

# 1.6.6

ee25btech11056 - Suraj.N

In each of the following, find the value of  $k$  for which the points are collinear:

- 1)  $(7, -2), (5, 1), (3, k)$
- 2)  $(8, 1), (k, -4), (2, -5)$

**Solution:** Three points  $A, B, C$  are collinear iff the collinearity matrix

$$M = (\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^\top$$

has  $\text{rank}(M) = 1$ .

(a)

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^\top = \begin{pmatrix} -2 & 3 \\ -4 & k+2 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 3 \\ -4 & k+2 \end{pmatrix} \xrightarrow{R_2=R_2-2R_1} \begin{pmatrix} -2 & 3 \\ 0 & k-4 \end{pmatrix}$$

For collinearity,  $\text{rank}(M) = 1 \iff k - 4 = 0 \implies \boxed{k = 4}$ .

(b)

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^\top = \begin{pmatrix} k-8 & -5 \\ -6 & -6 \end{pmatrix}$$

$$\begin{pmatrix} k-8 & -5 \\ -6 & -6 \end{pmatrix} \xrightarrow{R_2=(k-8)R_2+6R_1} \begin{pmatrix} k-8 & -5 \\ 0 & 18-6k \end{pmatrix}$$

For collinearity,  $\text{rank}(M) = 1 \iff 18 - 6k = 0 \implies \boxed{k = 3}$ .

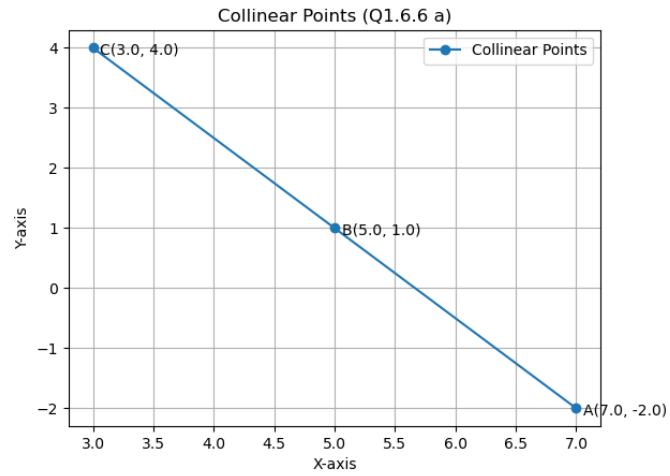


Fig 1 : Line through the given points

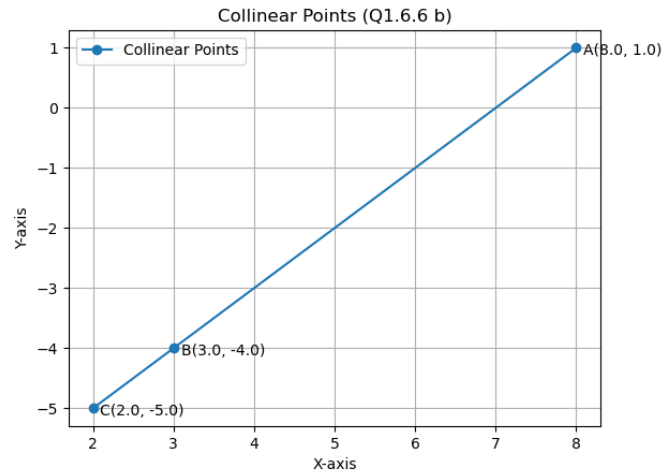


Fig 2 : Line through the given points