Control Systems

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Abstract—**This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Routh Hurwitz Criterion

2 Bode Plot

2.1. For an LTI system, the Bode plot for its gain is as illustrated in the Fig. 2.1 The number of system poles N_p and number of system zeros N_z in the frequency range 1 Hz ≤ f ≤ 10^7 Hz is

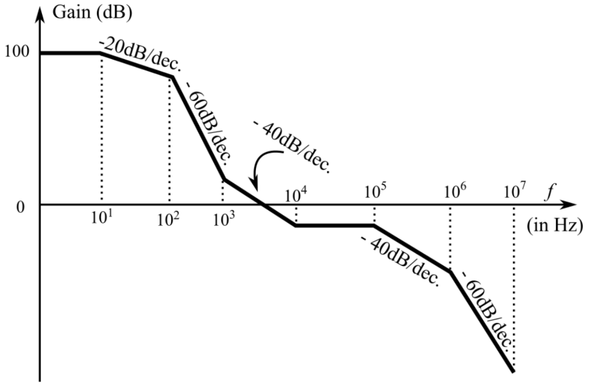

Fig. 2.1

Solution:- Let us consider a generalized transfer gain

$$H(s) = k\frac{(s-z_1)(s-z_2)...(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)....(s-p_{n-1})(s-p_n)}$$

$$\begin{aligned} Gain = 20\log|H(s)| = 20\log|k| + 20\log|s-z_1| \\ +20\log|s-z_2| + \cdots + 20\log|s-z_m| - 20\log|s-p_1| \\ -20\log|s-p_2| - \cdots - 20\log|s-z_n| \quad (2.1.1) \end{aligned}$$

Let us consider a $20\log|s-z_1|$
Let $s = j\omega$
$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right|$
Based on log scale plot approximations,to the left of z_1 $\omega << z_1$ and towards right $\omega >> z_1$

For $\omega < z_1$
$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|z_1|$
$= constant$ i.e. Slope = 0

For $\omega > z_1$
$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|\omega|$
i.e Slope = 20
When a zero is encountered the slope always increases by 20 dB/decade

Doing similar analysis for $-20\log|s-p_1|$
We conclude
When a pole is encountered the slope always decreases by 20 dB/decade

$$Slope = \frac{d(20\log H(f))}{df}$$

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$$Slope = \begin{cases} 0 & 0 < f < 10^1 \\ -20 & 10 < f < 10^2 \\ -60 & 10^2 < f < 10^3 \\ -40 & 10^3 < f < 10^4 \\ 0 & 10^4 < f < 10^5 \\ -40 & 10^5 < f < 10^6 \\ -60 & 10^6 < f < 10^7 \end{cases}$$

$\Delta Slope$ = Change in slope at f

$$\Delta Slope = \begin{cases} -20 & f = 10^1 \\ -40 & f = 10^2 \\ +20 & f = 10^3 \\ +40 & f = 10^4 \\ -40 & f = 10^5 \\ -20 & f = 10^6 \end{cases}$$

Final Transfer function is

$$H(f) = \frac{K(f + 10^3)(f + 10^4)^2}{(f + 10^1)(f + 10^2)^2(f + 10^5)^2(f + 10^6)}$$

$\boxed{\text{N}_p = 6 \ N_z = 3}$

Python plot of the obtained transfer function is shown in fig 2.1

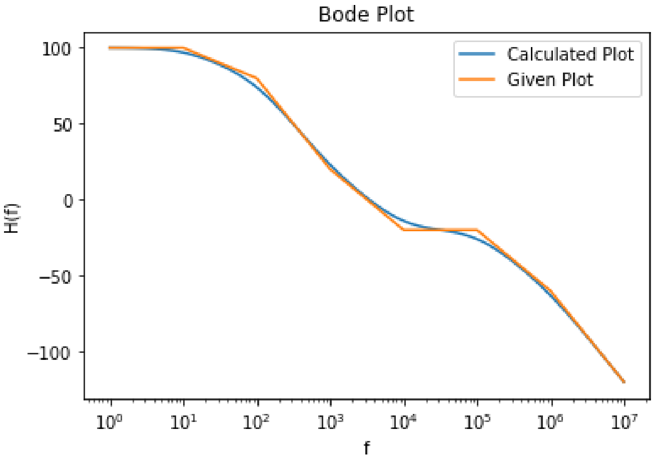

Fig. 2.1

3 Compensators

4 Nyquist Plot