

Sr. No: 100476

Roll No \_\_\_\_\_

**B. SC. (HONS) MATHEMATICS – 5<sup>TH</sup> SEMESTER EXAMINATION; DECEMBER - 2017**

[SUB: - REAL ANALYSIS; PAPER CODE: 09050501]

**Time: 3 Hrs.**

**Max. Marks: 75**

**Instructions:-**

1. Write your Roll No. on the Question paper.
2. Candidates should ensure that they have been provided correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter.
3. Attempt Five (5) Questions in all, Question No. 1 is compulsory. Students are required to attempt Four (4) questions selecting one question from each unit. Marks are indicated against each Question.
4. Draw diagram wherever required.

**Unit-1**

1. (a) Show that  $f(x) = x^3$  is R-integrable on  $[0, a]$ .  
(b) Discuss the convergence of improper integral  $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$   
(c) If  $d(x, y) = \min \{ 2, |x-y| \}$ . Show that  $d$  is a metric for  $\mathbb{R}$ .  
(d) Prove that in a metric space, the union of arbitrary collection of open sets is open.  
(e) Define  $\epsilon$  - net and Total Boundedness for a metric space. (3\*5=15)

**Unit-2**

2. (a) State and Prove Darboux Theorem. (8)  
(b) If  $f$  is continuous on  $[a, b]$  and  $F(x) = \int_a^x f(t) dt$ , then  $F$  is differentiable on  $[a, b]$  and  $F' = f$ . (7)
3. (a) State and Prove First Mean Value theorem of Integral Calculus. (7)  
(b) Prove that if  $f$  is integrable on  $[a, b]$  and ' $c$ ' is a real no., then  $cf$  is integrable on  $[a, b]$ . Moreover,

$$\int_a^b cf dx = c \int_a^b f dx \quad (8)$$

**Unit -3**

4. (a) Discuss the convergence of gamma function (7)  
(b) Show that 
$$\int_0^{\infty} \frac{\sin ax \sin bx}{x} dx = \frac{1}{2} \log \frac{a+b}{a-b} \quad (8)$$
5. (a) Prove that 
$$\int_0^{\pi} \frac{\log(1 + \cos x \sin bx)}{\sin x} dx = \frac{\pi^2 - 4\alpha^2}{4} \quad (7)$$
  
(b) State and prove Dirichlet's Test for Convergence of the integral of a product of two functions. (8)

**Unit-4**

6. (a) State and prove Cantor's intersection Theorem. (7)

- (b) Prove that the metric space  $(\mathbb{R}, d)$  is complete, where  $\mathbb{R}$  is the set of real numbers and  $d$  denotes the usual metric for  $\mathbb{R}$ . (8)
7. (a) Prove that in a Metric space every convergent sequence is a Cauchy sequence but the converse need not be true. (9)
- (b) Show that in a metric space  $(X, d)$ , a subset of  $X$  is closed if and only if its complement is open. (6)

#### Unit-5

8. (a) A metric space is compact iff it is closed and Totally Bounded. (8)
- (b) Show that every compact metric space is complete. (7)
9. (a) A metric space  $(X, d)$  is disconnected iff there exists a non empty proper subset of  $X$  which is both  $d$ -open and  $d$ -closed. (8)
- (b) Prove that a Metric Space is sequentially compact iff it has BWP. (7)

Sr. No: 166477

Roll No \_\_\_\_\_

**B. Sc. (Hons.) MATHEMATICS- 5<sup>th</sup> SEMESTER EXAMINATION; DECEMBER - 2017**

**[SUB: - GROUPS & RINGS: 09050502]**

**Time: 3 Hrs.**

**Max. Marks: 75**

**Instructions:-**

1. Write your Roll No. on the Question paper.
2. Candidates should ensure that they have been provided correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter.
3. Attempt Five (5) Questions in all, Question No. 1 is compulsory. Attempt other 4 questions selecting one question from each unit. All question carry equal marks
4. Draw diagram wherever required.

**Q1. (a)** Let 'a' be an element of the group and let  $|a| = 15$ . Then find the order of the following element of the groups.

**(3 x 5 = 15)**

1.  $a^2$
2.  $a^9$
3.  $a^{12}$

**(b)** Show that the set  $Q_0$  of non zero rational numbers becomes a commutative group under binary operation  $*$  where  $a * b = \frac{ab}{2}$

**(c)** Define the following

1. Principal Ideal
2. Maximal Ideal
3. Prime Ideal.

**(d)** Show that the Integral Domain  $\langle \mathbb{Z}, +, \rangle$  of integers is an Euclidean domain.

**(e)** Let 'a' be a fixed element of a group G, then the mapping  $T_a: G \rightarrow G$ , such that  $T_a(x) = a^{-1}xa$  is an automorphism of G

**UNIT-I**

**Q2. (a)** Define normal subgroup of a group G and centre of a group. Also prove that centre of a group G is a normal subgroup of G.

**(8)**

**(b)** Define a subgroup. Also prove that a non empty subset H of a group G is subgroup of G if  $ab^{-1} \in H, \forall a, b \in H$

**(7)**

**Q3. (a)** State and prove Lagrange's Theorem.

**(7)**

**(b)** Prove that if N is a finite cyclic subgroup of a group G, then a proper Subgroup H of N is normal in G.

**(8)**

## UNIT-II

Q4. (a) The set  $\text{Inn}(G)$  of all inner automorphisms of a group  $G$  is a normal subgroup of the group  $\text{Aut}(G)$  of its automorphisms. (11)

(b) Let  $G$  be a cyclic group of degree 4. Show that the groups of automorphisms of  $G$  is of order 2. (4)

Q5. (a) Let  $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ ,  $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$ , then compute each of the following: (6)

1.  $\alpha^{-1}$
2.  $\alpha\beta$
3.  $\beta\alpha$

(b) State and prove Cayley's Theorem. (9)

## UNIT-III

Q6. (a) If  $f: R \rightarrow R'$  is an onto homomorphism, then  $R'$  is isomorphic to the quotient ring  $R/S$  of  $R$ , where  $S = \text{Ker } f$  i.e.  $R \cong R/S$  (8)

(b) Let  $R$  be a ring of  $2 \times 2$  matrices over integers. Let  $S = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} : a, b \text{ are integers} \right\}$ , then  $S$  is left ideal but not right ideal. (7)

Q7. (a) An Ideal  $S$  of a commutative ring  $R$  with unity is maximal if  $R/S$  is a field. (9)

(b) Let  $S_1$  and  $S_2$  be two ideals of Ring  $R$ . then Show that  $S_1 + S_2 = \{a_1 + a_2 : a_1 \in S_1, a_2 \in S_2\}$  is also an ideal of  $R$  (6)

## UNIT-IV

Q8. (a) If  $F$  is a field, then  $F[x]$  is an Euclidean domain. (9)

(b) Let  $R$  be an Euclidean ring. Then any two elements  $a$  and  $b$  in  $R$  have a greatest common divisor. (6)

Q9. (a) An Ideal  $S$  of an Euclidean ring  $R$  is maximal if it is generated by some prime element of  $R$ . (8)

(b) Prove that every Principal ideal domain is a Unique Factorizations domain. (7)

\*\*\*\*\*

**B.Sc. (HONS.) MATHEMATICS – 5<sup>TH</sup> SEM. EXAMINATIONS; DECEMBER 2017**  
**(SUB.: NUMERICAL ANALYSIS ; PAPER CODE: 09050503)**

**TIME: 03:00 Hrs.****Max. Marks: 75****Instructions:-**

1. Write your Roll no. on the Question paper.
2. Candidate should ensure that they have been provided with the correct question paper. Complaints in this regards, If any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter.
3. Attempt FIVE questions in all, Q.1 is compulsory. Students are required to attempt FOUR questions selecting one question from each Unit in addition to Q.No. 1. Marks are indicated against each question. All question carry equal marks.
4. Draw Diagram wherever required.

**Q.1. Attempt all the questions:****(5x3=15)**

- a) Define the difference operators  $\Delta$ . Also evaluate  $\Delta^n \left( \frac{1}{x} \right)$ .
- b) Show that  $\Delta \equiv \frac{1}{2} \delta^2 + \delta \sqrt{1 + \frac{\delta^2}{4}}$ .
- c) Show that  $f(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$ .
- d) By means of Lagrange's formula prove that  $y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} - y_{-5})$ .
- e) Define Interpolation. Differentiate equal and unequal interpolation. Also discuss various formulae for equal interpolation.

**UNIT-I****Q.2. a) Find out the missing term for the following table. (7)****b) Evaluate  $\Delta^2 \left[ \frac{a^{2x} + a^{4x}}{(a^2 - 1)^2} \right]$  (8)****Q.3. a) Given  $\log_{10} 654 = 2.8156$ ,  $\log_{10} 658 = 2.8182$ ,  $\log_{10} 659 = 2.8189$ ,  $\log_{10} 661 = 2.8202$ . Find  $\log_{10} 656$ . (7)****b) The populations of a Country in the decimal census were as under. Estimate the population for the year 1925 using appropriate interpolation formula (8)**

Year (x)	1891	1901	1911	1921	1931
Population (y)	46	66	81	93	101

**UNIT-II****Q.4. a) If the third differences are constant, then using Bessel's formula proves that**

$$y_{x+1/2} = \frac{1}{2}(y_x + y_{x+1}) - \frac{1}{16}(\Delta^2 y_{x-1} + \Delta^2 y_x). \quad (7)$$

**b) The mean and variance of a Binomial distribution are 4 and 4/3 respectively. Find the following: (8)**

- i) Probability of 2 successes
- ii) Probability of more than 3 successes
- iii) Probability of 3 or more than 3 successes

- Q.5. a) Apply Stirling's formula to find the value of  $\tan 16$  from the following data: (7)

$\theta$ (degree)	0	5	10	15	20	25	30
$\tan \theta$	0.0000	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

- b) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience shows that 2 percent of such fuses are defective. (8)

### UNIT-III

- Q.6. a) Find the first derivative of the function tabulated below at  $x = 4$ . (7)

$x$	1	2	4	8	10
$f(x)$	0	1	5	21	27

- b) Use Jacobi method to find Eigen values and Eigen vectors of the matrix (8)

$$\begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & -1.5 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}.$$

- Q.7. a) Given  $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ . Find the smallest Eigen value of  $A$  using Power method. (7)

- b) Use Given's method to find the Eigen values of the matrix  $\begin{pmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 5 \end{pmatrix}$ . (8)

### UNIT-IV

- Q.8. a) Derive General Quadrature formulae for equidistant ordinates. Also derive Simpson's  $\frac{3}{8}$  rule. (7)

- b) Use Taylor's series method to obtain approximate value of  $y$  at  $x = 0.2$   $\frac{dy}{dx} = 2y + 2e^x$ ,  $y(0) = 0$ . Compare the solution obtained with the exact solution. (8)

- Q.9. a) Apply Milne's predictor-corrector method to obtain a solution of the equation  $\frac{dy}{dx} = x - y^2$ ,  $y(0) = 0$  in the range of  $0 \leq x \leq 1$ . (7)

- b) Use Runge-Kutta method of fourth order to find an approximate value of  $y$  for  $x = 0.2$  in steps of 0.1 if  $f(x, y) = x + y^2$ ,  $y(0) = 1$ . (8)

\*\*\*\*\*

Sr. No: 100679

Roll No. \_\_\_\_\_

B. Sc. (Hons.) MATHEMATICS- 5<sup>th</sup> SEMESTER EXAMINATION; DECEMBER - 2017

[SUB: - INTEGRAL EQUATIONS: 09050504]

Time: 3 Hrs.

Max. Marks: 75

Instructions:-

1. Write your Roll No. on the Question paper.
2. Candidates should ensure that they have been provided correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter.
3. Attempt Five (5) Questions in all, Question No. 1 is compulsory. Attempt other 4 questions selecting one question from each unit. All question carry equal marks
4. Draw diagram wherever required.

1. Answer all the question.

(5 × 3 = 15)

(a) State Fredholm Third Fundamental Theorem .

(b) Write down the four basic properties of Green's function.

(c) For the following Kernels

(i) Show that  $K(x, t) = \sin(x - t)$  is skew-symmetric.

(ii) Show that  $K(x, t) = \begin{cases} \cos x \sin t & 0 \leq x \leq t \\ \sin x \cos t & t < x \leq l \end{cases}$  is symmetric.

(iii) Show that  $K(x, t) = e^{|x-t|}$  is not separable

(d) Find the resolvent kernel of the volterra integral equation with kernel :  $K(x, t) = 1$

(e) Find the iterated kernel upto second order of the Fredholm integral equation with

the following kernel  $K(x, t) = e^{|x+t|}$ , on the interval  $[-1, 1]$ .

#### UNIT-1

2. (a) Form an integral equation corresponding to the differential equation given by:

$$\frac{d^2 y}{dx^2} + xy = 1, \quad y(0) = y'(0) = 0. \quad (8)$$

(b) Find the solution of integral equation :

P.T.O

$$y(x) = 1 + \int_0^1 (1 + e^{x+t}) y(t) dt. \quad (7)$$

3. (a) Find the resolvent kernel for  $y(x) = x + \int_1^x x y(t) dt.$  (6)

(b) Solve the integral equation  $y(x) = f(x) + \lambda \int_1^{-1} (xt + x^2 t^2) y(t) dt$ . Also find its resolvent kernel. (9)

## UNIT-2

4. (a) Find the eigen values and eigen functions of the homogeneous Fredholm Integral

$$\text{Equation } y(x) = \lambda \int_{-1}^1 (5xt^3 + 4x^2t + 3tx) y(t) dt. \quad (8)$$

(b) Prove that eigen values of a symmetric kernel are real. (7)

5. (a) With the help of resolvent kernel solve the integral equation:

$$y(x) = \frac{5x}{6} + \frac{1}{2} \int_0^1 xty(t) dt. \quad (8)$$

(b) Using the method of successive approximation solve the integral equation

$$y(x) = 1 + \int_0^x (x-t)y(t) dt \text{ when } y_0(x) = 0. \quad (7)$$

## UNIT- 3

6. (a) Solve the integral equation using Laplace Transform

$$y(x) = x + 2 \int_0^x \cos(x-t) y(t) dt. \quad (8)$$

(b) Solve:  $x = \int_0^x 3^{x-t} y(t) dt.$  (7)

7. (a) With the help of Neumann series solve the Volterra integral equation

$$y(x) = 1 + x + \lambda \int_0^x (x-t) y(t) dt. \quad (8)$$

(b) Solve:  $\int_0^x e^{x-t} y(t) dt = \sin x.$  (7)

P.T.O



#### UNIT-4

8. (a) Using Green's function solve the BVP  $y'' + xy - 1 = 0, y(0) = y(1) = 0$ . (8)

(b) State and prove Fredholm's First Fundamental Theorem. (7)

9. (a) Using Green's function reduce  $y'' + y = x, y(0), y'(1) = 0$  to a Fredholm Integral equation. (8)

(b) (b) Solve the Abel integral equation.

(i)  $f(x) = \int_0^x \frac{u(t)}{(x-t)^\alpha} dt$  where  $0 < \alpha < 1$ .

(ii)  $f(x) = \int_0^x \frac{u(t)}{\sqrt{x-t}} dt$ . (7)

Sr. No: 100480

Roll No \_\_\_\_\_

**B. SC. (HONS) MATHEMATICS – 5<sup>TH</sup> SEMESTER EXAMINATION; DECEMBER - 2017**

**[SUB: - METHODS OF APPLIED MATHEMATICS; PAPER CODE: 09050505]**

**Time: 3 Hrs.**

**Max. Marks: 75**

**Instructions:-**

1. Write your Roll No. on the Question paper.
2. Candidates should ensure that they have been provided correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter.
3. Attempt Five (5) Questions in all, Question No. 1 is compulsory. Students are required to attempt Four (4) questions selecting one question from each unit. Marks are indicated against each Question.
4. Draw diagram wherever required.

**Q1. Attempt all the Questions:**

**(15)**

(a) Reduce the nonhomogeneous problem

$$\left( \frac{\partial^2 u}{\partial x^2} \right) = \frac{\delta u}{\delta t}, u(x, 0) = f(x), u(0, t) = T_0, u(l, t) = T_1, \forall 0 \leq x \leq l, t > 0 \text{ into homogeneous}$$

problems so that one can use method of separation of variables.

(b) Define the following terms:

- (i). Moment of Inertia
- (ii). Product of Inertia.

(c) Reduce the nonhomogeneous problem:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, u(x, 0) = f_1(x), u(x, b) = f_2(x), u(0, y) = g(y), u(a, y) = g_2(y) \text{ into homogeneous problems so that one can use method of separation of variables.}$$

(d) Find the Fourier transform of  $\frac{\partial^2 u(x, t)}{\partial x^2}$  with respect to  $x$  with the condition that  $u$

and its first derivative  $\frac{\delta u(x, t)}{\delta x}$  vanish as  $x \rightarrow \pm\infty$ .

(e) Write a short note on:

- (i). Kinetic energy of a body rotating about a fixed axis.
- (ii). Principal axes.

(f) Find the Fourier sine transform of  $e^{(-x)}$

## UNIT-I

**Q2.** Write the axisymmetric solution of Laplace equation in spherical coordinate system using separation of variable method. (15)

**Q3. (a)** By using separation of variable method solve the problem: (15)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b \quad \text{with } u(0, y) = u(a, y) = 0 \quad \text{and} \\ u(x, 0) = f(x), u(x, b) = 0$$

(b) Find the moment of Inertia of the rod of mass M, length 2a about a perpendicular axis through an end.

## UNIT-II

**Q4. (a)** By using separation of variable method solve the problem: (15)

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \frac{\partial u}{\partial t}, \quad 0 < x < a, \quad 0 < t, \quad u(0, t) = 0, u(a, t) = 0, 0 < t, u(x, 0) = T_1 \frac{x}{a}.$$

(b) By using separation of variable method solve the following problem on unbounded domain:

$$\frac{\partial^2 u}{\partial x^2} = \alpha \frac{\partial u}{\partial t}, u(x, 0) = f(x), -\infty < x < \infty, t > 0$$

**Q5.** Using Fourier transform method solve the following problem: (15)

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}, u(x, 0) = f(x), u_t(x, 0) = g(x), 0 < x < \infty, 0 < t$$

## UNIT-III

**Q6. (a)** If  $\mathcal{H}_n$  denotes  $n^{\text{th}}$  order Hankel transform and  $\mathcal{H}_n f(r) = \bar{f}_n(k)$  then prove that (15)

$$\mathcal{H}_n (f'(r)) = \frac{k}{2n} \left[ (n-1) \bar{f}_{n+1}(k) - (n+1) \bar{f}_{n-1}(k) \right], n > 1$$

$$\mathcal{H}_1 (f'(r)) = -k \bar{f}_0(k)$$

Provided  $[rf(r)]$  vanishes as  $r \rightarrow 0$  and  $r \rightarrow \infty$

(b) Write the definition of:

- (i). Fourier transform
- (ii). Finite Fourier transform
- (iii). Hankel transform

**Q7. (a)** Using finite Fourier sine transforms method; solve Laplace equation  $u_{xx} + u_{yy} = 0$   
 $0 \leq x \leq \pi$  with boundary conditions  $u(0, y) = u(\pi, y) = 0; u(x, 0) = u(x, \pi) = u_0$   
where  $u_0$  is constant. (15)

(b) Find the Fourier transform of  $f(x) = e^{-a|x|}, -\infty < x < \infty$

#### UNIT-IV

**Q8. (a)** Find moment of inertia of the rod of mass  $M$ , length  $2a$  about perpendicular axis through centre of gravity  $G$ . (15)

(b) Find the moment of Inertia of a rectangle lamina about an axis through its centre and perpendicular to the lamina.

**Q9.** Show that moments of an uniform triangle about any lines are same as the moments, about the same lines of three particles placed at the middle points of the sides each equal to one third the mass of the triangle. (15)

\*\*\*\*\*

Sr. No: 100481

Roll No \_\_\_\_\_

**B. SC: (HONS) MATHEMATICS – 5<sup>TH</sup> SEMESTER EXAMINATION; DECEMBER - 2017**

**[SUB: - OPERATION RESEARCH-I; PAPER CODE: 09050506]**

**Time: 3 Hrs.**

**Max. Marks: 75**

**Instructions:-**

1. Write your Roll No. on the Question paper.
2. Candidates should ensure that they have been provided correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter.
3. Attempt Five (5) Questions in all, Question No. 1 is compulsory. Students are required to attempt Four (4) questions selecting one question from each unit. Marks are indicated against each Question.
4. Draw diagram wherever required.

**Q1. Attempt all the Questions:**

**(15)**

- (a) What is meant by balanced problem in case of transportation problem and assignment problem?
- (b) Explain the limitations of a graphical method.
- (c) Define slack and surplus variable in LPP with suitable examples.
- (d) Define basic, basic feasible and optimal solutions of a linear programming problem (LPP).
- (e) Explain in brief the two-person zero-sum-game.
- (f) Write the dual of the following linear programming problem(LPP)

$$\text{Maximize } z = 15x_1 + 29x_2 + 11x_3 + 37x_4$$

Subject to the constraints

$$10x_1 + 15x_2 + 14x_4 \leq 250$$

$$22x_1 + 205x_2 + 20x_3 \geq 540$$

$$15x_1 + 13x_2 + 12x_3 + 13x_4 = 600$$

And  $x_2, x_3, x_4 \geq 0$  and  $x_1$  unrestricted.

**UNIT-I**

**Q2. (a) What do you mean by the term "Operations Research?" Explain the applications of Operations Research in various fields.**

**(10)**

**(b) Explain briefly the different types of models used in Operations Research.**

**(5)**

OR

Q3. (a) Solve the following linear programming problem (LPP) by graphical method. (5)

$$\text{Maximize } z = 3x + 2y$$

Subject to the constraints

$$-2x + y \leq 1$$

$$x \leq 2$$

$$x + y \leq 3$$

$$x, y \geq 0$$

(b) Consider a calculator company which produces a scientific calculator and a graphing calculator. Long-term projections indicate an expected demand of at least 1000 scientific and 800 graphing calculators each month. Because of limitations on production capacity, no more than 2000 scientific and 1700 graphing calculators can be made monthly. To satisfy a supplying contract, a total of at least 2000 calculators must be supplied each month. If each scientific calculator sold results in Rs. 120/- profit and each graphing calculator sold produces a Rs. 150/- profit. How many of each type of calculators should be made monthly to maximize the net profit. (10)

Unit-II

Q4. Use the simplex method to solve the following linear programming problem (LPP). (15)

$$\text{Maximize } z = 3x_1 + 5x_2 + 4x_3$$

Subject to the constraints

$$2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$\text{And } x_1, x_2, x_3 \geq 0$$

OR

Q5. Define artificial variables. Use the Big-M method to solve the following LPP. (15)

$$\text{Maximize } z = 4x + 3y$$

Subject to the constraints

$$2x + y \geq 10$$

$$-3x + 2y \leq 6$$

$$x + y \geq 6$$

$$\text{And } x, y \geq 0$$

### Unit-III

Q6. What do you understand by 'Duality' in linear programming? Explain the primal-dual relationships. Write the dual of the following LPP and Solve.

(15)

Maximize  $z = 2x + 3y + 4z$

Subject to the constraints

$$x + 2y + z \geq 3$$

$$2x - y + 3z \geq 4$$

And  $x, y, z \geq 0$

Hence or otherwise find the solution of the primal problem.

OR

Q7. (a) Describe the transportation problem with its general mathematical formulation.

(5)

(b) A Company has three plants P, Q and R, four warehouses A, B, C and D. The number of units available at the plants, demand of the warehouses and the unit cost of transportation are given below.

(10)

	A	B	C	D	Supply
P	10	12	15	08	130
Q	14	11	9	10	150
R	20	5	7	18	170
Demand	90	100	140	120	

Determine an optimum allocation for the company in order to minimize the total transportation cost.

### Unit-IV

Q8. (a) State the assignment problem. Describe an algorithm for the solution of the assignment problem.

(6)

(b) A student has to select one and only one elective in each semester and the same elective should not be selected in different semesters. Due to various reasons, the expected grades in each subject if selected in different semesters vary and they are given below:

(9)

Semesters	Advance Q.R	Advance Statistics	Graph Theory	Discrete Mathematics
I	F	E	D	C
II	E	E	C	C
III	C	D	C	A
IV	B	A	H	H

The grade points are H=10, A = 9, B = 8, C = 7, D = 6, E = 5 and F = 4

How will the student select the electives in order to maximize the total expected points and what will be his maximum expected total points?

OR

Q9. Explain the "Principle of Dominance" in a game problem. Use the rule of dominance to solve the game with the following pay off matrix. (15)

		Player B			
		B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>
Player A	A <sub>1</sub>	3	2	4	0
	A <sub>2</sub>	3	4	2	4
	A <sub>3</sub>	4	2	4	0
	A <sub>4</sub>	1	4	0	8

\*\*\*\*\*

25



**Faculty of Physical Sciences**  
**SGT University**  
**B.Sc. (Hons) V Semester**

Subject: Mathematics (Numerical Analysis)

Subject Code: 09050503

Time: 3 Hours

Maximum Marks: 75

- Attempt one question from each unit. Question 1 is compulsory.
- All questions carry equal marks.
- Use of Scientific Calculator is allowed.

**Q 1:****[3\*5]**

- a) Define the difference operators  $\Delta$ . Also evaluate  $\Delta^n \left( \frac{1}{x} \right)$ .
- b) Show that  $\Delta \equiv \frac{1}{2} \delta^2 + \delta \sqrt{1 + \frac{\delta^2}{4}}$ .
- c) Show that  $f(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$ .
- d) By means of Lagrange's formula prove that  $y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} - y_{-5})$ .
- e) Define Interpolation. Differentiate equal and unequal interpolation. Also discuss various formulae for equal interpolation.

**Unit I****Q 1:**

- a) Find out the missing term for the following table.

**[7]**

$x$	1	2	3	4	5	6	7	8
$f(x)$	1	8	?	64	?	216	343	512

- b) Evaluate
- $\Delta^2 \left[ \frac{a^{2x} + a^{4x}}{(a^2 - 1)^2} \right]$
- .

**[8]****Q 2:**

- a) Given  $\log_{10} 654 = 2.8156$ ,  $\log_{10} 658 = 2.8182$ ,  $\log_{10} 659 = 2.8189$ ,  $\log_{10} 661 = 2.8202$ . Find  $\log_{10} 656$ . **[7]**
- b) The populations of a Country in the decimal census were as under. Estimate the population for the year 1925 using appropriate interpolation formula. **[8]**

Year (x)	1891	1901	1911	1921	1931
Population (y)	46	66	81	93	101

**Unit II****Q 1:**