Matgeo-q.1.4.14

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August 29, 2025

Question

Points $A(-6, 10)$, $B(-4, 6)$ and $C(3, -8)$ are collinear such that

$$AB = \frac{2}{9}AC.$$

Solution

Given: $\mathbf{A} = \begin{pmatrix} -6 \\ 10 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 3 \\ -8 \end{pmatrix}$.

Assume: $\mathbf{B}$ divides $\mathbf{AC}$ in the ratio $k : 1$.

$$\mathbf{B} = \frac{k\mathbf{C} + \mathbf{A}}{k+1}$$

Compute k:

$$\begin{aligned} k(\mathbf{B} - \mathbf{C}) &= \mathbf{A} - \mathbf{B} \\ \Rightarrow \quad k &= \frac{(\mathbf{A} - \mathbf{B})^\top (\mathbf{B} - \mathbf{C})}{\|\mathbf{B} - \mathbf{C}\|^2} \end{aligned}$$

$$\begin{aligned} \mathbf{A} - \mathbf{B} &= \begin{pmatrix} -6 \\ 10 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \\ \mathbf{B} - \mathbf{C} &= \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ -8 \end{pmatrix} = \begin{pmatrix} -7 \\ 14 \end{pmatrix}, \\ \|\mathbf{B} - \mathbf{C}\|^2 &= (-7)^2 + 14^2 = 245. \end{aligned}$$

Solution

Finish k:

$$k = \frac{\begin{pmatrix} -2 \\ 4 \end{pmatrix}^{\top} \begin{pmatrix} -7 \\ 14 \end{pmatrix}}{245} = \frac{(-2)(-7) + (4)(14)}{245} = \frac{70}{245} = \boxed{\tfrac{2}{7}}$$

Ratios:

$$AB : BC = \boxed{2:7}, \qquad \frac{AB}{AC} = \frac{2}{2+7} = \boxed{\tfrac{2}{9}}.$$

Check:

$$\begin{aligned} \mathbf{B} &= \frac{7\mathbf{A} + 2\mathbf{C}}{9} = \frac{7\begin{pmatrix} -6 \\ 10 \end{pmatrix} + 2\begin{pmatrix} 3 \\ -8 \end{pmatrix}}{9} \\ &= \frac{\begin{pmatrix} -42 \\ 70 \end{pmatrix} + \begin{pmatrix} 6 \\ -16 \end{pmatrix}}{9} = \frac{\begin{pmatrix} -36 \\ 54 \end{pmatrix}}{9} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}. \end{aligned}$$

Graphical Representation

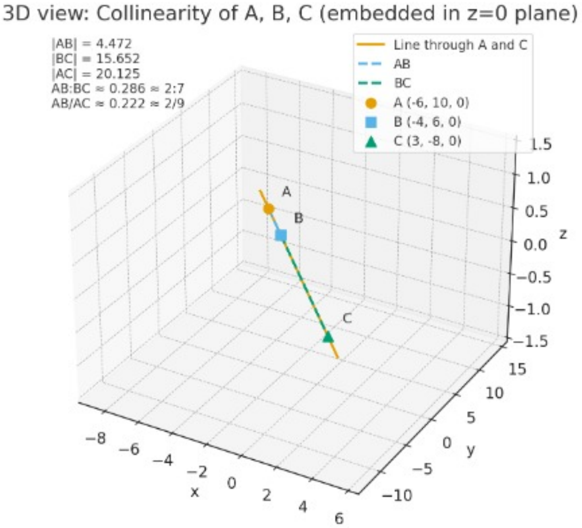

Figure: 3D view (embedded in $z = 0$ plane) confirming collinearity