Assignment 2

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Problem 11.16.3.5(exemplar):-

A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution:-

Let *X* be a random variable denoting the number obtained on the die.

Let m be any natural number such that $m \in \{1, 2, 3\}$.

$$\Pr(X = k) = \begin{cases} 2p, & \text{if } k = 2m - 1 \\ p, & \text{if } k = 2m \end{cases} \tag{1}$$

Let $F_X(k)$ be the cumulative distribution function such that,

$$F_X(k) = \Pr\left(X \le k\right) \tag{2}$$

If k = 2m - 1,

$$F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i)$$
 (3)

$$= \sum_{i=1}^{m} \Pr(2i-1) + \sum_{i=1}^{m-1} \Pr(2i)$$
 (4)

$$=\sum_{i=1}^{m} 2p + \sum_{i=1}^{m-1} p \tag{5}$$

$$= (m)(2p) + (m-1)(p)$$
 (6)

$$= p(3m-1) \tag{7}$$

$$=\frac{p(3k+1)}{2}\tag{8}$$

If k = 2m,

$$F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i)$$
 (9)

$$= \sum_{i=1}^{m} \Pr(2i - 1) + \sum_{i=1}^{m} \Pr(2i) \quad (10)$$

$$=\sum_{i=1}^{m}2p+\sum_{i=1}^{m}p$$
(11)

$$= (m)(2p) + (m)(p)$$
 (12)

$$= p(3m) \tag{13}$$

$$=\frac{3pk}{2}\tag{14}$$

So,

$$F_X(k) = \begin{cases} \frac{p(3k+1)}{2}, & \text{if } k = 2m - 1\\ \frac{3pk}{2}, & \text{if } k = 2m \end{cases}$$
 (15)

Since $1 \le X \le 6$,

$$F_X(6) = 1 \tag{16}$$

$$\implies \frac{3p(6)}{2} = 1 \tag{17}$$

$$\implies p = \frac{1}{9} \tag{18}$$

$$Pr(G) = Pr(X > 3) \tag{19}$$

$$= F_X(6) - F_X(3) \tag{20}$$

$$=\frac{3p(6)}{2} - \frac{p\{3(3)+1\}}{2} \tag{21}$$

$$= 9p - 5p = 4p \tag{22}$$

$$=\frac{4}{9}\tag{23}$$

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