Problem 2.4.18

ee25btech11023-Venkata Sai

August 27, 2025

Problem Statement

Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

Variable	**Description**
$\mathbf{m_1}$	Direction vector of Line 1
$\mathbf{m_2}$	Direction vector of line 2

Table: Variables given

Requirement

To show that two lines are at right angles

$$(\mathbf{m_1})^\top (\mathbf{m_2}) = 0 \tag{3.1}$$

Transformation of given lines

Line 1:

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \implies \frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2} \tag{3.2}$$

Line 2:

$$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5} \implies \frac{x-1}{-\frac{3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} \tag{3.3}$$

Direction Vectors

Direction vector for line 1:

$$\mathbf{m_1} = \begin{pmatrix} -3 \\ \frac{2p}{7} \\ 2 \end{pmatrix} \tag{3.4}$$

Direction vector for line 2:

$$\mathbf{m_2} = \begin{pmatrix} -\frac{3p}{7} \\ 1 \\ -5 \end{pmatrix} \tag{3.5}$$

Answer

$$(\mathbf{m_1})^{\top} (\mathbf{m_2}) = 0 \tag{3.6}$$

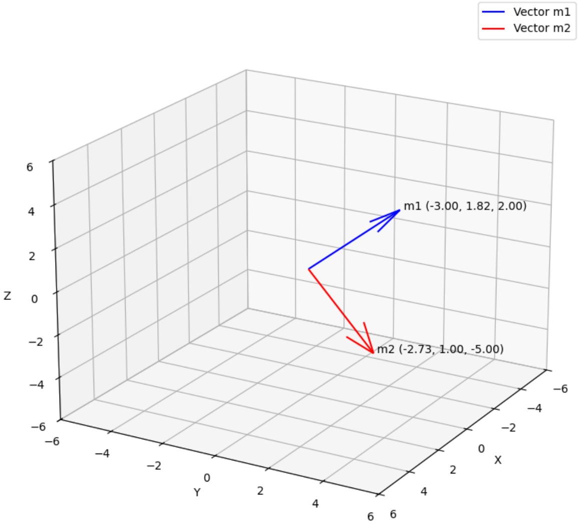
$$\begin{pmatrix} -3 & \frac{2p}{7} & 2 \end{pmatrix} \begin{pmatrix} -\frac{3p}{7} \\ 1 \\ -5 \end{pmatrix} = 0 \tag{3.7}$$

$$(-3)\left(-\frac{3p}{7}\right) + \left(\frac{2p}{7}\right)(1) + (2)(-5) = 0 \tag{3.8}$$

$$p = \frac{70}{11} \tag{3.9}$$

Hence the value of p is $\frac{70}{11}$

Plot



C Code for Finding p value

```c
include <math.h>
#include <stdio.h>
#include <stdlib.h>

int main() {

    double p;
    double m1_p_coeffs[3] = {0.0, 2.0/7.0, 0.0};
    double m1_consts[3] = {-3.0, 0.0, 2.0};

    double m2_p_coeffs[3] = {-3.0/7.0, 0.0, 0.0};
    double m2_consts[3] = {0.0, 1.0, -5.0};

    // Calculate A (the total coefficient for 'p') from the dot
        product expansion.
    double p_coefficient = (m1_consts[0] * m2_p_coeffs[0]) + (
        m1_p_coeffs[0] * m2_consts[0]) +(m1_consts[1] *
        m2_p_coeffs[1]) + (m1_p_coeffs[1] * m2_consts[1]) + (
        m1_consts[2] * m2_p_coeffs[2]) + (m1_p_coeffs[2] *
```

C Code for Finding p value

```
  m2_consts[2]);
// Calculate B (the total constant term) from the dot product
    expansion.
    double constant_term = (m1_consts[0] * m2_consts[0]) +
                           (m1_consts[1] * m2_consts[1]) +
                           (m1_consts[2] * m2_consts[2]);


    p = -constant_term / p_coefficient;

    double m1_final[3];
    double m2_final[3];

    m1_final[0] = m1_consts[0] + m1_p_coeffs[0] * p;
    m1_final[1] = m1_consts[1] + m1_p_coeffs[1] * p;
```

C Code for Finding p value

```c
 m1_final[2] = m1_consts[2] + m1_p_coeffs[2] * p;
m2_final[0] = m2_consts[0] + m2_p_coeffs[0] * p;
m2_final[1] = m2_consts[1] + m2_p_coeffs[1] * p;
m2_final[2] = m2_consts[2] + m2_p_coeffs[2] * p;


FILE *file = fopen("values.dat", "w");
if (file == NULL) {
    printf("Error opening file!\n");
    return 1;
}
```

C Code for Finding p value

```
    fprintf(file, "%.4f %.4f %.4f\n", m1_final[0], m1_final[1],
        m1_final[2]);
    fprintf(file, "%.4f %.4f %.4f\n", m2_final[0], m2_final[1],
        m2_final[2]);

    fclose(file);
    printf("Direction vectors calculated and written to values.
        dat\n");
    printf("(The calculated value of p was: %.4f)\n", p);

    return 0;
}
```

Python Code for Plotting

```
# Code by /sdcard/github/matgeo/codes/CoordGeoVV Sharma
# September 12, 2023
# Revised July 21, 2024
# Released under GNU GPL
# Section Formula

import sys
sys.path.insert(0, '/workspaces/urban-potato/matgeo/codes/
    CoordGeo/') # path to my scripts
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
# Local imports
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen
```

Python Code for Plotting

```
# Read data
data = np.loadtxt("values.dat", skiprows=1)

xc = data[0] # Extract x-coordinate (e.g., -1)
yc = data[1] # Extract y-coordinate (e.g., 4.5)

# Given points
A = np.array([-6, 7]).reshape(-1, 1)
B = np.array([-1, -5]).reshape(-1, 1)
P = np.array([xc, yc]).reshape(-1, 1)

# Generating line AB
x_AB = line_gen(A, B)

# Plotting
plt.plot(x_AB[0, :], x_AB[1, :], label='$AB$')
```

Python Code for Plotting

```
# Labeling the coordinates
tri_coords = np.block([[A, B, P]])
plt.scatter(tri_coords[0, :], tri_coords[1, :])

vert_labels = ['A', 'B', 'P']

# Helper function: format number with decimal only if needed
def fmt(val):
    return f"{val:.1f}" if abs(val - round(val)) > 1e-6 else f"{int(val)}"

for i, txt in enumerate(vert_labels):
    x = tri_coords[0, i].item()
    y = tri_coords[1, i].item()
    plt.annotate(f'{txt}\n({fmt(x)}, {fmt(y)})',
                 (x, y),
                 textcoords="offset points",
                 xytext=(20, -10),
                 ha='center')
```

Python Code for Plotting

```
ax = plt.gca()
ax.spines['left'].set_visible(False)
ax.spines['right'].set_visible(False)
ax.spines['top'].set_visible(False)
ax.spines['bottom'].set_visible(False)

plt.legend(loc='best')
plt.grid()

# Increase y-axis from -8 to 8 to show full range
plt.ylim(-7, 8)
plt.xlim(-7,7)

# Save and open
plt.show()
plt.savefig('../figs/fig1.png')
```