

1.6.16

AI25BTECH11001 - ABHISEK MOHAPATRA

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Question:

Find the values of k if the points $\mathbf{A}(k + 1, 2k)$, $\mathbf{B}(3k, 2k + 3)$ and $\mathbf{C}(5k - 1, 5k)$ are collinear.

Solution: From the given information,

$$\mathbf{A} = \begin{pmatrix} k+1 \\ 2k \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3k \\ 2k+3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5k-1 \\ 5k \end{pmatrix} \quad (0.1)$$

To check if the points are collinear, we can use

$$\text{rank}(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A}) = 1 \quad (0.2)$$

So,

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T = \begin{pmatrix} 2k-1 & 3 \\ 4k-2 & 3k \end{pmatrix} \quad (0.3)$$

$$\xleftrightarrow{R_2 = R_2 - 2R_1} \begin{pmatrix} 2k-1 & 3 \\ 0 & 3k-6 \end{pmatrix} \quad (0.4)$$

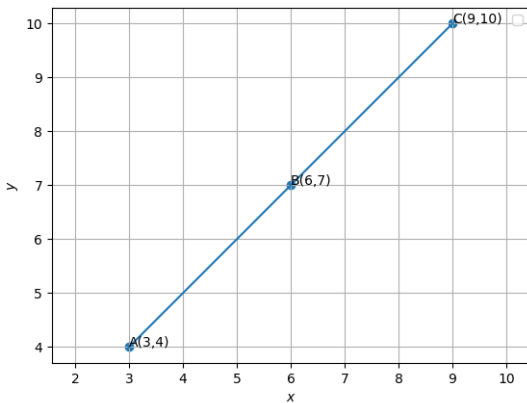
The rank of the matrix will be 1 when

$$3k - 6 = 0 \quad (0.5)$$

$$\Rightarrow k = 2$$

(0.6)

Graph:



Therefore, $k = 2$.