Roll No._______________

B. Sc (Hons) Mathematics – 6^{th} SEMESTER
SPECIAL EXAMINATION, AUGUST-2018
Linear Algebra – 9050602

TIME: 03: 00 Hrs. **Max. Marks: 75**

INSTRUCTIONS:

1. Write Roll No. on the Question Paper.
2. Candidate should ensure that they have been provided with the correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter;
3. Attempt five (5) questions in all and Question No. 1 is compulsory.
4. Students are required to attempt four questions selecting one question from each unit in addition to Q. No.1. All question carry equal marks.
5. Draw diagram whenever required.

Q.1 Attempt all the following questions: **(2.5 X 6 =15)**

a) Find the co-ordinates of the vector (a,b,c) of $\mathbf{R}^3$ relative to the basis $\{(1,0,0),(1,1,0),(1,1,1)\}$ of $\mathbf{R}^3$.

b) Show that the mapping T defined by $T(\alpha,\beta)=(\alpha+\beta,\alpha-\beta,\beta)$ for all $(\alpha,\beta)\ \varepsilon\ V_2(R)$ is a linear transformation.

c) If x and y are vectors in a real inner product space and if $\|x\|=\|y\|$, then show that x-y and x + y are orthogonal.

d) Define homomorphism and isomorphism of a vector space.

e) Show that (1,1,1), (0,1,1) and (0,1, -1) generate $\mathbf{R}^3$.

f) If W_1 and W_2 are subspaces of a vector space V over a field F and $W_1 \subset W_2$ then show $W_2^0 \subset W_1^0$.

UNIT-I

Q.2 (a) Show that every finitely generated vector space has a finite basis.

(b) Under what conditions on the scalar 'a' are the vectors (a,1,0), (1,a,1) and (0,1,a) in R^3 linearly dependent? **(15)**

OR

Q.3 (a) If V is a vector space over a field F and if W is a subspace of V, then show the set $\frac{V}{W}=\{\alpha+W:\alpha\in V\}$. Is a vector space over the field F with respect to the linear compositions:

(i) $(\alpha + W) + (\beta + W) = (\alpha + \beta) + W$ for every $\alpha,\beta\in V$. **(15)**

(b) If a finite –dimensional vector space V(F) be the direct sum of its two subspaces W_1 and W_2 then show that $\dim.V = \dim.W_1 + \dim.W_2$. **(15)**

UNIT-II

Q.4 Let U and V be vector spaces over the same field F and let T be a linear transformation from U into V. Suppose U is finite dimensional. Then show rank (T)+ nullity (T)= dim. U. **(15)**

OR

Q.5 (a) If U(F) and V(F) are two vector spaces and T is a linear transformation from U into V , then show that range of T is A subspace of V.

(b) Find the dual basis of the basis set B = {(1,-1,3) , (0,1,-1) , (0,3,-2) for **V_3(R).** **(15)**

UNIT-III

Q.6 (a) Let U and V be vector spaces over the same field F and let T be a linear transformation from U into V. If T is invertible, then show that T^{-1} is a linear transformation from V into U.

(b) A linear transformation T from U(F) into vector space V(F). Then show T is non-singular if and only if T is one-one. **(15)**

OR

Q.7 (a) Find the matrix representation of a linear map **T: $\mathbf{R}^3 \to \mathbf{R}^3$** defined by T(x,y,z)=(z, y+z, x+ y+ z) relative to the basis{ (1,0,1), (-1,2,1), (2,1,1)}.

(b) Describe explicitly the linear transformation T: $R^2 \to R^2$ such that T(2,3)=(4,5) and T(1,0)= (0,0) **(15)**

UNIT-IV

Q.8 (a) If in an inner product space $\|x+y\| = \|x\| + \|y\|$ then prove that x,y are linearly dependent but converse is not true.

(b) Let V_1=(3,0,4), V_2= (-1,0,7), V_3=(2,9,11) be vectors in R^3 equipped with the standard inner product. Obtain an orthogonal basis. **(15)**

OR

Q.9 (a) Show that any orthonormal set of vectors in an inner product space is linearly independent.

(b) Show that every finite –dimensional inner product space has an orthonormal basis. **(15)**

Roll No.______________

B. Sc (Hons) Mathematics – 6th SEMESTER
SPECIAL EXAMINATION, AUGUST-2018
Elementary Topology – 9050604

TIME: 03: 00 Hrs. **Max. Marks: 75**

INSTRUCTIONS:

1. Write Roll No. on the Question Paper.
2. Candidate should ensure that they have been provided with the correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter;
3. Attempt five (5) questions in all and Question No. 1 is compulsory.
4. Students are required to attempt four questions selecting one question from each unit in addition to Q. No.1. All question carry equal marks.
5. Draw diagram whenever required.

Q1.

a) Write all the topologies on set $X=(a,b,c)$ which contain exactly 6 elements.

b) If U is open in Y and Y is open in X, then show that U is open in X.

c) Show that the lower limit topology is strictly finer than the Standard topology.

d) Define Neighborhood of a point with an example.

e) Define Adherent point and Limit point of a subset. **(15)**

UNIT - I

Q2.

a) Show that the topological space (X,T) is T_2 if and only if the diagonal $\Delta=\{x\times x: x\in X\}$ is closed in $X\times X$.

b) Define a co-finite topology. Show that the collection $T_c=\{U: X-U \text{ is finite or is all of } X\}$ is a topology. **(15)**

Q3.

a) Define Closure of a set. If A and B are two sets then show that $\overline{A\cup B}=\overline{A}\cup\overline{B}$.

b) Show that the collection $T_y=\{G\cap Y: G\in T\}$, forms a topology on Y. **(15)**

UNIT – II

Q4. If (X,T) and (Y,V) be two topological spaces then show the following

a) A mapping $f: X\to Y$ is continuous if and only if $\overline{f^{-1}(B)}\subset f^{-1}(\overline{B})$ for every $B\subset Y$.

b) A mapping $f: X\to Y$ is continuous if and only if inverse image of every open set in Y is open in X. **(15)**

Q5.

a) Show that the union of a collection of connected subspaces of X, that have a point in common, is connected.

b) Show that the components of a totally disconnected space are only the singleton subsets of X. **(15)**

UNIT- III

Q6.

a) Define a compact space. Show that every closed subspace of a compact space is compact.

b) Show that every compact subset of a T_2 space is closed. **(15)**

Q7.

a) A topological space then (X,T) is compact if and only if for every collection C of closed sets in X, having FIP, the intersection $\bigcap_{c\in C} C$ is non empty.

b) Show that continuous image of a sequentially compact space is sequentially compact.

(15)

UNIT-IV

Q8.

a) Show that the property of being second countable space is topological. **(15)**

b) A topological space is Normal if and only if for a given closed set A and an open set U containing A, there exists an open set V containing A such that $\overline{V} \subset U$.

Q9.

a) Show that the property of being a T_2 space is topological.

b) Every compact Hausdorff space is regular.

(15)

Roll No.________________

B. Sc (Hons) Mathematics – 5th SEMESTER
SPECIAL EXAMINATION, AUGUST-2018
Real Analysis – 9050501

TIME: 03: 00 Hrs. **Max. Marks: 75**

INSTRUCTIONS:

1. Write Roll No. on the Question Paper.
2. Candidate should ensure that they have been provided with the correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter;
3. Question No. 1 is Compulsory. Attempt **Five** Questions in all.
4. Students are required to attempt other **FOUR** selecting one from each **Section**. All questions carry equal marks.
5. Draw diagram whenever required.

Q.1 **a)** Define Subset and Matric space. **(15)**

b) State Baire's category theorem.

c) Discuss Dirichlet's tests.

d) State mean value theorem.

e) Define Continuous and monotonic function.

SECTION –A

Q.2 Show that a constant function k is integrable and $\int_a^b \text{kdx} = \text{k(b-a)}$. **(15)**

Q.3 A function f is defined on [0, 1] by $\text{f(x)} = \frac{1}{\text{n}} \text{for} \frac{1}{\text{n}} \geq x \succ \frac{1}{\text{n}+1}, \text{n} = 1,2,3.....$ **(15)**

SECTION –B

Q.4 Show that the function f defined by $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) \; if \; x \neq 0 \\ 0 \;\; if \; x = 0 \end{cases}$ is differentiable at x =0 but

$\lim_{x \to 0} f'(x) \neq f'(0)$. **(15)**

Q.5 Test the convergence of the integral $\int_1^2 \frac{dx}{\sqrt{x^4 - 1}}$. **(15)**

SECTION –C

Q.6 Let X = R^n denote the set of all ordered n- tuples of real numbers for a fixed $\text{n} \in N$. **(15)**

Q.7 Let X = [0, 1]. Consider the usual metric d(x, y) = $|x - y|$ defined for the set X = [0, 1]. **(15)**

SECTION – D

Q.8 State and prove Bolzano-Weierstrass Theoram. **(15)**

Q.9 If (X, d) is a discrete metric space and (Y, ρ) is any metric space, then show that every function f: X $\rightarrow Y$ is continuous on X. **(15)**

Roll No.________________

B. Sc (Hons) Mathematics – 5th SEMESTER
SPECIAL EXAMINATION, AUGUST-2018
Groups & Rings – 9050502

TIME: 03: 00 Hrs. **Max. Marks: 75**

INSTRUCTIONS:

1. Write Roll No. on the Question Paper.
2. Candidate should ensure that they have been provided with the correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter;
3. Question No. 1 is Compulsory. Attempt **Five** Questions in all.
4. Students are required to attempt other **FOUR** selecting one from each **Section**. All questions carry equal marks.
5. Draw diagram whenever required.

Q.1 Attempt all the following questions: **(3 X 5 = 15)**

a) Show that the set G = {1, 2, 3, 4, 5} is not a group w.r.t. multiplication modulo 6.
b) Define a Quotient group.
c) Find the inverse of cycles: i) (1 2 3 4) ii) (2 3 4 5 6).
d) Define a subring.
e) Show that every field is a Eucidean ring.

SECTION - A

Q.2 Prove that in an abelian group (G, .), $(a.b)^n = a^n.b^n$ for all integers n and all a, b $\in G$. **(15)**

Q.3 Prove that every subgroup of a cyclic group is cyclic. **(15)**

SECTION – B

Q.4 State and prove Cayley's Theorem. **(15)**

Q.5 State and prove The Fundamental Theorem of group homomorphism. **(15)**

SECTION – C

Q.6 Prove that a finite non-zero integral domain is a field. **(15)**

Q.7 Define Principal Ideal Domain and show that ring of integers is Principal Ideal Domain. **(15)**

SECTION – D

Q.8 Find all units of $Z[\sqrt{-5}]$. **(15)**

Q.9 Every principal ideal domain is a unique factorization domain. **(15)**