1.5.33

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Question:
Find the ratio in which the Y-axis divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$. Also find the coordinates of the point of intersection.

Solution:
Let the given points be A and B

$$\mathbf{A} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

Let the Y-axis divide the line segment $\mathbf{AB}$ at point $\mathbf{P}$ in the ratio $k : 1$. Since $\mathbf{P}$ lies on Y-axis, let

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix}$$

The point $\mathbf{A}$, $\mathbf{B}$, $\mathbf{P}$ are collinear.

$$\implies \text{rank}\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{P} - \mathbf{A} \end{pmatrix} = 1 \tag{1}$$

$$\begin{pmatrix} -6 & -5 \\ 2 & y+6 \end{pmatrix} \xleftrightarrow{R_2 \to \frac{1}{3}R_1 + R_2} \begin{pmatrix} -6 & -5 \\ 0 & y+\frac{13}{3} \end{pmatrix} \tag{2}$$

The number of nonzero rows in the row reduced matrix (also known as *echelon form*) is defined as the rank. For above matrix to be of rank 1,

$$y + \frac{13}{3} = 0 \tag{3}$$
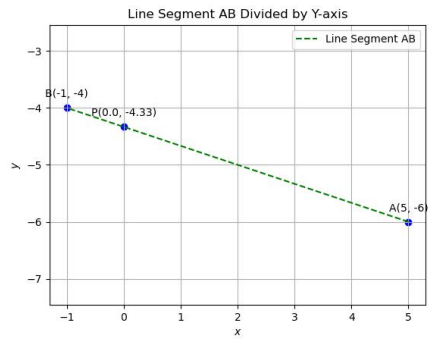
$$y = \frac{-13}{3} \tag{4}$$

$\therefore$ The coordinates of the point of intersection are

$$\mathbf{P} = \begin{pmatrix} 0 \\ \frac{-13}{3} \end{pmatrix}$$

Substituting the values of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{P}$,

$$k = \frac{\begin{pmatrix} 5 & \frac{-5}{3} \end{pmatrix}\begin{pmatrix} 1 \\ \frac{-1}{3} \end{pmatrix}}{\left\|\begin{pmatrix} 1 \\ \frac{-1}{3} \end{pmatrix}\right\|^2} = 5 \tag{5}$$

Thus, the ratio in which the point $\mathbf{P}$ divides the line segment $\mathbf{AB}$ is **5:1**.

Plot of Intersection of AB by Y-axis