1.5.15

INDHIRESH S- EE25BTECH11027

Question The midpoint of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(1, 2a+1)$. Findthe values of a and b.
Solution:
Let us solve the given equation theoretically and then verify the solution computationally. From the given data,

$$\mathbf{A} = \begin{pmatrix} 2a \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 3b \end{pmatrix} \tag{1}$$

Let the midpoint of points A and B be C. where,

$$\mathbf{C} = \begin{pmatrix} 1 \\ 2a+1 \end{pmatrix} \tag{2}$$

We know that the midpoint formula for the points A and B is

$$\mathbf{C} = \frac{\mathbf{A}+\mathbf{B}}{2} \tag{3}$$

$$\begin{pmatrix} 1 \\ 2a+1 \end{pmatrix} = \frac{\begin{pmatrix} 2a \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 3b \end{pmatrix}}{2} \tag{4}$$

$$\begin{pmatrix} 1 \\ 2a+1 \end{pmatrix} = \frac{\begin{pmatrix} 2a-2 \\ 4+3b \end{pmatrix}}{2} \tag{5}$$

$$\begin{pmatrix} 1 \\ 2a+1 \end{pmatrix} = \begin{pmatrix} a-1 \\ 2+\frac{3b}{2} \end{pmatrix} \tag{6}$$

From Eq.6 we can say that:

$$2a+1 = 2 + \frac{3b}{2} \tag{7}$$

$$2a = 1 + \frac{3b}{2} \tag{8}$$

$$4a = 2 + 3b \tag{9}$$

$$4a - 3b = 2 \tag{10}$$

Let P=(C-A B-A) . A,B and C lies in the same line so they are collinear. So,

$$rank(C - A \quad B - A) = 1 \tag{11}$$

$$rank\begin{pmatrix} 1-2a & -2-2a \\ 2a-3 & 3b-4 \end{pmatrix} = 1 \tag{12}$$

Now by applying the row operation for the matrix P
$R_2 \longrightarrow R_2 + R_1$

$$P = \begin{pmatrix} 1-2a & -2-2a \\ -2 & 3b-2a-6 \end{pmatrix} \tag{13}$$

Now applying another row operation for the matrix P
$R_2 \longrightarrow -\frac{1}{2}R_2$

$$P = \begin{pmatrix} 1-2a & -2-2a \\ 1 & \frac{-3b+2a+6}{2} \end{pmatrix} \tag{14}$$

Now killing the 1st entry of R_1 using the row operation:
$R_1 \longrightarrow R_1 + (2a-1)R_2$

$$P = \begin{pmatrix} 0 & -2-2a+(2a-1)(\frac{-3b+2a+6}{2}) \\ 1 & (\frac{-3b+2a+6}{2}) \end{pmatrix} \tag{15}$$

For the rank to be 1 , all entries of R_1 should be zero. so,

$$-2-2a+(2a-1)(\frac{-3b+2a+6}{2}) = 0 \tag{16}$$

$$4a^2 - 6ab + 6a + 3b - 10 = 0 \tag{17}$$

From Eq.10 we can get

$$b = \frac{4a-2}{3} \tag{18}$$

Now substituting 'b' in Eq.17, we get:

$$2a^2 - 7a + 6 = 0 \tag{19}$$

By solving the above quadratic equation we get:

$$a = 2, \frac{3}{2} \tag{20}$$

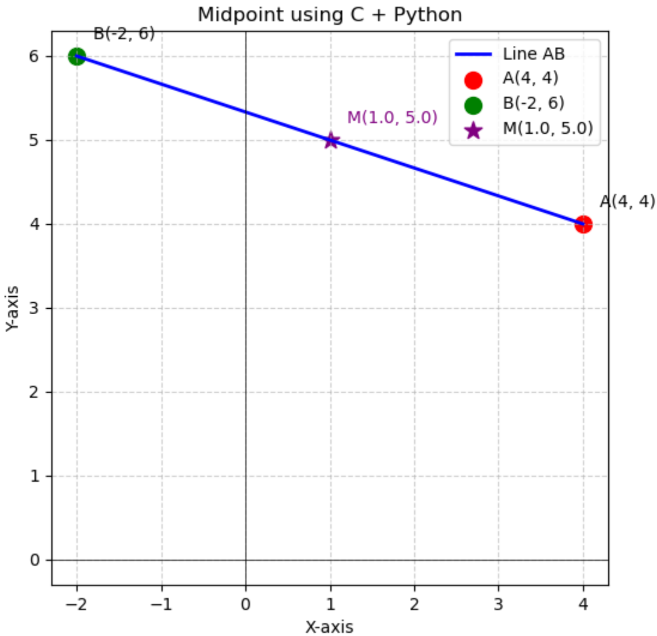
By substituting the value of 'a' in Eq.18 we get:

$$b = 2, \frac{4}{3} \tag{21}$$

But when $a = \frac{3}{2}$ and $b = \frac{4}{3}$ it does not satisfies the Eq.3
So the final value of a and b are:

$$a = 2 \; and \; b = 2 \tag{22}$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.


Midpoint using C + Python
B(-2, 6)
M(1.0, 5.0)
A(4, 4)
Line AB
A(4, 4)
B(-2, 6)
M(1.0, 5.0)
Y-axis
X-axis
6
5
4
3
2
1
0
−2
−1
0
1
2
3
4