2.4.21

EE25BTECH11026-Harsha

Question:
The number of vectors of unit length perpendicular to the vectors $a = 2\hat{i} + \hat{j} + 2\hat{k}$ and $b = \hat{j} + \hat{k}$ is
Solution:
Let us solve the given equation theoretically and then verify the solution computationally
According to the question,
Given the two vectors,

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \tag{0.1}$$

we need to find the unit vector which is perpendicular to the vectors **a** and **b**.The vector perpendicular to **a** and **b** is given by their cross-product.
Let the perpendicular vector be $\mathbf{x}^T = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}$

$$\because \mathbf{a}^T\mathbf{x} = 0 \tag{0.2}$$

$$\mathbf{b}^T\mathbf{x} = 0 , \tag{0.3}$$

$$\therefore \begin{pmatrix} \mathbf{a}^T \\ \mathbf{b}^T \end{pmatrix} \mathbf{x} = 0 \tag{0.4}$$

$$\begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \tag{0.5}$$

This can be represented as,

$$\begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \tag{0.6}$$

yielding,

$$2x_1 + x_3 = 0 \tag{0.7}$$

$$x_2 + x_3 = 0 \tag{0.8}$$

$$\mathbf{x} = x_3 \begin{pmatrix} \frac{-1}{2} \\ -1 \\ 1 \end{pmatrix} = \frac{x_3}{2} \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} \tag{0.9}$$

As we know that the vector can be in both the directions i.e, into and out of the plane containing **a** and **b**, so the vector perpendicular to vectors **a** and **b** would be ± (**a**×**b**).

The desired output is

$$\mathbf{x} = \pm\frac{1}{3}\begin{pmatrix}-1\\-2\\2\end{pmatrix} \tag{0.10}$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.