GATE MA 2009

EE25BTECH11030-AVANEESH

1 - 20 carry one mark each.

1) The dimension of the vector space $V = \{A = (a_{ij})_{n\times n} : a_{ij} \in \mathbb{C},\ a_{ij} = -a_{ji}\}$ over $\mathbb{R}$ is

a) n^2 b) $n^2 - 1$ c) $n^2 - n$ d) $n^2/2$

(GATE MA 2009)

2) The minimal polynomial associated with the matrix $\begin{pmatrix} 0 & 0 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix}$ is

a) $x^3 - x^2 - 2x - 3$
b) $x^3 - x^2 + 2x - 3$
c) $x^3 - x^2 - 3x - 3$
d) $x^3 - x^2 + 3x - 3$

(GATE MA 2009)

3) For the function $f(z) = \sin\left(\frac{1}{\cos(1/z)}\right)$, the point $z = 0$ is

a) a removable singularity
b) a pole
c) an essential singularity
d) a non-isolated singularity

(GATE MA 2009)

4) Let $f(z) = \sum_{n=0}^{\infty} z^n$ for $z \in \mathbb{C}$. If $C : |z - i| = 2$, then

$$\oint_C \frac{f(z)}{(z-i)^{16}} dz =$$

a) $2\pi i(1 + 15i)$ b) $2\pi i(1 - 15i)$ c) $4\pi i(1 + 15i)$ d) $2\pi i$

(GATE MA 2009)

5) For what values of α and β, the quadrature formula $\int_{-1}^{1} f(x)\,dx = \alpha f(-1) + f(\beta)$ is exact for all polynomials of degree ≤ 1?

a) $\alpha = 1, \beta = 1$ b) $\alpha = -1, \beta = 1$ c) $\alpha = 1, \beta = -1$ d) $\alpha = -1, \beta = -1$

(GATE MA 2009)

6) Let $f : [0, 4] \to \mathbb{R}$ be a three times continuously differentiable function. Then the value of $f[1, 2, 3, 4]$ is

a) $\frac{f''(\xi)}{3}$, for some $\xi \in (0, 4)$
b) $\frac{f''(\xi)}{6}$, for some $\xi \in (0, 4)$
c) $\frac{f'''(\xi)}{3}$, for some $\xi \in (0, 4)$
d) $\frac{f'''(\xi)}{6}$, for some $\xi \in (0, 4)$

(GATE MA 2009)

7) Which one of the following is **TRUE**?

a) Every linear programming problem has a feasible solution.
b) If a linear programming problem has an optimal solution then it is unique.
c) The union of two convex sets is necessarily convex.
d) Extreme points of the disk $x^2 + y^2 \leq 1$ are the points on the circle $x^2 + y^2 = 1$.

(GATE MA 2009)

8) The dual of the linear programming problem: Minimize $c^T x$ subject to $Ax \geq b$ and $x \geq 0$ is
   a) Maximize $b^T w$ subject to $A^T w \geq c$, $w \geq 0$
   b) Maximize $b^T w$ subject to $A^T w \leq c$, $w \geq 0$
   c) Maximize $b^T w$ subject to $A^T w \leq c$, w unrestricted
   d) Maximize $b^T w$ subject to $A^T w \geq c$, w unrestricted

(GATE MA 2009)

9) The resolvent kernel for the integral equation $u(x) = F(x) + \int e^{t-x} u(t) dt$ is

   a) $\cos(x - t)$ b) 1 c) e^{-x} d) $e^{2(t-x)}$

(GATE MA 2009)

10) Consider the metrics $d_2(f, g) = \left(\int |f(t) - g(t)|^2 dt\right)^{1/2}$ and $d_\infty(f, g) = \sup |f(t) - g(t)|$ on the space $X = C[a, b]$ of all real-valued continuous functions on $[a, b]$. Which is TRUE?
   a) Both (X, d_2) and (X, d_∞) are complete.
   b) (X, d_2) is complete, but (X, d_∞) is not complete.
   c) (X, d_∞) is complete, but (X, d_2) is not complete.
   d) Both are NOT complete.

(GATE MA 2009)

11) A function $f : \mathbb{R} \to \mathbb{R}$ need NOT be Lebesgue measurable if
   a) f is monotone
   b) $\{x : f(x) \geq a\}$ is measurable for all $a \in \mathbb{Q}$
   c) $\{x : f(x) = a\}$ is measurable for all $a \in \mathbb{R}$
   d) For each open set $G \subset \mathbb{R}$, $f^{-1}(G)$ is measurable

(GATE MA 2009)

12) Let $\{e_n\}$ be an orthonormal sequence in a Hilbert space H, and let $x \neq 0 \in H$. Then
   a) $\lim_{n\to\infty} \langle x, e_n \rangle$ does not exist
   b) $\lim_{n\to\infty} \langle x, e_n \rangle = ||x||$
   c) $\lim_{n\to\infty} \langle x, e_n \rangle = 1$
   d) $\lim_{n\to\infty} \langle x, e_n \rangle = 0$

(GATE MA 2009)

13) The subspace $\mathbb{Q} \times [0, 1]$ of $\mathbb{R}^2$ (with the usual topology) is

   a) dense in $\mathbb{R}^2$
   b) connected
   c) separable
   d) compact

(GATE MA 2009)

14) $\mathbb{Z}_2[x]/\langle x^2 + x^2 + 1 \rangle$ is

   a) a field with 8 elements
   b) a field with 9 elements
   c) an infinite field
   d) NOT a field

(GATE MA 2009)

15) The number of elements of a principal ideal domain can be

   a) 15 b) 25 c) 35 d) 36

(GATE MA 2009)

16) Let F, G, H be pairwise independent with $P(F) = P(G) = P(H) = 1/3$ and $P(F \cap G \cap H) = 1/4$. The probability that at least one event among F, G and H occurs is

a) $11/12$ b) $7/12$ c) $5/12$ d) $3/4$

(GATE MA 2009)

17) Let X be a random variable such that $E(X^2) = E(X) = 1$. Then $E(X^{100}) =$

a) 0 b) 1 c) 2^{100} d) $2^{100} + 1$

(GATE MA 2009)

18) For which of the following distribution, the weak law of large numbers NOT hold?

a) Normal b) Gamma c) Beta d) Cauchy

(GATE MA 2009)

19) If $D = \frac{d}{dx}$, then the value of $\frac{1}{(xD+1)}(x^{-1})$ is

a) $\log x$ b) $\frac{\log x}{x}$ c) $\frac{\log x}{x^2}$ d) $\frac{\log x}{x^3}$

(GATE MA 2009)

20) The equation $(\alpha xy^3 + y\cos x)dx + (x^2y^2 + \beta\sin x)dy = 0$ is exact for

a) $\alpha = \frac{3}{2}, \beta = 1$
b) $\alpha = 1, \beta = \frac{3}{2}$
c) $\alpha = 1, \beta = 1$
d) $\alpha = 1, \beta = \frac{2}{3}$

(GATE MA 2009)

21 - 60 carry two marks each.

21) If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 + i\sqrt{3} & 0 \\ 0 & 0 & 1 + 2i \end{pmatrix}$, then the trace of A^{102} is

a) 0 b) 1 c) 2 d) 3

(GATE MA 2009)

22) Which of the following matrices is NOT diagonalizable?

a) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$ c) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

(GATE MA 2009)

23) Let V be the column space of $A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -1 \end{pmatrix}$. The orthogonal projection of $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ on V is

a) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ b) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ c) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

(GATE MA 2009)

24) Let $\sum_{n=-\infty}^{\infty} a_n(z+1)^n$ be the Laurent series expansion of $f(z) = \sin\left(()\,\frac{z}{z+1}\right)$. Then $a_2 =$

a) 1 b) 0 c) $\cos(1)$ d) $\frac{-1}{2}\sin(1)$

(GATE MA 2009)

25) Let $u(x,y)$ be the real part of an entire function $f(z) = u(x,y) + iv(x,y)$ for $z = x + iy \in \mathbb{C}$.If C is the positively oriented boundary of a rectangular region R in $\mathbb{R}^2$. Then

$$\oint_C \frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy =$$

a) 1 b) 0 c) 2π d) π

(GATE MA 2009)

26) Let $\varphi : [0,1] \to \mathbb{R}$ be three times continuously differentiable. Suppose that the iterates $x_{n+1} = \varphi(x_n)$, $n \geq 0$ converge to the fixed point ξ of φ. If the order of convergence is three then

a) $\varphi'(\xi) = 0$, $\varphi''(\xi) = 0$
b) $\varphi'(\xi) \neq 0$, $\varphi''(\xi) = 0$
c) $\varphi'(\xi) = 0$, $\varphi''(\xi) \neq 0$
d) $\varphi'(\xi) \neq 0$, $\varphi''(\xi) \neq 0$

(GATE MA 2009)

27) Let $f : [0,2] \to \mathbb{R}$ be twice continuously differentiable. If $\int_0^2 f(x)dx = 2f(1)$, the error in the approximation is

a) $\frac{f'(5)}{12}$ for some $\xi \in (0,2)$
b) $\frac{f'(5)}{2}$ for some $\xi \in (0,2)$
c) $\frac{f''(\xi)}{3}$ for some $\xi \in (0,2)$
d) $\frac{f''(\xi)}{6}$ for some $\xi \in (0,2)$

(GATE MA 2009)

28) For fixed $t \in \mathbb{R}$, consider: Max $z = 3x + 4y,\ x + y \leq 100,\ x + 3y \leq t,\ x, y \geq 0$. The maximum value $z = 400$ for $t =$

a) 50 b) 100 c) 200 d) 300

(GATE MA 2009)

29) Minimize $z = 2x_1 - x_2 + x_3 + 5x_4 + 2x_5$, subject to:

$$\begin{aligned} x_1 - 2x_4 + x_5 &= 6 \\ x_2 + x_4 - 4x_5 &= 3 \\ x_3 + 3x_1 + 2x_5 &= 10 \\ x_j \geq 0,\ j &= 1, \ldots, 5 \end{aligned}$$

is

a) 28 b) 19 c) 10 d) 9

(GATE MA 2009)

30) Using the Hungarian method, the optimal value of the assignment problem whose cost matrix is given by

$$\begin{pmatrix} 5 & 23 & 14 & 8 \\ 10 & 25 & 1 & 23 \\ 35 & 16 & 15 & 12 \\ 16 & 23 & 11 & 7 \end{pmatrix}$$

is

a) 29 b) 52 c) 26 d) 44

(GATE MA 2009)

31) Which of the following sequence $\{f_n\}_{n=1}^{\infty}$ of functions does NOT converge uniformly on $[0,1]$?

a) $f_n(x) = e^{-x}/n$
b) $f_n(x) = (1-x)^n$
c) $f_n(x) = (x^2 + nx)/n$
d) $f_n(x) = \sin(()\, nx + n)/n$

(GATE MA 2009)

32) Let $E = \{(x, y) \in \mathbb{R}^2 : 0 < x < y\}$. Then

$$\iint_E y e^{-(x+y)} dx\, dy =$$

a) $\frac{1}{4}$
b) $\frac{3}{2}$
c) $\frac{4}{3}$
d) $\frac{3}{4}$

(GATE MA 2009)

33) Let

$$f_n(x) = \frac{1}{n} \sum_{k=0}^{n} \sqrt{k(n-k)} \binom{n}{k} x^k (1-x)^{n-k}$$

on $x \in [0, 1]$, $n = 1, 2, ...$ If $\lim_{n\to\infty} f_n(x) = f(x)$ for $x \in [0, 1]$, then the maximum of $f(x)$ on $[0, 1]$ is

a) 1
b) $\frac{1}{2}$
c) $\frac{1}{3}$
d) $\frac{1}{4}$

(GATE MA 2009)

34) Let $f : (()\, c_{00}, \|\cdot\|_1) \to \mathbb{C}$ be a non-zero continuous linear functional. The number of Hahn-Banach extensions to $(\ell^1, \|\cdot\|_1)$ is

a) one
b) two
c) three
d) infinite

(GATE MA 2009)

35) If $I : (()\, \ell^1, \|\cdot\|_2) \to (()\, \ell^1, \|\cdot\|_1)$ is the identity map, then

a) both I and I^{-1} are continuous
b) I is continuous but I^{-1} is not continuous
c) I^{-1} is continuous but I is not continuous
d) neither I nor I^{-1} is continuous

(GATE MA 2009)

36) Consider the topology $\tau = \{G \subseteq \mathbb{R} : \mathbb{R} \setminus G \text{ is compact in } (()\, \mathbb{R}, \tau_u)\} \cup \{\phi, \mathbb{R}\}$ on $\mathbb{R}$, where τ_u is the usual topology on $\mathbb{R}$ and ϕ is the empty set. Then $(\mathbb{R}, \tau)$ is

a) a connected Hausdorff space
b) connected but NOT Hausdorff
c) Hausdorff but NOT connected
d) neither connected nor Hausdorff

(GATE MA 2009)

37) Let
$\tau_1 = \{G \subseteq \mathbb{R} : G \text{ is finite or } \mathbb{R} \setminus G \text{ is finite}\}$
and
$\tau_2 = \{G \subseteq \mathbb{R} : G \text{ is countable or } \mathbb{R} \setminus G \text{ is countable}\}$.
Then:

a) neither τ_1 not τ_2 is a topology on $\mathbb{R}$
b) τ_1 is a topology on $\mathbb{R}$ but τ_2 is NOT a topology on $\mathbb{R}$
c) τ_2 is a topology on $\mathbb{R}$ but τ_1 is NOT a topology on $\mathbb{R}$
d) both are topologies on $\mathbb{R}$

(GATE MA 2009)

38) Which one of the following ideals of the ring $\mathbb{Z}[i]$ of Gaussian integers is NOT maximal?

a) $\langle 1+i\rangle$ b) $\langle 1-i\rangle$ c) $\langle 2+i\rangle$ d) $\langle 3+i\rangle$

(GATE MA 2009)

39) If $Z(G)$ denotes the centre of a group G , then the order of the quotient group $G/Z(G)$ cannot be

a) 4 b) 6 c) 15 d) 25

(GATE MA 2009)

40) Which group is NOT cyclic?

a) $\mathrm{Aut}(\mathbb{Z}_4)$ b) $\mathrm{Aut}(\mathbb{Z}_6)$ c) $\mathrm{Aut}(\mathbb{Z}_8)$ d) $\mathrm{Aut}(\mathbb{Z}_{10})$

(GATE MA 2009)

41) Let X be a non-negative integer valued random variable with $E(X^2) = 3$ and $E(X) = 1$. Then

$$\sum_{i=1}^{\infty} iP(X \geq i) =$$

a) 1 b) 2 c) 3 d) 4

(GATE MA 2009)

42) Let X be a random variable with probability density function $f \in \{f_o, f_1\}$, where

$$f_o(x) = \begin{cases} 2x, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

and

$$f_1(x) = \begin{cases} 3x^2, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

For testing the null hypothesis $H_o : f \equiv f_1$ at level of significance $\alpha = 0.19$, the power of the most powerful test is

a) 0.729 b) 0.271 c) 0.615 d) 0.385

(GATE MA 2009)

43) Let X and Y be independent and identically distributed $U(0, 1)$ random variables. Then $P\left(Y < (X - 1/2)^2\right) =$

a) $1/12$ b) $1/4$ c) $1/3$ d) $2/3$

(GATE MA 2009)

44) Let X and Y be Banach spaces and let $T : X \to Y$ be a linear map. Consider the statements:
P: If $x_n \to x$ in X then $Tx_n \to Tx$ in Y.
Q: If $x_n \to x$ in X and $Tx_n \to y$ in Y then $Tx = y$.
Then

a) P implies Q and Q implies P
b) P implies Q but Q does not imply P
c) Q implies P but P does not imply Q
d) neither P implies Q nor Q implies P

(GATE MA 2009)

45) If $y(x) = x$ is a solution of the differential equation $y'' - \left(\frac{2}{x^2} + \frac{1}{x}\right)\left(xy^1 - y\right) = 0$, $0 < x < \infty$, then its general solution is

a) $(\alpha+\beta e^{-2x})x$ b) $(\alpha+\beta e^{2x})x$ c) $\alpha x+\beta e^{x}$ d) $(\alpha e^{x}+\beta)x$

(GATE MA 2009)

46) Let $P_n(x)$ be the Legendre polynomial of degree n such that $P_n(1) = 1, n = 1, 2, ...$ If

$$\int_{-1}^{1}\left(\sum_{j=1}^{n}\sqrt{j(2j+1)}P_j(x)\right)^2 dx = 20$$

then $n =$

a) 2 b) 3 c) 4 d) 5

(GATE MA 2009)

47) The integral surface satisfying the equation $y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y} = x^2+y^2$ and passing through the curve $x = 1-t$, $y = 1+t$, $z = 1+t^2$ is

a) $z = xy + \frac{1}{2}(x^2-y^2)^2$
b) $z = xy + \frac{1}{4}(x^2-y^2)^2$
c) $z = xy + \frac{1}{8}(x^2-y^2)^2$
d) $z = xy + \frac{1}{16}(x^2-y^2)^2$

(GATE MA 2009)

48) For the diffusion problem $u_t = u_{xx}$, $0 < x < \pi$, $t > 0$, $u(0,t) = 0$, $u(\pi,t) = 0$ and $u(x,0) = 3\sin 2x$ the solution is given by

a) $3e^{-t}\sin 2x$ b) $3e^{-4t}\sin 2x$ c) $3e^{-9t}\sin 2x$ d) $3e^{-2t}\sin 2x$

(GATE MA 2009)

49) A simple pendulum, consisting of a bob of mass m connected with a string of length a, is oscillating in a vertical plane. If the string is making an angle θ with the vertical, then the expression for the Lagrangian is given as

a) $ma^2\left(\dot{\theta}^2-\frac{2g}{a}\sin^2\frac{\theta}{2}\right)$
b) $2mga\sin^2\frac{\theta}{2}$
c) $ma^2\left(\frac{\dot{\theta}^2}{2}-\frac{2g}{a}\sin^2\frac{\theta}{2}\right)$
d) $\frac{ma^2}{2}\left(\dot{\theta}^2-\frac{2g}{a}\cos\theta\right)$

(GATE MA 2009)

50) The extremal of the functional $\int_0^1\left(y+x^2+\frac{y'^2}{4}\right)dx$, $y(0) = 0$, $y(1) = 0$ is

a) $4(x^2-x)$ b) $3(x^2-x)$ c) $2(x^2-x)$ d) x^2-x

(GATE MA 2009)

Common Data Questions

Common Data Questions 51 and 52:

Let T:$\mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(x_1,x_2,x_3) = (x_1+3x_2+2x_3, 3x_1+4x_2+x_3, 2x_1+x_2-x_3)$$

51) The dimension of the range space of T^2 is

a) 0 b) 1 c) 2 d) 3

(GATE MA 2009)

52) The dimension of the null space of T^3 is

a) 0 b) 1 c) 2 d) 3

(GATE MA 2009)

Common Data for Questions 53 and 54:

Let $y_1(x) = 1 + x$ and $y_2(x) = e^x$ be two solutions of $y''(x) + P(x)y'(x) + Q(x)y(x) = 0$.

53) $P(x) =$

a) $1 + x$ b) $-1 - x$ c) $\frac{1+x}{x}$ d) $\frac{-1-x}{x}$

(GATE MA 2009)

54) The set of initial conditions for which there is NO solution is:

a) $y(0) = 2,\ y'(0) = 1$
b) $y(1) = 0,\ y'(1) = 1$
c) $y(1) = 1,\ y'(1) = 0$
d) $y(2) = 1,\ y'(2) = 2$

(GATE MA 2009)

Common Data for Questions 55 and 56:

Let X and Y be random variables having the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2}, & \text{if } -\infty < x < \infty, 0 < y < 1 \\ 0, & otherwise \end{cases}$$

55) The variance of X is

a) 1/12 b) 1/4 c) 7/12 d) 5/12

(GATE MA 2009)

56) The covariance between X and Y is

a) 1/3 b) 1/4 c) 1/16 d) 1/12

(GATE MA 2009)

Linked Answer Questions

Statement for Linked Answer Questions 57 and 58:

Consider the function $f(z) = \frac{e^{iz}}{z(z^2+1)}$

57) The residue of f at the isolated singular point in the upper half-plane $\{z = x + iy \in \mathbb{C} : y > 0\}$ is:

a) $-\frac{1}{2}e$ b) $\frac{1}{2}e$ c) e^{-2} d) 1

(GATE MA 2009)

58) The Cauchy Principal Value of the integral $PV \int_{-\infty}^{\infty} \frac{\sin x}{x^2+1}\, dx$ is

a) $-2\pi(1 + 2e^{-1})$ b) $\pi(1 - e^{-1})$ c) $2\pi(1 + e)$ d) $-\pi(1 + e^{-1})$

(GATE MA 2009)

Statement for Linked Answer Questions 59 and 60 :

Let $f(x, y) = kxy - x^3y - xy^3$ for $(x, y) \epsilon \mathbb{R}^2$, where k is a real constant. The directional derivative of f at the point $(1, 2)$ in the direction of the unit vector $u = \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is $\frac{15}{\sqrt{2}}$

59) The value of k is

a) 2 b) 4 c) 1 d) -2

(GATE MA 2009)

60) The value of f at a local minimum in the rectangular region $R = \left\{(x, y) \in \mathbb{R}^2 : |x| \leq \frac{3}{2}, |y| \leq \frac{3}{2}\right\}$ is

a) -2 b) -3 c) -7 d) 0

(GATE MA 2009)

END OF QUESTION PAPER