1.9.22

EE25BTECH11033 - Kavin

Question:
Find the value of y for which the distance between the points $\mathbf{P}(2,-3)$ and $\mathbf{Q}(10,y)$ is 10 units.

Solution:
Given the points,

$$\mathbf{P} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 10 \\ y \end{pmatrix} \tag{1}$$

The distance between the points P and Q is given as,

$$d = \|\mathbf{P} - \mathbf{Q}\| = 10 \tag{2}$$

$$\tag{3}$$

We know that, the length of a vector is defined as

$$\|\mathbf{P} - \mathbf{Q}\| \triangleq \sqrt{(\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q})} \tag{4}$$

$$(\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q}) = \|\mathbf{P} - \mathbf{Q}\|^2 \tag{5}$$

$$(\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q}) = 10^2 \tag{6}$$

$$\tag{7}$$

$$\because \mathbf{P} - \mathbf{Q} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 10 \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ -3 - y \end{pmatrix}, \tag{8}$$

$$\implies \begin{pmatrix} -8 \\ -3 - y \end{pmatrix}^\top \begin{pmatrix} -8 \\ -3 - y \end{pmatrix} = 10^2 \tag{9}$$

$$\implies \begin{pmatrix} -8 & -3 - y \end{pmatrix} \begin{pmatrix} -8 \\ -3 - y \end{pmatrix} = 100 \tag{10}$$

$$\implies 8^2 + (3 + y)^2 = 100 \tag{11}$$

$$\implies (3 + y)^2 = 36 \tag{12}$$

$$\implies 3 + y = \pm 6 \tag{13}$$

$$\implies y = 3, -9 \tag{14}$$

Therefore the points $(10, 3)$ and $(10, -9)$ are at a distance of 10 units from the point P.

See Fig. 0 ,

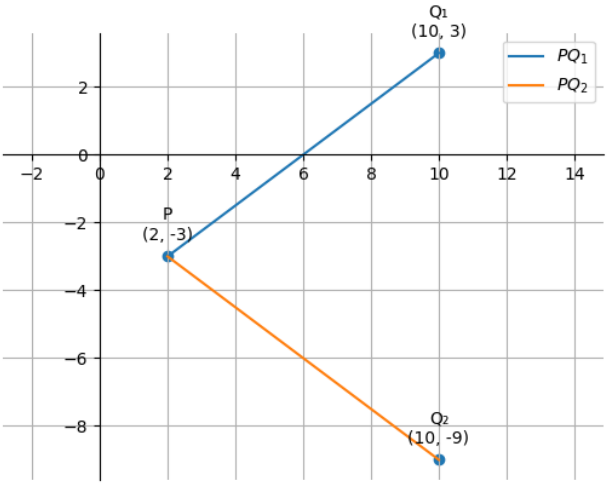

Fig. 0