1.3.8

SAMYAK GONDANE - AI25BTECH11029

Question

ABCD is a rectangle formed by the points $\mathbf{A}(-1,-1)$, $\mathbf{B}(-1,6)$, $\mathbf{C}(3,6)$ and $\mathbf{D}(3,-1)$. **P**, **Q**, **R** and **S** are mid-points of sides AB, BC, CD and DA respectively. Show that diagonals of the quadrilateral PQRS bisect each other.

Solution

Step 1: Represent Points as Column Vectors

Let's define the points as column vectors:

$$A = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 \\ 6 \end{pmatrix}, \quad C = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \quad D = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

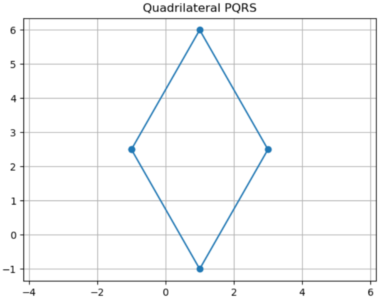

Figure:

Solution

Step 2: Midpoints as Matrix Expressions

So,

$$P = \frac{1}{2}(A + B) = \frac{1}{2}\begin{pmatrix} -1 + (-1) \\ -1 + 6 \end{pmatrix} = \begin{pmatrix} -1 \\ 2.5 \end{pmatrix} \tag{1}$$

$$Q = \frac{1}{2}(B + C) = \frac{1}{2}\begin{pmatrix} -1 + 3 \\ 6 + 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} \tag{2}$$

$$R = \frac{1}{2}(C + D) = \frac{1}{2}\begin{pmatrix} 3 + 3 \\ 6 + 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3.5 \end{pmatrix} \tag{3}$$

$$S = \frac{1}{2}(D + A) = \frac{1}{2}\begin{pmatrix} 3 + (-1) \\ 1 + (-1) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4}$$

Solution

Step 3: Diagonals PR and QS

Diagonal PR:

$$\text{Midpoint}_{PR} = \frac{1}{2}(P + R) = \frac{1}{2}\left(\begin{pmatrix} -1 \\ 2.5 \end{pmatrix} + \begin{pmatrix} 3 \\ 3.5 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Diagonal QS:

$$\text{Midpoint}_{QS} = \frac{1}{2}(Q + S) = \frac{1}{2}\left(\begin{pmatrix} 1 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Conclusion

Since both diagonals PR and QS have the same midpoint $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ they bisect each other.