EE22BTECH11050 - Snehil Singh

Question 10.13.3.22

Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is less than 9

Solution: Let the X and Y be random variables denoting the roll of first dice snd second dice respectively. Assuming both dice rolls and equally likely:

$$p_X(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$p_Y(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The probability mass function is:

$$p_{XY}(k) = \Pr(XY = k) \tag{3}$$

$$= \Pr\left(X = \frac{k}{Y}\right) \tag{4}$$

$$= E\left(p_X\left(\frac{k}{Y}\right)\right) \tag{5}$$

$$= \sum_{i=1}^{6} p_X\left(\frac{k}{i}\right) p_Y(i) \tag{6}$$

$$= \frac{1}{6} \sum_{i=1}^{6} p_X\left(\frac{k}{i}\right) \tag{7}$$

$$= \frac{1}{6} \sum_{i=1}^{6} \frac{[k \bmod i = 0]}{6} \left[\frac{k}{i} \leq 6\right] \tag{8}$$

$$= \frac{1}{36} \sum_{i=1}^{6} [k \bmod i = 0] \left[\frac{k}{i} \leq 6\right] \tag{9}$$

Thus, the probability of getting product less than 9 is:

$$\Pr(XY < 9) = \sum_{i=1}^{8} \left(\frac{1}{36} \sum_{i=1}^{6} [j \bmod i = 0] \left[\frac{j}{i} \leq 6\right]\right) \tag{10}$$

$$= \frac{1}{36}(1 + 2 + 2 + 3 + 2 + 4 + 2) \tag{11}$$

$$= \frac{16}{36} \tag{12}$$

$$= \frac{4}{9} \tag{13}$$

TABLE 1: Table

Parameters	Values	Description
X	$1 \leq X \leq 6$	First dice roll
Y	$1 \leq Y \leq 6$	Second dice roll