

Question 11.16.4.9

If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when:

1. The digits are repeated?
2. The repetition of digits is not allowed?

solution: Let X be a random variable such that:

$$X = \begin{cases} 0 & n \not\equiv 0 \pmod{5} \\ 1 & n \equiv 0 \pmod{5} \end{cases} \quad (1)$$

Let N be a 4 digit number $X_1X_2X_3X_4$ where X_1, X_2, X_3, X_4 are digits of the number N .

Digit	Position
X_1	<i>Thousands's Digit</i>
X_2	<i>Hundred's Digit</i>
X_3	<i>Ten's Digit</i>
X_4	<i>One's Digit</i>

Table 1: Listing variables

Let's solve each part separately.

(i) Repetition of digits

Let number of favourable outcomes be $N(A)$ and total outcomes be $N(T)$.

For $N > 5000$,

Digit	Favourable
X_1	5, 7
X_2, X_3, X_4	0, 1, 3, 5, 7

Table 2: Conditions for N greater than 5000

We must also exclude the case of 5000. Hence,

$$N(T) = (2 \times 5 \times 5 \times 5) - 1 \quad (2)$$

$$\implies N(T) = 249 \quad (3)$$

Here also we must exclude the case of 5000.

$$N(A) = (2 \times 5 \times 5 \times 2) - 1 \quad (4)$$

$$\implies N(A) = 99 \quad (5)$$

Digit	Favourable
X_1	5, 7
X_2, X_3	0, 1, 3, 5, 7
X_4	0, 5

Table 3: Conditions for N greater than 5000 and divisible by 5

With this information we can find the required answer,

$$\Pr(X = 1) = \frac{N(A)}{N(T)} \quad (6)$$

$$\implies \Pr(X = 1) = \frac{33}{83} \quad (7)$$

(ii) No Repetition of Digits

Let number of favourable outcomes be N(B) and total outcomes be N(T).

For N > 5000,

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Digit	Favourable
X_1	5, 7
X_2, X_3, X_4	0, 1, 3, 5, 7

Table 4: Conditions for N greater than 5000

Hence,

$$N(T) = (2 \times 4 \times 3 \times 2) \quad (8)$$

$$\implies N(T) = 48 \quad (9)$$

For N > 5000 and also divisible by 5:

$$X_4 = \begin{cases} 0 & X_1 = 5 \\ 5, 0 & X_1 = 7 \end{cases} \quad (10)$$

Hence,

$$N(B) = (1 \times 3 \times 2 \times 1) + (1 \times 3 \times 2 \times 2) \quad (11)$$

$$\implies N(B) = 18 \quad (12)$$

With this information we can find the required answer,

$$\Pr(X = 1) = \frac{N(B)}{N(T)} \quad (13)$$

$$\implies \Pr(X = 1) = \frac{3}{8} \quad (14)$$