

Assignment 2

AI1110: Probability and Random Variables

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12.13.2.13 Question: Two balls are drawn at random with replacement from a box containing 10 black balls and 8 red balls. Find the probability that

- (i) both balls are red
- (ii) first ball is black and second is red
- (iii) one of them is black and other is red

Solution: Since balls are drawn with replacement, Each draw is independent of other draw. Probability of getting red ball p in any draw is

$$p = \frac{8}{18} = \frac{4}{9} \quad (1)$$

X , X_1 and X_2 are the random variables for binomial distribution and two bernoulli trials respectively.

$$X = X_1 + X_2 \quad (2)$$

$$p_{X_i}(k) = \begin{cases} 1-p, & k=0 \\ p, & k=1 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$i \in \{1, 2\} \quad (4)$$

$$p_X(r) = {}^nC_r p^r (1-p)^{n-r} \quad (5)$$

Here, $n = 2$ and $p = \frac{4}{9}$

$$X \in \{0, 1, 2\} \quad (6)$$

$$X \sim \text{Bin}\left(2, \frac{4}{9}\right) \quad (7)$$

From (1) and (5),

$$\therefore p_X(r) = {}^2C_r \left(\frac{4}{9}\right)^r \left(\frac{5}{9}\right)^{2-r} \quad (8)$$

$$p_X(k) = \begin{cases} \frac{25}{81}, & k=0 \\ \frac{40}{81}, & k=1 \\ \frac{16}{81}, & k=2 \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

- (i) Since both balls are red, the required probability is $p_X(2)$

$$\therefore p_X(2) = \frac{16}{81} \quad (10)$$

- (ii) Let E be the event that first ball is black and other is red. The required probability $\Pr(E)$ is

$$\Pr(E) = p_{X_1}(0) \times p_{X_2}(1) \quad (11)$$

$$= \frac{1}{2} \times p_X(1) \quad (12)$$

$$= \frac{1}{2} \times {}^2C_1 \left(\frac{4}{9}\right)^1 \left(\frac{5}{9}\right)^1 \quad (13)$$

$$= \frac{1}{2} \times 2 \times \frac{4}{9} \times \frac{5}{9} \quad (14)$$

$$= \frac{20}{81} \quad (15)$$

$$\therefore \Pr(E) = \frac{20}{81} \quad (16)$$

- (iii) Since only one of the two balls has to be red, The required probability is $p_X(1)$

$$\therefore p_X(1) = \frac{40}{81} \quad (17)$$