C Programming in Middle School

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About this Book

This book introduces C programming for middle school children based on the NCERT mathematics textbook of Class 7.

This book is dedicated to my Hindi teacher in middle school, Shri Mandavi.

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Github: https://github.com/gadepall/cprog

Contents

1 Integers

1.1 Formulae

1.1.1 Do the following addition through a C program

$$17 + 23$$

Solution:

```c
//Code by GVV Sharma
//Adding two integers
//April 14, 2025
#include <stdio.h>

//begin main function
int main(void)
{
//Declaring integers
int a = 17, b = 23;
//printing the sum
printf("%d\n",a+b);
        return 0;
}
//end main function
```

1.1.2 Do the following subtraction through a C program

$$7 - 9$$

Solution:

```c
//Code by GVV Sharma
//Adding negative integer
//April 14, 2025
#include <stdio.h>

//begin main function
int main(void)
{
//Declaring integers
int a = 7, b = 9;
//printing the difference
printf("%d\n",a-b);
        return 0;
}
//end main function
```

1.1.3 Mulitply the following through a C program

$$4 \times (-8)$$

Solution:

```
//Code by GVV Sharma
//April 14, 2025
//Multiplication of numbers
#include <stdio.h>

int main(void)
{
int a = 4, b = -8;
printf("%d\n",a*b);
        return 0;
}
```

1.1.4 Perform the following division

$$(-100) \div 5$$

Solution:

```
//Code by GVV Sharma
//April 15, 2025
//division of numbers
#include <stdio.h>

int main(void)
{
int a = -100, b = 5;
printf("%d\n",a/b);
        return 0;
}
```

1.2 NCERT

Compute the following

1.2.1 $(-75) + 18$
1.2.2 $19 + (-25)$
1.2.3 $27 + (-27)$
1.2.4 $(-20) + 0$
1.2.5 $(-35) + (-10)$
1.2.6 $(-10) + 3$
1.2.7 $17 - (-21)$
1.2.8 $8 \times (-2)$
1.2.9 $3 \times (-7)$
1.2.10 $10 \times (-1)$
1.2.11 $6 \times (-19)$
1.2.12 $12 \times (-32)$
1.2.13 $7 \times (-22)$
1.2.14 $15 \times (-16)$
1.2.15 $21 \times (-32)$
1.2.16 $(-42) \times 12$
1.2.17 $(-55) \times 15$
1.2.18 $(-5) \times (-6)$
1.2.19 $(-6) \times (-7)$
1.2.20 $3 \times (-1)$
1.2.21 $(-1) \times 225$
1.2.22 $(-21) \times (-30)$
1.2.23 $(-316) \times (-1)$
1.2.24 $(-81) \div 9$

1.2.25 $(-75) \div 5$
1.2.26 $(-32) \div 2$
1.2.27 $125 \div (-25)$
1.2.28 $80 \div (-5)$
1.2.29 $64 \div (-16)$
1.2.30 $(-30) \div 10$
1.2.31 $50 \div (-5)$
1.2.32 $(-36) \div (-9)$
1.2.33 $(-49) \div (-49)$

1.2.34 $13 \div [(-2) + 1]$
1.2.35 $(-31) \div [(-30) + (-1)]$
1.2.36 $[(-36) \div 12] \div (3)$
1.2.37 $[(-6) + 5] \div [(-2) + 1]$

Fill in the blanks

1.2.38 $20 \div$ ______ $= -2$
1.2.39 ______ $\div 4 = -3$

Find the values of the following expressions for $x = 2$.

1.2.40 $x + 4$
1.2.41 $4x–3$
1.2.42 $x–2$
1.2.43 $3x–5$
1.2.44 $9–5x$
1.2.45 $x + 7 + 4(x–5)$
1.2.46 $3(x + 2) + 5x–7$
1.2.47 $6x + 5(x–2)$
1.2.48 $4(2x–1) + 3x + 11$

If $x = –2$, find the value of

1.2.49 $5x–2$
1.2.50 $4p + 7$

Find the value of the following expressions for $a = 3, b = 2$.

1.2.51 $a + b$
1.2.52 $7a–4b$

Find the value of the following expressions, when $x = –1$

1.2.53 $2x–7$
1.2.54 $-x + 2$

When $a = 0, b = –1$, find the value of the given expressions

1.2.55 $2a + 2b$

Simplify these expressions and find their values if $x = 3, a = –1, b = –2$.

1.2.56 $2x + 4$
1.2.57 $6 – 4x$
1.2.58 $6–5a$
1.2.59 $6–8b$
1.2.60 $3a–2b – 9$
1.2.61 In a test (+5) marks are given for every correct answer and (-2) marks for every incorrect answer.

a) Radhika answered all the questions and scored 30 marks though she got 10 correct answers.

b) Jay also answered all the questions and scored (-12) marks though he got 4 correct answers. How many incorrect answers had they attempted?

1.2.62 A shopkeeper earns a profit of ₹1 by selling one pen and incurs a loss of 40 paise per pencil while selling pencils of her old stock.

a) In a particular month she incurs a loss of ₹5. In this period she sold 45 pens. How many pencils did she sell in this period?

b) In the next month she earns neither profit nor loss. If she sold 70 pens, how many pencils did she sell?

1.2.63 The temperature at 12 noon was 10°C above zero. If it decreases at the rate of 2°C per hour unitl midnight, at what time would the temperature be 8°C below zero?

1.2.64 In a class test (+3) marks are given for every correct answer and (-2) marks for every incorrect answer and no marks for not attempting any question.

a) Radhika scored 20 marks. If she got 12 correct answers, how many questions has she attempted incorrectly?

b) Mohini scores -5 marks in this test, though she has got 7 correct answers. How many questions has she attempted incorrectly?

1.2.65 An elevator descends a mine shaft at the rate of $6m/min$. If the descent starts from $10m$ above the ground, how long will it take to reach $-350m$.

1.2.66 What is the measure of the complement of each of the following angles?

a) 45° b) 65° c) 41° d) 54°

1.2.67 What will be the measure of the supplement of each one of the following angles?

a) 100° b) 90° c) 55° d) 125°

1.2.68 An exterior angle of a triangle is of measure 70° and one of its interior opposite angles is of measure 25°. Find the measure of the other interior opposite angle.

1.2.69 The two interior opposite angles of an exterior angle of a triangle are 60° and 80°. Find the measure of the exterior angle.

1.2.70 Two angles of a triangle are 30° and 80°. Find the third angle.

1.2.71 One of the angles of a triangle is 80° and the other two angles are equal. Find the measure of each of the equal angles.

1.2.72 The three angles of a triangle are in the ratio 1:2:1. Find all the angles of the triangle. Classify the triangle in two different ways.

1.2.73 One of the sides and the corresponding height of a parallelogram are 4 cm and 3 cm respectively. Find the area of the parallelogram.

1.2.74 Find the height x if the area of the parallelogram is 24 cm^2 and the base is 4 cm.

1.2.75 Find BC, if the area of the triangle ABC is 36 cm^2 and the height AD is 3 cm.

2 Decimal Numbers

2.1 Formulae

Find

2.1.1 $\frac{5}{3}+\frac{3}{5}$
Solution:

```
//Code by GVV Sharma
//Adding two decimals
//April 19, 2025
#include <stdio.h>

//begin main function
int main(void)
{
//Declaring decimals
float a = 5.0/3.0, b = 3.0/5.0;
//printing the sum
printf("%f\n",a+b);
        return 0;
}
//end main function
```

2.1.2 $\frac{2}{3}-\frac{3}{7}$
Solution:

```
//Code by GVV Sharma
//Subtracting two decimals/fractions
//April 19, 2025
#include <stdio.h>

//begin main function
int main(void)
{
//Declaring decimals
float a = 2.0/3.0, b = 3.0/7.0;
//printing the sum
printf("%f\n",a-b);
        return 0;
}
//end main function
```

2.1.3 5.6×1.4
Solution:

```
//Code by GVV Sharma
//April 19, 2025
//Multiplication of decimal numbers
#include <stdio.h>

```

```
int main(void)
{
float a = 5.6, b = 1.4;
printf("%f\n",a*b);
        return 0;
}
```

2.1.4 $37.8 \div 1.4$

Solution:

```
//Code by GVV Sharma
//April 15, 2025
//division of decimal numbers
#include <stdio.h>

int main(void)
{
float a = 37.8, b = 1.4;
printf("%f\n",a/b);
        return 0;
}
```

2.2 NCERT

Find

2.2.1 $\frac{2}{7} \times 3$

2.2.2 $\frac{9}{7} \times 6$

2.2.3 $\frac{1}{8} \times 3$

2.2.4 $\frac{13}{11} \times 6$

2.2.5 $\frac{2}{5} \times 2$

2.2.6 $3 \times 5\frac{1}{5}$

2.2.7 $5 \times 6\frac{3}{4}$

2.2.8 $7 \times 2\frac{1}{4}$

2.2.9 $4 \times 6\frac{1}{3}$

2.2.10 $6 \times 3\frac{1}{4}$

2.2.11 $8 \times 3\frac{2}{5}$

2.2.12 $\frac{1}{2} \times \frac{1}{7}$

2.2.13 $\frac{1}{5} \times \frac{1}{7}$

2.2.14 $\frac{1}{3} \times \frac{4}{5}$

2.2.15 $\frac{2}{3} \times \frac{1}{5}$

2.2.16 $\frac{8}{3} \times \frac{4}{7}$

2.2.17 $\frac{3}{4} \times \frac{2}{3}$

2.2.18 $\frac{2}{3} \times 2\frac{2}{3}$

2.2.19 $\frac{2}{7} \times \frac{7}{9}$

2.2.20 $\frac{3}{8} \times \frac{6}{4}$

2.2.21 $\frac{9}{5} \times \frac{3}{5}$

2.2.22 $\frac{1}{3} \times \frac{15}{8}$

2.2.23 $\frac{11}{2} \times \frac{3}{10}$

2.2.24 $\frac{4}{5} \times \frac{12}{7}$

2.2.25 $\frac{2}{5} \times 5\frac{1}{4}$

2.2.26 $6\frac{2}{5} \times \frac{7}{9}$

2.2.27 $\frac{3}{2} \times 5\frac{1}{3}$

2.2.28 $\frac{5}{6} \times 2\frac{3}{7}$

2.2.29 $3\frac{2}{5} \times \frac{4}{7}$

2.2.30 $2\frac{3}{5} \times 3$

2.2.31 $3\frac{4}{7} \times \frac{3}{5}$

2.2.32 $\frac{2}{3} \times \_\_\_\_ = \frac{10}{30}$

2.2.33 $\frac{3}{5} \times \_\_\_\_ = \frac{24}{75}$

2.2.34 $7 \div \frac{2}{5}$

2.2.35 $6 \div \frac{4}{7}$

2.2.36 $2 \div \frac{8}{9}$

2.2.37 $\frac{3}{5} \div \frac{1}{2}$

2.2.38 $\frac{1}{2} \div \frac{3}{5}$

2.2.39 $2\frac{1}{2} \div \frac{3}{5}$

2.2.40 $5\frac{1}{6} \div \frac{9}{2}$

2.2.41 $12 \div \frac{3}{4}$

2.2.42 $14 \div \frac{5}{6}$

2.2.43 $8 \div \frac{7}{3}$

2.2.44 $4 \div \frac{8}{3}$

2.2.45 $3 \div 2\frac{1}{3}$

2.2.46 $5 \div 3\frac{4}{7}$

2.2.47 $\frac{7}{3} \div 2$

2.2.48 $\frac{4}{9} \div 5$

2.2.49 $\frac{6}{13} \div 7$

2.2.50 $4\frac{1}{3} \div 3$

2.2.51 $3\frac{1}{2} \div 4$

2.2.52 $4\frac{3}{7} \div 7$

2.2.53 $\frac{2}{5} \div \frac{1}{2}$

2.2.54 $\frac{4}{9} \div \frac{2}{3}$

2.2.55 $\frac{3}{7} \div \frac{8}{7}$

2.2.56 $2\frac{1}{3} \div \frac{3}{5}$

2.2.57 $3\frac{1}{2} \div \frac{8}{3}$

2.2.58 $\frac{2}{5} \div 1\frac{1}{2}$

2.2.59 $3\frac{1}{5} \div 1\frac{2}{3}$

2.2.60 $2\frac{1}{5} \div 1\frac{1}{5}$

2.2.61 0.2×6

2.2.62 8×4.6

2.2.63 2.71×5

2.2.64 20.1×4

2.2.65 0.05×7

2.2.66 211.02×4

2.2.67 2×0.86

2.2.68 2.5×0.3

2.2.69 0.1×51.7

2.2.70 0.2×316.8

2.2.71 1.3×3.1

2.2.72 0.5×0.05

2.2.73 11.2×0.15

2.2.74 1.07×0.02

2.2.75 10.05×1.05

2.2.76 101.01×0.01

2.2.77 100.01×1.1

2.2.78 7.75×0.25

2.2.79 42.8×0.02

2.2.80 $0.4 \div 2$

2.2.81 $0.35 \div 5$

2.2.82 $2.48 \div 4$

2.2.83 $65.4 \div 6$

2.2.84 $651.2 \div 4$

2.2.85 $14.49 \div 7$

2.2.86 $3.96 \div 4$

2.2.87 $0.80 \div 5$

2.2.88 $7 \div 3.5$

2.2.89 $36 \div 0.2$

2.2.90 $3.25 \div 0.5$

2.2.91 $30.94 \div 0.7$

2.2.92 $0.5 \div 0.25$

2.2.93 $7.75 \div 0.25$

2.2.94 $76.5 \div 0.15$

2.2.95 $2.73 \div 1.3$

2.2.96 $\frac{5}{4} + \frac{-11}{4}$

2.2.97 $\frac{-9}{10} + \frac{22}{15}$

2.2.98 $\frac{-3}{-11} + \frac{5}{9}$

2.2.99 $\frac{-8}{19} + \frac{-2}{57}$

2.2.100 $\frac{-2}{3} + 0$

2.2.101 $\frac{-13}{7} + \frac{6}{7}$

2.2.102 $\frac{19}{5} + \frac{-7}{5}$

2.2.103 $\frac{-5}{6} + \frac{-3}{11}$

2.2.104 $-2\frac{1}{3} + 4\frac{3}{5}$

2.2.105 $\frac{7}{24} - \frac{17}{36}$

2.2.106 $\frac{5}{63} - \frac{-6}{21}$

2.2.107 $\frac{-6}{13} - \frac{-7}{15}$

2.2.108 $\frac{-3}{8} - \frac{7}{11}$

2.2.109 $-2\frac{1}{9} - 6$

2.2.110 $\frac{7}{9} - \frac{2}{5}$

2.2.111 $2\frac{1}{5} - \frac{-1}{3}$

2.2.112 $\frac{9}{2} \times \frac{-7}{4}$

2.2.113 $\frac{3}{10} \times -9$

2.2.114 $\frac{-6}{5} \times \frac{9}{11}$

2.2.115 $\frac{3}{7} \times \frac{-2}{5}$

2.2.116 $\frac{3}{11} \times \frac{2}{5}$

2.2.117 $\frac{3}{-5} \times \frac{-5}{3}$

2.2.118 $-4 \div \frac{2}{3}$

2.2.119 $\frac{-3}{5} \div 2$

2.2.120 $\frac{-4}{5} \div -3$

2.2.121 $\frac{-1}{8} \div \frac{3}{4}$

2.2.122 $\frac{-2}{13} \div \frac{1}{7}$

2.2.123 $\frac{-7}{12} \div \frac{-2}{13}$

2.2.124 $\frac{3}{13} \div \frac{-4}{65}$

2.2.125 $\frac{-3}{5} \times 7$

2.2.126 $\frac{-6}{5} \times -2$

2.2.127 $\frac{-3}{4} \times \frac{1}{7}$

2.2.128 $\frac{2}{3} \times \frac{-5}{9}$

2.2.129 $\frac{2}{3} \times \frac{-7}{8}$

2.2.130 $\frac{-6}{7} \times \frac{5}{7}$

Find

2.2.131 $\frac{1}{2}$ of

a) $2\frac{3}{4}$

b) $4\frac{2}{9}$

2.2.132 $\frac{5}{8}$ of

a) $3\frac{5}{6}$

b) $9\frac{2}{3}$

Find

2.2.133 $\frac{1}{4}$ of

a) $\frac{1}{4}$

b) $\frac{3}{5}$

c) $\frac{4}{3}$

2.2.134 $\frac{1}{7}$ of

a) $\frac{2}{9}$

b) $\frac{6}{5}$

c) $\frac{3}{10}$

2.2.135 In a class of 40 students $\frac{1}{5}$ of the total number of students like to study English, $\frac{2}{5}$ of the total number like to study Mathematics and the remaining students like to study Science.

a) How many students like to study English?

b) How many students like to study Mathematics?

c) How many students like to study Science?

2.2.136 Vidya and Pratap went for a picnic. Their mother gave them a water bottle that contained 5 litres of water. Vidya consumed $\frac{2}{5}$ of the water. Pratap consumed the remaining water.

a) How much water did Vidya drink?

b) What fraction of the total quantity of water did Pratap drink?

2.2.137 Shaili plants 4 saplings in a row, in her garden. The distance between two adjacent saplings is $\frac{3}{4}m$. Find the distance between the first and the last sapling.

2.2.138 Lipika reads a book for $1\frac{3}{4}$ hours everyday. She reads the entire book in 6 days. How many hours in all were required by her to read the book.

2.2.139 A car runs $16km$ using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol.

2.2.140 The side of an equilateral triangle is $3.5cm$. Find its perimeter.

2.2.141 The length of a rectangle is $7.1cm$ and its breadth is $2.5cm$. What is its area?

2.2.142 Find the area of a rectangle whose length is $5.7cm$ and breadth is $3cm$.

2.2.143 A two wheeler covers a distance of $55.3km$ in one litre of petrol. How much will it cover in 10 litres of petrol?

2.2.144 Savita was preparing a design to decorate her classroom. She needed a few coloured strips of paper of length $1.9cm$ each. She had a strip of coloured paper of length $9.5cm$. How many pieces of the required length will she get out of this strip?

2.2.145 Each side of a regular polygon is $2.5cm$ in length. The perimeter of the polygon is $12.5cm$. How many sides does the polygon have?

2.2.146 A car covers a distance of $89.1km$ in 2.2 hours. What is the average distance covered by it in 1 hour?

2.2.147 A vehicle covers a distance of $43.2km$ in in 2.4 litres of petrol. How much will it cover in one litre of petrol?

2.2.148 Mala has a collection of bangles. She has 20 gold bangles and 10 silver bangles. What is the percentage of bangles of each type? Can you put it in the tabular form?

2.2.149 Out of 25 children in a class, 15 are girls. What is the percentage of girls?

2.2.150 Out of 32 students, 8 are absent. What per cent of the students are absent?

2.2.151 There are 25 radios, 16 of them are out of order. What per cent of radios are out of order?

2.2.152 A shop has 500 items, out of which 5 are defective. What per cent are defective?

2.2.153 There are 120 voters, 90 of them voted yes. What per cent voted yes?

2.2.154 A survey of 40 children showed that 25% liked playing football. How many children liked playing football?

2.2.155 Rahul bought a sweater and saved ₹200 when a discount of 25% was given. What was the price of the sweater before the discount?

2.2.156 9 is 25% of what number?

2.2.157 75% of what number is 15?

2.2.158 Out of 15,000 voters in a constituency, 60% voted. Find the percentage of voters who did not vote. Can you now find how many actually did not vote?

2.2.159 Meeta saves ₹4000 from her salary. If this is 10% of her salary. What is her salary?

2.2.160 A local cricket team played 20 matches in one season. It won 25% of them. How many matches did they win?

2.2.161 Reena's mother said, to make idlis, you must take two parts rice and one part urad dal. What percentage of such a mixture would be rice and what percentage would be urad dal?

2.2.162 If ₹250 is to be divided amongst Ravi, Raju and Roy, so that Ravi gets two parts, Raju three parts and Roy five parts. How much money will each get? What will it be in percentages?

2.2.163 Divide 15 sweets between Manu and Sonu so that they get 20 % and 80 % of them respectively.

2.2.164 If angles of a triangle are in the ratio 2 : 3 : 4. Find the value of each angle.

2.2.165 A school team won 6 games this year against 4 games won last year. What is the per cent increase?

2.2.166 The number of illiterate persons in a country decreased from 150 lakhs to 100 lakhs in 10 years. What is the percentage of decrease?

2.2.167 Find Percentage of increase or decrease

a) Price of shirt decreased from ₹280 to ₹210.

b) Marks in a test increased from 20 to 30.

2.2.168 My mother says, in her childhood petrol was ₹1 a litre. It is ₹52 per litre today. By what Percentage has the price gone up?

2.2.169 The cost of a flower vase is ₹120. If the shopkeeper sells it at a loss of 10%, find the price at which it is sold.

2.2.170 Selling price of a toy car is ₹540. If the profit made by shopkeeper is 20%, what is the cost price of this toy?

2.2.171 A shopkeeper bought a chair for ₹375 and sold it for ₹400. Find the gain Percentage.

2.2.172 Cost of an item is ₹50. It was sold with a profit of 12%. Find the selling price.

2.2.173 An article was sold for ₹250 with a profit of 5%. What was its cost price?

2.2.174 An item was sold for ₹540 at a loss of 5%. What was its cost price?

2.2.175 Anita takes a loan of ₹5,000 at 15% per year as rate of interest. Find the interest she has to pay at the end of one year.

2.2.176 ₹10,000 is invested at 5% interest rate p.a. Find the interest at the end of one year.

2.2.177 ₹3,500 is given at 7% p.a. rate of interest. Find the interest which will be received at the end of two years.

2.2.178 ₹6,050 is borrowed at 6.5% rate of interest p.a.. Find the interest and the amount to be paid at the end of 3 years.

2.2.179 ₹7,000 is borrowed at 3.5% rate of interest p.a. borrowed for 2 years. Find the amount to be paid at the end of the second year.

2.2.180 If Manohar pays an interest of ₹750 for 2 years on a sum of ₹4,500, find the rate of interest.

2.2.181 You have ₹2,400 in your account and the interest rate is 5%. After how many years would you earn ₹240 as interest.

2.2.182 On a certain sum the interest paid after 3 years is ₹450 at 5% rate of interest per annum. Find the sum.

2.2.183 Tell what is the profit or loss in the following transactions. Also find profit per cent or loss per cent in each case.

a) Gardening shears bought for ₹250 and sold for ₹325.

b) A refrigerater bought for ₹12,000 and sold at ₹13,500.

c) A cupboard bought for ₹2,500 and sold at ₹3,000.

d) A skirt bought for ₹250 and sold at ₹150.

2.2.184 Convert each part of the ratio to percentage

a) 3 : 1

b) 2 : 3 : 5

c) 1:4

d) 1 : 2 : 5

2.2.185 The population of a city decreased from 25,000 to 24,500. Find the percentage decrease.

2.2.186 Arun bought a car for ₹3,50,000. The next year, the price went upto ₹3,70,000. What was the Percentage of price increase?

2.2.187 I buy a T.V. for ₹10,000 and sell it at a profit of 20%. How much money do I get for it?

2.2.188 Juhi sells a washing machine for ₹13,500. She loses 20% in the bargain. What was the price at which she bought it?

2.2.189 a) Chalk contains calcium, carbon and oxygen in the ratio 10:3:12. Find the percentage of carbon in chalk.

b) If in a stick of chalk, carbon is 3g, what is the weight of the chalk stick?

2.2.190 Aamani buys a book for ₹275 and sells it at a loss of 15%. How much does she sell it for?

2.2.191 Find the amount to be paid at the end of 3 years in each case

a) Principal = ₹1,200 at 12% p.a.

b) Principal = ₹7,500 at 5% p.a.

2.2.192 What rate gives ₹280 as interest on a sum of ₹56,000 in 2 years?

2.2.193 If Meena gives an interest of ₹45 for one year at 9% rate p.a.. What is the sum she has borrowed?

2.2.194 Find the missing values in Table 2.2.194

S.No	**Base**	**Height**	**Area of the Parallelogram**
a.	$20cm$		$246cm^2$
b.		$15cm$	$154.5cm^2$
c.		$8.4cm$	$48.72cm^2$
d.	$15.6cm$		$16.38cm^2$

TABLE 2.2.194

2.2.195 Find the missing values in Table 2.2.195

Base	**Height**	**Area of Triangle**
$15cm$		$87cm^2$
	$31.4mm$	$1256mm^2$
$22cm$		$170.5cm^2$

TABLE 2.2.195

3 Programming

3.1 Formulae

In a quiz, team A scored $a_1 = -40, a_2 = 10, a_3 = 0$ and team B scored $b_1 = 10, b_2 = 0, b_3 = -40$ in three successive rounds.

3.1.1 If the total scores are

$$a = a_1 + a_2 + a_3 \tag{3.1.1.1}$$

$$b = b_1 + b_2 + b_3 \tag{3.1.1.2}$$

which team scored more?

Solution:

```
//Code by Harini
//February 23, 2025
//Revised by GVV Sharma
//April 14, 2025
//add two sets of numbers and compare
#include <stdio.h>

//begin main function
int main() {
// first team scores
 int a1=-40,a2=10,a3=0;
// second team scores
 int b1=10,b2=0,b3=-40;

 //declaring scores variables
int a,b;
 //sum of scores
 a=a1+a2+a3;
 b=b1+b2+b3;
 //comparing scores
 if (a>b){
        printf("a scored more\n");
       }
 else if (a<b){
        printf("b scored more\n");
       }
 else {
        printf("they are equal\n");
}
//end comparison
 return 0;
}
//end main function
```

3.1.2 Write a function to compare the final scores. Check for the cases when $a = -40, b = -40; a = 30, b = 20; a = -20, b = -10$.

Solution:

```
//code by harini
//feb 23 2025
//code by GVV Sharma
//April 14 2025
//function to compare two numbers

#include <stdio.h>
```

```
//function to compare the numbers a and b
void compare(int a,int b){
        if (a>b){
        printf("a scored more\n");
}
 else if (a<b){
        printf("b scored more\n");
}
 else {
        printf("they are equal\n");
}
       }
//end function to compare the numbers a and b
//begin main function
int main() {
 int a=-40,b=-40;

 //call the function to compare the numbers
 compare(a,b);

 return 0;
}
//end main function
```

3.1.3 Use arrays and a for loop to evaluate

$$a = \sum_{i=0}^{2} a_i \tag{3.1.3.1}$$

$$b = \sum_{i=0}^{2} b_i \tag{3.1.3.2}$$

Solution:

```
//code by harini
//feb 23 2025
//revise by GVV Sharma
//April 14 2025
//compares sum of 2 arrays using a for loop
#include <stdio.h>

//compare function
void compare(int a,int b){
        if (a>b){
        printf("a scored more\n");
```

```
}
 else if (a<b){
            printf("b scored more\n");
}
 else {
            printf("they are equal\n");
}
            }
///end compare function
//begin main function
int main() {
            //Declaring arrays
int a1[]={-40,10,0};
int b1[]={10,0,-40};
//Initializing sums
int a=0,b=0;
   for (int i = 0; i <= 2; i++){
               a=a+a1[i];
               b=b+b1[i];
   }
//Call compare function
   compare(a,b);
 return 0;
}
//end main function
```

3.1.4 Revise the above code using only functions.
Solution:

```
//code by harini
//feb 23 2025
//revise by GVV Sharma
//April 14 2025
//using functions for arrays
#include <stdio.h>

//Declaring functions
void compare(int a,int b);
int sum(int a[]);

//begin main function
int main() {
            //Declaring arrays
int a1[]={-40,10,0};
int b1[]={10,0,-40};
//Initializing sums
```

```
int a=0,b=0;
//finding sum for A
a = sum(a1);
//finding sum for B
b = sum(b1);
//Call compare function
  compare(a,b);
 return 0;
}
//end main function

//compare function
void compare(int a,int b){
          if (a>b){
          printf("a scored more\n");
}
 else if (a<b){
          printf("b scored more\n");
}
 else {
          printf("they are equal\n");
}
         }
//end compare function
//sum function
int sum(int a1[]){
int a=0;
  for (int i = 0; i <= 2; i++){
           a=a+a1[i];
  }
  return a; //returning the sum to main
  }
//end sum function
```

3.1.5 Use files for the input data.

Solution:

```
//Code by GVV Sharma
//April 14 2025
//using files
#include <stdio.h>

//Declaring functions
void compare(int a,int b);
int sum(int a[]);
```

```
//begin main function
int main() {
            //Declaring arrays
int a1[3], b1[3];
//declare file pointer
FILE *fp;
int i;
//Initializing sums
int a=0,b=0;
            //Read a from file a.dat
            //Open file pointer
fp = fopen("a.dat", "r");

//load data from file to array a1
 for(i=0;i<=2;i++){
    fscanf(fp,"%d",&a1[i]);
   }

//Cose file pointer

fclose(fp);
            //Read a from file b.dat
            //Open file pointer
fp = fopen("b.dat", "r");

//load data from file to array b1
 for(i=0;i<=2;i++){
    fscanf(fp,"%d",&b1[i]);
   }
//Close file pointer
fclose(fp);

//finding sum for A
a = sum(a1);
//finding sum for B
b = sum(b1);
//Call compare function
   compare(a,b);
 return 0;
}
//end main function

//compare function
void compare(int a,int b){
             if (a>b){
```

```
        printf("a scored more\n");
}
 else if (a<b){
        printf("b scored more\n");
}
 else {
        printf("they are equal\n");
}
      }
//end compare function
//sum function
int sum(int a1[]){
int a=0;
  for (int i = 0; i <= 2; i++){
        a=a+a1[i];
  }
  return a; //returning the sum to main
  }
//end sum function
```

3.1.6 Revise the files program using pointer arrays
Solution:

```
//Code by GVV Sharma
//April 14 2025
//using pointer arrays
#include <stdio.h>
#include <stdlib.h>

//Declaring functions
void compare(int a,int b);
int sum(int a[], int m);

//begin main function
int main() {
//declare pointer arrays
int *a1,*b1,m = 3;
//Initializing sums
int a=0,b=0,i;
//File pointer
FILE *fp;

//Create a1
a1= (int *)malloc(m * sizeof( a1));
b1= (int *)malloc(m * sizeof( b1));
```

```
        //Read a from file a.dat
        //Open file pointer
fp = fopen(”a.dat”, ”r”);

//load data from file to array a1
 for(i=0;i<=2;i++){
   fscanf(fp,”%d”,&a1[i]);
  }

//Cose file pointer

fclose(fp);
        //Read a from file b.dat
        //Open file pointer
fp = fopen(”b.dat”, ”r”);

//load data from file to array b1
 for(i=0;i<=2;i++){
   fscanf(fp,”%d”,&b1[i]);
  }
//Close file pointer
fclose(fp);

//finding sum for A
a = sum(a1,m);
//finding sum for B
b = sum(b1,m);
//Call compare function
compare(a,b);

//free memory
free(a1);
free(b1);
 return 0;
}
//end main function

//compare function
void compare(int a,int b){
         if (a>b){
         printf(”a scored more\n”);
}
 else if (a<b){
         printf(”b scored more\n”);
}
```

```
 else {
            printf("they are equal\n");
}
            }
//end compare function
//sum function
int sum(int *vec,int m){
int a=0;
   for (int i = 0; i < m; i++){
            a=a+vec[i];
   }
   return a; //returning the sum to main
   }
//end sum function
```

3.1.7 Revise the files program using only functions

Solution:

```
//Code by GVV Sharma
//April 14 2025
//using functions for all
#include <stdio.h>
#include <stdlib.h>

//Declaring functions
void compare(int a,int b);
int sum(int a[], int m);
int *loadVec(char *str,int m);
int *createVec(int m);

//begin main function
int main() {
//Initializing sums
int a=0,b=0,m = 3;
//declare pointer arrays
int *a1,*b1;
            //Read a from file a.dat
a1= loadVec("a.dat",m);
b1= loadVec("b.dat",m);
            //Read b from file b.dat

//finding sum for A
a = sum(a1,m);
//finding sum for B
b = sum(b1,m);
//Call compare function
```

```
compare(a,b);
 return 0;
}
//end main function

//compare function
void compare(int a,int b){
        if (a>b){
        printf("a scored more\n");
}
 else if (a<b){
        printf("b scored more\n");
}
 else {
        printf("they are equal\n");
}
       }
//end compare function
//sum of vector elements
int sum(int *vec,int m){
int a=0;
  for (int i = 0; i < m; i++){
         a=a+vec[i];
  }
  return a; //returning the sum to main
  }
//end sum function
//loading file data into vector
int *loadVec(char *str,int m){
FILE *fp;
int i;
int *vec=createVec(m);
       //Open file pointer
fp = fopen(str, "r");

//load data from file to array a1
 for(i=0;i<m;i++){
   fscanf(fp,"%d",&vec[i]);
  }

//Cose file pointer

fclose(fp);
return vec;
}
```

```
//end loading file data into vector
//Defining the function for vector creation
int *createVec(int m)
{
 int *vec;

 //Allocate memory to the pointer
vec = (int *)malloc(m * sizeof( vec));
 return vec;
}
```

Reduce to standard form

3.1.8 $\frac{-45}{30}$

Solution:

```
//code by GVV Sharma
//April 23 2025
//reducing a fraction to lowest terms
//using the HCF
#include <stdio.h>

//Declaring functions
void swap(int *a,int *b);//swap two numbers
int hcf(int a,int b);//hcf of two numbers
int min(int a,int b);//min modulus of two numbers
int abs(int a);//absolute value of a number

//begin main function
int main() {

//define numerator and denominator
int num=-45,den=30;
int hcfval;

hcfval = hcf(num,den);
printf("%d/%d\n",num/hcfval,den/hcfval);
 return 0;
}
//end main function

//absolute value
int abs(int a){
        if (a < 0){
        return -a;
```

```
}
 else {
            return a;
}
            }
//end absolute value
//compare function
int min(int a,int b){
            a = abs(a);
            b = abs(b);
             if (a>=b){
             return b;
}
 else {
            return a;
}
            }
//end compare function


//hcf function
int hcf(int num, int den){

int temp;
           //taking absolute values
num = abs(num);
den = abs(den);


//Ensuring numerator > denominator
if(num < den)
swap(&num,&den);

//Remainder
temp=num%den;

   while (temp != 0){
              num = den;
              den = temp;
              temp=num%den;
   }
   return den; //returning the hcf to main
   }
//end hcf function
```

```
//Swap two numbers
void swap(int *a,int *b){
        int temp;
        temp = *a;
        *a = *b;
        *b = temp;
}
```

Use recursion for the following

3.1.9 9^3

Solution: For integer powers, the exponent can be computed as

$$x(n) = ax(n-1), x(0) = 1 \tag{3.1.9.1}$$

which results in the following C code.

```
//code by GVV Sharma
//April 22, 2025
//function to compute the nth power of an integer

#include <stdio.h>

//function to compute a to the power n
int power(int a,int n){
        if (n == 0){
        return(1);
        }
        else{
                return a*power(a,n-1);
        }
       }
//end function to compute a to the power n

//begin main function
int main(void) {
 int a=9,n=3;

 //call the function to compute 9 to the power 3
printf("%d\n",power(9,3));

 return 0;
}
//end main function
```

Use matrices for the following

3.1.10 The difference in the measures of two complementary angles is 12°. Find the measures of the angles.

Solution: Let the angles be x and y. Then we have the following equations

$$x + y = 180 \tag{3.1.10.1}$$
$$x - y = 12 \tag{3.1.10.2}$$

which can be expressed as the matrix equation

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 180 \\ 12 \end{pmatrix} \tag{3.1.10.3}$$
$$\text{or, } \mathbf{A}\mathbf{x} = \mathbf{b} \tag{3.1.10.4}$$

The solution can be obtained as

$$\mathbf{x} = \frac{\mathbf{A}^\top \mathbf{b}}{2} \tag{3.1.10.5}$$

using the following code

```
//Code by G V V Sharma
//April 23, 2025
//Find solution of simultaneous equations
//using matrices

#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include "libs/matfun.h"
#include "libs/geofun.h"


int main()
{
double **A,**b,**xtemp, **A_T,**x;
int m=2,n=2;//matrix rows and columns
        A = createMat(m,n);//Create A matrix array
        b = createMat(m,1);//Create b vector array
A = loadMat("A.dat",m,n);//load matrix from file
b = loadMat("b.dat",m,1);//load vector from file
A_T=transposeMat(A, m, n);//transpose of A
xtemp = Matmul(A_T, b, m, 2, 1);//A transpose times b
x= Matscale(xtemp, m, 1, 0.5);//divide by 2 to obtain solution vector

//print matrices

printf("Matrix A\n\n");
printMat(A,m,n);
```

```
printf("\nVector b\n");
printMat(b,m,1);
printf("\nSolution x\n");
//printMat(xtemp,m,1);
printMat(x,m,1);

//free all matrix and vector arrays
freeMat(A, m);
freeMat(b, m);
freeMat(A_T, m);
freeMat(xtemp, m);
freeMat(x, m);
    return 0;
}
```

by keeping the functions in .h files as below.

```
//Functions created by
// G V V Sharma
// October 27, 2023
// Revised November 13, 2023

//Function declaration

double **createMat(int m,int n);//create m x n matrix array
void printMat(double **p,int m,int n);//print matrix
double **loadMat(char *str,int m,int n);//load matrix from file
double **Matscale(double **a, int m, int n, double k);//scale matrix
double **Matmul(double **a, double **b, int m, int n, int p);//multiply matrices
    a and b
double **transposeMat(double **a, int m, int n);//transpose of a
//End function declaration

//scale matrix
double **Matscale(double **a, int m, int n, double k){
int i, j;
double **c;
c = createMat(m,n);

 for(i=0;i<m;i++)
 {
  for(j=0;j<n;j++)
  {
c[i][j]= k*a[i][j];
//printf("%lf\n",c[i][j]);
  }
```

```
 }
return c;
}

//Defining the function for matrix creation
double **createMat(int m,int n)
{
 int i;
 double **a;

 //Allocate memory to the pointer
a = (double **)malloc(m * sizeof( *a));
    for (i=0; i<m; i++)
          a[i] = (double *)malloc(n * sizeof( *a[i]));

 return a;
}
//End function for matrix creation


//Read matrix from file
double **loadMat(char *str,int m,int n)
{
FILE *fp;
double **a;
int i,j;


a = createMat(m,n);
fp = fopen(str, "r");

 for(i=0;i<m;i++)
 {
  for(j=0;j<n;j++)
  {
   fscanf(fp,"%lf",&a[i][j]);
  }
 }
//End function for reading matrix from file

fclose(fp);
 return a;

}
```

```
//Defining the function for printing
void printMat(double **p, int m,int n)
{
 int i,j;

 for(i=0;i<m;i++)
 {
  for(j=0;j<n;j++)
  printf("%lf ",p[i][j]);
 printf("\n");
 }
}
//End function for printing

//Defining the function for product of matrices

double **Matmul(double **a, double **b, int m, int n, int p)
{
int i, j, k;
double **c, temp =0;
c = createMat(m,p);

 for(i=0;i<m;i++)
 {
  for(k=0;k<p;k++)
  {
    for(j=0;j<n;j++)
    {
        temp= temp+a[i][j]*b[j][k];
    }
        c[i][k]=temp;
        temp = 0;
  }
 }
return c;

}

//Defining the function for transpose of matrix

double **transposeMat(double **a, int m, int n)
{
int i, j;
double **c;
```

```
//printf("I am here");
c = createMat(n,m);

 for(i=0;i<n;i++)
 {
  for(j=0;j<m;j++)
  {
c[i][j]= a[j][i];
// printf("%lf ",c[i][j]);
  }
 }
return c;

}
//End function for transpose of matrix
```

```
//Function declaration
void freeMat(double **matrix, int rows);
//End Function declaration

void freeMat(double **matrix, int rows) {
    for (int i = 0; i < rows; ++i) {
        free(matrix[i]);
    }
    free(matrix);
}
```

3.2 NCERT

Use ifelse for the following

3.2.1 $25 \times (-21) = (-21) \times 25$

Solution:

```
//Code by GVV Sharma
//April 22, 2025
//Check if two expressions are equal
#include <stdio.h>

//begin main function
int main() {
// declaring variables
 int a=25,b=-21;

 if (a*b==b*a){
```

```
            printf("correct\n");
        }
  else {
            printf("incorrect\n");
        }
//end comparison
  return 0;
}
//end main function
```

3.2.2 $(-48) \div (8) = 48 \div (-8)$

3.2.3 $(-23) \times 20 = 23 \times (-20)$

3.2.4 $90 \div (-45) = (-90) \div 45$

3.2.5 $(-136) \div 4 = 136 \div (-4)$

3.2.6 $10 \times [6 + (-2)] = 10 \times 6 + 10 \times (-2)$

3.2.7 $10 \times [6 - (-2)] = 10 \times 6 - 10 \times (-2)$

3.2.8 $(-15) \times [(-7) - (-1)] = (-15) \times (-7) - (-15) \times (-1)$

3.2.9 $18 \times [(7) + (-3)] = 18 \times (7) + 18 \times (-3)$

3.2.10 $(-21) \times [(-4) + (-6)] = (-21) \times (-4) + (-21) \times (-6)$

3.2.11 $(-15) \times [(-7) + (-1)] = (-15) \times (-7) + (-15) \times (-1)$

3.2.12 An angle is greater than 45°. Is its complementary angle greater than 45°or equal to 45°or less than 45°?

Is it possible to have a triangle with the following sides?

3.2.13 $2cm, 3cm, 5cm$

3.2.14 $3cm, 6cm, 7cm$

3.2.15 $6cm, 3cm, 2cm$

3.2.16 $10.2cm, 5.8cm, 4.5cm$?

Between which two numbers can length of the third side fall? The lengths of two sides of a triangle are

3.2.17 $6cm$ and $8cm$.

3.2.18 $12cm$ and $15cm$.

Which is greater?

3.2.19 $\frac{1}{2}$ of $\frac{3}{4}$ or $\frac{3}{5}$ of $\frac{5}{8}$

3.2.20 $\frac{1}{2}$ of $\frac{6}{7}$ or $\frac{2}{3}$ of $\frac{3}{7}$

Identify which of the following pairs of angles are complementary and which are supplementary.

3.2.21 65°, 115°

3.2.22 63°, 27°

3.2.23 130°, 50°

3.2.24 45°, 45°

3.2.25 112°, 68°

3.2.26 80°, 10°

Which of these are negative rational numbers?

3.2.27 $\frac{-2}{3}$

3.2.28 $\frac{5}{7}$

3.2.29 $\frac{3}{-5}$

3.2.30 $\frac{6}{11}$

3.2.31 $\frac{-2}{-9}$

3.2.32 0

Compare the following and fill in the blanks

3.2.33 $\frac{-5}{7}$ ___ $\frac{2}{3}$

3.2.34 $\frac{-8}{5}$ ___ $\frac{-7}{4}$

3.2.35 0 ___ $\frac{-7}{6}$

3.2.36 $\frac{-4}{5}$ ___ $\frac{-5}{7}$

3.2.37 $\frac{1}{-3}$ ___ $\frac{-1}{4}$

3.2.38 $\frac{-7}{8}$ ___ $\frac{14}{-16}$

3.2.39 $\frac{5}{-11}$ ___ $\frac{-5}{11}$

3.2.40 $\frac{2}{3}$ ___ $\frac{5}{7}$

3.2.41 $\frac{-3}{8}$ ___ $\frac{-2}{7}$

3.2.42 $\frac{-4}{3}$ ___ $\frac{-3}{2}$

3.2.43 $\frac{-7}{21}$ ___ $\frac{3}{9}$

3.2.44 $\frac{-3}{5}$ ___ $\frac{-12}{20}$

3.2.45 $\frac{-5}{-9}$ ___ $\frac{5}{-9}$

3.2.46 $\frac{-16}{20}$ ___ $\frac{20}{-25}$

3.2.47 $\frac{8}{-5}$ ___ $\frac{-24}{15}$

3.2.48 $\frac{-2}{-3}$ ___ $\frac{2}{3}$

3.2.49 $\frac{1}{3}$ ___ $\frac{-1}{9}$

3.2.50 $\frac{4}{-9}$ ___ $\frac{-16}{36}$

3.2.51 $\frac{2}{3}$ ___ $\frac{5}{2}$

3.2.52 $\frac{-1}{4}$ ___ $\frac{1}{4}$

3.2.53 $\frac{-5}{6}$ ___ $\frac{-4}{3}$

3.2.54 $-3\frac{2}{7}$ ___ $-3\frac{4}{5}$

3.2.55 $\frac{-3}{4}$ ___ $\frac{2}{-3}$

Write four more numbers in the following pattern

3.2.56 $\frac{-1}{3}, \frac{-2}{6}, \frac{-3}{9}, \frac{-4}{12}$

Solution:

```
//code by GVV Sharma
//April 23 2025
//generates terms of a
//sequence using a loop
#include <stdio.h>

//begin main function
int main() {

//printing the numbers
  for (int k = 5; k < 9; k++){
            printf("%d/%d\n",-k,3*k);
  }
 return 0;
}
//end main function
```

3.2.57 $\frac{-3}{5}, \frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}$

3.2.58 $\frac{-1}{4}, \frac{-2}{8}, \frac{-3}{12}$

3.2.59 $\frac{-1}{6}, \frac{2}{-12}, \frac{3}{-18}, \frac{4}{-24}$

3.2.60 $\frac{-2}{3}, \frac{2}{-3}, \frac{4}{-6}, \frac{6}{-9}$

Reduce to standard form

3.2.61 $\frac{36}{-24}$

3.2.62 $\frac{-3}{-15}$

3.2.63 $\frac{-18}{45}$

3.2.64 $\frac{-12}{18}$

3.2.65 $\frac{-8}{6}$

3.2.66 $\frac{25}{45}$

3.2.67 $\frac{-44}{72}$

3.2.68 $\frac{-8}{10}$

Use recursion for the following

3.2.69 $(-1) \times (-2) \times (-3) \times (-4)$

Solution: In this case,

$$x(n) = -nx(n-1), x(0) = 1 \tag{3.2.69.1}$$

Complete the code using the above equation.

3.2.70 2^6
3.2.71 11^2
3.2.72 5^4
3.2.73 $\left(6^2\right)^4$
3.2.74 $\left(2^2\right)^{100}$
3.2.75 $\left(7^{50}\right)^2$
3.2.76 $\left(5^3\right)^7$
3.2.77 $2^5 \times 2^3$
3.2.78 $4^3 \times 4^2$
3.2.79 $5^3 \times 5^7 \times 5^{12}$
3.2.80 $2^8 \div 2^3$
3.2.81 $9^{11} \div 9^7$
3.2.82 $7^{13} \div 7^{10}$
3.2.83 $10^8 \div 10^4$
3.2.84 $20^{15} \div 20^{13}$
3.2.85 $(-4)^3$
3.2.86 $\left(\frac{3}{5}\right)^4$
3.2.87 $\left(\frac{-4}{7}\right)^5$
3.2.88 $(-4)^{100} \times (-4)^{20}$

Use arrays for the following

3.2.89 $(-12) \times (-11) \times (10)$

Solution: The product can be expressed as

$$y = \prod_{k=0}^{2} a_k \tag{3.2.89.1}$$

where

$$\mathbf{a} = \begin{pmatrix} -12 \\ -11 \\ 10 \end{pmatrix} \tag{3.2.89.2}$$

The following code implements this

```
//code by GVV Sharma
//April 22 2025
//product of elements of a vector
#include <stdio.h>

//Declaring functions
int prod_vec(int *a);

//begin main function
int main() {
        //Declaring arrays
int a[]={-12,-11,10};
//Initializing products
int y;
//finding product for A
y = prod_vec(a);
printf("%d\n",y);
 return 0;
```

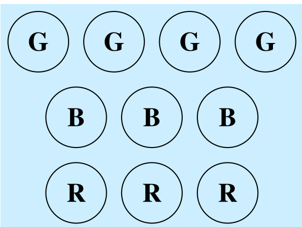
```
}
//end main function

//product function
int prod_vec(int *a){
int y=1;
   for (int i = 0; i <= 2; i++){
            y=y*a[i];
   }
   return y; //returning the product to main
   }
//end product function
```

3.2.90 $(9) \times (-3) \times (-6)$ 3.2.91 $(-18) \times (-5) \times (-4)$ 3.2.92 $(-3) \times (-6) \times (-2) \times (-1)$

Use matrices for the following

3.2.93 Among two supplementary angles the measure of the larger angle is 44° more than the measure of the smaller. Find their measures.

3.2.94 A collection of 10 chips with different colours is given in Fig. 3.2.94. Extend the table using a C program to include a column for the percentage of chips of each colour.



(a) Chips

Colour	Number
Green	
Blue	
Red	
Total	

(b) Table

Fig. 3.2.94

Solution:

```
//Code by G V V Sharma
//May 15, 2025
//Code to generate a table to compute percentage
#include <stdio.h>

int main() {
      int colour[] = {4,3,3};//chip colours
      char x[3][6] = {"Green", "Blue", "Red"};
      // Print table header
```

```
	printf("Colour\tNumber\tPercentage\n");
	// Print table
	for (int i = 0; i < 3; i++) {
		printf("%s\t%d\t%f\n", x[i],colour[i], (float)colour[i]*10);
	}
	return 0;
}
```

4 Data Handling

4.1 Formulae

4.1.1 A batsman scored the following number of runs in 6 innings.

$$36, 35, 50, 46, 60, 55$$

Calculate the mean runs scored by him in an inning.

Solution:

```
//code by GVV Sharma
//April 24 2025
//find the mean of given data
#include <stdio.h>
#include <stdlib.h>

//Declaring functions
float mean(int *a, int n);

//begin main function
int main() {
	//Declaring data array
int a[]={36,35,50,46,60,55};
int n = sizeof(a)/sizeof(a[0]);
printf("%f\n%d",mean(a,n),n);
//printf("%f\n",((float)(mean(a)))/n);
 return 0;
}
//end main function

//mean function
float mean(int *a, int n){
int avg=0;
  for (int i = 0; i < n; i++){
	avg=avg+a[i];
  }
  return ((float)avg)/n; //returning the mean to main
  }
```

```
//end mean function
```

4.1.2 The ages in years of 10 teachers of a school are

$$32, 41, 28, 54, 35, 26, 23, 33, 38, 40$$

a) What is the age of the oldest teacher and that of the youngest teacher?
Solution:

```
//code by GVV Sharma
//April 24 2025
//find the max of given data
#include <stdio.h>
#include <stdlib.h>

//Declaring functions
int max(int *a, int n);
int min(int *a, int n);

//begin main function
int main() {
        //Declaring data array
int a[]={32,41,28,54,35,26,23,33,38,40};
int n = sizeof(a)/sizeof(a[0]);
printf("%d,%d\n",max(a,n),min(a,n));
 return 0;
}
//end main function

//max function
int max(int *a, int n){
int temp=a[0];
  for (int i = 1; i < n; i++){
        if(temp < a[i])
                temp = a[i];
  }
  return temp; //returning the max to main
  }
//end max function
//min function
int min(int *a, int n){
int temp=a[0];
  for (int i = 1; i < n; i++){
        if(temp > a[i])
                temp = a[i];
  }
```

```
    return temp; //returning the min to main
    }
//end min function
```

b) What is the range of the ages of the teachers?
c) What is the mean age of these teachers?

4.1.3 Organize the following marks in a class assessment, in tabular form with columns as marks and frequency.

$$4, 6, 7, 5, 3, 5, 4, 5, 2, 6, 2, 5, 1, 9, 6, 5, 8, 4, 6, 7 \tag{4.1.3.1}$$

a) Which number is the highest?
b) Which number is the lowest?
c) What is the range of the data?
d) Find the arithmetic mean.

Solution:

```
//Code by G V V Sharma
//May 07, 2025
//Code to generate a table for data and frequency
#include <stdio.h>
#include <stdlib.h>
#include "libs/datafun.h"

int main() {
    int data[] = {4, 6, 7, 5, 3, 5, 4, 5, 2, 6, 2, 5, 1, 9, 6, 5, 8, 4, 6, 7};//data vector
    int datalen = sizeof(data) / sizeof(data[0]);
    int minval = min(data,datalen);//min value of data
    int maxval = max(data,datalen);// max value of data
    int freqlen = maxval-minval+1; // Range of data
    int *freq=createVec(freqlen); // Freqency vector

    // Count frequency of each mark
    for (int i = 0; i < datalen; i++) {
        freq[data[i]]++;
    }

    // Print table header
    printf("Mark\tFrequency\n");

    // Print frequencies
    for (int i = 0; i <= freqlen; i++) {
        if (freq[i] > 0) {
            printf("%d\t%d\n", i, freq[i]);
        }
    }
```

```
    return 0;
}
```

4.1.4 Find the median of the group of 17 students with the following heights (in cm)

$$106, 110, 123, 125, 117, 120, 112, 115,$$
$$110, 120, 115, 102, 115, 115, 109, 115, 101$$

Solution:

```
//code by GVV Sharma
//April 30 2025
//find the pivot index and median of given data
//using the quicksort algorithm
#include <stdio.h>

//Declaring functions
int Pindex(int *a, int low, int high);
void swap(int *a,int *b);//swap two numbers
void printData(int *a, int n);
void median(int *a, int n);
void quicksort(int data[], int low, int high);

//begin main function
int main() {
int data[]={106, 110, 123, 125, 117, 120, 112, 115, 110, 120, 115, 102, 115, 115,
    109, 115, 101};
    int n = sizeof(data)/sizeof(data[0]);

    printf("Original data:\n");
    printData(data, n);

    quicksort(data, 0, n - 1);

    printf("\nSorted data:\n");
    printData(data, n);

    //print median
    printf("\n%d,%d\n",n/2+1,data[n/2+1]);

    return 0;
}

//end main function
```

```
//Swap two numbers
void swap(int *a,int *b){
        int temp;
        temp = *a;
        *a = *b;
        *b = temp;
}
void printData(int *a, int n){
  for (int i = 0; i < n; i++){
          printf("%d ",a[i]);
  }
}

//pivot index function
int Pindex(int *a, int low, int high){
        int i;
  for ( i = low; i < high; i++){
          if(a[i] > a[high]){
                  swap(&a[i], &a[high]);
          }
  }
  return i; //returning the pivot index
  }
//end pivot index function

// Quicksort function
void quicksort(int data[], int low, int high) {
    if (low < high) {
        // Partition index
        int pi = Pindex(data, low, high);

        // Recursively sort elements before and after partition
        quicksort(data, low, pi - 1);
        quicksort(data, pi + 1, high);
    }
}
```

4.1.5 Sale of English and Hindi books in the years 1995, 1996, 1997 and 1998 are given below in Table 4.1.5.

Years	1995	1996	1997	1998
English	350	400	450	620
Hindi	500	525	600	650

TABLE 4.1.5

Draw a double bar graph and answer the following questions

a) In which year was the difference in the sale of the two language books least?.

b) Can you say that the demand for English books rose faster? Justify.

Solution:

```
//Revised by GVV Sharma
//May 9, 2025
// Drawing double bar graph
#include <stdio.h>
#include <stdlib.h>
#include "libs/datafun.h"

#define MAX_HEIGHT 10//bar graph max height

void bar(int val,int thresh);

void drawDoubleBarGraph(int y1[], int y2[], char x[][5], int n) {
    // Find maximum value
    int l = (max(y1, n)> max(y2, n)) ? max(y1, n) : max(y2, n);//ternary operator
    //bar graph scaling factor
    float scale = (l > MAX_HEIGHT) ? (float)MAX_HEIGHT / l: 1.0;

    // Draw the bars vertically
    for (int row = MAX_HEIGHT; row > 0; row--) {
        for (int i = 0; i < n; i++) {
            printf(" ");
            bar((int)(y1[i] * scale),row);
            bar((int)(y2[i] * scale),row);

        }
        printf("\n");
    }

    // Draw x-axis
    for (int i = 0; i < 2*n; i++) {
        printf("---");
    }
    printf("\n");
```

```
    // Show year labels
    for (int i = 0; i < n; i++) {
        printf("%s ", x[i]);
    }
    printf("\n");
}

int main() {
    int y1[] = {350, 400, 450, 620};//y1
    int y2[] = {500, 525, 600, 650};//y2
    int datalen= sizeof(y1) / sizeof(y1[0]);
    char x[4][5] = {"1995", "1996", "1997", "1998"};

    drawDoubleBarGraph(y1, y2, x, datalen);

    return 0;
}
//drawing bars
void bar(int val,int thresh){
        if (val >= thresh)
            printf("| ");
        else
            printf("  ");
}
```

4.1.6 Following are the margins of victory in the football matches of a league. Find the mode of this data.

$$1, 3, 2, 5, 1, 4, 6, 2, 5, 2, 2, 2, 4, 1, 2, 3, 1, 1, 2, 3, 2, 6, 4, 3, 2,$$
$$1, 1, 4, 2, 1, 5, 3, 3, 2, 3, 2, 4, 2, 1, 2.$$

Solution:

```
//code by GVV Sharma
//May 1 2025
//find the mode of given data

#include <stdio.h>
#include "libs/datafun.h"

//Declaring functions
int mode(int *a, int n);// the mode function

//begin main function
int main() {
```

```
            int ind;
int data
      []={1,3,2,5,1,4,6,2,5,2,2,2,4,1,2,3,1,1,2,3,2,6,4,3,2,1,1,4,2,1,5,3,3,2,3,2,4,2,1,2};
      int n = sizeof(data)/sizeof(data[0]);

      printf("Original data:\n");
      printData(data, n);



      //get mode
ind = mode(data,n);

      //print mode
printf("\n%d,%d\n",ind,data[ind]);

      return 0;
}

//end main function



//mode function
int mode(int *a, int n){
            int i,j,freq=0, maxfreq=0,modei;
   for ( i = 0; i < n; i++){
freq=0;
   for ( j = 0; j < n; j++){
            if(a[i]==a[j]){
                        freq++;
   }}

            if(freq > maxfreq){
            maxfreq=freq;
            modei = i;
            }
   }
   return modei; //returning the mode
   }
//end mode function
```

4.2 NCERT

4.2.1 Find the average of 4.2, 3.8 and 7.6.

4.2.2 Ashish studies for 4 hours, 5 hours and and 3 hours respectively on three consecutive days. How many hours does he study daily on an average?

4.2.3 A cricketer scores the following runs in eight innings.

$$58, 76, 40, 35, 46, 45, 0, 100$$

Find the mean score.

4.2.4 Generate Table 4.2.4 using a C program

Player	**Game 1**	**Game 2**	**Game 3**	**Game 4**
A	14	16	10	10
B	0	8	16	4
C	8	11	Did not play	13

TABLE 4.2.4

and answer the following questions.

a) Find the mean to determine $A's$ average number of points scored per game.
b) Who is the best performer?

4.2.5 The marks out of 100 obtained by a group of students in a science test are 85, 76, 90, 85, 39, 48, 56, 95, 81 and 75. Find the

a) Highest and lowest marks obtained by the students.
b) Range of marks obtained.
c) Mean marks obtained by the group.

4.2.6 The enrolment in a school during six consecutive years was as follows

$$1555, 1670, 1750, 2013, 2540, 2820$$

Find the mean enrolment of the school for this period.

4.2.7 The rainfall (in mm) in a city on 7 days a week was recorded as in Table 4.2.7. Generate this table using a C program.

Day	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
Rainfall (in mm)	0	12	2.1	0	21	5.5	1

TABLE 4.2.7

4.2.8 Find the range of the rainfall in the given data.

4.2.9 Find the mean rainfall for the week.

4.2.10 On how many days was the rainfall less than the mean rainfall

4.2.11 The height of 10 girls was measured in cm and result was as follows

$$135, 150, 139, 128, 151, 132, 146, 149, 143, 141.$$

a) What is the height of the tallest girl?
b) What is the height of the shortest girl?
c) What is the range of the data?
d) What is the mean height of the girls?
e) How many girls have heights more than the mean height?

4.2.12 To find out the weekly demand for different sizes of shirt, a shopkeeper kept records of sales of sizes as shown in Table 4.2.12. This is the record for a week. Find the mode of the data.

Size (in inches)	**90**	**95**	**100**	**105**	**110**	**Total**
Number of shirts sold	8	22	32	37	6	105

TABLE 4.2.12

4.2.13 Find the mode of the given set of numbers

$$1, 1, 1, 2, 2, 2, 2, 3, 4, 4$$

.

Find the mode of

4.2.14

$$2, 6, 5, 3, 0, 4, 3, 2, 4, 5, 2, 4$$

4.2.15

$$2, 4, 16, 12, 14, 14, 16, 14, 10, 14, 18, 14$$

4.2.16

$$2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 8$$

4.2.17

$$12, 14, 12, 16, 15, 13, 14, 18, 19, 12, 14, 15, 16, 15, 16, 16,$$
$$15, 17, 13, 16, 16, 15, 15, 13, 15, 17, 15, 14, 15, 13, 15, 14$$

4.2.18 Heights (in cm) of 25 children given below

$$168, 165, 163, 160, 163, 161, 162, 164, 163, 162, 164, 163, 160,$$
$$163, 160, 165, 163, 162, 163, 164, 163, 160, 165, 163, 162$$

What is the mode of their heights? What do we understand by mode here?

4.2.19 Your friend found the median and the mode of a given data. Describe and correct your friends error if any

$$35, 32, 35, 42, 38, 32, 34$$

Median = 42, Mode = 32.

4.2.20 Find the median of the data: 24, 36, 46, 17, 18, 25, 35.

4.2.21 The scores in mathematics test (out of 25) of 15 students is as follows

$$19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20$$

Find the mode and median of this data. Are they same?

4.2.22 The runs scored in a cricket match by 11 players is as follows

$$6, 15, 120, 50, 100, 80, 10, 15, 8, 10, 15$$

Find the mean, mode and median of this data. Are the three same?

4.2.23 The weights (in kg.) of 15 students of a class are

$$38, 42, 35, 37, 45, 50, 32, 43, 43, 40, 36, 38, 43, 38, 47$$

a) Find the mode and median of this data.
b) Is there more than one mode?

4.2.24 Find the mode and median of the data

$$13, 16, 12, 14, 19, 12, 14, 13, 14$$

4.2.25 The data

$$6, 4, 3, 8, 9, 12, 13, 9$$

has mean 9. True or False?

4.2.26 Two hundred students of 6th and 7th classes were asked to name their favourite colour so as to decide upon what should be the colour of their school building. The results are shown in the following table in Table 4.2.26. Represent the given data on a bar graph

Favourite Colour	**Red**	**Green**	**Blue**	**Yellow**	**Orange**
Number of Students	43	19	55	49	34

TABLE 4.2.26

Answer the following questions with the help of the bar graph

a) Which is the most preferred colour and which is the least preferred?
b) How many colours are there in all? What are they?

4.2.27 Following data in Table 4.2.27 gives total marks (out of 600) obtained by six children of a particular class. Represent the data on a bar graph.

Students	Ajay	Bali	Dipti	Falgun	Geetika	Hari
Marks Obtained	450	500	300	360	400	540

TABLE 4.2.27

4.2.28 Consider the following two collections of data in Table 4.2.28 giving the average daily hours of sunshine in two cities Aberdeen and Margate for all the twelve months of the year. These cities are near the south pole and hence have only a few hours of sunshine each day. In a particular month, which city has more sunshine hours? Explain through a double bar graph.

In Margate												
Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**July**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Average Hours of Sunshine	2	3.5	4	4	7.75	8	8	7	7	6	4	2
In Aberdeen												
Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**July**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Average Hours of Sunshine	1.5	3	3.5	6	5.5	6.5	6	5	5	4	3	2

TABLE 4.2.28

4.2.29 A mathematics teacher wants to see, whether the new technique of teaching she applied after quarterly test was effective or not. She takes the scores of the 5 weakest children in the quarterly test (out of 25) and in the half yearly test (out of 25) which are listed in Table 4.2.29. Is her technique effective?

Students	Ashish	Arun	Kavish	Maya	Ritu
Quarterly	10	15	12	20	9
Half Yearly	15	18	16	21	15

TABLE 4.2.29

4.2.30 Number of children in six different classes are given below in Table 4.2.30. Represent the data on a bar graph.

Class	Fifth	Sixth	Seventh	Eighth	Ninth	Tenth
Number of Children	135	120	95	100	90	80

TABLE 4.2.30

a) How would you choose a scale?
b) Which class has the maximum number of children? And the minimum?
c) Find the ratio of students of class sixth to the students of class eight.

4.2.31 The performance of a student in 1st Term and 2nd Term is given in Table 4.2.31. Draw a double bar graph choosing appropriate scale and answer the following

Subject	English	Hindi	Maths	Science	Social
First Term (MM 100)	67	72	88	81	73
Second Term (MM 100)	70	65	95	85	75

TABLE 4.2.31

a) In which subject, has the child improved his performance the most?
b) In which subject is the improvement the least?
c) Has the performance gone down in any subject?

4.2.32 Consider this data in Table 4.2.32 collected from a survey of a colony.

Favourite Sport	Cricket	Basketball	Swimming	Hockey	Athletics
Watching	1240	470	510	430	250
Participating	620	320	320	250	105

TABLE 4.2.32

a) Draw a double bar graph choosing an appropriate scale. What do you infer from the bar graph?
b) Which sport is most popular?
c) Which is more preferred, watching or participating in sports?

Write the following rational numbers in ascending order

4.2.33 $\frac{-3}{5}, \frac{-2}{5}, \frac{-1}{5}$ 4.2.34 $\frac{-1}{3}, \frac{-2}{9}, \frac{-4}{3}$ 4.2.35 $\frac{-3}{7}, \frac{-3}{2}, \frac{-3}{4}$

4.2.36 Find

a) 2.7×4 b) 1.8×1.2 c) 2.3×4.35

and arrange the products in descending order.

5 Math Library

5.1 $\triangle ABC$ is right-angled at C. If $AC = 5cm$ and $BC = 12cm$ find the length of AB.
Solution:

```
//Code by GVV Sharma
//May 9, 2025
//Using the sqrt to find
//the hypotenuse
#include <stdio.h>
#include <math.h>

int main(void)
{
int ac = 5, bc = 12;//sides
//hypotenuse
float ab = sqrt(pow(ac,2)+pow(bc,2));
printf("%f\n",ab);
        return 0;
}
```

5.2 Determine whether the triangle whose lengths of sides are $3cm, 4cm, 5cm$ is a right-angled triangle.
5.3 PQR is a triangle, right-angled at P. If $PQ = 10cm$ and $PR = 24cm$, find QR.
5.4 ABC is a triangle, right-angled at C. If $AB = 25cm$ and $AC = 7cm$, find BC.
5.5 A $15m$ long ladder reached a window $12m$ high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.
5.6 Which of the following can be the sides of a right triangle?

a) $2.5cm, 6.5cm, 6cm$.
b) $2cm, 2cm, 5cm$.
c) $1.5cm, 2cm, 2.5cm$.

5.7 A tree is broken at a height of $5m$ from the ground and its top touches the ground at a distance of $12m$ from the base of the tree. Find the original height of the tree.
5.8 Find the perimeter of the rectangle whose length is $40cm$ and a diagonal is $41cm$.
5.9 The diagonals of a rhombus measure $16cm$ and $30cm$. Find its perimeter.
5.10 Find the values of the following expressions for $x = 2$.

a) $19 - 5x^2$
b) $100–10x^3$

5.11 Find the value of the following expressions when $n = –2$.

a) $5n^2 + 5n–2$
b) $n^3 + 5n^2 + 5n–2$

5.12 Find the value of the following expressions for $a = 3, b = 2$.

a) $a^2 + 2ab + b^2$
b) $a^3 – b^3$

5.13 If $m = 2$, find the value of

a) $3m^2 – 2m – 7$
b) $\frac{5m^4}{2}$

5.14 If $p = –2$, find the value of

a) $–3p^2 + 4p + 7$
b) $–2p^3 – 3p^2 + 4p + 7$

5.15 Find the value of the following expressions, when $x = –1$

a) $2x^2 – x – 2$
b) $x^2 + 2x + 1$

5.16 If $a = 2, b = –2$, find the value of

a) $a^2 + b^2$
b) $a^2 + ab + b^2$
c) $a^2 – b^2$

5.17 When $a = 0, b = –1$, find the value of the given expressions

a) $2a^2 + b^2 + 1$
b) $2a^2b + 2ab^2 + ab$
c) $a^2 + ab + 2$

5.18 If $z = 10$, find the value of $z^3 – 3(z – 10)$.

5.19 If $p = –10$, find the value of $p^2 – 2p – 100$

5.20 Simplify the expression and find its value when $a = 5$ and $b = –3$

$$2a^2 + ab + 3$$

5.21 What is the circumference of a circle of diameter 10 cm?

Solution:

```c
//Code by GVV Sharma
//May 9, 2025
//Using pi to find
//the circumference
#include <stdio.h>
#include <math.h>

int main(void)
{
int d = 10;//diameter
//circumference
float circum = M_PI*d;
printf("%f\n",circum);
        return 0;
}
```

5.22 What is the circumference of a circular disc of radius 14 cm?

5.23 The radius of a circular pipe is 10 cm. What length of a tape is required to wrap once around the pipe?

5.24 Sudhanshu divides a circular disc of radius 7 cm in two equal parts. What is the perimeter of each semicircular shape disc?

5.25 Find the area of a circle of radius 30 cm?

5.26 Diameter of a circular garden is 9.8 m. Find its area.

5.27 Find the circumference of the circles with the following radius

a) 28 mm
b) 14 cm

5.28 Find the area of the following circles, given that the radius is

a) 14 mm
b) 5 cm
c) 21 cm
d) diameter = 49 m

5.29 If the circumference of a circular sheet is 154 m, find its radius. Also find the area of the sheet.

5.30 A gardener wants to fence a circular garden of diameter 21m. Find the length of the rope he needs to purchase, if he makes 2 rounds of fence. Also find the cost of the rope, if it costs ₹4 per meter.

5.31 From a circular sheet of radius 4 cm, a circle of radius 3 cm is removed. Find the area of the remaining sheet.

5.32 Seema wants to put a lace on the edge of a circular table cover of diameter 1.5 m. Find the length of the lace required and also find its cost if one meter of the lace costs ₹15.

5.33 Find the cost of polishing a circular table-top of diameter 1.6 m, if the rate of polishing is ₹$15/m^2$.

5.34 Shalya took a wire of length 44 cm and bent it into the shape of a circle. Find the radius of that circle. Also find its area. If the same wire is bent into the shape of a square, what will be the length of each of its sides? Which figure encloses more area, the circle or the square?

5.35 From a circular card sheet of radius 14 cm, two circles of radius 3.5 cm and a rectangle of length 3 cm and breadth 1cm are removed. Find the area of the remaining sheet.

5.36 A circle of radius 2 cm is cut out from a square piece of an aluminium sheet of side 6 cm. What is the area of the left over aluminium sheet?

5.37 The circumference of a circle is 31.4 cm. Find the radius and the area of the circle.

5.38 A circular flower bed is surrounded by a path 4 m wide. The diameter of the flower bed is 66 m. What is the area of this path?

5.39 A circular flower garden has an area of $314m^2$. A sprinkler at the centre of the garden can cover an area that has a radius of 12 m. Will the sprinkler water the entire garden?

5.40 How many times a wheel of radius 28 cm must rotate to go 352 m?

5.41 The minute hand of a circular clock is 15 cm. How far does the tip of the minute hand move in 1 hour?

5.42 The two sides of the parallelogram ABCD are 6 cm and 4 cm. The height corresponding to the base CD is 3 cm. Find the

a) area of the parallelogram.
b) the height corresponding to the base AD.

Find the value of

5.3 2×10^3

5.4 $7^2 \times 2^2$

5.5 $4^3 \times 2^3$

5.6 $5^6 \times (-2)^6$

5.7 $(-2)^4 \times -3^4$

5.8 $2^3 \times 5$

5.9 3×4^4

5.10 $5^2 \times 3^3$

5.11 $2^4 \times 3^2$

5.12 $3^2 \times 10^4$

5.13 $(-3) \times -2^3$

5.14 $(-3)^2 \times -5^2$

5.15 $-2^3 \times -10^3$

5.16 $4^5 \div 3^5$

5.17 $5^6 \div (-2)^6$

5.18 $\left(\frac{3^7}{3^2}\right) \times 3^5$

5.19 $2^3 \times 2^2 \times 5^5$

5.20 $6^2 \times 6^4 \div 6^3$

5.21 $8^2 \div 2^3$

5.22 $\left(2^2\right)^3 \times 3^6 \times 5^6$

5.23 $\frac{(12)^4 \times 9^3 \times 4}{(6)^3 \times 8^2 \times 27}$

5.24 $\frac{(2) \times 3^4 \times 2^5}{9 \times 4^2}$

5.25 $3^2 \times 3^4 \times 3^8$

5.26 $6^{15} \div 6^{10}$

5.27 $\left(5^2\right)^3 \div 5^3$

5.28 $2^5 \times 5^5$

5.29 $\left(3^4\right)^3$

5.30 $\left(2^{20} \div 2^{15}\right) \times 2^3$

5.31 $\frac{(2)^3 \times 3^4 \times 4}{3 \times 32}$

5.32 $\left(5^{2^3} \times 5^2\right) \div 5^7$

5.33 $25^4 \div 5^3$

5.34 $\frac{(3) \times 7^2 \times 11^8}{21 \times 11^3}$

5.35 $\frac{(3)^7}{3^4 \times 3^3}$

5.36 $\left(2^3 \times 2\right)^2$

5.37 $\frac{\left(2^5\right)^2 \times 7^3}{8^3 \times 7}$

5.38 $\frac{(3)^5 \times 10^5 \times 25}{(5)^7 \times 6^5}$

Find the logarithms

5.39 512 base 2

Solution:

```
//Code by GVV Sharma
//May 9, 2025
//Finding the logarithm
#include <stdio.h>
#include <math.h>

int main(void)
{
int num = 512, base=2;//number and base
//finding the logarithm
float lognum= log(512)/log(2);
printf("%f\n",lognum);
        return 0;
}
```

5.40 256 base 2

5.41 343 base 7

5.42 729 base 3

5.43 3125 base 5

Identify the greater number, wherever possible, in each of the following

5.44 4^3 or 3^4
5.45 5^3 or 3^5
5.46 100^2 or 2^{100}
5.47 2^{10} or 10^2
5.48 2^3 or 3^2
5.49 8^2 or 2^8
5.50 2.7×10^{12} or 1.5×10^8
5.51 4×10^{14} or 3×10^{17}

Express each of the following as product of powers of their prime factors

5.52 648
5.53 405
5.54 540
5.55 3600
5.56 72
5.57 432
5.58 1000
5.59 16000
5.60 270
5.61 768
5.62 108×192
5.63 629×65

Use ifelse for the following

5.64 $10 \times 10^{11} = 100^{11}$
5.65 $2^3 > 5^2$
5.66 $2^3 \times 3^2 = 6^5$
5.67 $3^0 = 1000^0$

6 Random Numbers

6.1 Take a board marked from -104 to 104 as shown in Table 6.1.

104	103	102	101	100	99	98	97	96	95	94
83	84	85	86	87	88	89	90	91	92	93
82	81	80	79	78	77	76	75	74	73	72
61	62	63	64	65	66	67	68	69	70	71
60	59	58	57	56	55	54	53	52	51	50
39	40	41	42	43	44	45	46	47	48	49
38	37	36	35	34	33	32	31	30	29	28
17	18	19	20	21	22	23	24	25	26	27
16	15	14	13	12	11	10	9	8	7	6
-5	-4	-3	-2	-1	0	1	2	3	4	5
-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16
-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17
-28	-29	-30	-31	-32	-33	-34	-35	-36	-37	-38
-49	-48	-47	-46	-45	-44	-43	-42	-41	-40	-39
-50	-51	-52	-53	-54	-55	-56	-57	-58	-59	-60
-71	-70	-69	-68	-67	-66	-65	-64	-63	-62	-61
-72	-73	-74	-75	-76	-77	-78	-79	-80	-81	-82
-93	-92	-91	-90	-89	-88	-87	-86	-85	-84	-83
-94	-95	-96	-97	-98	-99	-100	-101	-102	-103	-104

TABLE 6.1

6.2 Take a bag containing two blue and two red dice. Number of dots on the blue dice indicate positive integers and number of dots on the red dice indicate negative integers.
6.3 Every player will place his/her counter at zero.
6.4 Each player will take out two dice at a time from the bag and throw them.
6.5 After every throw, the player has to multiply the numbers marked on the dice.
6.6 If the product is a positive integer then the player will move his counter towards 104; if the product is a negative integer then the player will move his counter towards -104.
6.7 The player who reaches either -104 or 104 first is the winner.
6.8 Write a program to simulate the game. Give the inputs manually.
Solution:

```c
//Code by GVV Sharma
//May 9, 2025
//Simulating the board game
//using two dice
#include <stdio.h>
#include <stdlib.h>

int main(void)
{
int num1,num2,score1=0;//numbers on the dice
int score2=0,temp;//numbers on the dice
while(1){
label1: printf("\n\nPlayer 1 inputs\n");
printf("\n Die 1:");
scanf("%d",&num1);
if(num1< -6 || num1 > 6 || num1 == 0){
        printf("Wrong input\n");
        goto label1;
}
label2: printf("\n Die 2:");
scanf("%d",&num2);
if(num2< -6 || num2 > 6|| num2 == 0){
        printf("Wrong input\n");
        goto label2;
}
score1=(num1*num2 > 0)?score1+1:score1-1;
printf("Player 1 score is %d",score1);
if(score1 == 104 || score1 == -104){
        printf("Player 1 wins\n");
        exit(0);
}
label3:printf("\n\nPlayer 2 inputs\n");
```

```
printf("\n Die 1:");
scanf("%d",&num1);
if(num1< −6 || num1 > 6|| num1 == 0){
        printf("Wrong input\n");
        goto label3;
}
label4: printf("\n Die 2:");
scanf("%d",&num2);
if(num2< −6 || num2 > 6|| num2 == 0){
        printf("Wrong input\n");
        goto label4;
}
score2=(num1∗num2 > 0)?score2+1:score2−1;
printf("Player 2 score is %d",score2);
if(score2 == 104 || score2 == −104){
        printf("Player 2 wins\n");
        exit(1);
}
                }
        return 0;
}
```

6.9 Revise the program by replacing the second player with the computer. The computer generates the inputs randomly as follows

a) Generate the numbers on all the dice using a uniform distribution ranging from 1 to 6.
b) Simulate the blue and red dice through a Bernoulli distribution having values 1 and -1.

Solution:

```
//Code by GVV Sharma
//May 10, 2025
//Simulating the board game
//using two dice
//player vs computer
//computer generates values
//using uniform and bernoulli
//random variables
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int dice();
int bernoulli();
int main(void)
```

```
{
        srand(time(0));
int num1,num2,score1=0;//numbers on the dice
int score2=0,temp;//numbers on the dice
while(1){
label1: printf("\n\nPlayer 1 inputs\n");
printf("\n Die 1:");
scanf("%d",&num1);
if(num1< -6 || num1 > 6 || num1 == 0){
        printf("Wrong input\n");
        goto label1;
}
label2: printf("\n Die 2:");
scanf("%d",&num2);
if(num2< -6 || num2 > 6|| num2 == 0){
        printf("Wrong input\n");
        goto label2;
}
score1=(num1*num2 > 0)?score1+1:score1-1;
printf("Player 1 score is %d",score1);
if(score1 == 104 || score1 == -104){
        printf("Player 1 wins\n");
        exit(0);
}
printf("\n\nComputer inputs\n");
num1 = dice()*bernoulli();
printf("\n Die 1:%d",num1);
num2 = dice()*bernoulli();
printf("\n Die 2:%d",num2);
score2=(num1*num2 > 0)?score2+1:score2-1;
printf("\nComputer score is %d",score2);
if(score2 == 104 || score2 == -104){
        printf("Computer wins\n");
        exit(1);
}
                }
        return 0;
}
//Defining the function for generating uniform random numbers
//between 1 to 6
int dice()
{
return rand()%6+1;
}
//End function for generating uniform random numbers
```

```
int bernoulli(){
return (rand() % 2 == 0) ? -1 : 1;
}
```

6.10 Now revise the program so that both players are simulated by the computer.

Appendix A
Triangle

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{10.1}$$

A.1 Sides

A.1.1. The direction vector of AB is defined as

$$\mathbf{B} - \mathbf{A} \tag{A.1.1.1}$$

Find the direction vectors of AB, BC and CA.

Solution:

a) The Direction vector of AB is

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 - 1 \\ 6 - (-1) \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{A.1.1.2}$$

b) The Direction vector of BC is

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 - (-4) \\ -5 - 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -11 \end{pmatrix} \tag{A.1.1.3}$$

c) The Direction vector of CA is

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} = \begin{pmatrix} 1 - (-3) \\ -1 - (-5) \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{A.1.1.4}$$

A.1.2. The length of side BC is

$$c = \|\mathbf{B} - \mathbf{A}\| \triangleq \sqrt{(\mathbf{B} - \mathbf{A})^{\top} (\mathbf{B} - \mathbf{A})} \tag{A.1.2.1}$$

where

$$\mathbf{A}^{\top} \triangleq \begin{pmatrix} 1 & -1 \end{pmatrix} \tag{A.1.2.2}$$

Similarly,

$$b = \|\mathbf{C} - \mathbf{B}\|, \; a = \|\mathbf{A} - \mathbf{C}\| \tag{A.1.2.3}$$

Find a, b, c.

a) From (A.1.1.2),

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -7 \end{pmatrix}, \tag{A.1.2.4}$$

$$\implies c = \|\mathbf{B} - \mathbf{A}\| = \|\mathbf{A} - \mathbf{B}\| \tag{A.1.2.5}$$

$$= \sqrt{\begin{pmatrix} 5 & -7 \end{pmatrix} \begin{pmatrix} 5 \\ -7 \end{pmatrix}} = \sqrt{(5)^2 + (7)^2} \tag{A.1.2.6}$$

$$= \sqrt{74} \tag{A.1.2.7}$$

b) Similarly, from (A.1.1.3),

$$a = \|\mathbf{B} - \mathbf{C}\| = \sqrt{\begin{pmatrix}-1 & 11\end{pmatrix}\begin{pmatrix}-1\\11\end{pmatrix}} \tag{A.1.2.8}$$

$$= \sqrt{(1)^2 + (11)^2} = \sqrt{122} \tag{A.1.2.9}$$

and from (A.1.1.4),

c)

$$b = \|\mathbf{A} - \mathbf{C}\| = \sqrt{\begin{pmatrix}4 & 4\end{pmatrix}\begin{pmatrix}4\\4\end{pmatrix}} \tag{A.1.2.10}$$

$$= \sqrt{(4)^2 + (4)^2} = \sqrt{32} \tag{A.1.2.11}$$

A.1.3. The parametric form of the equation of AB is

$$\mathbf{x} = \mathbf{A} + k\mathbf{m} \quad k \neq 0, \tag{A.1.3.1}$$

where

$$\mathbf{m} = \mathbf{B} - \mathbf{A} \tag{A.1.3.2}$$

is the direction vector of AB. Find the parameteric equations of AB, BC and CA.
Solution:

a) From (A.1.3.1) and (A.1.1.2), the parametric equation for AB is given by

$$AB : \mathbf{x} = \begin{pmatrix}1\\-1\end{pmatrix} + k\begin{pmatrix}-5\\7\end{pmatrix} \tag{A.1.3.3}$$

Similarly, from (A.1.1.3) and (A.1.1.4),

$$BC : \mathbf{x} = \begin{pmatrix}-4\\6\end{pmatrix} + k\begin{pmatrix}1\\-11\end{pmatrix} \tag{A.1.3.4}$$

$$CA : \mathbf{x} = \begin{pmatrix}-3\\-5\end{pmatrix} + k\begin{pmatrix}4\\4\end{pmatrix} \tag{A.1.3.5}$$

b) The normal form of the equation of AB is

$$\mathbf{n}^{\top}(\mathbf{x} - \mathbf{A}) = 0 \tag{A.1.3.6}$$

where

$$\mathbf{n}^{\top}\mathbf{m} = \mathbf{n}^{\top}(\mathbf{B} - \mathbf{A}) = 0 \tag{A.1.3.7}$$

$$\text{or, } \mathbf{n} = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\mathbf{m} \tag{A.1.3.8}$$

Find the normal form of the equations of AB, BC and CA.
Solution:

i) From (A.1.1.3), the direction vector of side **BC** is

$$\mathbf{m} = \begin{pmatrix} 1 \\ -11 \end{pmatrix} \tag{A.1.3.9}$$

$$\implies \mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -11 \end{pmatrix} = \begin{pmatrix} -11 \\ -1 \end{pmatrix} \tag{A.1.3.10}$$

from (A.1.3.8). Hence, from (A.1.3.6), the normal equation of side BC is

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{B}) = 0 \tag{A.1.3.11}$$

$$\implies \begin{pmatrix} -11 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -11 & -1 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{A.1.3.12}$$

$$\implies BC : \quad \begin{pmatrix} 11 & 1 \end{pmatrix} \mathbf{x} = -38 \tag{A.1.3.13}$$

ii) Similarly, for AB, from (A.1.1.2),

$$\mathbf{m} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{A.1.3.14}$$

$$\implies \mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \tag{A.1.3.15}$$

and

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \tag{A.1.3.16}$$

$$\implies AB : \quad \mathbf{n}^\top \mathbf{x} = \begin{pmatrix} 7 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{A.1.3.17}$$

$$\implies \begin{pmatrix} 7 & 5 \end{pmatrix} \mathbf{x} = 2 \tag{A.1.3.18}$$

iii) For CA, from (A.1.1.4),

$$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{A.1.3.19}$$

$$\implies \mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{A.1.3.20}$$

$$\tag{A.1.3.21}$$

$$\implies \mathbf{n}^\top (\mathbf{x} - \mathbf{C}) = 0 \tag{A.1.3.22}$$

$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} = 2 \tag{A.1.3.23}$$

codes/msoft/sides.c

c) Find the angles A, B, C if

$$\cos A \triangleq \frac{(\mathbf{B} - \mathbf{A})^\top \mathbf{C} - \mathbf{A}}{\|\mathbf{B} - \mathbf{A}\| \, \|\mathbf{C} - \mathbf{A}\|} \tag{A.1.3.24}$$

Solution:

i) From (A.1.1.2), (A.1.1.4), (A.1.2.7) and (A.1.2.11)

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}-5 & 7\end{pmatrix}\begin{pmatrix}-4\\-4\end{pmatrix} \tag{A.1.3.25}$$

$$= -8 \tag{A.1.3.26}$$

$$\implies \cos A = \frac{-8}{\sqrt{74}\sqrt{32}} = \frac{-1}{\sqrt{37}} \tag{A.1.3.27}$$

$$\implies A = \cos^{-1}\frac{-1}{\sqrt{37}} \tag{A.1.3.28}$$

ii) From (A.1.1.2), (A.1.1.3), (A.1.2.7) and (A.1.2.9)

$$(\mathbf{C}-\mathbf{B})^\top(\mathbf{A}-\mathbf{B}) = \begin{pmatrix}1 & -11\end{pmatrix}\begin{pmatrix}5\\-7\end{pmatrix} \tag{A.1.3.29}$$

$$= 82 \tag{A.1.3.30}$$

$$\implies \cos B = \frac{82}{\sqrt{74}\sqrt{122}} = \frac{41}{\sqrt{2257}} \tag{A.1.3.31}$$

$$\implies B = \cos^{-1}\frac{41}{\sqrt{2257}} \tag{A.1.3.32}$$

iii) From (A.1.1.3), (A.1.1.4), (A.1.2.9) and (A.1.2.11)

$$(\mathbf{A}-\mathbf{C})^\top(\mathbf{B}-\mathbf{C}) = \begin{pmatrix}4 & 4\end{pmatrix}\begin{pmatrix}-1\\11\end{pmatrix} \tag{A.1.3.33}$$

$$= 40 \tag{A.1.3.34}$$

$$\implies \cos C = \frac{40}{\sqrt{32}\sqrt{122}} = \frac{5}{\sqrt{61}} \tag{A.1.3.35}$$

$$\implies C = \cos^{-1}\frac{5}{\sqrt{61}} \tag{A.1.3.36}$$

```
codes/msoft/ang.c
```

A.1.4. The area of $\triangle ABC$ is defined as

$$\frac{1}{2}\left\|(\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{C})\right\| \tag{A.1.4.1}$$

where

$$\mathbf{A}\times\mathbf{B} \triangleq \begin{vmatrix}1 & -4\\-1 & 6\end{vmatrix} \tag{A.1.4.2}$$

Find the area of $\triangle ABC$.

Solution: From (A.1.1.2) and (A.1.1.4),

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix} 5 \\ -7 \end{pmatrix}, \mathbf{A}-\mathbf{C} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{A.1.4.3}$$

$$\implies (\mathbf{A}-\mathbf{B}) \times (\mathbf{A}-\mathbf{C}) = \begin{vmatrix} 5 & 4 \\ -7 & 4 \end{vmatrix} \tag{A.1.4.4}$$

$$= 5 \times 4 - 4 \times (-7) \tag{A.1.4.5}$$

$$= 48 \tag{A.1.4.6}$$

$$\implies \frac{1}{2} \left\|(\mathbf{A}-\mathbf{B}) \times (\mathbf{A}-\mathbf{C})\right\| = \frac{48}{2} = 24 \tag{A.1.4.7}$$

which is the desired area.

```
codes/msoft/area.c
```

A.2 Formulae

A.1. The equation of a line is given by

$$y = mx + c \tag{A.1.1}$$

$$\implies \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix} + x \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{A.1.2}$$

yielding (**??**).

A.2. (A.1.1) can also be expressed as

$$y - mx = c \tag{A.2.1}$$

$$\implies \begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = c \tag{A.2.2}$$

yielding (**??**).

A.3. The direction vector is

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{A.3.1}$$

and the normal vector is

$$\mathbf{n} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \tag{A.3.2}$$

A.4. From (**??**), if $\mathbf{A}, \mathbf{D}$ and $\mathbf{C}$ are on the same line,

$$\mathbf{D} = \mathbf{A} + q\mathbf{m} \tag{A.4.1}$$

$$\mathbf{C} = \mathbf{D} + p\mathbf{m} \tag{A.4.2}$$

$$\implies p(\mathbf{D}-\mathbf{A}) + q(\mathbf{D}-\mathbf{C}) = 0, \quad p, q \neq 0 \tag{A.4.3}$$

$$\implies \mathbf{D} = \frac{p\mathbf{A} + q\mathbf{C}}{p+q} \tag{A.4.4}$$

yielding (??) upon substituting

$$k = \frac{p}{q}. \tag{A.4.5}$$

$(\mathbf{D} - \mathbf{A}), (\mathbf{D} - \mathbf{C})$ are then said to be *linearly dependent*.

A.5. If $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are collinear, from (??),

$$\mathbf{n}^\top \mathbf{A} = c \tag{A.5.1}$$

$$\mathbf{n}^\top \mathbf{B} = c \tag{A.5.2}$$

$$\mathbf{n}^\top \mathbf{C} = c \tag{A.5.3}$$

which can be expressed as

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}^\top \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{A.5.4}$$

$$\equiv \begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \tag{A.5.5}$$

$$\implies \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}^\top \begin{pmatrix} \mathbf{n} \\ -1 \end{pmatrix} = \mathbf{0} \tag{A.5.6}$$

yielding

$$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix} = 2 \tag{A.5.7}$$

Rank is defined to be the number of linearly indpendent rows or columns of a matrix.

A.6. The equation of a line can also be expressed as

$$\mathbf{n}^\top \mathbf{x} = 1 \tag{A.6.1}$$

A.3 Median

A.3.1. If $\mathbf{D}$ divides BC in the ratio $k : 1$,

$$\mathbf{D} = \frac{k\mathbf{C} + \mathbf{B}}{k + 1} \tag{A.3.1.1}$$

Find the mid points $\mathbf{D}, \mathbf{E}, \mathbf{F}$ of the sides BC, CA and AB respectively.
Solution: Since $\mathbf{D}$ is the midpoint of BC,

$$k = 1, \tag{A.3.1.2}$$

$$\implies \mathbf{D} = \frac{\mathbf{C} + \mathbf{B}}{2} = \frac{1}{2} \begin{pmatrix} -7 \\ 1 \end{pmatrix} \tag{A.3.1.3}$$

Similarly,

$$\mathbf{E} = \frac{\mathbf{A}+\mathbf{C}}{2} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \tag{A.3.1.4}$$

$$\mathbf{F} = \frac{\mathbf{A}+\mathbf{B}}{2} = \frac{1}{2}\begin{pmatrix} -3 \\ 5 \end{pmatrix} \tag{A.3.1.5}$$

A.3.2. Find the equations of AD, BE and CF.

Solution: :

a) The direction vector of AD is

$$\mathbf{m} = \mathbf{D} - \mathbf{A} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -9 \\ 3 \end{pmatrix} \equiv \begin{pmatrix} -3 \\ 1 \end{pmatrix} \tag{A.3.2.1}$$

$$\implies \mathbf{n} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{A.3.2.2}$$

Hence the normal equation of median AD is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{A}\right) = 0 \tag{A.3.2.3}$$

$$\implies \begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -2 \tag{A.3.2.4}$$

b) For BE,

$$\mathbf{m} = \mathbf{E} - \mathbf{B} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \end{pmatrix} \equiv \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{A.3.2.5}$$

$$\implies \mathbf{n} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{A.3.2.6}$$

Hence the normal equation of median BE is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{B}\right) = 0 \tag{A.3.2.7}$$

$$\implies \begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} = -6 \tag{A.3.2.8}$$

c) For median CF,

$$\mathbf{m} = \mathbf{F} - \mathbf{C} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \equiv \begin{pmatrix} 1 \\ 5 \end{pmatrix} \tag{A.3.2.9}$$

$$\implies \mathbf{n} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \tag{A.3.2.10}$$

Hence the normal equation of median CF is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{C}\right) = 0 \tag{A.3.2.11}$$

$$\implies \begin{pmatrix} 5 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 & -1 \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} = -10 \tag{A.3.2.12}$$

A.3.3. Find the intersection $\mathbf{G}$ of BE and CF.

Solution: From (A.3.2.8) and (A.3.2.12), the equations of BE and CF are, respectively,

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -6 \end{pmatrix} \tag{A.3.3.1}$$

$$\begin{pmatrix} 5 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -10 \end{pmatrix} \tag{A.3.3.2}$$

From (A.3.3.1) and (A.3.3.2) the augmented matrix is

$$\begin{pmatrix} 3 & 1 & -6 \\ 5 & -1 & -10 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 + R_2} \begin{pmatrix} 8 & 0 & -16 \\ 5 & -1 & -10 \end{pmatrix} \tag{A.3.3.3}$$

$$\xleftrightarrow{R_1 \leftarrow R_1/8} \begin{pmatrix} 1 & 0 & -2 \\ 5 & -1 & -10 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 5R_1} \begin{pmatrix} 1 & 0 & -2 \\ 0 & -1 & 0 \end{pmatrix} \tag{A.3.3.4}$$

$$\xleftrightarrow{R_2 \leftarrow -R_2} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix} \tag{A.3.3.5}$$

using Gauss elimination. Therefore,

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{A.3.3.6}$$

A.3.4. Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2 \tag{A.3.4.1}$$

Solution:

a) From (A.3.1.4) and (A.3.3.6),

$$\mathbf{G} - \mathbf{B} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}, \mathbf{E} - \mathbf{G} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{A.3.4.2}$$

$$\implies \mathbf{G} - \mathbf{B} = 2\left(\mathbf{E} - \mathbf{G}\right) \tag{A.3.4.3}$$

$$\implies \left\|\mathbf{G} - \mathbf{B}\right\| = 2\left\|\mathbf{E} - \mathbf{G}\right\| \tag{A.3.4.4}$$

$$\text{or, } \frac{BG}{GE} = 2 \tag{A.3.4.5}$$

b) From (A.3.1.5) and (A.3.3.6),

$$\mathbf{F} - \mathbf{G} = \frac{1}{2}\begin{pmatrix} 1 \\ 5 \end{pmatrix}, \mathbf{G} - \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \tag{A.3.4.6}$$

$$\implies \mathbf{G} - \mathbf{C} = 2\left(\mathbf{F} - \mathbf{G}\right) \tag{A.3.4.7}$$

$$\implies \left\|\mathbf{G} - \mathbf{C}\right\| = 2\left\|\mathbf{F} - \mathbf{G}\right\| \tag{A.3.4.8}$$

$$\text{or, } \frac{CG}{GF} = 2 \tag{A.3.4.9}$$

c) From (A.3.1.3) and (A.3.3.6),

$$\mathbf{G}-\mathbf{A}=\begin{pmatrix}-3\\1\end{pmatrix},\ \mathbf{D}-\mathbf{G}=\frac{1}{2}\begin{pmatrix}-3\\1\end{pmatrix} \tag{A.3.4.10}$$

$$\mathbf{G}-\mathbf{A}=2\left(\mathbf{D}-\mathbf{G}\right) \tag{A.3.4.11}$$

$$\implies \left\|\mathbf{G}-\mathbf{A}\right\|=2\left\|\mathbf{D}-\mathbf{G}\right\| \tag{A.3.4.12}$$

$$\text{or, } \frac{AG}{GD}=2 \tag{A.3.4.13}$$

From (A.3.4.5), (A.3.4.9), (A.3.4.13)

$$\frac{BG}{GE}=\frac{CG}{GF}=\frac{AG}{GD}=2 \tag{A.3.4.14}$$

A.3.5. Show that $\mathbf{A},\mathbf{G}$ and $\mathbf{D}$ are collinear.

Solution: Points $\mathbf{A},\mathbf{D},\mathbf{G}$ are defined to be collinear if

$$\text{rank}\begin{pmatrix}1 & 1 & 1\\ \mathbf{A} & \mathbf{D} & \mathbf{G}\end{pmatrix}=2 \tag{A.3.5.1}$$

$$\implies \begin{pmatrix}1 & 1 & 1\\ 1 & -\frac{7}{2} & -2\\ -1 & \frac{1}{2} & 0\end{pmatrix} \xleftrightarrow{R_3\leftarrow R_3+R_2} \begin{pmatrix}1 & 1 & 1\\ 1 & -\frac{7}{2} & -2\\ 0 & -3 & -2\end{pmatrix} \tag{A.3.5.2}$$

$$\xleftrightarrow{R_2\leftarrow R_2-R_1} \begin{pmatrix}1 & 1 & 1\\ 0 & -\frac{9}{2} & -3\\ 0 & -3 & -2\end{pmatrix} \xleftrightarrow{R_3\leftarrow R_3-\frac{2}{3}R_2} \begin{pmatrix}1 & 1 & 1\\ 0 & -\frac{9}{2} & -3\\ 0 & 0 & 0\end{pmatrix} \tag{A.3.5.3}$$

Thus, the matrix (A.3.5.1) has rank 2 and the points are collinear. Thus, the medians of a triangle meet at the point $\mathbf{G}$. See Fig. A.3.5.

A.3.6. Verify that

$$\mathbf{G}=\frac{\mathbf{A}+\mathbf{B}+\mathbf{C}}{3} \tag{A.3.6.1}$$

$\mathbf{G}$ is known as the *centroid* of $\triangle ABC$.

Solution:

$$\mathbf{G}=\frac{\begin{pmatrix}1\\-1\end{pmatrix}+\begin{pmatrix}-4\\6\end{pmatrix}+\begin{pmatrix}-3\\-5\end{pmatrix}}{3} =\begin{pmatrix}-2\\0\end{pmatrix} \tag{A.3.6.2}$$

A.3.7. Verify that

$$\mathbf{A}-\mathbf{F}=\mathbf{E}-\mathbf{D} \tag{A.3.7.1}$$

The quadrilateral $AFDE$ is defined to be a parallelogram.

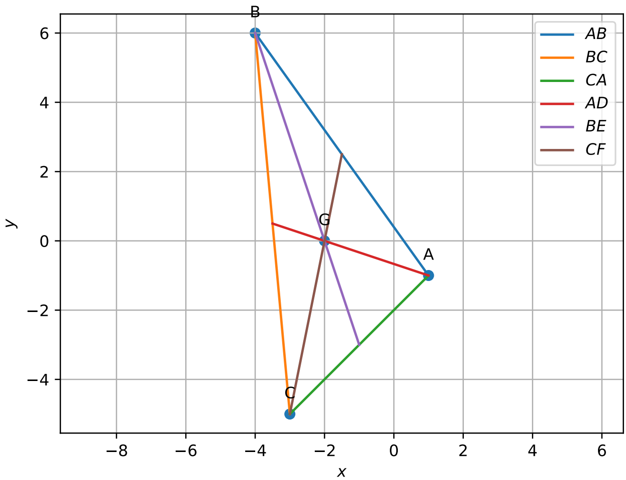


Fig. A.3.5: Medians of $\triangle ABC$ meet at $\mathbf{G}$.

Solution:

$$\mathbf{A}-\mathbf{F}=\begin{pmatrix}1\\-1\end{pmatrix}-\begin{pmatrix}\frac{-3}{2}\\\frac{5}{2}\end{pmatrix}=\begin{pmatrix}\frac{5}{2}\\\frac{-7}{2}\end{pmatrix} \tag{A.3.7.2}$$

$$\mathbf{E}-\mathbf{D}=\begin{pmatrix}-1\\-3\end{pmatrix}-\begin{pmatrix}\frac{-7}{2}\\\frac{1}{2}\end{pmatrix}=\begin{pmatrix}\frac{5}{2}\\\frac{-7}{2}\end{pmatrix} \tag{A.3.7.3}$$

$$\implies \mathbf{A}-\mathbf{F}=\mathbf{E}-\mathbf{D} \tag{A.3.7.4}$$

See Fig. A.3.7,
All codes for this section are available in

```
codes/triangle/medians.py
codes/triangle/pgm.py
```

A.4 Altitude

A.4.1. $\mathbf{D}_1$ is a point on BC such that

$$AD_1 \perp BC \tag{A.4.1.1}$$

and AD_1 is defined to be the altitude. Find the normal vector of AD_1.
Solution: The normal vector of AD_1 is the direction vector BC and is obtained from (A.1.1.3) as

$$\mathbf{n}=\begin{pmatrix}1\\-11\end{pmatrix} \tag{A.4.1.2}$$

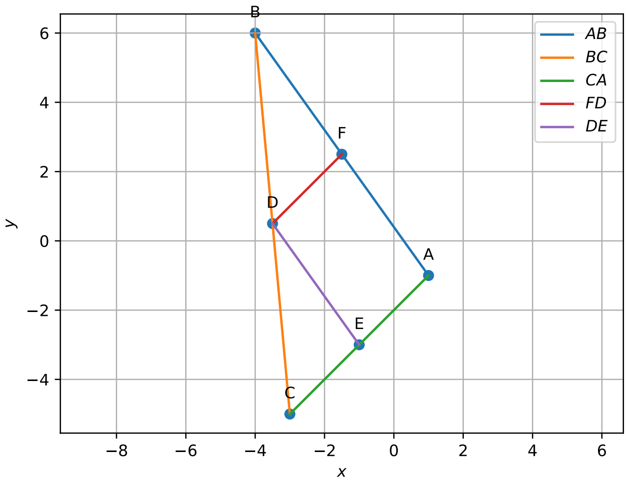


Fig. A.3.7: $AFDE$ forms a parallelogram in triangle ABC

A.4.2. Find the equation of AD_1.
Solution: The equation of AD_1 is

$$\mathbf{n}^\top(\mathbf{x} - \mathbf{A}) = 0 \tag{A.4.2.1}$$

$$\implies \begin{pmatrix} -1 & 11 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -1 & 11 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -12 \tag{A.4.2.2}$$

A.4.3. Find the equations of the altitudes BE_1 and CF_1 to the sides AC and AB respectively.

Solution:

a) From (A.1.1.4), the normal vector of CF_1 is

$$\mathbf{n} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{A.4.3.1}$$

and the equation of CF_1 is

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{C}) = 0 \tag{A.4.3.2}$$

$$\implies \begin{pmatrix} -5 & 7 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} -3 \\ -5 \end{pmatrix}\right) = 0 \tag{A.4.3.3}$$

$$\implies \begin{pmatrix} 5 & -7 \end{pmatrix} \mathbf{x} = 20, \tag{A.4.3.4}$$

b) Similarly, from (A.1.1.2), the normal vector of BE_1 is

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{A.4.3.5}$$

and the equation of BE_1 is

$$\mathbf{n}^\top(\mathbf{x}-\mathbf{B}) = 0 \tag{A.4.3.6}$$

$$\implies \begin{pmatrix}1 & 1\end{pmatrix}\left(\mathbf{x}-\begin{pmatrix}-4\\6\end{pmatrix}\right) = 0 \tag{A.4.3.7}$$

$$\implies \begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 2, \tag{A.4.3.8}$$

A.4.4. Find the intersection $\mathbf{H}$ of BE_1 and CF_1.

Solution: The intersection of (A.4.3.8) and (A.4.3.4), is obtained from the matrix equation

$$\begin{pmatrix}1 & 1\\5 & -7\end{pmatrix}\mathbf{x} = \begin{pmatrix}2\\20\end{pmatrix} \tag{A.4.4.1}$$

which can be solved as

$$\begin{pmatrix}1 & 1 & 2\\5 & -7 & 20\end{pmatrix} \xleftrightarrow{R_2\leftarrow R_2-5R_1} \begin{pmatrix}1 & 1 & 2\\0 & -12 & 10\end{pmatrix} \tag{A.4.4.2}$$

$$\xleftrightarrow{R_2\leftarrow \frac{R_2}{-12}} \begin{pmatrix}1 & 1 & 2\\0 & 1 & \frac{-5}{6}\end{pmatrix} \xleftrightarrow{R_1\leftarrow R_1-R_2} \begin{pmatrix}1 & 0 & \frac{17}{6}\\0 & 1 & \frac{-5}{6}\end{pmatrix} \tag{A.4.4.3}$$

yielding

$$\mathbf{H} = \frac{1}{6}\begin{pmatrix}17\\-5\end{pmatrix}, \tag{A.4.4.4}$$

See Fig. A.4.4.1

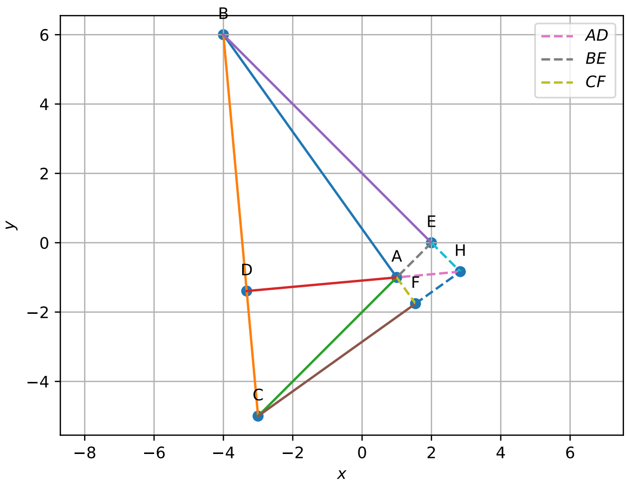


Fig. A.4.4.1: Altitudes BE_1 and CF_1 intersect at $\mathbf{H}$

A.4.5. Verify that

$$(\mathbf{A}-\mathbf{H})^{\top}(\mathbf{B}-\mathbf{C})=0 \tag{A.4.5.1}$$

Solution: From (A.4.4.4),

$$\mathbf{A}-\mathbf{H}=-\frac{1}{6}\begin{pmatrix}11\\1\end{pmatrix}, \mathbf{B}-\mathbf{C}=\begin{pmatrix}-1\\11\end{pmatrix} \tag{A.4.5.2}$$

$$\implies (\mathbf{A}-\mathbf{H})^{\top}(\mathbf{B}-\mathbf{C})=\frac{1}{6}\begin{pmatrix}11 & 1\end{pmatrix}\begin{pmatrix}-1\\11\end{pmatrix}=0 \tag{A.4.5.3}$$

A.4.6. Find the length of the altitude AD_1.
Solution: If the equation of BC be $\mathbf{n}^{\top}\mathbf{x}=c$, from (**??**),

$$\mathbf{D}_1=\mathbf{A}+k\mathbf{n} \tag{A.4.6.1}$$

$$\implies AD_1=\|\mathbf{D}_1-\mathbf{A}\|=|k|\,\|\mathbf{n}\| \tag{A.4.6.2}$$

From (A.4.6.1),

$$\mathbf{n}^{\top}\mathbf{D}_1=\mathbf{n}^{\top}\mathbf{A}+k\|\mathbf{n}\|^2 \tag{A.4.6.3}$$

$$\implies |k|=\frac{\left|\mathbf{n}^{\top}(\mathbf{D}_1-\mathbf{A})\right|}{\|\mathbf{n}\|^2} \tag{A.4.6.4}$$

$$\implies AD_1=|k|\,\|\mathbf{n}\|=\frac{\left|\mathbf{n}^{\top}\mathbf{A}-c\right|}{\|\mathbf{n}\|} \tag{A.4.6.5}$$

upon substituting from (A.4.6.2).

A.4.7. Find $\mathbf{D}_1$.
Solution: $\because$ $\mathbf{D}_1$ lies on BC,

$$\mathbf{n}^{\top}\mathbf{D}_1=c \tag{A.4.7.1}$$

and $\because AD_1 \perp BC$,

$$\mathbf{m}^{\top}(\mathbf{A}-\mathbf{D}_1)=0 \tag{A.4.7.2}$$

$$\implies \mathbf{m}^{\top}\mathbf{D}_1=\mathbf{m}^{\top}\mathbf{A} \tag{A.4.7.3}$$

Clubbing (A.4.7.1) and (A.4.7.3),

$$\begin{pmatrix}\mathbf{m} & \mathbf{n}\end{pmatrix}^{\top}\mathbf{D}_1=\begin{pmatrix}\mathbf{m}^{\top}\mathbf{A}\\c\end{pmatrix} \tag{A.4.7.4}$$

A.4.8. All codes for this section are available at

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codes/triangle/altitude.py
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A.5 Perpendicular Bisector

A.5.1. The equation of the perpendicular bisector of BC is

$$\left(\mathbf{x}-\frac{\mathbf{B}+\mathbf{C}}{2}\right)^{\top}(\mathbf{B}-\mathbf{C})=0 \tag{A.5.1.1}$$

Substitute numerical values and find the equations of the perpendicular bisectors of AB, BC and CA.

Solution: From (A.1.1.2), (A.1.1.3), (A.1.1.4), (A.3.1.3), (A.3.1.4) and (A.3.1.5),

$$\frac{\mathbf{B}+\mathbf{C}}{\mathbf{2}} = \frac{1}{2}\begin{pmatrix}-7\\1\end{pmatrix}, \mathbf{B}-\mathbf{C} = \begin{pmatrix}-1\\11\end{pmatrix} \tag{A.5.1.2}$$

$$\frac{\mathbf{A}+\mathbf{B}}{\mathbf{2}} = \frac{1}{2}\begin{pmatrix}-3\\5\end{pmatrix}, \mathbf{A}-\mathbf{B} = \begin{pmatrix}5\\-7\end{pmatrix} \tag{A.5.1.3}$$

$$\frac{\mathbf{C}+\mathbf{A}}{\mathbf{2}} = \begin{pmatrix}-1\\-3\end{pmatrix}, \mathbf{C}-\mathbf{A} = \begin{pmatrix}-4\\-4\end{pmatrix} \tag{A.5.1.4}$$

yielding

$$(\mathbf{B}-\mathbf{C})^\top\left(\frac{\mathbf{B}+\mathbf{C}}{2}\right) = \begin{pmatrix}-1 & 11\end{pmatrix}\begin{pmatrix}-\frac{7}{2}\\\frac{1}{2}\end{pmatrix} = 9 \tag{A.5.1.5}$$

$$(\mathbf{A}-\mathbf{B})^\top\left(\frac{\mathbf{A}+\mathbf{B}}{2}\right) = \begin{pmatrix}5 & -7\end{pmatrix}\begin{pmatrix}-\frac{3}{2}\\\frac{5}{2}\end{pmatrix} = -25 \tag{A.5.1.6}$$

$$(\mathbf{C}-\mathbf{A})^\top\left(\frac{\mathbf{C}+\mathbf{A}}{2}\right) = \begin{pmatrix}-4 & -4\end{pmatrix}\begin{pmatrix}-1\\-3\end{pmatrix} = 16 \tag{A.5.1.7}$$

Thus, the perpendicular bisectors are obtained from (A.5.1.1) as

$$BC: \quad \begin{pmatrix}-1 & 11\end{pmatrix}\mathbf{x} = 9 \tag{A.5.1.8}$$

$$CA: \quad \begin{pmatrix}5 & -7\end{pmatrix}\mathbf{x} = -25 \tag{A.5.1.9}$$

$$AB: \quad \begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = -4 \tag{A.5.1.10}$$

A.5.2. Find the intersection $\mathbf{O}$ of the perpendicular bisectors of AB and AC.

Solution:

The intersection of (A.5.1.9) and (A.5.1.10), can be obtained as

$$\begin{pmatrix}5 & -7 & -25\\1 & 1 & -4\end{pmatrix} \xleftrightarrow{R_2\leftarrow 5R_2-R_1} \begin{pmatrix}5 & -7 & -25\\0 & 12 & 5\end{pmatrix} \tag{A.5.2.1}$$

$$\xleftrightarrow{R_1\leftarrow \frac{12}{7}R_1+R_2} \begin{pmatrix}\frac{60}{7} & 0 & \frac{-265}{7}\\0 & 12 & 5\end{pmatrix} \xleftrightarrow[R_1\leftarrow\frac{7}{60}R_1]{R_2\leftarrow\frac{1}{12}R_2} \begin{pmatrix}1 & 0 & \frac{-53}{12}\\0 & 1 & \frac{5}{12}\end{pmatrix} \tag{A.5.2.2}$$

$$\implies \mathbf{O} = \begin{pmatrix}\frac{-53}{12}\\\frac{5}{12}\end{pmatrix} \tag{A.5.2.3}$$

A.5.3. Verify that $\mathbf{O}$ satisfies (A.5.1.1). $\mathbf{O}$ is known as the circumcentre.

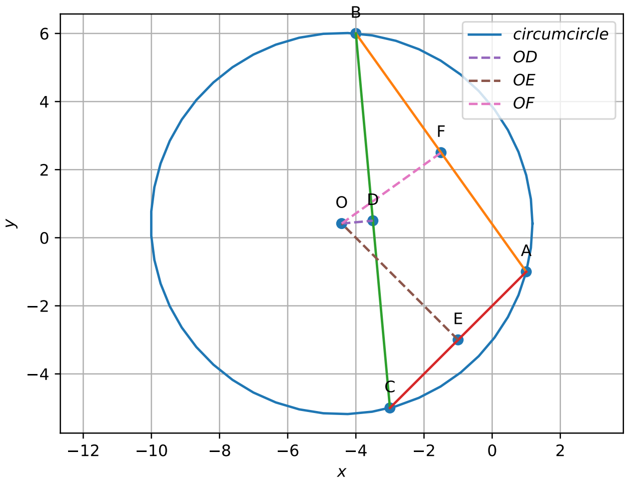

Fig. A.5.5.1: Circumcircle of $\triangle ABC$ with centre $\mathbf{O}$.

Solution: Substituing from (A.5.2.3) in (A.5.1.1),

$$\left(\mathbf{O} - \frac{\mathbf{B}+\mathbf{C}}{2}\right)^{\top} (\mathbf{B}-\mathbf{C})$$

$$= \left(\frac{1}{12}\begin{pmatrix}-53\\5\end{pmatrix} - \frac{1}{2}\begin{pmatrix}-7\\1\end{pmatrix}\right)^{\top}\begin{pmatrix}-1\\11\end{pmatrix}$$

$$= \frac{1}{12}\begin{pmatrix}-11 & -1\end{pmatrix}\begin{pmatrix}-1\\11\end{pmatrix} = 0 \quad \text{(A.5.3.1)}$$

A.5.4. Verify that

$$OA = OB = OC \quad \text{(A.5.4.1)}$$

A.5.5. Draw the circle with centre at $\mathbf{O}$ and radius

$$R = OA \quad \text{(A.5.5.1)}$$

This is known as the *circumradius*.
Solution: See Fig. A.5.5.1.

A.5.6. Verify that

$$\angle BOC = 2\angle BAC. \quad \text{(A.5.6.1)}$$

Solution:

a) To find the value of $\angle BOC$:

$$\mathbf{B}-\mathbf{O}=\begin{pmatrix}\frac{5}{12}\\\frac{67}{12}\end{pmatrix}, \mathbf{C}-\mathbf{O}=\begin{pmatrix}\frac{17}{12}\\\frac{-65}{12}\end{pmatrix} \tag{A.5.6.2}$$

$$\implies (\mathbf{B}-\mathbf{O})^\top(\mathbf{C}-\mathbf{O})=\frac{-4270}{144} \tag{A.5.6.3}$$

$$\implies \|\mathbf{B}-\mathbf{O}\|=\frac{\sqrt{4514}}{12}, \|\mathbf{C}-\mathbf{O}\|=\frac{\sqrt{4514}}{12} \tag{A.5.6.4}$$

Thus,

$$\cos BOC=\frac{(\mathbf{B}-\mathbf{O})^\top(\mathbf{C}-\mathbf{O})}{\|\mathbf{B}-\mathbf{O}\|\,\|\mathbf{C}-\mathbf{O}\|}=\frac{-4270}{4514} \tag{A.5.6.5}$$

$$\implies \angle BOC=\cos^{-1}\left(\frac{-4270}{4514}\right) \tag{A.5.6.6}$$

$$=161.07536^\circ \text{ or } 198.92464^\circ \tag{A.5.6.7}$$

b) To find the value of $\angle BAC$:

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}-5\\7\end{pmatrix}, \mathbf{C}-\mathbf{A}=\begin{pmatrix}-4\\-4\end{pmatrix} \tag{A.5.6.8}$$

$$\implies (\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A})=-8 \tag{A.5.6.9}$$

$$\|\mathbf{B}-\mathbf{A}\|=\sqrt{74}\,\|\mathbf{C}-\mathbf{A}\|=4\sqrt{2} \tag{A.5.6.10}$$

Thus,

$$\cos BAC=\frac{(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{C}-\mathbf{A}\|}=\frac{-8}{4\sqrt{148}} \tag{A.5.6.11}$$

$$\implies \angle BAC=\cos^{-1}\left(\frac{-8}{4\sqrt{148}}\right) \tag{A.5.6.12}$$

$$=99.46232^\circ \tag{A.5.6.13}$$

From (A.5.6.13) and (A.5.6.7),

$$2\times\angle BAC=\angle BOC \tag{A.5.6.14}$$

A.5.7. Let

$$\mathbf{P}=\begin{pmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{pmatrix} \tag{A.5.7.1}$$

where

$$\theta=\angle BOC \tag{A.5.7.2}$$

Verify that

$$\mathbf{B}-\mathbf{O}=\mathbf{P}(\mathbf{C}-\mathbf{O}) \tag{A.5.7.3}$$

All codes for this section are available at

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codes/triangle/perp-bisect.py
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A.6 Angle Bisector

A.6.1. Let $\mathbf{D}_3, \mathbf{E}_3, \mathbf{F}_3$, be points on AB, BC and CA respectively such that

$$BD_3 = BF_3 = m, CD_3 = CE_3 = n, AE_3 = AF_3 = p. \tag{A.6.1.1}$$

Obtain m, n, p in terms of a, b, c obtained in Problem A.1.2.
Solution: From the given information,

$$a = m + n, \tag{A.6.1.2}$$

$$b = n + p, \tag{A.6.1.3}$$

$$c = m + p \tag{A.6.1.4}$$

which can be expressed as

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{A.6.1.5}$$

$$\implies \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{A.6.1.6}$$

Using row reduction,

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \tag{A.6.1.7}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right) \tag{A.6.1.8}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 - R_2]{R_3 \leftarrow R_3 + R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{array}\right) \tag{A.6.1.9}$$

$$\xleftrightarrow[R_1 \leftarrow 2R_1 + R_3]{R_2 \leftarrow 2R_2 - R_3} \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & -1 & 1 \\ 0 & 2 & 0 & 1 & 1 & -1 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{array}\right) \tag{A.6.1.10}$$

yielding

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix} \tag{A.6.1.11}$$

Therefore,

$$\begin{aligned} p &= \frac{c+b-a}{2} = \frac{\sqrt{74}+\sqrt{32}-\sqrt{122}}{2} \\ m &= \frac{a+c-b}{2} = \frac{\sqrt{74}+\sqrt{122}-\sqrt{32}}{2} \\ n &= \frac{a+b-c}{2} = \frac{\sqrt{122}+\sqrt{32}-\sqrt{74}}{2} \end{aligned} \tag{A.6.1.12}$$

upon substituting from (A.1.2.7), (A.1.2.9) and (A.1.2.11).

A.6.2. Using section formula, find

$$\mathbf{D}_3 = \frac{m\mathbf{C}+n\mathbf{B}}{m+n}, \mathbf{E}_3 = \frac{n\mathbf{A}+p\mathbf{C}}{n+p}, \mathbf{F}_3 = \frac{p\mathbf{B}+m\mathbf{A}}{p+m} \tag{A.6.2.1}$$

A.6.3. Find the circumcentre and circumradius of $\triangle D_3E_3F_3$. These are the *incentre* and *inradius* of $\triangle ABC$.

A.6.4. Draw the circumcircle of $\triangle D_3E_3F_3$. This is known as the *incircle* of $\triangle ABC$.
Solution: See Fig. A.6.4.1

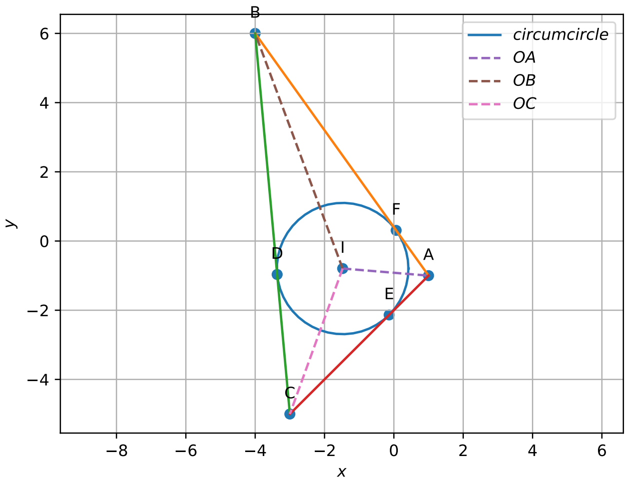

Fig. A.6.4.1: Incircle of $\triangle ABC$

A.6.5. Using (A.1.3.24) verify that

$$\angle BAI = \angle CAI. \tag{A.6.5.1}$$

AI is the bisector of $\angle A$.

A.6.6. Verify that BI, CI are also the angle bisectors of $\triangle ABC$. All codes for this section are available at

```
codes/triangle/ang−bisect.py
```

A.7 Eigenvalues and Eigenvectors

A.7.1. The equation of a circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{A.7.1.1}$$

for

$$\mathbf{u} = -\mathbf{O}, f = \|\mathbf{O}\|^2 - r^2, \tag{A.7.1.2}$$

$\mathbf{O}$ being the incentre and r the inradius.

A.7.2. Compute

$$\mathbf{\Sigma} = (\mathbf{V}\mathbf{h} + \mathbf{u})(\mathbf{V}\mathbf{h} + \mathbf{u})^\top - g(\mathbf{h})\mathbf{V} \tag{A.7.2.1}$$

for $\mathbf{h} = \mathbf{A}$.

A.7.3. Find the roots of the equation

$$\left|\lambda\mathbf{I} - \mathbf{\Sigma}\right| = 0 \tag{A.7.3.1}$$

These are known as the eigenvalues of $\mathbf{\Sigma}$.

A.7.4. Find $\mathbf{p}$ such that

$$\mathbf{\Sigma p} = \lambda\mathbf{p} \tag{A.7.4.1}$$

using row reduction. These are known as the eigenvectors of Σ.

A.7.5. Define

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \tag{A.7.5.1}$$

$$\mathbf{P} = \begin{pmatrix} \frac{\mathbf{p}_1}{\|\mathbf{p}_1\|} & \frac{\mathbf{p}_2}{\|\mathbf{p}_2\|} \end{pmatrix} \tag{A.7.5.2}$$

A.7.6. Verify that

$$\mathbf{P}^\top = \mathbf{P}^{-1}. \tag{A.7.6.1}$$

$\mathbf{P}$ is defined to be an orthogonal matrix.

A.7.7. Verify that

$$\mathbf{P}^\top \mathbf{\Sigma} \mathbf{P} = \mathbf{D}, \tag{A.7.7.1}$$

This is known as the spectral (eigenvalue) decomposition of a symmetric matrix

A.7.8. The direction vectors of the tangents from a point $\mathbf{h}$ to the circle in (**??**) are given by

$$\mathbf{m} = \mathbf{P}\begin{pmatrix} \sqrt{|\lambda_2|} \\ \pm\sqrt{|\lambda_1|} \end{pmatrix} \tag{A.7.8.1}$$

A.7.9. The points of contact of the pair of tangents to the circle in (**??**) from a point $\mathbf{h}$ are given by

$$\mathbf{x} = \mathbf{h} + \mu \mathbf{m} \tag{A.7.9.1}$$

where

$$\mu = -\frac{\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \tag{A.7.9.2}$$

for $\mathbf{m}$ in (A.7.8.1). Compute the points of contact. You should get the same points that you obtained in the previous section.
All codes for this section are available at

```
codes/triangle/tangpair.py
```