1.9.12

EE25BTECH11023 - Venkata Sai

Question:
Find the length of the segment joining $\mathbf{A}(-6, 7)$ and $\mathbf{B}(-1, -5)$. Also, find the midpoint of AB.
Solution:

Variable	Description
x	x coordinate of P
y	y coordinate of P

TABLE 0: Variables Used

Let the given points be

$$\mathbf{A} = \begin{pmatrix} -6 \\ 7 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \tag{0.1}$$

The direction vector of the segment joining A and B is given by:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 - (-6) \\ -5 - 7 \end{pmatrix} = \begin{pmatrix} 5 \\ -12 \end{pmatrix} \tag{0.2}$$

The length of the segment is the magnitude of the direction vector:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 - (-6) \\ -5 - 7 \end{pmatrix} = \begin{pmatrix} 5 \\ -12 \end{pmatrix} \tag{0.3}$$

$$d = \|\mathbf{B} - \mathbf{A}\| = \sqrt{(\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A})} \tag{0.4}$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 5 & -12 \end{pmatrix} \begin{pmatrix} 5 \\ -12 \end{pmatrix} \tag{0.5}$$

$$= 5 \times 5 + -12 \times -12 = 169 \tag{0.6}$$

$$d = \sqrt{169} = 13 \tag{0.7}$$

Hence the length of the segment is 13 units.
To find midpoint of segment $\mathbf{AB}$:
Let the required point be $\mathbf{P}$

$$\mathbf{P} = \frac{k(\mathbf{B}) + (\mathbf{A})}{k + 1} \tag{0.8}$$

Here according to problem value of k is 1

$$P = \frac{B + A}{2} = \frac{\begin{pmatrix} -6 \\ 7 \end{pmatrix} + \begin{pmatrix} -1 \\ -5 \end{pmatrix}}{2} = \frac{\begin{pmatrix} -7 \\ 2 \end{pmatrix}}{2} \qquad (0.9)$$

$$(0.10)$$

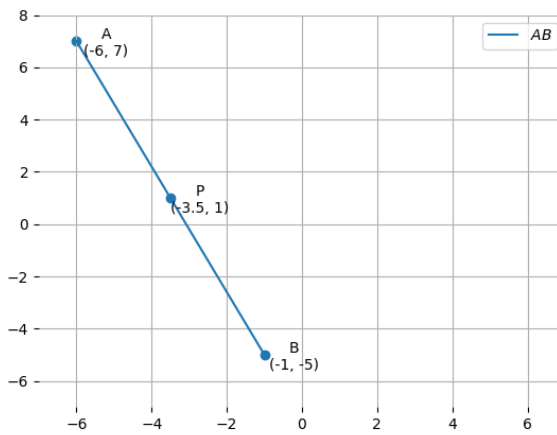$$P = \begin{pmatrix} \frac{-7}{2} \\ 1 \end{pmatrix} \qquad (0.11)$$

Hence the coordinates of P are $\left(\frac{-7}{2}, 1\right)$

Fig. 0.1: Stem Plot of y(n)