1.4.19

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Question

Find the position vector of a point **R** which divides the line joining two points **P** and **Q** whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio **2:1**.

- externally
- internally

Given Information

Given vector P is:

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \tag{1}$$

Given vector Q is:

$$\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \tag{2}$$

Continued

Let the point which divides **PQ** internally be **R**.

Let the point which divides **PQ** externally be **S**.

Required Formulae

The formula to calculate the coordinates of the point which divides a line segment internally in the ratio m:n is

$$\mathbf{R} = \frac{\frac{m}{n}\mathbf{P} + \mathbf{Q}}{\frac{m}{n} + 1} \tag{3}$$

and to calculate the coordinates of the point which divides a line segment externally in the ratio m:n is

$$\mathbf{S} = \frac{\frac{m}{n}\mathbf{P} - \mathbf{Q}}{\frac{m}{n} - 1} \tag{4}$$

Solution

Substituting $P \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $Q \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ in the first formula, we get

$$\mathbf{R} = \frac{2\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}}{\frac{2}{1}+1} = \frac{\begin{pmatrix} 2-1 \\ 4+1 \\ -2+1 \end{pmatrix}}{3} = \begin{pmatrix} 1/3 \\ 5/3 \\ -1/3 \end{pmatrix} \tag{5}$$

Solution

Substituting $P \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $Q \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ in the second formula, we get

$$\mathbf{S} = \frac{2\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}}{\frac{2}{1} - 1} = \frac{\begin{pmatrix} 2-(-1) \\ 4-1 \\ -2-1 \end{pmatrix}}{1} = \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix} \tag{6}$$

Python Code

```
import sys

import numpy as np

import numpy.linalg as LA
import matplotlib.pyplot as plt

P = np.array([1,2,-1]).reshape(-1,1)
#Defining vector P from the given information

Q = np.array([-1,1,1]).reshape(-1,1)
#Defining vector Q from the given information

ratio = 2
#Defining the ratio as given in the question
```

Python Code

```
R = (ratio*Q + P) / (ratio + 1)
#Calculating vector R with the first formula

S = (ratio*Q - P) / (ratio - 1)
#Calculating vector S with the second formula
```

Python Code

```
x_PQ = np.block([P,Q,R,S])
fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection='3d')

ax.plot(x_PQ[0,:],x_PQ[1,:], x_PQ[2,:],label='$BC$')
#Plotting all lines

all_coords = np.block([P, Q, R, S]) # Stack A, B, C vertically
ax.scatter(all_coords[0, :],all_coords[1, :],all_coords[2, :])
vert_labels = ['P', 'Q', 'R', 'S']

for i, txt in enumerate(vert_labels):
    ax.text(all_coords[0, i], all_coords[1, i], all_coords[2, i],
         f'{txt}\n({all_coords[0, i]:.0f}, {all_coords[1, i]:.0f
        }, {all_coords[2, i]:.0f})',
            fontsize=12, ha='center', va='bottom')

#Plotting the points and labelling them
```

Python Code

```
ax.spines['top'].set_color('none')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.spines['bottom'].set_position('zero')

plt.grid() # minor
plt.axis('equal')

plt.show()
```

Plot