2.5.19

ee25btech11056 - Suraj.N

Question : Find the value of p for which the lines $\frac{1-x}{3} = \frac{2y-14}{2p} = \frac{z-3}{2}$ and $\frac{1-x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular.

Solution:

Symbol	Line
A	$\frac{1-x}{3} = \frac{2y-14}{2p} = \frac{z-3}{2}$
B	$\frac{1-x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$

Table : Lines

These lines can also be written in the vector form $\mathbf{x} = \mathbf{h} + k\mathbf{m}$.

$$\text{Line A:} \quad \mathbf{x} = \begin{pmatrix} 1 \\ 7 \\ 3 \end{pmatrix} + k_1 \begin{pmatrix} -3 \\ p \\ 2 \end{pmatrix}$$

$$\text{Line B:} \quad \mathbf{x} = \begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix} + k_2 \begin{pmatrix} -3p \\ 1 \\ -5 \end{pmatrix}$$

Hence, the direction vectors are

$$\mathbf{m_1} = \begin{pmatrix} -3 \\ p \\ 2 \end{pmatrix}, \qquad \mathbf{m_2} = \begin{pmatrix} -3p \\ 1 \\ -5 \end{pmatrix}.$$

For the lines to be perpendicular, we require $\mathbf{m_1}^\top \mathbf{m_2} = 0$.

$$\begin{aligned} \mathbf{m_1}^\top \mathbf{m_2} &= \begin{pmatrix} -3 & p & 2 \end{pmatrix} \begin{pmatrix} -3p \\ 1 \\ -5 \end{pmatrix} \\ &= (-3)(-3p) + p(1) + 2(-5) \\ &= 9p + p - 10 \\ &= 10p - 10. \end{aligned}$$

Thus,

$$10p - 10 = 0 \;\Rightarrow\; p = 1.$$

Final answer: $p = 1$.

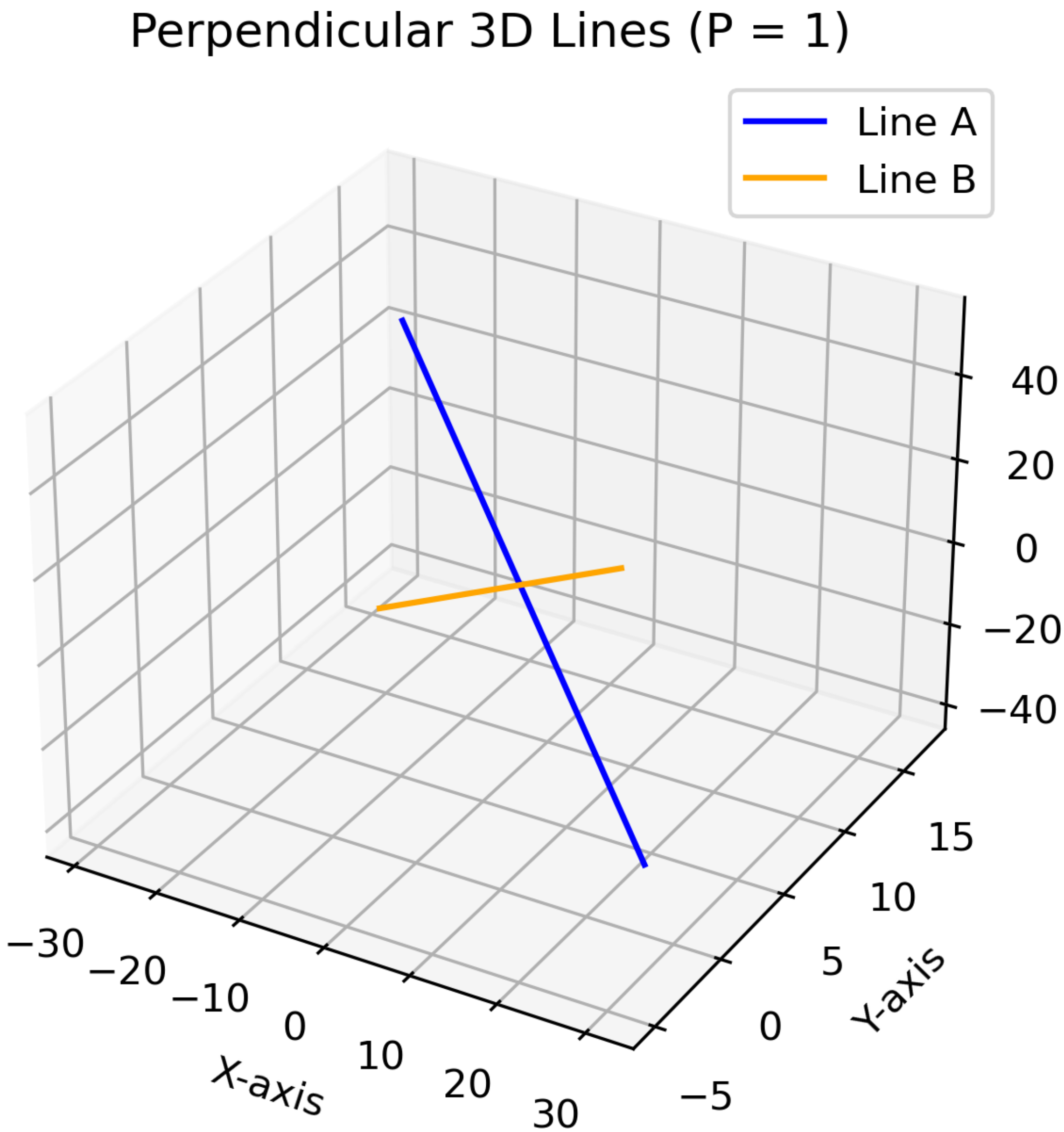

Fig : Lines A and B