

1.4.17

EE25BTECH11002 - Achat Parth Kalpesh

Question:

Find the coordinates of the points of trisection (i.e. points dividing to three equal parts) of the line segment joining the points **A** (2, -2) and **B** (-7, 4).

Solution:

Let the given points be represented by the vectors **A** and **B**.

$$\mathbf{A} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -7 \\ 4 \end{pmatrix}$$

The points of trisection, let's call them **P** and **Q**, divide the line segment **AB** into three equal parts. This means that point **P** divides **AB** in the ratio 1:2, and point **Q** divides **AB** in the ratio 2:1

The position vector of a point dividing the line segment joining points **A** and **B** in the ratio $m : n$ is given by the section formula:

$$\frac{n\mathbf{A} + m\mathbf{B}}{m + n}$$

For the first point of trisection, **P** (ratio 1:2)

Here, $m = 1$ and $n = 2$.

$$\begin{aligned} \mathbf{P} &= \frac{2\mathbf{A} + 1\mathbf{B}}{1 + 2} \\ &= \frac{1}{3} \left(2 \begin{pmatrix} 2 \\ -2 \end{pmatrix} + 1 \begin{pmatrix} -7 \\ 4 \end{pmatrix} \right) \\ &= \frac{1}{3} \left(\begin{pmatrix} 4 \\ -4 \end{pmatrix} + \begin{pmatrix} -7 \\ 4 \end{pmatrix} \right) \\ &= \frac{1}{3} \begin{pmatrix} 4 - 7 \\ -4 + 4 \end{pmatrix} \\ &= \frac{1}{3} \begin{pmatrix} -3 \\ 0 \end{pmatrix} \\ \Rightarrow \mathbf{P} &= \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{aligned}$$

So, the coordinates of the first point of trisection are **P** (-1, 0)

For the second point of trisection, **Q** (ratio 2:1)

Here, $m = 2$ and $n = 1$.

$$\begin{aligned}
 \mathbf{Q} &= \frac{1\mathbf{A} + 2\mathbf{B}}{2 + 1} \\
 &= \frac{1}{3} \left(1 \begin{pmatrix} 2 \\ -2 \end{pmatrix} + 2 \begin{pmatrix} -7 \\ 4 \end{pmatrix} \right) \\
 &= \frac{1}{3} \left(\begin{pmatrix} 2 \\ -2 \end{pmatrix} + \begin{pmatrix} -14 \\ 8 \end{pmatrix} \right) \\
 &= \frac{1}{3} \begin{pmatrix} 2 - 14 \\ -2 + 8 \end{pmatrix} \\
 &= \frac{1}{3} \begin{pmatrix} -12 \\ 6 \end{pmatrix} \\
 \Rightarrow \mathbf{Q} &= \begin{pmatrix} -4 \\ 2 \end{pmatrix}
 \end{aligned}$$

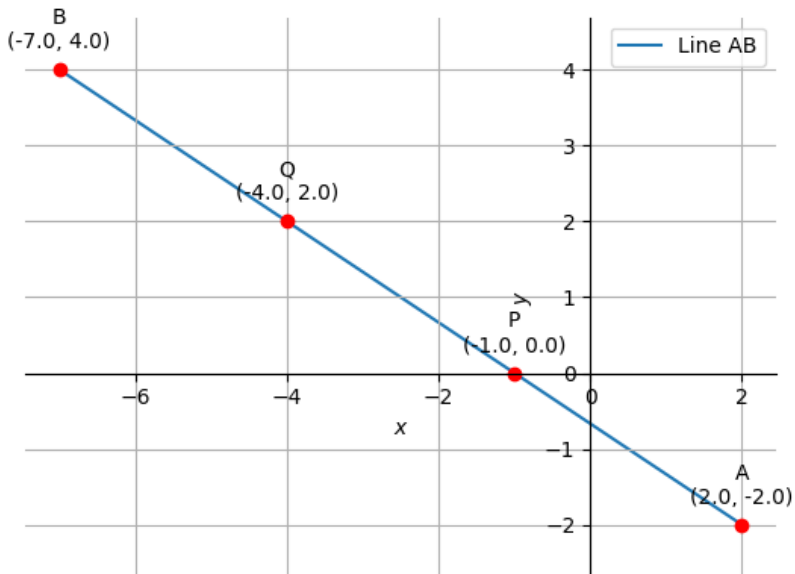


Fig. 0.1: Graph

So, the coordinates of the second point of trisection are $\mathbf{Q}(-4, 2)$
 Therefore, the coordinates of the points of trisection are $(-1, 0)$ and $(-4, 2)$