Question

ABCD is a rectangle formed by the points **A**$(-1,-1)$, **B**$(-1,6)$, **C**$(3,6)$ and **D**$(3,-1)$. **P**, **Q**, **R** and **S** are mid-points of sides AB, BC, CD and DA respectively. Show that diagonals of the quadrilateral PQRS bisect each other.

Let's define the points as column vectors:

$A = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 \\ 6 \end{pmatrix}, \quad C = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \quad D = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

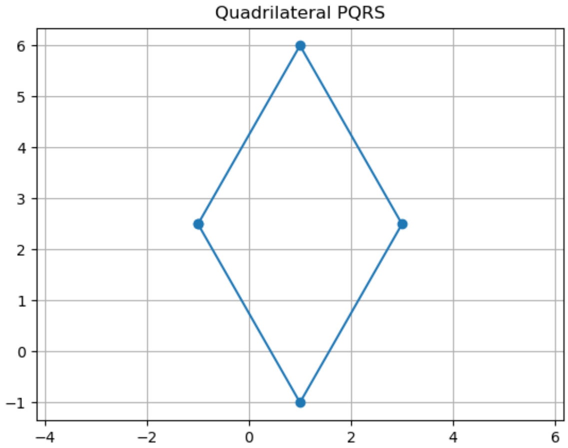

Figure 1

So,

$$P = \frac{1}{2}(A+B) = \frac{1}{2}\begin{pmatrix} -1+(-1) \\ -1+6 \end{pmatrix} = \begin{pmatrix} -1 \\ 2.5 \end{pmatrix} \tag{1}$$

$$Q = \frac{1}{2}(B+C) = \frac{1}{2}\begin{pmatrix} -1+3 \\ 6+6 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} \tag{2}$$

$$R = \frac{1}{2}(C+D) = \frac{1}{2}\begin{pmatrix} 3+3 \\ 6+1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3.5 \end{pmatrix} \tag{3}$$

$$S = \frac{1}{2}(D+A) = \frac{1}{2}\begin{pmatrix} 3+(-1) \\ 1+(-1) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4}$$

Diagonal PR:

$$\text{Midpoint}_{PR} = \tfrac{1}{2}(P+R) = \tfrac{1}{2}\left(\begin{pmatrix} -1 \\ 2.5 \end{pmatrix} + \begin{pmatrix} 3 \\ 3.5 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Diagonal QS:

$$\text{Midpoint}_{QS} = \frac{1}{2}(Q + S) = \frac{1}{2}\left(\begin{pmatrix}1\\6\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\3\end{pmatrix}$$

Conclusion

Since O is the midpoint of both diagonals PR and QS, the diagonals of quadrilateral PQRS bisect each other.