1.9.30

EE25BTECH11042 - Nipun Dasari

Question:
If the distances of $\mathbf{P} = (x, y)$ from $\mathbf{A} = (5, 1)$ and $\mathbf{B} = (\hat{a}1, 5)$are equal, then prove that $3x = 2y$.

Solution:
Consider the matrices A, B and P as follows:

$$\mathbf{A} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}$$

The condition for distances from $\mathbf{B}$ to $\mathbf{P}$ and $\mathbf{A}$ to $\mathbf{P}$ to be equal is

$$\|\mathbf{P} - \mathbf{A}\| = \|\mathbf{P} - \mathbf{B}\| \equiv \|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2$$

Using inner products:

$$(\mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{B})^T (\mathbf{P} - \mathbf{B})$$

Expanding on both sides:

$$\mathbf{P}\mathbf{P}^T - 2\mathbf{A}^T\mathbf{P} + \mathbf{A}^T\mathbf{A} = \mathbf{P}\mathbf{P}^T - 2\mathbf{B}^T\mathbf{P} + \mathbf{B}^T\mathbf{B}$$

On simplification:

$$\left(-2\mathbf{A}^T + 2\mathbf{B}^T\right)\mathbf{P} = \mathbf{B}^T\mathbf{B} - \mathbf{A}^T\mathbf{A}$$

LHS constant matrix:

$$2\left(\mathbf{B} - \mathbf{A}\right)^T = 2\begin{pmatrix} -1 - 5 \\ 5 - 1 \end{pmatrix} = \begin{pmatrix} -12 & 8 \end{pmatrix}$$

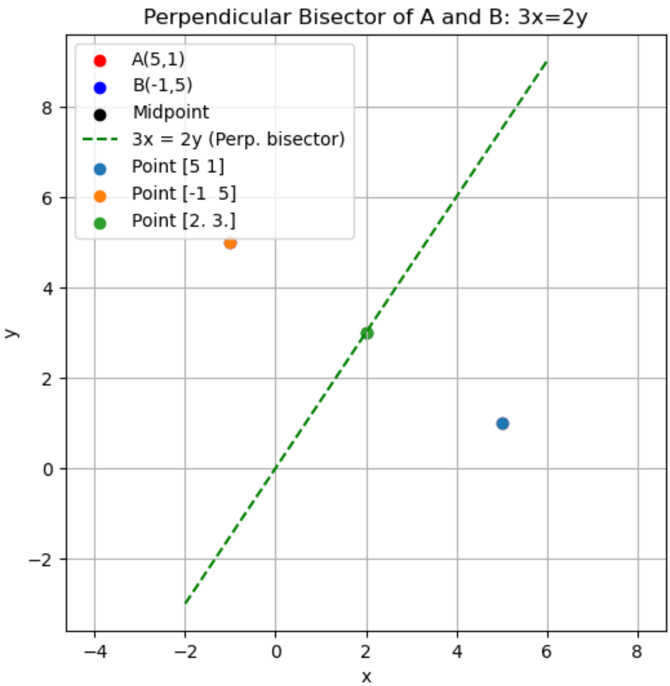
RHS constant matrix:

$$\mathbf{B}^T\mathbf{B} - \mathbf{A}^T\mathbf{A} = \left((-1)^2 + 5^2\right) - \left(1^2 + 5^2\right) = 0$$

From the above:

$$\begin{pmatrix} -12 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \implies -12x + 8y = 0 \implies 3x = 2y$$

From the plot we can infer that the locus is perpendicular bisector of the line joining the 2 vectors



Perpendicular Bisector of A and B: 3x=2y
A(5,1)
B(-1,5)
Midpoint
3x = 2y (Perp. bisector)
Point [5 1]
Point [-1 5]
Point [2. 3.]
y
x
8
6
4
2
0
−2
−4
−2
0
2
4
6
8