1.6.16

AI25BTECH11001 - ABHISEK MOHAPATRA

Question:
Find the values of k if the points $\mathbf{A}(k+1, 2k)$, $\mathbf{B}(3k, 2k+3)$ and $\mathbf{C}(5k-1, 5k)$ are collinear.

Solution: From the given information,

$$\mathbf{A} = \begin{pmatrix} k+1 \\ 2k \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3k \\ 2k+3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5k-1 \\ 5k \end{pmatrix} \tag{1}$$

To check if the points are collinear, we can use

$$rank\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} \end{pmatrix} = 1 \tag{2}$$

So,

$$\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} \end{pmatrix}^T = \begin{pmatrix} 2k-1 & 3 \\ 4k-2 & 3k \end{pmatrix} \tag{3}$$

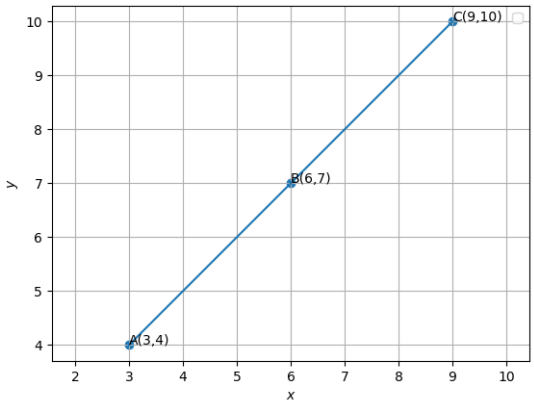
$$\xleftrightarrow{R_2=R_2-2R_1} \begin{pmatrix} 2k-1 & 3 \\ 0 & 3k-6 \end{pmatrix} \tag{4}$$

The rank of the matrix will be 1 when

$$3k - 6 = 0 \tag{5}$$

$$\Rightarrow k = 2 \tag{6}$$

Graph:



Therefore, k = 2.