**Problem 1.4.2 .** Find the coordinates of the point $R$ on the line segment joining P(1,3) and Q(2,5) such that $\mathbf{PR} = \frac{3}{5}\mathbf{PQ}$

**Solution.**

| Input variable | Value |
|---|---|
| $\mathbf{P}$ | $\begin{pmatrix}1\\3\end{pmatrix}$ |
| $\mathbf{Q}$ | $\begin{pmatrix}2\\5\end{pmatrix}$ |
| $\frac{PR}{PQ}$ | $\frac{3}{5}$ |

Table 1

Let the position vectors be

$$\mathbf{P} = \begin{pmatrix}1\\3\end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix}2\\5\end{pmatrix}.$$

If $\mathbf{R}$ is the position vector of $R$, then

$$\mathbf{R} - \mathbf{P} = \frac{3}{5}(\mathbf{Q} - \mathbf{P}) \implies \mathbf{R} = \mathbf{P} + \frac{3}{5}(\mathbf{Q} - \mathbf{P}).$$

So,

$$\mathbf{R} = \begin{pmatrix}1\\3\end{pmatrix} + \frac{3}{5}\left(\begin{pmatrix}2\\5\end{pmatrix} - \begin{pmatrix}1\\3\end{pmatrix}\right) = \begin{pmatrix}1\\3\end{pmatrix} + \frac{3}{5}\begin{pmatrix}1\\2\end{pmatrix}.$$

Hence,

$$\mathbf{R} = \begin{pmatrix}1 + \frac{3}{5}\\3 + \frac{6}{5}\end{pmatrix} = \begin{pmatrix}\frac{8}{5}\\\frac{21}{5}\end{pmatrix}.$$

Therefore, the required point is

$$\boxed{\mathbf{R} = \begin{pmatrix}\frac{8}{5}\\\frac{21}{5}\end{pmatrix}}$$

which indeed satisfies $\mathbf{R} - \mathbf{P} = \frac{3}{5}(\mathbf{Q} - \mathbf{P})$.

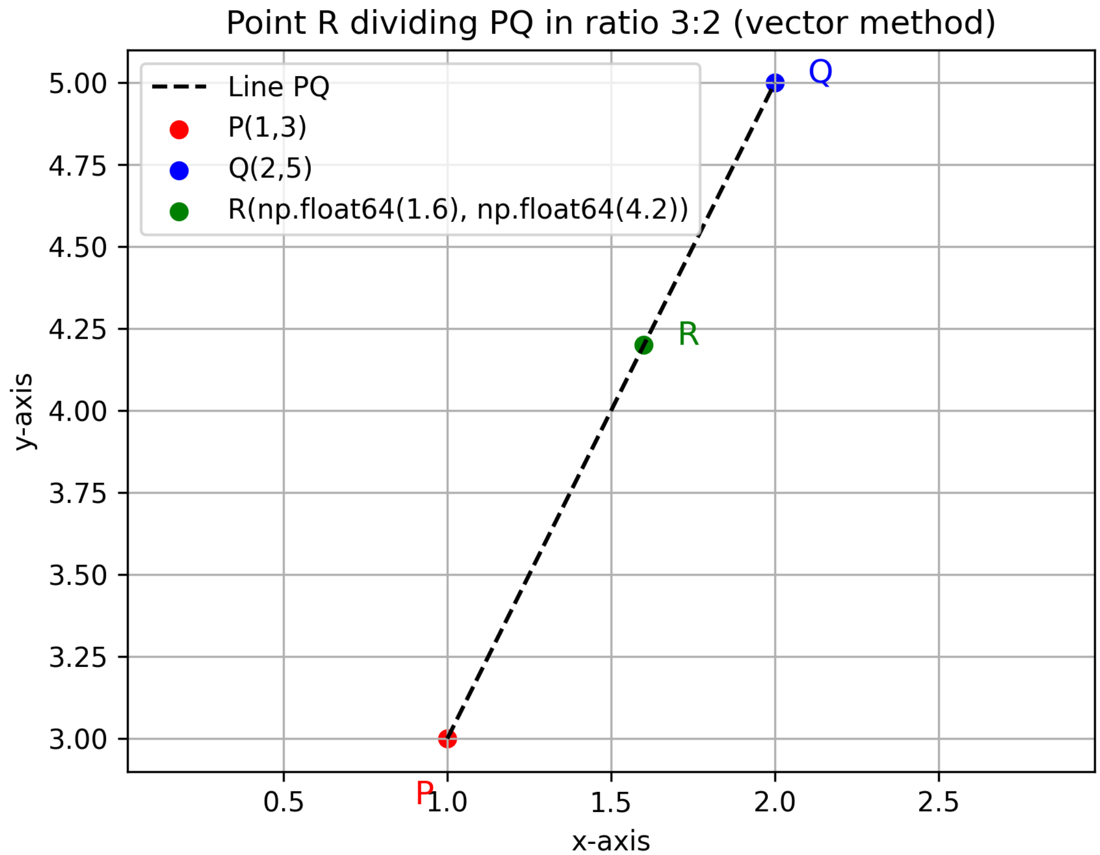
Point R dividing PQ in ratio 3:2 (vector method)
Line PQ
P(1,3)
Q(2,5)
R(np.float64(1.6), np.float64(4.2))
Q
R
P
5.00
4.75
4.50
4.25
4.00
3.75
3.50
3.25
3.00
y-axis
0.5
1.0
1.5
2.0
2.5
x-axis

Figure 1