

2.2.5.31

ee25btech11063-vejith

Question:

If two vertices of an equilateral triangle are (3,0) and (6,0), find the third vertex

Solution

vector	Name
$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$	Vector A
$\begin{pmatrix} 6 \\ 0 \end{pmatrix}$	Vector B
$\begin{pmatrix} x \\ y \end{pmatrix}$	Vector C

TABLE 0: Variables Used

$$\text{The vector joining from } \mathbf{A} \text{ to } \mathbf{B} \text{ is given by } \mathbf{B} - \mathbf{A} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (1)$$

$$\Rightarrow \mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}. \quad (2)$$

(3)

An equilateral triangle can be obtained by rotating $\mathbf{B} - \mathbf{A}$ by \mathbf{A} about $+60^\circ$ or -60° . The rotation matrix \mathbf{P} at angle θ is defined as

$$\mathbf{P}(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (4)$$

(5)

$$\mathbf{P}(60^\circ) = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \quad \mathbf{P}(-60^\circ) = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \quad (6)$$

(7)

Apply $\mathbf{P}(60^\circ)$ or $\mathbf{P}(-60^\circ)$ to $\mathbf{B} - \mathbf{A}$ and add it to \mathbf{A} to get \mathbf{C}

$$\mathbf{C} = \mathbf{A} + \mathbf{P}(60^\circ)(\mathbf{B} - \mathbf{A}) \quad \text{or} \quad \mathbf{C} = \mathbf{A} + \mathbf{P}(-60^\circ)(\mathbf{B} - \mathbf{A}) \quad (8)$$

$$\mathbf{P}(60^\circ) \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix} \quad \text{or} \quad \mathbf{P}(-60^\circ) \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ -\frac{3\sqrt{3}}{2} \end{pmatrix} \quad (9)$$

$$\mathbf{C} = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix} \quad \text{or} \quad \mathbf{C} = \begin{pmatrix} \frac{3}{2} \\ -\frac{3\sqrt{3}}{2} \end{pmatrix} \quad (10)$$

