

Assignment

Karthikeya hanu prakash kanithi (EE22BTECH11026)

Question : If two events are independent, then

- A) they must be mutually exclusive
- B) the sum of their probabilities must be equal to 1
- C) (A) and (B) both are correct
- D) None of the above is correct

Solution: Let X, Y be bernoulli random variables as defined in Table I, Lets us consider the pmf's as follows:

RV	Value	Description
X	$\{0, 1\}$	Event A
Y	$\{0, 1\}$	Event B

TABLE I
RANDOM VARIABLE X DECLARATION

$$p_X(k) = \begin{cases} x & \text{if } k = 1 \\ 1 - x & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$p_Y(k) = \begin{cases} y & \text{if } k = 1 \\ 1 - y & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Two events A and B are independent if and only if:

$$\Pr(X = 1, Y = 1) = p_X(1)p_Y(1) = xy \quad (3)$$

- A) Two events A and B are mutually-exclusive if and only if:

$$\Pr(X = 1, Y = 1) = 0 \quad (4)$$

from (3) and (4), We can say that X and Y are not mutually exclusive as

$$\Pr(X = 1, Y = 1) \neq 0 \quad (5)$$

Hence Option A is false.

- B) The sum of the probabilities of two events A and B can be represented by:

$$\Pr(X = 1) + \Pr(Y = 1) = x + y \quad (6)$$

Here, It is not always true that $x + y$ should be 1 rather

$$0 \leq x + y \leq 2 \quad (7)$$

Hence Option B is false.

- C) Option C cant be true as Both A and B are false.

- D) So option D none of these is the most appropriate answer

For example: Let event X and Y be independent and defined as follows: Then the pmf's as follows:

RV	Value	Description
X	$\{0, 1\}$	Getting Head in a coin toss
Y	$\{0, 1\}$	Getting 4 by a dice roll

TABLE II
RANDOM VARIABLE X DECLARATION

$$p_X(k) = \begin{cases} \frac{1}{2} & \text{if } k = 1 \\ \frac{1}{2} & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

$$p_Y(k) = \begin{cases} \frac{1}{6} & \text{if } k = 1 \\ \frac{5}{6} & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Then

A)

$$\Pr(X = 1, Y = 1) = p_X(1)p_Y(1) \quad (10)$$

$$= \frac{1}{12} \neq 0 \quad (11)$$

From (11), we can say that X and Y are not mutually exclusive and

B)

$$\Pr(X = 1) + \Pr(Y = 1) = \frac{2}{3} \quad (12)$$

From (12), the sum of their probabilities may not be equal to 1

Therefore, Option A and B are wrong and Option D is correct