Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

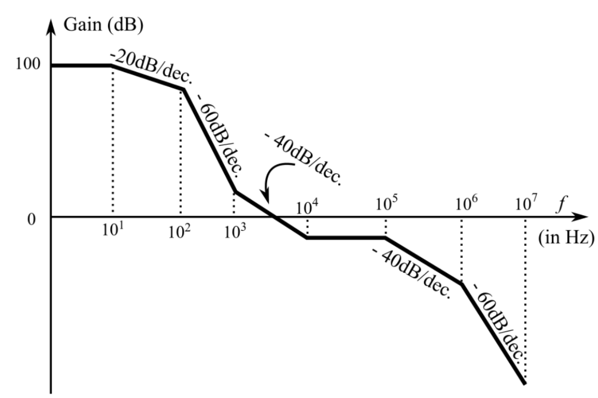
Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Routh Hurwitz Criterion

2 Bode Plot

2.1. For an LTI system, the Bode plot for its gain is as illustrated in the Fig. ?? The number of system poles N_p and number of system zeros N_z in the frequency range 1 Hz ≤ f ≤ 10^7 Hz is



Solution:-

Let us consider a generalized transfer gain

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$H(s) = k\frac{(s-z_1)(s-z_2)...(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)....(s-p_{n-1})(s-p_n)}$

$$\text{Gain} = 20\log|H(s)| = 20\log|k|+20\log|s-z_1| +20\log|s-z_2|+\dots+20\log|s-z_m|-20\log|s-p_1| -20\log|s-p_2|-\dots-20\log|s-z_n| \quad (2.1.1)$$

Let us consider a $20\log|s-z_1|$

Let $s = j\omega$

$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right|$

Based on log scale plot approximations,to the left of z_1 $\omega << z_1$ and towards right $\omega >> z_1$

For $\omega < z_1$

$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|z_1|$
$= constant$ i.e. Slope = 0

For $\omega > z_1$

$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|\omega|$
i.e Slope = 20

When a zero is encountered the slope always increases by 20 dB/decade

Doing similar analysis for $-20\log|s-p_1|$
We conclude

When a pole is encountered the slope always decreases by 20 dB/decade

- $f = 10Hz$
  Slope($f < 10$) = 0 dB/dec
  Slope($f > 10$) = −20 dB/dec
  $\Delta Slope = -20$ dB/dec
  $n_z = 0$ $n_p = 1$

- $f = 10^2$ Hz
  Slope($f < 10^2$) = -20 dB/dec
  Slope($f > 10^2$) = -60 dB/dec
  $\Delta Slope$ = -40 dB/dec

$n_z = 0\ n_p = 2$

- $f = 10^3 Hz$
  Slope($f < 10^3$) = -60 dB/dec
  Slope($f > 10^3$) = -40 dB/dec
  $\Delta Slope$ = +20 dB/dec
  $n_z = 1\ n_p = 0$

- $f = 10^4 Hz$
  Slope($f < 10^4$) = -40 dB/dec
  Slope($f > 10^4$) = 0 dB/dec
  $\Delta Slope$ = +40 dB/dec
  $n_z = 2\ n_p = 0$

- $f = 10^5$ Hz
  Slope($f < 10^5$) = 0 dB/dec
  Slope($f > 10^5$) = -40 dB/dec
  $\Delta Slope$ = -40 dB/dec
  $n_z = 0\ n_p = 2$

- $f = 10^6$ Hz
  Slope($f < 10^2$) = -40 dB/dec
  Slope($f > 10^2$) = -60 dB/dec
  $\Delta Slope$ = -20 dB/dec
  $n_z = 0\ n_p = 1$

$\boxed{\mathrm{N}_p = 6\ N_z = 3}$

3 Compensators

4 Nyquist Plot