2.7.25

EE25BTECH11013 - Bhargav

Question:
Find the area of quadrilateral $ABCD$ whose vertices are $A(-3,-1)$, $B(-2,-4)$, $C(4,-1)$ and $D(3,4)$.
Solution:
The area of the quadrilateral can be found by dividing it into 2 triangles and adding them to find the area of the quadrilateral. The area of triangle ABC and ACD can be computed separately.

$$\mathbf{A} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{B} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \tag{0.2}$$

$$\mathbf{C} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \tag{0.3}$$

$$\mathbf{D} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{0.4}$$

Choose **A** as a common vertex and form vectors for triangles ABC and ACD.

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -3+2 \\ -1+4 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{0.5}$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -3-4 \\ -1+1 \end{pmatrix} = \begin{pmatrix} -7 \\ 0 \end{pmatrix} \tag{0.6}$$

$$(\triangle ABC) = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C})\| \tag{0.7}$$

$$(\triangle ABC) = \frac{1}{2} \left\| \begin{pmatrix} -1 \\ 3 \end{pmatrix} \times \begin{pmatrix} -7 \\ 0 \end{pmatrix} \right\| = \frac{1}{2} \cdot 21 = \frac{21}{2}. \tag{0.8}$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} -3 - 3 \\ -1 - 4 \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \end{pmatrix} \tag{0.9}$$

$$(\triangle ACD) = \frac{1}{2} \left\| \mathbf{A} - \mathbf{C}) \times (\mathbf{A} - \mathbf{D}) \right\| = \frac{1}{2} \cdot 35 = \frac{35}{2} \tag{0.10}$$

Therefore, the area of the quadrilateral is

$$(ABCD) = (\triangle ABC) + (\triangle ACD) = \tfrac{21}{2} + \tfrac{35}{2} = \tfrac{56}{2} = 28. \tag{0.11}$$

Therefore, the Area of the Quadrilateral $ABCD$ is 28