

1.7.1 – Matgeo Assignment

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Question

Show that the points $(0, 0)$, $(2m, -4)$, and $(3, 6)$ are collinear, and hence find m , using the rank method.

Step 1: Form vectors

$$A = (0, 0), \quad B = (2m, -4), \quad C = (3, 6)$$

$$\mathbf{AB} = \begin{pmatrix} 2m \\ -4 \end{pmatrix}, \quad \mathbf{AC} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

Step 2: Matrix form

Form the matrix with **AB** and **AC** as columns:

$$M = \begin{pmatrix} 2m & 3 \\ -4 & 6 \end{pmatrix}$$

For collinearity, $\text{rank}(M) = 1$, i.e. $\det(M) = 0$.

Step 3: Using RREF

$$M = \begin{pmatrix} 2m & 3 \\ -4 & 6 \end{pmatrix}.$$

We use RREF of M and look for when its rank drops below 2.

$$\begin{aligned} \begin{pmatrix} 2m & 3 \\ -4 & 6 \end{pmatrix} &\xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} -4 & 6 \\ 2m & 3 \end{pmatrix} \xrightarrow{R_1 \leftarrow -\frac{1}{4}R_1} \begin{pmatrix} 1 & -\frac{3}{2} \\ 2m & 3 \end{pmatrix} \\ &\xrightarrow{R_2 \leftarrow R_2 - 2m R_1} \begin{pmatrix} 1 & -\frac{3}{2} \\ 0 & 3(m+1) \end{pmatrix}. \end{aligned}$$

If $m \neq -1$, the second row has a pivot, so the RREF is I_2 and $\text{rank}(M) = 2$. For the rank to drop we need

$$3(m+1) = 0 \Rightarrow m = -1.$$

When $m = -1$,

$$\begin{pmatrix} 1 & -\frac{3}{2} \\ 0 & 0 \end{pmatrix}$$

is the reduced row-echelon form (rank = 1).

Conclusion

The given points are collinear when

$$m = -1$$

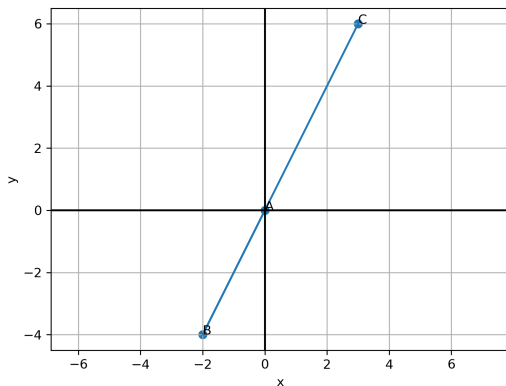


Figure: Graph