1.6.11

EE25BTECH11061 - Vankudoth Sainadh

Question: If the points $A(1, 2)$, $O(0, 0)$ and $C(a, b)$ are collinear, find the relation between a and b.

Solution (Rank & Row-Reduction Method).
Step 1: Write the points.

$$A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \qquad O = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad C = \begin{pmatrix} a \\ b \end{pmatrix}.$$

Step 2: Form difference vectors relative to A.

$$O - A = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix},$$

$$C - A = \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} a - 1 \\ b - 2 \end{pmatrix}.$$

Step 3: Build the matrix and state the rank condition for collinearity.

$$M = \begin{pmatrix} O - A & C - A \end{pmatrix}^T = \begin{pmatrix} -1 & -2 \\ a - 1 & b - 2 \end{pmatrix},$$

$$\text{Collinearity} \iff \text{rank}(M) = 1.$$

Step 4: Row elimination (make the $(2, 1)$ entry zero).

$$R_2 \longrightarrow R_2 - \frac{a - 1}{-1} R_1 \quad (\text{note: } -1 \neq 0),$$

$$\begin{pmatrix} -1 & -2 \\ a - 1 & b - 2 \end{pmatrix} \longrightarrow \begin{pmatrix} -1 & -2 \\ 0 & b - 2a \end{pmatrix}.$$

Step 5: Rank condition.

$$\text{rank}(M) = 1 \iff \text{second row is the zero row} \iff b - 2a = 0.$$
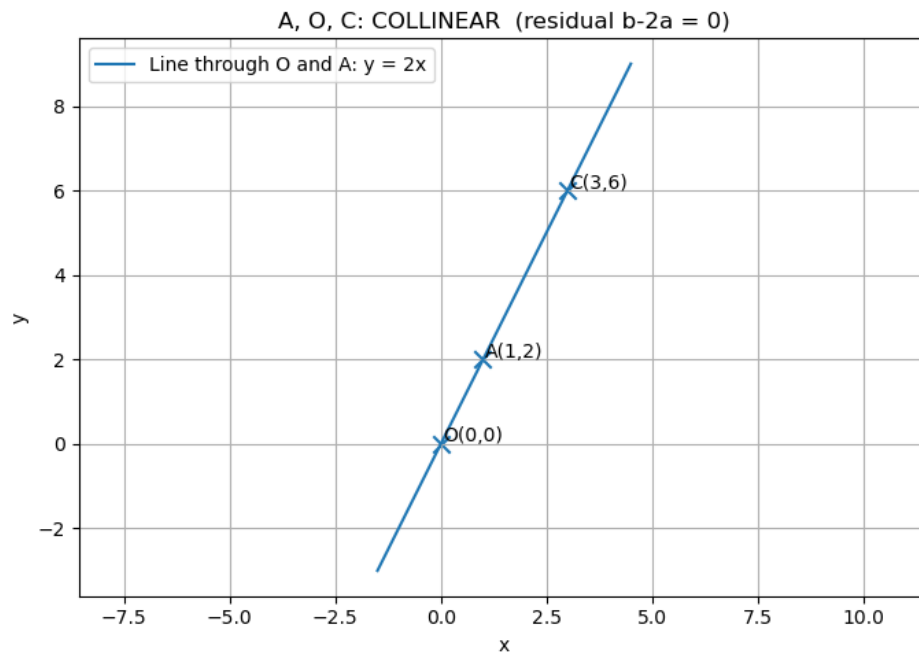
Step 6: Final relation.

$$\boxed{b = 2a}.$$

Fig. 0