Assignment

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Question:- The random variable X can take only the values $0, 1, 2$. Given that $\Pr(X = 0) = \Pr(X = 1) = p$ and that $E\left(X^2\right) = E(X)$, find the value of p.

Solution: Given that X is a random variable such that

$$X = \{0, 1, 2\} \tag{1}$$

$$\Pr(X = k) = p_X(k) \tag{2}$$

$$p_X(0) = p_X(1) = p \tag{3}$$

Then,

$$p_X(0) + p_X(1) + p_X(2) = 1 \tag{4}$$

$$\implies p + p + p_X(2) = 1 \tag{5}$$

$$\implies p_X(2) = 1 - 2p \tag{6}$$

Expectation is defined as:

$$E(X) = \sum_{k=0}^{2} k p_X(k) \tag{7}$$

$$= 0p_X(0) + 1p_X(1) + 2p_X(2) \tag{8}$$

$$= 2 - 3p \tag{9}$$

And

$$E\left(X^2\right) = \sum_{k=0}^{2} k^2 p_X(k) \tag{10}$$

$$= 0p_X(0) + 1p_X(1) + 4p_X(2) \tag{11}$$

$$= 4 - 7p \tag{12}$$

Given,

$$E(X) = E\left(X^2\right) \tag{13}$$

using (9) and (12)

$$\implies 2 - 3p = 4 - 7p \tag{14}$$

$$\implies p = \frac{1}{2} \tag{15}$$