

1.2.21

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QUESTION

The centroid of triangle $\triangle ABC$ is at the point $\mathbf{G} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. The coordinates of points \mathbf{A} and \mathbf{B} are given by:

$$\mathbf{A} = \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -1 \\ 7 \\ -6 \end{pmatrix}$$

Find the coordinates of point \mathbf{C} .

SOLUTION

The centroid \mathbf{G} of triangle $\triangle ABC$ is given by the formula:

$$\mathbf{G} = \frac{1}{3}(\mathbf{A} + \mathbf{B} + \mathbf{C})$$

Multiplying both sides by 3:

$$3\mathbf{G} = \mathbf{A} + \mathbf{B} + \mathbf{C}$$

Solving for \mathbf{C} :

$$\mathbf{C} = 3\mathbf{G} - \mathbf{A} - \mathbf{B}$$

Symbolic Expression

Let $\mathbf{A} = \mathbf{a}$, $\mathbf{B} = \mathbf{b}$, and $\mathbf{G} = \mathbf{g}$, then:

$$\mathbf{C} = 3\mathbf{g} - \mathbf{a} - \mathbf{b}$$

Numerical Substitution

Substitute the given values:

$$\mathbf{C} = 3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \\ -6 \end{pmatrix}$$

Compute step-by-step:

$$3\mathbf{G} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 0 \\ 8 \\ -4 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \\ -6 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

Final Answer

The coordinates of point **C** are:

$$\mathbf{C} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

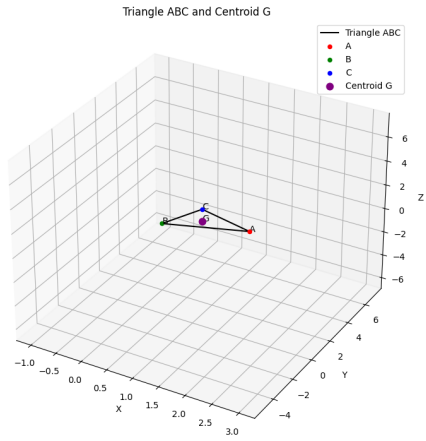


Fig. 0.1: 3D plot of triangle ABC and centroid G