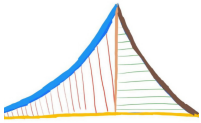
Discrete Mathematics



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About this Book

This book introduces progressions, binomial theorem, limits and sequences. All problems in the book are from NCERT mathematics textbooks from Class 9-12. Exercises are from CBSE, JEE and Olympiad exam papers.

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Github: https://github.com/gadepall/algebra

Contents

1 Arithmetic Progression

1.1 Formulae

1.1.1 Find the sum

$$S = 1 + 2 + \cdots + 10 \tag{1.1.1.1}$$

Solution: Reversing the sum in (1.1.1.1) as

$$S = 10 + 9 + \cdots + 1 \tag{1.1.1.2}$$

and adding (1.1.1.1) and (1.1.1.2),

$$2S = 11 + 11 + \cdots + 11 \quad 10 \text{ times} \tag{1.1.1.3}$$

$$\implies S = \frac{11 \times 10}{2} = 55 \tag{1.1.1.4}$$

1.1.2 The sum of the first n natural numbers is

$$S_n = \sum_{k=1}^{n} k = 1 + 2 + \cdots + n \tag{1.1.2.1}$$

$$= \frac{n(n+1)}{2} \tag{1.1.2.2}$$

1.1.3 The nth term of an arithmetic progression (AP) is

$$a_n = a_0 + nd, \quad n = 0, 1, \ldots \tag{1.1.3.1}$$

1.1.4 The sum to $n + 1$ terms of an AP is given by

$$S_n = \sum_{k=0}^{n} a_k = \frac{n+1}{2}\left[a_0 + \frac{dn}{2}\right] \tag{1.1.4.1}$$

Solution: From (1.1.3.1) and (1.1.4.1),

$$S_n = \sum_{k=0}^{n} a_k = \sum_{k=0}^{n} (a_0 + kd) \tag{1.1.4.2}$$

$$= \sum_{k=0}^{n} a_0 + d\sum_{k=1}^{n} k = (n+1)\,a_0 + d\frac{n(n+1)}{2} \tag{1.1.4.3}$$

upon substituting from (1.1.2.2), yielding (1.1.4.2) upon simplification.

1.2 NCERT

1.2.1 For the AP

$$\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \cdots$$

write the first term a and the common difference d.

1.2.2 Which of the following list of numbers form an AP? If they form an AP, write the next two terms

a) $4, 10, 16, 22, \ldots$
b) $1, -1, -3, -5, \ldots$
c) $-2, 2, -2, 2, -2, \ldots$
d) $1, 1, 1, 2, 2, 2, 3, 3, 3, \ldots$

1.2.3 Find the 10th term of the AP : $2, 7, 12, \ldots$

1.2.4 Which term of the AP : $21, 18, 15, \ldots$ is - 81? Also, is any term 0? Give reason for your answer.

1.2.5 Determine the AP whose 3rd term is 5 and the 7th term is 9.

1.2.6 Check whether 301 is a term of the list of numbers $5, 11, 17, 23, \ldots$

1.2.7 How many two-digit numbers are divisible by 3?

1.2.8 Find the 11th term from the last term (towards the first term) of the AP : $10, 7, 4, \ldots, -62$.

1.2.9 A sum of ₹1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.

1.2.10 In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

1.2.11 Find the sum of the first 22 terms of the AP : $8, 3, -2, \ldots$

1.2.12 If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.

1.2.13 How many terms of the AP : $24, 21, 18, \ldots$ must be taken so that their sum is 78?

1.2.14 Find the sum of

a) the first 1000 positive integers.
b) the first n positive integers.

1.2.15 Find the sum of first 24 terms of the list of numbers whose n^{th} term is given by $a_n = 3 + 2n$

1.2.16 A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find

a) the production in the 1st year
b) the production in the 10th year
c) the total production in first 7 years.

1.2.17 In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

a) The taxi fare after each km when the fare is ₹15 for the first km and ₹8 for each additional km.
b) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.
c) The cost of digging a well after every metre of digging, when it costs ₹150 for the first metre and rises by ₹50 for each subsequent metre.
d) The amount of money in the account every year, when ₹10000 is deposited at compound interest at 8 % per annum.

1.2.18 Write first four terms of the AP, when the first term a and the common difference d are given as follows

a) $a = 10, d = 10$
b) $a = 4, d = -3$
c) $a = -2, d = 0$
d) $a = -1, d = \frac{1}{2}$
e) $a = -1.25, d = -0.25$

1.2.19 For the following APs, write the first term and the common difference

a) $3, 1, -1, -3, \ldots$
b) $-5, -1, 3, 7, \ldots$
c) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$
d) $0.6, 1.7, 2.8, 3.9, \ldots$

1.2.20 Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

a) $2, 4, 8, 16, \ldots$
b) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$
c) $-1.2, -3.2, -5.2, -7.2, \ldots$
d) $-10, -6, -2, 2, \ldots$
e) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \ldots$
f) $0.2, 0.22, 0.222, 0.2222, \ldots$
g) $0, -4, -8, -12, \ldots$
h) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots$
i) $1, 3, 9, 27, \ldots$
j) $a, 2a, 3a, 4a, \ldots$
k) $a, a^2, a^3, a^4, \ldots$
l) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$
m) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$
n) $1^2, 3^2, 5^2, 7^2, \ldots$
o) $1^2, 5^2, 7^2, 73, \ldots$

1.2.21 Fill in the blanks in Table 1.2.21, given that a is the first term, d the common difference and a_n the n^{th} term of the AP.

	a	d	n	a_n
(i)	7	3	8	...
(ii)	-18	...	10	0
(iii)	...	-3	18	-5
(iv)	-18.9	2.5	...	3.6
(v)	3.5	0	105	...

TABLE 1.2.21

1.2.22 Choose the correct choice in the following and justify

a) 30^{th} term of the AP: $10, 7, 4, \ldots$ is

i) 97 ii) 77 iii) -77 iv) -87

b) 11^{th} term of the AP: $-3, -\frac{1}{2}, 2, \ldots$ is

i) 28 ii) 22 iii) -38 iv) $-48\frac{1}{2}$

c) In the following APs, find the missing terms in the blanks

i) $2, \ldots, 26$
ii) $\ldots, 13, \ldots, 3$
iii) $5, \ldots, \ldots, 9\frac{1}{2}$
iv) $-4, \ldots, \ldots, \ldots, \ldots, 6$
v) $\ldots, 38, \ldots, \ldots, \ldots, -22$

1.2.23 Which term of the AP : $3, 8, 13, 18, \ldots$ is 78?

1.2.24 Find the number of terms in each of the following APs:

a) $7, 13, 19, \ldots, 205$.
b) $18, 15\frac{1}{2}, 13, \ldots, -47$

1.2.25 Check whether -150 is a term of the AP : $11, 8, 5, 2 \ldots$

1.2.26 Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

1.2.27 An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

1.2.28 If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

1.2.29 The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

1.2.30 Which term of the AP : $3, 15, 27, 39, \ldots$ will be 132 more than its 54th term?

1.2.31 How many three-digit numbers are divisible by 7?

1.2.32 How many multiples of 4 lie between 10 and 250?

1.2.33 For what value of n, are the n^{th} terms of two APs: $63, 65, 67, \ldots$ and $3, 10, 17, \ldots$ equal?

1.2.34 Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

1.2.35 Find the 20th term from the last term of the AP : $3, 8, 13, \ldots, 253$.

1.2.36 The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

1.2.37 Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In which year did his income reach ₹7000?

1.2.38 Ramkali saved ₹5 in the first week of a year and then increased her weekly savings by ₹1.75. If in the n^{th} week, her weekly savings become ₹20.75, find n.

1.2.39 Find the sum of the following APs

a) $2, 7, 12, \ldots$, to 10 terms.
b) $-37, -33, -29, \ldots$, to 12 terms.
c) $0.6, 1.7, 2.8, \ldots$, to 100 terms.
d) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms.

1.2.40 Find the sums given below

a) $7 + 10\frac{1}{2} + 14 + \cdots + 84$
b) $34 + 32 + 30 + \cdots + 10$
c) $-5 + (-8) + (-11) + \cdots + (-230)$

1.2.41 In an A.P

a) given $a = 5, d = 3, a_n = 50$, find n and S_n.

b) given $a = 7, a_{13} = 35$, find d and S_{13}.
c) given $a_{12} = 37, d = 3$, find a and S_{12}.
d) given $a_3 = 15, S_{10} = 125$, find d and a_{10}.
e) given $d = 5, S_9 = 75$, find a and a_9.
f) given $a = 2, d = 8, S_n = 90$, find n and a_n.
g) given $a = 8, a_n = 62, S_n = 210$, find n and d.
h) given $a_n = 4, d = 2, S_n = -14$, find n and a.
i) given $a = 3, n = 8, S = 192$, find d.
j) given $l = 28, S = 144$, and there are total 9 terms. Find a.

1.2.42 How many terms of the AP : $9, 17, 25, \ldots$ must be taken to give a sum of 636?

1.2.43 The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

1.2.44 The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

1.2.45 Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

1.2.46 Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

1.2.47 If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

1.2.48 Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where a_n is defined as below

a) $a_n = 3 + 4n$
b) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

1.2.49 If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

1.2.50 Find the sum of the first 40 positive integers divisible by 6.

1.2.51 Find the sum of the first 15 multiples of 8.

1.2.52 Find the sum of the odd numbers between 0 and 50.

1.2.53 A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹250 for the second day, ₹300 for the third day, etc., the penalty for each succeeding day being ₹50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

1.2.54 A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

1.2.55 In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

1.2.56 A spiral is made up of successive semicircles, with centres alternately at A and B,

starting with centre at A, of radii $0.5cm, 1.0cm, 1.5cm, 2.0cm, \ldots$ as shown in Fig. 1.2.1 What is the total length of such a spiral made up of thirteen consecutive 22 semicircles? (Take $\pi = \frac{22}{7}$)

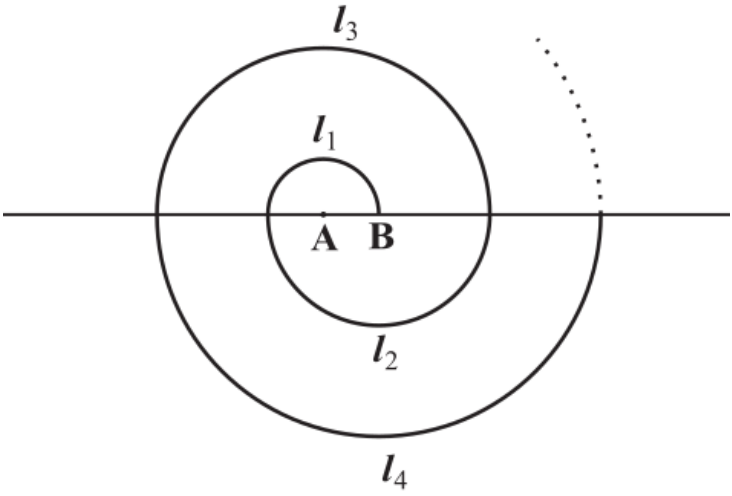

Fig. 1.2.1

Hint: Length of successive semicircles is $l_1, l_2, l_3, l_4, \ldots$ with centres at A, B, A, B, ..., respectively.

1.2.57 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig 1.2.2). In how many rows are the 200 logs placed and how many logs are in the top row?

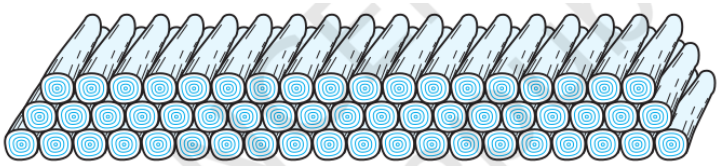

Fig. 1.2.2

1.2.58 In a potato race, a bucket is placed at the starting point, which is $5m$ from the first potato, and the other potatoes are placed $3m$ apart in a straight line. There are ten potatoes in the line as shown in Fig. 1.2.3.

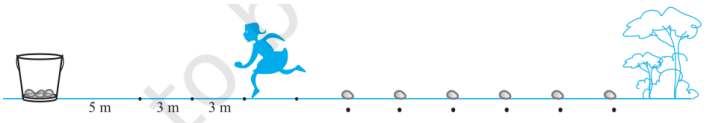

Fig. 1.2.3

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run? [*Hint:* To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$].

1.2.59 Which term of the AP : $121, 117, 113, \ldots$ is its first negative term? [*Hint:* Find n for $a_n < 0$]

1.2.60 The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

1.2.61 The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.[*Hint:* $S_{x-1} = S_{49} - S_x$]

1.2.62 A small terrace at a football ground comprises of 15 steps each of which is $50m$ long and built of solid concrete. Each step has rise of $\frac{1}{4}m$ and a tread of $\frac{1}{2}m$ (see Fig. 1.2.4). Calculate the total volume of concrete required to build the terrace. [*Hint:* Volume of concrete required to build the first step = $\frac{1}{4} \times \frac{1}{2} \times 50m^3$]

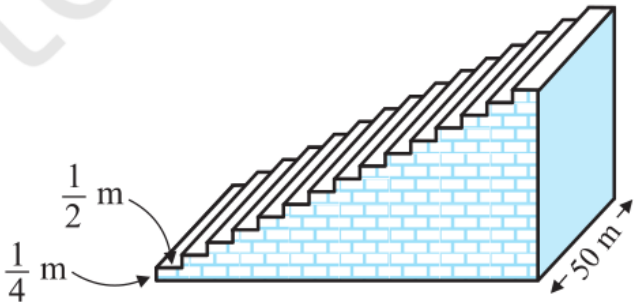

Fig. 1.2.4

1.2.63 Find the sum to n terms of the following series

a) $a_n = 2n + 5$ b) $a_n = \frac{n-3}{4}$ c) $a_n = \frac{2n-3}{6}$ d) $a_n = 4n - 3$

1.2.64 Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

1.2.65 In an AP, the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

1.2.66 How many terms of the AP $-6, -\frac{11}{2}, -5, \ldots$ are needed to give the sum -25?

1.2.67 In an AP, If p^{th} term is $\frac{1}{q}, q^{th}$ term is $\frac{1}{p}$, prove that the sum of first pq terms is $\frac{1}{2}(pq+1)$, where $p \neq q$.

1.2.68 If the sum of a certain number of terms of the AP: $25, 22, 19, \ldots$ is 116, find the last term.

1.2.69 Find the sum to n terms of the AP, whose k^{th} term is $5k+1$.

1.2.70 If the sum of n terms of an AP is $pn + qn^2$, where p and q are constants, find the common difference.

1.2.71 The sums of n terms of two arithmetic progressions are in the ratio $5n+4 : 9n+6$. Find the ratio of their 18^{th} terms.

1.2.72 If the sum of first p terms of an AP is equal to the sum of the first q terms, then find the sum of the first $p+q$ terms.

1.2.73 Sum of the first p, q and r terms of an AP are a, b and c, respectively. Prove that

$$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$$

1.2.74 The ratio of the sums of m and n terms of an AP is $m^2 : n^2$. Show that the ratio of m^{th} and n^{th} term is $(2m-1) : (2n-1)$.

1.2.75 If the sum of n terms of an AP is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m.

1.2.76 Insert five numbers between 8 and 26 such that the resulting sequence is an AP

1.2.77 Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an AP and the ratio of 7^{th} and $(m-1)^{th}$ numbers is $5 : 9$. Find the value of m.

1.2.78 A man starts repaying a loan as first instalment of ₹100. If he increases the instalment by Rs 5 every month, what amount he will pay in the 30^{th} instalment?

1.2.79 The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of the sides of the polygon.

1.2.80 Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an AP is equal to twice the m^{th} term.

1.2.81 If the sum of three numbers in AP is 24 and their product is 440, find the numbers.

1.2.82 Let the sum of $n, 2n, 3n$ terms of an AP be S_1, S_2 and S_3, respectively, show that

$$S_3 = 3(S_2 - S_1)$$

1.2.83 Find the sum of all numbers between 200 and 400 which are divisible by 7.

1.2.84 Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

1.2.85 The sum of the first four terms of an AP is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.

1.2.86 The p^{th}, q^{th} and r^{th} terms of an AP are a, b, c, respectively. Show that

$$(q-r)\,a + (r-p)\,b + (p-q)\,c = 0.$$

1.2.87 If

$$a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$$

are in AP, prove that a, b, c are in AP.

1.2.88 In an AP if the m^{th} is n and the n^{th} term is m, where $m \neq n$, find the p^{th} term.

1.2.89 If the sum of n terms of an AP is

$$nP + \frac{1}{2}n(n-1)Q,$$

where P and Q are constants, find the common difference.

1.2.90 The sum of n terms of two arithmetic progressions are in the ratio $(3n + 8) : (7n + 15)$. Find the ratio of their 12^{th} terms.

1.2.91 The income of a person is ₹3,00,000 in the first year and he receives an increase of ₹10,000 to his income per year for the next 19 years. Find the total amount he received in 20 years.

1.2.92 Insert 6 numbers between 3 and 24 such that the resulting sequence is an AP.

1.2.93 Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

1.2.94 Find the sum of odd integers from 1 to 2001.

1.2.95 If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the AM between a and b, then find the value of n.

1.2.96 Find the sum of all two digit numbers which when divided by 4, yields 1 as remainder.

1.2.97 A farmer buys a used tractor for Rs 12000. He pays Rs 6000 cash and agrees to pay the balance in annual instalments of Rs 500 plus 12% interest on the unpaid amount. How much will the tractor cost him?

1.2.98 Shyam Anand buys a scooter for Rs 22000. He pays Rs 4000 cash and agrees to pay the balance in annual instalment of Rs 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

1.2.99 A man deposited Rs 10000 in a bank at the rate of 5% simple interest annually. Find the amount in the 15^{th} year since he deposited the amount and also calculate the total amount after 20 years.

2 Geometric Progression

2.1 Formulae

2.2 NCERT

2.2.1. Find the 20^{th} and n^{th} terms of the GP: $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots,$

2.2.2. The 5^{th}, 8^{th} and 11^{th} terms of a GP are p, q and s, respectively. Show that $q^2 = ps$.

2.2.3. The 4^{th} term of a GP is square of its second term, and the first term is -3. Determine its 7^{th} term.

2.2.4. Find the sum to n terms of the following series

a) $a_n = 2^n$

b) $n^2 + 2^n$

2.2.5. Which term of the following sequences

a) 2, 2, $\sqrt{2}$, 4,..., is 128 ?

b) $\sqrt{3}, 3, 3\sqrt{3}, \ldots,$ is 729?

c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots,$ is $\frac{1}{19683}$?

2.2.6. For what values of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in GP?

Find the sum to indicated number of terms in each of the geometric progressions.

2.2.7. 0.15, 0.015, 0.0015,..., 20 terms.

2.2.8. $\sqrt{7}, \sqrt{21}, \sqrt[3]{7}, \ldots, n$ terms.

2.2.9. $1, -a, a^2, -a^2, a^3, \ldots, n$ terms $(a \neq -1)$.

2.2.10. $x^3, x^5, x^7, \ldots, n$ terms $(x \neq \pm 1)$.

2.2.11. Evaluate

$$\sum_{k=1}^{11} \left(2 + 3^k\right).$$

2.2.12. The sum of first three terms of a GP is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

2.2.13. How many terms of GP 3, $3^2, 3^3, \ldots$, are needed to give the sum 120?

2.2.14. The sum of first three terms of a GP is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the GP

2.2.15. Given a GP with $a = 729$ and 7^{th} term 64, determine S_7.

2.2.16. If the 4^{th}, 10^{th} and 16^{th} terms of a GP are x, y and z, respectively. Prove that x, y, z are in GP.

2.2.17. Find the sum to n terms of the sequence, $8, 88, 888, 8888, \ldots$.

2.2.18. Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32, and 128, 32, 8, 2, $\frac{1}{2}$.

2.2.19. Show that the products of the corresponding terms of the sequences $a, ar, ar^2, \ldots, ar^{n-1}$ and $A, AR, AR^2, \ldots, AR^{n-1}$ form a GP and find the common ratio.

2.2.20. If the p^{th}, q^{th} and r^{th} terms of a GP are a, b and c, respectively. Prove that

$$a^{q-r} b^{r-p} c^{p-q} = 1.$$

2.2.21. If the first and the n^{th} term of a GP are a and b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

2.2.22. If a, b, c and d are in GP show that $\left(a^2 + b^2 + c^2\right)\left(b^2 + c^2 + d^2\right) = (ab + bc + cd)^2$.

2.2.23. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour ?

2.2.24. What will Rs 500 amount to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

2.2.25. If AM and GM of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

2.2.26. If f is a function satisfying

$$f(x + y) = f(x) f(y) \; \forall x, y \in N$$

such that

$$f(1) = 3 \text{ and } \sum_{x=1}^{n} f(x) = 120,$$

find the value of n.

2.2.27. The sum of some terms of GP is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number of terms.

2.2.28. The first term of a GP is 1. The sum of the third term and fifth term is 90. Find the common ratio of the GP.

2.2.29. The sum of three numbers in GP is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

2.2.30. A GP consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

2.2.31. The sum of the first four terms of an AP is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.

2.2.32. If

$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}\,(x \neq 0),$$

then show that a, b, c and d are in GP.

2.2.33. Let S be the sum, P the product and R the sum of reciprocals of n terms in a GP. Prove that $P^2R^n = S^n$.

2.2.34. The p^{th}, q^{th} and r^{th} terms of an AP are a, b, c, respectively. Show that

$$(q-r)\,a + (r-p)\,b + (p-q)\,c = 0.$$

2.2.35. If

$$a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$$

are in AP, prove that a, b, c are in AP

2.2.36. If a, b, c, d are in GP prove that

$$(a^n + b^n), (b^n + c^n), (c^n + d^n)$$

are in GP.

2.2.37. If a and b are the roots of

$$x^2 - 3x + p = 0$$

and c, d are roots of

$$x^2 - 12x + q = 0,$$

where a, b, c, d form a GP, prove that

$$(q+p) : (q-p) = 17 : 15.$$

2.2.38. The ratio of the AM and GM of two positive numbers a and b, is $m : n$. Show that

$$a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right).$$

2.2.39. If a, b, c are in AP; b, c, d are in GP and

$$\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$$

are in AP prove that a, c, e are in GP.

2.2.40. Find the sum of the following series up to n terms

a) $5 + 55 + 555 + \ldots$,
b) $.6 + .66 + .666 + \ldots$

2.2.41. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8^{th} set of letter is mailed.

2.2.42. A manufacturer reckons that the value of a machine, which costs him Rs. 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

2.2.43. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on.It took 8 more days to finish the work. Find the number of days in which the work was completed.

2.2.44. Find the n^{th} and n^{th} terms of the GP: $5, 25, 125, \ldots$

2.2.45. Which term of the GP: $2, 8, 32, \ldots$ upto n terms is 131072.

2.2.46. In a GP the 3^{rd} term is 24 and the 6^{th} term is 192. Find the 10^{th} term.

2.2.47. Find the sum of the first n terms and the sum of the first 5 terms of the series

$$1 + \frac{2}{3} + \frac{4}{9} + \ldots$$

2.2.48. How many terms of the GP:

$$3 + \frac{3}{2} + \frac{3}{4} + \ldots$$

are needed to give the sum $\frac{3069}{512}$.

2.2.49. The sum of the first 3 terms of a GP is $\frac{13}{12}$ and their product is -1. Find the common ratio and the terms.

2.2.50. Find the sum of the sequence $7, 77, 777, \ldots$ to n terms.

2.2.51. A person has 2 parents, 4 grandparents, 8 great grand parents and so on. Find the number of his ancestors during the ten generations preceeding his own.

2.2.52. Insert 3 numbers between 1 and 256 so that the resulting sequence is a GP.

2.2.53. If the AM and GM of two positive numbers a and b are 10 and 8 respectively, find the numbers.

2.2.54. Find the 12^{th} term of a GP whose 8^{th} term is 192 and the common ratio is 2.

2.2.55. Find a GP for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

2.2.56. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4^{th} by 18.

2.2.57. Show that the ratio of the sum of first n terms of a GP to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

2.2.58. Insert two numbers between 3 and 81 so that the resulting sequence is GP

2.2.59. Find the value of n so that

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$$

may be the geometric mean between a and b.

2.2.60. The sum of two numbers is 6 times their geometric mean, show that numbers are in

the ratio

$$\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right).$$

2.2.61. If A and G be AM and GM, respectively between two positive numbers, prove that the numbers are

$$A \pm \sqrt{(A+G)(A-G)}.$$

2.2.62. If the p^{th}, q^{th}, r^{th} and s^{th} terms of an AP are in GP, then show that $(p-q), (q-r), (r-s)$ are also in GP.

2.2.63. If a, b, c are in GP and $a^{\frac{1}{x}} b^{\frac{1}{y}} c^{\frac{1}{z}}$, prove that x, y, z are in AP.

2.2.64. If a, b, c, d, p are different real numbers such that

$$\left(a^2+b^2+c^2\right)p^2 - 2(ab+bc+cd)p + \left(b^2+c^2+d^2\right) \leq 0,$$

then show that a, b, c and d are in GP.

2.2.65. If p, q, r are in GP and the equations

$$px^2 + qx + r = 0, \quad dx^2 + 2ex + f = 0$$

have a common root, then show that

$$\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$$

are in AP.

3 Z Transform

3.1 Formulae

3.2 NCERT

3.2.1 Find the sum to n terms of the series: $5 + 11 + 19 + 29 + 41 + \ldots$

3.2.2 Find the sum to n terms of the series whos n^{th} term is $n(n+3)$.

3.2.3 Find the sum to n terms of each of the series

a) $n(n+2)$

b) $n\left(\frac{n^2+5}{4}\right)$

c) $1\times 2 + 2\times 3 + 3\times 4 + 4\times 5 + \ldots$

d) $1\times 2\times 3 + 2\times 3\times 4 + 3\times 4\times 5 + \ldots$

e) $3\times 1^2 + 5\times 2^2 + 7\times 3^2 + \ldots$

f) $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots$

g) $5^2 + 6^2 + 7^2 + \cdots + 20^2$

h) $3\times 8 + 6\times 11 + 9\times 14 + \ldots$

i) $1^2 + (1^2+2^2) + (1^2+2^2+3^2) + \ldots$

j) $n(n+1)(n+4)$.

k) $(2n-1)^2$

l) $(n-1)(2-n)(3+n)$

3.2.4 Find the sum to n terms of the following series

a) $a_n = (-1)^{n-1} 5^{n+1}$

b) $a_n = (-1)^{n-1} n^3$

c) $a_n = \frac{n^2}{2^n}$

3.2.5 Obtain the closed form expression for the following

a) $a_1 = 1, a_n = a_{n-1} + 2 \quad \forall n > 1$

b) $a_1 = 3, a_n = 3a_{n-1} + 2 \quad \forall n > 1$
c) $a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$
d) The Pingala sequence, defined by

$$a_n = a_{n-1} + a_{n-2}, \quad n > 2, a_1 = a_2 = 1$$

3.2.6 Find the sum of the following series up to n terms

$$\frac{1^3}{1} + \frac{1^3 + 2^2}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \dots$$

3.2.7 Show that

$$\frac{1 \times 2^2 + 2 \times 3^2 + \dots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + \dots + n^2 \times (n+1)} = \frac{3n + 5}{3n + 1}.$$

3.2.8 Find the 20^{th} term of the series: $2 \times 4 + 4 \times 6 + 6 \times 8 + \dots$
3.2.9 Find the sum of the first n terms of the series: $3 + 7 + 13 + 21 + 31 + \dots$,
3.2.10 If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes, respectively, show that

$$9S_2^2 = S_3 (1 + 8S_1).$$