2.4.18

EE25BTECH11023 - Venkata Sai

Question:
Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

Solution:

Variable	Description
$\mathbf{m_1}$	Direction vector of Line 1
$\mathbf{m_2}$	Direction vector of line 2

TABLE 0: Variables Used

Line 1:

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2} \implies \frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2} \tag{1}$$

Line 2:

$$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5} \implies \frac{x-1}{-\frac{3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} \tag{2}$$

Direction vector for line 1:

$$\mathbf{m_1} = \begin{pmatrix} -3 \\ \frac{2p}{7} \\ 2 \end{pmatrix} \tag{3}$$

Direction vector for line 2:

$$\mathbf{m_2} = \begin{pmatrix} -\frac{3p}{7} \\ 1 \\ -5 \end{pmatrix} \tag{4}$$

Since the lines are at right angles

$$\left(\mathbf{m_1}\right)^{\top} \left(\mathbf{m_2}\right) = 0 \tag{5}$$

$$\begin{pmatrix} -3 & \frac{2p}{7} & 2 \end{pmatrix} \begin{pmatrix} -\frac{3p}{7} \\ 1 \\ -5 \end{pmatrix} = 0 \tag{6}$$

$$(-3)\left(-\frac{3p}{7}\right) + \left(\frac{2p}{7}\right)(1) + (2)(-5) = 0 \tag{7}$$

$$p = \frac{70}{11} \tag{8}$$

Hence the value of p is $\frac{70}{11}$

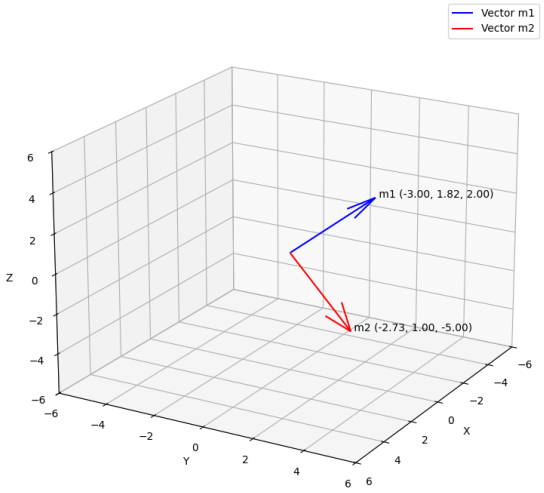

Fig. 0.1: Stem Plot of y(n)