

1.5.15

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Question

The midpoint of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(1, 2a + 1)$. Find the values of a and b .

$$\mathbf{A} = \begin{pmatrix} 2a \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 3b \end{pmatrix} \quad (1)$$

Let the midpoint of points A and B be C. where,

$$\mathbf{C} = \begin{pmatrix} 1 \\ 2a + 1 \end{pmatrix} \quad (2)$$

Theoretical Solution

We know that the midpoint formula for the points A and B is

$$\mathbf{C} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (3)$$

$$\begin{pmatrix} 1 \\ 2a + 1 \end{pmatrix} = \frac{\begin{pmatrix} 2a \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 3b \end{pmatrix}}{2} \quad (4)$$

$$\begin{pmatrix} 1 \\ 2a + 1 \end{pmatrix} = \frac{\begin{pmatrix} 2a - 2 \\ 4 + 3b \end{pmatrix}}{2} \quad (5)$$

$$\begin{pmatrix} 1 \\ 2a + 1 \end{pmatrix} = \begin{pmatrix} a - 1 \\ 2 + \frac{3b}{2} \end{pmatrix} \quad (6)$$

Theoretical Solution

From Eq.6 we can say that:

$$2a + 1 = 2 + \frac{3b}{2} \quad (7)$$

$$2a = 1 + \frac{3b}{2} \quad (8)$$

$$4a = 2 + 3b \quad (9)$$

$$4a - 3b = 2 \quad (10)$$

Theoretical Solution

And also A,B and C lies in the same line so they are collinear. So,

$$\text{rank}(C - A \quad B - A) = 1 \quad (11)$$

$$\text{rank} \begin{pmatrix} 1 - 2a & -2 - 2a \\ 2a - 3 & 3b - 4 \end{pmatrix} = 1 \quad (12)$$

When the rank of the (2×2) matrix is 1, it's determinant is 0 .So,

$$\text{Det} \begin{pmatrix} 1 - 2a & -2 - 2a \\ 2a - 3 & 3b - 4 \end{pmatrix} = 0 \quad (13)$$

$$(1 - 2a)(3b - 4) - (2a - 3)(-2 - 2a) = 0 \quad (14)$$

$$4a^2 + 6a + 3b - 6ab - 10 = 0 \quad (15)$$

Theoretical Solution

From Eq.10 we can get

$$b = \frac{4a - 2}{3} \quad (16)$$

Now substituting 'b' in Eq.15, we get:

$$2a^2 - 7a + 6 = 0 \quad (17)$$

By solving the above quadratic equation we get:

$$a = 2, \frac{3}{2} \quad (18)$$

By substituting the value of 'a' in Eq.16, we get:

$$b = 2, \frac{4}{3} \quad (19)$$

But when $a = \frac{3}{2}$ and $b = \frac{4}{3}$ it does not satisfies the Eq.3

So the final value of a and b are:

$$a = 2 \text{ and } b = 2 \quad (20)$$

C Code - Midpoint formula

```
#include <stdio.h>

// Function to calculate midpoint
void midpoint(float x1, float y1, float x2, float y2, float *mx,
             float *my) {
    *mx = (x1 + x2) / 2.0;
    *my = (y1 + y2) / 2.0;
}
```


Python Code

```
import numpy as np
import ctypes
import matplotlib.pyplot as plt

# Load the shared library
lib = ctypes.CDLL(./midpoint.so) # use midpoint.dll on Windows

# Define function signature
lib.midpoint.argtypes = [
    ctypes.c_float, ctypes.c_float, # x1, y1
    ctypes.c_float, ctypes.c_float, # x2, y2
    ctypes.POINTER(ctypes.c_float), # mx
    ctypes.POINTER(ctypes.c_float) # my
]
```

```
# Given values from problem
a, b = 2, 2
A = (2*a, 4) # (4,4)
B = (-2, 3*b) # (-2,6)

# Prepare variables to hold midpoint
mx, my = ctypes.c_float(), ctypes.c_float()

# Call the C function
lib.midpoint(A[0], A[1], B[0], B[1], ctypes.byref(mx), ctypes.
    byref(my))
M = (mx.value, my.value)

print(fMidpoint from C: {M})
```

```
# --- Plot ---
plt.figure(figsize=(6,6))
plt.plot([A[0], B[0]], [A[1], B[1]], 'b-', linewidth=2, label='
    Line AB')

# Scatter points
plt.scatter(*A, color='red', s=100, label=fA{A})
plt.scatter(*B, color='green', s=100, label=fB{B})
plt.scatter(*M, color='purple', s=120, marker='*', label=fM{M})

# Annotate
plt.text(A[0]+0.2, A[1]+0.2, fA{A}, fontsize=10)
plt.text(B[0]+0.2, B[1]+0.2, fB{B}, fontsize=10)
plt.text(M[0]+0.2, M[1]+0.2, fM{M}, fontsize=10, color=purple)
```

```
# Axes formatting
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.grid(True, linestyle='--', alpha=0.6)
plt.legend()
plt.title(Midpoint using C + Python)
plt.xlabel(X-axis)
plt.ylabel(Y-axis)
plt.show()
```

