1.5.1

EE25BTECH11051 - Shreyas Goud Burra

Question

Find the ratio in which the Y axis divides the line segment joining the points (6, -4) and (-2, -7). Also find the point of intersection.

Solution:

Let us solve the given equation theoretically and then verify the solution computationally.

Assume the two points to be position vectors $\mathbf{A} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 \\ -7 \end{pmatrix}$ To find the ratio in which the Y axis divides the line segment. We can use the section formula

$$\mathbf{C} = \left(\frac{\frac{m}{n}A + B}{\frac{m}{n} + 1} \right) \tag{0.1}$$

Here, $\mathbf{C}$ is the vector on the Y axis that intersects the line segment joining the position vectors $\mathbf{A}$ and $\mathbf{B}$ and divides it in the ratio of $m : n$, where m and n are integers in their lowest form.

Here we can assume some constant $k = \frac{m}{n}$. This gives us

$$\mathbf{C} = \left(\frac{kA + B}{k + 1} \right) \tag{0.2}$$

We know that the point which divides the line segment lies on the Y axis and therefore its x coordinate is zero.

We can write $\mathbf{C}$ as

$$\mathbf{C} = \begin{pmatrix} 0 \\ y \end{pmatrix} \tag{0.3}$$

Where y_1 is the y coordinate of the point of intersection of the Y axis and the line segment AB.
We know that these three points are collinear, so by using rank method we get. Rank of matrix

$$\mathbf{P} = (\mathbf{B} - \mathbf{A} \ \ \mathbf{C} - \mathbf{A}) = 1 \tag{0.4}$$

$$\implies \text{Rank of} \begin{pmatrix} -8 & -6 \\ -3 & y+4 \end{pmatrix} = 1 \tag{0.5}$$

On applying row transformations

$$R_2 \to R_2 - \frac{3}{8}R_1$$

$$\mathbf{C} = \begin{pmatrix} -8 & -6 \\ 0 & y + \frac{25}{4} \end{pmatrix} \tag{0.6}$$

$$\tag{0.7}$$

If rank $= 0$

$$\implies y + \frac{25}{4} = 0 \tag{0.8}$$

$$y = -\frac{25}{4} \tag{0.9}$$
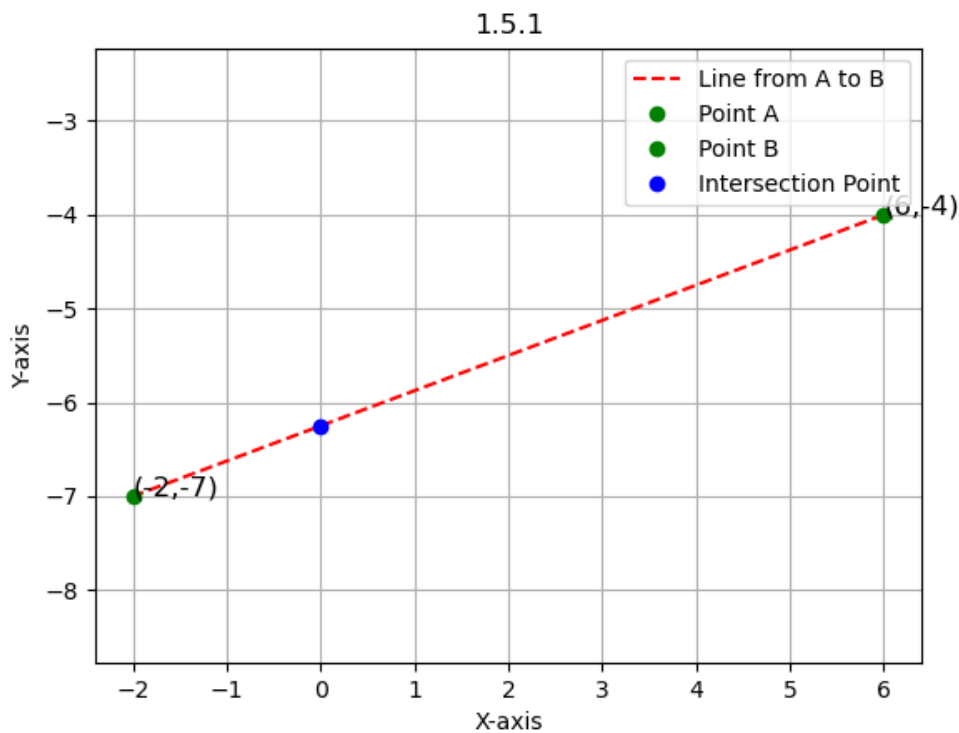
We get the same result by plotting the graph

Fig. 0.1: 2D Plot