Question

Problem

If the distance between the points $(3, -5)$ and $(x, -5)$ is 15 units, then find the values of x using matrices.

Solution Process

Step 1: Represent points

$$\mathbf{A} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} x \\ -5 \end{bmatrix}$$

Step 2: Distance formula using matrices

$$d^2 = (\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{A})$$

Step 3: Substitution

$$d^2 = \begin{bmatrix} x-3 \\ 0 \end{bmatrix}^T \begin{bmatrix} x-3 \\ 0 \end{bmatrix} = (x-3)^2$$

Final Answer

Given $d = 15$, we get

$$(x-3)^2 = 225$$

$$x - 3 = \pm 15$$

$$\therefore \quad x = 18 \quad \text{or} \quad x = -12$$

C Code

```c
#include <stdio.h>
#include <math.h>

// Distance formula
double distance(double A[2], double B[2]) {
    return sqrt((B[0]-A[0])*(B[0]-A[0]) +
                (B[1]-A[1])*(B[1]-A[1]));
}

int main() {
    double A[2] = {3, -5};
    double B[2];
    double d;

    printf("Enter x-coordinate of B: ");
    scanf("%lf", &B[0]);
    B[1] = -5;

    d = distance(A, B);
```

Python Code (1/2)

```python
import matplotlib.pyplot as plt

# Given point
A = (3, -5)
d = 15 # distance

# Solve for x
x1 = 18
x2 = -12

B1 = (x1, -5)
B2 = (x2, -5)

print("Solutions for x:", x1, "and", x2)
```

Python Code (2/2 - Plotting)

```
plt.figure(figsize=(8,6))
plt.axhline(0, color='black')
plt.axvline(0, color='black')

# Plot points
plt.scatter(*A, color='black', s=80)
plt.text(A[0]+0.3, A[1]+0.5, "A(3,-5)")

plt.scatter(*B1, color='blue', s=80)
plt.text(B1[0]+0.3, B1[1]+0.5, "B1(18,-5)")

plt.scatter(*B2, color='red', s=80)
plt.text(B2[0]+0.3, B2[1]+0.5, "B2(-12,-5)")

# Plot lines
plt.plot([A[0], B1[0]], [A[1], B1[1]], 'b-', lw=2, label="15
    units")
plt.plot([A[0], B2[0]], [A[1], B2[1]], 'r--', lw=2, label="15
    units")
```

Graphical Representation