1.4.19

EE25BTECH11004 - Aditya Appana

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Question

Find the acute angle between the planes $\mathbf{r}\cdot(\hat{i}-2\hat{j}-2\hat{k}) = 1$ and $\mathbf{r}\cdot(3\hat{i}-6\hat{j}+2\hat{k}) = 0$

Solution

Let the normal vectors be

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \tag{1}$$

$$\mathbf{n_2} = \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} \tag{2}$$

The formula to calculate the angle between the two planes is

$$\theta = \frac{\pi}{2} - \cos^{-1}\left(\frac{\mathbf{n_1}^T\mathbf{n_2}}{|\mathbf{n_1}||\mathbf{n_2}|}\right) = \sin^{-1}\left(\frac{\mathbf{n_1}^T\mathbf{n_2}}{|\mathbf{n_1}||\mathbf{n_2}|}\right) \tag{3}$$

Substituting $\mathbf{n_1}, \mathbf{n_2}$ in this formula :

$$\theta = \sin^{-1}\left(\frac{\begin{pmatrix}1\\-2\\-2\end{pmatrix}^T\begin{pmatrix}3\\-6\\2\end{pmatrix}}{|\begin{pmatrix}1\\-2\\-2\end{pmatrix}||\begin{pmatrix}3\\-6\\2\end{pmatrix}|}\right) = \sin^{-1}\left(\frac{19}{|3||7|}\right) = \sin^{-1}\left(\frac{11}{21}\right) \tag{4}$$

This is 31.58906757233914°

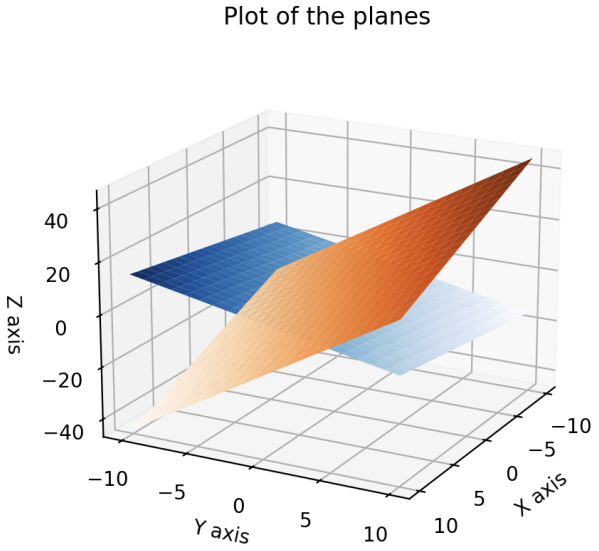

Figure 1: Plot

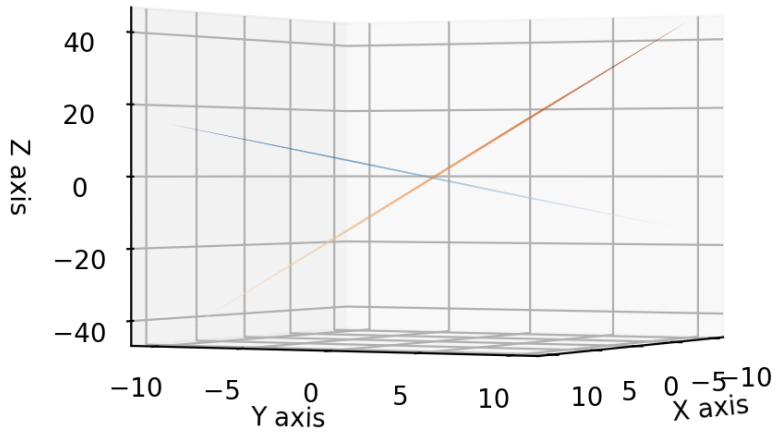

Figure 2: Plot