

1.9.2

EE25BTECH11013 - Bhargav

Question:

The point on the X axis which is equidistant from $(-4, 0)$ and $(10, 0)$ is

Solution:

Let the 2 points be \mathbf{A} and \mathbf{B} and let the desired point equidistant from both \mathbf{A} and \mathbf{B} be \mathbf{O} :

$$\mathbf{A} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \quad (0.1)$$

$$\mathbf{B} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \quad (0.2)$$

$$\mathbf{O} = x \mathbf{e}_1 = \begin{pmatrix} x \\ 0 \end{pmatrix} \quad (0.3)$$

If \mathbf{O} lies on X axis and is equidistant from \mathbf{A} and \mathbf{B}

$$\|\mathbf{O} - \mathbf{A}\| = \|\mathbf{O} - \mathbf{B}\| \quad (0.4)$$

$$\implies \|\mathbf{O} - \mathbf{A}\|^2 = \|\mathbf{O} - \mathbf{B}\|^2 \quad (0.5)$$

$$\implies (\mathbf{O} - \mathbf{A})^\top (\mathbf{O} - \mathbf{A}) = (\mathbf{O} - \mathbf{B})^\top (\mathbf{O} - \mathbf{B}) \quad (0.6)$$

$$\implies \mathbf{O}^\top \mathbf{O} - 2\mathbf{O}^\top \mathbf{A} + \mathbf{A}^\top \mathbf{A} = \mathbf{O}^\top \mathbf{O} - 2\mathbf{O}^\top \mathbf{B} + \mathbf{B}^\top \mathbf{B} \quad (0.7)$$

$$\implies \|\mathbf{O}\|^2 - 2\mathbf{O}^\top \mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{O}\|^2 - 2\mathbf{O}^\top \mathbf{B} + \|\mathbf{B}\|^2 \quad (0.8)$$

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{O} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}. \quad (0.9)$$

$$\mathbf{O} = x\mathbf{e}_1, \quad (0.10)$$

$$x = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A} - \mathbf{B})^\top \mathbf{e}_1}. \quad (0.11)$$

Solving for x , we get $x = 3$

$$\therefore \mathbf{O} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (0.12)$$

