Solutions: Linear Algebra by Hoffman and Kunze

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***Abstract*—This book provides solutions to the Linear Algebra book by Hoffman and Kunze.**

1 Fields and Linear Equations

1.1. Let $\mathbb{F}$ be a set which contains exactly two elements,0 and 1.Define an addition and multiplication by tables. Verify that the set $\mathbb{F}$,

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

together with these two operations, is a field.
Solution:
To prove that ($\mathbb{F}$,+,·) is a field we need to satisfy the following,

a) + and · should be closed
   - For any a and b in $\mathbb{F}$, a+b ∈ $\mathbb{F}$ and a · b ∈ $\mathbb{F}$.For example 0+0=0 and 0·0=0.
b) + and · should be commutative
   - For any a and b in $\mathbb{F}$, a+b = b+a and a · b = b· a. For example 0+1=1+0 and 0· 1=1· 0.
c) + and · should be associative
   - For any a and b in $\mathbb{F}$, a+(b+c) = (a+b)+c and a· (b · c) = (a · b) · c. For example 0+(1+0)=(0+1)+0 and 0·(1·0)=(0·1)·0.
d) + and · operations should have an identity element
   - If we perform a + 0 then for any value of a from $\mathbb{F}$ the result will be a itself. Hence 0 is an identity element of + operation.If we perform a · 1 then for any value of a from $\mathbb{F}$ the result will be a itself. Hence 1 is an identity element of · operation.
e) ∀ a ∈ $\mathbb{F}$ there exists an additive inverse
   - For additive inverse to exist, ∀ a in $\mathbb{F}$ a+(-a)=0. For example. 1-1=0 and 0-0=0.
f) ∀ a ∈ $\mathbb{F}$ such that a is non zero there exists a multiplicative inverse
   - For multiplicative inverse to exist, ∀ a such that a is non zero in $\mathbb{F}$, $a \cdot a^{-1}$=1.For example $1 \cdot 1^{-1} = 1$.
g) + and · should hold distributive property
   - For any a,b and c in $\mathbb{F}$ the property a·(b+c)=a·b+a·c should always hold

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true.For example $0\cdot(1+1)=0\cdot1+0\cdot1$.

Since the above properties are satisfied we can say that $(\mathbb{F},+,\cdot)$ is a field.