1.4.19

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August 28, 2025

Question

Find the acute angle between the planes $\mathbf{r} \cdot (\hat{i} - 2\hat{j} - 2\hat{k}) = 1$ and $\mathbf{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 0$.

Given Information

Let vector $\mathbf{n_1}$ be:

$$\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \tag{1}$$

Let vector $\mathbf{n_2}$ be:

$$\begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} \tag{2}$$

Formula

The formula to calculate the angle between the two planes is

$$\theta = \frac{\pi}{2} - \cos^{-1}\left(\frac{\mathbf{n_1}^T\mathbf{n_2}}{|\mathbf{n_1}||\mathbf{n_2}|}\right) = \sin^{-1}\left(\frac{\mathbf{n_1}^T\mathbf{n_2}}{|\mathbf{n_1}||\mathbf{n_2}|}\right) \tag{3}$$

Solution

Substituting $\mathbf{P}, \mathbf{Q}$ in this formula :

$$= \sin^{-1}\left(\frac{\begin{pmatrix}1\\-2\\-2\end{pmatrix}^{T}\begin{pmatrix}3\\-6\\2\end{pmatrix}}{\left|\begin{pmatrix}1\\-2\\-2\end{pmatrix}\right|\left|\begin{pmatrix}3\\-6\\2\end{pmatrix}\right|}\right) = \sin^{-1}\left(\frac{19}{|3||7|}\right) = \sin^{-1}\left(\frac{11}{21}\right)$$

This is 31.589067572339°

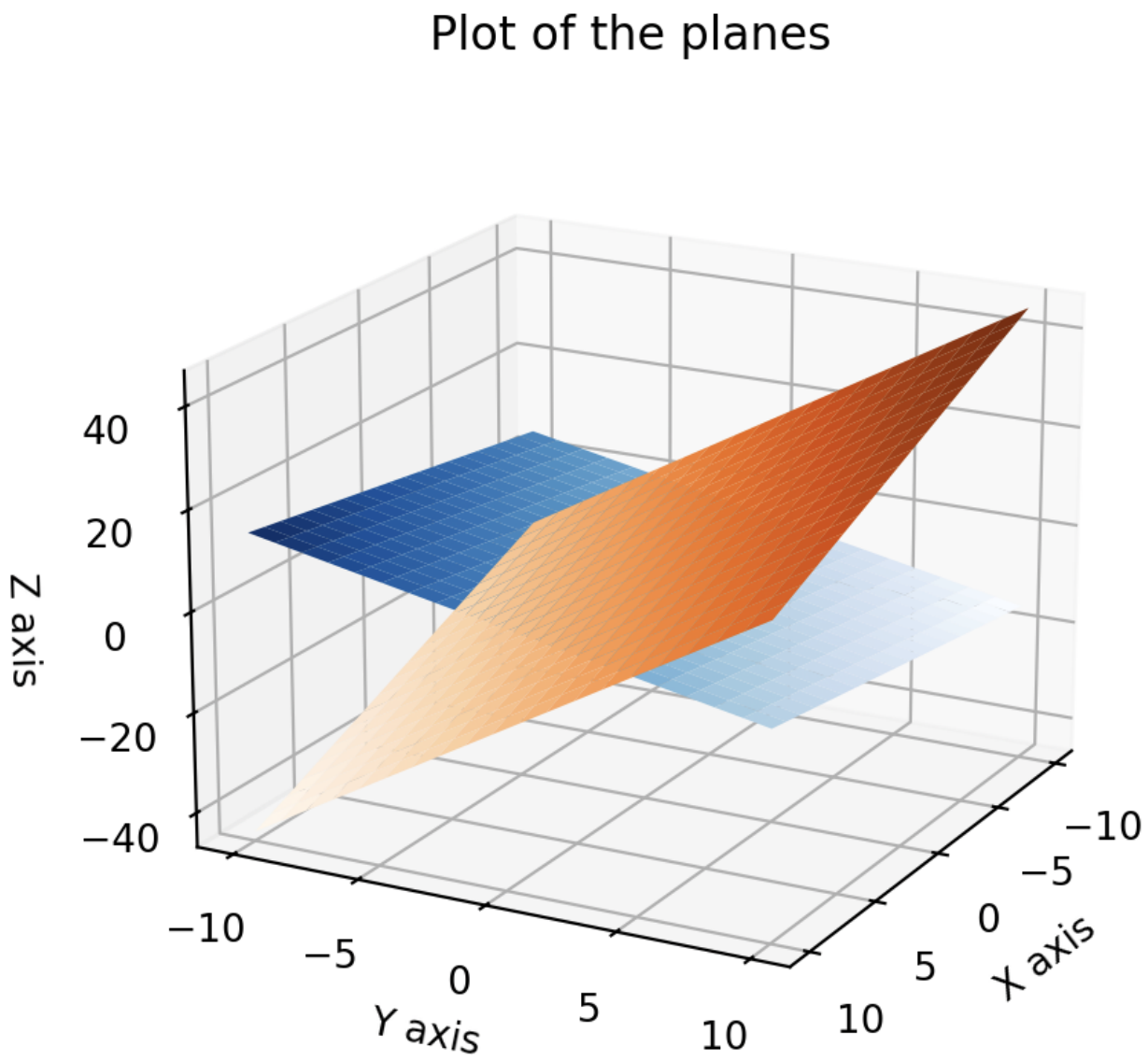
Python Code

```
import numpy as np
import matplotlib.pyplot as plt
import sys
import numpy.linalg as LA
import math

vec1 = np.array([1,-2,-2])
vec2 = np.array([3,-6,2])

dot_product = vec1@vec2

norm1 = np.linalg.norm(vec1)
norm2 = np.linalg.norm(vec2)

cos = dot_product/(norm1*norm2)
angle = math.asin(cos)
print(angle*180/3.1415)
```

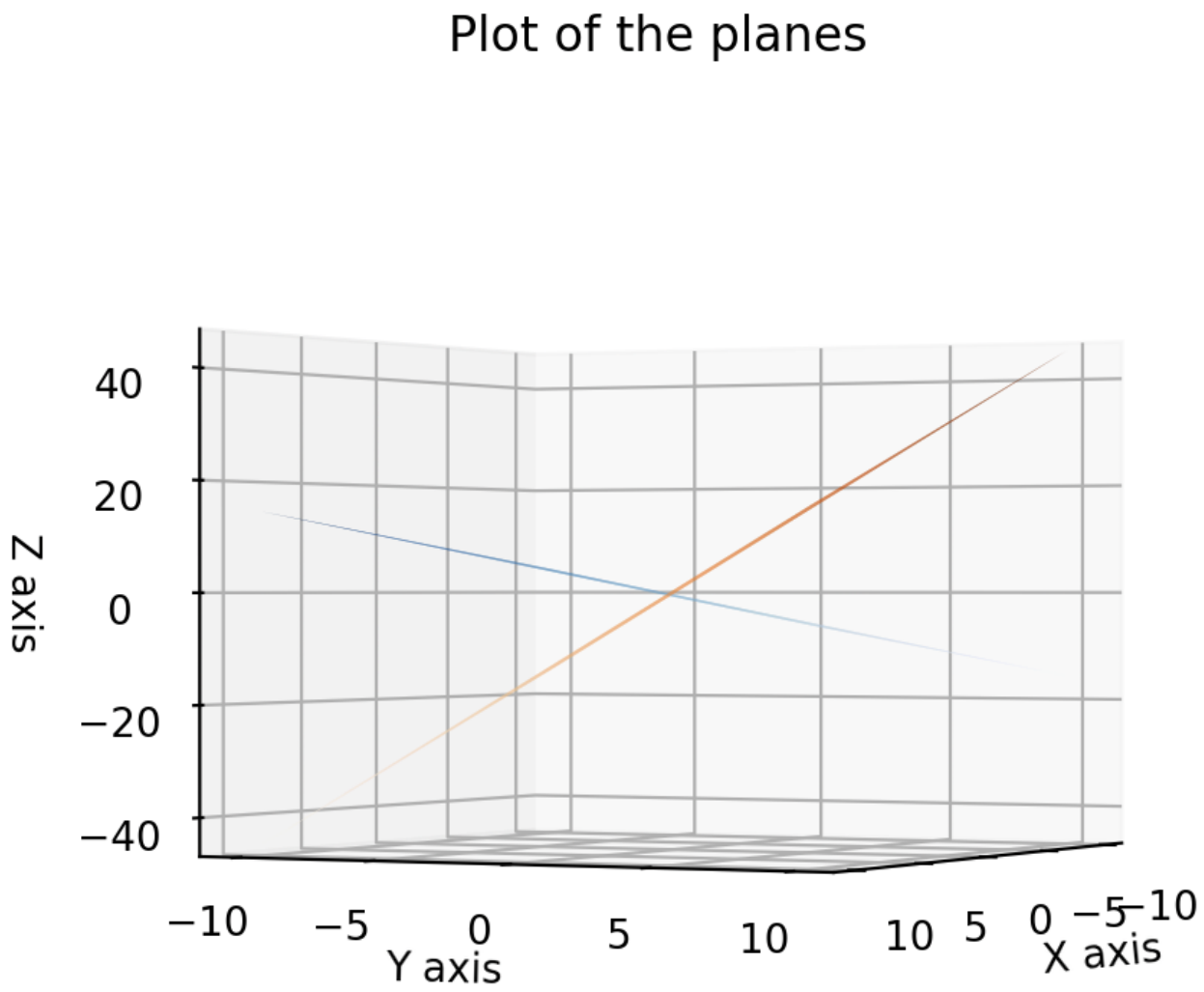
Python Code

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

x = np.linspace(-10, 10, 20)
y = np.linspace(-10, 10, 20)
X, Y = np.meshgrid(x, y)

Z = (X-2*Y- 1)/2
z1 = (6*Y - 3*X)/2

ax.plot_surface(X, Y, Z, alpha=1, cmap='Blues')
ax.plot_surface(X, Y, z1, alpha=1, cmap='Oranges')

ax.set_xlabel('X axis')
ax.set_ylabel('Y axis')
ax.set_zlabel('Z axis')
ax.set_title('Plot of the planes')
plt.show()
```

Plot 1


Plot of the planes
Z axis
40
20
0
−20
−40
−10
−5
0
5
10
Y axis
−10
−5
0
5
10
X axis

Plot 2


Plot of the planes
Z axis
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0
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−40
−10
−5
0
5
10
Y axis
10
5
0
−5
−10
X axis

C Code

```c
#include<stdio.h>
#include<math.h>


float anglefinder(float x1, float y1, float z1, float x2, float
    y2, float z2){

float dot_product;
float mod1;
float mod2;
float cosval;
float angle;

dot_product = x1*x2 + y1*y2 + z1*z2;
```

C Code

```
mod1 = pow(x1,2) + pow(y1,2) + pow(z1,2);
mod2 = pow(x2,2) + pow(y2,2) + pow(z2,2);

mod1 = sqrt(mod1);
mod2 = sqrt(mod2);

cosval = dot_product/(mod1 * mod2);

angle= asin(cosval);

return angle;
}
```

Python and C Code

```
import numpy as np
import matplotlib.pyplot as plt
import sys
import ctypes

c_lib=ctypes.CDLL('./3c.so')

c_lib.anglefinder.argtypes = [ctypes.c_float, ctypes.c_float,
    ctypes.c_float, ctypes.c_float, ctypes.c_float, ctypes.
    c_float]
c_lib.anglefinder.restype = ctypes.c_float
```

Python and C Code

```
v1 = np.array([1,2,-2])
v2 = np.array([3,-6,2])

angle = c_lib.anglefinder(
    ctypes.c_float(v1[0]),
    ctypes.c_float(v1[1]),
    ctypes.c_float(v1[2]),
    ctypes.c_float(v2[0]),
    ctypes.c_float(v2[1]),
    ctypes.c_float(v2[2])
)

print(angle*180/3.1415)
```

Python and C Code

```
c_lib=ctypes.CDLL('./main.so')

# Define the argument types for the x function
c_lib.xfinder.argtypes = [ctypes.c_float, ctypes.c_float,ctypes.
    c_float, ctypes.c_float]
# Define the return type of the x function
c_lib.xfinder.restype = ctypes.c_float
# --- Define Points and Calculate 'm' using C function ---

v1 = np.array([7,6])
v2 = np.array([3,4])

xcoord = c_lib.xfinder(
    ctypes.c_float(v1[0]),
    ctypes.c_float(v1[1]),
    ctypes.c_float(v2[0]),
    ctypes.c_float(v2[1])
)
```

Python Code

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ax.set_zlabel('Z axis')
ax.set_title('Plot of the planes')
plt.show()
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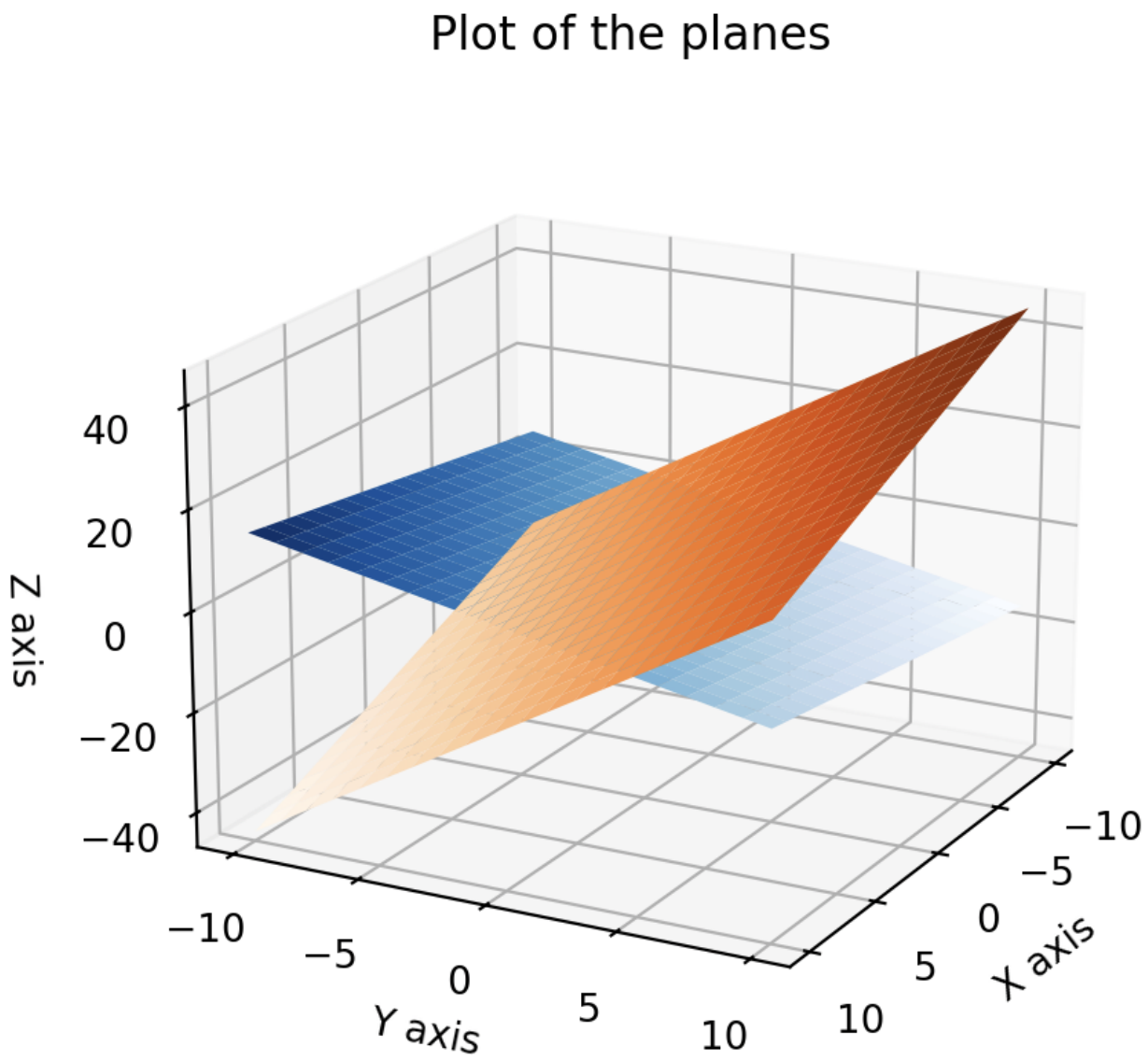
Plot 1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−10
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5
10
Y axis
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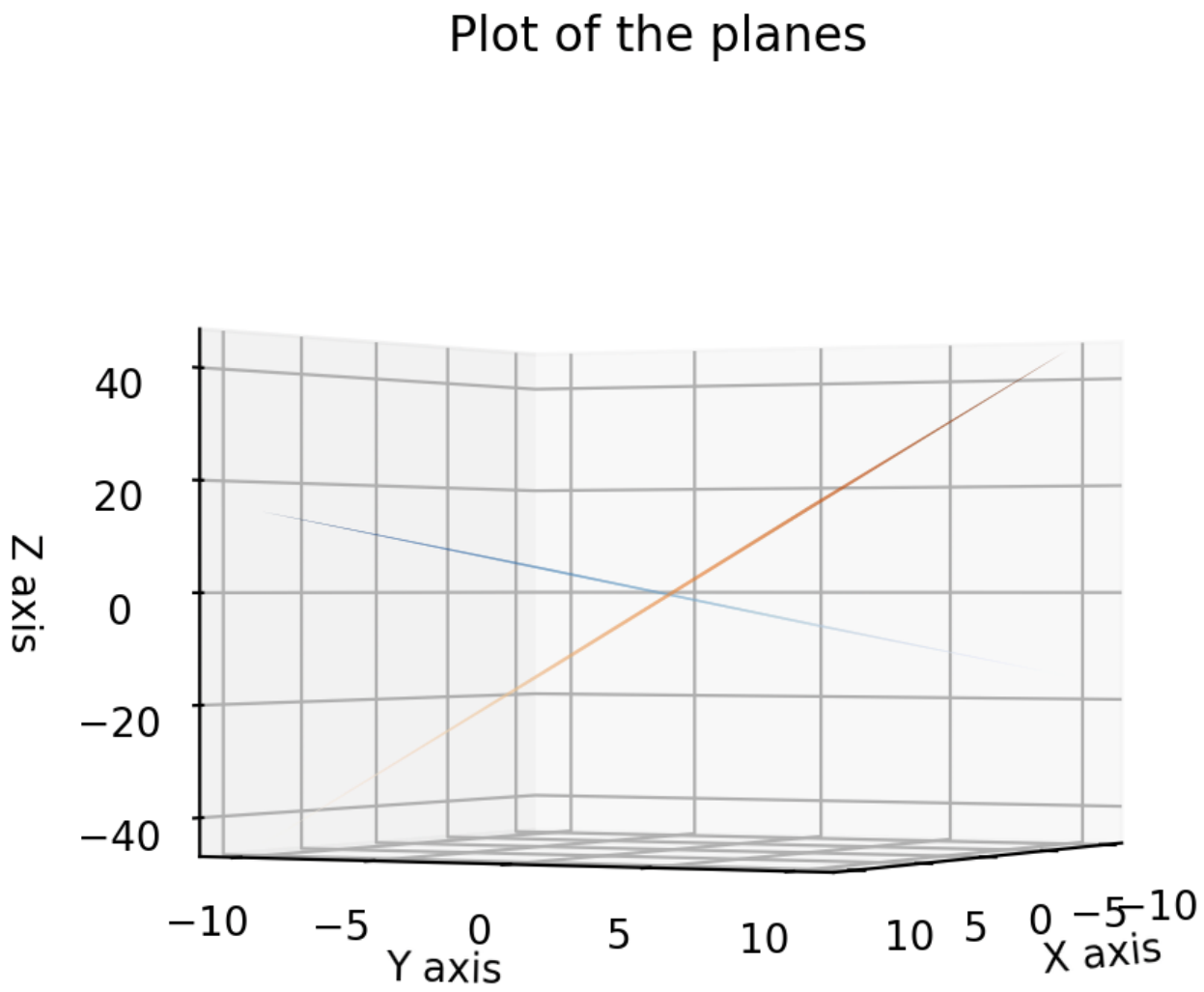
−5
0
5
10
X axis

Plot 2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