

Exemplar - 12.13.3.50

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Question: The probability distribution of a random variable X is given as under:

$$p_X(x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3 \\ 2kx & \text{for } x = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. Calculate:

- 1) $E(X)$
- 2) $E(3X^2)$
- 3) $\Pr(X \geq 4)$

Solution: From the axiom of total probability,

$$\sum_{i=1}^6 p_X(i) = 1 \quad (1)$$

$$\Rightarrow \sum_{i=1}^3 ki^2 + \sum_{i=4}^6 2ki = 1 \quad (2)$$

$$\Rightarrow k + 4k + 9k + 8k + 10k + 12k = 1 \quad (3)$$

$$\Rightarrow k = \frac{1}{44} \quad (4)$$

Thus, the probability distribution of X is

$$p_X(x) = \begin{cases} \frac{x^2}{44} & \text{for } x = 1, 2, 3 \\ \frac{2x}{44} & \text{for } x = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

1) Calculating $E(X)$:

$$E(X) = \sum_{i=1}^6 ip_X(i) \quad (5)$$

$$= 1\left(\frac{1}{44}\right) + 2\left(\frac{4}{44}\right) + 3\left(\frac{9}{44}\right) + 4\left(\frac{8}{44}\right) + 5\left(\frac{10}{44}\right) + 6\left(\frac{12}{44}\right) \quad (6)$$

$$= \frac{95}{22} \quad (7)$$

$$= 4.32 \quad (8)$$

2) Calculating $E(3X^2)$:

$$E(3X^2) = 3E(X^2) \quad (9)$$

$$= 3 \sum_{i=1}^6 i^2 p_X(i) \quad (10)$$

$$= 3 \left(1\left(\frac{1}{44}\right) + 4\left(\frac{4}{44}\right) + 9\left(\frac{9}{44}\right) + 16\left(\frac{8}{44}\right) + 25\left(\frac{10}{44}\right) + 36\left(\frac{12}{44}\right) \right) \quad (11)$$

$$= \frac{2724}{44} \quad (12)$$

$$= 61.91 \quad (13)$$

3) Firstly, calculating the CDF:

$$F_X(x) = \sum_{i=1}^x p_X(i) \quad (14)$$

$$= \begin{cases} \sum_{i=1}^x \frac{i^2}{44} & \text{if } x \leq 3 \\ \sum_{i=1}^3 \frac{i^2}{44} + \sum_{i=4}^x \frac{2i}{44} & \text{if } x \geq 4 \end{cases} \quad (15)$$

$$= \begin{cases} \frac{x(x+1)(2x+1)}{6 \times 44} & \text{if } x \leq 3 \\ \frac{14}{44} + \frac{x(x+1)}{44} - \frac{3 \times 4}{44} & \text{if } x \geq 4 \end{cases} \quad (16)$$

$$= \begin{cases} \frac{x(x+1)(2x+1)}{264} & \text{if } x \leq 3 \\ \frac{x(x+1)+2}{44} & \text{if } x \geq 4 \end{cases} \quad (17)$$

Calculating $\Pr(X \geq 4)$:

$$\Pr(X \geq 4) = 1 - \Pr(X \leq 3) \quad (18)$$

$$= 1 - F_X(3) \quad (19)$$

$$= 1 - \frac{3 \times 4 \times 7}{264} \quad (20)$$

$$= \frac{15}{22} \quad (21)$$

$$= 0.68 \quad (22)$$