GATE 2024 Data Science and Artificial Intelligence (DA)

General Aptitude (GA) Questions

1) If '→' denotes increasing order of intensity, then the meaning of the words [sick → infirm → moribund] is analogous to [silly → _____ → daft]. Which one of the given options is appropriate to fill the blank?

   a) frown   b) fawn   c) vein   d) vain

   (GATE 2024 DA)

2) The 15 parts of the given figure are to be painted such that no two adjacent parts with shared boundaries (excluding corners) have the same color. The minimum number of colors required is _____.

   a) 4   b) 3   c) 5   d) 6

   (GATE 2024 DA)

3) How many 4-digit positive integers divisible by 3 can be formed using only the digits $\{1, 3, 4, 6, 7\}$, such that no digit appears more than once in a number?

   a) 24   b) 48   c) 72   d) 12

   (GATE 2024 DA)

4) The sum of the following infinite series is $2 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{8} + \frac{1}{9} + \frac{1}{16} + \frac{1}{27} + \ldots$

   a) 11/3   b) 7/2   c) 13/4   d) 9/2

   (GATE 2024 DA)

5) In an election, the share of valid votes received by the four candidates A, B, C, and D is represented by the pie chart shown. The total number of votes cast in the election were $1,15,000$, out of which $5,000$ were invalid.

   Share of valid votes: A 40%, B 25%, C 20%, D 15%

   Based on the data provided, the total number of valid votes received by the candidates B and C is _____.

   a) 45,000   b) 49,500   c) 51,750   d) 54,000

   (GATE 2024 DA)

6) Thousands of years ago, some people began dairy farming. This coincided with a number of mutations in a particular gene that resulted in these people developing the ability to digest dairy milk. Based on the given passage, which of the following can be inferred?
   a) All human beings can digest dairy milk.
   b) No human being can digest dairy milk.
   c) Digestion of dairy milk is essential for human beings.
   d) In human beings, digestion of dairy milk resulted from a mutated gene.

   (GATE 2024 DA)

7) The probability of a boy or a girl being born is 1/2. For a family having only three children, what is the probability of having two girls and one boy?

a) 3/8   b) 1/8   c) 1/4   d) 1/2

<div align="right">(GATE 2024 DA)</div>

Person 1 and Person 2 invest in three mutual funds A, B, and C. The amounts they invest in each of these mutual funds are given in the table below.

	Mutual fund A	Mutual fund B	Mutual fund C
Person 1	$20,000	$20,000	$10,000
Person 2	$20,000	$15,000	$15,000

8) At the end of one year, the total amount that Person 1 gets is $500 more than Person 2. The annual rate of return for the mutual funds B and C is 15% each. What is the annual rate of return for the mutual fund A?

a) 7.5%   b) 10%   c) 15%   d) 20%

<div align="right">(GATE 2024 DA)</div>

9) Three different views of a dice are shown in the figure below.

Fig. 1. for q-9

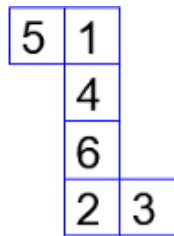The piece of paper that can be folded to make this dice is _ _ _ _ _.
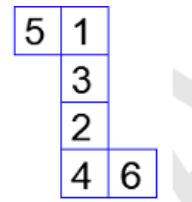

Fig. 2. option a for q-9

a)

Fig. 3. option b for q-9

b)

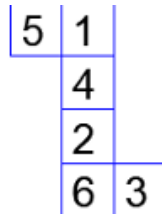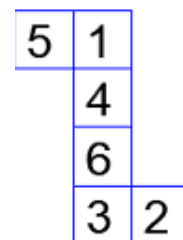Fig. 4. option c for q-9

c)

Fig. 5. option d for q-9

d)

<div align="right">(GATE 2024 DA)</div>

10) Visualize two identical right circular cones such that one is inverted over the other and they share a common circular base. If a cutting plane passes through the vertices of the assembled cones, what shape does the outer boundary of the resulting cross-section make?

a) A rhombus   b) A triangle   c) An ellipse   d) A hexagon

(GATE 2024 DA)

Data Science and Artificial Intelligence (DA) Questions

11) Consider the following statements:

   (i) The mean and variance of a Poisson random variable are equal.

   (ii) For a standard normal random variable, the mean is zero and the variance is one.

   Which ONE of the following options is correct?

   a) Both (i) and (ii) are true

   b) (i) is true and (ii) is false

   c) (ii) is true and (i) is false

   d) Both (i) and (ii) are false

(GATE 2024 DA)

12) Three fair coins are tossed independently. T is the event that two or more tosses result in heads. S is the event that two or more tosses result in tails. What is the probability of the event T ∩ S?

   a) 0   b) 0.5   c) 0.25   d) 1

(GATE 2024 DA)

13) Consider the matrix $\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$. Which ONE of the following statements is TRUE?

   a) The eigenvalues of M are non-negative and real.

   b) The eigenvalues of M are complex conjugate pairs.

   c) One eigenvalue of M is positive and real, and another eigenvalue of M is zero.

   d) One eigenvalue of M is non-negative and real, and another eigenvalue of M is negative and real.

(GATE 2024 DA)

14) Consider performing depth-first search (DFS) on an undirected and unweighted graph G starting at vertex s. For any vertex u in G, $d[u]$ is the length of the shortest path from s to u. Let (u, v) be an edge in G such that $d[u] < d[v]$. If the edge (u, v) is explored first in the direction from u to v during the above DFS, then (u, v) becomes a _____ edge.

   a) tree   b) cross   c) back   d) gray

(GATE 2024 DA)

15) For any twice differentiable function $f : \mathbb{R} \to \mathbb{R}$, if at some $x^* \in \mathbb{R}$, $f'(x^*) = 0$ and $f''(x^*) > 0$, then the function f necessarily has a _____ at $x = x^*$. Note: $\mathbb{R}$ denotes the set of real numbers.

   a) local minimum   b) global minimum   c) local maximum   d) global maximum

(GATE 2024 DA)

16) Match the items in Column 1 with the items in Column 2 in the following table:

Column 1	Column 2
(p) First In First Out	(i) Stacks
(q) Lookup Operation	(ii) Queues
(r) Last In First Out	(iii) Hash Tables

   a) (p) - (ii), (q) - (iii), (r) - (i)   c) (p) - (i), (q) - (ii), (r) - (iii)

   b) (p) - (ii), (q) - (i), (r) - (iii)   d) (p) - (i)

(GATE 2024 DA)

17) Consider the dataset with six datapoints: $\{(x_1, y_1), (x_2, y_2), \ldots, (x_6, y_6)\}$, where $x_1 = [1 \quad 0]^T$, $x_2 = [0 \quad 1]^T$, $x_3 = [0 \quad -1]^T$, $x_4 = [-1 \quad 0]^T$, $x_5 = [2 \quad 2]^T$, $x_6 = [-2 \quad -2]^T$ and the labels are given by $y_1 = y_2 = y_5 = 1$, and $y_3 = y_4 = y_6 = -1$. A hard margin linear support vector machine is trained on the above dataset. Which ONE of the following sets is a possible set of support vectors?

a) $\{x_1, x_2, x_5\}$ b) $\{x_3, x_4, x_5\}$ c) $\{x_4, x_5\}$ d) $\{x_1, x_2, x_3, x_4\}$

(GATE 2024 DA)

18) Match the items in Column 1 with the items in Column 2 in the following table:

Column 1	Column 2
(p) Principal Component Analysis	(i) Discriminative Model
(q) Naive Bayes Classification	(ii) Dimensionality Reduction
(r) Logistic Regression	(iii) Generative Model

a) (p) - (iii), (q) - (i), (r) - (ii)
b) (p) - (ii), (q) - (i), (r) - (iii)
c) (p) - (ii), (q) - (iii), (r) - (i)
d) (p) - (iii), (q) - (ii), (r) - (i)

(GATE 2024 DA)

19) Euclidean distance based k-means clustering alogorithm was run on a dataset of 100 points with k=3 .If the points $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\end{pmatrix}$ are both part of cluster 3,then which one of the following point is neccesarily also part of cluster 3 ?

a) $\begin{pmatrix}0\\0\end{pmatrix}$ b) $\begin{pmatrix}0\\2\end{pmatrix}$ c) $\begin{pmatrix}2\\0\end{pmatrix}$ d) $\begin{pmatrix}0\\1\end{pmatrix}$

(GATE 2024 DA)

20) Given a dataset with K binary-valued attributes (where $K > 2$) for a two-class classification task, the number of parameters to be estimated for learning a naÃ¯ve Bayes classifier is _____.

a) $2K + 1$ b) $2K + 1$ c) $2K + 1 + 1$ d) $K^2 + 1$

(GATE 2024 DA)

21) Consider performing uniform hashing on an open address hash table with load factor $\alpha = n/m < 1$, where n elements are stored in the table with m slots. The expected number of probes in an unsuccessful search is at most $\frac{1}{1-\alpha}$. Inserting an element in this hash table requires at most _____ probes, on average.

a) $\ln(\frac{1}{1-\alpha})$ b) $\frac{1}{1-\alpha}$ c) $1 + \frac{\alpha}{2}$ d) $\frac{1}{1+\alpha}$

(GATE 2024 DA)

22) For any binary classification dataset, let $S_B \in \mathbb{R}^{d \times d}$ and $S_W \in \mathbb{R}^{d \times d}$ be the between-class and within-class scatter (covariance) matrices, respectively. The Fisher linear discriminant is defined by $u^* \in \mathbb{R}^d$, that maximizes $J(u) = \frac{u^T S_B u}{u^T S_W u}$. If $\lambda = J(u^*)$, S_W is non-singular and $S_B \neq 0$, then (u^*, λ) must satisfy which ONE of the following equations? Note: $\mathbb{R}$ denotes the set of real numbers.

a) $S_W^{-1} S_B u^* = \lambda u^*$ b) $S_W u^* = \lambda S_B u^*$ c) $S_B S_W u^* = \lambda u^*$ d) $u^{*T} u^* = \lambda^2$

(GATE 2024 DA)

23) Let h_1 and h_2 be two admissible heuristics used in A^* search. Which ONE of the following expressions is always an admissible heuristic?

a) $h_1 + h_2$ b) $h_1 \times h_2$ c) $h_1/h_2, (h_2 \neq 0)$ d) $|h_1 - h_2|$

<div align="right">(GATE 2024 DA)</div>

24) Consider five random variables U, V, W, X, and Y whose joint distribution satisfies: $P(U, V, W, X, Y) = P(U)P(V)P(W|U, V)P(X|W)P(Y|W)$. Which ONE of the following statements is FALSE?

   a) Y is conditionally independent of V given W
   b) X is conditionally independent of U given W
   c) U and V are conditionally independent given W
   d) Y and X are conditionally independent given W

<div align="right">(GATE 2024 DA)</div>

25) Consider the following statement: In adversarial search, $\alpha - \beta$ pruning can be applied to game trees of any depth where α is the (m) —————— value choice we have formed so far at any choice point along the path for the MAX player and β is the (n) —————— value choice we have formed so far at any choice point along the path for the MIN player. Which ONE of the following choices of (m) and (n) makes the above statement valid?

   a) (m) = highest, (n) = highest   c) (m) = highest, (n) = lowest
   b) (m) = lowest, (n) = highest   d) (m) = lowest, (n) = lowest

<div align="right">(GATE 2024 DA)</div>

26) Consider a database that includes the following relations: Defender(name, rating, side, goals)
   Forward(name, rating, assists, goals)
   Team(name, club, price)
   Which ONE of the following relational algebra expressions checks that every name occurring in Team appears in either Defender or Forward, where ϕ denotes the empty set?

   a) $\Pi_{name}(\text{Team}) \setminus (\Pi_{name}(\text{Defender}) \cap \Pi_{name}(\text{Forward})) = \phi$
   b) $(\Pi_{name}(\text{Defender}) \cap \Pi_{name}(\text{Forward})) \setminus \Pi_{name}(\text{Team}) = \phi$
   c) $\Pi_{name}(\text{Team}) \setminus (\Pi_{name}(\text{Defender}) \cup \Pi_{name}(\text{Forward})) = \phi$
   d) $(\Pi_{name}(\text{Defender}) \cup \Pi_{name}(\text{Forward})) \setminus \Pi_{name}(\text{Team}) = \phi$

<div align="right">(GATE 2024 DA)</div>

27) Let the minimum, maximum, mean and standard deviation values for the attribute income of data scientists be \$46000, \$170000, \$96000, and \$21000, respectively. The z-score normalized income value of \$106000 is closest to which ONE of the following options?

   a) 0.217   b) 0.476   c) 0.623   d) 2.304

<div align="right">(GATE 2024 DA)</div>

28) Consider the following tree traversals on a full binary tree: (i) Preorder (ii) Inorder (iii) Postorder Which of the following traversal options is/are sufficient to uniquely reconstruct the full binary tree?

   a) (i) and (ii)   b) (ii) and (iii)   c) (i) and (iii)   d) (ii) only

<div align="right">(GATE 2024 DA)</div>

29) Let x and y be two propositions. Which of the following statements is a tautology /are tautologies?

   a) $(\neg x \wedge y) \implies (y \implies x)$ c) $(\neg x \wedge y) \implies (\neg x \implies y)$
   b) $(x \wedge \neg y) \implies (\neg x \implies y)$ d) $(x \wedge \neg y) \implies (y \implies x)$

<div align="right">(GATE 2024 DA)</div>

30) Consider sorting the following array of integers in ascending order using an in-place Quicksort algorithm that uses the last element as the pivot. [60 70 80 90 100] The minimum number of swaps performed during this Quicksort is _ _ _ _ _ _ .   (GATE 2024 DA)

the following two tables named Raider and Team in a relational database maintained by a Kabaddi league. The attribute ID in table Team references the primary key of the Raider table, ID.

Raider					Team		
ID	Name	Raids	RaidPoints		City	ID	BidPoints
1	Arjun	200	250		Jaipur	2	200
2	Ankush	190	219		Patna	3	195
3	Sunil	150	200		Hyderabad	5	175
4	Reza	150	190		Jaipur	1	250
5	Pratham	175	220		Patna	4	200
6	Gopal	193	215		Jaipur	6	200

The SQL query described below is executed on this database:

```
SELECT *
FROM Raider, Team
WHERE Raider.ID=Team.ID AND City="Jaipur" AND RaidPoints > 200;
```

The number of rows returned by this query is _ _ _ _ _ _ .   (GATE 2024 DA)

31) fundamental operations in a double-ended queue D are:

- `insertFirst(e)` – Insert a new element e at the beginning of D.
- `insertLast(e)` – Insert a new element e at the end of D.
- `removeFirst()` – Remove and return the first element of D.
- `removeLast()` – Remove and return the last element of D.

In an empty double-ended queue, the following operations are performed:

```
insertFirst(10)
insertLast(32)
a <- removeFirst()
insertLast(28)
insertLast(17)
a <- removeFirst()
a <- removeLast()
```

The value of a is _____ .   (GATE 2024 DA)

32) Let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) = \frac{1}{1+e^{-x}}$. The value of the derivative of f at x where $f(x) = 0.4$ is _____ (rounded off to two decimal places). Note: $\mathbb{R}$ denotes the set of real numbers. (GATE 2024 DA)

33) Let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) = \frac{1}{1+e^{-x}}$. The value of the derivative of f at x where $f(x) = 0.4$ is _____ (rounded off to two decimal places). Note: $\mathbb{R}$ denotes the set of real numbers. (GATE 2024 DA)

34) The sample average of 50 data points is 40. The updated sample average after including a new data point taking the value of 142 is _____ .

(GATE 2024 DA)

35) Consider the 3×3 matrix $M = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \\ 4 & 3 & 6 \end{pmatrix}$. The determinant of $\left(M^2 + 12M\right)$ is _____ .

(GATE 2024 DA)

36) A fair six-sided die (with faces numbered 1, 2, 3, 4, 5, 6) is repeatedly thrown independently. What is the expected number of times the die is thrown until two consecutive throws of even numbers are seen?

a) 2   b) 4   c) 6   d) 8

<div align="right">(GATE 2024 DA)</div>

37) Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Note: $\mathbb{R}$ denotes the set of real numbers. $f(x) = \begin{cases} -x, & \text{if } x < -2 \\ ax^2 + bx + c, & \text{if } x \in [-2, 2] \\ x, & \text{if } x > 2 \end{cases}$

Which ONE of the following choices gives the values of a, b, c that make the function f continuous and differentiable?

a) $a = 1/4, b = 0, c = 1$
b) $a = 1/2, b = 0, c = 0$
c) $a = 0, b = 0, c = 0$
d) $a = 1, b = 1, c = -4$

<div align="right">(GATE 2024 DA)</div>

38) Consider the following Python code:

```python
def count(child_dict, i):
    if i not in child_dict.keys():
        return 1
    ans = 1
    for j in child_dict[i]:
        ans += count(child_dict, j)
    return ans


child_dict = dict()
child_dict[0] = [1,2]
child_dict[1] = [3,4,5]
child_dict[2] = [6,7,8]
print(count(child_dict,0))
```

Which ONE of the following is the output of this code?

a) 6   b) 1   c) 8   d) 9

<div align="right">(GATE 2024 DA)</div>

39) Consider the function computeS(X) whose pseudocode is given below:

```
computeS(X)
S[1] ← 1
for i ← 2 to length(X) do
    S[i] ← 1
    if X[i − 1] ≤ X[i] then
        S[i] ← S[i] + S[i - 1]
    end if
end for
return  S
```

Which ONE of the following values is returned by the function computeS(X) for $X = [6, 3, 5, 4, 10]$?

a) $[1, 1, 2, 3, 4]$ b) $[1, 1, 2, 3, 3]$ c) $[1, 1, 2, 1, 2]$ d) $[1, 1, 2, 1, 5]$

<div align="right">(GATE 2024 DA)</div>

40) Let $F(n)$ denote the maximum number of comparisons made while searching for an entry in a sorted array of size n using binary search. Which ONE of the following options is TRUE?

a) $F(n) = F(\lfloor n/2 \rfloor) + 1$
b) $F(n) = F(\lfloor n/2 \rfloor) + F(\lceil n/2 \rceil)$
c) $F(n) = F(\lfloor n/2 \rfloor)$
d) $F(n) = F(n - 1) + 1$

(GATE 2024 DA)

41) Consider the following Python function:

```
def fun(D, s1, s2):
    if s1 < s2:
        D[s1], D[s2] = D[s2], D[s1]
        fun(D, s1+1, s2−1)
```

What does this Python function fun() do? Select the ONE appropriate option below.

a) It finds the smallest element in D from index s1 to s2, both inclusive.
b) It performs a merge sort in-place on this list D between indices s1 and s2, both inclusive.
c) It reverses the list D between indices s1 and s2, both inclusive.
d) It swaps the elements in D at indices s1 and s2, and leaves the remaining elements unchanged.

(GATE 2024 DA)

42) Consider the table below, where the $(i, j)^{th}$ element of the table is the distance between points x_i and x_j. Single linkage clustering is performed on data points, x_1, x_2, x_3, x_4, x_5.

	x_1	x_2	x_3	x_4	x_5
x_1	0	1	4	3	6
x_2	1	0	3	5	3
x_3	4	3	0	2	5
x_4	3	5	2	0	1
x_5	6	3	5	1	0

Which ONE of the following is the correct representation of the clusters produced?

a) (Image of dendrogram A)
b) (Image of dendrogram B)
c) (Image of dendrogram C)
d) (Image of dendrogram D)

(GATE 2024 DA)

43) Consider the two neural networks (NNs) shown in Figures 1 and 2, with *ReLU* activation ($ReLU(z) = \max\{0, z\}, \forall z \in \mathbb{R}$). $\mathbb{R}$ denotes the set of real numbers. The connections and their corresponding weights are shown in the Figures. The biases at every neuron are set to 0. For what values of p, q, r in Figure 2 are the two NNs equivalent, when x_1, x_2, x_3 are positive?

a) $p = 36, q = 24, r = 24$
b) $p = 24, q = 24, r = 36$
c) $p = 18, q = 36, r = 24$
d) $p = 36, q = 36, r = 36$

(GATE 2024 DA)

44) Consider a state space where the start state is number 1. The successor function for the state numbered n returns two states numbered n+1 and n+2. Assume that the states in the unexpanded state list are expanded in the ascending order of numbers and the previously expanded states are not added to the unexpanded state list. Which ONE of the following statements about breadth-first search (BFS) and depth-first search (DFS) is true, when reaching the goal state number 6?

a) BFS expands more states than DFS.
b) DFS expands more states than BFS.
c) Both BFS and DFS expand equal number of states.
d) Both BFS and DFS do not reach the goal state number 6.

(GATE 2024 DA)

45) Consider the following sorting algorithms: (i) Bubble sort (ii) Insertion sort (iii) Selection sort. Which ONE among the following choices of sorting algorithms sorts the numbers in the array [4, 3, 2, 1, 5] in increasing order after exactly two passes over the array?

    a) (i) only   b) (iii) only   c) (i) and (iii) only   d) (ii) and (iii) only

<div align="right">(GATE 2024 DA)</div>

46) Given the relational schema $R = (U, V, W, X, Y, Z)$ and the set of functional dependencies: $\{U \to V, U \to W, WX \to Y, WX \to Z, V \to X\}$. Which of the following functional dependencies can be derived from the above set?

    a) $VW \to YZ$ b) $WX \to YZ$ c) $VW \to U$ d) $VW \to Y$

<div align="right">(GATE 2024 DA)</div>

47) Select all choices that are subspaces of $\mathbb{R}^3$. Note: $\mathbb{R}$ denotes the set of real numbers.
    a) $\{\mathbf{x} = [x_1\ x_2\ x_3]^T \in \mathbb{R}^3 : \mathbf{x} = \alpha[1\ 1\ 0]^T + \beta[1\ 0\ 0]^T, \alpha, \beta \in \mathbb{R}\}$
    b) $\{\mathbf{x} = [x_1\ x_2\ x_3]^T \in \mathbb{R}^3 : \mathbf{x} = \alpha^2[1\ 2\ 0]^T + \beta^2[1\ 0\ 1]^T, \alpha, \beta \in \mathbb{R}\}$
    c) $\{\mathbf{x} = [x_1\ x_2\ x_3]^T \in \mathbb{R}^3 : 5x_1 + 2x_3 = 0, 4x_1 - 2x_2 + 3x_3 = 0\}$
    d) $\{\mathbf{x} = [x_1\ x_2\ x_3]^T \in \mathbb{R}^3 : 5x_1 + 2x_3 + 4 = 0\}$

<div align="right">(GATE 2024 DA)</div>

48) Which of the following statements is/are TRUE? Note: $\mathbb{R}$ denotes the set of real numbers.
    a) There exist $M \in \mathbb{R}^{3\times3}$, $p \in \mathbb{R}^3$, and $q \in \mathbb{R}^3$ such that $Mx = p$ has a unique solution and $Mx = q$ has infinite solutions.
    b) There exist $M \in \mathbb{R}^{3\times3}$, $p \in \mathbb{R}^3$, and $q \in \mathbb{R}^3$ such that $Mx = p$ has no solutions and $Mx = q$ has infinite solutions.
    c) There exist $M \in \mathbb{R}^{2\times3}$, $p \in \mathbb{R}^2$, and $q \in \mathbb{R}^2$ such that $Mx = p$ has a unique solution and $Mx = q$ has infinite solutions.
    d) There exist $M \in \mathbb{R}^{3\times2}$, $p \in \mathbb{R}^3$, and $q \in \mathbb{R}^3$ such that $Mx = p$ has a unique solution and $Mx = q$ has no solutions.

<div align="right">(GATE 2024 DA)</div>

49) Let $\mathbb{R}$ be the set of real numbers, U be a subspace of $\mathbb{R}^3$ and $M \in \mathbb{R}^{3\times3}$ be the matrix corresponding to the projection on to the subspace U. Which of the following statements is/are TRUE?
    a) If U is a 1-dimensional subspace of $\mathbb{R}^3$, then the null space of M is a 1-dimensional subspace.
    b) If U is a 2-dimensional subspace of $\mathbb{R}^3$, then the null space of M is a 1-dimensional subspace.
    c) $M^2 = M$
    d) $M^3 = M$

<div align="right">(GATE 2024 DA)</div>

50) Consider the function $f : \mathbb{R} \to \mathbb{R}$ where $\mathbb{R}$ is the set of all real numbers. $f(x) = \frac{x^4}{4} - \frac{2x^3}{3} - \frac{3x^2}{2} + 1$ Which of the following statements is/are TRUE?

    a) $x = 0$ is a local maximum of f   c) $x = -1$ is a local maximum of f
    b) $x = 3$ is a local minimum of f   d) $x = 0$ is a local minimum of f

<div align="right">(GATE 2024 DA)</div>

51) Consider the directed acyclic graph (DAG) below: Which of the following is/are valid vertex orderings that can be obtained from a topological sort of the DAG?

    a) P Q R S T U V   b) P R Q V S U T   c) P Q R S V U T   d) P R Q S V T U

<div align="right">(GATE 2024 DA)</div>

52) Let H, I, L, and N represent height, number of internal nodes, number of leaf nodes, and the total number of nodes respectively in a rooted binary tree. Which of the following statements is/are always TRUE?

   a) $L \leq I + 1$ b) $H + 1 \leq N \leq 2^{H+1} - 1$ c) $H \leq I \leq 2^H - 1$ d) $H \leq L \leq 2^{H-1}$

<div align="right">(GATE 2024 DA)</div>

53) Consider the following figures representing datasets consisting of two-dimensional features with two classes denoted by circles and squares. Which of the following is/are TRUE?

   a) (i) is linearly separable.   c) (iii) is linearly separable.
   b) (ii) is linearly separable.   d) (iv) is linearly separable.

<div align="right">(GATE 2024 DA)</div>

54) Let game(ball, rugby) be true if the ball is used in rugby and false otherwise. Let shape(ball, round) be true if the ball is round and false otherwise. Consider the following logical sentences: s1: $\forall$ball¬game($ball, rugby$) $\implies$ shape(ball, round) s2: $\forall$ball¬shape(($ball, round$)) $\implies$ game(ball, rugby) s3: $\forall$ballgame($ball, rugby$)) $\implies$ ¬shape(ball, round) s4: $\forall$ballshape($ball, round$)) $\implies$ ¬game(ball, rugby) Which of the following choices is/are logical representations of the assertion, "All balls are round except balls used in rugby"?

   a) $s1 \wedge s3$ b) $s1 \wedge s2$ c) $s2 \wedge s3$ d) $s3 \wedge s4$

<div align="right">(GATE 2024 DA)</div>

55) An OTT company is maintaining a large disk-based relational database of different movies with the following schema: Movie(ID, CustomerRating) Genre(ID, Name) Movie_Genre(MovieID, GenreID) Consider the following SQL query on the relation database above:

```
SELECT *
FROM Movie, Genre, Movie_Genre
WHERE
Movie.CustomerRating > 3.4 AND
Genre.Name = ''Comedy'' AND
Movie_Genre.MovieID = Movie.ID AND
Movie_Genre.GenreID = Genre.ID;
```

This SQL query can be sped up using which of the following indexing options?
   a) A B+ tree on all the attributes.
   b) Hash index on Genre.Name and B+ tree on the remaining attributes.
   c) Hash index on Movie.CustomerRating and B+tree on the remaining attributes.
   d) Hash index on all the attributes.

<div align="right">(GATE 2024 DA)</div>

56) Let X be a random variable uniformly distributed in the interval [1, 3] and Y be a random variable uniformly distributed in the interval [2, 4]. If X and Y are independent of each other, the probability $P(X \geq Y)$ is _____ (rounded off to three decimal places).   (GATE 2024 DA)

57) Let X be a random variable exponentially distributed with parameter $\lambda > 0$. The probability density function of X is given by: $f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$. If $5E(X) = Var(X)$, where $E(X)$ and $Var(X)$ indicate the expectation and variance of X, respectively, the value of λ is _____ (rounded off to one decimal place).   (GATE 2024 DA)

58) Consider two events T and S. Let $\bar{T}$ denote the complement of the event T. The probability associated with different events are given as follows: $P(\bar{T}) = 0.6$, $P(S|T) = 0.3$, $P(S|\bar{T}) = 0.6$. Then, $P(T|S)$ is _____ (rounded off to two decimal places). (GATE 2024 DA)

59) Consider a joint probability density function of two random variables X and Y $f_{X,Y}(x, y) = \begin{cases} 2xy, & 0 < x < 2, 0 < \\ 0, & \text{otherwise} \end{cases}$
Then, $E[Y|X = 1.5]$ is _____. (GATE 2024 DA)

60) Evaluate the following limit: $\lim_{x \to 0} \frac{\ln((x^2+1)\cos x)}{x^2} = $ _____. (GATE 2024 DA)

61) Let $\boldsymbol{u} = [1\ 2\ 3\ 4\ 5]^T$, and let $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$ be the singular values of the matrix $M = \boldsymbol{u}\boldsymbol{u}^T$ (where $\boldsymbol{u}^T$ is the transpose of $\boldsymbol{u}$). The value of $\sum_{i=1}^{5} \sigma_i$ is _____. (GATE 2024 DA)

62) Details of ten international cricket games between two teams "Green" and "Blue" are given in Table C. This table consists of matches played on different pitches, across formats along with their winners. The attribute Pitch can take one of two values: spin-friendly (represented as S) or pace-friendly (represented as F). The attribute Format can take one of two values: one-day match (represented as O) or test match (represented as T). A cricket organization would like to use the information given in Table C to develop a decision-tree model to predict outcomes of future games between these two teams. To develop such a model, the computed Information Gain(C, Pitch) with respect to the Target is _____ (rounded off to two decimal places).

Table C

Match Number	Pitch	Format	Winner (Target)
1	S	T	Green
2	S	T	Blue
3	F	O	Blue
4	S	O	Blue
5	F	T	Green
6	F	O	Blue
7	S	O	Green
8	F	T	Blue
9	F	O	Blue
10	S	O	Green

(GATE 2024 DA)

63) Given the two-dimensional dataset consisting of 5 data points from two classes (circles and squares) and assume that the Euclidean distance is used to measure the distance between two points. The minimum odd value of k in k-nearest neighbor algorithm for which the diamond ($\diamond$) shaped data point is assigned the label square is _____. (GATE 2024 DA)

64) Given the following Bayesian Network consisting of four Bernoulli random variables and the associated conditional probability tables:

$P(\cdot)$	
$U = 0$	0.5
$U = 1$	0.5

	$P(V|\cdot)$	
	$V = 0$	$V = 1$
$U = 0$	0.5	0.5
$U = 1$	0.5	0.5

	$P(W|\cdot)$	
	$W = 0$	$W = 1$
$U = 0$	1	0
$U = 1$	0	1

		$P(Z|\cdot)$	
		$Z = 0$	$Z = 1$
$V = 0$	$W = 0$	0.5	0.5
$V = 0$	$W = 1$	1	0
$V = 1$	$W = 0$	1	0
$V = 1$	$W = 1$	0.5	0.5

The value of $P(U = 1, V = 1, W = 1, Z = 1) = $ _____ (rounded off to three decimal places). (GATE 2024 DA)

65) Two fair coins are tossed independently. X is a random variable that takes a value of 1 if both tosses are heads and 0 otherwise. Y is a random variable that takes a value of 1 if at least one of the tosses is heads and 0 otherwise. The value of the covariance of X and Y is _____ (rounded off to three decimal places). (GATE 2024 DA)