Assignment

Aryan Jain - EE22BTECH11011*

Question: Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is

1) $\frac{1}{2}$
2) $\frac{1}{3}$
3) $\frac{2}{3}$
4) $\frac{4}{7}$

Solution:

Let X_0, X_1, X_2 be the random variables which denotes the three children, where X_0 is the eldest child and X_2 is the youngest child.

RV	Value	Description
X_i	0	child is boy
	1	child is girl

TABLE 4
RV DESCRIPTION TABLE

so the required probability is,

$$\Pr\left(X_0 = 1 \middle| X_0 + X_1 + X_2 \geq 1\right) = \frac{\Pr\left(X_0 = 1, X_0 + X_1 + X_2 \geq 1\right)}{\Pr\left(X_0 + X_1 + X_2 \geq 1\right)} \tag{1}$$

$$= \frac{\Pr\left(X_0 = 1\right) \times \Pr\left(X_1 + X_2 \geq 0\right)}{\Pr\left(X_0 + X_1 + X_2 \geq 1\right)} \tag{2}$$

$$= \frac{\frac{1}{2} \times \sum_{k=0}^{2} {}^{2}C_k \times \frac{1}{2}^{k} \times \frac{1}{2}^{2-k}}{\sum_{k=1}^{3} {}^{3}C_k \times \frac{1}{2}^{k} \times \frac{1}{2}^{3-k}} \tag{3}$$

$$= \frac{\frac{1}{2} \times 1}{\frac{3}{8} + \frac{3}{8} + \frac{1}{8}} \tag{4}$$

$$= \frac{4}{7} \tag{5}$$

Therefore, the probability that the eldest child is a girl given that the family has atleast one girl is $\frac{4}{7}$