Exemplar - 12.13.3.28

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Question: A die is thrown three times. Let X be 'the number of twos seen'. Find the expectation of X.

Solution: Let the random variables be:

RV	Values	Description
X	{0, 1, 2, 3}	The number of twos rolled in three dice rolls
X_1	{0, 1}	The number of twos rolled on first die
X_2	{0, 1}	The number of twos rolled on second die
X_3	{0, 1}	The number of twos rolled on third die

TABLE 0
Random Variables

For a single die roll, since the probability of rolling a two is $\frac{1}{6}$ the probability distribution function of X_i is:

$$p_{X_i}(k) = \begin{cases} \frac{1}{6} & \text{if } k = 1 \\ \frac{5}{6} & \text{if } k = 0 \end{cases} \tag{1}$$

Thus,

$$E(X_1) = E(X_2) = E(X_3) = \sum_{k=0}^{1} k p_{X_i}(k) \tag{2}$$

$$= \frac{5}{6}(0) + \frac{1}{6}(1) \tag{3}$$

$$= \frac{1}{6} \tag{4}$$

But, as all three dice rolls are independent, and expectation is linear:

$$X = X_1 + X_2 + X_3 \tag{5}$$

$$\therefore E(X) = E(X_1 + X_2 + X_3) \tag{6}$$

$$= E(X_1) + E(X_2) + E(X_3) \tag{7}$$

$$= \frac{1}{2} \tag{8}$$