2.8.16

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Question

Prove that the lines $x = py + q, z = ry + s$ and $x = p'y + q', z = r'y + s'$ are perpendicular if $pp' + rr' + 1 = 0$..

Theoretical Solution

Let line L_1 be the intersection of the planes,

$$x - py - q = 0 \ , \ z - ry - s = 0$$

Let line L_2 be the intersection of the planes,

$$x - p'y - q' = 0 \ , \ z - r'y - s' = 0$$

Theoretical Solution

Let $\mathbf{n_1}$, $\mathbf{n_2}$ be the normals for the planes
$x - py - q = 0$ and $z - ry - s = 0$ respectively.

$$\text{direction vector of } \mathbf{n_1} = \begin{pmatrix} 1 \\ -p \\ 0 \end{pmatrix} \tag{1}$$

$$\text{direction vector of } \mathbf{n_2} = \begin{pmatrix} 0 \\ -r \\ 1 \end{pmatrix} \tag{2}$$

Let $\mathbf{n_3}$, $\mathbf{n_4}$ be the normals for the planes
$x - p'y - q' = 0$ and $z - r'y - s' = 0$ respectively.

$$\text{direction vector of } \mathbf{n_3} = \begin{pmatrix} 1 \\ -p' \\ 0 \end{pmatrix} \tag{3}$$

Theoretical Solution

To find the direction vectors, the equations of the lines can be expressed as,

$$\mathbf{x} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \kappa \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{4}$$

where $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the direction vector.

Theoretical Solution

The equations for the line L_1 are:

$$x = py + q \tag{5}$$

$$z = ry + s \tag{6}$$

We can rearrange these to isolate y:

$$x - q = py \implies y = \frac{x - q}{p} \tag{7}$$

$$z - s = ry \implies y = \frac{z - s}{r} \tag{8}$$

By equating these expressions for y, and including the term for y itself, the line equation of L_1 to be:

$$\mathbf{x} = \begin{pmatrix} q \\ 0 \\ s \end{pmatrix} + \kappa_1 \begin{pmatrix} p \\ 1 \\ r \end{pmatrix} \tag{9}$$

Theoretical Solution

From this, the direction vectors of the line L_1 are $\begin{pmatrix} p \\ 1 \\ r \end{pmatrix}$.

The equations for the line L_2 are:

$$x = p'y + q' \tag{10}$$

$$z = r'y + s' \tag{11}$$

Similarly, we rearrange to isolate y:

$$x - q' = p'y \implies y = \frac{x - q'}{p'} \tag{12}$$

$$z - s' = r'y \implies y = \frac{z - s'}{r'} \tag{13}$$

Theoretical Solution

By equating these expressions for y, and including the term for y itself, the line equation of L_2 to be:

$$\mathbf{x} = \begin{pmatrix} q' \\ 0 \\ s' \end{pmatrix} + \kappa_2 \begin{pmatrix} p' \\ 1 \\ r' \end{pmatrix} \tag{14}$$

From this, the direction vectors of the line L_2 are $\begin{pmatrix} p' \\ 1 \\ r' \end{pmatrix}$.

Theoretical Solution

If the lines are perpendicular, then their dot product of direction vectors must be zero.

$$\implies (direction\ vector\ of\ L_1)^\top (direction\ vector\ of\ L_2) = 0 \tag{15}$$

$$\implies \begin{pmatrix} p & 1 & r \end{pmatrix} \begin{pmatrix} p' \\ 1 \\ r' \end{pmatrix} = 0 \tag{16}$$

$$\implies pp' + rr' + 1 = 0 \tag{17}$$

$\therefore$ The lines $x = py + q, z = ry + s$ and $x = p'y + q', z = r'y + s'$ are

perpendicular if $pp' + rr' + 1 = 0$

Hence proved.

C Code - A function to check whether they are perpendicular

```c
#include <stdio.h>

int is_perpendicular(double p, double r, double p_prime, double
    r_prime) {
    if ((p * p_prime) + (r * r_prime) + 1 == 0) {
        return 1; // True, the lines are perpendicular
    }
    return 0; // False, the lines are not perpendicular
}
```

Python Code

```python
import ctypes
import os

# Load the shared library
lib = ctypes.CDLL('./code.so')
# Define the argument types for the C function
lib.is_perpendicular.argtypes = [ctypes.c_double, ctypes.c_double
    , ctypes.c_double, ctypes.c_double]
# Define the return type for the C function
lib.is_perpendicular.restype = ctypes.c_int

def check_perpendicular(p, r, p_prime, r_prime):
    result = lib.is_perpendicular(p, r, p_prime, r_prime)
    return bool(result)
```