Matrices in Geometry 1.5.25

EE25BTECH11037 - Divyansh

Question: In what ratio does the point $\begin{pmatrix}\frac{24}{11}\\ y\end{pmatrix}$ divide the line segment joining the points $\mathbf{P}=\begin{pmatrix}2\\-2\end{pmatrix}$ and $\mathbf{Q}=\begin{pmatrix}3\\7\end{pmatrix}$? Also find the value of y.

Given: $\mathbf{P} = \begin{pmatrix}2\\-2\end{pmatrix}$, $\mathbf{Q}\begin{pmatrix}3\\7\end{pmatrix}$ and a point $\mathbf{R}\begin{pmatrix}\frac{24}{11}\\ y\end{pmatrix}$ on PQ.

Let R divide PQ internally in the ratio $k : 1$.

Therefore, they are defined to be collinear if rank of the collinearity matrix is 1

$$\text{Collinearity matrix is } \begin{pmatrix}\mathbf{P}-\mathbf{R} & \mathbf{Q}-\mathbf{R}\end{pmatrix}^{\top} = 1$$

$$\mathbf{P}-\mathbf{R} = \begin{pmatrix}\frac{-2}{11}\\ -y-2\end{pmatrix}$$

$$\mathbf{Q}-\mathbf{R} = \begin{pmatrix}\frac{9}{11}\\ 7-y\end{pmatrix}$$

$$\implies \operatorname{rank}\begin{pmatrix}\frac{-2}{11} & -y-2\\ \frac{9}{11} & 7-y\end{pmatrix} = 1$$

$$\begin{pmatrix}\frac{-2}{11} & -2-y\\ \frac{9}{11} & 7-y\end{pmatrix} \xrightarrow{R_2 \to R_2 + \frac{9}{2}R_1} \begin{pmatrix}\frac{-2}{11} & -2-y\\ 0 & \frac{-11-4y}{2}\end{pmatrix}$$

for rank of this matrix to be 1, all the elements in the lower row have to be zero

$$\therefore -11-4y = 0 \implies y = \frac{-4}{11}$$

We know that k is the ratio in which $\mathbf{R}$ divides $\mathbf{P}$ and $\mathbf{Q}$,

$$\mathbf{R} = \frac{k\mathbf{Q}+\mathbf{P}}{1+k}$$

$$k(\mathbf{R}-\mathbf{Q}) = \mathbf{P}-\mathbf{R}$$

$$\implies k = \frac{(\mathbf{P}-\mathbf{R})^{\top}(\mathbf{R}-\mathbf{Q})}{\|\mathbf{R}-\mathbf{Q}\|^2}$$

$$(\mathbf{P}-\mathbf{R})^{\top} = \begin{pmatrix}\frac{-2}{11} & \frac{-18}{11}\end{pmatrix}$$

$$(\mathbf{R}-\mathbf{Q}) = \begin{pmatrix}\frac{-9}{11}\\ \frac{-81}{11}\end{pmatrix}$$

$$\|\mathbf{R}-\mathbf{Q}\|^2 = \left(\mathbf{R}-\mathbf{Q}\right)^{\top}\left(\mathbf{R}-\mathbf{Q}\right)$$

$$= \begin{pmatrix}\frac{-9}{11} & \frac{-81}{11}\end{pmatrix}\begin{pmatrix}\frac{-9}{11}\\ \frac{-81}{11}\end{pmatrix} = \frac{81}{121}+\frac{6561}{121} = \frac{6642}{121}$$

$$\therefore k = \frac{\begin{pmatrix}\frac{-2}{11} & \frac{-18}{11}\end{pmatrix}\begin{pmatrix}\frac{-9}{11}\\ \frac{-81}{11}\end{pmatrix}}{\frac{6642}{121}}$$

$$\implies k = \frac{\frac{18}{121} + \frac{1458}{121}}{\frac{6642}{121}} \implies k = \frac{1476}{6624} = \frac{2}{9}$$

Hence, the final answer is $\boxed{\text{k} = \frac{2}{9}}$ and $\boxed{\text{y} = \frac{-4}{11}}$

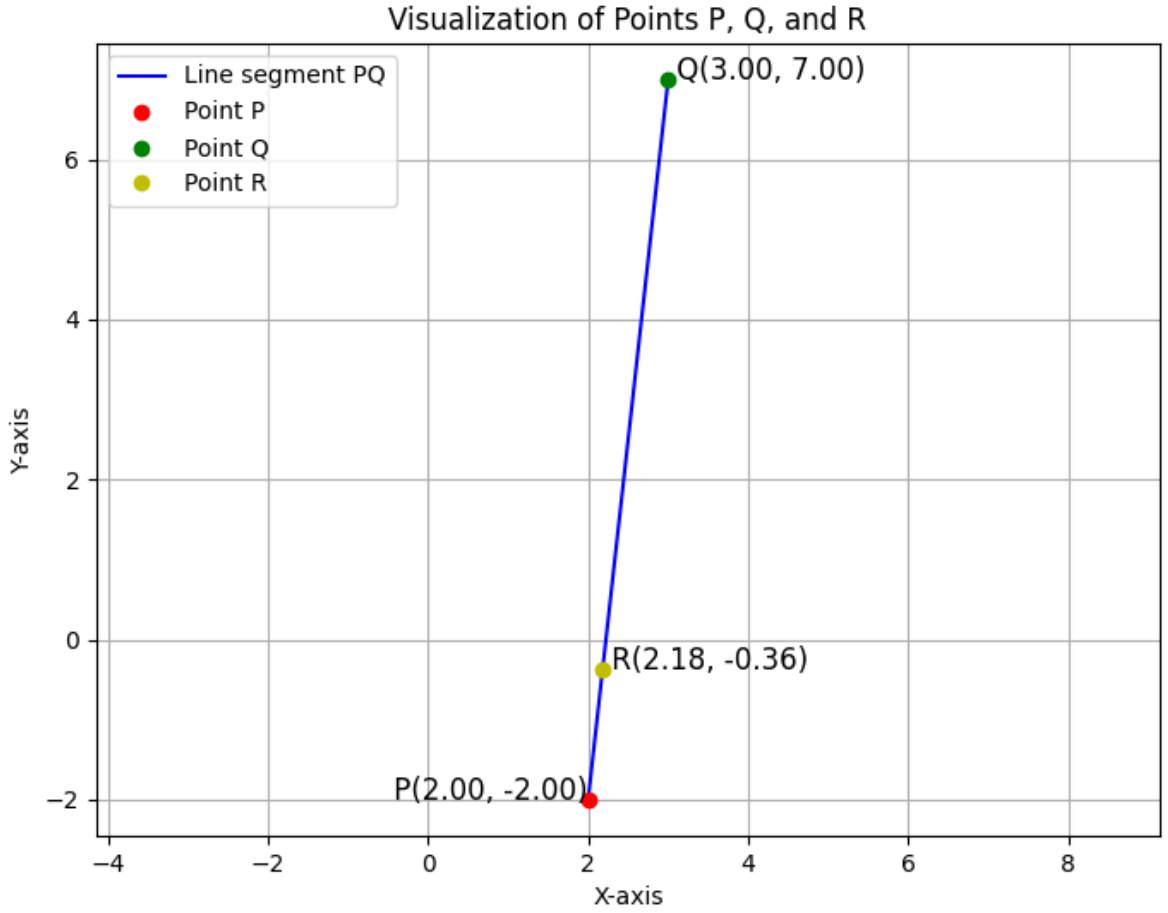

Fig. 1: Plot for 1.5.25