GATE 2016 MA

EE25BTECH11001 - AARUSH DILAWRI

1) An apple costs 10. An onion costs 8.
   Select the most suitable sentence with respect to grammar and usage.
   a) The price of an apple is greater than an onion.
   b) The price of an apple is more than onion.
   c) The price of an apple is greater than that of an onion.
   d) Apples are more costlier than onions.

(GATE MA 2016)

2) The Buddha said, "Holding on to anger is like <u>grasping</u> a hot coal with the intent of throwing it at someone else; you are the one who gets burnt."
   Select the word below which is closest in meaning to the word underlined above.

   a) burning b) igniting c) clutching d) flinging

(GATE MA 2016)

3) M has a son Q and a daughter R. He has no other children. E is the mother of P and daughter-in-law of M. How is P related to M?

   a) P is the son-in-law of M.
   b) P is the grandchild of M.
   c) P is the daughter-in-law of M.
   d) P is the grandfather of M.

(GATE MA 2016)

4) The number that least fits this set $(324, 441, 97, 64)$ is .

   a) 324 b) 441 c) 97 d) 64

(GATE MA 2016)

5) It takes 10, s and 15, s, respectively, for two trains travelling at different constant speeds to completely pass a telegraph post. The length of the first train is 120, m and that of the second train is 150, m. The magnitude of the difference in the speeds of the two trains (in m/s) is .

   a) 2.0 b) 10.0 c) 12.0 d) 22.0

(GATE MA 2016)

6) The velocity V of a vehicle along a straight line is measured in m/s and plotted as shown with respect to time in seconds. At the end of the 7 seconds, how much will the odometer reading increase by (in m)?

   a) 0 b) 3 c) 4 d) 5

(GATE MA 2016)

7) The overwhelming number of people infected with rabies in India has been flagged by the World Health Organization as a source of concern. It is estimated that inoculating 70% of pets and stray dogs against rabies can lead to a significant reduction in the number of people infected with rabies.
Which of the following can be logically inferred from the above sentences?
   a) The number of people in India infected with rabies is high.
   b) The number of people in other parts of the world who are infected with rabies is low.
   c) Rabies can be eradicated in India by vaccinating 70% of stray dogs.
   d) Stray dogs are the main source of rabies worldwide.

(GATE MA 2016)

8) A flat is shared by four first year undergraduate students. They agreed to allow the oldest of them to enjoy some extra space in the flat. Manu is two months older than Sravan, who is three months younger than Trideep. Pavan is one month older than Sravan. Who should occupy the extra space in the flat?

   a) Manu   b) Sravan   c) Trideep   d) Pavan

(GATE MA 2016)

9) Find the area bounded by the lines $3x + 2y = 14$, $2x - 3y = 5$ in the first quadrant.

   a) 14.95   b) 15.25   c) 15.70   d) 20.35

(GATE MA 2016)

10) A straight line is fit to a data set $(\ln x, y)$. This line intercepts the abscissa at $\ln x = 0.1$ and has a slope of -0.02. What is the value of y at $x = 5$ from the fit?

   a) -0.030 b) -0.014 c) 0.014 d) 0.030

(GATE MA 2016)

11) Let X, Y, Z be a basis of $\mathbb{R}^3$. Consider the following statements P and Q:
(P) $\{X + Y, Y + Z, X - Z\}$ is a basis of $\mathbb{R}^3$
(Q) $\{X + Y + Z, X + 2Y - Z, X - 3Z\}$ is a basis of $\mathbb{R}^3$

   a) both P and Q   c) only Q
   b) only P   d) neither P nor Q

(GATE MA 2016)

12) Consider the following statements P and Q:
(P) : If $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}$, then M is singular.
(Q) : Let S be a diagonalizable matrix. If T is a matrix such that $S + S^T = Id$, then T is diagonalizable.
Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) neither P nor Q

(GATE MA 2016)

13) Consider the following statements P and Q:
(P) : If M is an $n \times n$ complex matrix, then $\mathcal{R}(M) = (N(M^*))^{\perp}$.
(Q) : There exists a unitary matrix with an eigenvalue λ such that $|\lambda| < 1$.
Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) neither P nor Q

(GATE MA 2016)

14) Consider a real vector space V of dimension n and a non-zero linear transformation $T : V \to V$. If dimension$(T(V)) < n$ and $T^2 = \lambda T$, for some $\lambda \in \mathbb{R} \setminus (0)$, then which of the following statements is TRUE?
a) determinant$(T) = |\lambda|^n$
b) There exists a non-trivial subspace V_1 of V such that $T(X) = 0$ for all $X \in V_1$
c) T is invertible
d) λ is the only eigenvalue of T

(GATE MA 2016)

15) Let $S = [0, 1) \cup [2, 3]$ and $f : S \to \mathbb{R}$ be a strictly increasing function such that $f(S)$ is connected. Which of the following statements is TRUE?

a) f has exactly one discontinuity
b) f has exactly two discontinuities
c) f has infinitely many discontinuities
d) f is continuous

(GATE MA 2016)

16) Let $a_1 = 1$ and $a_n = a_{n-1} + 4,\ n \geq 2$. Then,

$$\lim_{n\to\infty} \left(\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \cdots + \frac{1}{a_{n-1} a_n} \right)$$

is equal to (GATE MA 2016)

17) Maximum $\left(x + y : (x, y) \in \overline{B(0, 1)}\right)$ is equal to (GATE MA 2016)

18) Let $a, b, c, d \in \mathbb{R}$ such that $c^2 + d^2 \neq 0$. Then, the Cauchy problem

$$au_x + bu_y = e^{x+y}, \quad x, y \in \mathbb{R},$$

$$u(x, y) = 0 \quad \text{on} \quad cx + dy = 0$$

has a unique solution if

a) $ac + bd \neq 0$
b) $ad - bc \neq 0$
c) $ac - bd \neq 0$
d) $ad + bc \neq 0$

(GATE MA 2016)

19) Let $u(x,t)$ be the d'Alembert's solution of the initial value problem for the wave equation

$$u_{tt} - c^2 u_{xx} = 0$$
$$u(x,0) = f(x), \quad u_t(x,0) = g(x),$$

where c is a positive real number and f, g are smooth odd functions. Then, $u(0,1)$ is equal to (GATE MA 2016)

20) Let the probability density function of a random variable X be

$$f(x) = \begin{cases} x & 0 \le x < \frac{1}{2} \\ c(2x-1)^2 & \frac{1}{2} \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Then, the value of c is equal to (GATE MA 2016)

21) Let V be the set of all solutions of the equation $y'' + ay' + by = 0$ satisfying $y(0) = y(1)$, where a, b are positive real numbers. Then, dimension(V) is equal to (GATE MA 2016)

22) Let $y'' + p(x)y' + q(x)y = 0, \ x \in (-\infty, \infty)$, where $p(x)$ and $q(x)$ are continuous functions. If $y_1(x) = \sin(x) - 2\cos(x)$ and $y_2(x) = 2\sin(x) + \cos(x)$ are two linearly independent solutions of the above equation, then $|4p(0) + 2q(1)|$ is equal to (GATE MA 2016)

23) Let $P_n(x)$ be the Legendre polynomial of degree n and $I = \int_{-1}^{1} x^k P_n(x) dx$, where k is a non-negative integer. Consider the following statements P and Q:
(P) : $I = 0$ if $k < n$
(Q) : $I = 0$ if $n - k$ is an odd integer.
Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) neither P nor Q

(GATE MA 2016)

24) Consider the following statements P and Q:
(P) : $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$ has two linearly independent Frobenius series solutions near $x = 0$.
(Q) : $x^2y'' + 3\sin(x)y' + y = 0$ has two linearly independent Frobenius series solutions near $x = 0$.
Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) neither P nor Q

(GATE MA 2016)

25) Let the polynomial x^4 be approximated by a polynomial of degree ≤ 2, which interpolates x^4 at $x = -1, 0$ and 1. Then, the maximum absolute interpolation error over the interval $(-1, 1)$ is equal to (GATE MA 2016)

26) Let (z_n) be a sequence of distinct points in $D(0,1) = (z \in \mathbb{C} : |z| < 1)$ with $\lim_{n\to\infty} z_n = 0$. Consider the following statements P and Q:
(P) : There exists a unique analytic function f on $D(0,1)$ such that $f(z_n) = \sin(z_n)$ for all n.
(Q) : There exists an analytic function f on $D(0,1)$ such that $f(z_n) = 0$ if n is even and $f(z_n) = 1$ if n is odd.
Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) neither P nor Q

(GATE MA 2016)

27) Let $(\mathbb{R}, \tau)$ be a topological space with the cofinite topology. Every infinite subset of $\mathbb{R}$ is

a) Compact but NOT connected
b) Both compact and connected
c) NOT compact but connected
d) Neither compact nor connected

(GATE MA 2016)

28) Let $c_0 = ((x_n) : x_n \in \mathbb{R},\ x_n \to 0)$ and $M = ((x_n) \in c_0 : x_1 + x_2 + \cdots + x_{10} = 0)$. Then, dimension$(c_0/M)$ is equal to (GATE MA 2016)

29) Consider $(\mathbb{R}^2, \|\cdot\|_\infty)$, where $\|(x,y)\|_\infty = \max(|x|,|y|)$. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \frac{x+y}{2}$ and $\tilde{f}$ the norm preserving linear extension of f to $(\mathbb{R}^3, \|\cdot\|_\infty)$. Then, $\tilde{f}(1,1,1)$ is equal to (GATE MA 2016)

30) $f : [0,1] \to [0,1]$ is called a shrinking map if $|f(x) - f(y)| < |x - y|$ for all $x, y \in [0,1]$ and a contraction if there exists an $a < 1$ such that $|f(x) - f(y)| \le a|x - y|$ for all $x, y \in [0,1]$.
Which of the following statements is TRUE for the function $f(x) = x - \frac{x^2}{2}$?

a) is both a shrinking map and a contraction
b) is a shrinking map but NOT a contraction
c) is NOT a shrinking map but a contraction
d) is neither a shrinking map nor a contraction

(GATE MA 2016)

31) Let $\mathbb{M}$ be the set of all $n \times n$ real matrices with the usual norm topology. Consider the following statements P and Q:
(P) : The set of all symmetric positive definite matrices in $\mathbb{M}$ is connected.
(Q) : The set of all invertible matrices in $\mathbb{M}$ is compact.
Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) neither P nor Q

(GATE MA 2016)

32) Let $X_1, X_2, \ldots, X_n$ be a random sample from the following probability density function for $0 < \mu < \infty,\ 0 < \alpha < 1$,

$$f(x;\mu,\alpha) = \begin{cases} \frac{1}{\Gamma(\alpha)}(x-\mu)^{\alpha-1}e^{-(x-\mu)}, & x > \mu \\ 0, & \text{otherwise} \end{cases}$$

Here α and μ are unknown parameters. Which of the following statements is TRUE?

a) Maximum likelihood estimator of only μ exists
b) Maximum likelihood estimator of only α exists
c) Maximum likelihood estimators of both μ and α exist
d) Maximum likelihood estimator of neither μ nor α exists

(GATE MA 2016)

33) Suppose X and Y are two random variables such that $aX + bY$ is a normal random variable for all $a, b \in \mathbb{R}$. Consider the following statements P, Q, R, and S:
(P) : X is a standard normal random variable.
(Q) : The conditional distribution of X given Y is normal.
(R) : The conditional distribution of X given $X + Y$ is normal.
(S) : $X - Y$ has mean 0.
Which of the above statements ALWAYS hold TRUE?

a) both P and Q
b) both Q and R
c) both Q and S
d) both P and S

(GATE MA 2016)

34) Consider the following statements P and Q:
(P) : If H is a normal subgroup of order 4 of the symmetric group S_4, then S_4/H is abelian.
(Q) : If $Q = (\pm 1, \pm i, \pm j, \pm k)$ is the quaternion group, then $Q/(-1, 1)$ is abelian.
Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) neither P nor Q

(GATE MA 2016)

35) Let F be a field of order 32. Then the number of non-zero solutions $(a, b) \in F \times F$ of the equation $x^2 + xy + y^2 = 0$ is equal to (GATE MA 2016)

36) Let $\gamma = (z \in \mathbb{C} : |z| = 2)$ be oriented in the counter-clockwise direction. Let

$$I = \frac{1}{2\pi i} \oint_\gamma z^7 \cos\left(\frac{1}{z^2}\right) dz.$$

Then, the value of I is equal to (GATE MA 2016)

37) Let $u(x, y) = x^3 + ax^2y + bxy^2 + 2y^3$ be a harmonic function and $v(x, y)$ its harmonic conjugate. If $v(0, 0) = 1$, then $|a + b + v(1, 1)|$ is equal to (GATE MA 2016)

38) Let γ be the triangular path connecting the points $(0,0),(2,2)$ and $(0,2)$ in the counter-clockwise direction in $\mathbb{R}^2$. Then

$$I = \oint_{\gamma} \sin(x^3)dx + 6xy\,dy$$

is equal to (GATE MA 2016)

39) Let y be the solution of

$$y' + y = |x|, \quad x \in \mathbb{R}$$
$$y(-1) = 0.$$

Then $y(1)$ is equal to

a) $\frac{2}{e} - \frac{2}{e^2}$
b) $\frac{2}{e} - 2e^2$
c) $2 - \frac{2}{e}$
d) $2 - 2e$

(GATE MA 2016)

40) Let X be a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \le x < \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

Then $P\left(\frac{1}{4} < X < 1\right)$ is equal to (GATE MA 2016)

41) Let y be the curve which passes through $(0,1)$ and intersects each curve of the family $y = cx^2$ orthogonally. Then y also passes through the point

a) $(\sqrt{2}, 0)$
b) $(0, \sqrt{2})$
c) $(1,1)$
d) $(-1,1)$

(GATE MA 2016)

42) Let $S(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$ be the Fourier series of the 2π periodic function defined by $f(x) = x^2 + 4\sin(x)\cos(x)$, $-\pi \le x \le \pi$. Then

$$\left| \sum_{n=0}^{\infty} a_n - \sum_{n=1}^{\infty} b_n \right|$$

is equal to (GATE MA 2016)

43) Let $y(t)$ be a continuous function on $[0,\infty)$. If

$$y(t) = t\left(1 - 4\int_0^t y(x)dx\right) + 4\int_0^t xy(x)dx,$$

then $\int_0^{\pi/2} y(t)\,dt$ is equal to (GATE MA 2016)

44) Let $S_n = \sum_{k=1}^{n} \frac{1}{k}$ and $I_n = \int_1^n \frac{x - \lfloor x \rfloor}{x^2} dx$. Then, $S_{10} + I_{10}$ is equal to

a) $\ln 10 + 1$
b) $\ln 10 - 1$
c) $\ln 10 - \frac{1}{10}$
d) $\ln 10 + \frac{1}{10}$

(GATE MA 2016)

45) For any $(x, y) \in \mathbb{R}^2 \setminus \overline{B(0,1)}$, let

$$f(x,y) = \text{distance}\left((x,y), \overline{B(0,1)}\right) = \inf\left(\sqrt{(x-x_1)^2 + (y-y_1)^2} : (x_1, y_1) \in \overline{B(0,1)}\right)$$

Then, $\|\nabla f(3,4)\|$ is equal to (GATE MA 2016)

46) Let $f(x) = \left(\int_0^x e^{-t^2} dt\right)^2$ and $g(x) = \int_0^x \frac{e^{-t^2}(1+t^2)}{1+t^2} dt$. Then $f'(\sqrt{\pi}) + g'(\sqrt{\pi})$ is equal to (GATE MA 2016)

47) Let $M = \begin{pmatrix} a & b & c \\ l & d & e \\ l & e & f \end{pmatrix}$ be a real matrix with eigenvalues $1, 0$ and 3. If the eigenvectors corresponding to 1 and 0 are $(1,1,1)^T$ and $(1,-1,0)^T$ respectively, then the value of $3f$ is equal to (GATE MA 2016)

48) Let $M = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ and $e^M = Id + M + \frac{1}{2!}M^2 + \frac{1}{3!}M^3 + \dots$. If $e^M = [b_{ij}]$, then

$$\frac{1}{e}\sum_{i=1}^{3}\sum_{j=1}^{3} b_{ij}$$

is equal to (GATE MA 2016)

49) Let the integral $I = \int_0^4 f(x)\, dx$, where

$$f(x) = \begin{cases} x & 0 \le x \le 2 \\ 4 - x & 2 < x \le 4 \end{cases}$$

Consider the following statements P and Q:
(P) : If I_2 is the value of the integral obtained by the composite trapezoidal rule with two equal sub-intervals, then I_2 is exact.
(Q) : If I_3 is the value of the integral obtained by the composite trapezoidal rule with three equal sub-intervals, then I_3 is exact.
Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) neither P nor Q

(GATE MA 2016)

50) The difference between the least two eigenvalues of the boundary value problem

$$y'' + \lambda y = 0, \qquad 0 < x < \pi$$
$$y(0) = 0, \qquad y'(\pi) = 0,$$

is equal to (GATE MA 2016)

51) The number of roots of the equation $x^2 - \cos(x) = 0$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is equal to (GATE MA 2016)

52) For the fixed point iteration $x_{k+1} = g(x_k),\ k = 0, 1, 2, \ldots$, consider the following statements P and Q:
(P) : If $g(x) = 1 + \frac{2}{x}$ then the fixed point iteration converges to 2 for all $x_0 \in [1, 100]$.
(Q) : If $g(x) = \sqrt{2 + x}$ then the fixed point iteration converges to 2 for all $x_0 \in [0, 100]$.
Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) neither P nor Q

(GATE MA 2016)

53) Let $T : \ell_2 \to \ell_2$ be defined by

$$T\left((x_1, x_2, ..., x_n, \ldots)\right) = (x_2 - x_1, x_3 - x_2, ..., x_{n+1} - x_n, ...)$$

Then

a) $\|T\| = 1$
b) $\|T\| \geq 2$ but bounded
c) $1 < \|T\| \leq 2$
d) $\|T\|$ is unbounded

(GATE MA 2016)

54) Minimize $w = x + 2y$ subject to

$$\begin{cases} 2x + y \geq 3 \\ x + y \geq 2 \\ x \geq 0, y \geq 0 \end{cases}$$

Then, the minimum value of w is equal to (GATE MA 2016)

55) Maximize $w = 11x - z$ subject to

$$\begin{cases} 10x + y - z \leq 1 \\ 2x - 2y + z \leq 2 \\ x, y, z \geq 0 \end{cases}$$

Then, the maximum value of w is equal to (GATE MA 2016)

56) Let $X_1, X_2, X_3, \ldots$ be a sequence of i.i.d. random variables with mean 1. If N is a geometric random variable with the probability mass function $P(N = k) = \frac{1}{2^k}$, $k = 1, 2, 3, \ldots$, and it is independent of the X_i's, then $E(X_1 + X_2 + \cdots + X_N)$ is equal to (GATE MA 2016)

57) Let X_1 be an exponential random variable with mean 1 and X_2 a gamma random variable with mean 2 and variance 2. If X_1 and X_2 are independently distributed, then $P(X_1 < X_2)$ is equal to (GATE MA 2016)

58) Let $X_1, X_2, X_3, \ldots$ be a sequence of i.i.d. uniform $(0, 1)$ random variables. Then, the value of

$$\lim_{n\to\infty} P\left(-\ln(1 - X_1) - \cdots - \ln(1 - X_n) \geq n\right)$$

is equal to (GATE MA 2016)

59) Let X be a standard normal random variable. Then, $P(X < 0 \mid |X| = 1)$ is equal to

a) $\frac{\Phi(1)-\frac{1}{2}}{\Phi(2)-\frac{1}{2}}$
b) $\frac{\Phi(1)+\frac{1}{2}}{\Phi(2)+\frac{1}{2}}$
c) $\frac{\Phi(1)-\frac{1}{2}}{\Phi(2)+\frac{1}{2}}$
d) $\frac{\Phi(1)+\frac{1}{2}}{\Phi(2)+1}$

(GATE MA 2016)

60) Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from the probability density function

$$f(x) = \theta\, a\, e^{-ax} + (1-\theta)2a\, e^{-2ax}, \quad x \geq 0;\ 0 \text{ otherwise}$$

where $a > 0,\ 0 \leq \theta \leq 1$ are parameters. Consider the following testing problem: $H_0 : \theta = 1,\ a = 1$ versus $H_1 : \theta = 0,\ a = 2$.
Which of the following statements is TRUE?

a) Uniformly Most Powerful test does NOT exist
b) Uniformly Most Powerful test is of the form $\sum_{i=1}^{n} X_i > c$, for some $0 < c < \infty$
c) Uniformly Most Powerful test is of the form $\sum_{i=1}^{n} X_i < c$, for some $0 < c < \infty$
d) Uniformly Most Powerful test is of the form $c_1 < \sum_{i=1}^{n} X_i < c_2$, for some $0 < c_1 < c_2 < \infty$

(GATE MA 2016)

61) Let $X_1, X_2, X_3, \ldots$ be a sequence of i.i.d. $N(\mu, 1)$ random variables. Then,

$$\lim_{n\to\infty} \frac{\sqrt{\pi}}{2n} \sum_{i=1}^{n} E\,|X_i - \mu|$$

is equal to (GATE MA 2016)

62) Let $X_1, X_2, X_3, ..., X_n$ be a random sample from uniform $[1, \theta]$, for some $\theta > 1$. If $X_{(n)} = \text{Maximum}(X_1, X_2, ..., X_n)$, then the UMVUE of θ is

a) $\frac{n+1}{n}X_{(n)} + \frac{1}{n}$
b) $\frac{n+1}{n}X_{(n)} - \frac{1}{n}$
c) $\frac{n}{n+1}X_{(n)} + \frac{1}{n}$
d) $\frac{n}{n+1}X_{(n)} + \frac{n+1}{n}$

(GATE MA 2016)

63) Let $x_1 = x_2 = x_3 = 1,\ x_4 = x_5 = x_6 = 2$ be a random sample from a Poisson random variable with mean θ, where $\theta \in (1, 2)$. Then, the maximum likelihood estimator of θ is equal to (GATE MA 2016)

64) The remainder when 98! is divided by 101 is equal to (GATE MA 2016)

65) Let G be a group whose presentation is

$$G = \left(x, y \mid x^5 = y^2 = e, \quad x^2 y = yx\right)$$

Then G is isomorphic to

a) $\mathbb{Z}_5$
b) $\mathbb{Z}_{10}$
c) $\mathbb{Z}_2$
d) $\mathbb{Z}_{30}$

(GATE MA 2016)