GATE 2015 MA

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GATE MA 2015

AI25BTECH11012 - UNNATHI GARIGE

Q.1-Q.5 carry one mark each.

_	Lang one in			1 1	
1) Choose the appropriate word/phrase, out of the four options given below, to corthe following sentence:					
	pparent lifelessnes		dormant life.	GATE MA 2015	
	11		•		
a)	harbours	b) leads to	c) supports	d) affects	
	ll in the blank wit wn was a			A 2015 That boy from the GATE MA 2015	
	dog out of herd sheep from the he	eap	c) fish out of vd) bird from th		
3) C	hoose the statemer	nt where underli	ned word is used corre	ectly. GATE MA 2015	
a)b)c)	When the teacher When the thief ke Matters that are d	eludes to differ eeps eluding the lifficult to under	rent authors, he is being elected as the police, he is being elected as the police, the is being elected as the police and the police are the police as the police are the police as the police are the p	g <u>elusive</u> . <u>usive</u> . ember are <u>allusive</u> .	
If	the first two state		der than Tanya. Eric is then the third statemer	older than Cliff. at is: GATE MA 2015	
	True False				
	Uncertain Data insufficient				
5) Fi	ve teams have to	e going to the n	next round. How many	playing every other team matches will have to be GATE MA 2015	
a)	20	b) 10	c) 8	d) 5	
Q	.6-Q.10 carry two	mark each.			
6) Se	elect the appropria	te option in plac	ce of underlined part of	f the sentence.	
In	creased productivi	ty necessary ref	lects greater efforts ma	nde by the employees.	
,	Increase in produ			GATE MA 2015	
,	Increase productiv	•			
	Increase in produ	•	ly		
	No improvement	•	6.11		
7) G	iven below are ty	wo statements b	followed by two cond	clusions. Assuming these	

statements to be true, decide which one logically follows.

Statements:

- I. No manager is a leader.
- II. All leaders are executives.

Conclusions:

- I. No manager is an executive.
- II. No executive is a manager.
- a) Only conclusion I follows.
- b) Only conclusion II follows.
- c) Neither conclusion I nor II follows.
- d) Both conclusions I and II follow.
- 8) In the given figure angle Q is a right angle. PS : QS = 3 : 1, RT : QT = 5 : 2 and PU : UR = 1 : 1. If area of triangle QTS is $20 \, \text{cm}^2$, then the area of triangle PQR in cm² is ______. GATE MA 2015

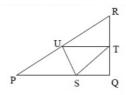


Fig. 8.1

- 9) constructed in the *xy*-plane so that the right angle is at *P* and line *PR* is parallel to the *x*-axis. The *x* and *y* coordinates of *P*, *Q*, and *R* are to be integers that satisfy the inequalities: $-4 \le x \le 5$ and $6 \le y \le 16$. How many different triangles could be constructed with these properties?

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 - a) 110
- b) 1100
- c) 9900
- d) 10000
- 10) A coin is tossed thrice. Let *X* be the event that head occurs in each of the first two tosses. Let *Y* be the event that a tail occurs on the third toss. Let *Z* be the event that two tails occur in three tosses. Based on the above information, which one of the following statements is TRUE?

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 - a) X and Y are not independent
- c) Y and Z are independent

b) Y and Z are dependent

d) X and Z are independent

Q.11 to Q.35 carry one mark each.

11) Let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be a linear map defined by

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$$T(x, y, z, w) = (x + z, 2x + y + 3z, 2y + 2z, w).$$

Then the rank of T is equal to _____.

12) Let M be a 3×3 matrix and suppose that 1,2 and 3 are the eigenvalues of M.

If GATE MA 2015

$$M^{-1} = \frac{M^2 - M + I_3}{\alpha}$$

for some scalar $\alpha \neq 0$, then α is equal to ...

- 13) Let M be a 3×3 singular matrix and suppose that 2 and 3 are eigenvalues of M. Then the number of linearly independent eigenvectors of $M^3 + 2M + I_3$ is equal to GATE MA 2015
- 14) Let M be a 3×3 matrix such that $M \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$ and suppose that $M^3 \begin{pmatrix} 1 \\ -1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ for some $\alpha, \beta, \gamma \in \mathbb{R}$. Then $|\alpha|$ is equal to ______. GATE MA 2015
- 15) Let $f:[0,\infty)\to\mathbb{R}$ be defined by

$$f(x) = \int_0^x \sin^2(t^2) dt.$$

Then the function f is:

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- a) uniformly continuous on [0,1] but NOT on $(0,\infty)$
- b) uniformly continuous on $(0, \infty)$ but NOT on [0, 1]
- c) uniformly continuous on both [0, 1] and $(0, \infty)$
- d) neither uniformly continuous on [0,1] nor uniformly continuous on $(0,\infty)$
- 16) Consider the power series $\sum_{n=0}^{\infty} a_n z^n$, where

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$$a_n = \begin{cases} \frac{1}{3^n} & \text{if } n \text{ is even} \\ \frac{1}{5^n} & \text{if } n \text{ is odd} \end{cases}$$

The radius of convergence of the series is equal to _____

17) Let $C = \{z \in \mathbb{C} : |z - i| = 2\}$. Then

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$$\frac{1}{2\pi i} \oint_C \frac{z^2 - 4}{z^2 + 4} dz$$

is equal to ______.

18) Let $X \sim B(5, \frac{1}{2})$ and $Y \sim U(0, 1)$. Then

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$$\frac{P(X+Y\leq 2)}{P(X+Y\geq 5)}$$

is equal to ______.

19) Let the random variable X have the distribution function

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$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{2} & \text{if } 0 \le x < 1 \\ \frac{3}{5} & \text{if } 1 \le x < 2 \\ \frac{1}{2} + \frac{x}{8} & \text{if } 2 \le x < 3 \\ 1 & \text{if } x \ge 3 \end{cases}$$

Then $P(2 \le X < 4)$ is equal to

20) Let X be a random variable having the distribution function

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$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{4} & \text{if } 0 \le x < 1 \\ \frac{1}{3} & \text{if } 1 \le x < 2 \\ \frac{1}{2} & \text{if } 2 \le x < \frac{11}{3} \\ 1 & \text{if } x \ge \frac{11}{3} \end{cases}$$

Then E(X) is equal to

21) In an experiment, a fair die is rolled until two sixes are obtained in succession. The probability that the experiment will end in the fifth trial is equal to GATE MA 2015

a)
$$\frac{125}{6^5}$$

b)
$$\frac{150}{6^5}$$

c)
$$\frac{175}{6^5}$$

b)
$$\frac{150}{6^5}$$
 c) $\frac{175}{6^5}$ d) $\frac{200}{6^5}$

22) Let $x_1 = 2.2$, $x_2 = 4.3$, $x_3 = 3.1$, $x_4 = 4.5$, $x_5 = 1.1$, $x_6 = 5.7$ be the observed values of a random sample of size 6 from a $U(0 - \theta, 0 + \theta)$ distribution, where $\theta \in (0, \infty)$ is unknown. Then a maximum likelihood estimate of θ is equal to

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23) Let $\Omega = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$ be the open unit disc in \mathbb{R}^2 with boundary $\partial \Omega$. If u(x, y) is the solution of the Dirichlet problem

$$\begin{cases} u_{xx} + u_{yy} = 0 & \text{in } \Omega \\ u(x, y) = 1 - 2y^2 & \text{on } \partial\Omega \end{cases}$$

then $u(\frac{1}{2},0)$ is equal to

	a) -1	$-\frac{1}{4}$	c) \(\frac{1}{4}\)	d) 1
24)	Let $c \in \mathbb{Z}$ be such that	ıt		
			$\frac{\mathbb{Z}(x)}{+x+c}$	
	is a field. Then c is e	qual to	<u></u> ·	GATE MA 2015
25)	Let $V = C^1[0, 1]$, $X =$: (C[0_1] · ∞) a	nd $Y = (C[0, 1], \ \cdot\ _2)$)

- 25) Let $V = C^1[0,1]$, $X = (C[0,1], \|\cdot\|\infty)$ and $Y = (C[0,1], \|\cdot\|2)$. Then V is
 - GATE MA 2015

- a) dense in X but NOT in Y
- b) dense in Y but NOT in X
- c) dense in both X and Y
- d) neither dense in X nor dense in Y
- 26) Let $T: (C[0,1], \|\cdot\|\infty) \to \mathbb{R}$ be defined by $T(f) = \int_0^1 2x f(x) dx$ for all $f \in C[0,1]$. GATE MA 2015
- 27) Let τ_1 be the usual topology on \mathbb{R} . Let τ_2 be the topology on \mathbb{R} generated by

$$\mathcal{B} = \{ [a, b) \subset \mathbb{R} : -\infty < a < b < \infty \}.$$

Then the set $\left\{x \in \mathbb{R} : 4\sin^2 x \le 1\right\} \cup \left\{\frac{\pi}{2}\right\}$ is

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- a) closed in (\mathbb{R}, τ_1) but NOT in (\mathbb{R}, τ_2)
- b) closed in (\mathbb{R}, τ_2) but NOT in (\mathbb{R}, τ_1)
- c) closed in both (\mathbb{R}, τ_1) and (\mathbb{R}, τ_2)
- d) neither closed in (\mathbb{R}, τ_1) nor closed in (\mathbb{R}, τ_2)
- 28) Let X be a connected topological space such that there exists a non-constant continuous function $f: X \to \mathbb{R}$, where \mathbb{R} is equipped with the usual topology. Let $f(X) = \{f(x) : x \in X\}$. Then
 - a) X is countable but f(X) is uncountable
 - b) closed in (\mathbb{R}, τ_2) but NOT in (\mathbb{R}, τ_1)
 - c) both f(X) and X are countable
 - d) both f(X) and X are uncountable
- 29) Let d_1 and d_2 denote the usual metric and the discrete metric on \mathbb{R} , respectively. Let $f: (\mathbb{R}, d_1) \to (\mathbb{R}, d_2)$ be defined by f(x) = x, $x \in \mathbb{R}$. Then GATE MA 2015
 - a) f is continuous but f^{-1} is NOT continuous
 - b) f^{-1} is continuous but f is NOT continuous
 - c) both f and f^{-1} are continuous
 - d) neither f nor f^{-1} is continuous
- 30) If the trapezoidal rule with single interval [0, 1] is exact for approximating the integral

$$\int_0^1 (x^3 - cx^2) \, dx,$$

then the value of c is equal to _____

31)	Suppose that the Newton-Raphson method is applied to the equation $2x^2 + 1 - e^{x^2} = 0$ with an initial approximation x_0 sufficiently close to zero. Then, for the root $x = 0$, the order of convergence of the method is equal to
	GATE MA 2015
32)	The minimum possible order of a homogeneous linear ordinary differential equation with real constant coefficients having $x^2 \sin(x)$ as a solution is equal to GATE MA 2015
33)	The Lagrangian of a system in terms of polar coordinates (r, θ) is given by
	$L = \frac{1}{2}mr^2 + \frac{1}{2}m(r^2 + r^2\dot{\theta}^2) - mgr(1 - \cos(\theta)),$
	where m is the mass, g is the acceleration due to gravity and \dot{s} denotes the derivative of s with respect to time. Then the equations of motion are GATE MA 2015

- a) $2\ddot{r} = r\dot{\theta}^2 g(1 \cos(\theta)), \frac{d}{dt}(r^2\dot{\theta}) = -gr\sin(\theta)$ b) $2\ddot{r} = r\dot{\theta}^2 + g(1 \cos(\theta)), \frac{d}{dt}(r^2\dot{\theta}) = -gr\sin(\theta)$ c) $\ddot{r} = r\dot{\theta}^2 g(1 \cos(\theta)), \frac{d}{dt}(r^2\dot{\theta}) = gr\sin(\theta)$ d) $\ddot{r} = r\dot{\theta}^2 + g(1 \cos(\theta)), \frac{d}{dt}(r^2\dot{\theta}) = gr\sin(\theta)$

- 34) If y(x) satisfies the initial value problem

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$$(x^2 + y)dx = x dy,$$
 $y(1) = 2,$

then y(2) is equal to

35) It is known that Bessel functions $I_n(x)$, for $n \ge 0$, satisfy the identity

$$e^{\frac{x}{2}(t-\frac{1}{t})} = J_0(x) + \sum_{n=1}^{\infty} J_n(x) \left(t^n + \frac{(-1)^n}{t^n}\right)$$

for all t > 0 and $x \in \mathbb{R}$. The value of $J_0\left(\frac{\pi}{2}\right) + 2\sum_{n=1}^{\infty} J_{2n}\left(\frac{\pi}{2}\right)$ is equal to GATE MA 2015

Q.36 to Q.65 carry two marks each.

36) Let X and Y be two random variables having the joint probability density function

$$f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then the conditional probability $P(X \le \frac{2}{3} \mid Y = \frac{3}{4})$ is equal to GATE MA 2015

a) $\frac{5}{9}$

- b) $\frac{2}{3}$
- c) $\frac{7}{9}$
- d) $\frac{8}{9}$

37) Let $\Omega = [0, 1]$ be the sample space and let $P(\cdot)$ be a probability function defined by

$$P((0, x]) = \begin{cases} \frac{1}{2} & \text{if } 0 \le x \le \frac{1}{2} \\ x & \text{if } \frac{1}{2} < x \le 1 \end{cases}$$

Then $P((0,\frac{1}{2}))$ is equal to _____

		/
38)	Let X_1, X_2, X_3 be independent and identically distributed random variables v	vith
	$E(X_1) = 0$ and $E(X_1^2) = \frac{15}{4}$. If $\psi: (0, \infty) \to (0, \infty)$ is defined through the condition	nal
	expectation	
	$\mathbf{r}(\mathbf{v}) = \mathbf{r}(\mathbf{v}^2 + \mathbf{v}^2 + \mathbf{v}^2 + \mathbf{v}^2)$	

$$\psi(t) = E(X_1^2 \mid X_1^2 + X_2^2 + X_3^2 = t), \quad t > 0,$$

then $E\left(\psi((X_1^2+X_2^2))\right)$ is equal to

GATE MA 2015

39) Let $X \sim \text{Poisson}(\lambda)$, where $\lambda > 0$ is unknown. If f(X) is the unbiased estimator of $g(\lambda) = e^{-\lambda}(3\lambda^2 + 2\lambda + 1)$, then $\sum_{k=0}^{\infty} \delta(k)$ is equal to _____

40) Let x_1, \ldots, x_n be a random sample from $N(\mu, 1)$ distribution, where $\mu \in \{0, \frac{1}{2}\}$. For testing the null hypothesis $H_0: \mu = 0$ against the alternative hypothesis $H_1: \mu = \frac{1}{2}$, consider the critical region

$$R = \left\{ (x_1, x_2, \dots, x_n) : \sum_{i=1}^n x_i > c \right\}$$

where c is some real constant. If the critical region R has size 0.025 and power 0.7054, then the value of the sample size n is equal to ____ GATE MA 2015

41) Let X and Y be independently distributed central chi-squared random variables with degrees of freedom $m \ge 3$ and $n \ge 3$ respectively. If $E\left(\frac{X}{X+Y}\right) = \frac{3}{7}$ and m+n=14, then $E\left(\frac{Y}{X+Y}\right)$ is equal to GATE MA 2015

a) $\frac{2}{7}$

c) $\frac{4}{7}$

d) $\frac{5}{7}$

42) Let X_1, X_2, \ldots be a sequence of independent and identically distributed random variables with

$$P(X_1 = 1) = \frac{1}{4}$$
 and $P(X_1 = 2) = \frac{3}{4}$.

If $X_n = \frac{1}{n} \sum_{i=1}^n X_i$, for n = 1, 2, ..., then $\lim_{n \to \infty} P(X_n)$ \leq 1.8) is equal to GATE MA 2015

43) Let $u(x, y) = 2f(y)\cos(x - 2y)$, $(x, y) \in \mathbb{R}^2$, be a solution of the initial value problem

$$2u_x + u_y = u,$$

$$u(x, 0) = \cos(x).$$

Then f(1) is equal to

GATE MA 2015

a) $\frac{1}{2}$

b) $\frac{i}{2}$

c) e

d) $\frac{3e}{2}$

44) Let $u(x,t), x \in \mathbb{R}, t \ge 0$, be the solution of the initial value problem

$$u_{tt} = u_{xx}$$
$$u(x,0) = x$$
$$u_t(x,0) = 1$$

		8
	Then $u(2,2)$ is equal to	ATE MA 2015
45)	Then $u(2,2)$ is equal to G. Let $W = \text{Span}\left\{\frac{1}{\sqrt{2}}(0,0,1,1), \frac{1}{\sqrt{2}}(1,-1,0,0)\right\}$ be a subspace of the E	uclidean space
	\mathbb{R}^4 . Then the square of the distance from the point $(1, 1, 1, 1)$ to the	subspace W is TE MA 2015
46)	Let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be a linear map such that the null space of T is	
	$\{(x, y, z, w) \in \mathbb{R}^4 : x + y + z + w = 0\}$	
	and the rank of $(T - 4I_4)$ is 3. If the minimal polynomial of T is $x(t_1, t_2, t_3)$	$(x-4)^2$, then α
	is equal to GA	TE MA 2015
		_

- 47) Let M be an invertible Hermitian matrix and let $x, y \in \mathbb{R}$ be such that $x^2 < 4y$. Then **GATE MA 2015**
 - a) both $M^2 + xM + yI$ and $M^2 xM + yI$ are singular
 - b) $M^2 + xM + yI$ is singular but $M^2 xM + yI$ is non-singular
 - c) $M^2 + xM + yI$ is non-singular but $M^2 xM + yI$ is singular
 - d) both $M^2 + xM + yI$ and $M^2 xM + yI$ are non-singular
- 48) Let $G = \{e, x, x^2, x^3, y, xy, x^2y, x^3y\}$ with o(x) = 4, o(y) = 2 and $xy = yx^3$. Then the number of elements in the center of the group G is equal to **GATE MA 2015**
 - a) 1 b) 2 c) 4 d) 8
- 49) The number of ring homomorphisms from $\mathbb{Z}_2 \times \mathbb{Z}_2$ to \mathbb{Z}_4 is equal to _____ GATE MA 2015
- 50) Let $p(x) = 9x^5 + 10x^3 + 5x + 15$ and $q(x) = x^3 x^2 x 2$ be two polynomials in $\mathbb{O}[x]$. Then, over \mathbb{O} , **GATE MA 2015**
 - a) p(x) and q(x) are both irreducible
 - b) p(x) is reducible but q(x) is irreducible
 - c) p(x) is irreducible but q(x) is reducible
 - d) p(x) and q(x) are both reducible
- 51) Consider the linear programming problem Maximize 3x + 9y, subject to

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$$2y - x \le 2$$
$$3y - x \ge 0$$
$$2x + 3y \le 10$$
$$x, y \ge 0$$

Then the maximum value of the objective function is equal to

52) Let $S = \{(x, \sin \frac{1}{x}) : 0 < x \le 1\}$ and $T = S \cup \{(0, 0)\}$. Under the usual metric on \mathbb{R}^2 ,

a) S is closed but T is NOT closed

- b) T is closed but S is NOT closed
- c) both S and T are closed

d) neither S nor T is closed

53) Let

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$$H = \left\{ (x_n) \in \ell^2 : \sum_{n=1}^{\infty} \frac{x_n}{n} = 1 \right\}.$$

Then H

- a) is bounded
- b) is closed
- c) is a subspace
- d) has an interior point
- 54) Let V be a closed subspace of $L^2[0,1]$ and let $f,g \in L^2[0,1]$ be given by f(x) = x and $g(x) = x^2$. If $V = \text{Span}\{f\}$ and Pg is the orthogonal projection of g on V, then

$$(g - Pg)(x), x \in [0, 1], is:$$

Options:

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- a) $\frac{3}{4}x$
- b) $\frac{1}{4}x$
- c) $\frac{3}{4}x^2$
- d) $\frac{1}{4}x^2$

55) Let p(x) be the polynomial of degree at most 3 that passes through the points (-2, 12), (-1, 1), (0, 2) and (2, -8). Then the coefficient of x^3 in p(x) is equal to GATE MA 2015

56) If, for some $\alpha, \beta \in \mathbb{R}$, the integration formula

$$\int_0^2 p(x) \, dx = p(\alpha) + p(\beta)$$

holds for all polynomials p(x) of degree at most 3, then the value of $3(\alpha - \beta)^2$ is equal to ______. GATE MA 2015

57) Let y(t) be a continuous function on $[0, \infty)$ whose Laplace transform exists. If y(t) satisfies GATE MA 2015

$$\int_0^t (1 - \cos(t - \tau)) y(\tau) d\tau = t^4,$$

then y(1) is equal to _____.

58) Consider the initial value problem

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$$x^2y'' - 6y = 0, y(1) = a, y'(1) = 6.$$

If $y(x) \to 0$ as $x \to 0^+$, then Then a is equal to _____.

59) Define $f_1, f_2 : [0, 1] \to \mathbb{R}$ by

$$f_1(x) = \sum_{n=1}^{\infty} \frac{\sin(n^2 x)}{n^2}$$

$$f_2(x) = \sum_{n=1}^{\infty} x(1-x^2)^n.$$

Then GATE MA 2015

- a) f_1 is continuous but f_2 is NOT continuous
- b) f_2 is continuous but f_1 is NOT continuous
- c) Both f_1 and f_2 are continuous
- d) Neither f_1 nor f_2 is continuous
- 60) Consider the unit sphere $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ and the unit normal vector $\hat{n} = (x, y, z)$ at each point (x, y, z) on S. The value of the surface integral

$$\iint_{S} \left(\frac{2x}{\pi} + \sin(y^2) \right) x + \left(e^x - \frac{y}{\pi} \right) y + \left(\frac{2z}{\pi} + \sin^2(y)z \right) d\sigma$$

is equal to _____

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61) Let $D = \{(x, y) \in \mathbb{R}^2 : 1 \le x \le 1000, 1 \le y \le 1000\}$. Define

$$f(x,y) = \frac{xy}{2} + \frac{500}{x} + \frac{500}{y}$$

Then the minimum value of f on D is equal to GATE MA 2015

- 62) Let $D = \{z \in \mathbb{C} : |z| < 1\}$. Then there exists a non-constant analytic function f on D such that for all $n = 2, 3, 4, \dots$ GATE MA 2015
 - a) $f(\sqrt[n]{-1}) = 0$ b) $f(\frac{1}{n}) = 0$

c) $f\left(1 - \frac{1}{n}\right) = 0$ Selected answer: \boxed{C} d) $f\left(\frac{1}{2} - \frac{1}{n}\right) = 0$

- 63) Let $\sum_{n=-\infty}^{\infty} a_n z^n$ be the Laurent series expansion of

$$f(z) = \frac{1}{z^2 - 13z + 15}$$

in the annulus $\frac{3}{2} < |z| < 5$. Then $\frac{a_4}{a_2}$ is equal to ______. GATE MA 2015

64) The value of

$$\frac{i}{4-\pi} \oint_{|z|=4} \frac{dz}{z \cos(z)}$$

is equal to _____

65) Suppose that among all continuously differentiable functions y(x), $x \in \mathbb{R}$ with y(0) = 0 and $y(1) = \frac{1}{2}$, the function $y_0(x)$ minimizes the functional

$$\int_0^1 \left(e^{-y'(x)-x} + (1+y)y'(x)^2 \right) dx.$$

Then $y_0\left(\frac{1}{2}\right)$ is equal to

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a) 0 b) $\frac{1}{8}$

c) $\frac{1}{4}$ d) $\frac{1}{3}$

END OF THE QUESTION PAPER