## 1.2.23

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## Question

Represent graphically a displacement of 40 km, 30° west of south.

## Solution (Matrix Method)

Coordinate convention. Let the x-axis point East and the y-axis point North. Thus:

East 
$$\equiv +x$$
, West  $\equiv -x$ , North  $\equiv +y$ , South  $\equiv -y$ .

**Rotation matrix.** For a counterclockwise rotation by an angle  $\theta$ , use

$$R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}.$$

**Direction setup.** The unit direction for *South* is the column matrix

$$\mathbf{s} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

"30° west of south" means rotate the south direction towards west by 30°. this is a anti-clockwise rotation of 330°,

Hence the required unit direction column is

$$\mathbf{u} = R(330^{\circ})\,\mathbf{s} = \begin{bmatrix} \cos 330^{\circ} & -\sin 330^{\circ} \\ \sin 330^{\circ} & \cos 330^{\circ} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\sin 30^{\circ} \\ -\cos 30^{\circ} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix}.$$

**Displacement column.** With magnitude  $40 \,\mathrm{km}$ , the displacement (as a  $2 \times 1 \,\mathrm{matrix}$ ) is

$$\mathbf{d} = 40 \,\mathbf{u} = 40 \begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} -20 \\ -20\sqrt{3} \end{bmatrix} \text{ (km)}.$$

So the endpoint relative to the origin is

$$(x,y) = (-20, -20\sqrt{3}) \text{ km},$$

which lies in the third quadrant (west and south components), consistent with the description.

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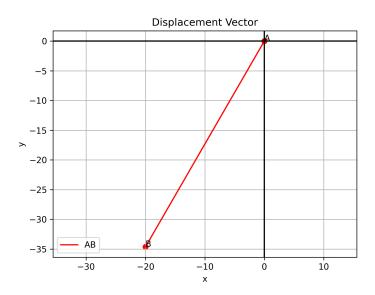


Figure 1: Displacement vector: 40 km,  $30^\circ$  west of south