1.6.7

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Problem Statement

Find a relation between x and y if the points (x,y) ,(1,2) and (7,0) are collinear.

Variable	Description	Values
A	Point	(x, y)
B	Point	$(1, 2)$
C	Point	$(7, 0)$

Table: Variables Used

Collinearity of Matrix

Let the three points be $\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}.$

For collinearity,

$$\text{rank}\Big(\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^T \Big) = 1. \tag{3.1}$$

Now,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 - x \\ 2 - y \end{pmatrix}, \quad \mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 - x \\ -y \end{pmatrix}. \tag{3.2}$$

So the matrix is

$$\mathbf{M} = \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^T = \begin{pmatrix} 1 - x & 2 - y \\ 7 - x & -y \end{pmatrix}. \tag{3.3}$$

Echelon Form and Row Operations

Step 1: Start with

$$\mathbf{M} = \begin{pmatrix} 1-x & 2-y \\ 7-x & -y \end{pmatrix}. \tag{3.4}$$

Step 2: Eliminate the first entry of the second row:

$$R_2 \longrightarrow R_2 - \frac{7-x}{1-x} R_1 \quad \text{(assuming } x \neq 1\text{)}. \tag{3.5}$$

$$\begin{pmatrix} 1-x & 2-y \\ 7-x & -y \end{pmatrix} \longrightarrow \begin{pmatrix} 1-x & 2-y \\ 0 & -y - \frac{7-x}{1-x}(2-y) \end{pmatrix}. \tag{3.6}$$

Rank Condition

For rank($\mathbf{M}$) $= 1$, the second row must vanish:

$$-y - \frac{7-x}{1-x}(2-y) = 0. \tag{3.7}$$

Multiply through by $(1-x)$:

$$-y(1-x) - (7-x)(2-y) = 0. \tag{3.8}$$

Expand:

$$-y + xy - (14 - 2x - 7y + xy) = 0. \tag{3.9}$$

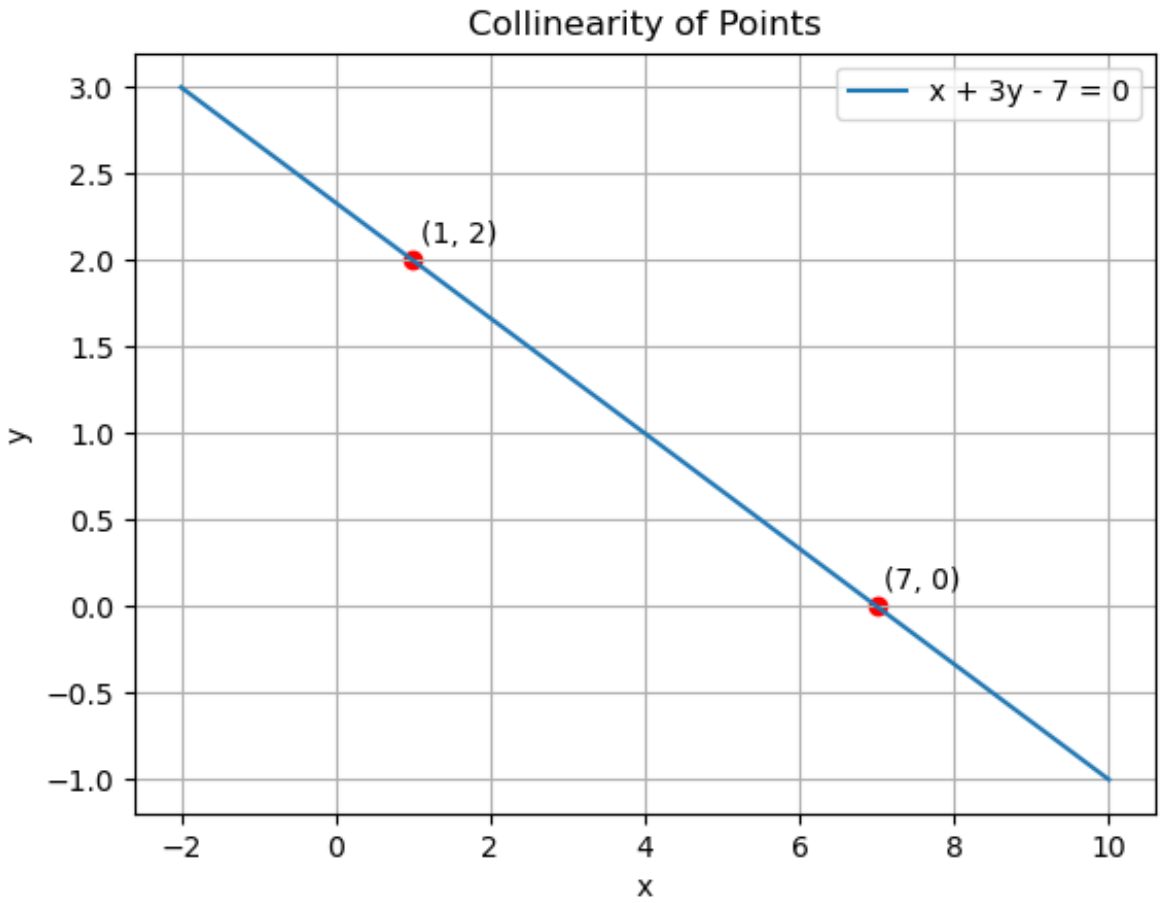
$$-y + xy - 14 + 2x + 7y - xy = 0. \tag{3.10}$$

$$2x + 6y - 14 = 0. \tag{3.11}$$

Thus, the condition for collinearity is

$$\boxed{x + 3y = 7}. \tag{3.12}$$

Plots



Figure

C Code

```c
#include <stdio.h>

// Function to return relation value
int relation(int x, int y) {
    return x + 3*y - 7;
}

int main() {
    // Given points
    int x1 = 1, y1 = 2;
    int x2 = 7, y2 = 0;

    // Step 1: Compute slope
    float m = (float)(y2 - y1) / (x2 - x1);
```

```
 printf("m=(y2-y1)/(x2-x1)=(%d-%d)/(%d-%d)=%.2f\n\n",
                y2, y1, x2, x1, m);
    // Step 2: Point-slope form
    printf("Step 2: Equation using point-slope form:\n");
    printf("(y-%d)=m(x-%d)\n\n", y1, x1);

    // Final Relation
    printf("Final Relation: x+3y-7=0\n");

    return 0;
}
```

Python Code for Plotting

```
import numpy as np
import matplotlib.pyplot as plt

# Equation: x + 3y − 7 = 0
y = (7 − x)/3
x_vals = np.linspace(−2, 10, 100)
y_vals = (7 − x_vals) / 3

plt.plot(x_vals, y_vals, label="x+3y−7=0")

# Given points
points = [(1,2), (7,0)]
for p in points:
    plt.scatter(p[0], p[1], color='red')
    plt.text(p[0]+0.1, p[1]+0.1, f'{p}")

plt.xlabel("x")
```

```
plt.title("Collinearity-of-Points")
plt.legend()
plt.grid(True)
plt.savefig('../figs/fig1.png')
plt.show()
```