1.9.21

EE25BTECH11032 - Kartik Lahoti

Question:

Given vertices of a parallelogram $\mathbf{A}(-2,1), \mathbf{B}(a,0), \mathbf{C}(4,b)$, and $\mathbf{D}(1,2)$. Find the values of a and b. Hence, find the lengths of its sides.

Solution:

Given : A parallelogram $ABCD$ with ,

$$\mathbf{A} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ b \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.1}$$

Theory :

In a Parallelogram $PQRS$ the opposite side are parallel and equal , i.e. $PQ \parallel RS$ and $PQ = RS$ and similarly $QR \parallel PS$ and $QR = PS$

$\therefore$ we can say,

$$\mathbf{AB} = \mathbf{DC} \tag{0.2}$$

Calculating $\mathbf{AB}$,

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} \tag{0.3}$$

$$\mathbf{AB} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} a+2 \\ -1 \end{pmatrix} \tag{0.4}$$

Similarly,

$$\mathbf{DC} = \begin{pmatrix} 3 \\ b-2 \end{pmatrix} \tag{0.5}$$

From Eqn (0.2) we get :

$$\begin{pmatrix} a+2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ b-2 \end{pmatrix} \tag{0.6}$$

$$\tag{0.7}$$

$$\implies a = 1 \text{ and } b = 1 \tag{0.8}$$

$\therefore a = 1$ and $b = 1$

Calculating the side lengths ,

$$\because \mathbf{A}-\mathbf{B}=\begin{pmatrix}-3\\1\end{pmatrix}, \tag{0.9}$$

$$(\mathbf{A}-\mathbf{B})^{\top}(\mathbf{A}-\mathbf{B})=10 \tag{0.10}$$

Thus, the desired length **AB** is

$$d_1=\|\mathbf{A}-\mathbf{B}\|=\sqrt{10} \tag{0.11}$$

Similarly,

$$\because \mathbf{B}-\mathbf{C}=\begin{pmatrix}-3\\-1\end{pmatrix}, \tag{0.12}$$

$$(\mathbf{B}-\mathbf{C})^{\top}(\mathbf{B}-\mathbf{C})=10 \tag{0.13}$$

Thus, the desired length **BC** is

$$d_2=\|\mathbf{B}-\mathbf{C}\|=\sqrt{10} \tag{0.14}$$

Hence: The length of the sides of the parallelogram is $\sqrt{10}$

Fig:1.9.21