Question-1.5.10

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Question

Find the ratio in which the line segment joining the points $\mathbf{A}(1, -5)$ and $\mathbf{B}(-4, 5)$ is divided by X-axis. Also,find the coordinates of the point of division.

Solution

Let the given points be A and B

$$\mathbf{A} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

Let the X-axis divide the line segment $\overline{\mathbf{AB}}$ at point $\mathbf{P}$ in the ratio $k : 1$. Since $\mathbf{P}$ lies on X-axis, let

$$\mathbf{P} = \begin{pmatrix} x \\ 0 \end{pmatrix}$$

The point $\mathbf{A}$, $\mathbf{B}$, $\mathbf{P}$ are collinear.

$$\implies \text{rank} \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{P} - \mathbf{A} \end{pmatrix} = 1 \tag{1}$$

Solution

$$\begin{pmatrix} -5 & x-1 \\ 10 & 5 \end{pmatrix} R_1 \to R_1 + \frac{1}{2}R_2 \begin{pmatrix} 0 & x-\frac{3}{2} \\ 10 & 5 \end{pmatrix} R_1 \leftrightarrow R_2 \begin{pmatrix} 10 & 5 \\ 0 & x-\frac{3}{2} \end{pmatrix} \quad (2)$$

The number of nonzero rows in the row reduced matrix (also known as *echelon form*) is defined as the rank. For above matrix to be of rank 1,

$$x + \frac{3}{2} = 0 \quad (3)$$

$$x = \frac{-3}{2} \quad (4)$$

$\therefore$ The coordinates of the point of intersection are

$$\mathbf{P} = \begin{pmatrix} \frac{-3}{2} \\ 0 \end{pmatrix}$$

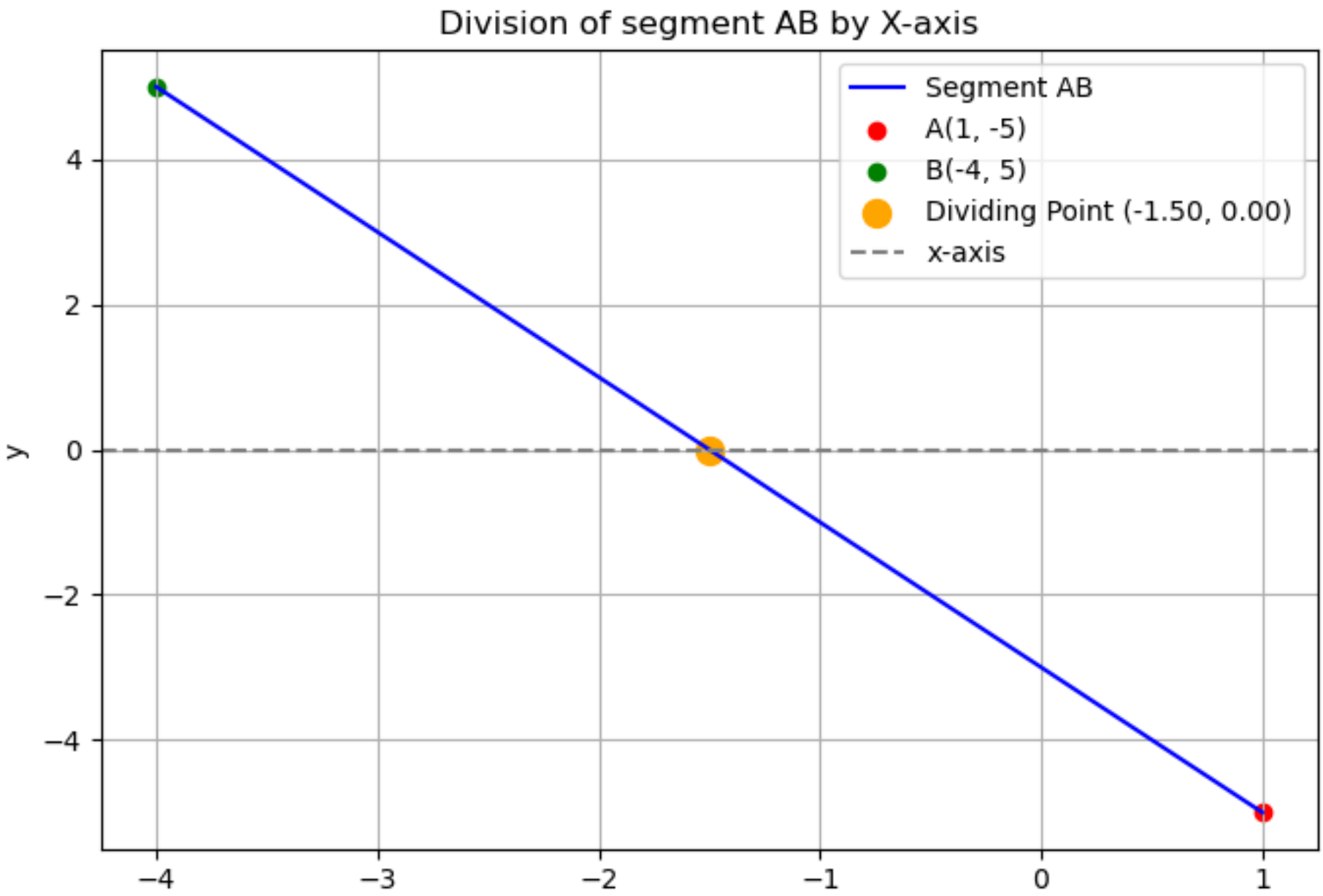
Solution

Substituting the values of **A**, **B** and **P**,

$$k = \frac{\begin{pmatrix} \frac{5}{2} & -5 \end{pmatrix} \begin{pmatrix} \frac{5}{2} \\ -5 \end{pmatrix}}{\left\| \begin{pmatrix} \frac{5}{2} \\ -5 \end{pmatrix} \right\|^2} = 1 \tag{5}$$

Thus, the ratio in which the point **P** divides the line segment **AB** is **1:1**.

graph


Division of segment AB by X-axis
Segment AB
A(1, -5)
B(-4, 5)
Dividing Point (-1.50, 0.00)
x-axis
y
4
2
0
−2
−4
−4
−3
−2
−1
0
1

C-Code

```c
#include <stdio.h>

void division_point(double *A, double *B, double *P, double *k) {
    *k = -A[1] / B[1];
    P[0] = ((*k) * B[0] + A[0]) / ((*k) + 1);
    P[1] = 0;
}

int main() {
    double A[2] = {1, -5};
    double B[2] = {-4, 5};
        double P[2];
    double k;
     division_point(A, B, P, &k);
    printf("Ratio: %f : 1\n", k);
    printf("Division Point: (%f, %f)\n", P[0], P[1]);
    return 0;
}
```

Python-Code

```
# Code by GVV Sharma
# Modified for Problem Solution
# Released under GNU GPL
# Calculating area enclosed between curves
import ctypes
import numpy as np

lib = ctypes.cdll.LoadLibrary('./code.so')

lib.division_point.argtypes = [
    ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double)]
lib.division_point.restype = None
```

Python-Code

```
def get_points():
    A = np.array([1, -5], dtype=np.double)
    B = np.array([-4, 5], dtype=np.double)
    P = np.zeros(2, dtype=np.double)
    k = ctypes.c_double()
    lib.division_point(A.ctypes.data_as(ctypes.POINTER(ctypes.
        c_double)),
                       B.ctypes.data_as(ctypes.POINTER(ctypes.
                           c_double)),
                       P.ctypes.data_as(ctypes.POINTER(ctypes.
                           c_double)),
                       ctypes.byref(k))
    return P, k.value, A, B
```

Python-Code

```
import matplotlib.pyplot as plt
import numpy as np
from call import get_points

P, k, A, B = get_points()

plt.figure(figsize=(7,5))
plt.plot([A[0], B[0]], [A[1], B[1]], 'b-', label='Segment AB')
plt.scatter(A[0], A[1], color='red', label='A(1, -5)')
plt.scatter(B[0], B[1], color='green', label='B(-4, 5)')
plt.scatter(P[0], P[1], color='orange', s=100, label=f'Dividing
    Point ({P[0]:.2f}, {P[1]:.2f})')
```

Python-Code

```
plt.axhline(0, color='gray', ls='--', label='x-axis')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Division of segment AB by X-axis')
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()
```