

# 1.6.7

EE25BTECH11057 - Rushil Shanmukha Srinivas

**Question:** Find a relation between  $x$  and  $y$  if the points  $(x,y)$ ,  $(1,2)$  and  $(7,0)$  are collinear.

**Solution:** Let the three points be  $\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ,  $\mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$ .

For collinearity,

$$\text{rank}((\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T) = 1. \quad (0.1)$$

Now,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1-x \\ 2-y \end{pmatrix}, \quad \mathbf{C} - \mathbf{A} = \begin{pmatrix} 7-x \\ -y \end{pmatrix}. \quad (0.2)$$

So the matrix is

$$\mathbf{M} = (\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T = \begin{pmatrix} 1-x & 2-y \\ 7-x & -y \end{pmatrix}. \quad (0.3)$$

*Row Reduction*

Step 1: Start with

$$\mathbf{M} = \begin{pmatrix} 1-x & 2-y \\ 7-x & -y \end{pmatrix}. \quad (0.4)$$

Step 2: Eliminate the first entry of the second row:

$$R_2 \rightarrow R_2 - \frac{7-x}{1-x} R_1 \quad (\text{assuming } x \neq 1). \quad (0.5)$$

$$\begin{pmatrix} 1-x & 2-y \\ 7-x & -y \end{pmatrix} \rightarrow \begin{pmatrix} 1-x & 2-y \\ 0 & -y - \frac{7-x}{1-x}(2-y) \end{pmatrix}. \quad (0.6)$$

*Rank Condition*

For  $\text{rank}(\mathbf{M}) = 1$ , the second row must vanish:

$$-y - \frac{7-x}{1-x}(2-y) = 0. \quad (0.7)$$

Multiply through by  $(1-x)$ :

$$-y(1-x) - (7-x)(2-y) = 0. \quad (0.8)$$

Expand:

$$-y + xy - (14 - 2x - 7y + xy) = 0. \quad (0.9)$$

$$-y + xy - 14 + 2x + 7y - xy = 0. \quad (0.10)$$

$$2x + 6y - 14 = 0. \quad (0.11)$$

Thus, the condition for collinearity is

$$\boxed{x + 3y = 7}. \quad (0.12)$$

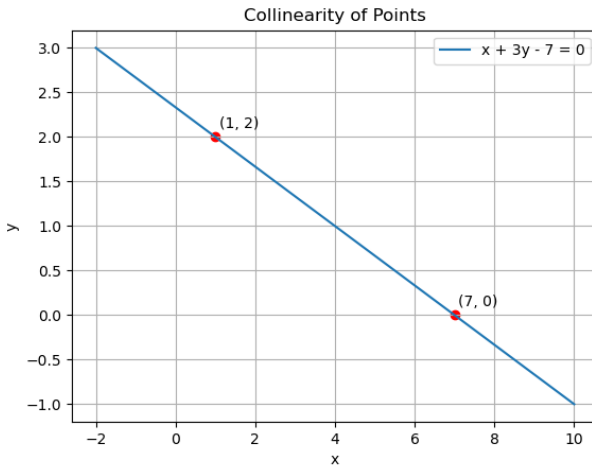


Fig. 0.1