GATE 2024 STATISTICS

EE25BTECH11022 - SANKEERTHAN

General Aptitude

1) If "→" denotes increasing order of intensity, then the meaning of the words [walk → jog → sprint] is analogous to [bothered → _______ → daunted]. Which one of the given options is appropriate to fill the blank?

a) phased b) phrased c) fazed d) fused

(GATE ST 2024)

2) Two wizards try to create a spell using all the four elements: water, air, fire, and earth. They decide to mix all these elements in *all possible orders* and work independently. After exhausting all possible combinations, they conclude the spell does not work. How many attempts does each wizard make independently before concluding?

a) 24 b) 48 c) 16 d) 12

(GATE ST 2024)

3) In an engineering college of $10,000$ students, 1500 like neither their core branches nor other branches. The number of students who like their core branches is $\frac{1}{4}$ of those who like other branches. Number of students who like both core and other branches is 500. The number of students who like their core branches is:

a) 1800 b) 3500 c) 1600 d) 1500

(GATE ST 2024)

4) For positive non-zero real variables x and y, if $\ln\left(x+\frac{y}{2}\right)=\frac{1}{2}\left[\ln(x)+\ln(y)\right]$ then the value of $\frac{x}{y}+\frac{y}{x}$ is:

a) 1 b) $\frac{1}{2}$ c) 2 d) 4

(GATE ST 2024)

5) In the sequence $6, 9, 14, x, 30, 41$, a possible value of x is:

a) 25 b) 21 c) 18 d) 20

(GATE ST 2024)

6) Sequence the following sentences in a coherent passage. P: This fortuitous geological event generated a colossal amount of energy and heat that resulted in the rocks rising to an average height of 4 km across the contact zone. Q: Thus, the geophysicists tend to think of the Himalayas as an active geological event rather than as a static geological feature. R: The natural process of cooling of this massive edifice absorbed large quantities of atmospheric carbon dioxide, altering the Earth's atmosphere and making it better suited for life. S: Many millennia ago, a breakaway chunk of bedrock from the Antarctic Plate collided with the massive Eurasian Plate.

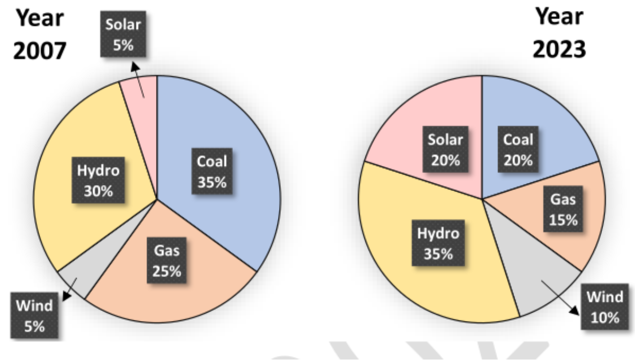

Fig. 1.

a) QPSR b) QSPR c) SPRQ d) SRPQ

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7) A person sold two different items at the same price. He made 10% profit in one item, and 10% loss in the other. Overall, the person made:

a) 1% profit b) 2% profit c) 1% loss d) 2% loss

(GATE ST 2024)

8) The pie charts show the shares of different power generation technologies in total electricity generation for years 2007 and 2023.
The renewable sources of electricity generation consist of Hydro, Solar and Wind. Assume total electricity generation is same for both years. Find the % increase in renewable share from 2007 to 2023.

a) 25% b) 50% c) 77.5% d) 62.5%

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9) A cube is to be cut into 8 equal pieces of identical size and shape. Each cut should be straight and extend fully through the cube. Minimum number of such cuts required is:

a) 3 b) 4 c) 7 d) 8

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10) In the 4×4 array below, each cell of the first three rows has either a cross (X) or a number. A number in a cell shows count of its immediate neighbours (adjacent or diagonal) *not* having a cross. Given last row has no crosses, sum of four numbers to be filled in last row equals:

a) 11 b) 10 c) 12 d) 9

(GATE ST 2024)

11) Let D be the region bounded by the line $y = x$ and the parabola $y = 4x - x^2$. Then $\iint_D x\,dx\,dy$ equals:

a) $\frac{27}{4}$ b) $\frac{29}{4}$ c) 7 d) 6

(GATE ST 2024)

12) Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = \sqrt{6}$ and $a_{n+1} = \sqrt{6 + a_n}$, $n \geq 1$. Consider the following statements: (I) $\{a_n\}_{n\geq 1}$ is an increasing sequence. (II) $\lim_{n\to\infty} a_n = 2$.
Which of the above statements is/are true?

a) Only (I) b) Only (II) c) Both (I) and (II) d) Neither (I) nor (II)

(GATE ST 2024)

13) Let A be a 3×3 real matrix and I_3 be the 3×3 identity matrix. Which of the following statements is NOT true?
   a) If the row-reduced echelon form of A is I_3, then zero is not an eigenvalue of A
   b) If zero is not an eigenvalue of A, then the row-reduced echelon form of A is I_3
   c) If A has three distinct eigenvalues, then the row-reduced echelon form of A is I_3
   d) If the system $A\mathbf{x} = \mathbf{b}$ has a solution for every 3×1 real vector $\mathbf{b}$, then the row-reduced echelon form of A is I_3

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14) Let $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be vectors in $\mathbb{R}^4$. Let U be the span of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and V be the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.
   Statements: (I) If $\dim(U \cap V) = 2$ and $\dim(U) = 3$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent. (II) If $U + V = \mathbb{R}^4$, then either $(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$ is linearly independent or $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ is linearly independent.
   Which is/are true?

   a) Only (I) b) Only (II) c) Both (I) and (II) d) Neither (I) nor (II)

(GATE ST 2024)

15) Consider $\mathbb{R}^2$ with standard inner product. If $\mathbf{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ is such that $\langle \mathbf{u}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \rangle = 2$ and $\langle \mathbf{u}, \begin{pmatrix} 4 \\ -2 \end{pmatrix} \rangle = -1$, then which statement is true?

   a) $\langle \mathbf{u}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \rangle = \frac{1}{2}$ b) $\langle \mathbf{u}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \rangle = \frac{3}{5}$ c) $\langle \mathbf{u}, \begin{pmatrix} 2 \\ -\frac{1}{2} \end{pmatrix} \rangle = -\frac{6}{5}$ d) $\langle \mathbf{u}, \begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix} \rangle = \frac{4}{5}$

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16) Let $A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}$ be a 2×3 real matrix with $(a_1, a_2, a_3) \neq (0,0,0)$, $(b_1, b_2, b_3) \neq (0,0,0)$, and $\operatorname{rank}(A) = 1$. Define: $W = \left\{ \mathbf{x} \in \mathbb{R}^3 : A\mathbf{x} = \mathbf{0} \right\}$ $W_1 = \left(\mathbf{x} \in \mathbb{R}^3 : a_1x_1 + a_2x_2 + a_3x_3 = 0 \right)$ $W_2 = \left(\mathbf{x} \in \mathbb{R}^3 : b_1x_1 + b_2x_2 + b_3x_3 = 0 \right)$
   Statements: (I) $W = W_1 \cap W_2$ (II) $W_1 = W_2$
   Which is/are true?

   a) Only (I) b) Only (II) c) Both (I) and (II) d) Neither (I) nor (II)

(GATE ST 2024)

17) Let X take values 1 and 2. Let $M_X(\cdot)$ be its moment generating function. If $E(x) = \frac{10}{7}$, then the fourth derivative of $M_X(\cdot)$ at 0 equals:

   a) $\frac{52}{7}$ b) $\frac{67}{7}$ c) $\frac{48}{7}$ d) $\frac{60}{7}$

(GATE ST 2024)

18) Two fair dice (red and blue) are tossed. Let A = red die shows 5 or 6. Let B = sum of outcomes = 7. Let C = sum of outcomes = 8. Which is true?
   a) A and B are independent, and A and C are independent
   b) A and B are independent, but A and C are not
   c) A and C are independent, but A and B are not
   d) Neither A and B nor A and C are independent

(GATE ST 2024)

19) Let X have PDF $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1$ where $\alpha > 0, \beta > 0$. If $E(X) = \frac{1}{3}$ and $E\left(X^2\right) = \frac{1}{6}$, then $\alpha + 3\beta =$:

a) 7 b) 5 c) 4 d) 8

(GATE ST 2024)

20) Let X and Y have CDFs $F_X(\cdot)$ and $F_Y(\cdot)$. Which is NOT true?
a) There exist X, Y with $F_X(u) = F_Y(u)$ for all $u \in \mathbb{R}$ and $P(X \neq Y) > 0$
b) There exist X, Y with $F_X(u) = F_Y(u)$ for all $u \in \mathbb{R}$ and $P(X = Y) = 0$
c) If X and Y are independent then X^2 and Y^2 are independent
d) If X^2 and Y^2 are independent then X and Y are independent

(GATE ST 2024)

21) Let $(F_n)_{n\geq1}$ be a sequence of cumulative distribution functions given by $F_n(x) = 0, x < -n$,
$\frac{x+n}{2n}, -n \leq x < n$,
$1, x \geq n$. Which statement is true?
a) $F_n(x)$ converges for all $x \in \mathbb{R}$ and the limiting function is a CDF
b) $F_n(x)$ converges for all $x \in \mathbb{R}$, but the limiting function is not a CDF
c) $F_n(x)$ does not converge for any $x \in \mathbb{R}$
d) There exist $x, y \in \mathbb{R}$ such that $F_n(x)$ converges but $F_n(y)$ does not

(GATE ST 2024)

22) Let $\{W(t)\}_{t\geq0}$ be a standard Brownian motion. Which one is **NOT** true?

a) $E[w(7)] = 0$ c) $2w(1)$ is $\mathcal{N}(0, 4)$ 3
b) $E[w(5)\,w(9)] = 7$ d) $E[w(5) \mid w(3) = 3] =$

(GATE ST 2024)

23) Let X_1, X_2, X_3 be i.i.d. Binomial($n = 100, p$) random variables, with $p \in (0, 1)$ unknown. Define $T_1 = (X_1 + X_2, X_3), \quad T_2 = X_1 + X_2 + X_3$. Consider: (I) Distribution of T_2 given $T_1 = t_1$ is independent of p (II) Distribution of T_1 given $T_2 = t_2$ is independent of p

a) Only (I) b) Only (II) c) Both (I) and (II) d) Neither (I) nor (II)

(GATE ST 2024)

24) Let $X_1, \ldots, X_n$ be a random sample from $f(x;\theta) = \theta(2x)^{\theta-1}, 0 < x \leq \frac{1}{2}$,
$\theta(2-2x)^{\theta-1}, \frac{1}{2} < x \leq 1$,
0, otherwise, where $\theta > 0$. Which is an MLE of θ?
a) $n\left[\sum_{i:X_i\leq1/2} \log_e(2X_i) + \sum_{i:X_i>1/2} \log_e(2-2X_i)\right]^{-1}$
b) $-n\left[\sum_{i:X_i\leq1/2} \log_e(2X_i) + \sum_{i:X_i>1/2} \log_e(2-2X_i)\right]^{-1}$
c) $n\left[\sum_{i=1}^{n} \log_e(2X_i) + \sum_{i=1}^{n} \log_e(2-2X_i)\right]^{-1}$
d) $-n\left[\sum_{i=1}^{n} \log_e(2X_i) + \sum_{i=1}^{n} \log_e(2-2X_i)\right]^{-1}$

(GATE ST 2024)

25) In hypothesis testing, which statement is true?
a) Type-I error probability cannot be higher than Type-II error probability
b) Type-II error occurs when the test accepts H_0 when H_0 is false
c) Type-I error occurs when the test rejects H_0 when H_0 is false
d) Sum of probabilities of Type-I and Type-II errors must be 1

(GATE ST 2024)

26) A sample of size $n = 40$ from 4 categories has:

Category	1	2	3	4
Observed Freq	5	8	12	15

Test $H_0:\ \theta_i = \frac{1}{4}$ for all i using χ^2 GOF test. Which statement is true?

a) d.f. = 3, test statistic = 5.8
b) d.f. = 3, test statistic = 1.4
c) d.f. = 4, test statistic = 5.8
d) d.f. = 4, test statistic = 1.4

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27) Let $X_1, \ldots, X_n$ be i.i.d. from a continuous distribution having unknown median M. Test $H_0 : M = 10$ vs $H_1 : M > 10$ at level α using T = number of observations > 10. If t_0 is observed T, then p-value is:

a) $\sum_{i=t_0}^{n} \binom{n}{i} (0.5)^n$
b) $\sum_{i=10}^{n} \binom{n}{i} (0.5)^n$
c) $\sum_{i=0}^{10} \binom{n}{i} (0.5)^n$
d) $\sum_{i=0}^{t_0} \binom{n}{i} (0.5)^n$

(GATE ST 2024)

28) Consider $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, ϵ_i i.i.d. mean 0, variance σ^2. Let $\bar{y}$ be sample mean, $\hat{\beta}_1$ LSE. Which is true?

a) $Cov\left(\bar{y}, \hat{\beta}_1\right) < 0$
b) $Cov\left(\bar{y}, \hat{\beta}_1\right) > 0$
c) $Cov\left(\bar{y}, \hat{\beta}_1\right) = 0$
d) $Cov\left(\bar{y}, \hat{\beta}_1\right)$ does not exist

(GATE ST 2024)

29) Consider same linear model as Q.28 but with statistics: $T_1 = \frac{1}{n-2} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2, \quad T_2 = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$ Which is true?

a) Both T_1 and T_2 are unbiased estimators of σ^2
b) T_1 unbiased, T_2 not
c) T_1 not unbiased, T_2 unbiased
d) Neither is unbiased

(GATE ST 2024)

30) Power series $\sum_{n=0}^{\infty} a_n x^n$ with $a_{2n+1} = \frac{1}{2^{2n+1}}$, $a_{2n} = \frac{1}{3^{2n}}$ has radius of convergence equal to ___ (integer).
31) Let X be a random variable having Poisson(λ) distribution with $\lambda > 0$ such that $P(X = 4) = 2P(X = 5)$. If $p_k = P(X = k)$ for $k = 0, 1, 2, \ldots$, and $p_\alpha = \max_k p_k$, then $\alpha =$ ______ (integer). (GATE ST 2024)
32) Let X_1, X_2, X_3 be i.i.d. random variables with PDF $f(x) = 2x, 0 < x < 1,$ $0,$ otherwise. Then $P(\min\{X_1, X_2, X_3\}) \geq E(X_1) =$ ______ (round to two decimal places). (GATE ST 2024)
33) Let (x, y) have a bivariate normal distribution with $E(x) = E(Y) = 0$. Let $\mathrm{Var}(X \mid Y = 1)$ be the conditional variance of X given $Y = 1$ and similarly for $\mathrm{Var}(Y \mid X = 2)$. If $\frac{E(Y|X=2)}{E(X|Y=1)} = 8$, then $\frac{\mathrm{Var}(Y|X=2)}{\mathrm{Var}(X|Y=1)} =$ ______ (integer). (GATE ST 2024)
34) Let X be a random sample of size one from $N0, \sigma^2$ with $\sigma > 0$ unknown. Let $\Phi(\cdot)$ be the CDF of $\mathcal{N}(0, 1)$. Let $\chi^2_{\nu,\alpha}$ be the $(1 - \alpha)$-quantile of the central chi-square with ν degrees of freedom. Given: $\Phi(1.96) = 0.975$, $\Phi(1.64) = 0.95$, $\chi^2_{1,0.05} = 3.841$, $\chi^2_{2,0.05} = 5.991$. Test $H_0 : \sigma^2 = 1$ vs $H_1 : \sigma^2 = 2$ using NP most powerful test of size 0.05: reject when $\lambda(x) > c$, where $\lambda(x) = \frac{f\left(x;\sigma^2=2\right)}{f\left(x;\sigma^2=1\right)}$. Find $c =$ ______ (round to two decimal places). (GATE ST 2024)
35) Linear regression: $y_i = \beta_1 x_i + \epsilon_i$, $i = 1, \ldots, n$, with β_1 unknown, ϵ_i uncorrelated mean 0, variance $\sigma^2 > 0$. 5 data points: $(x_1, y_1) = (2, 5)$, $(x_2, y_2) = (1, 6)$, $(x_3, y_3) = (3, 4)$, $(x_4, y_4) = (2, 3)$, $(x_5, y_5) = (4, 6)$. LSE of $\beta_1 =$ ______ (round to two decimal places).

(GATE ST 2024)

36) $f : \mathbb{R}^2 \to \mathbb{R}$ given by: $f(x, y) = 108xy - 2x^2 y - 2xy^2$. Which is NOT true?

a) f has four critical points
b) f has a local minimum at $(0,0)$
c) f has a local maximum at $(18,18)$
d) f has two or more saddle points

(GATE ST 2024)

37) $f:\mathbb{R}^2 \to \mathbb{R}$ given by: $f(x,y) = \frac{x^3-y^3}{x^2+y^2}, (x,y) \neq (0,0)$
$0, (x,y) = (0,0)$ Let f_x and f_y denote partial derivatives. Which is NOT true?

a) f is continuous at $(0,0)$
b) $f_x(0,0) \neq f_y(0,0)$
c) f_x is continuous at $(0,0)$
d) f_y is not continuous at $(0,0)$

(GATE ST 2024)

38) X has PDF: $f(x) = \frac{3}{8}(x+1)^2, -1 < x < 1,$
$0,$ otherwise. If $Y = 1 - X^2$, find $P(Y \leq 3/4)$.

a) $\frac{19}{32}$
b) $\frac{9}{16}$
c) $\frac{15}{32}$
d) $\frac{5}{8}$

(GATE ST 2024)

39) X has PDF: $f(x) = \frac{c_1}{\sqrt{x}}, 0 < x \leq 1,$
$\frac{c_2}{x^2}, 1 < x < \infty,$
$0,$ otherwise, where c_1, c_2 are constants. If $P(X \in [1/4, 4]) = 5/8$, consider: (I) $P(X \in [3,5]) = 1/12$ (II) Both X and $1/X$ do not have finite expectations. Which are true?

a) Only (I)
b) Only (II)
c) Both (I) and (II)
d) Neither (I) nor (II)

(GATE ST 2024)

40) Checkout counter service time T (in minutes) has PDF: $f(t) = \frac{1}{10}e^{-t/10}, t \geq 0,$
$0,$ otherwise. On arrival, you see 1 person in service for already 5 minutes. Assume independence of service times. Find $P(\text{your total time} > 15)$.

a) $\frac{5}{2}e^{-3/2}$
b) $\frac{3}{2}e^{-3/2}$
c) $\frac{3}{2}e^{-5/2}$
d) $\frac{5}{2}e^{-5/2}$

(GATE ST 2024)

41) X has a discrete uniform distribution on $\{1,3,5,\ldots,99\}$. Then $E(X \mid X \text{is not a multiple of } 15)$ equals:

a) $\frac{2365}{47}$
b) $\frac{2365}{50}$
c) 50
d) 47

(GATE ST 2024)

42) Let $X_1,\ldots,X_n$ be i.i.d. $\mathcal{N}\left(\mu,\sigma^2\right)$, $\mu \in \mathbb{R}$, $\sigma > 0$. Statements: (I) If $\dfrac{\sqrt{5}}{\sigma(2n+1)} \sum_{i=1}^{n}(X_i - \mu) \sim \mathcal{N}(0,1)$, then $n = 2$. (II) $E\left[\left(-\log_e \Phi\left(\frac{X_1-\mu}{\sigma}\right)\right)^3\right] = 6$, where $\Phi(\cdot)$ is CDF of $\mathcal{N}(0,1)$.
Which is/are true?

a) Only (I)
b) Only (II)
c) Both (I) and (II)
d) Neither (I) nor (II)

(GATE ST 2024)

43) Let X_n be i.i.d. with PDF $f(x) = e^{-(x-\theta)}$ for $x \geq \theta$, $\theta > 0$. (I) $\frac{1}{n}\sum_{i=1}^{n} X_i \to_p \frac{\theta+1}{2}$ as $n \to \infty$. (II) $\lim_{n\to\infty} E[\min\{X_1,\ldots,X_n\}] = \theta$.
Which is/are true?

a) Only (I) b) Only (II) c) Both (I) and (II) d) Neither (I) nor (II)

(GATE ST 2024)

44) Let $X_n \sim \text{Poisson}(\lambda_n)$ where $\lambda_n = \lambda + \frac{1}{2n}$, $\lambda > 0$. Which statement +is true?

a) $\frac{1}{n}\sum X_i$ is an unbiased estimator of λ
b) $\frac{1}{n}\sum X_i$ is a consistent estimator of λ
c) $\sum X_i$ is a consistent estimator of λ
d) $\frac{1}{n^2}\sum X_i$ is an unbiased estimator of λ

(GATE ST 2024)

45) Let $\mathbf{X}_1, \ldots, \mathbf{X}_{25} \overset{i.i.d.}{\sim} N_3(\mu, \Sigma)$, Σ nonsingular unknown. Let $S = \frac{1}{24}\sum_{j=1}^{25}\left(\mathbf{X_j} - \bar{\mathbf{X}}\right)\left(\mathbf{X_j} - \bar{\mathbf{X}}\right)^{\top}$, and $B = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{pmatrix}$. Which is true?

a) $24BSB^{\top}$ is Wishart of order 3, df=24
b) $24BSB^{\top}$ is Wishart of order 2, df=25
c) $24BSB^{\top}$ is Wishart of order 2, df=24
d) $24BSB^{\top}$ is Wishart of order 3, df=25

(GATE ST 2024)

46) Regression $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, ϵ_i uncorrelated mean 0 var σ^2. Statements: (I) The 95% joint confidence region for (β_0, β_1) is bounded by an ellipse. (II) Covariance between $\hat{\beta}_0$ and $\hat{\beta}_1$ does not involve σ^2. Which is/are true?

a) Only (I) b) Only (II) c) Both (I) and (II) d) Neither (I) nor (II)

(GATE ST 2024)

47) Let $f : [-2, 2] \to \mathbb{R}$ continuous. Which are true?
a) $F(x) = \int_0^x f(t)\, dt$ is differentiable on $(0, 2)$
b) For any $x_1, \ldots, x_{10} \in [-2, 2]$, there exists $x_0 \in [-2, 2]$ such that $f(x_0) = \frac{1}{10}\sum_{i=1}^{10} f(x_i)$
c) f is bounded on $[-2, 2]$
d) If f differentiable at 0 and $f(0) = 0$, then $\lim_{x\to 0} \frac{f(x)+f\left(x^2\right)+\cdots+f(\left(x^{10}\right)}{x} = 10f'(0)$

(GATE ST 2024)

48) Let A be an $n \times n$ real matrix. Which are true?
a) If A symmetri (GATE ST 2024)c and $A + \epsilon I_n$ PSD for all $\epsilon > 0$, then A is PSD
b) If n odd, then $A - A^{\top}$ not invertible
c) If A symmetric and all singular values > 0, then A positive definite
d) If 1 is the only singular value of A, then A is orthogonal

(GATE ST 2024)

49) Which statements are true?
a) If A is 3×3 real with 3 dis (GATE ST 2024)tinct eigenvalues, then A is diagonalizable
b) If A^2 is diagonalizable, then A is diagonalizable
c) For real a, b, c, if $\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$ is diagonalizable, then $a = b = c = 0$
d) If $A = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$ is diagonalizable, then $AA^{\top} = A^{\top}A$

(GATE ST 2024)

50) $\Omega = [1, 2, 3, \ldots]$. $\mathcal{H}$ all subsets of Ω, $P(k)) = 1/2^k$. Let $X(\omega) = \omega$.
Which are true?
a) $\exists k$ with $P(X = k) < 10^{-6}$
b) $\lim_{n\to\infty} P(X \geq 4 + 1/n) = 1/16$

c) $\lim_{n\to\infty} P\left(4 + 1/n^2 \leq X < 5 - 1/n\right) = 1/16$
d) If $x_n = 3 + (-1)^n/n$, then $\lim_{n\to\infty} P(X \leq x_n) = 7/8$

(GATE ST 2024)

51) Let (x, y) have joint PDF $f_{X,Y}(x, y) = \frac{3}{4}, x^2 \leq y < 1,\ -1 \leq x \leq 1,$
0, otherwise. Which statements are true?

a) X has the same distribution as $-X$
b) $E(Y \mid X = 0) = \frac{1}{2}$
c) $\text{Corr}(x, y) = 0$
d) X and Y are independent

(GATE ST 2024)

52) Let $[X_n]_{n\geq 1}$ be independent with PDF $f_n(x) = \frac{1}{\lambda_n} e^{-x/\lambda_n}, x \geq 0,$
0, otherwise, where $\lambda_n = 10 - \sum_{i=1}^{n} \frac{5}{2^{i-1}}$. Which statements are true?
a) $\{X_n\}$ converges in distribution to the zero random variable
b) $\{X_n\}$ converges in probability to the zero random variable
c) $\{X_n\}$ converges in distribution to a Poisson(10) random variable
d) $\{X_n\}$ converges in probability to a Poisson(10) random variable

(GATE ST 2024)

53) A Markov chain with state space $(0, 1, 2)$ has transition matrix $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$. Which statements are true?

a) 0 and 1 are recurrent
b) 2 is transient
c) Chain has unique stationary distribution
d) Chain is irreducible

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54) Let $X_1, \ldots, X_n$ be i.i.d. Poisson (λ), $\lambda > 0$ unknown. $T_1 = \bar{X}$, $T_2 = \sqrt{\frac{1}{n-1} \sum \left(X_i - \bar{X}\right)^2}$. Which statements are true?

a) T_1 is unbiased for λ
b) T_2 is unbiased for $\sqrt{\lambda}$
c) T_2^2 is unbiased for λ
d) Both T_1 and T_2 estimate λ and λ^2

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55) Bernoulli(p) sample X_1, X_2, X_3, $p \in (0, 1)$ unknown. Define $T_1\left(X_i, X_j, X_k\right) = X_i - X_j(1 - X_k)$, $T_2\left(X_i, X_j, X_k\right) = \frac{1}{2}\left(X_i X_j + X_j X_k\right)$. Which statements are true?
a) $T_1(X_1, X_2, X_3)$ has same distribution as $T_1(X_2, X_3, X_1)$, but they may differ for realizations
b) $T_2(X_1, X_2, X_3)$ and $T_2(X_3, X_2, X_1)$ are both unbiased for p^2
c) $T_1(\cdot)$ forms unbiased estimators for p^2 and always equal for cyclic permutations
d) $T_2(X_1, X_2, X_3) = T_2(X_3, X_2, X_1)$ for all realizations

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56) $X_1, \ldots, X_n$ i.i.d. Exp (λ), $\lambda > 0$ unknown. $T_1 = \sum X_i$, $T_2 = 1/\sum X_i$. Testing $H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda > \lambda_0$, which tests are UMP level α?

a) Reject if $\frac{2}{\lambda_0} T_1 > \chi^2_{n,\alpha}$ b) Reject if $\frac{2}{\lambda_0} T_1 > \chi^2_{n,1-\alpha}$ c) Reject if $\frac{\lambda_0}{2} T_2 > \chi^2_{n,\alpha}$ d) Reject if $\frac{\lambda_0}{2} T_2 > \chi^2_{n,1-\alpha}$

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57) Two samples: $\{1, 6, 5, 3\}$ and $\{11, 7, 15, 4\}$. Mann-Whitney U_{MW} statistic has probabilities given: $P(U_{MW} > 12) \leq 0.10$, $P(U_{MW} > 14) \leq 0.05$, $P(U_{MW} > 15) \leq 0.025$, $P(U_{MW} > 16) \leq 0.01$. Which statements are true?

a) H_0 rejected at $\alpha = 0.10$ c) H_0 rejected at $\alpha =$ d) H_0 rejected at $\alpha = 0.01$
b) H_0 rejected at $\alpha = 0.05$ 0.025

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58) $(X_1, X_2, X_3) \sim N_3(\mu, \Sigma)$ with $\mu = [\,2, -3, 1\,]^\top$ and $\Sigma = \begin{pmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{pmatrix}$ For which $\mathbf{a}$ are X_2 and $X_2 - \mathbf{a}^\top [X_1, X_2]^\top$ independent?

a) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ b) $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ c) $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ d) $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

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59) A is 2×2 with $\operatorname{tr}(A) = 5$, $\det(A) = 6$. Let the characteristic polynomial of $(A + I_2)^{-1}$ be $x^2 - bx + c$. Find $b/c =$ ______ (integer).

60) Markov chain, states $\{0, 1, 2\}$, transition $\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$, $P(X_0 = 0) = P(X_0 = 1) = \frac{1}{4}$. Find $32\,E(X_2) =$ ______ (integer). (GATE ST 2024)

61) (x, y) has MGF $M_{X,Y}(u, v) = \frac{e^{u^2/2}}{(1-2v)^3}, \quad v < \frac{1}{2}$. Find $E\left(\frac{6X^2}{Y}\right) =$ ______ $(2 decimals)$. (GATE ST 2024)

62) $\{N(t)\}$ Poisson process with rate λ. Potholes at distances: $0.9, 1.3, 1.8, 2.7, 3.4, 4.1, 4.7, 5.5, 6.2, 6.8, 7.4, 8.1, 8.9, 9$ MoM estimate of $\lambda =$ ______ $(2 decimals)$. (GATE ST 2024)

63) Sample size 4 from $\mathrm{U}(0, \theta)$. Let $X_{(4)}$ be max. Test $H_0 : \theta = 1$ vs $H_1 : \theta = 0.1$; reject if $X_{(4)} < 0.3$. Let p be Type-I error. Find $100p =$ ______ $(2 decimals)$. (GATE ST 2024)

64) Sample mean 0.16 from size 4 normal with unknown mean, variance 1. $\Phi(2.28) = 0.989$, $\Phi(1.96) = 0.975$, $\Phi((1.64) = 0.95$. LRT at $\alpha = 0.05$ for $H_0 : \mu = 0$ vs $H_1 : \mu \neq 0$. Find power at $\bar{x} = 0.16 =$ ______ $(3 decimals)$. (GATE ST 2024)

65) Multiple regression $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i$, $n = 25$. Test $H_0 : \beta_1 = \beta_2 = 0$ with $F_0 = \frac{1}{1} \cdot \frac{R^2}{1-R^2}$. Reject if $F_0 > F_{\alpha,2,22}$. Given $F_{0.025,2,22} = 4.38$, $F_{0.05,2,22} = 3.44$. Find smallest R^2 to reject at $\alpha = 0.05 =$ (GATE ST 2024)______ $(2 decimals)$. (GATE ST 2024)