

1.6.18

AI25BTECH11003 - Bhavesh Gaikwad

Question: Prove that points A(2,1), B(0,5) and C(-1,2) are not collinear.

Solution:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 0 - 2 \\ 5 - 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \quad \mathbf{C} - \mathbf{A} = \begin{pmatrix} -1 - 2 \\ 2 - 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$\mathbf{M} = (\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T = \begin{pmatrix} -2 & -3 \\ 4 & 1 \end{pmatrix}$$

Row-reduce to compute the rank:

$$\begin{pmatrix} -2 & -3 \\ 4 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + 2R_1} \begin{pmatrix} -2 & -3 \\ 0 & -5 \end{pmatrix}$$

The echelon form has two nonzero rows, hence $\text{rank}(\mathbf{M}) = 2 \neq 1$

Therefore, The points A(2, 1), B(0, 5) and C(-1, 2) are not collinear.

(0.1)

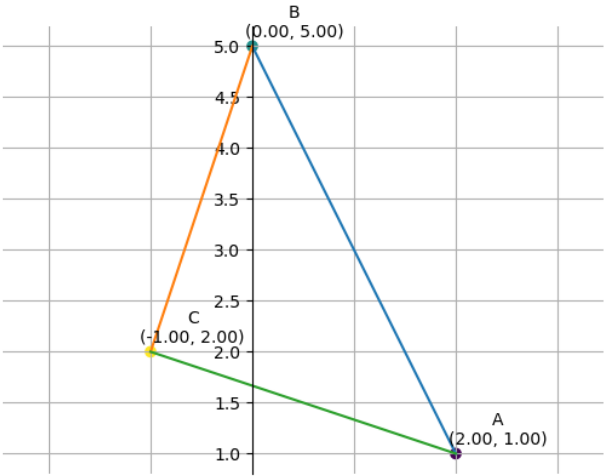


Fig. 0.1: Graph