1.4.19

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1 Question

Find the position vector of a point **R** which divides the line joining two points **P** and **Q** whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio **2:1**

(a) externally

(b) internally

2 Solution

Given vector **P** is

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \tag{1}$$

and vector **Q** is

$$\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \tag{2}$$

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We need to find the points which divide line segment **PQ** internally and externally in the ratio **2:1**.

Let the point which divides **PQ** internally be **R**.

Let the point which divides **PQ** externally be **S**.

The formula to calculate the coordinates of the point which divides a line segment internally in the ratio m:n is

$$\mathbf{R} = \frac{\frac{m}{n}\mathbf{P} + \mathbf{Q}}{\frac{m}{n} + 1} \tag{3}$$

and to calculate the coordinates of the point which divides a line segment externally in the ratio m:n is

$$\mathbf{S} = \frac{\frac{m}{n}\mathbf{P} - \mathbf{Q}}{\frac{m}{n} - 1} \tag{4}$$

Substituting $P\begin{pmatrix}1\\2\\-1\end{pmatrix}$ and $Q\begin{pmatrix}-1\\1\\1\end{pmatrix}$ in the first formula, we get

$$\mathbf{R} = \frac{2\begin{pmatrix}1\\2\\-1\end{pmatrix} + \begin{pmatrix}-1\\1\\1\end{pmatrix}}{\frac{2}{1} + 1} = \frac{\begin{pmatrix}2-1\\4+1\\-2+1\end{pmatrix}}{3} = \begin{pmatrix}1/3\\5/3\\-1/3\end{pmatrix} \tag{5}$$

Substituting $P\begin{pmatrix}1\\2\\-1\end{pmatrix}$ and $Q\begin{pmatrix}-1\\1\\1\end{pmatrix}$ in the second formula, we get

$$\mathbf{S} = \frac{2\begin{pmatrix}1\\2\\-1\end{pmatrix} - \begin{pmatrix}-1\\1\\1\end{pmatrix}}{\frac{2}{1} - 1} = \frac{\begin{pmatrix}2-(-1)\\4-1\\-2-1\end{pmatrix}}{1} = \begin{pmatrix}3\\3\\-3\end{pmatrix} \tag{6}$$

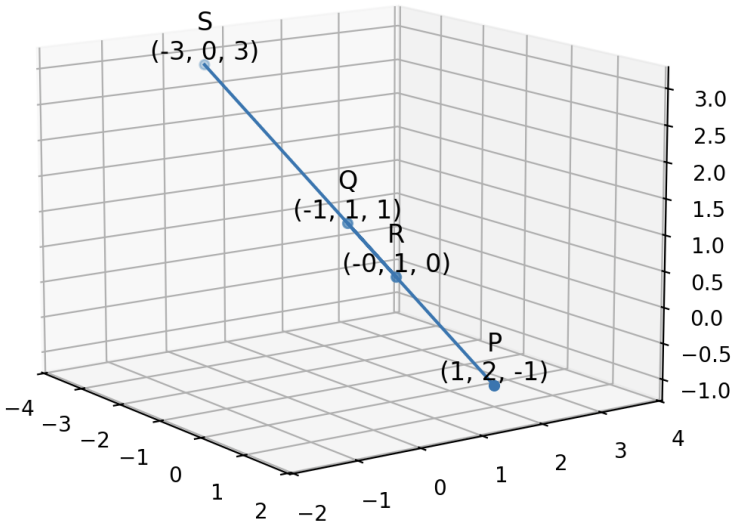

Figure 1: 3D Plot