

Question: Two probability distributions of the discrete random variable X and Y are given below.

TABLE 0

TABLE-1

X	0	1	2	3
$P(X)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

TABLE 0

TABLE-2

Y	0	1	2	3
$P(Y)$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

Prove that $E(Y^2) = 2E(X)$

Solution:

The probability distribution function of X is:

$$p_Y(k) = \begin{cases} \frac{1}{5} & k = 0 \\ \frac{3}{10} & k = 1 \\ \frac{2}{5} & k = 2 \\ \frac{1}{10} & k = 3 \end{cases} \quad (1)$$

$$E(Y^2) = \sum_k (k)^2 \times p_Y(k) \quad (2)$$

$$= 0 \times \frac{1}{5} + 1^2 \times \frac{3}{10} + (2)^2 \times \frac{2}{5} + (3)^2 \times \frac{1}{10} \quad (3)$$

$$= \frac{14}{5} \quad (4)$$

The probability distribution function of Y is:

$$p_Y(k) = \begin{cases} \frac{1}{5} & k = 0 \\ \frac{2}{5} & k = 1 \\ \frac{1}{5} & k = 2 \\ \frac{1}{5} & k = 3 \end{cases} \quad (5)$$

$$E(X) = \sum_k k \times p_X(k) \quad (6)$$

$$= 0 \times \frac{1}{5} + 1 \times \frac{2}{5} + 2 \times \frac{1}{5} + 3 \times \frac{1}{5} \quad (7)$$

$$= \frac{7}{5} \quad (8)$$

From above;

$$\frac{14}{5} = 2 \times \frac{7}{5} \quad (9)$$

$$\therefore E(Y^2) = 2E(X) \quad (10)$$

$$\text{Hence proved} \quad (11)$$