1.5.14

EE25BTECH11026-Harsha

Question:
Points P and Q trisect the line segment joining the points A $(-2, 0)$ and B$(0, 8)$ such that P is nearer to A. Find the coordinates of points P and Q.
Solution:
Let us solve the given equation theoretically and then verify the solution computationally
According to the question,
Let the vectors $\mathbf{P}$ and $\mathbf{Q}$ be

$$\mathbf{P} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \tag{0.1}$$

Given the points,

$$\mathbf{A} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 0 \\ 8 \end{pmatrix} \tag{0.2}$$

we can use the internal division formula to find the vectors $\mathbf{P}$ and $\mathbf{Q}$.
Internal division formula for a vector R which divides the line formed by vectors A and B in the ratio m:n is given by

$$\mathbf{R} = \frac{m\mathbf{B} + n\mathbf{A}}{m + n} \tag{0.3}$$

To find vector $\mathbf{P}$, as it is near the point A, it divides the line formed by line A and B in ratio 1:2. Therefore,

$$\mathbf{P} = \frac{2 \times \begin{pmatrix} -2 \\ 0 \end{pmatrix} + 1 \times \begin{pmatrix} 0 \\ 8 \end{pmatrix}}{1 + 2} \tag{0.4}$$

$$\mathbf{P} = \begin{pmatrix} \frac{-4}{3} \\ \frac{8}{3} \end{pmatrix} \tag{0.5}$$

To find vector $\mathbf{Q}$, as it is near the point B, it divides the line formed by line A and B in ratio 2:1.Therefore,

$$\mathbf{Q} = \frac{1 \times \begin{pmatrix} -2 \\ 0 \end{pmatrix} + 2 \times \begin{pmatrix} 0 \\ 8 \end{pmatrix}}{2+1} \tag{0.6}$$

$$\mathbf{Q} = \begin{pmatrix} \frac{-2}{3} \\ \frac{16}{3} \end{pmatrix} \tag{0.7}$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.