1.5.15

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Question The midpoint of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(1, 2a+1)$. Findthe values of a and b.

Solution:

Let us solve the given equation theoretically and then verify the solution computationally. From the given data,

$$\mathbf{A} = \begin{pmatrix} 2a \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 3b \end{pmatrix} \tag{1}$$

Let the midpoint of points A and B be C. where,

$$\mathbf{C} = \begin{pmatrix} 1 \\ 2a + 1 \end{pmatrix} \tag{2}$$

We know that the midpoint formula for the points A and B is

$$\mathbf{C_x} = \frac{\mathbf{A_x} + \mathbf{B_x}}{2} \tag{3}$$

Where $C_X, A_X \; and \; B_X$ are x coordinates of point C,A and B

And also A,B and C lies in the same line so they are collinear. So,

$$rank(C - A \quad B - A) = 1 \tag{4}$$

$$rank\begin{pmatrix} 1 - 2a & -2 - 2a \\ 2a - 3 & 3b - 4 \end{pmatrix} = 1 \tag{5}$$

From eq.3:

$$\mathbf{C_x} = \frac{2a - 2}{2} \tag{6}$$

$$1 = \frac{2a - 2}{2} \tag{7}$$

$$1 = a - 1 \tag{8}$$

$$a = 2 \tag{9}$$

Now substituiting the value of a in Eq.5, we get:

$$rank\begin{pmatrix} 1 - 2(2) & -2 - 2(2) \\ 2(2) - 3 & 3b - 4 \end{pmatrix} = 1 \tag{10}$$

$$rank\begin{pmatrix} -3 & -6 \\ 1 & 3b - 4 \end{pmatrix} = 1 \tag{11}$$

By applying row operation for the matrix
$R_2 \longrightarrow 3R_2 + R_1$
We get:

$$\begin{pmatrix} C - A & B - A \end{pmatrix} = \begin{pmatrix} -3 & -6 \\ 0 & 9b - 18 \end{pmatrix} \tag{12}$$

For the rank to be 1, the second row must be a zero vector. Therefore:

$$9b - 18 = 0 \tag{13}$$

$$9b = 18 \tag{14}$$

$$b = 2 \tag{15}$$

Therefore the final values of a and b are:

$$a = 2 \ and \ b = 2 \tag{16}$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.