1.5.21

EE25BTECH11033 - Kavin

Question:
Find the ratio in which $\mathbf{P}(4, m)$ divides the line segment joining the points $\mathbf{A}(2, 3)$ and $\mathbf{B}(6, -3)$. Hence, find m.

Solution:
Let the vector $\mathbf{P}$ be

$$\mathbf{P} = \begin{pmatrix} 4 \\ m \end{pmatrix}, \tag{1}$$

Given the points,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 6 \\ -3 \end{pmatrix} \tag{2}$$

The points A, P, B are collinear.

Points $\mathbf{A}, \mathbf{P}, \mathbf{B}$ are defined to be collinear if

$$\text{rank}\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix} = 1 \tag{3}$$

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} 2 \\ m - 3 \end{pmatrix} \tag{4}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ m - 3 & -6 \end{pmatrix} \tag{6}$$

$$R_2 \to 2R_2 + 3R_1 \implies \begin{pmatrix} 2 & 4 \\ 2m & 0 \end{pmatrix}$$

For rank 1, the second row must be zero:

$$2m = 0 \implies m = 0 \tag{7}$$

$$\therefore \mathbf{P} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

Section formula for a vector $\mathbf{P}$ which divides the line formed by vectors $\mathbf{A}$ and $\mathbf{B}$ in the ratio k:1 is given by

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \tag{8}$$

$$k(\mathbf{P}-\mathbf{B}) = \mathbf{A}-\mathbf{P} \tag{9}$$

$$\implies k = \frac{(\mathbf{A}-\mathbf{P})^{\top}(\mathbf{P}-\mathbf{B})}{\|\mathbf{P}-\mathbf{B}\|^2} \tag{10}$$

$$(\mathbf{A}-\mathbf{P})^{\top}(\mathbf{P}-\mathbf{B}) = \begin{pmatrix}-2 & 3\end{pmatrix}\begin{pmatrix}-2\\3\end{pmatrix} = 13 \tag{11}$$

$$\|\mathbf{P}-\mathbf{B}\|^2 = \left(\sqrt{2^2+3^2}\right)^2 = 13 \tag{12}$$

$$\implies k = 1 \tag{13}$$

Therefore the ratio in which $\mathbf{P}$ divides the line segment joining the points $\mathbf{A}$ and $\mathbf{B}$ is $1:1$

See Fig. 0 ,

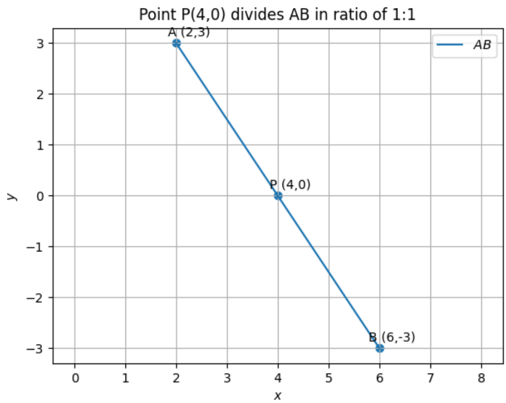

Fig. 0