Matrices in Geometry - 1.5.25

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Problem Statement

In what ratio does the point $\mathbf{R} = \begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$ divide the line segment joining the points $\mathbf{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$? Also find the value of y.

Solution

$\mathbf{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and a point $\mathbf{R} = \begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$ on PQ.
Let R divide PQ internally in the ratio $k : 1$.
Therefore, they are defined to be collinear if rank of the collinearity matrix is 1

$$\text{Collinearity matrix is } \begin{pmatrix} \mathbf{P} - \mathbf{R} & \mathbf{Q} - \mathbf{R} \end{pmatrix}^{\top} = 1$$

$$\mathbf{P} - \mathbf{R} = \begin{pmatrix} \frac{-2}{11} \\ -y - 2 \end{pmatrix}$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} \frac{9}{11} \\ 7 - y \end{pmatrix}$$

$$\implies \text{rank} \begin{pmatrix} \frac{-2}{11} & -y - 2 \\ \frac{9}{11} & 7 - y \end{pmatrix} = 1$$

Solution

$$\begin{pmatrix} \frac{-2}{11} & -2-y \\ \frac{9}{11} & 7-y \end{pmatrix} \xrightarrow{R_2 \to R_2 + \frac{9}{2} R_1} \begin{pmatrix} \frac{-2}{11} & -2-y \\ 0 & \dfrac{-11-4y}{2} \end{pmatrix}$$

for rank of this matrix to be 1, all the elements in the lower row have to be zero

$$\therefore -11-4y = 0 \implies y = \frac{-4}{11}$$

We know that k is the ratio in which $\mathbf{R}$ divides $\mathbf{P}$ and $\mathbf{Q}$,

$$\mathbf{R} = \frac{k\mathbf{Q}+\mathbf{P}}{1+k}$$

$$k\left(\mathbf{R}-\mathbf{Q}\right) = \mathbf{P}-\mathbf{R}$$

Solution

$$\implies k = \frac{(\mathbf{P}-\mathbf{R})^\top (\mathbf{R}-\mathbf{Q})}{\|\mathbf{R}-\mathbf{Q}\|^2}$$

$$(\mathbf{P}-\mathbf{R})^\top = \begin{pmatrix} \frac{-2}{11} & \frac{-18}{11} \end{pmatrix}$$

$$(\mathbf{R}-\mathbf{Q}) = \begin{pmatrix} \frac{-9}{11} \\ \frac{-81}{11} \end{pmatrix}$$

$$\|\mathbf{R}-\mathbf{Q}\|^2 = \left(\mathbf{R}-\mathbf{Q}\right)^\top \left(\mathbf{R}-\mathbf{Q}\right)$$

$$= \begin{pmatrix} \frac{-9}{11} & \frac{-81}{11} \end{pmatrix} \begin{pmatrix} \frac{-9}{11} \\ \frac{-81}{11} \end{pmatrix} = \frac{81}{121} + \frac{6561}{121} = \frac{6642}{121}$$

Solution

$$\therefore k = \frac{\begin{pmatrix} \frac{-2}{11} & \frac{-18}{11} \end{pmatrix} \begin{pmatrix} \frac{-9}{11} \\ \frac{-81}{11} \end{pmatrix}}{\frac{6642}{121}}$$

$$\implies k = \frac{\dfrac{18}{121} + \dfrac{1458}{121}}{\dfrac{6642}{121}}$$

$$\implies k = \frac{1476}{6624} = \frac{2}{9}$$

Final Answer

Hence, the final answer is $\boxed{k = \frac{2}{9}}$ and $\boxed{y = \frac{-4}{11}}$