

1.9.15

EE25BTECH11026-Harsha

Question:

If \mathbf{a} , \mathbf{b} , \mathbf{c} are position vectors of the points $A(2, 3, -4)$, $B(3, -4, -5)$, and $C(3, 2, -3)$ respectively, then $\|\mathbf{a} + \mathbf{b} + \mathbf{c}\|$ is equal to

Solution:

Let us solve the given equation theoretically and then verify the solution computationally

According to the question,

Given the position vectors,

$$\mathbf{a} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} \quad (0.1)$$

To find the magnitude of $\|\mathbf{a} + \mathbf{b} + \mathbf{c}\|$, we can add these three vectors to find their sum, say \mathbf{S} , and find their magnitude.

$$\mathbf{S} = \mathbf{a} + \mathbf{b} + \mathbf{c} \quad (0.2)$$

$$\mathbf{S} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} \quad (0.3)$$

$$\therefore \mathbf{S} = \begin{pmatrix} 8 \\ 1 \\ -12 \end{pmatrix} \quad (0.4)$$

The magnitude of \mathbf{S} is given by

$$\|\mathbf{S}\|^2 = \mathbf{S}^T \mathbf{S} \quad (0.5)$$

$$\therefore \|\mathbf{S}\|^2 = \begin{pmatrix} 8 & 1 & -12 \end{pmatrix} \begin{pmatrix} 8 \\ 1 \\ -12 \end{pmatrix} \quad (0.6)$$

$$\|\mathbf{S}\|^2 = (209) \quad (0.7)$$

$$\therefore \|\mathbf{S}\| = (14.457) \text{ units} \quad (0.8)$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.

