1.5.17

Jnanesh Sathisha Karmar- EE25BTECH11029

QuestionThe midpoint of the line segment joining $\mathbf{A}\,(2a, 4)$ and $\mathbf{B}\,(-2, 3b)$ is $(1, 2a + 1)$. Find the values of a and b.

SolutionThe midpoint M of line segment AB, with $\mathbf{A}\,(x_1, y_1)$ and $\mathbf{B}\,(x_2, y_2)$, is:

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}}{2} \tag{1}$$

Given details:

$$\mathbf{A} = \begin{pmatrix} 2a \\ 4 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -2 \\ 3b \end{pmatrix} \mathbf{M} = \begin{pmatrix} 1 \\ 2a + 1 \end{pmatrix} \tag{2}$$

Substituting the points:

$$\frac{\begin{pmatrix} 2a \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 3b \end{pmatrix}}{2} = \begin{pmatrix} \frac{2a-2}{2} \\ \frac{(4+3b)}{2} \end{pmatrix} \tag{3}$$

Equating coordinates, we get two equations:

$$\frac{2a - 2}{2} = 1 \tag{4}$$

$$\frac{4 + 3b}{2} = 2a + 1 \tag{5}$$

Using (3)

$$a = 2 \tag{6}$$

Using (3) and (6)

$$b = 2 \tag{7}$$

Therefore Values of a and b are both 2

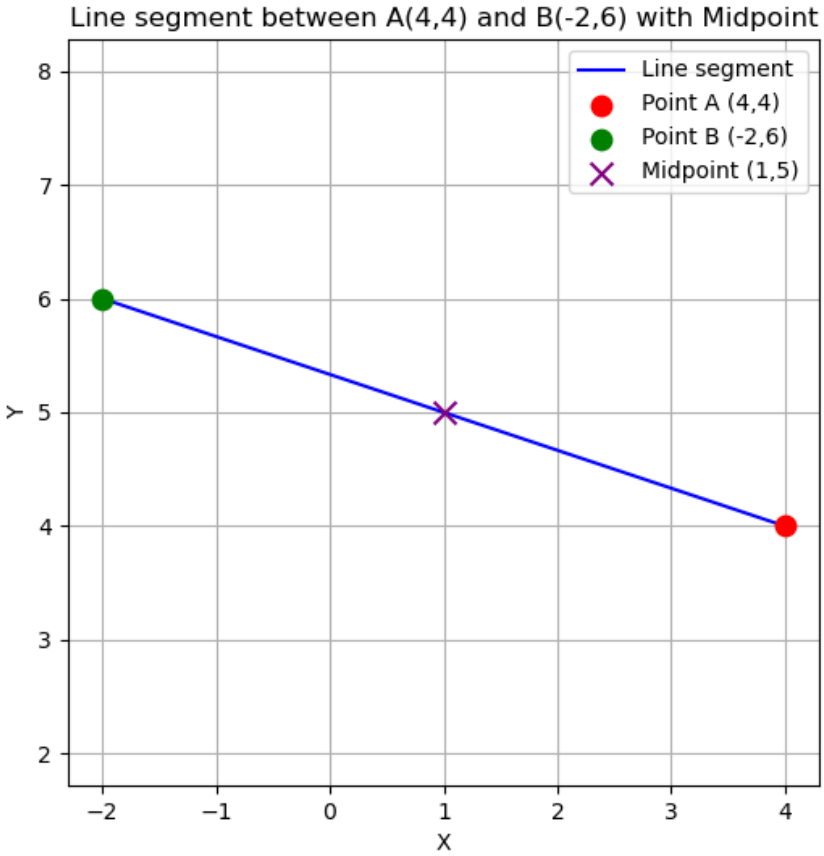

Fig. 0. linesegment with 2 points and its midpoint