1.6.15

EE25BTECH11065 - Yoshita

Question:
Find the value of m if the points $(5, 1)$, $(-2, -3)$ and $(8, 2m)$ are collinear.

Solution:
Let $\mathbf{A}(5, 1)$, $\mathbf{B}(-2, -3)$, $\mathbf{C}(8, 2m)$.

Point	Vector
A	$\begin{pmatrix}5\\1\end{pmatrix}$
B	$\begin{pmatrix}-2\\-3\end{pmatrix}$
C	$\begin{pmatrix}8\\2m\end{pmatrix}$

TABLE 0: Answers

Using the collinearity (*rank*) test, form the matrix with difference vectors:

$$(\mathbf{B}-\mathbf{A} \quad \mathbf{C}-\mathbf{A}) = \begin{pmatrix}-2-5 & 8-5\\-3-1 & 2m-1\end{pmatrix} = \begin{pmatrix}-7 & 3\\-4 & 2m-1\end{pmatrix}.$$

The three points are collinear $\iff$ this matrix has rank 1 (*its rows are linearly dependent*).

$$R_2 \leftarrow 7R_2 - 4R_1 \implies \begin{pmatrix}-7 & 3\\0 & 14m-19\end{pmatrix}.$$

For rank 1, the second row must be zero:

$$14m - 19 = 0 \implies m = \frac{19}{14}$$

See Fig. 0 ,

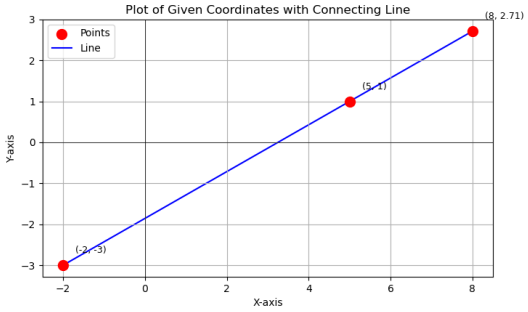
Plot of Given Coordinates with Connecting Line
Points
Line
(8, 2.71)
(5, 1)
(-2, -3)
Y-axis
X-axis
3
2
1
0
−1
−2
−3
−2
0
2
4
6
8

Fig. 0