then find the value of $\theta$

3. Find the discriminant of the quadratic equation $2x^2 - 5x - 6 = 0$.

4. In $\triangle ABC$, right-angled at $A$, if $AB = 7cm$ and $AC = 24cm$, then find $\sin B$ and $\tan C$.

5. (a) If $\sin(A + B) = \sqrt{3}/2, \sin(A - B) = 1/2$, Where $0° < A + B < 90°; A > B$, then find the values of $A$ and $B$.

   (b) Simplify :

$$\frac{\sin 30° + \tan 45° - \cos 60°}{\sec 30° + \cos 60° + \cot 45°} \tag{8.40}$$

6. The greater of two supplementary angles exceeds the smaller by 18°. Find the two angles.

7. Prove that $7\sqrt{2}$ is an irrational number ,given that $\sqrt{2}$ is an irrational number.

8. (a) Prove that :

$$\sec\theta(1 - \sin\theta)(\sec\theta + \tan\theta) = 1 \tag{8.41}$$

(b) Prove that :

$$\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A} \tag{8.42}$$

9. If $\alpha, \beta$ are the zeroes of the quadratic polynomial $x^2 + 9x + 20$, from a quadratic polynomial whose zeroes are $(\alpha + 1)$ and $(\beta + 1)$.

10. (a) The diagonal of a rectangular field is 60 meters more than the shorter side, find the sides of the field.

    (b) The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present ages

11. Write a quadratic polynomial sum of Whose zeroes is $-5$ and product is 6.

12. If the sum of the zeroes of the polynomial $2x^2 - 3ax + 4$ is 6, then the value of a

    (a) 4

    (b) $-4$

    (c) 2

    (d) $-2$

13. The common zero of the polynomials $x^3 + 1, x^2 - 1$ and $x^2 + 2x + 1$ is

    (a) $-2$

    (b) $-1$

(c) 1

(d) 2

14. If $\alpha, \beta$ are the zeroes of the polynomial $x^2 - 4x + 6$, then the value of $\alpha\beta$ is

(a) 4

(b) $-4$

(c) 6

(d) $-6$

15. The zeroes of the polynomial $3x^2 - 5x - 2$ are

(a) $\frac{1}{3}$,2

(b) $-\frac{1}{3}$,2

(c) $\frac{1}{3}$,-2

(d) $-\frac{1}{3}$,-2

16. If is a zero of the polynomial $p(x) = ax^2 - 3(a-1)x - 1$ then the value of a is

(a) $\frac{1}{3}$,2

(b) $-\frac{1}{3}$,2

(c) $\frac{1}{3}$,-2

(d) $-\frac{1}{3}$,-2

17. If $\tan\theta = 4/3$, find the value $\frac{2\sin\theta - 3\cos\theta}{2\sin\theta + 3\cos\theta}$

18. If x= $a\cos\theta$ and y=$b\sin\theta$, then find the value of $b^2x^2 + a^2y^2$

19. A number consists of two digits whose sum is 9. if 27 is added to the number, the digits are reversed. Find the number

20. Prove that :

$$\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \tan^2\theta - \cot^2\theta \tag{8.43}$$

21. Prove that:

$$(\sec\theta - \tan\theta)^2 = \frac{1 + \sin\theta}{1 - \sin\theta} \tag{8.44}$$

22. The sum of the squares of three consecutive positive integers is 110. Find the positive integers.

23. Ram can row a boat at the rate of 4 km/hour in still water. If he takes 8 hours in going 12 km upstream and 12 km downstream, find the speed of the stream.

24. Write the quadratic equation in $x$ whose roots are 2 and $-5$.

25. If $\alpha$ and $\beta$ are zeros of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

26. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then find the value of $k$.

27. If $3\sin A = 1$, then find the value of $\sec A$.

28. Show that: $\frac{1+\cot^2\theta}{1+\tan^2\theta} = \cot^2\theta$.

29. Simplify :

$$\csc^2 60^\circ \sin^2 30^\circ - \sec^2 60^\circ$$

30. If $\tan\theta + \cot\theta = \frac{4\sqrt{3}}{3}$, then find the value of $\tan^2\theta + \cot^2\theta$.

31. Divide the polynomial $f(x) = 5x^3 + 10x^2 - 30x - 15$ by the polynomial $g(x) = x^2 + 1 + x$ and hence, find the quotient and the remainder.

32. Prove:

$$\frac{1}{(\cot A)(\sec A) - \cot A} - \csc A = \csc A - \frac{1}{(\cot A)(\sec A) + \cot A}$$

33. Prove:

$$\sin^6 A + 3\sin^2 A\cos^2 A = 1 - \cos^6 A$$

34. One of the root of the quadratic equation $2x^2 - 8x - k = 0$ is $\frac{5}{2}$. Find the value of $k$, Also find the root.

35. Using quadratic formula, solve the following equation for x:

$$abx^2 + (b^2 - ac)x - bc = 0$$

.

36. With vertices A,B and C of a triangle ABC as centers, arcs are drawn with radii 2 cm each as/ shown in the figure. If AB = 6 cm, BC = 8 cm and AC = 10 cm, find the area of the shaded region.

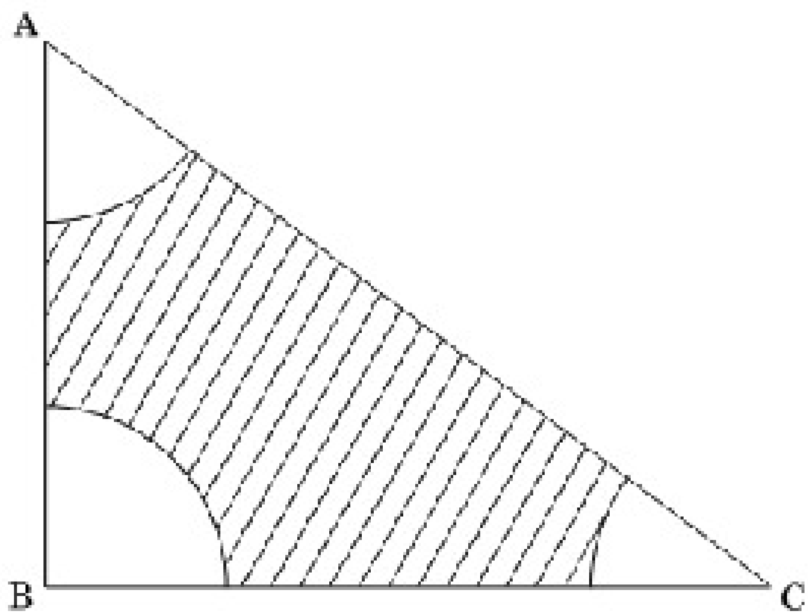



Figure 8.6:

37. Water is being pumped out through a circular pipe whose internal diameter is 8 cm. If the rate of flow of water is 80 cm/s, then how many liters of water is being pumped out through this pipe in one hour ?

38. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 18 minutes for the angle of depression to change from $30^\circ$ to $60^\circ$, how soon after this will the car reach the tower ?

39. A girl on a ship standing on a wooden platform, which is 50 m above water level, observes the angle of elevation of a top of a hill as $30^\circ$ and

the angle of depression of the base of the hill as 60°. Calculate the distance of the hill from the platform and the height of the hill.

40. If one zero of the polynomial $p(x) = (a^2+4)X^2+20X+4a$ is reciprocal of the other, find the value of $a$.

41. Find the roots of the quadratic equation

$$X^2 + X - (a+1)(a+2) = 0 \tag{8.45}$$

42. Solve for x :

$$10X - \frac{1}{X} = 3, X \neq 0 \tag{8.46}$$

43. In $\triangle ABC$ , $\angle B = 90°$ and $\tan A = \frac{1}{\sqrt{3}}$.Then find the value of $\sin A \cos C + \cos A \sin C$

44. If $X = a\sin\theta + b\cos\theta$ and $y = a\cos\theta - b\sin\theta$,then find the value of $(X^2 + Y^2)$.

45. Answer any **four** of the following questions:

    (a) The sum and the product of the zeroes of a quadratic polynomial are -1 and -12 respectively. The polynomial is

        i. $X^2 - X - 12$

        ii. $X^2 + X - 12$

        iii. $X^2 - X + 12$

        iv. $X^2 + X + 12$

(b) The zeroes of the quadratic polynomial $x^2 + 20x + 91$ are

i. both positive.

ii. both equal.

iii. both negative.

iv. one positive and one negative.

(c) If the zeroes of the polynomial $5x^2 - 26x + k$ are reciprocal of each other, then the value of k is

i. 5

ii. -5

iii. $\frac{1}{5}$

iv. -$\frac{1}{5}$

(d) If $\alpha, \beta$ are the zeroes of the polynomial $x^2 - 5x - 14$, then the value of $\alpha\beta - \alpha - \beta$ is

i. -9

ii. 19

iii. 9

iv. -19

(e) What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of the resulting polynomial ?

i. 5

ii. 4

iii. 2

iv. 1

46. If $2\sin 2A = \sqrt{3}$ , then find the value of A.

47. If $7\sin^2\theta + 3\cos^2\theta = 4$, then show that $\tan\theta = \frac{1}{\sqrt{3}}$, $0^\circ < \theta < 90^\circ$

48. Find the quadratic polynomial whose zeroes are $(\sqrt{5}-4)$and$(\sqrt{5}+4)$.

49. If the sum of $LCM$ and $HCF$ of two numbers is 1260 and the $LCM$ is 900 more than their $HCF$, find their $LCM$.

50. Find the values of m and n for which x=2 and x=3 are the roots of the quadratic equation $3x^2 - 2mx + 2n = 0$.

51. Divide 19 into two parts such that sum of their squares is 193.

52. The angles of depression of the top and bottom of an 8 m tall building from the top of a multi-storeyed building are $30^\circ$ and $45^\circ$ respectively. Find the height of the multi-storeyed building.

53. From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed on the top of a 20m high building are $45^\circ$ and $60^\circ$ respectively. Find the height of the tower.

54. As observed from the top of 75m high lighthouse from the sea-level,the angles of depression of two ships are $30^\circ$ and $45^\circ$. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

55. It takes 12 hours to fill a swimming pool using two pipes together. If the larger pipe is used for 4 hours and smaller pipe is used for 9 hours, only half of the pool is filled. How long will it take for each pipe alone to fill the pool?

## 8.6. 2021

### 8.6.1. 12

1. Two angles of a triangle are $\cot^{-1} 2$ and $\cot^{-1} 3$.The third angle of the triangle is ______

2. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

3. Akbar invested ₹6060 in the shares of face value ₹100 each of a company.At the end of the year,the company declared dividend of 15 % which gave him an income of ₹600.At what price was the share quoted if the brokerage was 1% ?

4. sin [[] $\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})$] is equal to:

   (a) $\frac{1}{2}$

   (b) $\frac{1}{3}$

   (c) -1

   (d) 1

5. $\sin(\tan^{-1} x)$,where $|x| \leq 1$,is equal to:

   (a) $\frac{x}{\sqrt{1-x^2}}$

   (b) $\frac{1}{\sqrt{1-x^2}}$

   (c) $\frac{1}{\sqrt{1+x^2}}$

   (d) $\frac{x}{\sqrt{1+x^2}}$

6. Simplest form of $\tan^{-1}(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}), \pi < x < \frac{3\pi}{2}$ is:

   (a) $\frac{\pi}{4} - \frac{x}{2}$

   (b) $\frac{3\pi}{2} - \frac{x}{2}$

   (c) $-\frac{x}{2}$

   (d) $\pi - \frac{x}{2}$

# 8.7. 2019

## 8.7.1. 12

1. Find the value of $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$.

2. Solve for x:

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$$

3. Prove that :

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

4. If $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right), x > 0$ ,find the value of $x$ and hence find the value of $\sec^{-1}\left(\frac{2}{x}\right)$.

5. If

$$\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2}$$

then find the value of $x$.

6. Find the value of $x$, if $\tan\left(\sec^{-1}\left(\frac{1}{x}\right)\right) = \sin\left(\tan^{-1} 2\right), x > 0$.

7. Prove that

$$\sin^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{12} + \cos^{-1}\frac{63}{65} = \frac{\pi}{2}$$

# 8.8. 2019

## 8.8.1. 10

1. Obtain all the zeroes of the polynomial $2x^4 - 5x^3 - 11x^2 + 20x + 12$ when 2 and $-2$ are two zeroes of the above polynomial.

2. Find the quadratic polynomial, sum and product of whose zeroes are $-1$ and $-20$ respectively. Also find the zeroes of the polynomial so obtained.

3. Sum of the areas of two squares is $157m^2$. If the sum of their perimeters is $68m$, find the sides of the two squares.

4. A plane left 30 minutes later than the scheduled time and in order to reach its destination $1500km$ away on time, it has to increase its speed

by $250 km/hr$ from its usual speed. Find the usual speed of the plane.

5. A motorboat whose speed is 18 km/hr in still water takes one hour more to go $24km$ upstream than to return downstream to the same spot. Find the speed of the stream.

6. Solve for $x$ :

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}; x \neq 0, x \neq \frac{-2a-b}{2}, a, b \neq 0$$

7. The sum of the areas of two squares is $640m^2$. If the difference of their perimeters is $64m$, find the sides of the square.

8. For what values of $k$ does the quadratic equation $4x^2 - 12x - k = 0$ have no real roots ?

9. Evaluate:

$$\frac{\tan 65^\circ}{\cot 25^\circ}$$

10. Express $(\sin 67^\circ + \cos 75^\circ)$ in terms of trigonometric ratios of the angle between $0^\circ$ and $45^\circ$.

11. Prove that :

$$(\sin\theta + 1 + \cos\theta)(\sin\theta - 1 + \cos\theta) . \sec\theta \csc\theta = 2$$

12. Prove that :

$$\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\csc\theta$$

13. If $\sec\theta + \tan\theta = m$, show that $\frac{m^2-1}{m^2+1} = \sin\theta$.

14. Prove that :

$$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$$

15. Find $A$ and $B$ if $\sin(A+2B) = \frac{\sqrt{3}}{2}$ and $\cos(A+4B) = 0$, where $A$ and $B$ are acute angles.

16. For what value of $k$, is the polynomial $f(x) = 3x^4 - 9x^3 + x^2 + 15x + k$ completely divisible by $3x^2 - 5$ ?

17. The total cost of a certain length of a piece of cloth is ₹200. If the piece was $5m$ longer and each metre of cloth costs ₹2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre ?

18. In a class test, the sum of Arun's marks in Hindi and English is 0. Had he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

19. Find the nature of the roots of the quadratic equation

$$4x^2 + 4\sqrt{3}x + 3 = 0$$

20. Write all the values of $p$ for which the quadratic equation $x^2+px+16 = 0$ has equal roots. Find the roots of the equation so obtained.

21. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

22. Solve for $x$ :

$$x^2 + 5x - \left(a^2 + a - 6\right) = 0$$

23. Find the nature of roots of the quadratic equation $2x^2 - 4x - 3 = 0$.

24. Evaluate :

$$\sin^2 60^\circ + 2\tan 45^\circ - \cos^2 30^\circ.$$

25. Evaluate :

$$\left(\frac{3\tan 41^\circ}{\cot 90^\circ}\right)^2 - \left(\frac{\sin 3^\circ \sec 55^\circ}{\tan 10^\circ \tan 20^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ}\right)^2$$

26. Prove that :

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\csc\theta$$

27. Prove that :

$$\frac{\sin\theta}{\cot\theta+\csc\theta} = 2 + \frac{\sin\theta}{\cot\theta-\csc\theta}$$

28. Evaluate :

$$\left(\frac{3\sin 43^\circ}{\cos 47^\circ}\right)^2 - \frac{\cos 37^\circ \csc 53^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$$

29. The larger of two supplementary angles exceeds the smaller by $18^\circ$. Find the angles.

30. If $\sin A = \frac{3}{4}$, calculate $\sec A$.

31. Write the discriminant of the quadratic equation $(x+5)^2 = 2\,(5x-3)$.

32. Using completing the square method, show that the equation $x^2-8x+18=0$ has no solution.

33. Check whether $g\,(x)$ is a factor of $p\,(x)$ by dividing polynomial $p\,(x)$ by polynomial $g\,(x)$,where $p\,(x) = x^5-4x^3+x^2+3x+1$ , $g\,(x) = x^3-3x+1$

34. If $\frac{2}{3}$ and $-3$ are the zeroes of the polynomial $ax^2+7x+b$, then find the values of $a$ and $b$.

35. If $\tan\alpha = \frac{5}{12}$, find the value of $\sec\alpha$.

36. $A$, $B$ and $C$ are interior angles of a triangle $ABC$. Show that

    (a) $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

    (b) If $\angle A = 90^\circ$ , then find the value of $\tan\left(\frac{B+C}{2}\right)$

37. If $\tan(A+B) = 1$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$ , $0° < A+B < 90°$, $A > B$, then find the values of $A$ and $B$.

38. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$ or $\tan\theta = \frac{1}{2}$

39. Prove that :

$$\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\csc\theta - 2\sin\theta\cos\theta$$

40. A motorboat whose speed in still water is $9km/h$, goes $15km$ downstream and comes back to the same spot, in a total time of $3hours$ $45minutes$. Find the speed of the stream.

41. Find all the zeroes of the polynomial $x^4 + x^3 - 14x^2 - 2x + 24$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$ .

42. Apply division algorithm to check if $g(x) = x^2 - 3x + 2$ is a factor of the polynomial $f(x) = x^4 - 2x^3 - x + 2$.

43. A train travels $360km$ at a uniform speed. If the speed had been $5km/hr$ more, it would have taken $1hr$ less for the same journey. Find the speed of the train.

44. Find the value of $k$ for which $x = 2$ is a solution of the equation $kx^2 + 2x - 3 = 0$.

45. Find the value/s of $k$ for which the quadratic equation $3x^2+kx+3 = 0$ has real and equal roots.

46. Solve for $x$ :

$$\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}; a\neq b\neq 0, x\neq 0, x\neq -(a+b)$$

47. If $\sin x+\cos y=1$ ; $x=30^\circ$ and $y$ is an acute angle, find the value of $y$ .

48. Find the value of $\cos 48^\circ-\sin 42^\circ$.

49. Prove that :

$$\frac{\tan\theta}{1-\tan\theta}-\frac{\cot\theta}{1-\cot\theta}=\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$$

50. If $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$, show that $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$.

51. Prove that :

$$\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{(\sec^3\theta-\csc^3\theta)}=\sin^2\theta\cos^2\theta$$

52. Evaluate :

$$\frac{\csc^2(90^\circ-\theta)-\tan^2\theta)}{2(\cos^2 37^\circ+\cos^2 53^\circ)}-\frac{2\tan^2 30^\circ\sec^2 37^\circ\cdot\sin^2 53^\circ}{\csc^2 63^\circ-\tan^2 27^\circ}$$

53. For what values of $k$, the roots of the equation $x^2+4x+k=0$ are real?

54. Find the value of $k$ for which the roots of the equation $3x^2-10x+k=0$ are reciprocal of each other.

55. How many two digits numbers are divisible by 3 ?

56. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

57. Find $c$ if the system of equations $cx+3y+(3-c)=0$; $12x+cy-c=0$ has infinitely many solutions.

58. Find the value of $k$ such that the polynomial $x^2-(k+6)\,x+2\,(2k-1)$ has sum of its zeros equal to half of their product.

59. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer 8 diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

60. Prove that $\sqrt{2}$ is an irrational number.

61. Prove that

$$(\sin\theta+\csc\theta)^2+(\cos\theta+\sec\theta)^2=7+\tan^2\theta+\cot^2\theta$$

.

62. Prove that

$$(1+\cot A-\csc A)\,(1+\tan A+\sec A)=2$$

.

63. Prove that

$$\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$$

64. Find $A$ if

$$\tan 2A = \cot(A - 24^\circ)$$

65. Find the value of

$$(\sin^2 33^\circ + \sin^2 57^\circ)$$

66. Find all zeros of the polynomial $3x^3 + 10x^2 - 9x - 4$ if one of its zero is 1.

67. if If $\sec\theta = x + \frac{1}{4x}$, where $x \neq 0$, find $(\sec\theta + \tan\theta)$.

68. prove that

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\csc^2 A}{\sec^2 A - \csc^2 A} = \frac{1}{1 - 2\cos^2 A}$$

# 8.9. 2018

## 8.9.1. 10

1. If $x = 3$ is one of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of $k$.

2. Find all zeroes of the polynomial $\left(2x^4 - 9x^3 + 5x^2 + 3x - 1\right)$ if two of its zeroes are $\left(2 + \sqrt{3}\right)$ and $\left(2 - \sqrt{3}\right)$.

3. If $4 \tan\theta = 3$, evaluate

$$\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$$

4. If $\tan 2A = \cot(A - 18°)$, where $2A$ is an acute angle, find the value of $A$.

5. What is the value of $\left(\cos^2 67° - \sin^2 23°\right)$ ?

6. Prove that: $\left(\frac{\sin A - 2sin^3 A}{2\cos^3 A - \cos A} = \tan A\right)$

7. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by $100km/h$ from the usual speed. Find its usual speed.

8. A motor boat whose speed is $18km/hr$ in still water takes $1hr$ more to go $24km$ upstream than to return downstream to the same spot. Find the speed of the stream.

9. A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of $6km/hr$ more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed?

10. $ABCD$ is a rectangle. Find the values of $x$ and $y$. Fig. 8.7

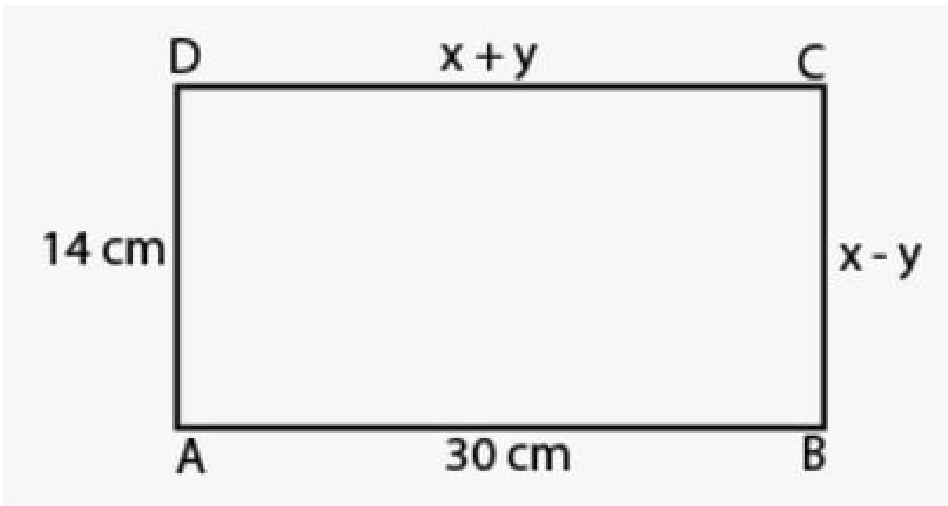



Figure 8.7: rect ABCD

## 8.10. Discrete

11. what is the HCF of smallest prime number and the smallest composite number?

12. Given that $\sqrt{2}$ is irrational, prove that $\left(5+3\sqrt{2}\right)$ is an irrational number.

13. Find the sum of 8 multiples of 3.

14. Find the HCF and LCM of 404 and 96 and verify that HCF*LCM = product of the given numbers.

15. In an $AP$, if the common difference $(d) = -4$, and the seventh term$(a_7)$ is 4, then find the first term.

16. The sum of four consecuive numbers in an AP is 32 and the ratio of

the product of the first and the last term to the product of two middle term is $7:15$. Find the numbers.

### 8.10.1. 12

1. Find the value of

$$\tan^{-1}\sqrt{3}-\cot^{-1}\left(\sqrt{-3}\right)$$

2. Prove that

$$3\sin^{-1}x=\sin^{-1}\left(3x-4x^{3}\right),x\in\left(\frac{-1}{2},\frac{1}{2}\right)$$

3. Prove that :

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$$

4. Prove that :

$$\sin^{-1}\left(\frac{8}{17}\right)+\cos^{-1}\left(\frac{4}{5}\right)=\cot^{-1}\left(\frac{36}{77}\right)\tag{8.47}$$

5. Let an operation $*$ on the set of natural numbers $N$ be defined by $a*b=a^{b}$. Find

   (a) whether $*$ is a binary or not, and

   (b) if it is a binary, then is it commutative or not.

6. Prove that :

$$\sin^{-1}\frac{4}{5}+\tan^{-1}\frac{5}{12}+\cos^{-1}\frac{63}{65}=\frac{\pi}{2}$$

7. Find the coordinates of the foot $Q$ of the perpendicular drawn from the point $P\,(1,3,4)$ to the plane $2x-y+z+3=0$. Find the distance $PQ$ and the image of $P$ treating the planes as a mirror.

8. If $\tan^{-1}x-\cot^{-1}x=tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $x>0$, find the value of $x$ and hence find the value of $\sec^{-1}\left(\frac{2}{x}\right)$.

9. If $\sin^{-1}\left(\frac{3}{x}\right)+\sin^{-1}\left(\frac{4}{x}\right)=\frac{\pi}{2}$, then find the value of $x$.

10. Find the value of $x$, if $\tan\left(\sec^{-1}\left(\frac{1}{x}\right)\right)=\sin\left(\tan^{-1}2\right),x>0.$

11. Find the value of $\sin\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$

12. Solve for $x$:

$$\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{8}{31}\right)$$

13. Solve : $\tan^{-1}4x+\tan^{-1}6x=\frac{\pi}{4}$

14. If $\log\left(x^2+y^2\right)=2\tan^{-1}\frac{y}{x}$,show that $\frac{dy}{dx}=\frac{x+y}{x-y}$

15. If $x^y-y^x=a^b$,find $\frac{dy}{dx}$

16. Find : $\int\frac{\tan^2x\sec^2x}{1-\tan^6x}dx$

17. Solve for x : $\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}$

18. If $y=\left(\sin^{-1}x\right)^2$,prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0$

## 8.11. 2017

### 8.11.1. 10

### 8.11.2. 12

1. if $\tan^{-1}\dfrac{x-3}{x-4}+\tan^{-1}\dfrac{x+3}{x+4}=\dfrac{\pi}{4}$, then find the value of $x$ .

2. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is $10m$. Find the dimensions of the window to admit maximum light through the whole opening.

## 8.12. 2016

### 8.12.1. 10

1. If $-5$ is a root of the quadratic equation $2x^2+px-15=0$ and the quadratic equation $p\left(x^2+x\right)+k=0$ has equal roots, find the value of $k$.

2. Solve for $x$ :

$$\sqrt{2x+9}+x=13$$

3. Solve for $x$ :

$$\sqrt{6x+7}-(2x-7)=0$$

4. If the roots of the quadratic equation $(a-b)\,x^2+(b-c)\,x+(c-a)=0$ are equal,prove that $2a=b+c$.

5. Solve for $x$ :

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}, x\neq 1,2,3$$

6. Solve for $x$ :

$$\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}, x\neq -1,-2,-4$$

7. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

8. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers.

9. Two pipes running together can fill a tank in $11\frac{1}{5}$ minutes. If one pipe

takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

### 8.12.2. 12

1. Prove that $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

2. Solve the equation for $x$: $\cos(\tan^{-1}) = \sin\left(\cot^{-1}\frac{3}{4}\right) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

3. Solve for

$$x : \tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x$$

4. Prove that

$$\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) - \tan^{-1}\left(\frac{4x}{1-4x^2}\right) = \tan^{-1} 2x;$$

$$|2x| < \frac{1}{\sqrt{3}}$$

5. The equation of tangent at $(2,3)$ on the curve $y^2 = ax^3 + b$ is $4x - 5$. Find the values of $a$ and $b$.

6. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of square bounded by $x = 0, x = 4, y = 4$ and $y = 0$ into three equal parts.

7. Find the equation of the tangent line to the curve $y = \sqrt{5x-3} - 5$, which is parallel to the line $4x - 2y + 5 = 0$.

## 8.13. 2015

### 8.13.1. 10

1. If the quadratic equation

$$px^2–2\sqrt{5}px + 15 = 0$$

has two equal roots,then find the value of $p$.

2. In an AP, if

$$S_5 + S_7 = 167$$
$$S_{10} = 235$$

then find the AP, where $S_n$ denotes the sum of its first $n$ terms.

3. Solve for x :

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$

4. Solve the following quadratic equation for $x$:

$$4x^2 + 4bx–(a^2–b^2) = 0$$

5. The $13^{th}$ term of an AP is four times its $3^{rd}$ term. If its fifth term is 16, then find the sum of its first ten terms.

6. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck.

7. An arithmetic progression $5, 12, 19, ...$ has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

# Chapter 9

# Geometry

## 9.1. 2023

### 9.1.1. 10

1. The hour-hand of a clock is 6 cm long.The angle swept by it between $7:20$ a.m. and $7:55$ a.m. is:

   (a) $\left(\frac{35}{4}\right)^\circ$

   (b) $\left(\frac{35}{2}\right)^\circ$

   (c) $35^\circ$

   (d) $70^\circ$

2. In the given Fig. 9.1, $AB \parallel PQ$.If $AB = 6$ cm,$PQ = 2$ cm and $OB = 3$ cm,then the length of $OP$ is:

   (a) 9cm

   (b) 3cm

   (c) 4cm

(d) 1cm

3. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower.Find the angle of elevation of the sun.

4. The angle of elevation of the top of a tower from a point on the ground which is 30 m away from the foot of the tower,is 30° .Find the height of the tower.

5. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle of 120°. Find the total area cleaned at each sweep of the two blades.

6. As observed from the top of a 75 m high lighthouse from the sea-level,the angles of depression of two ships are 30° and 60°.If one ship is exactly behind the other on the same side of the lighthouse,find the distance between two ships.$\left(Use\sqrt{3} = 1.73\right)$

7. From a point on the ground,the angle of elevation of the bottom and top of a transmission tower fixed at the top of 30 m high building are 30° and 60°, respectively.Find the height of the transmission tower.$\left(Use\sqrt{3} = 1.73\right)$

8. Sides $AB$ and $BC$ and median $AD$ of a triangle $ABC$ are respectively proportional to sides $PQ$ and $QR$ and median $PM$ of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.

9. Through the mid-point $M$ of the side $CD$ of a parallelogram $ABCD$,the line $BM$ is drawn intersecting $AC$ in $L$ and $AD$(produced) in $E$.Prove

that

$$EL = 2BL. \tag{9.1}$$

10. In an annual day function of a school,the organizers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in the Fig. 9.2 and its base $ABCD$ is shown from the front side. The rate of silver plating is ₹ 20 $per\ cm^2$.

    Based on the above, answer the following questions:

    (i) What is the area of the quadrant $ODCO$?

    (ii) Find the area of $\triangle AOB$.

    (iii) What is the total cost of silver plating the shaded part $ABCD$?

    (iv) what is the length of arc $CD$?

11. What is the length of the arc of the sector of a circle with radius 14 cm and of central angle 90°.

    (a) 22 cm

    (b) 44 cm

    (c) 88 cm

    (d) 11 cm

12. if$\triangle ABC \sim \triangle PQR$ with $\angle A = 32°$ and $\angle R = 65°$,then the measure of $\angle B$ is:

(a) $32^\circ$

(b) $65^\circ$

(c) $83^\circ$

(d) $97^\circ$

13. What is the total surface area of a solid hemisphere of diameter $'d'$?

(a) $3\pi d^2$

(b) $2\pi d^2$

(c) $\frac{1}{2}\pi d^2$

(d) $\frac{3}{4}\pi d^2$

14. In $\triangle ABC$,$DE \parallel BC$.if $AD = 2$ units,$DB = AE = 3$ units and $EC = x$ units,then the value of $x$ is :

(a) 2

(b) 3

(c) 5

(d) $\frac{9}{2}$

15. A straight highway leads to the foot of a tower.A man standing on the top of the 75 m high tower observes two cars at angles of depression of $30^\circ$ and $60^\circ$,Which are approaching the foot of the tower.If one car is exactly behind the other on the same side of the tower,find the distance between the two cars.

16. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is $60°$ and the angle of depression of its foot is $30°$.Determine the height of the tower.(take $\sqrt{3} = 1.73$)

17. Governing council of local public development authority of Dehradun decided to build and adventurous playground on the top of a hill,Which will have adequate space for parking. After survey,it was decided to build rectangular playground,with a semi-circular area allocated for parking at one end of the playground.The length and breadth of the rectangular playground are 14 units and 7 units,respectively.There are two quadrants of radius 2 units on one side for special seats:

    (a) What is the total perimeter of the parking area?

    (b) What is the total area of parking and the two quadrants?

    (c) What is the ratio of area of playground to the area of parking area?

    (d) Find the cost of fencing the playground and parking area at the rate of ₹2 per unit.

18. What is the total surface area of a solid hemisphere of diameter $'d'$?

    (a) $3\pi d^2$

    (b) $2\pi d^2$

    (c) $\frac{1}{2}\pi d^2$

    (d) $\frac{3}{4}\pi d^2$

19. In the given Fig. 9.5, $DE \parallel BC$. If $AD$=2 units, $DB = AE = 3$ units and $EC = x$ units, then the value of $x$ is:

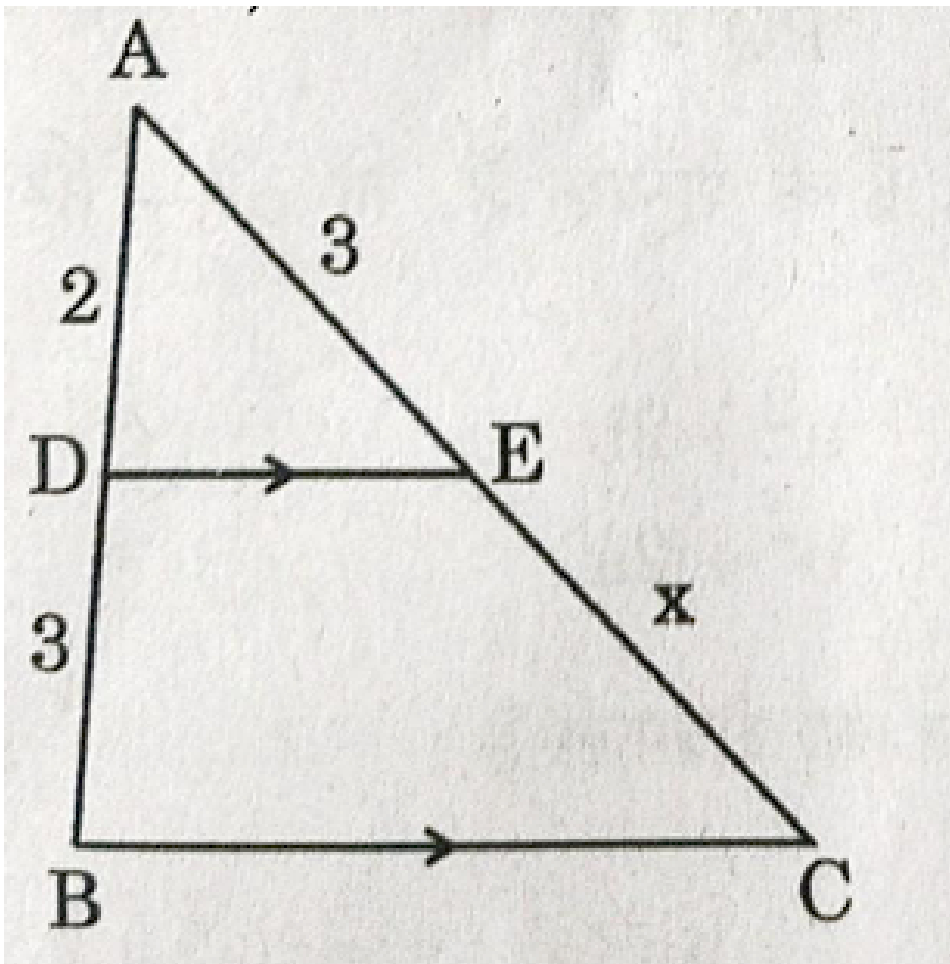



Figure 9.5:

(a) 2

(b) 3

(c) 5

(d) $\frac{9}{2}$

20. In the given Fig. 9.6, $XZ$ is parallel to $BC$. $AZ = 3$ cm, $ZC = 2$ cm, $BM = 3$ cm, and $MC = 5$ cm. Find the length of $XY$.

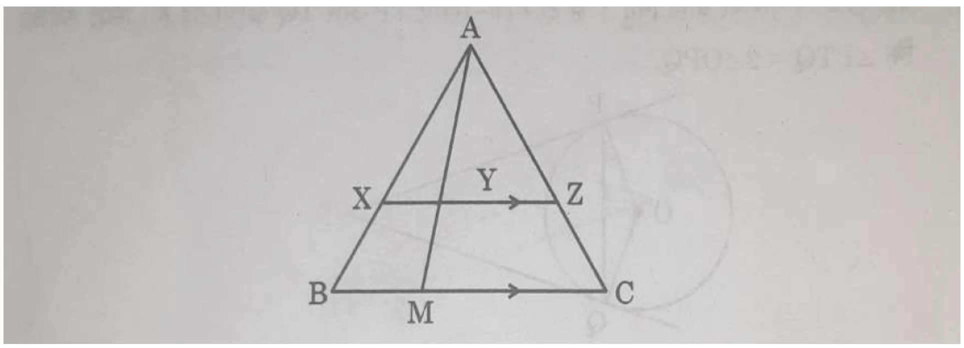



Figure 9.6:

21. A room is in the form of a cylinder surmounted by a hemi-spherical dome. The base radius of hemisphere is one-half the height of cylindrical part. Find total height of the room if it contains $\left(\frac{1408}{21}\right)$ $m^3$ of air.Take $\left(\pi = \frac{22}{7}\right)$

22. In the given Fig. 9.7, An empty cone is of radius 3 cm and height 12 cm. Ice-cream is filled so that lower part of the cone which is $\left(\frac{1}{6}\right)$th of the volume of the cone is unfilled but hemisphere is formed on the top. Find volume of the ice-cream.Take($\pi = 3.14$)

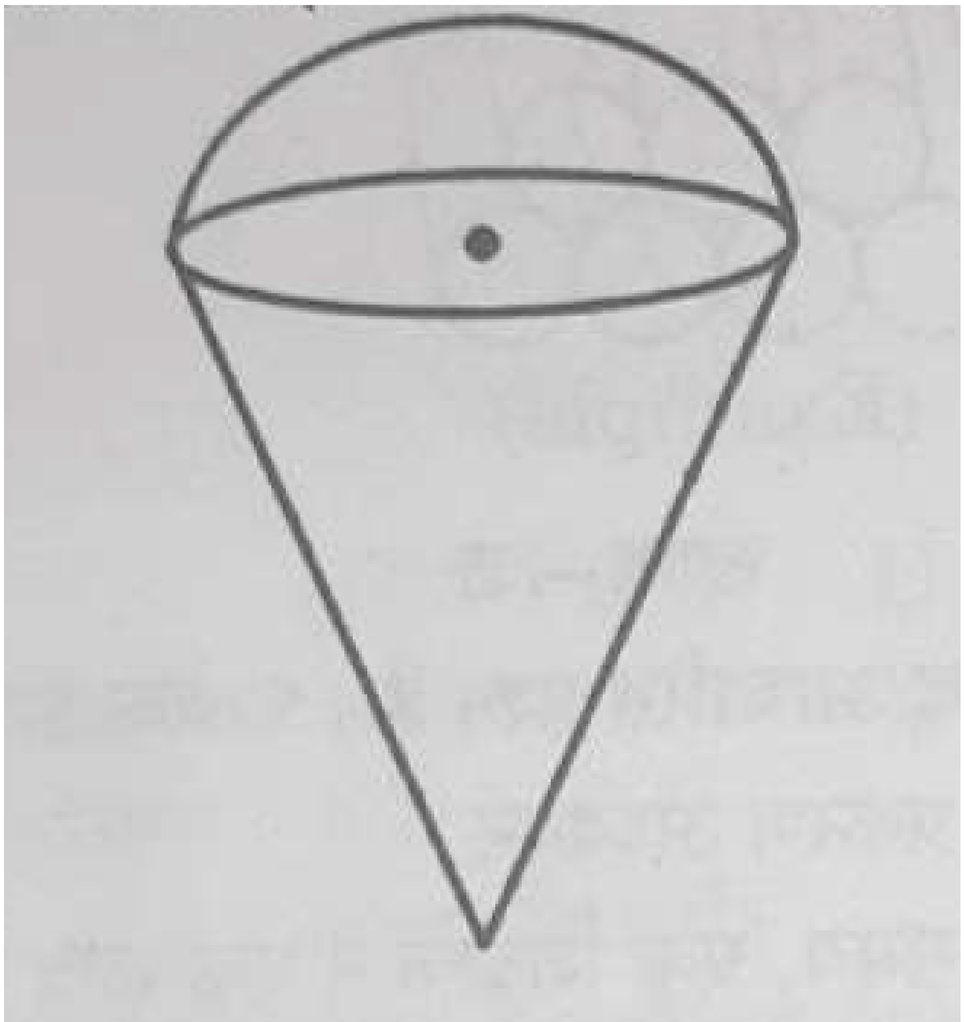


Figure 9.7:

23. If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio.

24. The angle of elevation of the top of a tower 24 m high from the foot of

another tower in the same plane is $60^\circ$. The angle of elevation of the top of second tower from the foot of the first tower is $30^\circ$. Find the distance between two towers and the height of the other tower. Also, find the length of the wire attached to the tops of both the towers.

25. A spherical balloon of radius $r$ subtends an angle of $60^\circ$ at the eye of an observer. If the angle of elevation of its centre is $45^\circ$ from the same point, then prove that height of the centre of the balloon is $\sqrt{2}$ times its radius.

26. A chord of a circle of radius 14 cm subtends an angle of $60^\circ$ at the centre. Find the area of the corresponding minor segment of the circle. Also find the area of the major segment of the circle.

## 9.2. 2022

### 9.2.1. 10

1. A solid spherical ball fits exactly inside the cubical box of side $2a$. The volume of the ball is

   (a) $\frac{16}{3}\pi a^3$

   (b) $\frac{1}{6}\pi a^3$

   (c) $\frac{32}{3}\pi a^3$

   (d) $\frac{4}{3}\pi a^3$

2. A frustum of a right circular cone which is of height $8cm$ with radii of its circular ends as $10cm$ and $4cm$, has its slant height equal to

   (a) $14cm$

   (b) $28cm$

   (c) $10cm$

   (d) $\sqrt{260}cm$

3. The capacity of a cylindrical glass tumbler is $125.6cm^3$. If the radius of the glass tumbler is $2cm$, then find its height. (Use $\pi = 3.14$)

4. A mint moulds four types of copper coins $A$,$B$,$C$ and $D$ whose diameters vary from $0.5cm$ to $5cm$. The first coin A has a diameter of $0.7cm$. The second coin $B$ has double the diameter of coin $A$ and from then onwards the diameters increase by 50%. Thickness of each coin is $0.25cm$. After reading the above, answer the following questions :

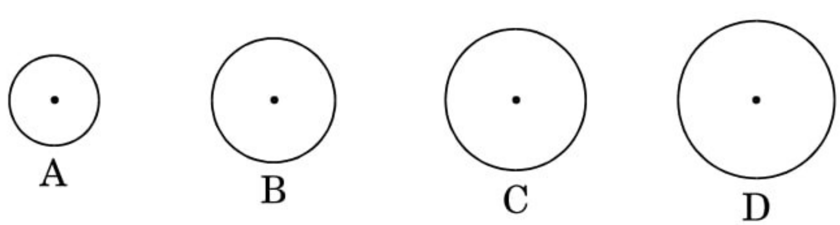



Figure 9.8: Cirls

- Fill in the diameters of the coins required in the following table :

| Type of Coin | Diameter (in cm) |
|---|---|
| A | 0·7 |
| B | --- |

Figure 9.9: table

- Complete the following table :

| Type of Coin | Area (in cm$^2$) of one face | Volume (in cm$^3$) |
|---|---|---|
| A | 0·385 | 0·09625 |
| B | --- | --- |

Figure 9.10: table2

5. A well of diameter $3m$ is dug $14m$ deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width $4m$ to form a platform. Find the height of the platform. (Take $\pi = \frac{22}{7}$)

6. A solid metallic sphere of radius $10.5cm$ is melted and recast into a number of smaller cones, each of radius $3.5cm$ and height $3cm$. Find the number of cones so formed.

7. In Figure 2, from a solid cube of side $7cm$, a cylinder of radius $2.1cm$

and height $7cm$ is scooped out. Find the total surface area of the remaining solid.

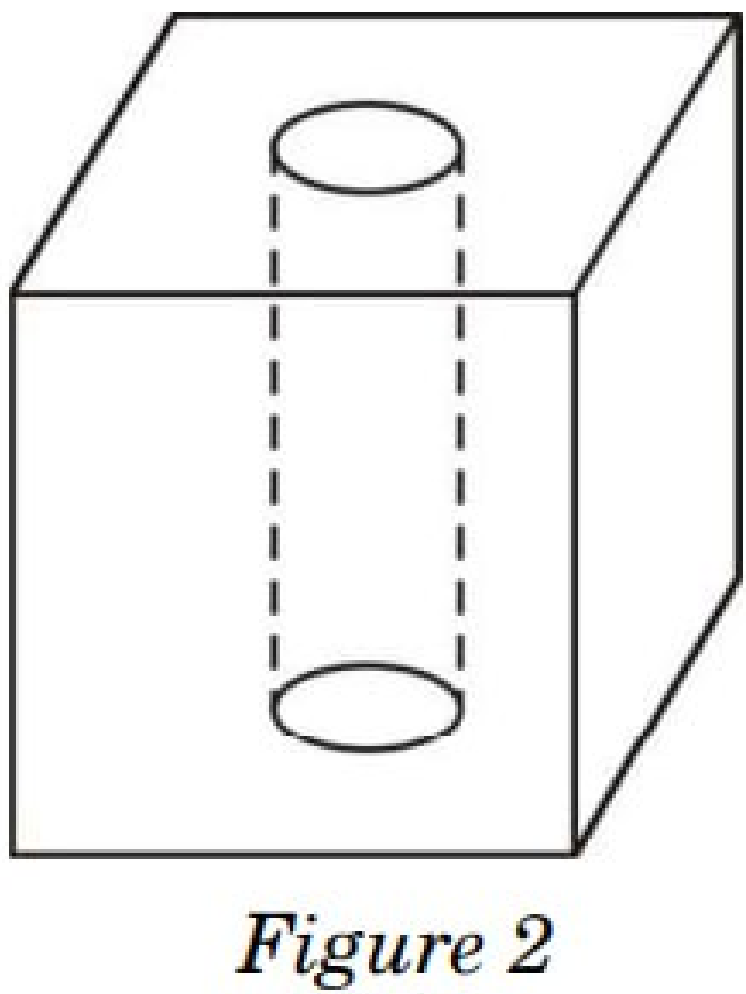



Figure 9.11: soildcube

8. A well of diameter $5m$ is dug $24m$ deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width

$3m$ to form an embankment. Find the height of the embankment.

9. A solid piece of metal in the form of a cuboid of dimensions $11cm \times 7cm \times 7cm$ is melted to form n number of solid spheres of radii $\frac{7}{2}cm$ each. Find the value of $n$.

10. A 'circus' is a company of performers who put on shows of acrobats, clowns etc. to entertain people started around 250 years back, in open fields, now generally performed in tents

    One such 'Circus Tent' is shown below.

Figure 9.12: circustent

The tent is in the shape of a cylinder surmounted by a conical top. if the height and diameter of cylinder part are $9m$ and $30m$ respectively and height of conical part is 8 cm with same diameter as that of the cylindrical part, then find.

(a) The area of the canvas used in making the tent

(b) The cost of the canvas bought for the tent at the rate ₹ 200 per sq. m, if 30 sq m canvas was wasted during stitching.

11. (a) 150 spherical marbles, each of diameter $1.4cm$ are dropped in a cylinder vessel of diameter $7cm$ containing some water, and are completely immersed in water.Find the rise in the level of water in the cylindrical vessel

(b) Three cubes of side $6cm$ each,are joined as shown in Figure 2

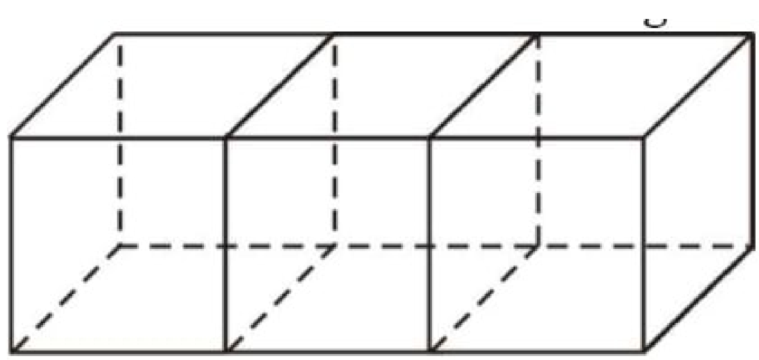


*Figure 2*

Figure 9.13: Cuboid

12. In the picture given below, one can see a rectangular in-ground swimming pool installed by a family in their backyard. There is a concrete sidewalk around the pool of width $xm$. The outside edges of the sidewalk measure $7m$ and $12m$. The area of the pool is $36sq.m$.

Figure 9.14: swimmingpool

(a) Based on the information given above, form a quadratic equation in terms of $x$

(b) Find the width of the sidewalk around the pool.

13. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions:

Cake : Cylindrical shape with diameter $24cm$ and height $14cm$.

Cap : Conical shape with base circumference $44cm$ and height $24cm$.

Figure 9.15: birthdaycake

Based on the above information, answer the following questions :

(a) How many square cm paper would be used to make 4 such caps ?

(b) The bakery shop sells cakes by weight ($0.5kg, 1kg, 1.5kg, etc.$). To have the required dimensions, how much cake should they order, if $650cm^3$ equals $100g$ of cake ?

14. (a) The curved surface area of a right circular cylinder is $176sqcm$ and its volume is$1232cucm$ what is the height of the cylinder?

    (b) The largest sphere is carved out of a solid cube of side $21cm$ Find the volume of sphere

15. Khurja is a city in the Indian state of Uttar Pradesh famous for the pottery.Khurja pottery is traditional Indian pottery work which has at-

tracted Indians as well as foreighners with a variety of tea sets,crockery and ceramic tile works. A huge portion of the ceramics used in the country is supplied by Khurja and is also referred as 'The Ceramic Town' One of the private schools of Bulandshahr organised an Educational Tour for class 10 students to Khurja. Students were very excited about the trip. Following are the few pottery objects of Khurja

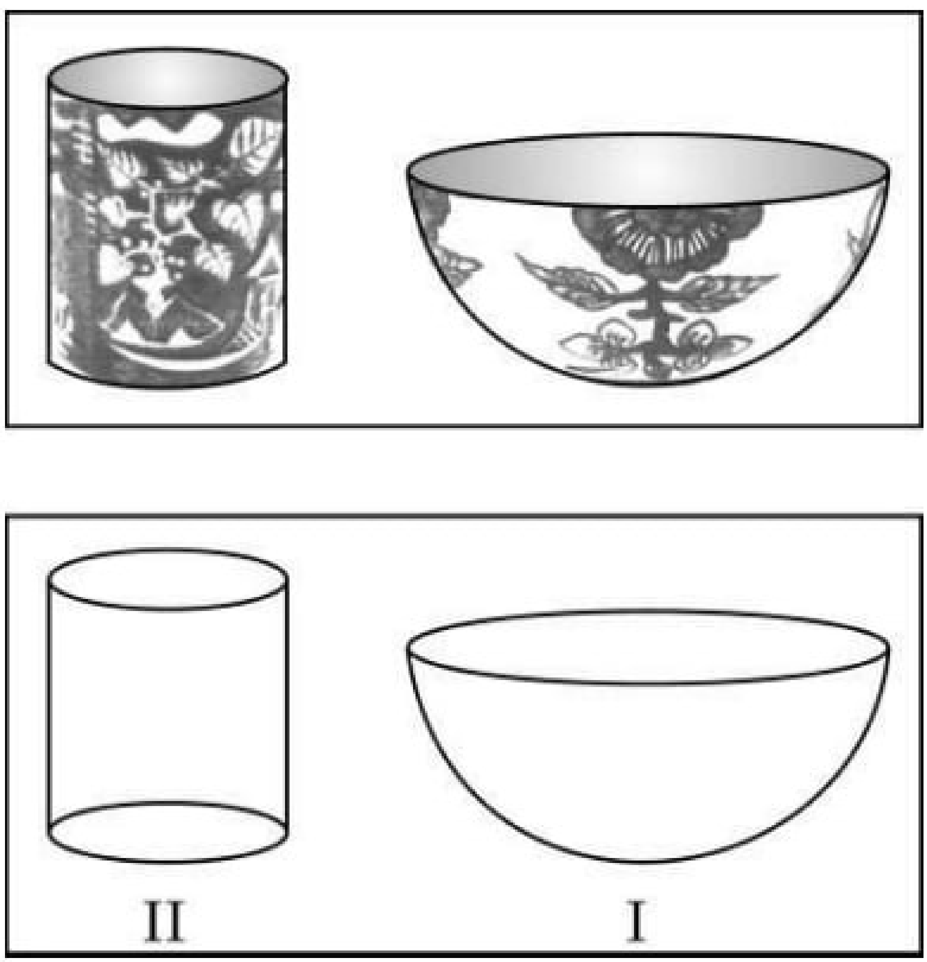



Figure 9.16: pottery

Students found the shapes of the objects very interesting and they could easily relate them with mathematical shapes viz sphere, hemisphere, cylinder etc. Maths teacher who was accompanying the students asked following question:

(a) The internal radius of hemispherical bowl(filled completely with water)in I is $9cm$ and radius and height of cylindrical jar in II is $1.5cm$ and $4cm$ respectively. If the hemisherical bowl is to be emptied in cylindrical jars, then how many cylindrical jars are required?

(b) If in the cylindrical jar full of water, a conical funnel of same height and same diameter is immersed, then how much water will flow out of the jar?

16. How many spherical shots each having diameter $3cm$ can be made by melting a cuboidal solid of dimensions $18cm \times 22cm \times 6cm$ ?

17. Conical bottom tanks in which an inverted cone at the bottom is surmounted by a cylinder of same diameter, are very advantageous in industry, specially where getting every last drop from the tank is important.

Vikas designed a conical bottom tank where the height of the conical part is equal to its radius and the height of the cylindrical part is two times of its radius. The tank is closed from the top.

(a) If the radius of the cylindrical part is $3m$, then find the volume of the tank.

(b) Find the ratio of the volume of the cylindrical part to the volume of the conical part.

# 9.3. 2021

## 9.3.1. 10

1. (a) Write the expression for the volume of the cone of radius 'r' and height three times the radius 'r'.

   (b) Write the expression for total surface area of a solid hemisphere of radius 'r'.

2. A vertical pole is 100 metres high. Find the angle subtended by the pole at a point on the ground $100\sqrt{3}$ meters from the base of the pole.

3. (a) Find the area of that sector of a circle of radius 3.5cm whose central angle is $90°$.

   (b) The length of the minute hand of a clock is 14 cm.Find the area swept by the minute hand in 5 minutes.
   (Take $\pi = \frac{22}{7}$)

4. A semicircular ground of radius 1.5 m is to be fenced with wire. Find the cost of wiring at the rate of ₹30 per metre.

5. (a) The angle of elevation of the top of a tower from a point is found to be $60°$.At a point 40 m above the first point, the angle of

elevation of the top of the tower is 45°.Find the height of the tower.

(b) A statue 1.6m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°.Find the height of the pedestal.

6. The areas of two similar triangles are $121cm^2$ and $64cm^2$ respectively. If one median of the first triangle is 12.1 cm long, then find the length of the corresponding median of the other triangle.

7. (a) In a triangle $ABC$, a line is drawn parallel to base $BC$ meeting $AB$ in $D$ and $AC$ at $E$. If $\frac{AB}{BD} = 4$ and $CE = 2cm$, $BD$ then find the value of $AE$.

   (b) Two poles, 6m and 11 m high, stand vertically on the ground. If the distance between their feet is 12 m, find the distance between their tops.

8. Answer any **four** of the following questions :

   (i) If the sum of the areas of two circles with radii $r1$ and $r2$ is equal to the area of a circle of radius $r$, then

   (A) $r_1 + r_2 = r$

   (B) ${r_1}^2 + {r_2}^2 = r^2$

   (C) $r_1 + r_2 < r$

   (D) ${r_1}^2 + {r_2}^2 < r^2$

(ii) The area of a circle that can be inscribed in a square of side 8 cm is

(A) $64\pi cm^2$

(B) $24\pi cm^2$

(C) $16\pi cm^2$

(D) $8\pi cm^2$

(iii) The area of a square that can be inscribed in a circle of radius 6 cm is

(A) $36\ cm^2$

(B) $72\ cm^2$

(C) $18\ cm^2$

(D) $32\sqrt{2}\ cm^2$

(iv) The radius of a circle whose circumference is equal to the sum of the circumferences of two circles of diameters 36 cm and 20cm is

(A) $56cm$

(B) $42cm$

(C) $28cm$

(D) $16cm$

(v) If the circumference of a circle is equal to the perimeter of a square, then the ratio of their areas is

(A) 22 : 7

(B) 14 : 11

(C) 7 : 22

(D) 11 : 24

9. A solid right circular cone is 4.1cm high and the radius of its base is 2.1 cm. Another solid right circular cone is 4.3 cm high and radius of its base is 2.1 cm. Both the cones are melted and recast into a sphere. Find the diameter of the sphere.

10. Answer any **four** of the following questions :

(i) The radius of a solid hemisphere is 'r' cm.It is divided into two equal hemispherical parts. The whole surface area of one part is

(A) $2\pi r^2$sq.cm

(B) $3\pi r^2$sq.cm

(C) $\frac{2}{3}\pi r^3$sq.cm

(D) $\frac{1}{3}\pi r^3$sq.cm

(ii) The diameter of the largest sphere that can be carved out of a cube of side 21 cm is

(A) 42cm

(B) 7cm

(C) 21cm

(D) $\frac{21}{2}$cm

(iii) The total surface area of a solid right circular cylinder having the radius of the base as 7 cm and the height as 10 cm is

(A) 154sq.cm

(B) 440sq.cm

(C) 308sq.cm

(D) 748sq.cm

(iv) A cone and a cylinder are of the same height. If the radii of their bases are in the ratio 3 : 1, then the ratio of their volumes is

(A) 1 : 1

(B) 1 : 3

(C) 3 : 1

(D) 2 : 3

(v) The slant height of a cone of radius 5 cm and height 12 cm(in cm) is

(A) 12

(B) 13

(C) 5

(D) 17

11. The angle of elevation of the top of a tower from a point on the ground,which is 30 m away from the foot of the tower is $45^\circ$ .What is the height of the tower ?

12. Find the sun's altitude if the shadow of a 15 m high tower is $15\sqrt{3}$ m.

13. A circular piece of land is 40 m in diameter. A well of diameter 16 m has been dug to a depth of 28 m and the earth taken out has been spread evenly over the remaining area. How much has the level of ground been raised ?

14. From a point on the ground, 20 m away from the foot of vertical tower, the angle of elevation of the top of the tower is $60°$. Find the height of the tower.

15. (a) In a right triangle $ABC$, right-angled at $B$, $BC = 6cm$ and $AB = 8cm$. A circle is inscribed in the $\triangle ABC$. Find the radius of the incircle.

    (b) Two cricles touch externally at $P$ and $AB$ is a common tangent, touching one circle at $A$ and the other at $B$. Find the measure of $\angle APB$.

16. A solid sphere of radius $r$ is melted and cast into the shape of a solid cone of height $r$. What is the radius of the base of the cone in terms of $r$ ?

17. Answer any four of the following questions :

    (i) $ABC$ and $BDE$ are two equilateral triangles such that $D$ is the mid-point of $BC$. The ratio of the areas of the triangles $ABC$ and $BDE$ is

    (A) $2 : 1$

    (B) $1 : 2$

    (C) $4 : 1$

    (D) $1 : 4$

    (ii) In $\triangle$ $ABC$, $AB = 4\sqrt{3}$ cm, $AC = 8cm$ and $BC = 4cm$. The angle $B$ is

(A) $120°$

(B) $90°$

(C) $60°$

(D) $45°$

(iii) The perimeters of two similar triangles are $35cm$ and $21cm$ respectively. If one side of the first triangle is $9cm$,then the corresponding side of the second triangle is

(A) $5.4cm$

(B) $4.5cm$

(C) $5.6cm$

(D) $15cm$

(iv) In a $\triangle\ ABC, D$ and $E$ are points on the sides $AB$ and $AC$ respectively such that $DE||BC$ and $AD : DB = 3 : 1$. If $AE = 3.3cm$,then AC is equal to

(A) $4cm$

(B) $1.1cm$

(C) $4.4cm$

(D) $5.5cm$

(v) In an isosceles triangle $ABC$, if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C$ is equal to

(A) $30°$

(B) $45°$

(C) $60°$

(D) $90^\circ$

18. To explain how trignometry can be used measure the height of an inaccessible object, a teacher gave the following example to students :

A TV tower stands vertically n the bank of a canal. From a point on the other bank direct opposite the tower, the angle of the elevation of the top of the tower is $60^\circ$.From another point 20 m away from this point to the foot of the tower, the angle of elevation of the top of the tower is $30^\circ$ (as shown in Figure 1).

Based on the above, answer the following questions :

(i) The width of the canal is

(A) $10\sqrt{3}m$

(B) $20\sqrt{3}m$

(C) $10m$

(D) $20m$

(ii) Height of the tower is

(A) $10\sqrt{3}m$

(B) $10m$

(C) $20\sqrt{3}m$

(D) $20m$

(iii) Distance of the foot of the tower from the point $D$ is

(A) $20m$

(B) $30m$

(C) $10m$

(D) $20\sqrt{3}m$

(iv) The angle formed by the line of sight with the horizontal when it is above the horizontal line is known as

(A) angle of depression

(B) line of sight

(C) angle of elevation

(D) obtuse angle

(v) In above figure, measure of angle $XAC$ is

(A) $30°$

(B) $60°$

(C) $90°$

(D) $45°$

19. A children's park is in the triangular shape as shown in the below figure.In the middle of the park, there is a circular region for younger children to play. It is fenced with three layers of wire. The radius of the circular region is $3m$. Based on the above,answer the following questions:

(i) The perimeter(or circumference) of the circular region is

(A) $3\pi m$

(B) $18\pi m$

(C) $6\pi m$

(D) $9\pi m$

(ii) The Total length of wire used is

(A) $9\pi m$

(B) $18\pi m$

(C) $54\pi m$

(D) $27\pi m$

(iii) The area of the circular region is

(A) $54\pi m^2$

(B) $3\pi m^2$

(C) $18\pi m^2$

(D) $9\pi m^2$

(iv) If $BD = 6m$, $DC = 9m$ and ar $(\triangle ABC = 54\ m^2$,then the length of sides $AB$ and $AC$, respectively, are)

(A) $9m, 12m$

(B) $12m, 9m$

(C) $10m, 12m$

(D) $12m, 10m$

(v) The perimter of $\triangle ABC$ is

(A) $28m$

(B) $37m$

(C) $36m$

(D) $38m$

## 9.4. 2020

### 9.4.1. 10

1. The radius of a sphere (in cm) whose volume is $12\pi cm^3$, is

   (a) 3

   (b) $3\sqrt{3}$

   (c) $3^{\frac{2}{3}}$

   (d) $3^{\frac{1}{3}}$

2. In Fig. 9.19, the angle of elevation of the top of a tower from a point $C$ on the ground, which is $30m$ away from the foot of the tower, is $30°$. Find the height of the tower.

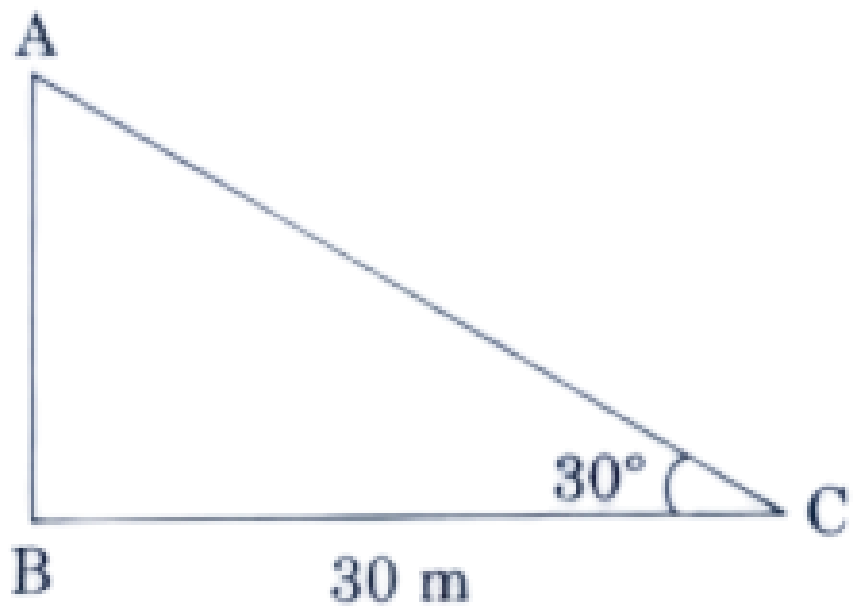



Figure 9.19:

3. A cone and a cylinder have the same radii but the height of the cone is 3 times that of the cylinder. Find the ratio of their volumes.

4. In Fig. 9.20, $ABCD$ is a parallelogram. A semicircle with centre $O$ and the diameter $AB$ has been drawn and it passes through $D$. If $AB = 12cm$ and $OD \perp AB$, then find the area of the shaded region. (Use $\pi = 3.14$)

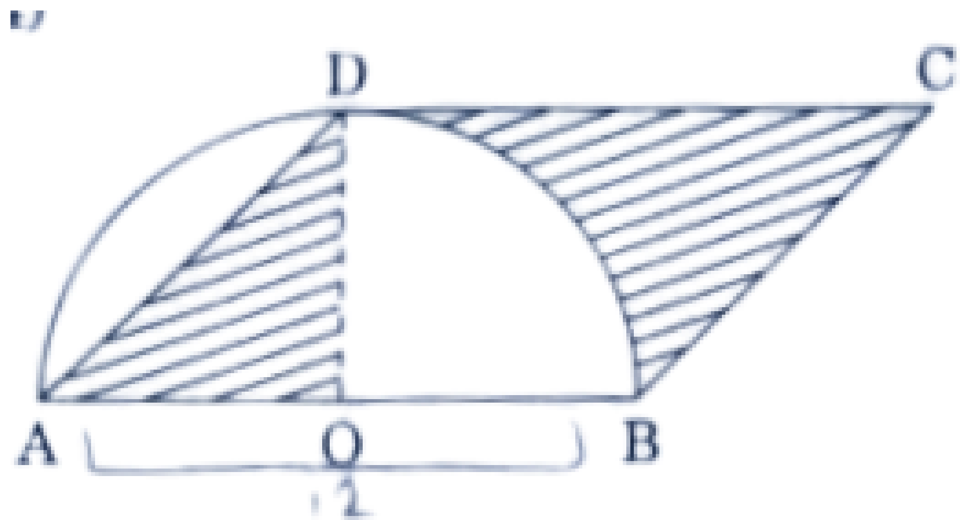



Figure 9.20:

5. A statue $1.6m$ tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is $60^\circ$ and from the same point the angle of elevation of the top of the pedestal is $45^\circ$. Find the height of the pedestal.(Use $\sqrt{3} = 1.73$)

6. In a cylindrical vessel of radius $10cm$, containing some water, 9000 small spherical balls are dropped which are completely immersed in water which raises the water level. If each spherical ball is of radius $0.5cm$, then find the rise in the level of water in the vessel.

7. If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio.

8. If $\tan^{-1}\left(\frac{y}{x}\right) = \log\sqrt{x^2+y^2}$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

## 9.5. 2019

### 9.5.1. 10

1. In Fig. 9.21, two concentric circles with centre $O$, have radii $21cm$ and $42cm$. If $\angle AOB = 60°$, find the area of the shaded region.

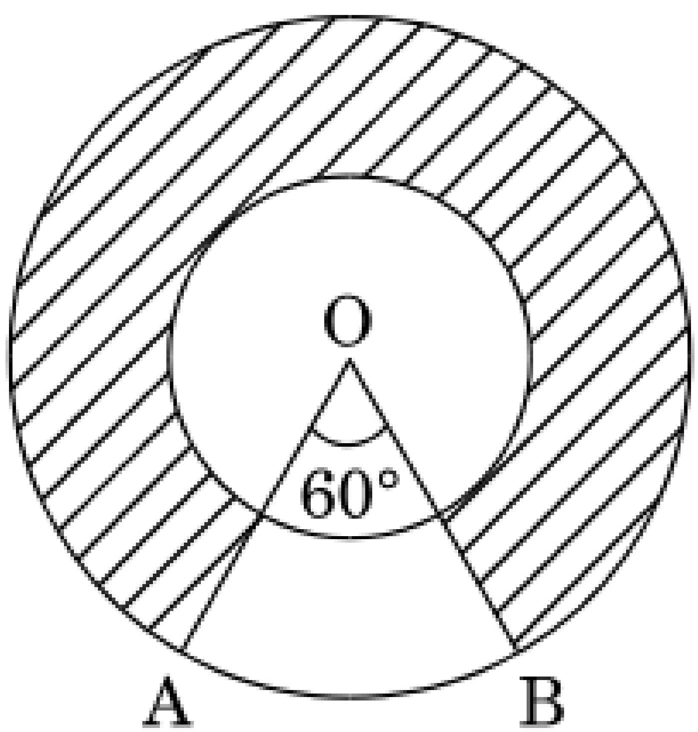



Figure 9.21: Circle $AOB$

2. A moving boat is observed from the top of a $150m$ high cliff moving away from the cliff. The angle of depression of the boat changes from $60°$ to $45°$ in 2 minutes. Find the speed of the boat in m/min.

3. There are two poles, one each on either bank of a river just opposite to each other. One pole is $60m$ high. From the top of this pole, the angle of depression of the top and foot of the other pole are $30^\circ$ and $60^\circ$respectively. Find the width of the river and height of the other pole.

4. A cone of height $24cm$ and radius of base $6cm$ is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere and hence find the surface area of this sphere.

5. A farmer connects a pipe of internal diameter $20cm$ from a canal into a cylindrical tank in his field which is $10m$ in diameter and $2m$ deep. If water flows through the pipe at the rate of $3km/hr$, in how much time will the tank be filled ?

6. Find the dimensions of a rectangular park whose perimeter is $60m$ and area $200m^2$.

7. A container opened at the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container, at the rate of ₹50 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹10 per $100cm^2$. Take($\pi = 3.14$)

8. Two poles of equal heights are standing opposite to each other on either side of the road which is $80m$ wide. From a point $P$ between them on the road, the angle of elevation of the top of a pole is $60^\circ$

and the angle of depression from the top of the other pole of point $P$ is $30°$. Find the heights of the poles and the distance of the point $P$ from the poles.

9. Amit, standing on a horizontal plane, finds a bird flying at a distance of $200m$ from him at an elevation of $30°$. Deepak standing on the roof of a $50m$ high building, finds the angle of elevation of the same bird to be $45°$. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

10. From a point $P$ on the ground, the angle of elevation of the top of a tower is $30°$ and that of the top of the flag-staff fixed on the top of the tower is $\sqrt{5}$. If the length of the flag-staff is $5m$, find the height of the tower. (Use $\sqrt{3} = 1.732$).

11. A solid iron pole consists of a cylinder of height $220cm$ and base diameter $24cm$, which is surmounted by another cylinder of height $60cm$ and radius $8cm$. Find the mass of the pole, given that $1cm^3$ of iron has approximately $8gm$ mass. (use $\pi = 3.14$)

12. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is $20cm$ and the diameter of the cylinder is $7cm$. Find the total volume of the solid. (use $\pi = \frac{22}{7}$).

13. Two spheres of same metal weigh $1kg$and $7kg$. The radius of the smaller sphere is $3cm$. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere.

14. A right cylindrical container of radius $6cm$ and height $15cm$ is full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone.

15. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.

16. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.

17. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

18. In Fig. 9.22, a square $OABC$ is inscribed in a quadrant $OPBQ$. If $OA = 15cm$, find the area of the shaded region. (Use $\pi = 3.14$)

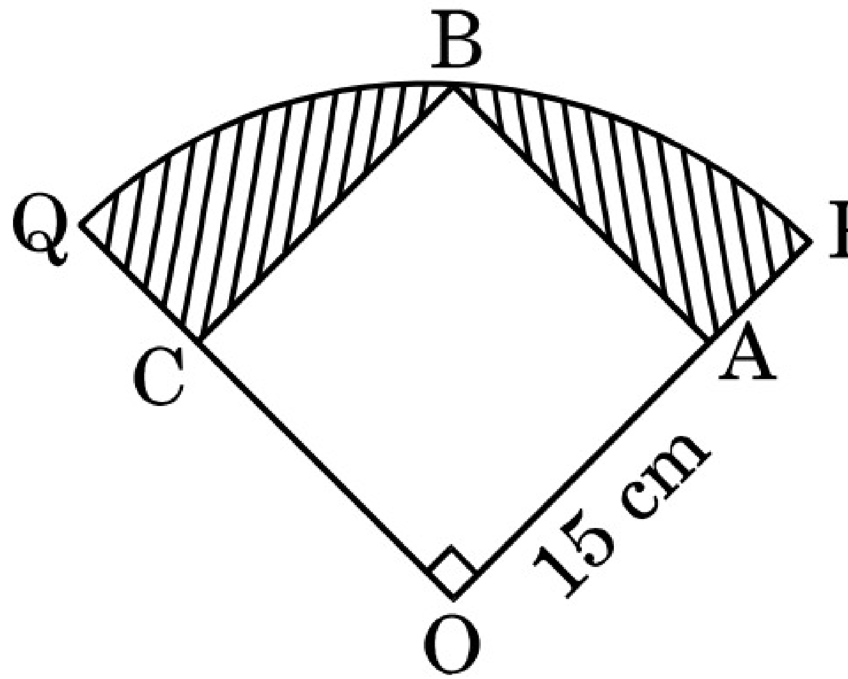



Figure 9.22: Square $OABC$

19. In Fig. 9.23, $ABCD$ is a square with side $2\sqrt{2}cm$ and inscribed in a circle. Find the area of the shaded region. (Use $\pi = 3.14$).

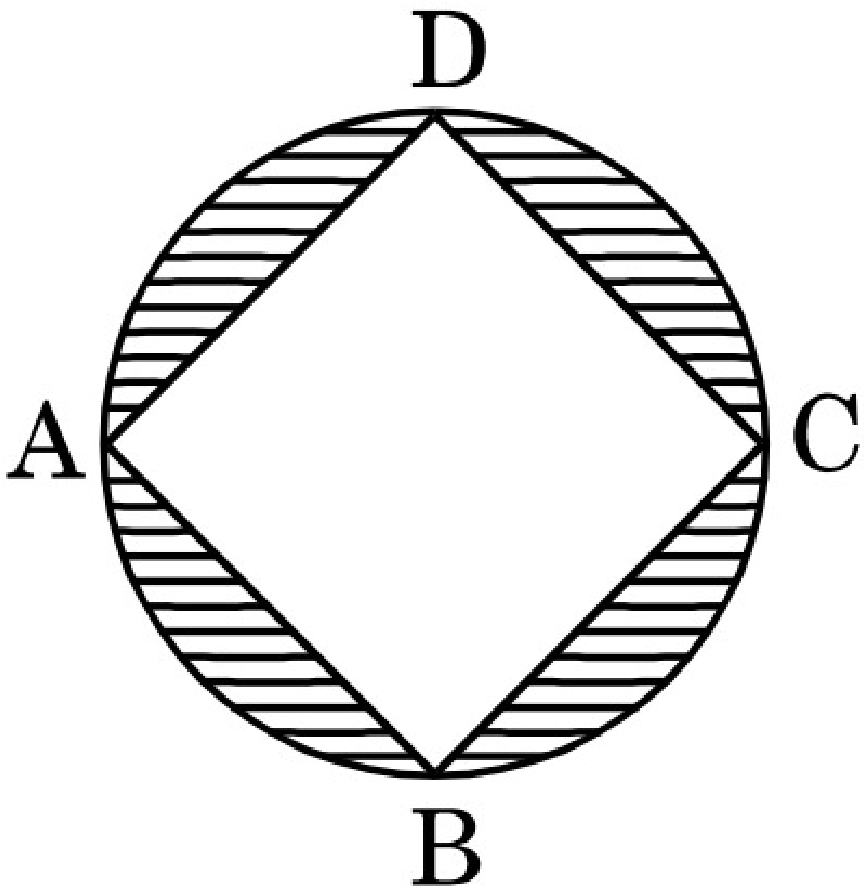



Figure 9.23: Square $ABCD$

20. The shadow of a tower standing on a level ground is found to be $40m$ longer when the Sun's altitude is $30°$ than when it was $60°$. Find the height of the tower. $(Given\sqrt{3} = 1.732)$

21. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

22. Prove that opposite sides of a quadrilateral circumscribing a circle

subtend supplementary angles at the centre of the circle.

23. A pole has to be erected at a point on the boundary of a circular park of diameter $13m$ in such a way that the difference of its distances from two diametrically opposite fixed gates $A$ and $B$ on the boundary is $7m$. Is it possible to do so ? If yes, at what distances from the two gates should the pole be erected ?

24. Water in a canal, $6m$ wide and $1.5m$ deep, is flowing with a speed of $10km/h$. How much area will it irrigate in 30 minutes if $8cm$ of standing water is needed?

25. A car has two wipers which do not overlap. Each wiper has a blade of length $21cm$ sweeping through an angle $120°$. Find the total area cleaned at each sweep of the blades. $(Take\pi = \frac{22}{7})$

26. In Fig. 9.24, a decorative block is shown which is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge $6cm$ and the hemisphere fixed on the top has a diameter of $4.2cm$. Find

    (a) the total surface area of the block.

    (b) the volume of the block formed.(Take $\pi = \frac{22}{7}$)

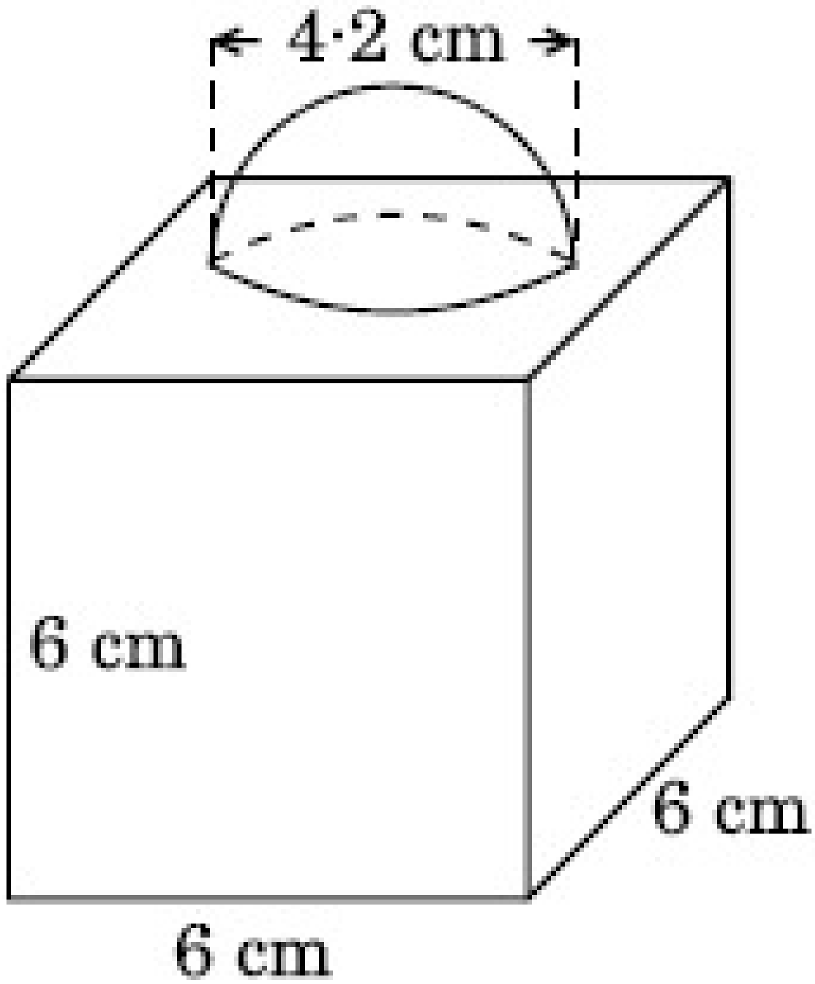



Figure 9.24: CUBE AND HEMISPHERE

27. A bucket open at the top is in the form of a frustum of a cone with a capacity of $12308.8cm^3$. The radii of the top and bottom circular ends are $20cm$ and $12cm$ respectively. Find the height of the bucket and the area of metal sheet used in making the bucket.$(Use \pi = 3.17)$

28. In Fig. 9.25, three sectors of a circle of radius $7cm$, making angles of $60°$, $80°$ and $40°$ at the centre are shaded. Find the the area of the shaded region.

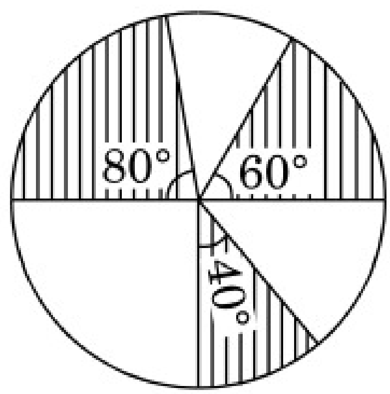



Figure 9.25: Circle

29. A juice seller was serving his customers using glasses as shown in Fig. 9.26 . The inner diameter of the cylindrical glass was $5cm$ but bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was $10cm$, find the apparent and actual capacity of the glass. (Use $\pi = 3.14$ )

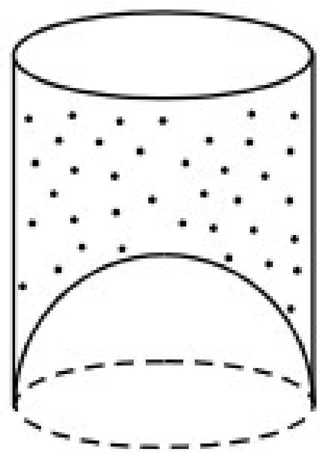


Figure 9.26: Hemisphere

30. A girl empties a cylindrical bucket full of sand, of base radius 18$cm$ and height 32$cm$ on the floor to form a conical heap of sand. If the height of this conical heap is 24$cm$, then find its slant height correct to one place of decimal.

31. An open metallic bucket is in the shape of a frustum of a cone. If the diameters of the two circular ends of the bucket are 45$cm$ and 25$cm$ and the vertical height of the bucket is 24$cm$, find the area of the metallic sheet used to make the bucket. Also find the volume of the water it can hold. (Use $\pi = \frac{22}{7}$)

32. Water in a canal, 6 m wide and 1.5 m deep,is flowing with a speed of 10 km/hr. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

33. A man in a boat rowing away from a light house 100$m$ high takes 2

minutes to change the angle of elevation of the top of the light house from $60^\circ$ to $30^\circ$. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$]

34. Two poles of equal heights are standing opposite each other on either side of the road, which is $80m$ wide. From a point between them on the road, the angles of elevation of the top of the poles are $60^\circ$ to $30^\circ$ respectively. Find the height of the poles and the distances of the point from the poles.

35. A bucket open at the top is in the form of a frustum of a cone with a capacity of $12308.8cm^3$. The radii of the top and bottom of circular ends of the bucket are $20cm$ and $12cm$ respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use $\pi = 3.14$)

36. Find the area of the shaded region in Fig. 9.27, if $ABCD$ is a rectangle with sides $8cm$ and $6cm$ and **O** is the centre of circle. ($\pi = 3.14$)

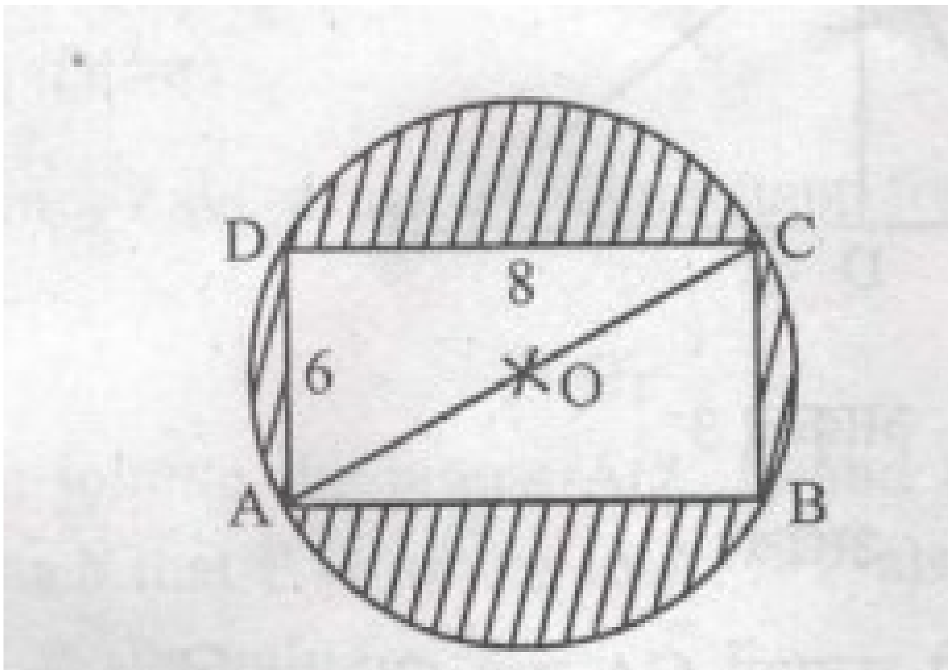



Figure 9.27:

37. If $P$ and $Q$ are the points on side $CA$ and $CB$ respectively of $ABC$, right angled at $C$, prove that $\left(AQ^2 + BP^2\right) = \left(AB^2 + PQ^2\right)$

# 9.6. 2018

## 9.6.1. 10

1. Find the area of the shaded region in Fig. 9.28, where arcs drawn with centres $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ intersect in pairs at mid-points $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$ of the sides AB, BC, CD and DA respectively of a square ABCD of side 12 cm. (**Use** $\pi = 3.14$)

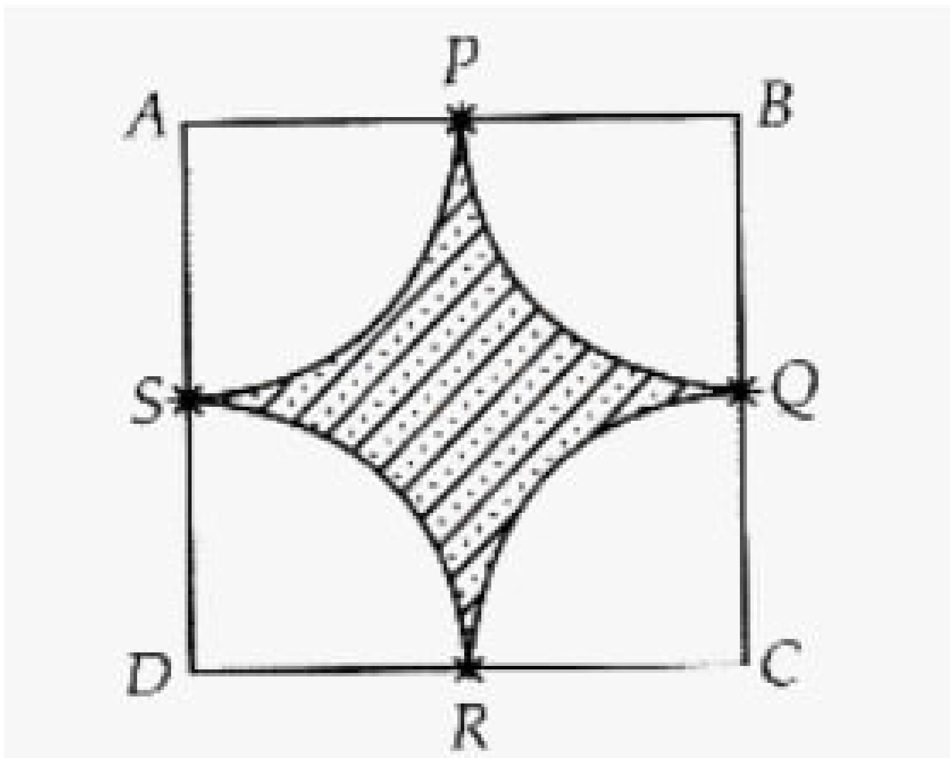



Figure 9.28: square ABCD

2. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 9.29. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.

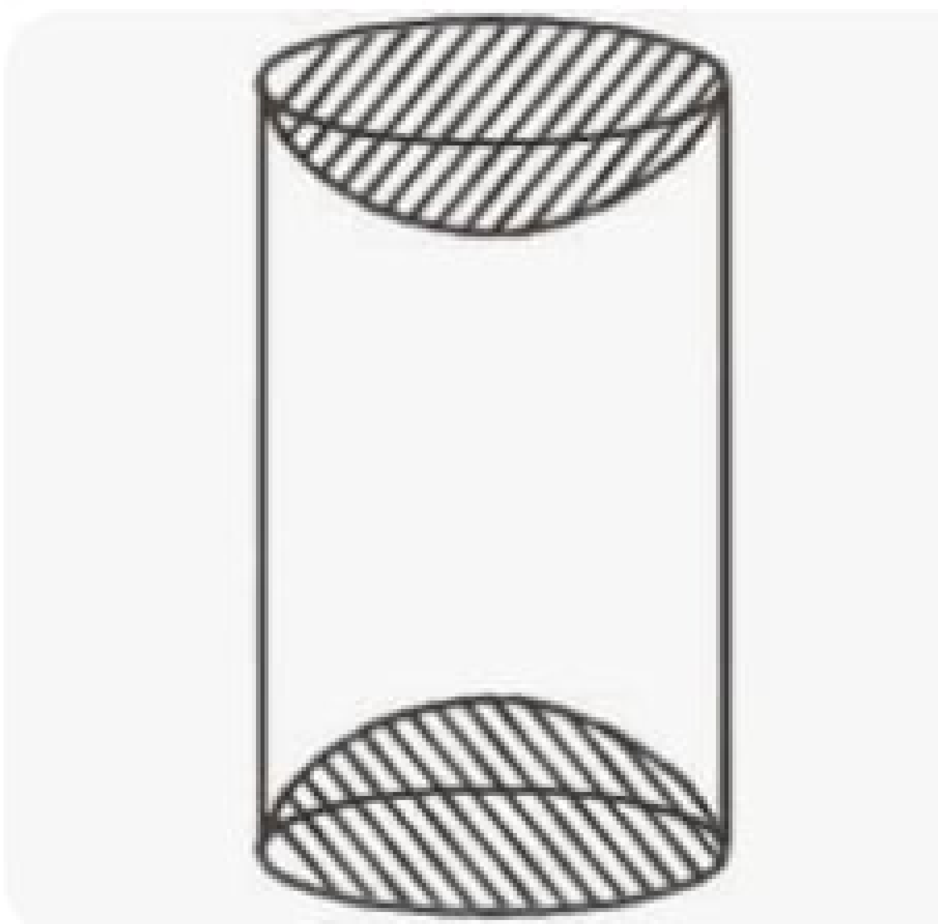


Figure 9.29: cylinder

3. A heap of rice is in the form of a cone of base diameter 24 and height $3.5m$. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

4. As observed from the top of a $100m$ high light house from the sea level, the angles of depression of two ships are $30°$ and $45°$. If one ship is exactly behind the other on the same side of the light house, find the

distance between the two ships. (**Use** $\sqrt{3} = 1.732$)

5. The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find:

   (i) The area of the metal sheet used to make the bucket.

   (ii) Why we should avoid the bucket made by ordinary plastic? (**Use** $\pi = 3.14$)

## 9.6.2. 12

1. Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.

2. An isosceles triangle of vertical angle $2\theta$ is inscribed in a circle of radius $a$. show that the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

3. Find the co-ordinates of the point, where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the yz-plane.

4. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

5. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius $R$ is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

6. Using the method of integration, find the area of the triangle whose vertices are $(1, 0)$, $(2, 2)$ and $(3, 1)$.

7. Using the method of integration, find the area of the region enclosed between two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

8. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of its edge is $12cm$ ?

9. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$,$y = 3x + 1$ and $x = 4$.

# 9.7. 2017

## 9.7.1. 10

## 9.7.2. 12

1. $AB$ is the diameter of a circle and $C$ is any point on the circle. Show that the area of triangle $ABC$ is maximum, when it is an isosceles triangle.

2. A variable plane which remains at a constant distance $3p$ from the origin cuts the coordinate axes at $A, B, C$ . Show that the locus of the centroid of triangle $ABC$ is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$ .

3. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube .

4. Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x–2y + 12 = 0$ .

## 9.8. 2016

### 9.8.1. 10

1. A ladder, leaning against a wall, makes an angle of $60°$ with the horizontal.If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

2. In 9.30, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of ₹500/sq.metre. $\left(\text{Use } \pi = \dfrac{22}{7}\right)$

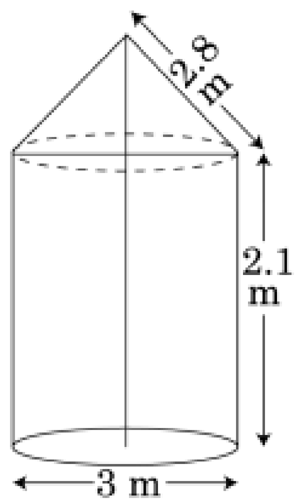



Figure 9.30:

3. In 9.31, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40°$ $\left(\text{Use } \pi = \frac{22}{7}\right)$.

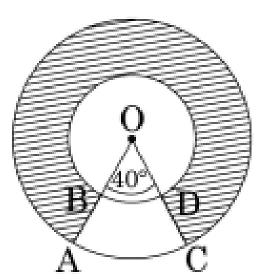



Figure 9.31:

4. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical

vessel. $\left(\text{Use } \pi = \frac{22}{7}\right)$

5. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If a sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$cm. Find the diameter of the cylindrical vessel.

6. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per sq.m, find the amount shared by each school to set up the tents. What value is generated by the above problem ? (Use $\pi = \frac{22}{7}$)

7. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as $60^\circ$ and the angle of depression of the base of hill as $30^\circ$. Find the distance of the hill from the ship and the height of the hill.

8. The angle of elevation of the top $Q$ of a vertical tower $PQ$ from a point

$X$ on the ground is $60°$. From a point $Y$, 40 m vertically above $X$, the angle of elevation of the top $Q$ of tower is $45°$. Find the height of the tower $PQ$ and the distance $PX$. $\left(\text{Use } \sqrt{3} = 1.73\right)$

9. A rectangular park is to be designed whose breadth is 3 m less than its length.Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.

10. In Fig 9.32 , is shown a sector $OAP$ of a circle with centre $O$, containing $\angle\theta$. $AB$ is perpendicular to the radius $OA$ and meets $OP$ produced at $B$. Prove that the perimeter of shaded region is $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$

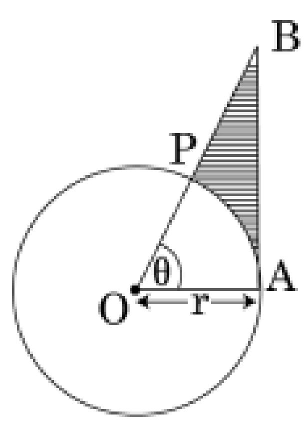



Figure 9.32:

## 9.8.2. 12

1. Show that the lines

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$$
$$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$$

intersect. Find their point of intersection.

2. Find the equation of the tangent line to the curve $y = \sqrt{5x-3} - 5$, which is parallel to line

$$4x - 2y + 5 = 0$$

3. Write the coordinates fo the point which is the reflection of the point $(\alpha, \beta, \gamma)$ in the $XZ$-plane.

4. The equation of tangent at $(2, 3)$ on the curve

$$y^2 = ax^3 + b \text{ is}$$

$$y = 4x - 5$$

Find the values of $a$ and $b$

5. Prove that the least perimeter of an isosceles triangle in which a circle of radius $r$ can be inscribed is $6\sqrt{3}r$

6. If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle

between them is $\frac{\pi}{3}$.

7. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of square bounded by $x = 0, y = 4$ and $y = 0$ into three equal parts.

8. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height $h$ and semi-vertical angle $\alpha$ is one third that of the cone and the greatest volume of the cyclinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

9. Find the coordinates of the foot of perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $A(2, 3, -1)$. Hence find the image of the point $A$ in the line $BC$.

10. Show that the four point $A(4, 5, 1), B(0, -1, -1), C(3, 9, 4)$ and $D(-4, 4, 4)$ are coplanar.

# 9.9. 2015

## 9.9.1. 10

1. In 1, $PQ$ is a chord of a circle with centre $O$ and $PT$ is a tangent. If $\angle QPT = 60°$, Find $\angle PRQ$.

2. The points

$$\mathbf{A} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} p \\ 3 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

are the vertices of a right triangle, right-angled at **B**. Find the value of $p$.

3. In Figure 3, two tangents $RQ$ and $RP$ are drawn from an external point $R$ to the circle with centre $O$. If $\angle PRQ = 120°$, then prove that $OR = PR + RQ$.

4. In Figure 4, a triangle $ABC$ is drawn to circumscribe a circle of radius 3 cm, such that the segments $BD$ and $DC$ are respectively of lengths 6 cm and 9 cm. If the area of $\triangle$ABC is $54cm^2$, then find the lengths of sides $AB$ and $AC$.

5. Find the relation between x and y if the points

$$\mathbf{A} = \begin{pmatrix} x \\ \\ y \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} -5 \\ \\ 7 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} -4 \\ \\ 5 \end{pmatrix}$$

are collinear.

6. Find the coordinates of a point $P$ on the line segment joining

$$\mathbf{A} = \begin{pmatrix} 1 \\ \\ 2 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 6 \\ \\ 7 \end{pmatrix}$$

such that AP $= \frac{2}{5}$AB.

7. Find the values of $k$ for which the points

$$\mathbf{A} = \begin{pmatrix} k+1 \\ 2k \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} 3k \\ 2k+3 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} 5k-1 \\ 5k \end{pmatrix}$$

are collinear.

8. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

9. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

10. Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but of height 2.8 m, and the canvas to be used costs ₹100 per sq. m, find the amount, the associations will have to pay. What values are shown by these associations ? [Use $\pi = \frac{22}{7}$]

11. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer.

12. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have ? Find the cost of painting the total surface area of the solid so formed, at the rate of ₹5 per 100 sq. cm. [Use $\pi = 3.14$]

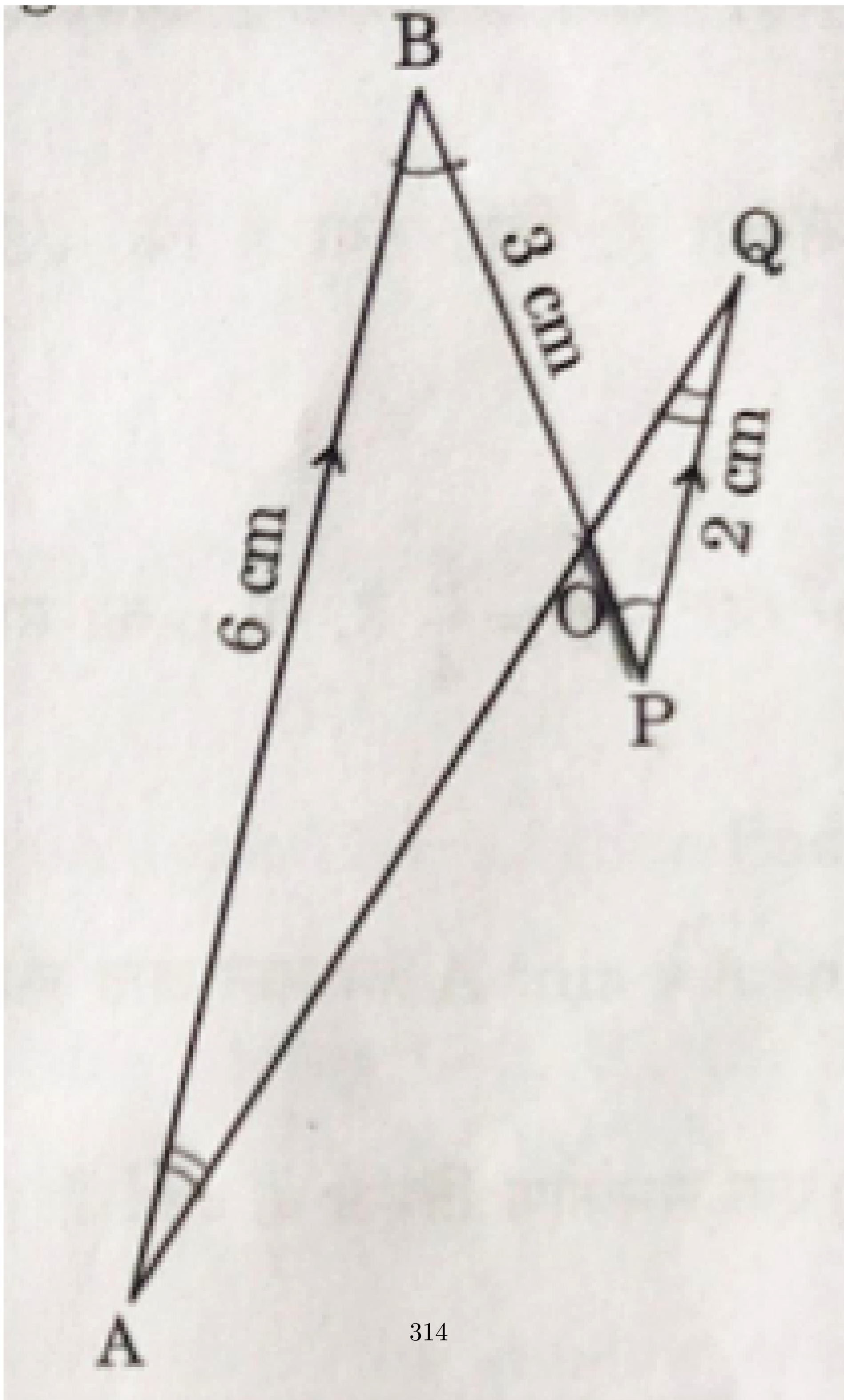
13. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. [Use $\pi = \frac{22}{7}$]

14. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is $60°$. Also find the area of the corresponding major segment. [Use $\pi = \frac{22}{7}$]

15. In 15, $PQRS$ is a square lawn with side $PQ = 42$ metres. Two circular flower beds are there on the sides $PS$ and $QR$ with centre at $O$, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).

16. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. [Use $\pi = \frac{22}{7}$]

17. The diagonal of a rectangular field is 16 metres more than the shorter side. If the longer side is 14 metres more than the shorter side, then find the lengths of the sides of the field.


B
3 cm
Q
6 cm
2 cm
O
P
A

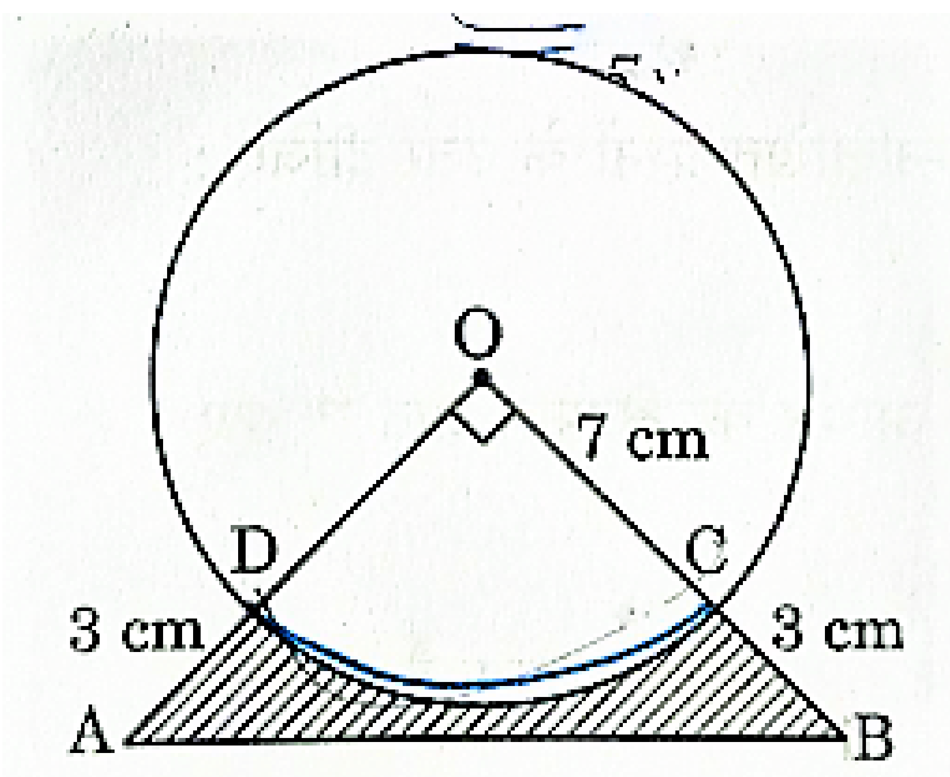



Figure 9.2: memento

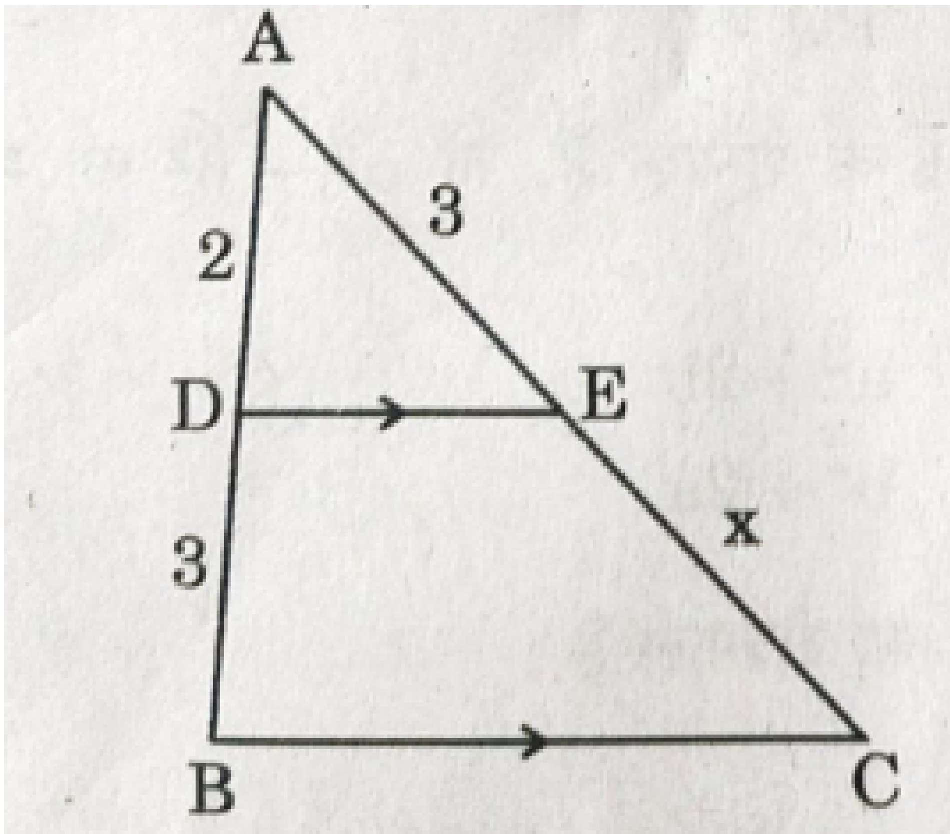



Figure 9.3: $\triangle ABC$

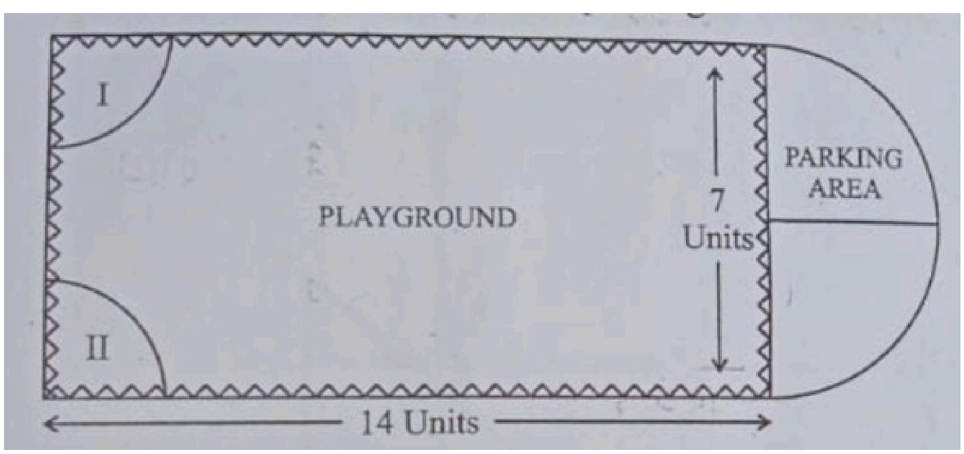



Figure 9.4: Playground

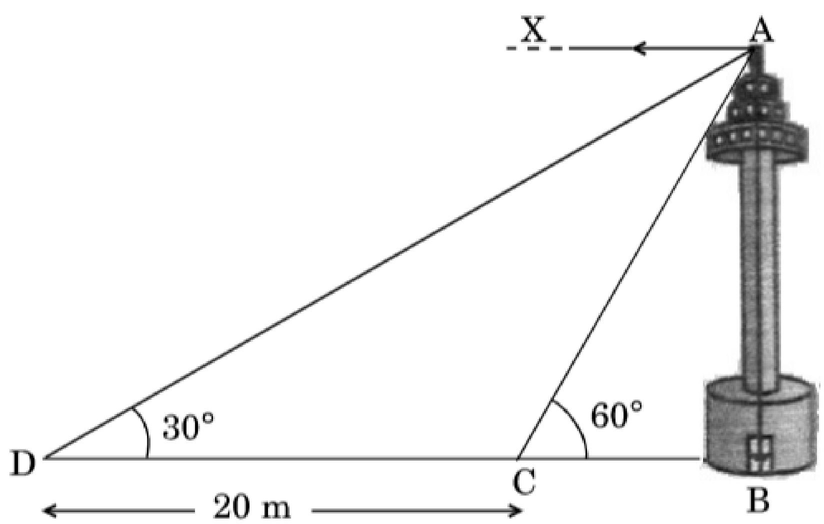



Figure 9.17: Projection of Tower

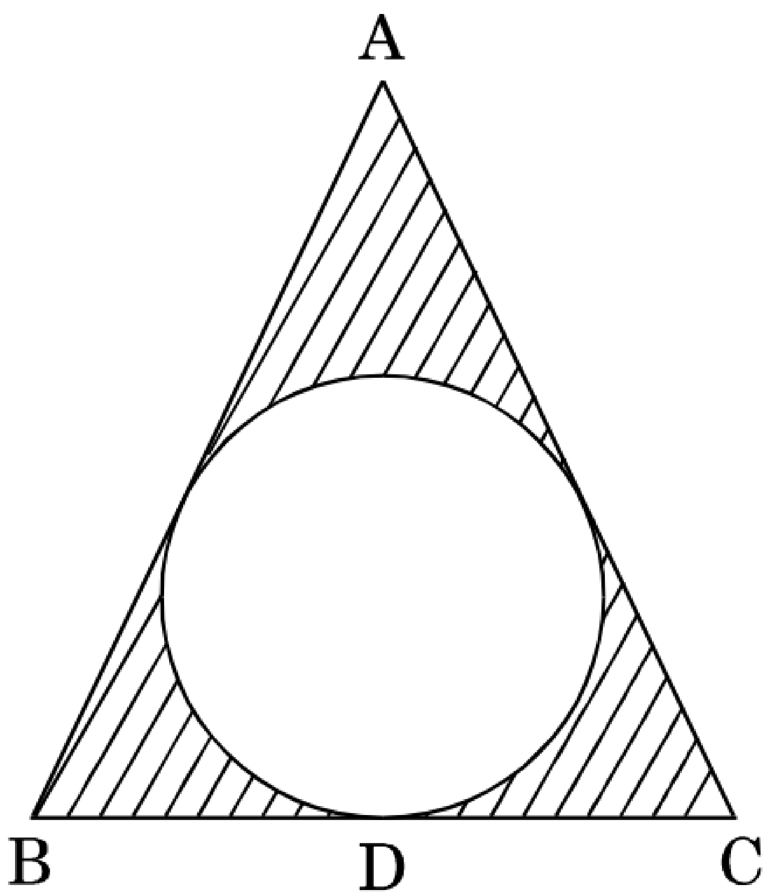
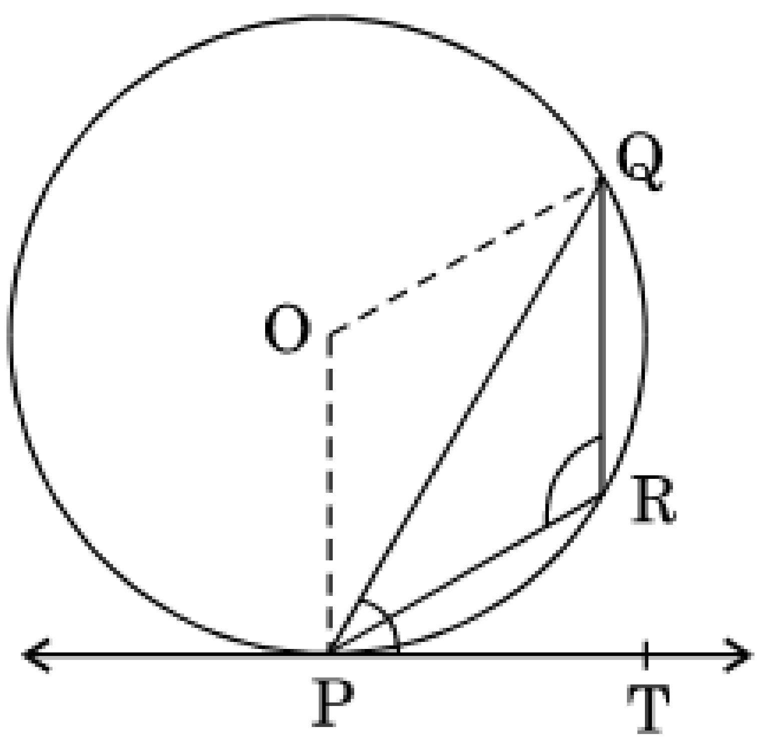



Figure 9.18: Children's Park in triangular shape


Q
O
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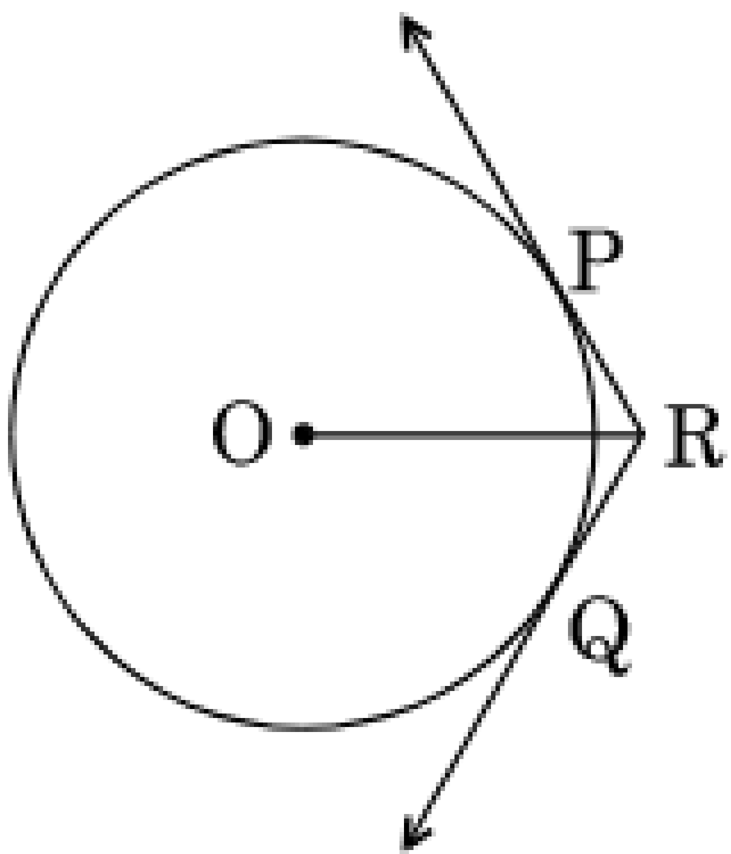
P
O
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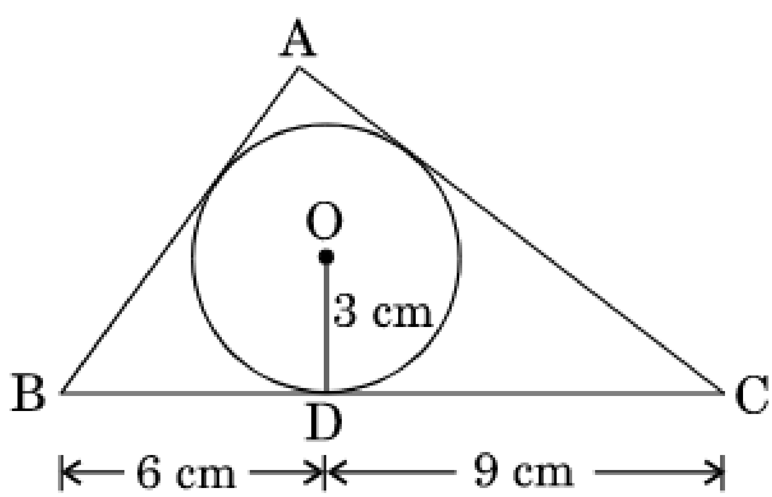
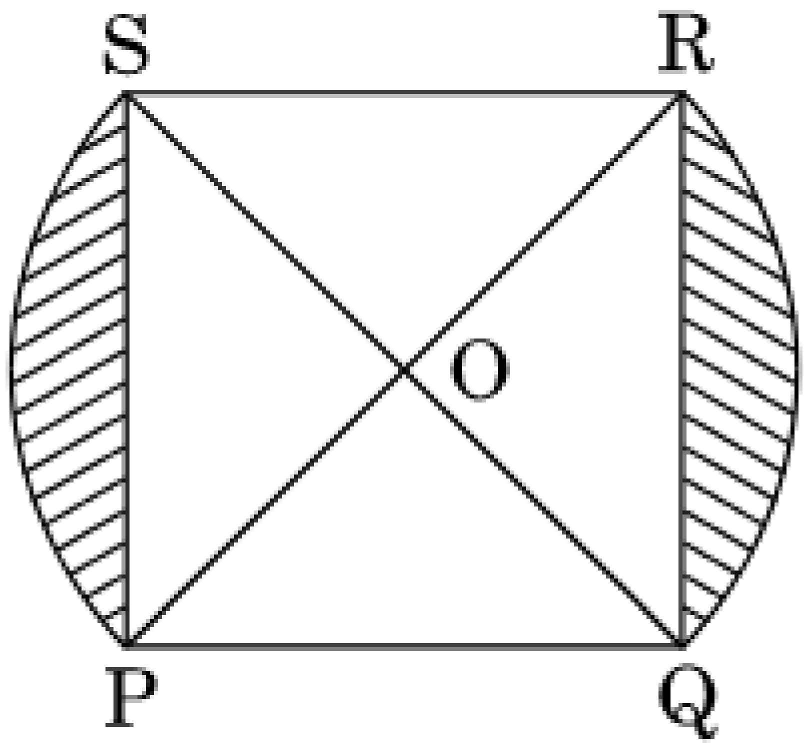

A
O
3 cm
B
D
C
6 cm
9 cm


S
R
O
P
Q

# Chapter 10

# Discrete

## 10.1. 2022

### 10.1.1. 10

1. If $-\frac{5}{7}$, $a$, 2 are consecutive terms in an Arthimetic Progression, then the value of $a$ is

   (a) $\frac{9}{7}$

   (b) $\frac{9}{14}$

   (c) $\frac{19}{7}$

   (d) $\frac{19}{14}$

2. If two positive integers $p$ and $q$ can be expressed as $p = ab^3$ and $q = a^2b$; $a$ and $b$ being prime numbers, then find LCM of $(p, q)$ .

3. Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$ for some integer $q$.

4. Prove that $\sqrt{5}$ is an irrational number.

5. (a) Find the sum of first 16 terms of an Arithmetic Progression whose $4^{\text{th}}$ and $9^{\text{th}}$ terms are $-15$ and $-30$ respectively.

   (b) If the sum of first 14 terms of an Arithmetic Progression is 1050 and its fourth term is 40, find its $20^{\text{th}}$ term.

6. (a) Find the sum of the first twelve 2-digit numbers which are multiples of 6.

   (b) In an AP, if $a_2 = 26$ and $a_{15} = -26$, then write the AP.

7. In Mathematics, relations can be expressed in various ways. The matchstick patterns are based on linear relations. Different strategies can be used to calculate the number of matchsticks used in different Fig. 10.1 One such pattern is shown below. Observe the pattern and answer the following questions using Arithmetic Progression :

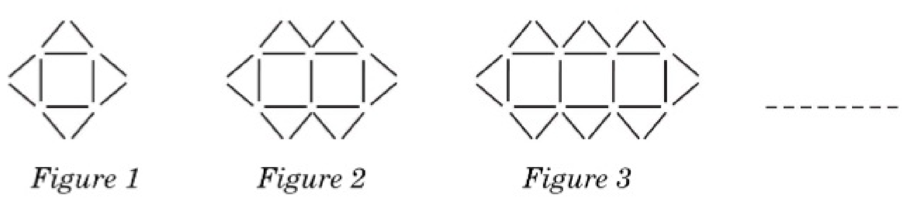



Figure 10.1: patterns of Figure1, figure2 ,figure3

   (a) Write the AP for the number of triangles used in the Fig. 10.1. Also, write the nth term of this AP.

   (b) Which figure has 61 matchsticks ?

8. (a) In an A.P. if the sum of third and seventh term is zero, find its $5^{\text{th}}$ term.

(b) Determine the AP whose third term is 5 and seventh term is 9.

9. Find the sum of the first 20 terms of an A.P. whose $n^{\text{th}}$ term is given as $a_n = 5 - 2n$

10. Find the common difference 'd' of an AP whose first term is 10 and the sum of the first 14 terms is 1505.

11. For what value of 'n', are the $n^{\text{th}}$ terms of the APs: $9, 7, 5, \ldots$ and $15, 12, 9, \ldots$ the same?

12. (a) The curved surface area of a right circular cylinder is $176 sq.cm$ and its volume is $1232 cu.cm$. What is the height of the cylinder?

    (b) The largest sphere is carved out of a soild cube of side $21 cm$. Find the volume of the sphere.

13. The sum of the first three terms of an A.P is 33. If the product of first and third term exceeds the second term by 29, find the A.P.

14. (a) Find the number of terms in the following A.P:

$$5, 11, 17, \ldots, 203 \tag{10.1}$$

    (b) Find the sum of the first 20 terms of an AP whose $n^{\text{th}}$ term is given as $a_n = 5 - 3n$

15. While buying an expensive item like a house or a car, it becomes easier for a middle-class person to take a loan from a bank and then repay the loan along with interest in easy instalments. Aman buys a car by

taking a loan of ₹2,36,000 from the bank and starts repaying the loan in monthly instalments. He pays ₹2,000 as the first instalment and then increases the instalment by ₹500 every month.

(a) Find the amount he pays in the $25^{\text{th}}$ installment.

(b) Find the total amount paid by him in the first 25 installments.

## 10.2. 2023

### 10.2.1. 10

1. The ratio of HCF to LCM of the least composite number and the least prime number is :

    (a) 1 : 2

    (b) 2 : 1

    (c) 1 : 1

    (d) 1 : 3

2. The next term of the A.P.: $\sqrt{7}$, $\sqrt{28}$, $\sqrt{63}$ is :

    (a) $\sqrt{70}$

    (b) $\sqrt{80}$

    (c) $\sqrt{97}$

    (d) $\sqrt{112}$

3. Two numbers are in the ratio 2 : 3 and their LCM is 180. what is the HCF of these numbers ?

4. How many terms are there in A.P whose first and fifth term are - 14 and 2, respectively and the last term is 62.

5. Which term of the A.P.:65, 61, 57, 53, ................ is the first negative term ?

6. Prove that $\sqrt{5}$ is an irrational number.

7. If $p$ and $q$ are natural numbers and $p$ is the multiple of $q$, then what is the HCF of $p$ and $q$ ?

   (a) $pq$

   (b) $p$

   (c) $q$

   (d) $p+q$

8. Prove that $2+\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

9. Find by prime factorisation the LCM of the numbers 18180 and 7575. Also, find the HCF of the two numbers

10. Three bells ring at intervals of 6, 12 and 18 minutes. If all the three bells rang at 6 a.m., when will they ring together again ?

11. How many terms of the arithmetic progression 45, 39, 33, ...... must be taken so that their sum is 180 ? Explain the double answer.

12. If $p-1$, $p+1$ and $2p+3$ are in A.P., then the value of $p$ is

    (a) $-2$

    (b) 4

    (c) 0

    (d) 2

13. Assertion (A): The perimeter of $\triangle ABC$ is a rational number.

    Reason (R): The sum of the squares of two rational numbers is always rational.

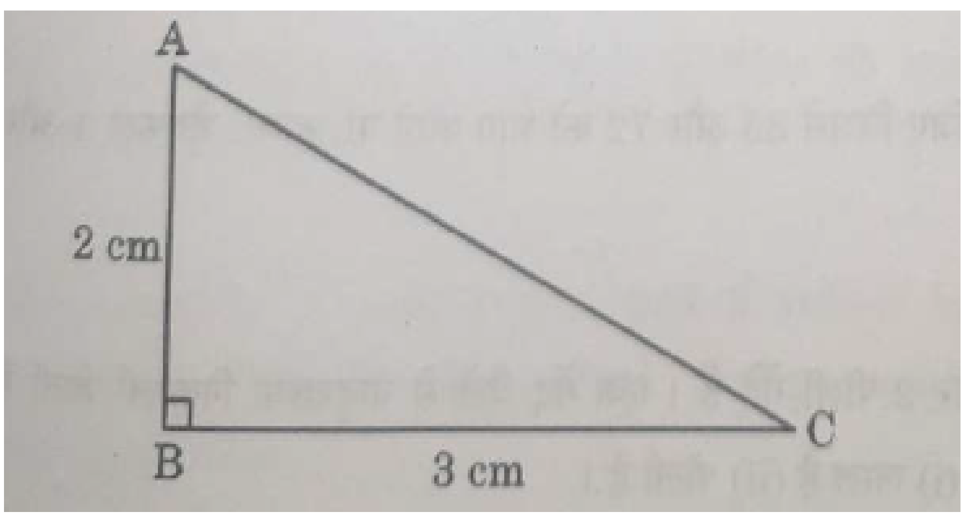



Figure 10.2: $\triangle ABC$

14. Find the greatest number which divides 85 and 72 leaving remainders 1 and 2 respectively.

15. Prove that $\sqrt{5}$ is an irrational number.

16. The ratio of the $11^{\text{th}}$ term to $17^{\text{th}}$ term of an A.P. is 3 : 4. Find the ratio of $5^{\text{th}}$ to $21^{\text{th}}$ of the same A.P. Also, find the ratio of the sum of first 5 terms to that of first 21 terms

17. 250 logs are stacked in the following manner:

    22 logs in the bottom row , 21 in the next row, 20 in the row next to it and so on(as shown by an example). In how many, are the 250 logs placed and how many logs are there in top row ?

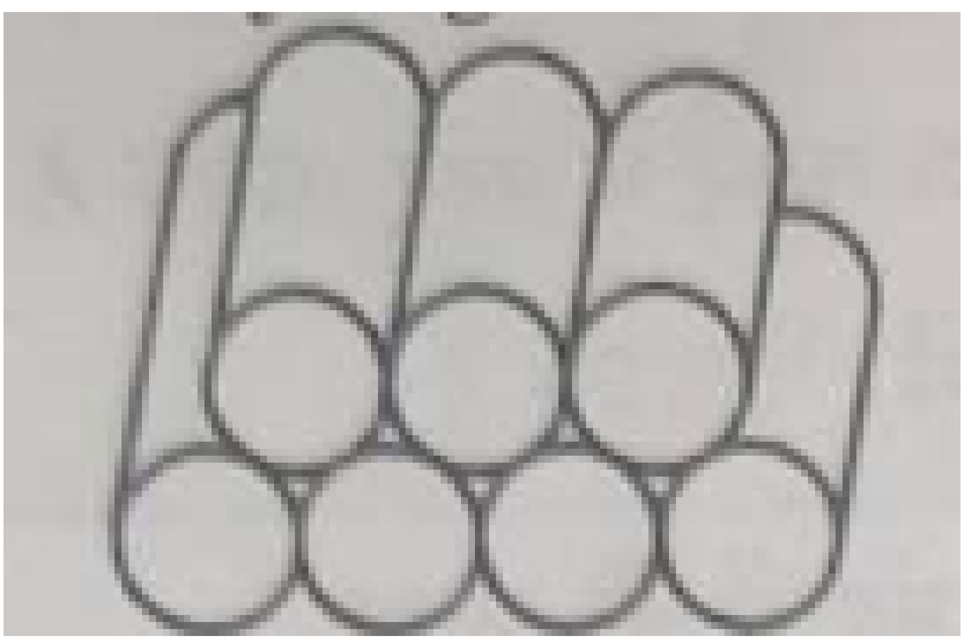


Figure 10.3: Pile of logs

## 10.3. 2021

### 10.3.1. 10

1. Write the common difference of the A.P. :$\frac{1}{5}, \frac{4}{5}, \frac{7}{5}, \frac{10}{5}, \cdots$

2. Find the $8^{th}$ term of the A.P. whose first term is $-2$ and common difference is 3.

3. Roshini being a plant lover decides to start a nursery. She bought few plants with pots.She placed the pots in such a way that the number of pots in the first row is 2, in the second is 5, in the third row is 8 and so on. Based on the above, answer the following questions :

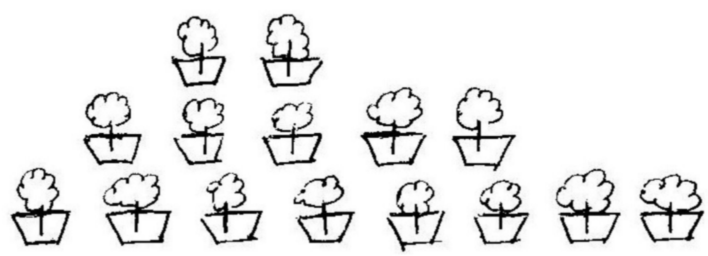


Figure 10.4: Plants

   (i) How many pots were placed in the $7^{th}$ row ?

      A 20

      B 23

      C 77

      D 29

   (ii) If Roshini wants to place 100 pots in total, then total number of rows formed in the arrangement will be ?

A 8

B 9

C 10

D 12

(iii) How many pots are placed in the last row ?

A 20

B 23

C 26

D 29

(iv) If Roshini ha sufficient space for 12 rows, then how many total number of pots are placed by her wih the same arrangement ?

A 222

B 155

C 187

D 313

4. Find the LCM and HCF of two numbers 26 and 91 by the method of prime factorization.

5. For two numbers $x$ and $y$, if $xy = 1344$ and HCF $(x, y) = 8$, then find LCM$(x, y)$.

6. Find the HCF of 96 and 404 by prime factorisation.

7. Express 792 as the product of its prime factors.

8. The sum of the first 4 terms of an A.P. is zero and its $4^{th}$ term is 2. Find the A.P.

9. If the sum of the first $n$ terms of an A.P. is given by $S_n = 4n - n^2$, then find its $n^{th}$ term. Hence, find the $25^{th}$ term and the sum if the first 25 terms of this A.P.

10. If $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

11. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then find the value of $k$.

12. Find the mean of first 10 composite numbers.

13. If $S_n$ denotes the sum of first $n$ terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$.

14. After how many decimal places will the decimal expansion of the rational number $\frac{14587}{1250}$ terminate ?

15. State giving reason whether $5 \times 7 \times 11 + 11$ is a composite number or a prime number.

16. If the $6^{th}$ and $14^{th}$ terms of an A.P. are 29 and 69 respectively, then find the $10^{th}$ term of the A.P.

17. If the first three consecutive terms of an A.P. are $3y - 1, 3y + 5$ and $5y + 1$ find the value of y.

## 10.4. 2020

### 10.4.1. 10

1. Which of the following is not an A.P.?

   (A) $-1.2, 0.8, 2.8, \ldots$

   (B) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

   (C) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$

   (D) $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \ldots$

2. Find the sum of the first 100 natural numbers.

3. The LCM of two numbers is 182 and their HCF is 13.If one of the numbers is 26, find the other.

4. Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

5. Find the Sum:

$$(-5)+(-8)+(-11)+\cdots+(-230)$$

6. Use Euclid Division Lemma to show that the square of any positive integer is either of the form $3q$ or $3q+1$ for some integer $q$.

## 10.5. 2019

### 10.5.1. 10

1. Write the number of zeroes in the end of a number whose prime factorization is $2^2 \times 5^3 \times 3^2 \times 17$.

2. Use Euclid's division algorithm to find the HCF of 255 and 867.

3. Find the number of terms in the A.P. :

$$18, 15\frac{1}{2}, 13, ..., -47.$$

4. Determine the A.P. whose third term is 16 and $7^{th}$ term exceeds the $5^{th}$ term by 12.

5. Find the value of $x$, when in the A.P. given below

$$2 + 6 + 10 + ... + x = 1800.$$

6. Which term of the A.P. $-4, -1, 2, ... is 101$?

7. In an A.P., the first term is $-4$, the last term is 29 and the sum of all its terms is 150. Find its common difference.

8. Prove that $2 + 3\sqrt{3}$ is an irrational number when it is given that $\sqrt{3}$ is an irrational number.

9. Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

10. Using Euclid's Algorithm, find the HCF of 2048 and 960.

11. If HCF $(336, 54) = 6$, find LCM $(336, 54)$.

12. Write the smallest number which is divisible by both 306 and 657.

13. Find the $21^{st}$ term of the A.P. $-4\frac{1}{2}, -3, -1\frac{1}{2}, ...$

14. Find the common difference of the Arithmetic Progression (A.P.)

$$\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, ..(a \neq 0)$$

15. Which term of the Arithmetic Progression $-7, -12, -17, -22, ...$ will be $-82$ ? Is $-100$ any term of the A.P. ? Give reason for your answer.

16. How many terms of the Arithmetic Progression $45, 39, 33, ...$must be taken so that their sum is 180 ? Explain the double answer.

17. Find after how many places of decimal the decimal form of the number $\frac{27}{2^3.5^4.3^2}$ will terminate.

18. Express 429 as a product of its prime factors.

19. If HCF of 65 and 117 is expressible in the form $65n - 117$, then find the value of $n$.

20. On a morning walk, three persons step out together and their steps measure $30cm$, $36cm$ and $40cm$ respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps ?

21. Prove that $\sqrt{3}$ is an irrational number.

22. Find the largest number which on dividing 1251, 9377 and 15628 leaves remainders 1, 2 and 3 respectively.

23. Find the sum of first 10 multiples of 6

24. If $m$ times the $m^{th}$ term of an Arithmetic Progression is equal to $n$ times its $n^{th}$ term and $m \neq n$, show that the $(m+n)^{th}$ term of the A.P is zero

25. The sum of the first three numbers in an Arithmetic Progression is 18. If the product of the first and the third term is 5 times the common difference, find the three numbers.

26. The HCF of two numbers $a$ and $b$ is 5 and their LCM is 200. Find the product $ab$.

27. Find the HCF of 612 and 1314 using prime factorisation.

28. Show that any positive odd integer is of the form $6m+1$ or $6m+3$ or $6m+5$, where $m$ is some integer.

29. Prove that $\sqrt{5}$ is an irrational number.

30. Prove that $\left(5-3\sqrt{2}\right)$ is an irrational number, given that $\sqrt{2}$ is irrational number.

31. Find the sum of all the two digit numbers which leave the remainder 2 when divided by 5.

32. If in an A.P ., $a = 15$,$d = -3$ and $a_n = 0$, then find the value of $n$.

33. If $S_n$, the sum of the first $n$ terms of an A.P. is given by $S_n = 2n^2 + n$,then find its $n^{th}$ term.

34. If the $17^{th}$ term of an A.P. exceeds its $10^{th}$ term by 7, find the common difference.

35. If the sum of the first $p$ terms of an A.P. is $q$ and the sum of the first $q$ terms is $p$; then show that the sum of the first $(p+q)$ terms is $\{-(p+q)\}$.

36. Write the common difference of the A.P.$\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$ , ...

37. In an A.P., the $n^{th}$ term is $\frac{1}{m}$ and the $m^{th}$ term is $\frac{1}{n}$. Find

    (a) $(mn)^{th}$term ,

    (b) sum of first $(mn)$ terms.

38. Find the HCF of 1260 and 7344 using Euclid's algorithm.

39. Show that every positive odd integer is of the form $(4q+1)$ or $(4q+3)$, where $q$ is some integer.

40. Which term of the $AP$ $3, 15, 27, 39, ....$ will be 120 more than its 21st term?

41. If $S_n$ ,the sum of first $n$ terms of an $AP$ is given by $S_n = 3n^2 - 4n$, find the $n$th term.

42. If the sum of first four terms of an $AP$ is 40 and that of first 14 terms is 280. Find the sum of its first $n$ terms.

43. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

44. The first term of an AP is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the AP.

45. Two positive integers $a$ and $b$ can be written as $a = x^3 * y^2$ and $b = x * y^3$. x,y are prime numbers. Find $LCM(a, b)$.

46. If the sum of first $n$ terms of an $AP$ is $n^2$, then find its 10th term.

## 10.6. 2018

### 10.6.1. 10

1. what is the HCF of smallest prime number and the smallest composite number?

2. Given that $\sqrt{2}$ is irrational, prove that $\left(5 + 3\sqrt{2}\right)$ is an irrational number.

3. Find the sum of 8 multiples of 3.

4. Find the HCF and LCM of 404 and 96 and verify that HCF*LCM = product of the given numbers.

5. In an $AP$, if the common difference $(d) = -4$, and the seventh term$(a_7)$ is 4, then find the first term.

6. The sum of four consecuive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle term is $7:15$. Find the numbers.

### 10.6.2. 12

1. Show that the relation $R$ on the set $Z$ of all integers, given by $R = \{(a,b) : 2 \ \textit{divides} \ (a-b)\}$ is an equivalence relation.

## 10.7. 2016

### 10.7.1. 10

1. For what value of $k$ will $k+9, 2k-1$ and $2k+7$ are the consecutive terms of an A.P ?

2. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term.

3. If the ratio of the sum of first $n$ terms of two A.P's is $(7n+1) : (4n+27)$, find the ratio of their $m$ th terms.

4. The sums of first $n$ terms of three arithmetic progressions are $S_1, S_2$ and $S_3$ respectively. The first term of each A.P. is 1 and their common

differences are $1, 2$ and $3$ respectively. Prove that $S_1 + S_3 = S_2$.

5. The digits of a positive number of three digits are in A.P. and their sum is 15.The number obtained by reversing the digits is 594 less than the original number. Find the number.

6. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of $X$ such that sum of numbers of houses preceeding the house numbered $X$ is equal to sum of the numbers of houses following $X$.

## 10.7.2. 12

1. Show that the lines

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$$
$$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$$

intersect. Find their point of intersection.

2. Find the equation of the tangent line to the curve $y = \sqrt{5x-3} - 5$, which is parallel to line

$$4x - 2y + 5 = 0$$

3. Write the coordinates fo the point which is the reflection of the point $(\alpha, \beta, \gamma)$ in the $XZ$-plane.

4. The equation of tangent at $(2, 3)$ on the curve

$$y^2 = ax^3 + b \text{ is}$$

$$y = 4x - 5$$

Find the values of $a$ and $b$

5. Prove that the least perimeter of an isosceles triangle in which a circle of radius $r$ can be inscribed is $6\sqrt{3}r$

6. If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

7. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of square bounded by $x = 0, y = 4$ and $y = 0$ into three equal parts.

8. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height $h$ and semi-vertical angle $\alpha$ is one third that of the cone and the greatest volume of the cyclinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

9. Find the coordinates of the foot of perpendicular drawn from the point $A\,(-1, 8, 4)$ to the line joining the points $B\,(0, -1, 3)$ and $A\,(2, 3, -1)$. Hence find the image of the point $A$ in the line $BC$.

10. Show that the four point $A\,(4,5,1)\,, B\,(0,-1,-1)\,, C\,(3,9,4)$ and $D\,(-4,4,4)$ are coplanar.

# Chapter 11

# Number Systems

## 11.1. 2019

### 11.1.1. 10

1. Find a rational number between $\sqrt{2}$ and $\sqrt{7}$.

2. How many multiples of 4 lie between 10 and 205?

# Chapter 12

# Differentiation

## 12.1. 2023

### 12.1.1. 12

1. If $\tan\left(\frac{x+y}{x-y}\right) = k$,then $\dfrac{dy}{dx}$ is equal to

2. $\frac{-y}{x}$

3. $\frac{y}{x}$

4. $\sec^2\left(\frac{y}{x}\right)$

5. $-\sec^2\left(\frac{y}{x}\right)$

6. **Assertion(A) :**Maximum value of $\left(\cos^{-1}\right)^2$ is $\pi^2$.

   **Reason(R):**Range of the principle value branch of $\cos^{-1} x$ is $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$.

7. If $y = \sqrt{ax+b}$ , prove that $y\left(\dfrac{d^2y}{dx^2}\right) + \left(\dfrac{dy}{dx}\right)^2 = 0$

8. If the circumference of circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius.

9. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine.The piston moves inside the cylinder bore


Figure 12.1:

10. The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi\ cm^2$.

    Based on the above information , answer the following questions:

    (i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

    (ii) Find $\dfrac{dv}{dr}$

    (iii) (a) Find the radius of cylinder when its volume is maximum.

    (b) For maximum volume, $h > r$.State true or false and justify.

11. The use of electric vehicles will curb air pollution in the long run.

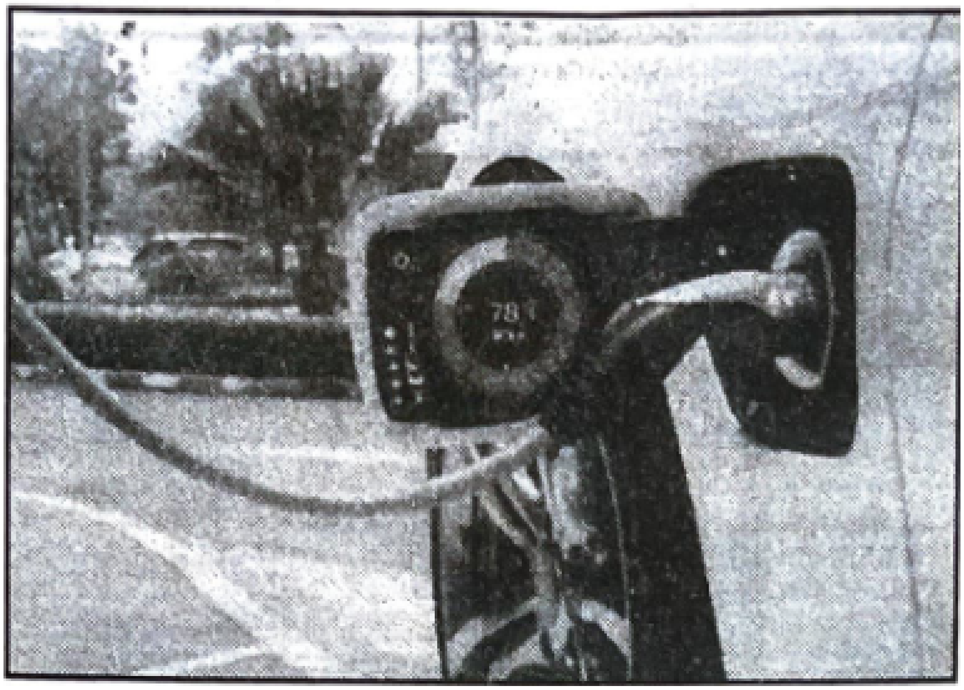


Figure 12.2:

12. The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time $t$ is given by the function $V$ :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2 \qquad (12.1)$$

Where $t$ represents the time and $t$=1,2,3... corresponds to year 2001,2002,2003... respectively. Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.

(ii) Prove that the function V($t$) is an increasing function.

13. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is ______.

14. The total revenue (in ₹) received from sale of x units of a product is $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 12$ is ______.

15. If $\sin y = x\sin(a+y)$, then prove that $\frac{dy}{dx} = \frac{\sin x^2(a+y)}{\sin a}$.

16. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point(3,22).

17. If $Y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$,$-\frac{1}{\sqrt{3}}$ $<$x $<\frac{1}{\sqrt{3}}$ then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

18. If $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$, then $\frac{dy}{dx}$ is equal to

19. The oder and degree of the differential equation of the family of parabolas having at organ and axis along positive x-axis is

20. If $y = \log x$, then $\frac{d^2y}{dx^2}$=

21. Find the intervals in which the function f defined as $f(x) = \sin(x) + \cos(x)$, $0 \leq x \leq 2\pi$ is strictly increasing or decreasing.

22. Prove that the radius of the right circular cylindar of greatest curved surface area which can be inscribed in a given cone is half of thatof the cone.

23. If $y = x^{\sin x} + \sin^{-1}(\sqrt{x})$, the find $\frac{dy}{dx}$.

24. The supply function of a commodity is $100p = (x+20)^2$. Find the Producer's Surplus(PS),whne the market price is ₹25.

25. Find: $\int \frac{2x^2+1}{x^2-3x+2}$dx

## 12.2. 2022

### 12.2.1. 12

1. The order and degree of the differential equation of the family of parabolas having vertex at origin and axis along positive x-axis is:

   (a) 1,1

   (b) 1,2

   (c) 2,1

   (d) 2,2

2. If $y = \log x$, then $\frac{d^2y}{dx^2} =$ ______

3. If $y = \sqrt{a + \sqrt{a+x}}$ :

4. Find the intervals in which the function f defined as $f(x) = \sin x + \cos x, 0 \leq x \leq 2$ is strictly increasing or decreasing.

5. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

6. If $y = x^{\sin x} + \sin^{-1} x$,then find $\frac{dy}{dx}$.

7. If $\sin y = x\sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

8. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point (3,22).

9. If $y = \tan^{-1}(\frac{3x-x^3}{1-3x^2}) - \frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ , then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ .

10. If $y = (\tan^{-1} x)^2$ , then show that $(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$.

11. Show that of all the rectangles inscribed in a given fixed circle, the square has maximum area.

12. Find the intervals in which the function f given by $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

13. If $\sec^{-1}(\frac{1+x}{1-y}) = a$ , then $\frac{dy}{dx}$ is equal to

    (a) $\frac{x-1}{y-1}$

    (b) $\frac{x-1}{y+1}$

    (c) $\frac{y-1}{x+1}$

    (d) $\frac{y+1}{x-1}$

## 12.3. 2021

### 12.3.1. 10

1. The order and degree of the differential equation of the family of parabolas having vertex at origin and axis along positive x-axis is

    (a) $1, 1$

(b) $1, 2$

(c) $2, 1$

(d) $2, 2$

2. If $y = \log x$, then $\frac{d^2y}{dx^2} =$ ______.

3. If $y = e^x + e^{-x}$, then show that $\frac{dy}{dx} = \sqrt{y^2 - 4}$.

4. If $y = x^{\sin x} + \sin^{-1}(\sqrt{x})$, the find $\frac{dy}{dx}$.

5. Find the intervals in which the function $f$ defined as $f(x) = \sin(x) + \cos(x)$, $0 \leq x \leq 2\pi$ is strictly increasing or decreasing.

6. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

7. $\lim\limits_{x \to 0} \frac{e^{-x} - e^x}{x}$ is equal to

(a) $2$

(b) $1$

(c) $-1$

(d) $2$

## 12.4. 2021

### 12.4.1. 12

1. The point at which the normal to the curve $y = x + \frac{1}{x}, X > 0$ is perpendicular to the line $3x - 4y7 = 0$ is:

   (a) $(2, \frac{5}{2})$

   (b) $(\pm 2, \frac{5}{2})$

   (c) $(-\frac{1}{2}, \frac{5}{2})$

   (d) $(\frac{1}{2}, \frac{5}{2})$

2. If $y = log(\cos e^x)$, then $\frac{dx}{dy}$ is:

   (a) $\cos e^{x-1}$

   (b) $e^{-x} \cos e^x$

   (c) $e^x \sin e^x$

   (d) $-e^x \tan e^x$

3. The least value of the function $f(x) = 2\cos x + x$ in the closed interval $[0, \frac{\pi}{2}]$ is:

   (a) 2

   (b) $\frac{\pi}{6} + \sqrt{3}$

   (c) $\frac{\pi}{2}$

   (d) The least value does not exist.

4. If $x = a\sec\theta, y = b\tan\theta$, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ is:

   (a) $\frac{-3\sqrt{3}b}{a^2}$

   (b) $\frac{-2\sqrt{3}b}{a}$

   (c) $\frac{-3\sqrt{3}b}{a}$

   (d) $\frac{-b}{3\sqrt{3}a^2}$

5. The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t $\sin^{-1}x, -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$, is:

   (a) 2

   (b) $\frac{\pi}{2} - 2$

   (c) $\frac{\pi}{2}$

   (d) $-2$

6. The point(s) on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$ is/are:

   (a) $(-2, 19)$

   (b) $(2, -9)$

   (c) $(\pm 2, 19)$

   (d) $(-2, 19) and (2, -9)$

7. For which value of m is the line $y = mx + 1$ a tangent to the curve $y^2 = 4x$ ?

   (a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 3

8. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$ is:

   (a) 0

   (b) $\frac{1}{2}$

   (c) 1

   (d) $\sqrt{3}\frac{1}{3}$

## 12.5. 2021

### 12.5.1. 12

1. A firm knows that the demand function for one of its product is linear. It also knows that it can sell 1400 units when the price is ₹4 per unit and it can sell 1800 units at a price ₹2 per unit.Find the marginal revenue function of this product.

2. Find the intervals in which the function $f(x) = x^4 – 4x^3 + 6x^2 – 4x + 1$ is increasing or decreasing.

3. If $\sqrt{1-x^2} + \sqrt{1-y^2} = 4(x-y)$,then show that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

4. If $f'(x) = 3x^2 - 4x - \frac{2}{x^3}$ and $f(1) = 0$, then find $f(2)$.

5. A window is in the form of a rectangle mounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the rectangular part of the window to admit maximum light through the whole opening.

6. Divide the number 8 into two positive numbers such that the sum of the cube of one and the square of the other is minimum.

7. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ is:

   (a) $e^{y-x}$

   (b) $e^{x+y}$

   (c) $-e^{y-x}$

   (d) $2e^{x-y}$

8. $I f y = 5\cos x - 3\sin x$, then $\frac{d^2y}{dx^2}$ is equal to

   (a) -y

   (b) y

   (c) 25y

   (d) 9y

9. The points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to y-axis are:

   (a) $(0, \pm 4)$

   (b) $(\pm 4, 0)$

   (c) $(\pm 3, 0)$

(d) $(0, \pm 3)$

(Differentiation 2021)

# 12.6. 2020

## 12.6.1. 12

1. If the radius of the circle is increasing at the rate of $0.5 cm/s$, then the rate of increase of its circumference is.

2. Differentiate $\sec^2(x^2)$ with respect to $x^2$.

3. If $y = f(x^2)$ and $f'(x) = e^{\sqrt{x}}$, then find $\frac{dy}{dx}$.

4. Find $f'(x)$ if $f(x) = (\tan x)^{\tan x}$.

5. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'\left(\frac{\pi}{3}\right)$

6. If $\tan^{-1}\left(\frac{y}{x}\right) = \log\sqrt{x^2 + y^2}$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

7. If $y = e^{a\cos^{-1}x}, -1 < x < 1$, then show that

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0 \tag{12.2}$$

## 12.7. 2019

### 12.7.1. 12

1. If $y = x\left|x\right|$, find $\frac{dy}{dx}$ for $x < 0$.

2. Find the order and degree (if defined) of the differential equation

$$\frac{d^2y}{d^2x} + x\left(\frac{dy}{dx}\right)^2 = 2x^2\log\left(\frac{d^2y}{dx^2}\right)$$

.

3. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where $A$ and $B$ are arbitrary constants.

4. If $x\sqrt{1+y}+ y\sqrt{1+x}=0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

5. If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

6. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of $a$ and $b$.

7. Solve the differential equation:

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

8. Solve the differential equation :

$$\frac{dy}{dx} = -\left[\frac{x + y\cos x}{1 + \sin x}\right]$$

9. Differentiate $e^{\sqrt{3x}}$, with respect to $x$.

10. Find the general solution of the differential equation $\dfrac{dy}{dx} = e^{x+y}$.

11. If $(a + bx)\, e^{\frac{y}{x}} = x$, then prove that $x^3 \dfrac{d^2y}{dx^2} = \left(x\dfrac{dy}{dx} - y\right)^2$.

12. Find the differential equation representing the family of curves $y = ae^{2x} + 5$, where $a$ is an arbitrary constant.

13. If $y = \cos\left(\sqrt{3x}\right)$, then find $\dfrac{dy}{dx}$.

14. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to $x$.

15. If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$, then prove that $\dfrac{dy}{dx} = \frac{x+y}{x-y}$.

16. A ladder $13m$ long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of $2cm/sec$. How fast is the height on the wall decreasing when the foot of the ladder is $5m$ away from the wall ?

17. If $y = (\log x)^x + x^{\log x}$, find $\frac{dy}{dx}$.

18. If $x = \sin t, y = \sin pt$ , prove that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

19. Differentiate

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$$

with respect to $\cos^{-1} x^2$.

20. Form the differential equation representing the family of curves $y = A\sin x$, by eliminating the arbitrary constant $A$ .

21. If

$$y = \sin^{-1} x + \cos^{-1} x,$$

find $\frac{dy}{dx}$.

22. Write the order and the degree of the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^2 = \left(x + \left(\frac{dy}{dx}\right)^2\right)^3.$$

23. If $\sin y = x\sin(a+y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

24. If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$.

25. If $y = \left(\sec^{-1} x^2\right), x > 0$ , show that

$$x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0$$

26. Find the integrating factor of the differential equation

$$x \ \frac{dy}{dx} - 2y = 2x^2$$

27. Find $\dfrac{dy}{dx}, \quad if \quad xy^2 - x^2 = 4$

28. If $y = \left(\cot^{-1} x\right)^2$, show that

$$\left(x^2+1\right)^2 \frac{d^2y}{dx^2} + 2x\left(x^2+1\right)\frac{dy}{dx} = 2.$$

29. y=$\sin^{-1} + \cos^{-1}$ , find $\dfrac{dy}{dx}$ .

30. If $e^y (x+1) = 1$ , then show that $\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$ .

31. Find $\dfrac{dy}{dx}$, if $y = \sin^{-1}\left(\dfrac{2^{x+1}}{1+4^x}\right)$.

32. Form the differential equation representing the family of curves $y^2 = m\left(a^2 - x^2\right)$ by eliminating the arbitrary constants $'m'$ and $'a'$ .

33. If $y = 2\sqrt{\sec\left(e^{2x}\right)}$, than find $\dfrac{dy}{dx}$.

34. If $y = (\sin x)^x + \sin^{-1}\left(\sqrt{1-x^2}\right)$ , then find $\dfrac{dy}{dx}$.

35. If

$$y = 5e^{7x} + 6e^{-7x}$$

show that $\dfrac{d^2y}{dx^2} = 49y$

36. Find the differential equation of the family of curves represented by $y^2 = a\left(b^2 - x^2\right)$.

37. Find the order of differential equation of the family of circles of radius 3 units.

38. Find the differential equation representing the family of curves

$$y = -A\cos 3x + B\sin 3x$$

39. Form the differential equation representing the family of curves $y = \frac{A}{x} + 5$, by eliminating the arbitrary constant $A$.

40. If $y = \csc\left(\cot\sqrt{x}\right)$, then find $\dfrac{dy}{dx}$.

41. If $y = \log(\cos e^x)$, then find $\dfrac{dy}{dx}$.

42. If $x = \sin t$, $y = \sin pt$, prove that

$$\left(1 - x^2\right)\frac{d^2x}{dx^2} - x\frac{dy}{dx} + p^2y = 0.$$

43. If

$$y = 5e^{7x} + 6e^{-7x}$$

show that $\dfrac{d^2y}{dx^2} = 49y$

44. if

$$x^p y^q = (x+y)^{p+q}$$

and prove that $\dfrac{dy}{dx} = \dfrac{y}{x}$ and $\dfrac{d^2y}{dx^2} = 0$

45. Differentiate

$$\tan^{-1} \frac{3x - x^3}{1 - 3x^2}$$

$|x| < \dfrac{1}{\sqrt{3}}$ w.r.t $\tan^{-1} \dfrac{x}{\sqrt{1-x^2}}$

46. If

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a\left(x-y\right) \quad |x| < 1, |y| < 1$$

show that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$

47. Find the differential equation of the function $\cos^{-1}(\sin 2x)$ w.r.t. $x$.

# 12.8. 2018

## 12.8.1. 12

1. Differentiate

$$\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

with respect to $x$.

2. Find the differential equation representing the family of curves $y = ae^{bx+5}$, where $a$ and $b$ are arbitrary constants.

3. If $y = \sin(\sin x)$, prove that

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$$

4. Find the particular solution of of the differential equation $e^x \tan y \, dx + (2 - e^x)\sec^2 y \, dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$.

5. Find the particular solution of the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$.

6. If $\left(x^2 + y^2\right)^2 = xy$, find $\frac{dy}{dx}$

7. If $x = a(20 - \sin 20)$ and $y = a(1 - \cos 20)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$

8. If $y = \sin^{-1} x + \cos^{-1} x$, find $\frac{dy}{dx}$.

9. Write the order and the degree of the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^2 = \left(x + \left(\frac{dy}{dx}\right)^2\right)^3.$$

10. If $\sin y = x\sin(a+y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

11. If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$.

12. Form the differential equation representing the family of curves $y^2 = \left(ma^2 - x^2\right)$ by eliminating the arbitrary constants '$m$' and '$a$'.

13. Find the integrating factor of the differential equation

$$x\frac{dy}{dx} - 2y = 2x^2.$$

14. If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

15. Find $\dfrac{dy}{dx}$, if $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$.

16. If $y = \left(\sec^{-1}x\right)^2$, $x > 0$, show that

$$x^2\left(x^2-1\right)\frac{d^2y}{dx^2} + \left(2x^3 - x\right)\frac{dy}{dx} - 2 = 0$$

.

17. Solve the differential equation :

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

18. Solve the differential equation :

$$\left(1+x^2\right)dy + 2xydx = \cot x dx$$

19. Find $\frac{dx}{dy}$, if $xy^2 - x^2 = 4$.

20. If $y = \left(\cot^{-1}x\right)^2$, show that $\left(x^2+1\right)^2\frac{d^2y}{dx^2} + 2x\left(x^2+1\right)\frac{dy}{dx} = 2$

21. Find the local maxima and local minima, if any, of the following function. Also find the local maximum and the local minimum values, as the case may be :

$$f(x) = \sin x + \frac{1}{2}\cos 2x, \qquad 0 \le x \le \frac{\pi}{2}$$

22. If $y = \log(\cos e^x)$, then find $\frac{dy}{dx}$.

23. If $y = (\log x)^x + x^{\log x}$, find $\frac{dy}{dx}$.

24. Form the differential equation representing the family of curves $y = A\sin x$, by eliminating the arbitrary constant $A$.

25. Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

26. Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{xy}{x^2+y^2}.$$

27. Solve the following differential equation: $\frac{dy}{dx} + y = \cos x - \sin x$

28. If

$$y = (x)^{\cos x} + (\cos x)^x, find \frac{dy}{dx}$$

29. If $X = \sin t$, $Y = \sin pt$, prove that $(1-x^2)\cdot\frac{d^2y}{dx^2} - x\cdot\frac{dy}{dx} + p^2y = 0$.

30. A ladder 13 m long is leaning against a vertical wall.The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall ?

31. Differentiate $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1}x^2$.

32. Write the integrating factor of the differential equation $(\tan^{-1}y - x)\,dy = (1+y^2)\,dx$.

33. Solve the following differential equation: $\dfrac{dx}{dy} + x = (\tan y + \sec^2 y)$.

34. If $y = (x)^{\cos x} + (\cos x)^{\sin x}$, find $\dfrac{dy}{dx}$.

35. Find the differential equation representing the family of curves $y = -A\cos 3x + B\sin 3x$.

36. Find the differential of the function $\cos^{-1}(\sin 2x)$ w.r.t. $x$.

37. Find the differential equation of the family of curves represented by $y^2 = a\left(b^2 - x^2\right)$.

38. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a\,(x-y)\,, \left|x\right| < 1, \left|y\right| < 1$, show that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$.

39. If $y = 2\sqrt{\sec\left(e^{2x}\right)}$, then find $\dfrac{dy}{dx}$.

40. Find the particular solution of the differential equation: $\left(1+e^{2x}\right)dy + \left(1+y^2\right)e^x dx = 0$, given that $y\,(0) = 1$.

41. If $X^p y^q = (x+y)^{p+q}$, prove that $\dfrac{dy}{dx} = \dfrac{y}{x}$ and $\dfrac{d^2y}{dx^2} = 0$.

42. Find the order of the differential equation of the family of circles of radius 3 units.

43. Find the particular solution of the differential equation: $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0$, given that $y\left(1\right)=\frac{\pi}{2}$.

44. If $y=5e^{7x}+6e^{-7x}$, show that $\frac{d^2y}{dx^2}=49y$.

45. Differentiate $\tan^{-1}\frac{3x-x^3}{1-3x^2}, |x|<\frac{1}{\sqrt{3}}$ w.r.t $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$.

46. Solve the following differential equation :

$$\left(y+3x^2\right)\frac{dx}{dy}=x$$

47. If $y=(\sin x^x)+\sin^{-1}\left(\sqrt{1-x^2}\right)$, then find $\frac{dy}{dx}$

48. Find the order and degree (if defined) of the differential equation

$$\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2=2x^2log\left(\frac{d^2y}{dx^2}\right)$$

49. Find the differential equation of the family of curves $y=Ae^{2x}+Be^{-2x}$, where $A$ and $B$ are arbitrary constants.

50. If $(x-a)^2+(y-b)^2=c^2$ , for some $c>0$, prove that $\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is constant independent of $a$ and $b$.

51. Solve the differential equation: $x\frac{dy}{dx}=y-x\tan\frac{y}{x}$

52. Solve the differential equation: $\frac{dy}{dx}=-\left(\frac{x+y\cos x}{1+\sin x}\right)$

53. If $y = x\left|x\right|$, find $\dfrac{dy}{dx}$ for $x < 0$.

54. If $(\cos x)^y = (\sin y)^x$, find $\dfrac{dy}{dx}$.

55. If $x = ae^t\left(\sin t + \cos t\right)$ and $y = ae^t\left(\sin t + \cos t\right)$, then prove that $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$.

56. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to $x$.

57. Find the general solution of the differential equation $\frac{dy}{dx}{=}e^{x+y}$.

58. If $(a+bx)e^{\frac{y}{x}}{=}x$ then prove that $x^3\frac{d^2y}{dx^2}{=}\left(x\frac{dy}{dx}-y\right)^2$.

59. If $x\sqrt{1+y}{+}\ y\sqrt{1+x}{=}0$ and $x \neq y$, prove that $\dfrac{dy}{dx} = -\frac{1}{(x+1)^2}$.

60. Find the differential equation representing the family of the curves $y = ae^{2x} + 5$, where $a$ is an arbitrary constant.

61. If $y = \cos\left(\sqrt{3x}\right)$, then find $\frac{dy}{dx}$.

62. Differentiate $e^{\sqrt{3x}}$, with respect to $x$

63. Find the order and the degree of the differential equation $x^2\frac{d^2y}{dx^2} = \left\{1\left(\frac{dy}{dx}\right)^2\right\}^4$

64. Form the differential equation representing the family of curves $y = e^{2x}\left(a+bx\right)$, where 'a' and 'b' are arbitrary constants.

65. Form the differential equation representing the family of curves $y = e^{2x}\left(a+bx\right)$, where 'a' and 'b' are arbitrary constants.

66. If $x = \cos t + log\tan\left(\frac{t}{2}\right), y = \sin t$,then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

67. write the order and degree of the following differential equation:

$$x^3\left(\frac{d^2y}{dx^2}\right)^2+x\left(\frac{dy}{dx}\right)^4=0$$

68. solve the differential equation: $\frac{dy}{dx}-\frac{2x}{1+x^2}y{=}x^2+2.$

69. Solve the differential equation: $(x+1)\,\frac{dy}{dx}{=}2e^{-y}-1;y\,(0){=}0.$

## 12.9. 2017

### 12.9.1. 10

### 12.9.2. 12

1. If $x^y+y^x=a^b$, then find $\frac{dy}{dx}$ .

2. If $e^y\,(x+1)=1$, then show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$ .

3. Solve the differential equation $\left(\tan^{-1}x-y\right)dx=\left(1+x^2\right)dy$ .

4. Find the particular solution of the differential equation $(x–y)\,\frac{dx}{dy}=(x+2y)$, given that $y=0$ when $x=1$ .

5. The length $x$, of a rectangle is decreasing at the rate of 5 $cm$ minute and the width $y$, is increasing at the rate of 4 $cm/$ minute. When $x=8\ cm$ and $y=6\ cm$, find the rate of change of the area of the rectangle.

6. Find the general solution of the differential equation

$$ydx - \left(x + 2y^2\right) dy = 0.$$

7. find the general solution of the differential equation

$$\frac{dy}{dx} - y = \sin x.$$

8. The volume of a sphere is increasing at the rate of $8cm^3/s$. Find the rate at which its surface area is increasing when the radius of the sphere is $12cm$.

9. The volume of a cube is increasing at the rate of $9cm^3/s$ . How fast is its surface area increasing when the length of an edge is $10cm$ ? .

# 12.10. 2016

## 12.10.1. 12

1. Differentiate $(\sin 2x)^x + \sin^{-1}\sqrt{3x}$ with respect to $x$.

2. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1} x^2$.

3. Determine the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is strictly increasing or strictly decreasing.

4. Find the maximum and minimum values of $f(x) = \sec x + \log\cos^2 x$, $0 < x < 2\pi$.

5. Find the eqaution of normal to the curve $ay^2 = x^3$ at the point whose $x$ coordinate is $am^2$

6. If $\cos(a+y) = \cos y$ then prove that $\dfrac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence show that $\sin a \dfrac{d^2y}{dx^2} + \sin 2(a+y)\dfrac{dy}{dx} = 0$.

7. Find $\dfrac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{6x-4\sqrt{1-4x^2}}{5}\right]$

8. Find the equation of the tangents to the curve $y = x^3 + 2x - 4$ which are perpendicular to line $x + 14y + 3 = 0$

9. Show that semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

10. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ ia an increasing function of $\theta$ on $\left[0, \frac{\pi}{2}\right]$.

11. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that

$$\frac{dy}{dx} = -\frac{y\log x}{x\log y}$$

12. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

13. If

$$y = \cos(\log x) + 2\sin(\log x)$$

prove that

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

14. If

$$x = a\sin 2t\,(1+\cos 2t) \text{ and}$$
$$y = b\cos 2t\,(1-\cos 2t)$$

find $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{4}$.

15. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

16. Form the differential equation of the family of circles in the second quadrant and touching the co-ordinate axes.

17. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to $x$.

18. If $x = a\sin 2t\,(1+\cos 2t)$ and $y = b\cos 2t\,(1-\cos 2t)$, find $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{4}$.

19. Solve the differential equation: $y + x\dfrac{dy}{dx} = x - y\dfrac{dy}{dx}$.

20. If $y = 2\cos(\log x) + 3\sin(\log x)$, prove that $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$.

21. Solve the differential equation:

$$\left(x^2 + 3xy + y^2\right)dx - x^2dy = 0$$

given that $y = 0$, when $x = 1$.

22. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\dfrac{dy}{dx} = -\dfrac{y\log x}{x\log y}$.

23. Solve the differential equation:

$$x\frac{dy}{dx} + y - x + xy\cot x = 0; x \neq 0.$$

24. Find the particular solution of the differential equation

$$2ye^{\frac{x}{y}}\,dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$$

given that $x = 0$ when $y = 1$.

25. If $x\cos(a+y) = \cos y$ then prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a+y)}{\sin a}$. Hence show that $\sin^2(a+y)\dfrac{dy}{dx} = 0$.

26. Find $\dfrac{dy}{dx}$ if $y = \sin^{-1}\left(\dfrac{6x - 4\sqrt{1-4x^2}}{5}\right)$

27. If

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \dfrac{\sqrt{1+bx} - 1}{x}, & x > 0 \end{cases}$$

is continuous at $x = 0$, then find the values of $a$ and $b$.

28. For what values of $k$, the system of linear equations

$$x + y + z = 2$$

$$2x + y + z = 3$$

$$3x + 2y + kz = 4$$

has a unique solution?

# Chapter 13

# Integration

## 13.1. 2023

### 13.1.1. 12

1. If

$$\frac{d}{dx}\left(f(x)\right)=2x+\frac{3}{x} \tag{13.1}$$

   and $f(1)=1$,then $f(x)$is

   (a) $x^2+3\log|x|+1$

   (b) $x^2+3\log|x|$

   (c) $2-\frac{3}{x^2}$

   (d) $x^2+3\log|x|-4$

2. The integral factor of the differential equation

$$\left(1-y^2\right)\frac{dx}{dy}+yx=ay,(-1<y<1) \tag{13.2}$$

is

(a) $\frac{1}{y^2-1}$

(b) $\frac{1}{\sqrt{y^2-1}}$

(c) $\frac{1}{1-y^2}$

(d) $\frac{1}{\sqrt{1-y^2}}$

3. Anti derivative of $\frac{\tan(x)-1}{\tan(x)+1}$ with respect to $x$ is:

   (a) $\sec^2(\frac{\pi}{4}-x)+c$

   (b) $-\sec^2(\frac{\pi}{4}-x)+c$

   (c) $\log\mid\sec(\frac{\pi}{4}-x)\mid+c$

   (d) $-\log\mid\sec(\frac{\pi}{4}-x)\mid+c$

4. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}}\left(\frac{1}{(e^x+e^{-x})(e^x-e^{-x})}\right)dx$

5. (a) Find the general solution of the differential equation:

$$\left(xy-x^2\right)dy=y^2dx \tag{13.3}$$

   (b) Find the general solution of the differential equation:

$$\left(x^2+1\right)\frac{dy}{dx}+2xy=\sqrt{x^2+4} \tag{13.4}$$

6. (a) Evaluate $\int_{-1}^{1}\left|x^4-x\right|dx$

   (b) Find $\int e^x\left(\frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}\right)dx$

7. Find $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

## 13.2. 2022

### 13.2.1. 12

1. Integration:

$$\int_0^1 x^2 e^x \, dx$$

2. Find the general solution for this differential equation:

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

3. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of $m$ using integration.

4. Find:

$$\int \frac{1}{e^x + 1} \, dx$$

5. Evaluate:

$$\int_1^4 \{|x| + |3 - x|\} \, dx$$

6. Evaluate:

$$\int_{-3}^3 \frac{x^4}{1 + e^x} \, dx$$

7. Find the particular solution of the differential equation:

$$x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$$

given that $y(1) = 0$.

8. Find the general solution of the differential equation:

$$x(y^3 + x^3)\,dy = (2y^4 + 5x^3y)\,dx$$

9. Find:

$$\int \frac{dx}{\sqrt{4x - x^2}}$$

10. Find the general solution of the following equation:

$$\frac{dy}{dx} = e^x - yx^2e^{-y}$$

11. Find:

$$\int e^x \sin(2x)\,dx$$

12. Find:

$$\int \frac{2x}{(x^2+1)(x^2+2)}\,dx$$

13. Evaluate:

$$\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\,dx$$

14. Solve the following differential equations:

$$(y-\sin^2 x)\,dx+\tan(x)\,dy=0$$

15. Find the general solution of the differential equation:

$$(x^3+y^3)\,dy=x^2y\,dx$$

16. Find:

$$\int \frac{1}{\sqrt{12+4x-x^2}}\,dx$$

17. Find:

$$\int \frac{xe^x}{(x+4)^5}\,dx$$

18. Find the general solution of the following differential equation:

$$(4+y^2)(3+\log x)\,dx+x\,dy=0$$

19. Evaluate:

$$\int_0^{\frac{\pi}{3}} |\cos(3x)| \, , dx$$

20. Find the general solution of the following differential equation:

$$2xe^{\frac{y}{x}}\,dy + (x - 2ye^{\frac{y}{x}})\,dx = 0$$

21. Find the particular solution of the differential equation:

$$(2x^2 + y)\frac{dx}{dy} = x$$

given that $y = 2$ when $x = 1$.

22. Find:

$$\int \frac{x^2 + x + 1}{(x+1)(x^2+4)}\,dx$$

23. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

24. Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration.

25. Find:

$$\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\cot x}}\,dx$$

is equal to:

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

26. Find:

$$\int \frac{(x+2)(x+2\log x)^3}{x}\,dx$$

27. Find:

$$\int_0^{\frac{\pi}{2}} \log(\tan x)\,dx$$

28. Find:

$$\int_{-1}^{2} |x| \; dx$$

29. Find:

$$\int x^2 \log x.dx$$

30. Find the general solution of the following differential equation :

$$\frac{dy}{dx} = (1+x)(1+y)$$

31. Find the integrating factor for the following differential equation:

$$\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x (x \neq 0)$$

32. Find:

$$\int \frac{x}{(x-1)^2(x+2)} dx$$

33. Find the following differential equation :

$$x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$$

34. Find the sum of the order and the degree of the differential equation : $\left(x + \frac{dx}{dy}\right)^2 = \left(\frac{dy}{dx}\right)^2 + 1$

35. If $\frac{d}{dx}(x) = \frac{\sec^4 x}{\csc^4 x}$ and $F\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$, then find $F(x)$

36. Find : $\int \frac{\log x - 3}{(\log x)^4} dx$.

37. Find : $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$.

38. Evaluate : $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(4+\sin x)} dx$

39. Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x} dx$.

40. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

41. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$, semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

42. Find the product of the order and the degree of the differential equation $\frac{d}{dx}\left(xy^2\right) \cdot \frac{dy}{dx} + y = 0$.

43. Find : $\int \frac{\sqrt{\cot x}}{\sin x \cos x} dx$

44. Find : $\int \frac{1}{x(x^2+4)} dx$

45. Evaluate : $\int_0^1 \tan^{-1} x dx$

46. Find : $\int \frac{2x}{x^2+3x+2} dx$

47. Solve the following differential equation : $\left(1 + e^{y/x}\right) dy + e^{y/x}(1 - \frac{y}{x})dx = 0$

48. Evaluate : $\int_0^1 x(1-x)^n dx$

49. Using integration, find the area of the smaller region enclosed the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$

50. If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq.units, then using integration, find the value of a, where $a > 0$.

51. Find the general solution of the differential equation : $\frac{dy}{dx} = \dfrac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$

52. Find : $\int \frac{dx}{x^2-6x+13}$

53. Find the particular solution of the differential equation $x\frac{dy}{dx} - y = x^2 \cdot e^x$, given $y(1) = 0$.

54. Find the general solution of the differential equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$

55. Evaluate : $\int_{-\pi/2}^{\pi/2} (\sin|x| + \cos|x|)\, dx$

56. Find : $\int \frac{x^2}{(x^2+1)(3x^2+4)} dx$

57. Evaluate : $\int_{-2}^{1} \sqrt{5-4x-x^2} dx$

58. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

59. $\int \frac{\cos 8x+1}{\tan 2x - \cot 2x} dx = \lambda \cos 8x + c$, then the value of $\lambda$ is

   (a) $\frac{1}{16}$

   (b) $\frac{1}{8}$

   (c) $-\frac{1}{16}$

   (d) $-\frac{1}{8}$

60. $\int_0^1 \tan(\sin^{-1} x) dx$ equals

   (a) 2

   (b) 0

   (c) -1

   (d) 1

61. The integrating factor of the differential equation $x\left(\frac{dy}{dx}\right) - y = \log x$ is ____________.

62. Find the solution of the differential equation $\log\left(\frac{dy}{dx}\right) = ax + by$

63. Solve the following homogeneous differential equation : $x\left(\frac{dy}{dx}\right) = x + y$

64. Evaluate $\int_1^3 (x^2 + 1 + e^x) dx$ as the limit of sums.

65. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a using integration.

66. Find : $\int \frac{x}{(x+1)^2(x+2)} dx$

67. Evaluate : $\int_0^1 \frac{xe^x}{(x+1)^2} dx$

68. Solve the following differential equation : $\left(\frac{dy}{dx}\right) = e^{x+y} + x^2e^y$

69. The supply function of a commodity is $100p = (x + 20)^2$. Find producer's surplus (PS), when the market price is ₹25.

70. Find : $\int \frac{2x^2+1}{x^2-3x+2} dx$

71. In a certain culture of bacteria, the rate of increase of bacteria is proportional to the number present. It is found that there are $10,000$ bacteria at the end of 3 hours and $40,000$ bacteria at the end of 5 hours . determine the number of bacteria present int the beginning

# 13.3. 2021

## 13.3.1. 12

1. If $f(x) = \frac{1-x}{1+x}$,then find $f \circ f(x)$.

2. Let W denote the set of words in the English dictionary. Define the relation R by

   $R = (x, y) \in W \times W$such that x and y  have at least one letter in common.

Show that this relation R is reflexive and symmetric, but not transitive.

3. Find the inverse of the function $f(x) = \frac{4x}{3x+4}$.

4. $\int x\sqrt{x+2}$ dx is equal to

   (a) $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{2}{3}(x+2)^{\frac{3}{2}} + C$

   (b) $\frac{5}{2}(x+2)^{\frac{5}{2}} + \frac{3}{2}(x+2)^{\frac{3}{2}} + C$

   (c) $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

   (d) $\frac{2}{5}(x+2)^{\frac{5}{2}} + \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

   where C is the constant of integration.

5. $\int_0^1 \tan(\sin^{-1} x)$ dx equals

   (a) 2

   (b) 0

   (c) $-1$

   (d) 1

6. $\int \frac{e^x}{x+1}|1 + (x+1)\log(x+1)|dx$ equals

   (a) $\frac{e^x}{x+1} + c$

   (b) $e^x \frac{e^x}{x+1} + c$

   (c) $e^x \log(x+1) + e^x + c$

   (d) $e^x \log(x+1) + c$

7. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}$dx

8. Find: $\int \frac{x}{(x-1)^2(x+2)}$dx

9. $\int_0^{\frac{\pi}{2}} (\sin^{100} x - \cos^{100} x)$ dx equals

   (a) $\frac{\pi}{100}$

   (b) 0

   (c) $\frac{1}{100}$

   (d) $\frac{\lfloor 100}{(100)^{100}}$

10. $\int \frac{\cos 8x+1}{\tan 2x-\cot 2x} dx = \lambda \cos 8x + c$,then the value of $\lambda$ is

   (a) $\frac{1}{16}$

   (b) $\frac{1}{8}$

   (c) $\frac{-1}{16}$

   (d) $\frac{-1}{8}$

# 13.4. 2020

## 13.4.1. 12

1. Evaluate:

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x dx \tag{13.5}$$

2. Find the integrating factor of the differential equation

$$x\frac{dy}{dx} = 2x^2 + y \tag{13.6}$$

3. Find:

$$\int \frac{\tan^3 x}{\cos^3 x} dx \tag{13.7}$$

4. Solve the following differential equation:

$$\left(1 + e^{\frac{y}{x}}\right) dy + e^{\frac{y}{x}} \left(1 - \frac{y}{x}\right) dx = 0 \, (x \neq 0) \tag{13.8}$$

5. Evaluate:

$$\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx \tag{13.9}$$

6. Using integration, find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.

7. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are $A\,(2, 0)$, $B\,(4, 5)$ and $C\,(6, 3)$.

# 13.5. 2019

## 13.5.1. 12

1. Find:

$$\int \sqrt{3 - 2x - x^2} dx$$

2. Find:

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$$

3. Find:

$$\int \frac{x-3}{(x-1)^3} e^x dx$$

4. Find:

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$$

5. prove that

$$\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$$

and hence evaluate

$$\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx$$

6. Find $\int_1^3 \left(x^2 + 2 + e^{2x}\right) dx$ as the limit of sums.

7. Find:

$$\int \frac{x-5}{(x-3)^3} e^x dx$$

8. Find:

$$\int \frac{2\cos x}{(1-\sin x)\left(2-\cos^2 x\right)} dx$$

9. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, given that $y = 1$ when $x = 0$.

10. Solve the differential equation $\frac{dy}{dx} = 1+x^2+y^2+x^2y^2$, given that $y = 1$ when $x = 0$.

11. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, and hence evaluate $\int_0^1 x^2 (1-x)^n\,dx$ .

12. Find

$$\int \frac{\sin 2x}{(\sin^2 x+1)(\sin^2 x+3)} dx$$

13. Prove that

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

and hence evaluate

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}.$$

14. Find

$$\int \frac{\sin x - \cos x}{\sqrt{1+2x}} dx, 0 < x < \pi/2.$$

15. Find

$$\int \frac{\sin (x-a)}{\sin (x+a)} dx.$$

16. Find

$$\int (\log x)^2 \, dx$$

17. Find

$$\int_a^b \frac{\log x}{x} dx.$$

18. Evaluate

$$\int_1^4 \left(1 + x + e^2 x\right) dx$$

as limit of sums.

19. Solve the differential equation :

$$\frac{dy}{dx} = \frac{x+y}{x-y}.$$

20. Solve the differential equation :

$$\left(1+x^2\right) dy + 2xydx = \cot x \ \ dx.$$

21. Using integration, find the area of following region :$\left\{(x, y) : x^2 + y^2 \leq 16a^2\right\}$ and $\left\{y^2 \leq 6ax\right\}$

22. Evaluate :

$$\int_{-1}^{2} |x^2 - x| \, dx.$$

23. Find

$$\int (\cos 2x \cos 4x \cos 6x) \ dx.$$

24. Find :

$$\int e^x \left(\frac{2 + \sin 2x}{2\cos^2 x}\right) \ dx$$

25. Solve the equation differential equation

$$\left(y + 3x^2\right) \frac{dx}{dy} = x \, dx$$

26. Find :

$$\int_{-\frac{\pi}{4}}^{0} \frac{1 + \tan x}{1 - \tan x} \, dx$$

27. Find :

$$\int x.\tan^{-1} x\,dx.$$

28. Find :

$$\int \frac{dx}{\sqrt{5-4x-2x^2}}.$$

29. Integrate the function

$$\frac{\cos(x+a)}{\sin(x+b)}$$

w. r. t. $x$.

30. Solve the following differential equation :

$$\frac{dy}{dx}+y=\cos x-\sin x.$$

31. Write the integrating factor of the differential equation

$$\left(\tan^{-1} y-x\right)dy=\left(1+y^2\right)dx.$$

32. Solve the differential equation $\dfrac{dy}{dx}=1+x^2+y^2+x^2y^2$, given that $y=1$ when $x=0$.

33. Find the particular solution of the differential equation $\dfrac{dy}{dx}=\frac{xy}{x^2+y^2}$, given that $y=1$ when $x=0$.

34. Solve the following equation :

$$\frac{dx}{dy} + x = \left(\tan y + \sec^2 y\right).$$

35. Find

$$\int e^x (\cos x - \sin x) \csc^2 x dx$$

36. Find

$$\int \frac{x-1}{(x-2)(x-3)} dx$$

37. Integrate

$$\frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}$$

with respect to $x$.

38. Find

$$\int (\sin x \sin 2x \sin 3x) \, dx$$

39. Evaluate

$$\int_0^1 (|x-1| + |x-2| + |x-4|) \, dx$$

40. Using integration, find the area of the following region: $(x, y) : x^2 + y^2 \leq 16a^2$

and $y^2 \leq 6ax$

41. Find the particular solution of the differential equation :

$$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0$$

given that $y(1)=\frac{\pi}{2}$

42. Find the particular solution of the differential equation :

$$\left(1+e^{2x}\right)dy+\left(1+y^2\right)e^x dx=0$$

given that $y(0)=1$

# 13.6. 2018

## 13.6.1. 12

1. Evaluate:

$$\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{16+9\sin 2x}.1$$

2. Evaluate

$$\int_1^3\left(x^2+3x+e^x\right)dx,$$

as the limit of the sum.

3. Evaluate:

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$

4. Find:

$$\int \frac{2\cos x}{(1-\sin x)\left(1+\sin^2 x\right)} dx$$

5. Find :

$$\int \frac{\sin(x-a)}{\sin(x+a)} dx$$

6. Find :

$$\int (\log x)^2 dx$$

7. Find :

$$\int \frac{\sin x - \cos x}{\sqrt{1+\sin 2x}} dx, 0 < x < \frac{\pi}{2}$$

8. Find:

$$\int \frac{\sin 2x}{\left(\sin^2 x + 1\right)\left(\sin^2 x + 3\right)} dx$$

9. Prove that

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

and hence evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}.$$

10. Find the area of the triangle whose vertices are $(-1,1)$,$(0,5)$ and $(3,2)$, using integration.

11. Find the area of the region bounded by the curves $(x-1)^2+y^2=1$ and $x^2+y^2=1$, using integration.

12. Find :

$$\int_a^b \frac{\log(x)}{x}\, dx.$$

13. Evaluate

$$\int_1^4 \left(1+x+e^{2x}\right) dx.$$

as limit of sums.

14. Find the area of the region

$$\left\{(x,y) : 0 \le y \le x^2, \quad 0 \le y \le x+2, \quad -1 \le x \le 3\right\}$$

15. Find :

$$\int_{\frac{-\pi}{4}}^{0} \frac{1+\tan x}{1-\tan x}\, dx$$

16. Find :

$$\int x.tan^{-1}x\, dx$$

17. Find :

$$\int \frac{dx}{\sqrt{5-4x-2x^2}}$$

18. integrate the function $\dfrac{\cos(x+a)}{\sin(x+b)}$ w.r.t. $x$.

19. Find :

$$\int \frac{1}{\sin(x-a)\cos(x-b)} \tag{13.10}$$

20. Prove that $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$, and hence evaluate $\int_0^1 x^2(1-x)^n\, dx$.

21. Find:

$$\int \frac{x-1}{(x-2)(x-3)} dx$$

22. Find:

$$\int e^x \left(\frac{2 + \sin 2x}{2\cos^2 x}\right) dx$$

23. Find :

$$\int e^x \left(\cos x - \sin x\right) \csc^2 x dx$$

24. Evaluate:

$$\int_{-1}^{2} \left| x^3 - x \right| dx$$

25. Find:

$$\int \left(\sin x . \sin 2x . sin 3x\right) dx$$

26. Using integration, find the area of triangle $ABC$ bounded by the lines

$$4x - y + 5 = 0, x + y - 5 = 0 \text{ and } x - 4y + 5 = 0.$$

27. Integrate:

$$\frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}$$

with respect to $x$.

28. Evaluate:

$$\int_1^5 (|x-1| + |x-2| + |x-4|) \ dx$$

29. Find:

$$\int \cos 2x \cos 4x \cos 6x \ dx$$

30. Find:

$$\int \sqrt{3-2x-x^2} dx$$

31. Find:

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$$

32. Find:

$$\int \frac{x-3}{(x-1)^3} e^x dx$$

33. Find :

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$$

34. Prove that

$$\int_0^a f(x)\ dx = \int_0^a f(a-x)$$

and hence evaluate

$$\int_0^{\frac{\pi}{2}} \frac{x}{sinx + cosx} dx$$

35. Find:

$$\int \frac{x-5}{(x-3)^3} e^x dx$$

36. Find $\int_1^3 \left(x^2 + 2 + e^{2x}\right) dx$ as the limit of sums.

37. Find: $\int \dfrac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$

38. Find: $\int \sin^{-1} 2x dx$

39. Find: $\int \sqrt{1 - \sin 2x} dx$ , $\frac{\pi}{4} < x < \frac{\pi}{2}$

40. Find: $\int \dfrac{3x+5}{x^2 + 3x - 18} dx$

41. Solve the differential equation :$\left(1+x^2\right) \frac{dy}{dx} + 2xy - 4x^2 = 0$ ,subject to the initial condition $y(0) = 0$

42. Solve the differential equation:$xdy - ydx = \sqrt{x^2 + y^2} dx$ ,given that $y = 0$ when $x = 1$.

43. Find: $\int \dfrac{\cos x}{(1+\sin x)(2+\sin x)} dx$.

44. Find: $\int \sin x \cdot \log \cos x \, dx$.

45. Evaluate$\int_{-\pi}^{\pi} \left(1 - x^2\right) \sin x \cos^2 x dx$.

46. Evaluate: $\int_{-1}^{2} \frac{|x|}{x} dx$.

47. prove that $\int_0^a f(x), dx = \int_0^a f(a-x) dx$ , hence evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$
.

# 13.7. 2017

## 13.7.1. 10

## 13.7.2. 12

1. Find: $\int \dfrac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$ .

2. Find: $\int \dfrac{dx}{5 - 8x - x^2}$ .

3. Find: $\int \dfrac{\cos\theta}{\left(4 + \sin^2\theta\right)\left(5 - 4\cos^2\theta\right)} d\theta$ .

4. Evaluate : $\int_0^{\pi} \dfrac{x \tan x}{\sec x + \tan x} dx$

5. Evaluate: $\int_1^4 \{|x-1| + |x-2| + |x-4|\} \, dx$ .

6. Using the method of integration, find the area of the triangle $ABC$, coordinates of whose vertices are $A(4,1), B(6,6) and C(8,4)$ .

7. Find :

$$\int \frac{\sin\theta d\theta}{\left(4+\cos^2\theta\right)\left(2-\sin^2\theta\right)}$$

8. Find:

$$\int \frac{e^x dx}{\left(e^x-1\right)^2\left(e^x+2\right)}$$

# 13.8. 2016

## 13.8.1. 12

1. Find: $\int \frac{1-\sin x}{\sin x(1+\sin x)}dx$

2. Find: $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$

3. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x}dx$

4. Evaluate : $\int_0^1 \cot^{-1}\left(1-x+x^2\right) dx$

5. Solve the differential equation: $(x+1)\dfrac{dy}{dx} - y = e^{3x}(x+1)^3$

6. Solve the differential equation : $2ye^{\frac{x}{y}} dx + \left(y - 2xe^{\frac{x}{y}}\right) dy = 0$

7. Using integration find the area of the region $(x, y) : y^2 \leq 6ax$ and $x^2 + y^2 \leq 16a^2$.

8. Find : $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}dx$

9. Find : $\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx$

10. Evaluate : $\int_{-2}^{2} \frac{x^2}{1+5^x}dx$

11. Find : $\int (x+3)\sqrt{3-4x-x^2}dx$

12. Find the particular solution of difference equation :

$$\frac{dy}{dx} = -\frac{x + y\cos x}{1+\sin x} \tag{13.11}$$

given that $y = 1$ when $x = 0$.

13. Find the particualr solution of the differential equation

$$2ye^{\frac{x}{y}}dx + (y - 2xe^{\frac{x}{y}})dy = 0$$

given that $x = 0$ when $y = 1$.

14. Using the method of integration, find the area of the triangular region whose vertices are $(2,2)$, $(4,3)$ and $(1,2)$.

15. Evaluate : $\int_0^{\frac{3}{2}} |x\cos\pi x|\ dx$

16. Find: $\int (3x+1)\sqrt{4-3x-2x^2}\,dx$.

17. Using integration, find the area of the triangle formed by negative $x$-axis and tangent and normal to the circle

$$x^2 + y^2 = 9$$

at $\left(-1, 2\sqrt{2}\right)$

18. Solve the differential equation

$$x\frac{dy}{dx}+y-x+xy\cot x=0,\quad x\neq 0$$

19. Solve the differential equation:

$$\left(x^2+3xy+y^2\right)dx-x^2dy=0$$

given that $y=0$, when $x=1$.

20. Find : $\int (3x+5)\sqrt{5+4x-2x^2}\,dx$.

21. Find : $\int \dfrac{2x+1}{(x^2+1)(x^2+4)}\,dx$.

22. Evaluate : $\int_0^\pi \dfrac{x\sin x}{1+3\cos^2 x}\,dx$.

23. Evaluate : $\int_1^5 \{|x-1|+|x-2|+|x-3|\}\,dx$.

24. Find :$\int \dfrac{x^2}{x^4+x^2-2}\,dx$.

25. Evaluate : $\int_0^{\frac{\pi}{2}} \dfrac{\sin^2 x}{\sin x+\cos x}\,dx$.

26. Solve the differential equation :

$$y+x\frac{dy}{dx}=x-y\frac{dy}{dx}$$

27. Find : $\int (3x+1)\sqrt{4-3x-2x^2}dx$

28. Evaluate: $\int_0^{\frac{\pi}{2}} \dfrac{\sin^2 x}{\sin x+\cos x}dx$

29. Evaluate: $\int_0^{\frac{3}{2}} |x \cos \pi x| dx$

30. Find: $\int \frac{x^2}{x^4 + x^2 - 2}$

31. Evaluate: $\int_1^5 |x-1| + |x-2| + |x-3| \, dx$

32. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1 + 3\cos^2 x} dx$

33. Find: $\int (3x+5) \sqrt{5 + 4x - 2x^2} dx$

34. Find: $\int \frac{2x+1}{(x^2+1)(x^2+4)} dx$

35. Using integration, find the area of the triangle formed by the negative $x$-axis and tangent and normal to the circle $x^2 + y^2 = 9$ at $\left(-1, 2\sqrt{2}\right)$.

36. Find: $\int (x+3) \sqrt{3 - 4x - x^2} dx$.

37. Find: $\int \frac{(2x-5)e^{2x}}{(2x-3)^3} dx$

38. Find: $\int \frac{x^2 + x + 1}{(x^2+1)(x+2)} dx$

39. Evaluate: $\int_{-2}^{2} \frac{x^2}{1 + 5^x} dx$.

40. Find the particular solution of differential equation: $\frac{dy}{dx} = -\frac{x + y \cos x}{1 + \sin x}$ given that $y = 1$ when $x = 0$.

41. Using the method of integration, find the area of the triangular region whose vertices are $(2, -2), (4, -3)$ and $(1, 2)$.

# Chapter 14

# Functions

## 14.1. 2023

### 14.1.1. 12

1. The function $f(x) = x\,|x|$ is

   (a) continuous and differentiable at $x = 0$.

   (b) continuous but not differentiable at $x = 0$.

   (c) differentiable but not continuous at $x = 0$.

   (d) neither differentiable nor continuous at $x = 0$.

2. If

$$f(x) = \begin{cases} ax + b & 0 < x \le 1 \\ 2x^2 - x & 1 < x < 2 \end{cases} \tag{14.1}$$

   is a differentiable function is (0,2), then find the values of $a$ and $b$.

3. A function $f : [-4, 4] \to [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that $f$ is an onto function but not a one-one function.Further,find all possible values of 'a' for which $f(a) = \sqrt{7}$.

## 14.2. 2022

### 14.2.1. 12

1. Let $R$ be the relation defined in $N$, as $R = \{(x,y) : 2x+3y = 15, x, y \in N\}$, then $R$= {_____,_____}.

2. If the function $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if} x \neq \frac{\pi}{2} \\ 2, & \text{if} x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of $k$ is _____.

3. Show that the relation $R$ in the set $\mathbb{R}$ of all real numbers,defined as $\mathbb{R} = \left\{(a,b) : a \leq b^2\right\}$ is neither reflexive nor symmetric.

4. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$

5. Let a function $f : \mathbb{R} - \left\{\frac{-4}{3}\right\} \to \mathbb{R}$ be defined as $f(x) = \frac{4x}{3x+4}$.To show that $f$ is one-one function. Hence, find the inverse of the function $f : \mathbb{R} - \left\{\frac{-4}{3}\right\} \to \text{Range of } f$.

6. If $f : R \to R$ be given by $f(x) = \left(3 - x^3\right)^{1/3}$, then find $(fof)(x)$.

7. Let $W$ denote the set of words in the English dictionary. Define the relation $R$ by $R = (x,y) \in W \times W$ such $x$ and $y$ have at least one letter in common. Show that this relation $R$ is reflexive and symmetric, but not transitive.

8. Find the inverse of the function $f(x) = \left(\frac{4x}{3x+4}\right)$

## 14.3. 2021

### 14.3.1. 10

1. The graph of $y = p(x)$ is shown in Figure 1 for some polynomial $p(x)$. Find the number of zeroes of $p(x)$.

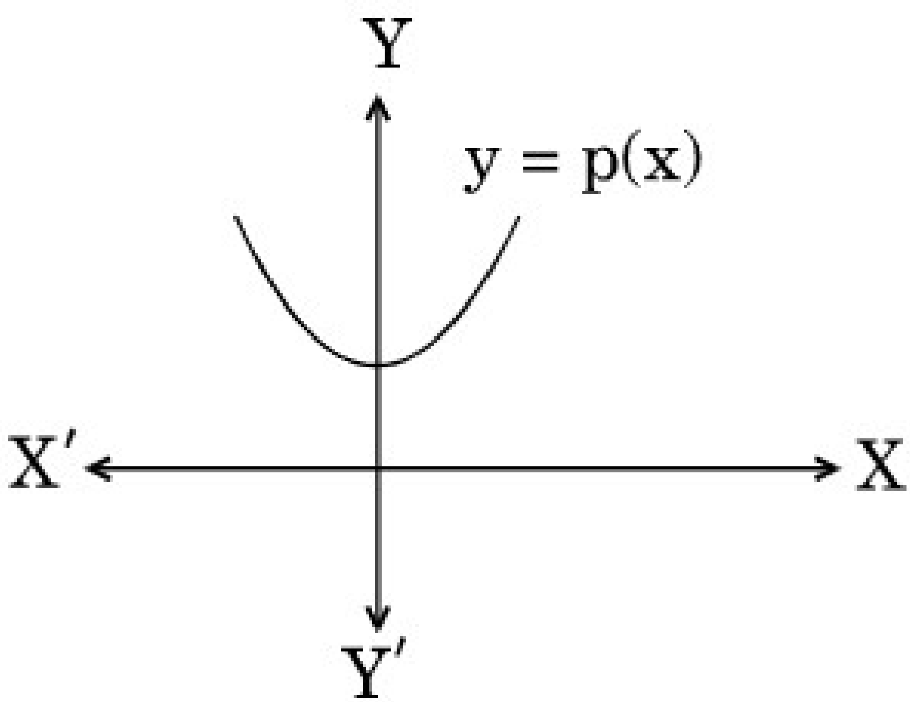



Figure 14.1:

### 14.3.2. 12

1. If $f(x) = \frac{1-x}{1+x}$, then find $(f \circ f)(x)$.

2. Let $W$ denote the set of words in the English dictionary. Define the

relation $R$ by $R = \{(x, y) \in W \times W \mid\} \, x$ and $y$ have at least one letter in common. Show that this relation $R$ is reflexive and symmetric, but not transitive.

3. Find the inverse of the function $f(x) = (\frac{4x}{3x+4})$.

4. The value of $k(k < 0)$ for which the function $f$ defined as

$$f(x) = \begin{cases} x^2, & \text{if } x < 0 \\ \sin(x), & \text{if } x \geq 0 \end{cases}$$

is continuous at $x = 0$ is:

(a) $\pm 1$

(b) $\pm 1$

(c) $\pm \frac{1}{2}$

(d) $\frac{1}{2}$

5. Find the intervals in which the function $f$ given by $f(x) = x^2–4x + 6$ is strictly increasing:

(a) $(-\infty, 2) \bigcup (2, \infty)$

(b) $(2, \infty)$

(c) $(-\infty, 2)$

(d) $(-\infty, 2) \bigcup (2, \infty)$

6. The real function $f(x) = 2x3–3x2–36x + 7$ is:

(a) Strictly increasing in $(-\infty, -2)$ and strictly decreasing in $(-2, \infty)$

(b) Strictly decreasing in $(-2, 3)$

(c) Strictly decreasing in $(-\infty, 3)$ and strictly increasing in $(3, \infty)$

(d) Strictly decreasing in $(-\infty, 2) \bigcup (3, \infty)$

7. The value of $b$ for which the function $f(x) = x + cosx + b$ is strictly decreasing over $\mathbf{R}$ is:

(a) $b < 1$

(b) No value of $b$ exists

(c) $b \leq 1$

(d) $b \geq 1$

8. The point(s), at which the function $f$ given by

$$f(x) = \begin{cases} \frac{x}{|x|}, x < 0 \\ -1, x \geq 0 \end{cases}$$

is continuous, is/are:

(a) $x \varepsilon R$

(b) $x = 0$

(c) $x \varepsilon R - \{0\}$

(d) $x = -1$ and $1$

9. The area of a trapezium is defined by function $f$ and given by $f(x) = (10+x)\sqrt{100-x^2}$ , then the area when it is maximised is:

   (a) $75cm^2$

   (b) $7\sqrt{3}cm^2$

   (c) $75\sqrt{3}cm^2$

   (d) $5cm^2$

10. If $tan^{-1}x = y$, then:

    (a) $-1 < y < 1$

    (b) $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$

    (c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

    (d) $y\varepsilon\{\frac{-\pi}{2}, \frac{\pi}{2}\}$

## 14.4. 2020

### 14.4.1. 12

1. The interval in which the function $f$ given by $f(x) = x^2e^{-x}$ is strictly increasing, is

   (a) $(-\infty, \infty)$

   (b) $(-\infty, 0)$

   (c) $(2, \infty)$

   (d) $(0, 2)$

2. The function $f(x) = \frac{x-1}{x(x^2-1)}$ is discontinuous at

   (a) exactly one point

   (b) exactly two points

   (c) exactly three points

   (d) no points

3. The function $f : \mathbb{R} \to [-1, 1]$ defined by $f(x) = \cos x$ is

   (a) both one-one and onto

   (b) not one-one, but onto

   (c) one-one, but onto

   (d) neither one-one, nor onto

4. The range of the principal value branch of the function $y = \sec^{-1} x$ is

5. The principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is

6. Find the value of $k$, so that the function $f(x) = \begin{cases} kx^2 + 5 & \text{if } x \leq 1, \\ 2 & \text{if } x > 1 \end{cases}$

   is continuous at $x = 1$.

7. Check whether the relation $\mathbb{R}$ in the set $\mathbb{N}$ of natural numbers given by

$$\mathbb{R} = \{(a, b) : \text{a is divisor of b}\} \tag{14.2}$$

   is reflexive, symmetric or transitive.Also determine whether $\mathbb{R}$ is an equivalence relation.

8. Prove that:

$$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right) \tag{14.3}$$

# 14.5. 2019

## 14.5.1. 12

1. Check whether the relation $R$ defined on the set $A=\{1,2,3,4,5,6\}$ as $R=\{(a,b):b=a+1\}$ is reflexive, symmetric or transitive.

2. Let $f:N\to Y$ be a function defined as $f(x)=4x+3$, where $Y=\{y\in N:y=4x+3,\ \text{for some } x\in N\}$. Show that $f$ is invertible. Find its inverse.

3. Examine whether the operation $*$ defined on $\mathbf{R}$, the set of all real numbers, by $a*b=\sqrt{a^2+b^2}$ is a binary operation or not, and if it is a binary operation, find whether it is associative or not.

4. Let $A=R-\{2\}$ and $B=R-\{1\}$. If $f:A\to B$ is a function defined by $f(x)=\frac{x-1}{x-2}$, show that $f$ is one-one and onto. Hence, find $f^{-1}$.

5. Show that the relation $S$ in the set $A=\{x\in Z:0\le x\le 12\}$ given by $S=\{(a,b):a,b\in Z, a-b \text{ is divisible by 3 }\}$ is an equivalence relation.

6. Show that the relation $R$ on the set $Z$ of all integers, given by $R$ $=\{(a,b):2\text{divides}\,(a-b)\}$ is an equivalence relation.

7. Let $*$ be a binary operation on $R - \{-1\}$ defined by $a * b = \dfrac{a}{b+1}$ ,for all $a, b \in R - \{-1\}$. Show that $*$ is neither commutative nor associative in $R - \{-1\}$.

8. Find the intervals in which the function $f$ given by $f(x) = 4x^3$ - $6\mathrm{x}^2 - 72x + 30$ is

   (a) strictly increasing,

   (b) strictly decreasing .

9. If $*$ is defined on the set $\mathbf{R}$ of all real numbers by :$a * b = \sqrt{a^2 + b^2}$ , find the identity element,if it exists in $\mathbf{R}$with respect to $*$.

10. If $f\ (x) = \dfrac{4x+3}{6x-4}, x \neq \dfrac{2}{3}$, show that $f\ (x) = x$ for all $x \neq \dfrac{2}{3}$.Also, find the inverse of $f$.

11. Let an operation *on the set of natural number $N$ be defined by $a*b = a^b$. Find $(i)$ where the *is a binary or not and $(ii)$ if it is a binary, then is it commutative or not.

12. Find whether the function $f\ (x) = \cos\left(2x + \dfrac{\pi}{4}\right)$; is increasing or decreasing in the interval $\dfrac{3\pi}{8} < x < \dfrac{5\pi}{8}$.

13. Find the interval in which the function $f$ given by

$$f\ (x) = \sin 2x + \cos 2x, 0 \leq x \leq \pi$$

is strictly decreasing.

14. Let $*$ be an operation defined as $*$: $\mathbf{R} \times \mathbf{R} \to \mathbf{R}$ such that $a * b = 2a + b$, $a$, $b \in \mathbf{R}$ . Check if * is a binary operation. If yes, find if it is associative too.

15. Let $A = R - \{2\}$ and $B = R\{1\}$. If $f : A \to B$ is a function defined by $f\,(x) = \dfrac{x-1}{x-2}$, show that $f$ is one-one and onto. Hence, find $f^{-1}$.

16. Show that the relation $S$ in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, \;\; b) : a, \, 6ptb \in Z, \;\; |a - b|\}$ is divisible by 3 is an equivalence relation.

17. Let $* : N \times N \to N$ be an operation defined as $a*b = a+ab$, $\forall a, \, b \in N$. Check if * is a binary operation. If yes, find if it is associative too.

18. Prove that the relation $R$ in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ given by $R{=}(a, b) : |a - b|$ is even, is an equivalence relation.

19. Show that the function $f$ in $A = R - \{\frac{2}{3}\}$ defined as $f(x) = \dfrac{4x+3}{6x-4}$ is one-one and onto. Hence find $f^{-1}$.

20. Find whether the function $f(x) = \cos\left(2x + \dfrac{\pi}{4}\right)$ is increasing or decreasing in the interval $\dfrac{3\pi}{8} < x < \dfrac{5\pi}{8}$.

21. If an operation on the set of integers $\mathbb{Z}$ is defined by $a * b = 2a^2 + b$, then find $(i)$ whether it is binary or not and $(ii)$ If a binary then is it commutative or not.

# 14.6. 2018

## 14.6.1. 12

1. Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is

   (a) strictly increasing.

   (b) strictly decreasing.

2. Let $A = \{x \in Z : 0 \leq x \leq 12\}$. Show that $R = \{(a, b) : a, b \in A, \left|a - b\right|$ is divisible by $4\}$ is equivalence relation.find the set of all elements related to 1. also write the equivalence class (2).

3. If is defined on the set $\mathbb{R}$ of all real numbers by : $a * b = \sqrt{a^2 + b^2}$, find the identity element, if it exists in $\mathbb{R}$ with respect to $*$.

4. Let $*$ be a binary operation on $\mathbf{R} - \{-1\}$ defined by $a * b = \frac{a}{b+1}$for all $a, b \in \mathbf{R} - \{-1\}$. show that $*$ is neither commutative nor associative in $\mathbf{R} - \{-1\}$ commutative nor associative in $\mathbf{R} - \{-1\}$.

5. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, Show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. Also, find the inverse of $f$.

6. Find the intervals in which the function $f$ given by $f(x) = 4x^3$-6x$^2 - 72x + 30$ is

   (a) Strictly increasing

   (b) Strictly decreasing.

7. Show that the relation $S$ in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by 3 $\}$ is an equivalence relation.

8. Let $*$ be an operation defined as $* : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ such that $a * b = 2a + b, a, \in \mathbb{R}$.Check if $*$ is a binary operation. If yes, find if it is associative too.

9. Let $A$=$R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x)$=$\dfrac{x-1}{x-2}$, show that $f$ is one-one and onto. Hence, find $f^{-1}$

10. Let $* : N \times N \to N$ be an operation defined as $a * b = a + ab, \forall a, b \in N$. Check if $*$ is binary operation. If yes, find if it is associative too.

11. Find the interval in which the function $f$ given by $f(x) = \sin 2x + \cos 2x, 0 \leq x \leq \pi$ is strictly decreasing.

12. Find whether the function $f(x) = \cos\left(2x + \dfrac{\pi}{4}\right)$; is increasing or decreasing in the interval $\dfrac{3\pi}{8} < x < \dfrac{5\pi}{8}$.

13. Prove that the relation $\mathbf{R}$ in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ given by $\mathbf{R} = \{a, b\} : \left|a - b\right|$ is even  is an equivalence relation.

14. Show that the function $f$ in $A = R - \{\dfrac{2}{3}\}$ defined as $f(x) = \dfrac{4x+3}{6x-4}$ is one-one and onto. Hence, find $f^{-1}$.

15. Check whether the relation $R$ defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric, or transitive.

16. Let $f : \mathbb{N} \to Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in \mathbb{N} : y = 4x + 3,\ \text{for some } x \in \mathbb{N}\}$. Show that $f$ is invertible. Find its inverse.

17. Examine whether the operation $*$ defined on $\mathbb{R}$, the set of all real numbers, by $a * b$=$\sqrt{a^2 + b^2}$ is a binary operation or not, and if it is a binary operation, find whether it is associative or not.

18. If $f(x) = x + 1$, find $\frac{d}{dx}(f \circ f)(x)$

19. Examine whether the operation $*$ defined on R by $a * b = ab + 1$ $(i)$is a binary operation.$(ii)$ If a binary operation, is it associative or not?

20. prove that the function$f : N \to N$,defined by $f(x) = x^2 + x + 1$ is one-one but not onto.Find the inverse of $f : N \to S$,where S is range of f.

21. Show that the relation $R$ on $R$ defined as $R = \{(a, b) : a \leq b\}$, is reflexive, and transitive but not symmetric.

## 14.7. 2017

### 14.7.1. 10

### 14.7.2. 12

1. Determine the value of '$k$' for which the following function is continuous at $x = 3$ :

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3} & , x \neq 3 \\ k & , x = 3 \end{cases}$$

2. Find the value of $c$ in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $\left[-\sqrt{3}, 0\right]$ .

3. Consider $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \dfrac{4x+3}{3x+4}$ . Show that $f$ is bijective . Find the inverse of $f$ and hence find $f^{-1}(0)$ and $x$ such that $f^{-1}(x) = 2$ .

4. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on $\mathbb{R}$ .

## 14.8. 2016

### 14.8.1. 12

1. Find $k$, if

$$f(x) = \begin{cases} k \sin \frac{\pi}{2}(x+1) & , x \leq 0 \\ \frac{\tan x - \sin x}{x^3} & , x > 0 \end{cases}$$

is continous at $x = 0$

2. Let $f : \mathrm{N} \to \mathrm{N}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : \mathrm{N} \to \mathrm{S}$ is invertible (where S is range of $f$). Find the inverse of $f$ and hence find $f^{-1}(31)$ and $f^{-1}(87)$.

3. If

$$f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin}{x} & , x < 0 \\ 2 & , x = 0 \\ \frac{\sqrt{1+bx}-1}{x} & , x > 0 \end{cases}$$

is continuous at $x = 0$, then find the values of $a$ and $b$.

4. Let $A = R \times R$ and $*$ be a binary operation on $A$ defined by

$$(a, b) * (c, d) = (a + c, b + d)$$

Show that $*$ is commutative and associative. Find the identity element for $*$ on $A$. Also find the inverse of every element $(a, b) \in A$.

5. Find the intervals in which the function

$$f\left(x\right)=\frac{4\sin x}{2+\cos x}-x,\quad 0\leq x\leq 2\pi$$

is strictly increasing or strictly decreasing.

6. Verify Mean Value theroem for the function

$$f\left(x\right)=2\sin x+\sin 2x$$

on $[0,\pi]$.

7. Show that the binary operation $*$ on $A=R-\{-1\}$ defined as

$$a*b=a+b+ab$$

for all $a,b\in A$ is commutative and associative on $A$. Also fid the identity element of $*$ in $A$ and prove that every element of $A$ is invertible.

8. Show that the function $f$ given by :

$$f\left(x\right)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & \text{if} x\neq 0\\ -1, & \text{if } x=0\end{cases}$$

is discontinuous at $x=0$.

9. Show that the binary operation $*$ on $A=\mathbf{R}-\{-1\}$ defined as $a*b=a+b+ab$ for all $a,b\in A$ is commutative and associative on $A$. Also

find the identity element of $*$ in $A$ and prove that every element of $A$ is invertible.

10. Show that the function f given by:

$$f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, & x \neq 0 \\ -1, & x = 0 \end{cases}$$

is discontinuous at $x = 0$.

11. Verify Mean Value theorem for the function $f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$.

12. Find the intervals in which the function $f(x) = \dfrac{4\sin x}{2 + \cos x} - x; 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

13. Let $A = R \times R$ and * be a binary operation on $A$ defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that * is commutative and associative. Find the identity elemeny for * on $A$. Also find the inverse of every element $(a, b) \in A$.

# Chapter 15

# Matrices

## 15.1. 2020

### 15.1.1. 10

1. The pair of linear equations $\frac{3x}{2} + \frac{5y}{3} = 7$ and $9x + 10y = 14$ is

   (a) consistent

   (b) inconsistent

   (c) consistent with one solution

   (d) consistent with many solutions

2. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

3. The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.

## 15.2. 2020

### 15.2.1. 12

1. If $A$ is an non-singular square matrix of order 3 such that $A^2 = 3A$, then value of $\begin{vmatrix} A \end{vmatrix}$ is

   (a) $-3$

   (b) 3

   (c) 9

   (d) 27

2. If $\begin{vmatrix} 2x & -9 \\ -2 & x \end{vmatrix} = \begin{vmatrix} -4 & 8 \\ 1 & -2 \end{vmatrix}$, then value of $x$ is.

3. If $A = \begin{vmatrix} -3 & 2 \\ 1 & -1 \end{vmatrix}$ and $I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$, find scalar $k$ so that $A^2 + I = kA$.

4. If $A = \begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix}$, find $A^{-1}$ and use it to solve the following system of equation:

$$5x - y + 4z = 5 \tag{15.1}$$

$$2x + 3y + 5z = 2 \tag{15.2}$$

$$5x - 2y + 6z = -1 \tag{15.3}$$

5. If $x, y, z$ are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^3 & 1+z^3 \end{vmatrix} = 0$, then using properties of determinants show that $1 + xyz = 0$.

## 15.3. 2022

### 15.3.1. 10

1. Solve the equation $x + 2y = 6$ and $2x - 5y = 12$ graphically.

2. Solve the following equations for $x$ and $y$ using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \tag{15.4}$$

$$(bx + ay) + (b - 4a) = 0 \tag{15.5}$$

### 15.3.2. 12

1. If $\begin{vmatrix} 3x & 3 \\ 13 & x \end{vmatrix} = \begin{vmatrix} 4 & -2 \\ 8 & 5 \end{vmatrix}$, then the value of $x$ is :
   (a) 3
   (b) $\pm 5$
   (c) 25
   (d) $\pm 1$

2. For $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, if $A + A' = O$, then the value of $\alpha$ is:

   (a) $\frac{\pi}{6}$

   (b) $\frac{\pi}{3}$

   (c) $\frac{\pi}{2}$

   (d) $\pi$

3. For the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \\ 2 & -1 & 3 \end{pmatrix}$,
   show that $A^3 - 6A^2 + 5A + 11I = 0$. Hence, find $A^{-1}$.

4. Using the properties of determinants, solve the following for $x$:

$$\begin{vmatrix} x+3 & x+7 & x-1 \\ x+7 & x-1 & x+3 \\ x-1 & x+3 & x+7 \end{vmatrix} = 0 \tag{15.6}$$

5. Find the value of $x$, if $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix}$=0.

6. If $A = \begin{pmatrix} 4x & 0 \\ 2x & 2x \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$, then $x =$ __________.

7. If $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, then $A^2$ equals

(a) $\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & -2 \\ -2 & -2 \end{pmatrix}$

(c) $\begin{pmatrix} -2 & -2 \\ -2 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$

8. The roots of the equation $\begin{vmatrix} x & 0 & 8 \\ 4 & 1 & 3 \\ 2 & 0 & x \end{vmatrix} = 0$ are

   (a) $-4, 4$

   (b) $2, -4$

   (c) $2, 4$

   (d) $2, 8$

9. A square matrix $A$ is said to be singular if __________.

10. If $A = \begin{pmatrix} 3 & -5 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 17 \\ 0 & -10 \end{pmatrix}$, then $\left|AB\right| =$ __________.

11. if $\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ is symmetric matrix, then find the value of $x$.

12. If $A$ is a square matrix such that $A^2 = A$, then find $(2+A)^3 - 19A$.

13. For the matrix $A = \begin{pmatrix} 2 & 3 \\ -4 & -6 \end{pmatrix}$, verify the following:

$$A(adjA) = (adjA)A = \left|A\right| I \tag{15.7}$$

14. Using properties of determinants show that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2) \tag{15.8}$$

15. Find the equation of the line join $A\,(1,3)$ and $B\,(0,0)$, using determinants. Also find $k$ if $D\,(k,0)$ is a point such that the area of $\triangle ABD$ is 3 square units.

## 15.4. 2023

### 15.4.1. 10

1. The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are :

   (a) intersecting

   (b) parallel

   (c) coincident

(d) either intersecting or parallel

2. Two schools $P$ and $Q$ decided to award prizes to their students for two games of Hockey ₹$x$ per students and cricket ₹$y$ per student. School $P$ decided to award a total of ₹$9,500$ for the two games to 5 and 4 students respectively; while school $Q$ decided to award ₹$7,370$ for the two games to 4 and 3 students respectively.

Figure 15.1: trophies

Based on the given information, answer the following questions :

(i) Represent the following information algebraically(in terms of $x$ and $y$).

(ii) (a) what is the prize amount for hockey ?

(b) Prize amount on which game is more and by how much ?

(iii) what will be the total prize amount if there are 2 students each from two games ?

3. If the pair of equations $3x - y + 8 = 0$ and $6x - ry + 16 = 0$ represents coincident lines,then the values of $r$ is :

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) 2

(d) $-2$

4. The pair of equations $x = a$ and $y = b$ graphically represents lines which are :

(a) parallel

(b) intersecting at $(b, a)$

(c) coincident

(d) intersecting at $(a, b)$

5. (a) If the system of linear equations $2x+3y = 7$ and $2ax+(a+b)y = 28$ have infinite number of solutions, then find the values of $a$ and $b$.

(b) If $217x + 131y = 913$ and $131x + 217y = 827$, then solve the equations for the values of $x$ and $y$.

6. Half of the difference between two numbers is 2. The sum of the greater number and twice the smaller number is 3.Find the numbers.

### 15.4.2. 12

1. If $(a, b)$,$(c, d)$ and $(e, f)$ are the vertices of $\triangle ABC$ and $\Delta$ denotes the area of $\triangle ABC$, then

$$\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2 \tag{15.9}$$

is equal to

(a) $2\Delta^2$

(b) $4\Delta^2$

(c) $2\Delta$

(d) $2\Delta$

2. If $\begin{pmatrix} 2 & 0 \\ 5 & 4 \end{pmatrix} = P+Q$ is a symmetric and $Q$ is a skew symmetric matrix, then $Q$ is equal to

(a) $\begin{pmatrix} 2 & \frac{5}{2} \\ \frac{5}{2} & 4 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & \frac{5}{2} \\ -\frac{5}{2} & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & -\frac{5}{2} \\ \frac{5}{2} & 4 \end{pmatrix}$

3. If $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{pmatrix}$ is non-singular matrix and $a \in A$, then the set $A$ is

(a) $\mathbb{R}$

(b) $\{0\}$

(c) $\{4\}$

(d) $\mathbb{R} - \{4\}$

4. If $\left|A\right| = \left|kA\right|$, where $A$ is a square matrix of order 2,then sum of all possible values of $k$ is

(a) 1

(b) $-1$

(c) 2

(d) 0

5. (a) If $A = \begin{pmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{pmatrix}$,then find $AB$

and use it to solve the following system of equations :

$$x - 2y = 3 \tag{15.10}$$

$$2x - y - z = 2 \tag{15.11}$$

$$-2y + z = 3 \tag{15.12}$$

(b) If $f(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ,then prove that

$$f(\alpha).f(-\beta) = f((\alpha - \beta)). \tag{15.13}$$

# 15.5. 2021

## 15.5.1. 12

1. If $\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ is a symmetric, find the value of $x$.

2. If $A$ is a square matrix such that $A^2 = A$, find $(2 + A)^3 - 19A$.

3. For the matrix $A = \begin{pmatrix} 2 & 3 \\ -4 & -6 \end{pmatrix}$, verify the fallowing $A(adjA) = (adjA)A = \left|A\right| I$.

4. Using properties of determinants shows that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

5. Find the equation of the line joining $A(1,3)$ and $B(0,0)$ using determinants. Also, find $k$ if $D(k,0)$ is a point such that the area of $\Delta ABD$ is 3 square units.

6. Solve the system of linear equations using the matrix method:

$$7x + 2y = 11$$
$$4x - 7y = 2$$

7. Find the value of $x$, if $\begin{pmatrix} x & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} x \\ 3 \end{pmatrix} = 0$

8. If $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then $A^4 =$ ____________.

9. Given $A = \begin{pmatrix} 1 & -1 & 1 \\ 3 & -2 & 1 \\ -2 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 1 & -2 \end{pmatrix}$, the order of the matrix $AB$ is ____________.

10. if $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $(i^2 = -1)$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, then $AB$ is equal to

(a) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

(b) $\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

(c) $\begin{pmatrix} i & -i \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 \\ i & 0 \end{pmatrix}$

11. If $A$ is a $5 \times p$ matrix, $B$ is a $2 \times q$ matrix, then the order of the matrix $AB$ is $5 \times 4$. What are the values of $p$ and $q$?

(a) $p = 2, q = 4$

(b) $p = 4, q = 2$

(c) $p = 2, q = 2$

(d) $p = 4, q = 4$

12. Value of $k$, for which $A = \begin{pmatrix} k & 8 \\ 1 & 2k \end{pmatrix}$ is a singular matrix is:

(a) 4

(b) $-4$

(c) $\pm 4$

(d) 0

13. If $A = [a_i j]$ is a square matrix of order 2 such that $a_i = \begin{cases} 1, & i+j \\ 0, & i-j \end{cases}$, then $A^2$ is:

(a) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

# 15.6. 2021

## 15.6.1. 12

1. If $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, then $A^2$ equals

(a) $\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & -2 \\ -2 & -2 \end{pmatrix}$

(c) $\begin{pmatrix} -2 & -2 \\ -2 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$

2. $\begin{vmatrix} 43 & 44 & 45 \\ 44 & 45 & 46 \\ 45 & 46 & 47 \end{vmatrix}$

   (a) 0

   (b) $-1$

   (c) 1

   (d) 2

3. A square matrix $A$ is said to be singular if ________.

4. If $A = \begin{pmatrix} 3 & -5 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 17 \\ 0 & -10 \end{pmatrix}$, then $\left|AB\right| =$ ________.

## 15.6.2. 10

1. Find whether the following pair of linear equations are consistent or inconsistent:

$$5x - 3y = 11, -10x + 6y = 22.$$

2. Solve for $x$ and $y$:

$$x + y = 6, 2x - 3y = 4.$$

3. Find out whether the pair of equations $2x + 3y = 0$ and $2x - 3y = 26$ is consistent or inconsistent.

4. For what values of $k$, does the pair of linear equations $kx - 2y = 3$ and $3x + y = 5$ have a unique solution?

5. What type of lines will you get by drawing the graph of the pair of equations $x - 2y + 3 = 0$ and $2x - 4y = 5$?

6. The sum of the numerator and the denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to $\frac{1}{3}$. Find the fraction.

7. Find the value of $k$ for which the system of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has no solution.

8. If 2 tables and 2 chairs cost ₹700 and 4 tables and 3 chairs cost ₹$1,250$, then find the cost of one table.

9. If the graph of a pair of lines $x - 2y + 3 = 0$ and $2x - 4y = 5$ be drawn, then what type of lines are drawn?

## 15.7. 2021

### 15.7.1. 12

1. Given that $A$ is a square matrix of order 3 and $|A| = -4$, then $|adj A|$ is equal to:

   (a) $-4$

   (b) $4$

   (c) $-16$

   (d) $16$

2. If $\begin{pmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix}$, then the value of $a+b-c+2d$ is:

   (a) $8$

   (b) $10$

   (c) $4$

   (d) $-8$

3. Given that matrices $A$ and $B$ are of order $3 \times n$ and $m \times 5$ respectively, then the order of matrix $C = 5A + 3B$ is:

   (a) $3 \times 5$

   (b) $5 \times 3$

   (c) $3 \times 3$

   (d) $5 \times 5$

4. For matrix $A = \begin{pmatrix} 2 & 5 \\ -11 & 7 \end{pmatrix}$, $(\text{adj}A)'$ is equal to:

(a) $\begin{pmatrix} -2 & -5 \\ 11 & -7 \end{pmatrix}$

(b) $\begin{pmatrix} 7 & 5 \\ 11 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 7 & 11 \\ -5 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 7 & -5 \\ 11 & 2 \end{pmatrix}$

5. Given that $A = [a_{ij}]$ is a square matrix of order $3 \times 3$ and $|A| = -7$, then the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$ ,where $A_{ij}$ denotes the cofactor of element $a_{ij}$ is:

(a) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

6. If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, then

   (a) $A^{-1} = B$

   (b) $A^{-1} = 6B$

   (c) $B^{-1} = B$

   (d) $B^{-1} = \frac{1}{6}A$

7. Given that $A$ is a non-singular matrix of order 3 such that $A^2 = 2A$, then the value of $|2A|$ is:

   (a) 4

   (b) 8

   (c) 64

   (d) 16

8. If $A = \begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$ and $kA = \begin{pmatrix} 0 & 3a \\ 2b & 24 \end{pmatrix}$, then the values of $k, a$ and $b$ respectively are:

   (a) $-6, -12, -18$

   (b) $-6, -4, -9$

   (c) $-6, 4, 9$

   (d) $-6, 12, 18$

9. If $A$ is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

(a) $A$

(b) $I + A$

(c) $I - A$

(d) $I$

10. For $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, then $14A^{-1}$ is given by:

(a) $14 \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & -2 \\ 2 & 6 \end{pmatrix}$

(c) $2 \begin{pmatrix} 2 & -1 \\ 1 & -3 \end{pmatrix}$

(d) $2 \begin{pmatrix} -3 & -1 \\ 1 & -2 \end{pmatrix}$

11. Given that $A = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$ and $A^2 = 3I$, then:

(a) $1 + \alpha^2 + \beta\gamma = 0$

(b) $1 - \alpha^2 - \beta\gamma = 0$

(c) $3 - \alpha^2 - \beta\gamma = 0$

(d) $3 + \alpha^2 + \beta\gamma = 0$

12. Let $A = \begin{pmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{pmatrix}$, where $0 \le \alpha \le 2\pi$, then:

   (a) $\left|A\right| = 0$

   (b) $\left|A\right| \in (2, \infty)$

   (c) $\left|A\right| \in (2, 4)$

   (d) $\left|A\right| \in \left|2, 4\right|$

# 15.8. 2019

## 15.8.1. 12

1. If **A** is a square matrix satisfying $A'A = I$, write the value of $\left|A\right|$.

2. $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$, show that$(\mathbf{A} - 2\mathbf{I})(\mathbf{A} - 3\mathbf{I}) = 0$.

3. Using properties of determinants, show that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

4. If $\mathbf{A} = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 1 \\ 5 & 1 & 1 \end{pmatrix}$, find $A^{-1}$. Hence solve the system of equations

$$\begin{aligned} x + 3y + 4z &= 8 \\ 2x + y + 2z &= 5 \\ \text{and } \; 5x + y + z &= 7 \end{aligned}$$

5. Find the inverse of the following matrix, using elementary transformations :

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

6. Find $\left|AB\right|$, if $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 5 \\ 0 & 0 \end{pmatrix}$.

7. If $\mathbf{A} = \begin{pmatrix} p & 2 \\ 2 & p \end{pmatrix}$ and $\left|A^3\right| = 125$, then find the values of $p$.

8. Show that for the matrix $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$, $A^3 - 6A^2 + 5A + 11I = 0$

Hence, find $\mathbf{A^{-1}}$.

9. Using matrix method, solve the following system of equations :

$$3x - 2y + 3z = 8$$
$$2x + y - z = 1$$
$$4x - 3y + 2z = 4$$

10. $A$ is a square matrix with $\left|A\right| = 4$. Then find the value of $\left|A.(\text{adj}A)\right|$.

11. For the matrix $A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$. Find $(A + A')$ and verify that it is a symmetric matrix.

12. Using elementary row transformations, find the inverse of the matrix $\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$.

13. Using matrices, solve the following system of linear equations :

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

14. Using properties of determinants, find the value of $k$ if $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = k\left|x^3 + y^3\right|$.

15. If $A$ is a square matrix of order 3 with $\left|A\right| = 4$, then write the value of $\left|-2A\right|$.

16. If $\mathbf{A} = \begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$ and $k\mathbf{A} = \begin{pmatrix} 0 & 3a \\ 2b & 24 \end{pmatrix}$, then find the values of $k$, $a$ and $b$.

17. Using properties of determinants, prove that

$$\begin{pmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{pmatrix} = 4abc.$$

18. Find the inverse of the following matrix, using elementary transformations:

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}.$$

19. If $\mathbf{A} = \begin{pmatrix} -3 & 6 \\ -2 & 4 \end{pmatrix}$, then show that $A^3 = A$.

20. Using properties of determinants, prove that

$$\begin{pmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{pmatrix} = 1 + a^2 + b^2 + c^2.$$

21. If $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ ,find $A^2$ and show that $A^2 = A^{-1}$ .

22. Using matrix method, solve the following system of equations :

$$2x - 3y + 5z = 13$$
$$3x + 2y - 4z = -2$$
$$x + y - 2z = -2.$$

23. If $\left(A\right) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix}$ ,find $A^{-1}$. Hence, solve the system of equations :

$$x + y + z = 6,$$
$$y + 3z = 11,$$
$$\text{and} \quad x - 2y + z = 0$$

24. If $A = \begin{pmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \\ 7 & 5 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \\ 2 & 2 & 6 \end{pmatrix}$ , then find the matrix $B'A'$.

25. If A=$\begin{pmatrix} 8 & 2 \\ 3 & 2 \end{pmatrix}$, then find $|\text{adj}.A|$.

26. Using properties of determinants, prove that following:

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

27. Using matrices solve the following system of linear equations:

$$\begin{aligned} 2x+3y+10z &= 4 \\ 4x+6y+5z &= 1 \\ 6x+9y-20z &= 2 \end{aligned}$$

28. Find the value of $(x-y)$ from the matrix equation

$$2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} -3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$$

29. If $A$ is a square matrix of order 3, with $|A| = 9$, then write the value of $|2.\,\mathrm{adj}A|$.

30. If $A$ and $B$ are symmertic matrices, such that $AB$ and $BA$ are both defined, then prove that $AB - BA$ is a skew matrix.

31. Using properties of determinants, find the value of $x$ for which

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0.$$

32. If $x, y, z$ are the different and $\triangle = \begin{vmatrix} x & x^2 & x^3-1 \\ y & y^2 & y^3-1 \\ z & z^2 & z^3-1 \end{vmatrix} = 0$ then using properties of determinants show that $xyz = 1$.

33. Using elementary row transformations find the inverse of the matrix $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

# 15.9. 2019

## 15.9.1. 10

1. Find the solution of the pair of equations :

$$\frac{3}{x} + \frac{8}{y} = -1; \frac{1}{x} - \frac{2}{y} = 2, x, y \neq 0$$

2. Find the value(s) of $k$ for which the pair of equations

$$kx + 2y = 3$$
$$3x + 6y = 10$$

has a unique solution.

3. Sumit is 3 times as old as his son. Five years later, he shall be two and a half times as old as his son. How old is Sumit at present ?

4. Find the value(s) of $k$ so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.

5. For what value of $k$, will the following pair of equations have infinitely many solutions :

$$2x + 3y = 7 \text{ and } (k+2)\,x - 3\,(1-k)\,y = 5k + 1$$

6. Solve the following pair of linear equations :

$$3x - 5y = 4$$
$$2y + 7 = 9x$$

7. Solve the following pair of linear equations :

$$3x + 4y = 10$$
$$2x - 2y = 2$$

8. Find the value of $k$ so that the area of triangle $ABC$ with $A(k+1, 1)$, $B(4, -3)$ and $C(7, -k)$ is 6 square units.

9. For what value of $k$, does the system of linear equations

$$2x + 3y = 7$$
$$(k-1)x + (k+2)y = 3k$$

have an infinite number of solutions ?

10. A part of monthly hostel charges in a college hostel are fixed and the remaining depends on the number of days one has taken food in the mess. When a student $A$ takes food for 25 days, he has to pay ₹$4,500$, whereas a student $B$ who takes food for 30 days, has to pay ₹$5,200$. Find the fixed charges per month and the cost of food per day.

11. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

12. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

13. A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator.Find the fraction.

14. Find the value of $k$ for which the following pair of linear equations have infinitely many solutions. $2x + 3y = 7$ , $(k+1)x + (2k-1)y = 4k+1$

# 15.10. 2018

## 15.10.1. 12

1. If the matrix $A = \begin{pmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{pmatrix}$ is skew symmetric, find the values of '$a$' and '$b$'.

2. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9\,(3xyz + xy + yz + zx)$$

3. If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, Find the $A^{-1}$. Use it ton solve the system of equations

$$2x - 3y + 5z = 11$$
$$3x - 2y - 4z = -5$$
$$x + y - 2z = -3$$

4. Using elementary row transformations, find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{pmatrix}$.

5. Given $A = \begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix}$, compute $A^{-1}$ and show that $2A^{-1} = 9I - A$.

6. If A is a square matrix of order 3 with $|A| = 4$,then write the value of $|-2A|$.

7. If $A = \begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$ and $KA = \begin{pmatrix} 0 & 3a \\ 2b & 24 \end{pmatrix}$ then find the values of $k$, $a$ and $b$.

8. Using properties of determinants, prove that

$$\begin{pmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{pmatrix} = 4abc$$

9. If $A = \begin{pmatrix} -3 & 6 \\ -2 & 4 \end{pmatrix}$, then show that $A^3 = A$.

10. Using properties of determinants, prove that

$$\begin{pmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{pmatrix} = 1 + a^2 + b^2 + c^2$$

11. If $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, find $A^2$ and show that $A^2 = A^{-1}$.

12. Using matrix method, solve the following system of equations:

$$\begin{aligned} 2x - 3y + 5z &= 13 \\ 3x + 2y - 4z &= -2 \\ x + y - 2z &= -2 \end{aligned}$$

13. If $A = \begin{pmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \\ 7 & 5 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \\ 2 & 2 & 6 \end{pmatrix}$, then find the matrix $B'A'$.

14. If $\begin{pmatrix} A \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix}$, find $A^{-1}$. Hence, solve the system of equations :

$$\begin{aligned} x + y + z &= 6, \\ y + 3z &= 11 \\ x - 2y + z &= 0 \end{aligned}$$

.

15. Find the inverse of the following matrix, using elementary transforma-

tions:

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$$

16. For the matrix $A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$. Find $(A + A')$ and verify that it is a symmetric matrix.

17. Using matrices, solve the following system of linear equations :

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

18. $A$ is a square matrix with $\left|A\right| = 4$. Then find the value of $\left|A.\,(\text{adj } A)\right|$.

19. If $A$ is a square matrix of order 3, with $|A| = 9$, then write the value of $\left|2\,.\text{ Adj } A\right|$.

20. Using properties of determinants, find the value of $x$ for which

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

21. Using elementary row transformations, find the inverse of the matrix

$$\begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

22. Using properties of determinants, find the value of $k$ if $\begin{vmatrix} x & -y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = k(x^3+y^3)$

23. If $A$ and $B$ are symmetric matrices, such that $AB$ and $BA$ are both defined, then prove that $AB - BA$ is a skew symmetric matrix.

24. Find the value of $(x-y)$ from the matrix equation $2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} -3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$

25. Using elementary row transformations, find the inverse of the matrix $\begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$.

26. If $A = \begin{pmatrix} 8 & 2 \\ 3 & 2 \end{pmatrix}$, then find $\left|adj\ A\right|$

27. Using the properties of determinants, prove the following :

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2 & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

28. If $x$, $y$, $z$ are different and $\Delta = \begin{vmatrix} x & x^2 & x^3-1 \\ y & y^2 & y^3-1 \\ z & z^2 & z^3-1 \end{vmatrix} = 0$, then using properties of determinants, show that $xyz = 1$.

29. Using matrices, solve the following system of linear equations:

$$2x + 3y + 10z = 4$$
$$4x - 6y + 5z = 1$$
$$6x + 9y - 20z = 2$$

30. Find the equation of the plane passing through $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

31. If $A$ is a square matrix of order 2 and $\left|A\right| = 4$, then find the value of $\left|2 \cdot AA'\right|$, where $A'$ is the transpose of matrix $A$.

32. If $A$ is a square matrix satisfying $A'A = I$, write the value of $|A|$.

33. If $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$, show that $(A - 2I)(A - 3I) = 0$.

34. Using properties of determinants, show that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3\left(a+b+c\right)\left(ab+bc+ca\right)$$

35. If $A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{pmatrix}$ ,find $A^{-1}$.Hence solve the system of equations :

$$x+3y+4z=8$$
$$2x+y+2z=5$$
$$5x+y+z=7$$

36. Find the inverse of the following matrix, using elementary transformations: $A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$

37. Show that for the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$, $A^3 - 6A^2 + 5A + 11I = 0$.

Hence, find $A^{-1}$

38. Using matrix, solve the following system of equation:

$$3x - 2y + 3z = 8$$
$$2x + y - z = 1$$
$$4x - 3y + 2z = 4$$

39. Find $|AB|$, if $A=\begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 5 \\ 0 & 0 \end{pmatrix}$.

40. If $A = \begin{pmatrix} p & 2 \\ 2 & p \end{pmatrix}$ and $\left|A^3\right| = 125$,then find the values of $p$.

41. If $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 3 & 1 & 1 \end{pmatrix}$, find $A{-}1$ . Hence solve the following system of equations $x + y + z = 6, x + 2z = 7, 3x + y + z = 12$.

42. Find the value of x-y,if

$$\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}.$$

43. If $A$ and $B$ are square matrices of the same order 3, such that $\left|A\right| = 2$ and $AB = 2I$, write the value of $\left|B\right|$.

44. Using properties of determinants,prove that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} =$ $(a-1)^3$

45. Using the properties of determinants,prove the following:

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+a)(c+a)$$

46. Find matrix A such that $2A-3B+5C=O$,where B = $\begin{pmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{pmatrix}$ and C= $\begin{pmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{pmatrix}$

47. Find the inverse of the following matrix using elementary operations

$$\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 3 & 1 & 1 \end{pmatrix}$$

48. If $3\mathbf{A}-\mathbf{B}=\begin{pmatrix} 5 & 0 \\ 1 & 1 \end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$, then find the matrix A.

49. Using properties of determinants, prove the following:

$$\begin{pmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{pmatrix} = a^3 + b^3 + c^3 - 3abc.$$

50. If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{pmatrix}$, find $A^{-1}$.Hence,solve the system of equations

$$x + y + z = 6,$$
$$x + 2z = 7,$$
$$3x + y + z = 12.$$

# 15.11. 2017

## 15.11.1. 10

## 15.11.2. 12

1. If for any $2 \times 2$ square matrix $A$, $A\,(Adj\,A) = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$,then write the value of $\det A$.

2. If $A$ is a skew-symmetric matrix of order 3, then prove that $\det A = 0$ .

3. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

4. Find matrix $A$ such that

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$

5. Determine the product $\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}$ and use it to solve the system of equations $x-y+z+4, x-2y-2z=9, 2x+y+3z=1$ .

6. Let $A = \mathbb{Q} \times \mathbb{Q}$ and let $*$ be a binary operation on $A$ defined by $(a,b)*(c,d) = (ac, b+ad)$ for $(a,b), (c,d) \in A$ . Determine,whether $*$ is commutative and associative . Then, with respect to $*$ on $A$

   (a) Find the identity element in $A$ .

   (b) Find the invertible elements of $A$ .

7. If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find $A^{-1}$. Hence using $A^{-1}$ solve the system of

equations

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

# 15.12. 2016

## 15.12.1. 12

1. If $A$ is a square matrix such that $|A| = 5$, write the value of $\left|AA^{\mathrm{T}}\right|$

2. $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -4 \\ 3 & -2 \end{pmatrix}$, find $|AB|$.

3. If $A = \begin{pmatrix} 0 & 3 \\ 2 & -5 \end{pmatrix}$ and $KA = \begin{pmatrix} 0 & 4a \\ -8 & 5b \end{pmatrix}$ find the values of $k$ and $a$.

4. Ishan wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by $50m$ and breadth is increased by $50m$, then its area will remain same, but if length is decreased by $10m$ and breadth is decreased by $20m$, then its area will decrease by $5300m^2$. Using matrices, find the dimensions of the plot. Also give reason why he wants to donate the plot for a school.

5. Using the properties of determinants, prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

6. Using elementary row operations, find the inverse of the following matrix :

$$A = \begin{pmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{pmatrix}$$

7. If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, find $\alpha$ satisfying $0 < \alpha < \frac{1}{2}$ when $A + A^{\mathrm{T}} = \sqrt{2} I_2$, where $A^{\mathrm{T}}$ is transpose of $A$

8. If $A$ is a $3 \times 3$ matrix and $|3A| = k\,|A|$ then write the value of $k$

9. Using properties of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

10. If

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

and $A^3 - 6A^2 + 7A + kI_3 = 0$ find $k$.

11. Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix equation:

$$\begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

12. Using elementary row operations find the inverse of matrix

$$A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$$

and hence solve thr following system of equations

$$\begin{aligned} 3x - 3y + 4z &= 21 \\ 2x - 3y + 4z &= 20 \\ -y + z &= 5. \end{aligned}$$

13. Write the number of all possible matrices of order $2 \times 3$ with each entry 1 or 2.

14. Write the number of all possible matrices of order $2 \times 2$ with each entry $1, 2$ or $3$.

15. A shopkeeper has 3 varieties of pens $A$, $B$ and $C$. Meenu purchased 1 pen of each variety for a total of ₹21. Jeevan purchased 4 pens of $A$ variety, 3 pens of $B$ variety and 2 pens of $C$ variety for ₹60. While Shikha purchased 6 pens of $A$ variety , 2 pens of $B$ variety and 3 pens of $C$ variety for ₹70. Using matrix method, find cost of each variety of pen.

16. If

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix} \text{ and}$$

$$B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix} \text{ and}$$

$$BA = (b_{ij})$$

find $b_{21} + b_{32}$.

17. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹10 more. However, if there were 16 children more, every one would have got ₹10 less. Using matrix method, find the number of children and the amount distributed by Seema. What values are reflected by Seema's decision ?

18. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹2, 800 as interest. However, if trust had interchanged money in bonds, they would have got ₹100 less as interest. Using matrix method, find the amount invested by the trust. Interst received on this amount will be given to Helpage India as donation. Which value is reflected in this question ?

19. Solve for x:

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

using properties of determinants.

20. If $x \in N$ and

$$\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$$

then find the value of $x$.

21. Using Properties of determinants, show that $\triangle ABC$ is isosceles if :

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{vmatrix} = 0$$

22. Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

23. Write the number of all possible matrices of order $2 \times 2$ with each entry 1, 2 or 3.

24. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of $x$.

25. Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix equation:

$$\begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

26. Using properties of determinants, show that $\triangle ABC$ is isosceles if:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

27. A trust invested some money in two types of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹2800 as interest. However if trust had interchanged money in bonds, they would have got ₹100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this

question?

28. A shopkeeper has 3 varieties of pens $A$, $B$ and $C$. Meenu purchased 1 pen of each variety for a total of ₹21. Jeevan purchased 4 pens of $A$ variety, 3 pens of $B$ variety and 2 pens of $C$ variety for ₹60. While Shikha purchased 6 pens of $A$ variety, 2 pens of $B$ variety and 3 pens of $C$ variety for ₹70. Using matrix method, find cost of each variety of pen.

29. Write the number of all possible matrices of order $2 \times 3$ with each entry 1 or 2.

30. If $A = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$ and $BA = (b_{ij})$, find $b_{21} + b_{32}$.

31. Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

32. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹10 more. However, if there were 16 children more, every one would have got ₹10 less. Using the matrix method, find the number of children and the amount distributed by Seema. What values are reflected by Seema's decision?

33. Solve for $x$ : $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$, using properties of determinants.

34. Using elementary row operations find the inverse of matrix $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ and hence solve the following system of equations $3x - 3y + 4z = 21$, $2x - 3y + 4z = 20$, $-y + z = 5$.

35. If $A$ is $3 \times 3$ matrix and $\left|3A\right| = k\left|A\right|$, then write the value of $k$.

36. If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, find $alpha$ satisfying $0 < \alpha < \frac{\pi}{2}$ when $A + A^T = \sqrt{2}I_2$; where $A^T$ is transpose of $A$.

37. Using properties of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz\,(x+y+z)^3$$

38. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = O$ find $k$.

# Chapter 16

# Trignometry

## 16.1. 2019

### 16.1.1. 10

1. A boy standing on a horizontal plane finds a bird flying at a distance of $100m$ from him at an elevation of $30°$. A girl standing on the roof of a $20m$ high building, finds the elevation of the same bird to be $45°$. The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl. (Given $\sqrt{2} = 1.414$)

2. The angle of elevation of an aeroplane from a point $A$ on the ground is $60°$. After a flight of 30seconds , the angle of elevation changes to $30°$. If the plane is flying at a constant height of $3600\sqrt{3}$ metres, find the speed of the aeroplane.

## 16.2. 2016

### 16.2.1. 12

1. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi-vertical angle $\alpha$ is one-third that of the cone and the greatest volume of the cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

2. Prove that

$$2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

3. Solve for $x$ :

$$\tan^{-1}\left(\frac{2-x}{2+x}\right) = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right), \quad x > 0$$

4. Solve the equation for

$$x : \sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$$

5. If

$$\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha \quad \text{prove that}$$
$$\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$$

6. Solve for $x$ : $\tan^{-1}\left(\dfrac{2-x}{2+x}\right) = \dfrac{1}{2}\tan^{-1}\dfrac{x}{2}, x > 0.$

7. Prove that $2\sin^{-1}\left(\dfrac{3}{5}\right) - \tan^{-1}\left(\dfrac{17}{31}\right) = \dfrac{\pi}{4}.$

8. Find the equation of tangents to the curve $y = x^3 + 2x - 4$, which are perpenicular to line $x + 14y + 3 = 0$.

9. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of $\theta$ on $\left[0, \frac{\pi}{2}\right]$.

10. Show that semi-vertical angle of a cone of a maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

11. Solve for x: $\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1}(3x)$.

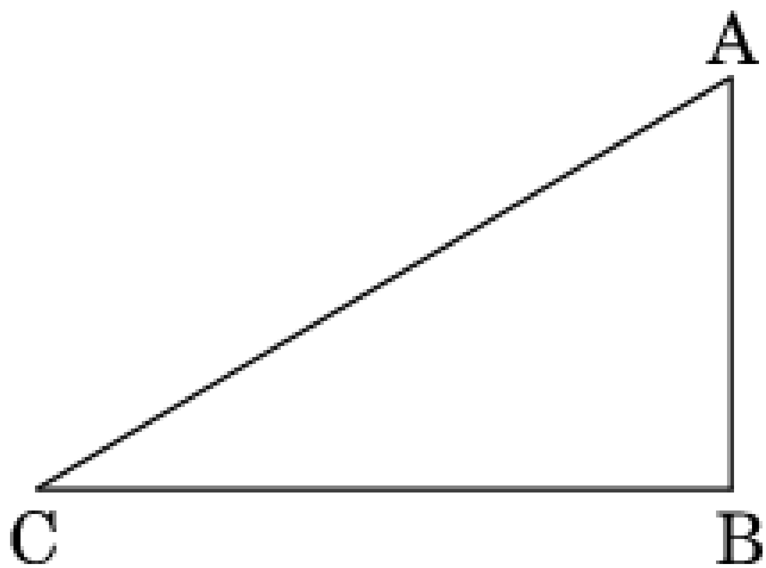
12. Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) + \tan^{-1}\left(\frac{4x}{1-4x^2}\right) = \tan^{-1}2x; \quad |2x| < \frac{1}{\sqrt{3}}$.

# 16.3. 2015

## 16.3.1. 10

1. At a point $A$, 20 metres above the level of water in a lake, the angle of elevation of a cloud is $30°$. The angle of depression of the reflection of the cloud in the lake, at $A$ is $60°$. Find the distance of the cloud from $A$.

2. In Figure 2, a tower $AB$ is 20 m high and $BC$, its shadow on the ground,is $20\sqrt{3}$ m long. Find the sun's altitude.




3. The angle of elevation of an aeroplane from a point $A$ on the ground is $60°$. After a flight of 15 seconds, the angle of elevation changes to $30°$. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr.