

# Control Systems

G V V Sharma\*

## CONTENTS

1	<b>Routh Hurwitz Criterion</b>	1
2	<b>Bode Plot</b>	1
3	<b>Compensators</b>	2
4	<b>Nyquist Plot</b>	2

**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

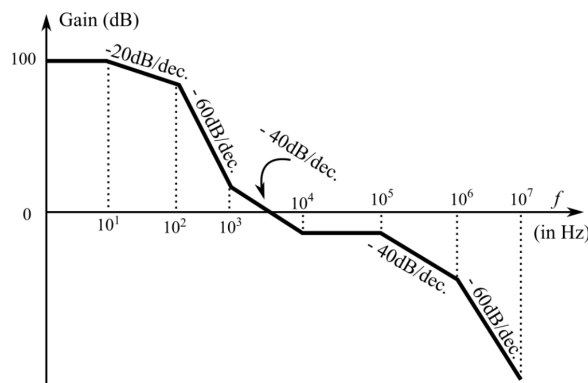
Download python codes using

```
svn co https://github.com/gadepall/school/trunk/control/codes
```

## 1 ROUTH HURWITZ CRITERION

## 2 BODE PLOT

2.1. For an LTI system, the Bode plot for its gain is as illustrated in the Fig. ?? The number of system poles  $N_p$  and number of system zeros  $N_z$  in the frequency range  $1 \text{ Hz} \leq f \leq 10^7 \text{ Hz}$  is



Solution:-

Let us consider a generalized transfer gain

\*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

$$H(s) = k \frac{(s-z_1)(s-z_2)\dots(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_{n-1})(s-p_n)}$$

$$\begin{aligned} \text{Gain} &= 20 \log |H(s)| = 20 \log |k| + 20 \log |s - z_1| \\ &+ 20 \log |s - z_2| + \dots + 20 \log |s - z_m| - 20 \log |s - p_1| \\ &- 20 \log |s - p_2| - \dots - 20 \log |s - p_n| \quad (2.1.1) \end{aligned}$$

Let us consider a  $20 \log |s - z_1|$

Let  $s = j\omega$

$$20 \log |s - z_1| = 20 \log \left| \sqrt{\omega^2 + z_1^2} \right|$$

Based on log scale plot approximations, to the left of  $z_1$   $\omega \ll z_1$  and towards right  $\omega \gg z_1$

For  $\omega < z_1$

$$\begin{aligned} 20 \log |s - z_1| &= 20 \log \left| \sqrt{\omega^2 + z_1^2} \right| = 20 \log |z_1| \\ &= \text{constant i.e. Slope} = 0 \end{aligned}$$

For  $\omega > z_1$

$$\begin{aligned} 20 \log |s - z_1| &= 20 \log \left| \sqrt{\omega^2 + z_1^2} \right| = 20 \log |\omega| \\ \text{i.e Slope} &= 20 \end{aligned}$$

**When a zero is encountered the slope always increases by 20 dB/decade**

Doing similar analysis for  $-20 \log |s - p_1|$   
We conclude

**When a pole is encountered the slope always decreases by 20 dB/decade**

$$\text{Slope} = \frac{d(20 \log H(f))}{df}$$

$$\text{Slope} = \begin{cases} 0 & 0 < f < 10^1 \\ -20 & 10 < f < 10^2 \\ -60 & 10^2 < f < 10^3 \\ -40 & 10^3 < f < 10^4 \\ 0 & 10^4 < f < 10^5 \\ -40 & 10^5 < f < 10^6 \\ -60 & 10^6 < f < 10^7 \end{cases}$$

$\Delta Slope =$  Change in slope at  $f$

$$\Delta Slope = \begin{cases} -20 & f = 10^1 \\ -40 & f = 10^2 \\ +20 & f = 10^3 \\ +40 & f = 10^4 \\ -40 & f = 10^5 \\ -20 & f = 10^6 \end{cases}$$

$$\boxed{N_p = 6 \quad N_z = 3}$$

### 3 COMPENSATORS

### 4 NYQUIST PLOT