

Assignment

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Question:- Let A and B be two events such that $\Pr(A) = \frac{3}{8}$, $\Pr(B) = \frac{5}{8}$ and $\Pr(A + B) = \frac{3}{4}$. Then $\Pr(A|B) \cdot \Pr(A'|B)$ is equal to

(a) $\frac{2}{5}$

(b) $\frac{3}{8}$

(c) $\frac{3}{20}$

(d) $\frac{6}{25}$

Solution: Given

$$\Pr(A) = \frac{3}{8} \quad (1)$$

$$\Pr(B) = \frac{5}{8} \quad (2)$$

$$\Pr(A + B) = \frac{3}{4} \quad (3)$$

As we know

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \quad (4)$$

$$\implies \Pr(AB) = \Pr(A) + \Pr(B) - \Pr(A + B) \quad (5)$$

$$\implies \Pr(AB) = \frac{1}{4} \quad (6)$$

Now,

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \quad (7)$$

$$\Pr(A'|B) = \frac{\Pr(A'B)}{\Pr(B)} \quad (8)$$

$$= \frac{\Pr(B) - \Pr(AB)}{\Pr(B)} \quad (9)$$

From (7) and (9)

$$\Pr(A|B) \cdot \Pr(A'|B) = \frac{\Pr(AB)}{\Pr(B)} \times \frac{\Pr(B) - \Pr(AB)}{\Pr(B)} \quad (10)$$

$$= \frac{\left(\frac{1}{4}\right)}{\frac{5}{8}} \times \frac{\left(\frac{5}{8} - \frac{1}{4}\right)}{\frac{5}{8}} \quad (11)$$

$$= \frac{6}{25} \quad (12)$$