Solution to 11.16.3.24

1

Sameer kendal - EE22BTECH11044

Question: If Pr(A + B) = Pr(AB) for any two events A and B, then

- A) Pr(A)=Pr(B)
- B) Pr(A) > Pr(B)
- C) Pr(A) < Pr(B)
- D) none of these

Solution:

$$Pr(A) + Pr(B) - Pr(AB) = Pr(A + B)$$

$$(1)$$

$$\Rightarrow Pr(A) + Pr(B) - Pr(AB) = Pr(AB)$$

$$(2)$$

$$\Rightarrow [Pr(A) - Pr(AB)] + [Pr(B) - Pr(AB)] = 0$$

$$(3)$$

But,

$$Pr(A|B) = \frac{Pr(AB)}{Pr(B)}$$
 (4)

Also,

$$0 \le \Pr(A|B) \le 1 \tag{5}$$

$$\implies 0 \le \frac{\Pr(AB)}{\Pr(B)} \le 1$$
 (6)

$$\implies 0 \le \Pr(AB) \le \Pr(B)$$
 (7)

Similarly,

$$0 \le \Pr(AB) \le \Pr(A) \tag{8}$$

$$\therefore \Pr(A) - \Pr(AB) \ge 0 \tag{9}$$

$$\Pr(B) - \Pr(AB) \ge 0 \tag{10}$$

$$\implies \Pr(A) - \Pr(AB) = 0$$
 (11)

$$\implies \Pr(A) = \Pr(AB)$$
 (12)

Also,

$$Pr(B) - Pr(AB) = 0 (13)$$

$$\implies \Pr(B) = \Pr(AB)$$
 (14)

From (12) and (14), it can be said that

$$Pr(A) = Pr(B) \tag{15}$$