GATE MA 2025

EE25BTECH11030-AVANEESH

1) Ravi had __________ younger brother who taught at __________ university. He was widely regarded as __________ honorable man. Select the option with the correct sequence of articles.

a) a; a; an
b) the; an; a
c) a; an; a
d) an; an; a

(GATE MA 2025)

2) The CEO's decision to downsize the workforce was considered myopic because it sacrificed long-term stability to accommodate short-term gains. Select the most appropriate replacement for "myopic".

a) visionary
b) shortsighted
c) progressive
d) innovative

(GATE MA 2025)

3) The average marks obtained by a class in an examination were 30.8. However, one student's marks were incorrectly entered as 24 instead of 42. After correction the average is 31.4. How many students does the class have?

a) 25
b) 28
c) 30
d) 32

(GATE MA 2025)

4) Consider relations(:) P is the brother of Q. S is the daughter of Q. T is the sister of S. R is the mother of Q.
Statements(:) 1. R is grandmother of S. 2. P is uncle of S and T. 3. R has only one son.4.Q$hasonlyonedaughter$.

a) Both 1 and 2 are true
b) Both 1 and 3 are true
c) Only 3 is true
d) Only 4 is true

(GATE MA 2025)

5) According to the map shown, which is correct?

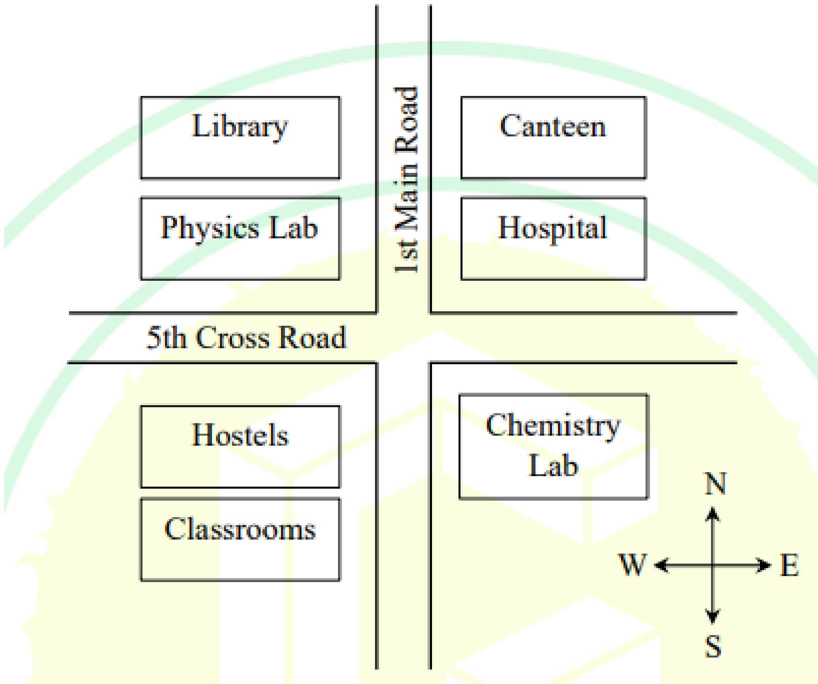


Fig. 1. *

Note:The figure shown is representative

a) The library is northwest of the canteen
b) The hospital is east of the chemistry lab
c) The chemistry lab is southeast of the physics lab
d) The classrooms and canteen are next to each other

(GATE MA 2025)

6) "I put the brown paper in my pocket along with the chalks, and possibly other things. I suppose every one must have reflected how primeval and how poetical are the things that one carries in one's pocket: the pocket-knife, for instance the type of all human tools, the infant of the sword. Once I planned to write a book of poems entirely about the things in my pocket. But I found it would be too long: and the age of the great epics is past." (From G.K. Chesterton's "A Piece of Chalk")

Based only on the information provided in the above passage, which one of the following statements is true?

a) The author of the passage carries a mirror in his pocket to reflect upon things.
b) The author of the passage had decided to write a poem on epics.
c) The pocket-knife is described as the infant of the sword.
d) Epics are described as too inconvenient to write.

(GATE MA 2025)

7) In the diagram, the lines QR and ST are parallel to each other. The shortest distance between these two lines is half the shortest distance between the point P and line QR. What is the ratio of the area of the triangle PST to the area of the trapezium SQRT?

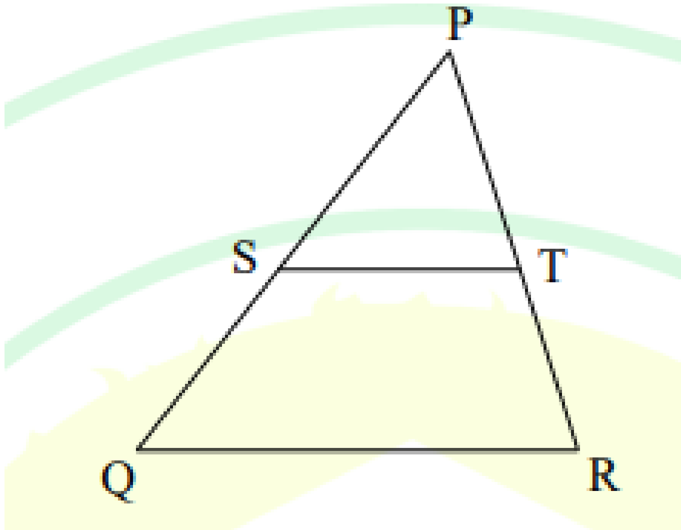


Fig. 2. *

Note:The figure shown is representative

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{2}{5}$ d) $\frac{1}{2}$

(GATE MA 2025)

8) A fair six-faced dice, with the faces labelled '1', '2', '3', '4', '5', and '6', is rolled thrice. What is the probability of rolling '6' exactly once?

a) $\frac{75}{216}$ b) $\frac{1}{6}$ c) $\frac{1}{18}$ d) $\frac{25}{216}$

(GATE MA 2025)

9) A square paper, shown in figure (I), is folded along the dotted lines as shown in the figures (II) and (III). Then a few cuts are made as shown in figure (IV). Which one of the following patterns will be obtained when the paper is unfolded?

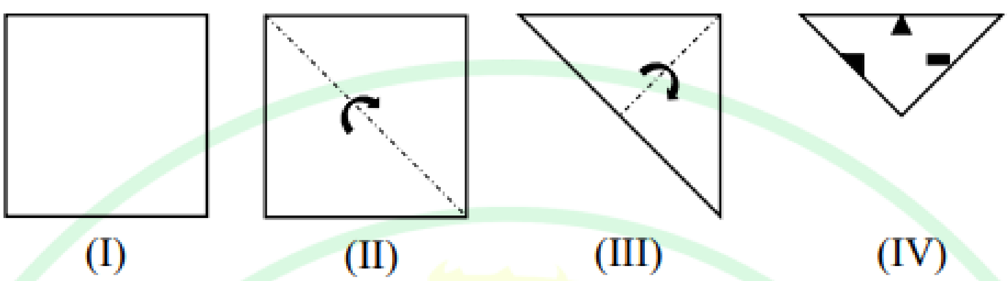
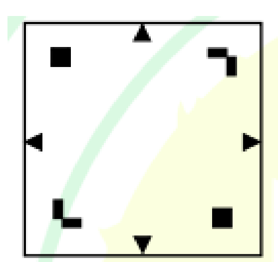
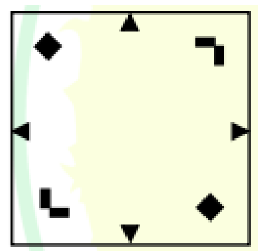


Fig. 3. *

Note:The figure shown are representative

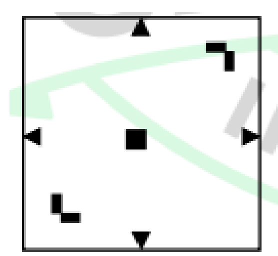


a)



b)

c)



d)

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10) A shop has 4 distinct flavors of ice-cream. One can purchase any number of scoops of any flavor. The order in which the scoops are purchased is inconsequential. If one wants to purchase 3 scoops of ice-cream, in how many ways can one make that purchase?

a) 4 b) 20 c) 24 d) 48

(GATE MA 2025)

11) Let $S = \left\{ w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \in \mathbb{R}^3 : \begin{pmatrix} 1 & 1 & 1 \\ -3 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \text{ diagonalizable, } |w| = 1 \right\}$. Which is true?

a) S is compact and connected
b) S is neither compact nor connected
c) S is compact but not connected
d) S is connected but not compact

(GATE MA 2025)

12) Given that the Laplace transforms of $J_0(x)$, $J_0'(x)$ and $J_0''(x)$ exist, where $J_0(x)$ is the Bessel function. Let $Y = Y(s)$ be the Laplace transform of the Bessel function $J_0(x)$. Then, which one of the following is TRUE?

a) $\frac{d^Y}{ds} + \frac{2sY}{s^2+1} = 0, s > 0$

b) $\frac{dY}{ds} - \frac{2sY}{s^2+1} = 0, s > 0$
c) $\frac{dY}{ds} - \frac{sY}{s^2+1} = 0, s > 0$
d) $\frac{dY}{ds} + \frac{sY}{s^2+1} + 1 = 0, s > 0$

(GATE MA 2025)

13) To find a real root of the equation $x^3 + 4x^2 - 10 = 0$ in the interval $(1, \frac{3}{2})$ by using the fixed-point iteration scheme, consider the following two statements:
$S_1 : x_{k+1} = \sqrt{\frac{10}{4+x_k}}, x_0 \in \left(1, \frac{3}{2}\right)$ converges. $S_2 : x_{k+1} = \sqrt{10 - x_k}$ diverges for some x_0 in the same range.
a) S1 true, S2 false
b) S2 true, S1 false
c) Both true
d) Neither true

(GATE MA 2025)

14) For the Linear programming problem:

$$\max Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

, Subject to $ax_1 + x_2 + x_3 = 4, x_1 + \beta x_2 + x_4 = 8, x_i \geq 0$.
consider the following Statements: S_1 : If $a = 2, \beta = 1$, then$(x_1, x_2)^T$ is optimal basis. S_2 : If $a = 1, \beta = 4$, then$(x_3, x_2)^T$ is optimal basis.
a) S1 true, S2 false
b) S2 true, S1 false
c) Both true
d) Neither true

(GATE MA 2025)

15) Consider the following subsets of the Euclidean space $\mathbb{R}^4$:

$$S = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \colon x_1^2 + x_2^2 + x_3^2 - x_4^2 = 0\}$$

,

$$T = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \colon x_1^2 + x_2^2 + x_3^2 - x_4^2 = 1\}$$

,

$$U = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \colon x_1^2 + x_2^2 + x_3^2 - x_4^2 = -1\}$$

. Which is true?
a) S connected, T,U not
b) T,U connected, S not
c) S,U connected, T not
d) S,T connected, U not

(GATE MA 2025)

16) Consider the system of ordinary differential equations

$$\frac{dX}{dt} = MX$$

, where M is 6×6 skew-symmetric in $\mathbb{R}$. Then the origin is stable critical point for:

a) any such matrix M
b) only such matrices M whose rank is 2
c) only such matrices M whose rank is 4
d) only such matrices M whose rank is 6

(GATE MA 2025)

17) Let $X = \{f \in C[0,1] : f(0) = 0 = f(1)\}$ with the norm $|f|_\infty = \sup_{0<t\leq 1} |f(t)|$, where $C[0,1]$ is the space of all real-valued continuous functions on $[0,1]$. Let $Y = C[0,1]$ with the norm $|f|_2 = \left(\int_0^1 |f(t)|^2\, dt\right)^{\frac{1}{2}}$. Let U_x and U_y be the closed unit balls in X and Y centred at the origin, respectively. Consider $T : X \to \mathbb{R}$ and $S : Y \to \mathbb{R}$ given by
$Tf = \int_0^1 f(t)dt$ and $Sf = \int_0^1 f(t)dt$.
Consider the following statements:
S1: $\sup_{f\in U_x} |Tf|$ is attained at a point of U_x. S2: $\sup_{f\in U_y} |Sf|$ is attained at a point of U_y.
Then, which one of the following is correct?

a) S1 is TRUE and S2 is FALSE
b) S2 is TRUE and S1 is FALSE
c) both S1 and S2 are TRUE
d) neither S1 nor S2 is TRUE

(GATE MA 2025)

18) Let $g(x,y) = f(x,y)e^{2x+3y}$ be defined in $\mathbb{R}^2$, where $f(x,y)$ is a continuously differentiable non-zero homogeneous function of degree 4. Then,

$$\frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} = 0$$

holds for

a) all points (x,y) in $\mathbb{R}^2$
b) all points (x,y) on the line given by $2x+3y+4 = 0$
c) all points (x,y) in the region of $\mathbb{R}^2$ except on the line $2x+3y+4 = 0$
d) all points (x,y) on the line given by $2x+3y = 0$

(GATE MA 2025)

19) The partial differential equation

$$\left(1+x^2\right)\frac{\partial^2 u}{\partial x^2} + 2x\left(1-y^2\right)\frac{\partial^2 u}{\partial y \partial x} + \left(1-y^2\right)\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + \left(1-y^2\right)\frac{\partial u}{\partial y} = 0$$

is

a) elliptic in the region $\{(x,y) \in \mathbb{R}^2 : |y| \leq 1\}$
b) hyperbolic in the region $\{(x,y) \in \mathbb{R}^2 : |y| > 1\}$
c) elliptic in the region $\{(x,y) \in \mathbb{R}^2 : |y| > 1\}$
d) hyperbolic in the region $\{(x,y) \in \mathbb{R}^2 : |y| < 1\}$

(GATE MA 2025)

20) Let $u(x,t)$ be the solution of the following initial-boundary value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad x \in (0,\pi),\, t > 0,$$

with $u(0,t) = u(\pi,t) = 0$, $u(x,0) = \sin 4x \cos 3x$.
Then, for each $t > 0$, the value of $u(0,t)$ is
a) $\frac{e^{-49t}}{2\sqrt{2}}\left(e^{48t} - 1\right)$
b) $\frac{e^{-49t}}{2\sqrt{2}}\left(1 - e^{48t}\right)$
c) $\frac{e^{-49t}}{2\sqrt{2}}\left(1 + e^{48t}\right)$
d) $\frac{e^{-49t}}{4\sqrt{2}}\left(1 - e^{48t}\right)$

(GATE MA 2025)

21) Consider the function F: $\mathbb{R}^2 \to \mathbb{R}^2$ given by

$$F(x,y) = (x–3xy–3x, 3xy–y–3y).$$

Then, for the function F, the inverse function theorem is

a) applicable at all points of R2
b) not applicable at exactly one point of R2
c) not applicable at exactly two points of R2
d) not applicable at exactly three points of R2

(GATE MA 2025)

22) Let the functions f: R2 → R and g: R2 → R be given by $f(x1, x2) = x + x2 - 2x1x2$, $g(x1, x2) = 2x + 2x - x_1x_2$.
Consider the following statements: S_1 : For every compact subset K of R, $f_{-1}(K)$ is compact. S_2 : For every compact subset K of R, $g_{-1}(K)$ is compact.
Then, which one of the following is correct?

a) S1 is TRUE and S2 is FALSE
b) S2 is TRUE and S1 is FALSE
c) both S1 and S2 are TRUE
d) neither S1 nor S2 is TRUE

(GATE MA 2025)

23) Let $pa(x)$ denote the characteristic polynomial of a square matrix A. Then, for which of the following invertible matrices M, the polynomial $pM(x)|pM_{-1}(x)$ is constant?

a) $M = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$
b) $M = \begin{pmatrix} 3 & 4 \\ 2 & 2 \end{pmatrix}$
c) $M = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$
d) $M = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

(GATE MA 2025)

24) Consider the balanced transportation problem with three sources S1, S2, S3, and four destinations D1, D2, D3, D4, for minimizing the total transportation cost whose cost matrix is as follows:

	D1	D2	D3	D4	Supply
S1	2	6	20	11	$\alpha + 10$
S2	12	7	4	10	$\alpha + \lambda + 10$
S3	8	14	16	11	5
Demand	$\alpha + 5$	10	$\lambda + 5$	$\alpha + \lambda$	

TABLE I
*

where $\alpha, \lambda > 0$. If the associated cost to the starting basic feasible solution obtained by using the North-West corner rule is 290, then which of the following is/are correct?

a) $\alpha^2 + \lambda^2 = 100$
b) $\alpha^2 + \alpha\lambda = 150$
c) The optimal cost of the transportation problem is 260
d) The optimal cost of the transportation problem is 290

(GATE MA 2025)

25) Consider the following regions:
$S_1 = \{(x_1, x_2) \in \mathbb{R}^2 : 2x_1 + x_2 \leq 4, x_1 + 2x_2 \leq 5, x_1, x_2 \geq 0\}$,
$S_2 = \{(x_1, x_2) \in \mathbb{R}^2 : 2x_1 - x_2 \leq 5, x_1 + 2x_2 \leq 5, x_1, x_2 \geq 0\}$.
Then, which of the following is/are TRUE?

a) The maximum value of $x1 + x2$ is 3 on the region S2
b) The maximum value of $x1 + x2$ is 5 on the region $S2 - S1$
c) The maximum value of $x1 + x2$ is 3 on the region $S1 \cap S2$
d) The maximum value of $x1 + x2$ is 4 on the region $S1 \cup S2$

(GATE MA 2025)

26) Let f: R2 – (0,0) → R be a function defined by

$f(x, y) = \dfrac{x^2 - y^2}{x^2 + y^2} + x \sin \frac{1}{(x^2+y^2)}$.
Consider the following three statements: S1: $\lim_{x\to 0} \lim_{y\to 0} f(x, y)$ exists. S2: $\lim_{y\to 0} \lim_{x\to 0} f(x, y)$ exists. S3: $\lim_{(x,y)\to(0,0)} f(x, y)$ exists.
Then, which of the following is/are correct?

a) S2 and S3 are TRUE and S1 is FALSE
b) S1 and S2 are TRUE and S3 is FALSE
c) S1 and S3 are TRUE and S2 is FALSE
d) S1, S2 and S3 all are TRUE

(GATE MA 2025)

27) Let M be a 7×7 matrix with entries in R and having the characteristic polynomial $c_M(x) = (x - 1)(x - 2)(x - 3)^2$.
Let $\text{rank}(M - I_7) = \text{rank}(M - 2I_7) = \text{rank}(M - 3I_7) = 5$, where I_7 is the 7×7 identity matrix. If $m_M(x)$ is the minimal polynomial of M, then $m_M(5)$ is equal to __________ (in integer).
(GATE MA 2025)

28) Let $y = P(x)$ be the unique polynomial of degree n satisfying the Legendre differential equation
$$(1 - x^2)y'' - 2xy' + n(n + 1)y = 0,$$
and $y(1) = 1$. Then, the value of $P_{11}(1)$ is equal to __________ (in integer).
(GATE MA 2025)

29) Let $\mathbf{a}$ be a unit vector parallel to the tangent at the point $P(1, 1, \sqrt{2})$ to the curve of intersection of the surfaces $2x^2 + 3y^2 - z^2 = 3$ and $x^2 + y^2 = z^2$. Then, the absolute value of the directional derivative of $f(x, y, z) = x^2 + 2y^2 - 2\sqrt{11}z$ at P in the direction of $\mathbf{a}$ is equal to __________ (in integer).
(GATE MA 2025)

30) The volume of the region bounded by the cylinders $x^2 + y^2 = 4$ and $x^2 + z^2 = 4$ is equal to __________ (rounded off to TWO decimal places).
(GATE MA 2025)

31) Let W be the vector space (over R) consisting of all bounded real-valued solutions of the differential equation
$$\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} + y = 0.$$
Then, the dimension of W is equal to __________ (in integer).
(GATE MA 2025)

32) Let $F = (y - z)\mathbf{i} + (z - x)\mathbf{j} + (x - y)\mathbf{k}$ be a vector field, and let S be the surface $x^2 + y^2 + (z - 1)^2 = 9$, $1 \le z \le 4$. If n denotes the unit outward normal vector to S, then the value of
$$\frac{1}{\pi}\left|\iint_S (\nabla \times F) \cdot n \, dS\right|$$
is equal to __________ (in integer).
(GATE MA 2025)

33) $I = \frac{1}{2\pi i} \int_C \frac{\sin z}{1-\cos(z^3)} dz$, where $C = \{z \in C : z = x + iy, |x| + |y| = 1, x, y \in R\}$ is oriented positively as a simple closed curve. Then the value of $120I$ is equal to __________ (in integer).
(GATE MA 2025)

34) Let $\alpha, \beta, \gamma, \delta \in R$ be such that the quadrature formula
$$\int_{-1}^{1} f(x)dx = \alpha f(-1) + \beta f(1) + \gamma f'(-1) + \delta f'(1)$$
is exact for all polynomials of degree less than or equal to 3. Then, $9(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to __________ (in integer).
(GATE MA 2025)

35) Let $y(x)$ be the solution of the initial value problem

$$\frac{dy}{dx} = \sin(\pi(x+y)), \quad y(0) = 0.$$

Using Euler's method with step-size $h = 0.5$, the approximate value of $y(1.5) + 2y(1)$ is equal to __________ (in integer).

(GATE MA 2025)

36) Consider the linear system $Ax = b$, where $A = [a_{ij}], i, j = 1, 2, 3$, and $a_{ii} \neq 0$ for $i = 1, 2, 3$, is a matrix with entries in $\mathbb{R}$. For

$$D = \begin{pmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{pmatrix},$$

let

$$D^{-1}A = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 1 & 1 \\ 4 & 0 & 0 \end{pmatrix}, \quad D^{-1}b = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}.$$

S1: The approximation of x after one iteration of the Jacobi scheme with initial vector $x^{(0)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is $x_1 = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix}$. S2: There exists an initial vector $x^{(0)}$ for which Jacobi iterative scheme diverges.

Then, which one of the following is correct?

a) S1 is TRUE and S2 is FALSE
b) S2 is TRUE and S1 is FALSE
c) both S1 and S2 are TRUE
d) neither S1 nor S2 is TRUE

(GATE MA 2025)

37) Let $y(x)$ be the solution of the differential equation

$$x^2y'' + 7xy' + 9y = x^3 \log x, \quad x > 0,$$

satisfying $y(1) = 0$ and $y'(1) = 0$. Then, the value of $y(e)$ is equal to

a) $\frac{1}{3}e^{-3}$
b) $\frac{1}{6}e^{-3}$
c) $\frac{2}{3}e^{-3}$
d) $\frac{1}{2}e^{-3}$

(GATE MA 2025)

38) Let $y_1(x)$ and $y_2(x)$ be the two linearly independent solutions of the differential equation

$$(1 + x^2)y'' - xy' + (\cos^2 x)y = 0,$$

satisfying the initial conditions $y_1(0) = 3, y_1'(0) = -1$ and $y_2(0) = -5, y_2'(0) = 2$.

Define $W(x) = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix}$.

Then, the value of $W(1)$ is

a) $\frac{\sqrt{5}}{4}$
b) $\frac{\sqrt{5}}{2}$
c) $\frac{2}{\sqrt{5}}$
d) $\frac{4}{\sqrt{5}}$

(GATE MA 2025)

39) Let C be the curve of intersection of the surfaces $z^2 = x^2 + y^2$ and $4x + z = 7$. If P is a point on C at minimum distance from the xy–plane, then the distance of P from the origin is

a) $\frac{7}{5}$
b) $\frac{7\sqrt{2}}{5}$
c) $\frac{14}{5}$
d) $\frac{14\sqrt{2}}{5}$

(GATE MA 2025)

40) Let $u(x,t)$ be the solution of the initial-value problem

$$\frac{\partial^2 u}{\partial t^2} - 9\frac{\partial^2 u}{\partial x^2} = 0, \quad x \in \mathbb{R},\ t > 0,$$

with initial conditions $u(x,0) = e^x$, $\frac{\partial u}{\partial t}(x,0) = \sin x$. Then, the value of $u(\pi,\pi)$ is

a) $\frac{1}{5}(e^\pi + \sin\pi)$
b) $\frac{1}{5}(e^\pi - \sin\pi)$
c) $\frac{1}{5}(e^\pi + 3)$
d) $\frac{1}{5}e^\pi$

(GATE MA 2025)

41) Let T be the Möbius transformation that maps the points $0, i, 1$ conformally onto the points $-3, \infty, 2$, respectively. If T maps the circle centred at 1 with radius k onto a straight line given by $ax+by+\gamma = 0$, then the value of

$$\frac{2k(\alpha+\beta)+\gamma}{\alpha+\beta-2k\gamma}$$

is equal to

a) $\frac{1}{7}$
b) $\frac{2}{7}$
c) $\frac{1}{3}$
d) $\frac{2}{13}$

(GATE MA 2025)

42) Let $U = \{z \in \mathbb{C} : \Im(z) > 0\}$ and $D = \{z \in \mathbb{C} : |z| < 1\}$. Let S be the set of all bijective analytic functions $f : U \to D$ such that $f(i) = 0$. Then, the value of $\sup_{f\in S} |f(4i)|$ is

a) 0
b) $\frac{1}{4}$
c) $\frac{1}{2}$
d) $\frac{3}{5}$

(GATE MA 2025)

43) Let Ω be a non-empty open connected subset of $\mathbb{C}$ and $f : \Omega \to \mathbb{C}$ a non-constant function. Define $f^2(z) = (f(z))^2$, $f^3(z) = (f(z))^3$.
S1: If f is continuous in Ω and f^2 is analytic in Ω, then f is analytic in Ω. S2: If f^2 and f^3 are analytic in Ω, then f is analytic in Ω.
Then, which one of the following is correct?

a) S1 is TRUE and S2 is FALSE
b) S2 is TRUE and S1 is FALSE
c) both S1 and S2 are TRUE
d) neither S1 nor S2 is TRUE

(GATE MA 2025)

44) Define on the sphere $S = \{(x_1,x_2,x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}$ the relation $(x_1,x_2,x_3) \sim (y_1,y_2,y_3)$ if $x_3 = y_3$.
Let $X = S/\sim$ with quotient topology. Then, which one of the following is TRUE?

a) X is homeomorphic to $\{x \in \mathbb{R} : -1 \le x \le 1\}$
b) X is homeomorphic to $\{(x_1,x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 = 1\}$
c) X is homeomorphic to $\{(x_1,x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 \le 1\}$
d) X is homeomorphic to $\{(x_1,x_2,x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 = 1, -1 \le x_3 \le 1\}$

(GATE MA 2025)

45) Consider $X = (C[-1,1], |\cdot|_\infty)$, $Y = (C[-1,1], |\cdot|_2)$ the spaces of continuous functions with sup- and L^2-norms.
Let W be the linear span of all Legendre polynomials.
Then which of the following is correct?

a) W is dense in X but not in Y
b) W is dense in Y but not in X
c) W is dense in both X and Y
d) W is dense neither in X nor in Y

(GATE MA 2025)

46) Consider metric spaces $X = (\mathbb{R}, d_1), Y = ([0,1], d_2)$ with $d_1(x,y) = |x-y|, d_2(x,y) = |x-y|$. Then which is TRUE?

a) $[0,1)$ is open in X but not in Y
b) $[0,1)$ is open in Y but not in X
c) $[0,1)$ is open in both X and Y
d) $[0,1)$ is open neither in X nor in Y

(GATE MA 2025)

47) Let K be an algebraically closed field containing a finite field F. Let L be the subfield of K consisting of elements of K algebraic over F.
S1: L is algebraically closed. S2: L is infinite.
Which one is correct?

a) S1 TRUE, S2 FALSE
b) S2 TRUE, S1 FALSE
c) both TRUE
d) neither TRUE

(GATE MA 2025)

48) Let $M_2(\mathbb{R})$ be the space of 2×2 reals. Define $T(X) = AXB$ with matrices
$A = \begin{pmatrix} 1 & -4 \\ 6 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 0 \\ -1 & -1 \end{pmatrix}$.
Let P be the representation matrix of T. Which of the following are TRUE?
a) P is invertible
b) The trace of P is 25
c) The rank of $(P^2 - 4I_4)$ is 4
d) The nullity of $(P - 2I_4)$ is 0

(GATE MA 2025)

49) Consider the LPP: Maximize $Z = 3x_1 + 5x_2$ subject to $x_1 + x_3 = 4, 2x_2 + x_4 = 12, 3x_1 + 2x_2 + x_5 = 18, x_i \geq 0$.
Given that (x_3, x_2, x_1) is optimal basis with $B^{-1} = \begin{pmatrix} \alpha & -\beta & \beta \\ \gamma & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.
If (p,q,r) is optimal dual solution, which are TRUE?
a) $\alpha + 3\beta + 2\gamma = 3$
b) $\alpha - 3\beta + 4\gamma = 1$
c) $p + q + r = \frac{5}{2}$
d) $p^2 + q^2 + r^2 = \frac{17}{4}$

(GATE MA 2025)

50) Let $0 < \alpha < 1$. Define

$$C^\alpha[0,1] = \{f : [0,1] \to \mathbb{R} : \sup_{s\neq t} \frac{|f(t) - f(s)|}{|t-s|^\alpha} < \infty\}.$$

This is Banach under norm $|f|_\alpha = |f(0)| + \sup_{s\neq t} \frac{|f(t)-f(s)|}{|t-s|^\alpha}$.

Let $C[0,1]$ with sup norm. Define $T : C^{\alpha}[0,1] \to C[0,1], Tf = f$. Then:

a) T is a compact linear map
b) Image of T is closed
c) Image of T is dense
d) T is not bounded

(GATE MA 2025)

51) PDE: $\frac{\partial u}{\partial t} + 3\frac{\partial u}{\partial x} = u,\ u(x,0) = \cos x$. Let v satisfy $\frac{\partial v}{\partial t} + 3\frac{\partial v}{\partial x} = v^2,\ v(x,0) = \cos x$. Which are TRUE?
a) $|u(x,t)| \le e^t$ for all x, t
b) $v(x,1)$ not defined for some x
c) $v(x,1)$ not defined for any x
d) $u(2\pi, \pi) = -e^{\pi}$

(GATE MA 2025)

52) Heat equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$; $u(0,t) = u(1,t) = 0, u(x,0) = 2x(1-x)$. Which are TRUE?
a) $0 \le u(x,t) \le 1/4$ all x, t
b) $u(x,t) = u(1-x,t)$
c) $\int_0^1 u(x,t)^2 dx$ decreasing in t
d) Same integral not decreasing

(GATE MA 2025)

53) $f(x,y) = (e^{2\pi x}\cos 2\pi y, e^{2\pi x}\sin 2\pi y)$. Which are TRUE?
a) If G open, then f(G) open
b) If G closed, then f(G) closed
c) If G dense, f(G) dense
d) f is surjective

(GATE MA 2025)

54) Let $\{x_k\}$ orthonormal in Hilbert space X. For fixed n, let $Y = \mathrm{span}\{x_1, \ldots, x_n\}$. Define $S_n(x) = \sum_{k=1}^{n} \langle x, x_k \rangle x_k$. Then:
a) $S_n(x)$ is orthogonal projection on Y
b) $S_n(x)$ is projection on $Y^{\perp}$
c) $x - S_n(x) \perp S_n(x)$
d) $\sum_{k=1}^{n} |\langle x, x_k \rangle|^2 = |x|^2$ for all x

(GATE MA 2025)

55) Sequence $f_1(x) = x/2,\ f_{n+1}(x) = f_n(x) - \frac{1}{2}(f_n(x)^2 - x)$. Then:
a) pointwise but not uniform convergence
b) uniform convergence
c) $\sqrt{x} - f_n(x) > \frac{2\sqrt{x}}{2+n\sqrt{x}}$
d) $0 \le f_n(x) \le \sqrt{x}$

(GATE MA 2025)

56) $u_n(x) = \sin(nx)/\sqrt{n}, x \in (0,\pi)$. Then:
a) $\sum u_n$ converges uniformly
b) $\sum u_n$ converges uniformly
c) converges pointwise not uniform
d) converges uniformly on compacts

(GATE MA 2025)

57) Let $S^1 = \{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 = 1\}$. For continuous non-constant $f : S^1 \to \mathbb{R}$:
a) maps closed sets to closed

b) is injective
c) is surjective
d) $\exists\lambda : f(\cos\lambda, \sin\lambda) = f(-\cos\lambda, -\sin\lambda)$

(GATE MA 2025)

58) Let X uncountable set with co-countable topology. Then:
a) compact subset closed
b) closed subset compact
c) X is T1 not T2
d) X is T2

(GATE MA 2025)

59) Statements about commutative rings:
P1: R is isomorphic to $R_1 \times R_2$ P2: $\exists r_1, r_2$ with $r_1^2 = r_1 \neq 0, r_2^2 = r_2 \neq 0, r_1 r_2 = 0, r_1 + r_2 = 1$ P3: $\exists I_1, I_2$ ideals, $R = I_1 + I_2, I_1 \cap I_2 = 0$ P4: $\exists a, b \neq 0$ with $ab = 0$
Which are TRUE?
a) P1 $\Leftrightarrow$ P2
b) P2 $\Rightarrow$ P3
c) P3 $\Rightarrow$ P4
d) P4 $\Rightarrow$ P1

(GATE MA 2025)

60) Let extensions $E \subset F \subset K$ not algebraic. Let $a \in K$ algebraic over F, not in F. Let L be subfield of K generated over E by coefficients of minimal polynomial of a over F. Then:
a) $F(a) \supset L(a)$ finite $\Leftrightarrow$ $F \supset L$ finite
b) $\dim L(a)/L > \dim F(a)/F$
c) $\dim L(a)/L < \dim F(a)/F$
d) $F(a) \supset L(a)$ alg. $\Leftrightarrow$ $F \supset L$ alg.

(GATE MA 2025)

61) Inner product $(f, g) = \int_{-1}^{1} f(x)g(x)dx$.
If $p(x) = \alpha+\beta x^2-30x^4$ is orthogonal to all polynomials degree ≤ 3, then $\alpha+5\beta$ is equal to __________ (in integer). (GATE MA 2025)

62) For $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, quadratic form

$$Q(X) = 2x_1^2 + 2x_2^2 + 3x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3.$$

Let M be symmetric matrix of Q. For $Y \in \mathbb{R}^3$ non-zero define

$$a_n = \frac{Y^T(M + I_3)^{n+1}Y}{Y^T(M + I_3)^nY}, \; n = 1, 2, ...$$

Then $\lim_{n\to\infty} a_n =$ __________ (in integer). (GATE MA 2025)

63) Let α, β distinct nonzero reals, Q(z) polynomial deg¡5. If

$$f(z) = \frac{\alpha^6 \sin(\beta z) - \beta^6(e^{2\alpha z} - Q(z))}{z^6}$$

satisfies Morera in $\mathbb{C} \setminus \{0\}$, then $\frac{\alpha}{4\beta} =$ __________. (GATE MA 2025)

64) Let G group with $g, h \in G$, $g \neq e, g^2 = e, h \neq e, h^2 \neq e, ghg^{-1} = h^2$. Then, least positive n with $h^n = e$ is __________. (GATE MA 2025)

65) Let $(\mathbb{R}^2, d_1), (\mathbb{R}^2, d_2)$ be metric spaces with

$$d_1((x_1, x_2), (y_1, y_2)) = |x_1 - y_1| + |x_2 - y_2|,$$

$$d_2((x_1,x_2),(y_1,y_2)) = \frac{d_1((x_1,x_2),(y_1,y_2))}{1+d_1((x_1,x_2),(y_1,y_2))}.$$

If the open ball centred at $(0,0)$ with radius $1/\alpha$ in $(\mathbb{R}^2, d_1)$ equals the ball with radius $1/7$ in $(\mathbb{R}^2, d_2)$, then $\alpha =$ __________. (GATE MA 2025)