DA24S1 - Data Science and Artificial Intelligence

EE25BTECH11009 - ANSHU KUMAR RAM

1. If '→' denotes increasing order of intensity, then the meaning of the words [sick → infirm → moribund] is analogous to [silly → _____ → daft]. Which one of the given options is appropriate to fill the blank?

   (A) frown
   (B) fawn
   (C) vein
   (D) vain

   [GATE EE 2025]

2. The 15 parts of the given figure are to be painted such that no two adjacent parts with shared boundaries (excluding corners) have the same color. The minimum number of colors required is:

   (A) 4
   (B) 3
   (C) 5
   (D) 6

   [GATE EE 2025]

3. How many 4-digit positive integers divisible by 3 can be formed using only the digits $\{1, 3, 4, 6, 7\}$, such that no digit appears more than once in a number?

   (A) 24
   (B) 48
   (C) 72
   (D) 12

   [GATE EE 2025]

4. The sum of the following infinite series is:

$$2 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{8} + \frac{1}{9} + \frac{1}{16} + \frac{1}{27} + \cdots$$

   (A) $\frac{11}{3}$
   (B) $\frac{7}{2}$
   (C) $\frac{13}{4}$
   (D) $\frac{9}{2}$

   [GATE EE 2025]

5. In an election, the share of valid votes received by the four candidates A, B, C, and D is represented by the pie chart shown. The total number of votes cast in the election were 115,000, out of which 5,000 were invalid. Based on the data provided, the total number of valid votes received by the candidates B and C is:
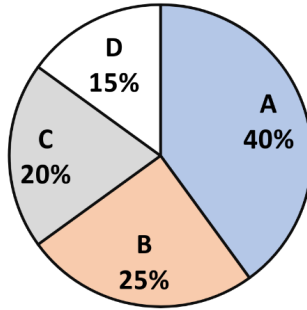
Share of valid votes

Fig. 1: Share of valid votes

a) 45,000   b) 49,500   c) 51,750   d) 54,000

6. Thousands of years ago, some people began dairy farming. This coincided with a number of mutations in a particular gene that resulted in these people developing the ability to digest dairy milk. Based on the given passage, which of the following can be inferred?

   a) All human beings can digest dairy milk.
   b) No human being can digest dairy milk.
   c) Digestion of dairy milk is essential for human beings.
   d) In human beings, digestion of dairy milk resulted from a mutated gene.

7. The probability of a boy or a girl being born is $\frac{1}{2}$. For a family having only three children, what is the probability of having two girls and one boy?

(A) $\frac{3}{8}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

8. Person 1 and Person 2 invest in three mutual funds A, B, and C. The amounts they invest in each are:

	Mutual fund A	Mutual fund B	Mutual fund C
Person 1	Rs. 10,000	Rs. 20,000	Rs. 20,000
Person 2	Rs. 20,000	Rs. 15,000	Rs. 15,000

At the end of one year, Person 1 gets Rs. 500 more than Person 2. Funds B and C earn 15% annual return. What is the annual rate of return for fund A?

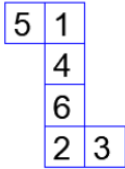a) 7.5%   b) 10%   c) 15%   d) 20%

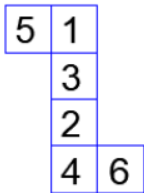9. Three different views of a dice are shown in the figure below.

Views of the dice

The piece of paper that can be folded to make this dice is
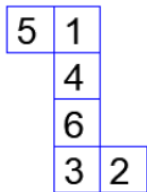

a)

b)

c)

d)

10. Visualize two identical right circular cones such that one is inverted over the other and they share a common circular base. If a cutting plane passes through the vertices of the assembled cones, what shape does the outer boundary of the resulting cross-section make?

   (A) A rhombus
   (B) A triangle
   (C) An ellipse
   (D) A hexagon

11. Consider the following statements:
   (i) The mean and variance of a Poisson random variable are equal.
   (ii) For a standard normal random variable, the mean is zero and the variance is one.
   Which ONE of the following options is correct?

   (A) Both (i) and (ii) are true
   (B) (i) is true and (ii) is false
   (C) (ii) is true and (i) is false
   (D) Both (i) and (ii) are false

12. Three fair coins are tossed independently. T is the event that two or more tosses result in heads. S is the event that two or more tosses result in tails. What is the probability of the event $T \cap S$?

   (A) 0
   (B) 0.5
   (C) 0.25
   (D) 1

13. Consider the matrix $\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$. Which ONE of the following statements is TRUE?
   (A) The eigenvalues of $\mathbf{M}$ are non-negative and real.
   (B) The eigenvalues of $\mathbf{M}$ are complex conjugate pairs.
   (C) One eigenvalue of $\mathbf{M}$ is positive and real, and another eigenvalue of $\mathbf{M}$ is zero.
   (D) One eigenvalue of $\mathbf{M}$ is non-negative and real, and another eigenvalue of $\mathbf{M}$ is negative and real.

14. Consider performing depth-first search (DFS) on an undirected and unweighted graph G starting at vertex s. For any vertex u in G, $d[u]$ is the length of the shortest path from s to u. Let (u, v) be an edge in G such that $d[u] < d[v]$. If the edge (u, v) is explored first in the direction from u to v during the above DFS, then (u, v) becomes a _____ edge.

   (A) tree
   (B) cross
   (C) back
   (D) gray

15. For any twice differentiable function $f : \mathbb{R} \to \mathbb{R}$, if at some $x^* \in \mathbb{R}$, $f'(x^*) = 0$ and $f''(x^*) > 0$, then the function f necessarily has a _____ at $x = x^*$.
   Note: $\mathbb{R}$ denotes the set of real numbers.

(A) local minimum   (C) local maximum

(B) global minimum   (D) global maximum

16. Match the items in Column 1 with the items in Column 2 in the following table:

Column 1	Column 2
(p) First In First Out	(i) Stacks
(q) Lookup Operation	(ii) Queues
(r) Last In First Out	(iii) Hash Tables

(A) (p)-(ii), (q)-(iii), (r)-(i)   (C) (p)-(i), (q)-(ii), (r)-(iii)

(B) (p)-(ii), (q)-(i), (r)-(iii)   (D) (p)-(i), (q)-(iii), (r)-(ii)

17. Consider the dataset with six datapoints $\{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_6, y_6)\}$, where
$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\mathbf{x}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\mathbf{x}_3 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$, $\mathbf{x}_4 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $\mathbf{x}_5 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\mathbf{x}_6 = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$,
and the labels are $y_1 = y_2 = y_5 = 1$ and $y_3 = y_4 = y_6 = -1$. A hard margin linear support vector machine is trained on the above dataset. Which ONE of the following sets is a possible set of support vectors?

(A) $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_5\}$ (C) $\{\mathbf{x}_4, \mathbf{x}_5\}$

(B) $\{\mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5\}$ (D) $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4\}$

18. Match the items in Column 1 with the items in Column 2 in the following table:

Column 1	Column 2
(p) Principal Component Analysis	(i) Discriminative Model
(q) Naïve Bayes Classification	(ii) Dimensionality Reduction
(r) Logistic Regression	(iii) Generative Model

(A) (p)-(iii), (q)-(i), (r)-(ii)   (C) (p)-(ii), (q)-(iii), (r)-(i)

(B) (p)-(ii), (q)-(i), (r)-(iii)   (D) (p)-(iii), (q)-(ii), (r)-(i)

19. Euclidean distance based k-means clustering algorithm was run on a dataset of 100 points with $k = 3$. If the points $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ are both part of cluster 3, then which ONE of the following points is necessarily also part of cluster 3?

(A) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ (C) $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ (D) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

20. Given a dataset with K binary-valued attributes (where $K > 2$) for a two-class classification task, the number of parameters to be estimated for learning a naïve Bayes classifier is

(A) $2K + 1$

(C) $2K + 1 + 1$

(B) $2K + 1$

(D) $K^2 + 1$

21. Consider performing uniform hashing on an open address hash table with load factor $\alpha = \frac{n}{m} < 1$, where n elements are stored in the table with m slots. The expected number of probes in an unsuccessful search is at most $\frac{1}{1-\alpha}$. Inserting an element in this hash table requires at most _____ probes, on average.

(A) $\ln\left(\frac{1}{1-\alpha}\right)$

(C) $1 + \frac{\alpha}{2}$

(B) $\frac{1}{1-\alpha}$

(D) $\frac{1}{1+\alpha}$

22. For any binary classification dataset, let $S_B \in \mathbb{R}^{d \times d}$ and $S_W \in \mathbb{R}^{d \times d}$ be the between-class and within-class scatter (covariance) matrices, respectively. The Fisher linear discriminant is defined by $u^* \in \mathbb{R}^d$, that maximizes

$$J(u) = \frac{u^T S_B u}{u^T S_W u}$$

If $\lambda = J(u^*)$, S_W is non-singular and $S_B \neq 0$, then (u^*, λ) must satisfy which ONE of the following equations?

Note: $\mathbb{R}$ denotes the set of real numbers.

(A) $S_W^{-1} S_B u^* = \lambda u^*$

(C) $S_B S_W u^* = \lambda u^*$

(B) $S_W u^* = \lambda S_B u^*$

(D) $u^{*T} u^* = \lambda^2$

23. Let h_1 and h_2 be two admissible heuristics used in A^* search. Which ONE of the following expressions is always an admissible heuristic?

(A) $h_1 + h_2$

(C) h_1/h_2, $(h_2 \neq 0)$

(B) $h_1 \times h_2$

(D) $|h_1 - h_2|$

24. Consider five random variables U, V, W, X, Y whose joint distribution satisfies:

$$P(U, V, W, X, Y) = P(U)P(V)P(W|U, V)P(X|W)P(Y|W)$$

Which ONE of the following statements is FALSE?

(A) Y is conditionally independent of V given W

(C) U and V are conditionally independent given W

(B) X is conditionally independent of U given W

(D) Y and X are conditionally independent given W

25. Consider the following statement: In adversarial search, α-β pruning can be applied to game trees of any depth where α is the (m) value choice we have formed so far at any choice point along the path for the MAX player and β is the (n) value choice we have formed so far at any choice point along the path for the MIN player. Which ONE of the following choices of (m) and (n) makes the above statement valid?

(A) (m) = highest, (n) = highest
(B) (m) = lowest, (n) = highest
(C) (m) = highest, (n) = lowest
(D) (m) = lowest, (n) = lowest

[GATE EE 2025]

26. Consider a database that includes the following relations:
    Defender(name, rating, side, goals)
    Forward(name, rating, assists, goals)
    Team(name, club, price)
    Which ONE of the following relational algebra expressions checks that every name occurring in Team appears in either Defender or Forward, where ϕ denotes the empty set?

(A) $\Pi_{name}(\text{Team}) \setminus (\Pi_{name}(\text{Defender}) \cap \Pi_{name}(\text{Forward})) = \phi$
(B) $(\Pi_{name}(\text{Defender}) \cap \Pi_{name}(\text{Forward})) \setminus \Pi_{name}(\text{Team}) = \phi$
(C) $\Pi_{name}(\text{Team}) \setminus (\Pi_{name}(\text{Defender}) \cup \Pi_{name}(\text{Forward})) = \phi$
(D) $(\Pi_{name}(\text{Defender}) \cup \Pi_{name}(\text{Forward})) \setminus \Pi_{name}(\text{Team}) = \phi$

[GATE EE 2025]

27. Let the minimum, maximum, mean and standard deviation values for the attribute income of data scientists be ₹46,000, ₹1,70,000, ₹96,000, and ₹21,000, respectively. The z-score normalized income value of ₹1,06,000 is closest to which ONE of the following options?

(A) 0.217
(B) 0.476
(C) 0.623
(D) 2.304

[GATE EE 2025]

28. Consider the following tree traversals on a full binary tree: (i) Preorder, (ii) Inorder, (iii) Postorder. Which of the following traversal options is/are sufficient to uniquely reconstruct the full binary tree?

(A) (i) and (ii)
(B) (ii) and (iii)
(C) (i) and (iii)
(D) (ii) only

[GATE EE 2025]

29. Let x and y be two propositions. Which of the following statements is a tautology/are tautologies?

(A) $(\neg x \wedge y) \implies (y \implies x)$ (C) $(\neg x \wedge y) \implies (\neg x \implies y)$

(B) $(x \wedge \neg y) \implies (\neg x \implies y)$ (D) $(x \wedge \neg y) \implies (y \implies x)$

[GATE EE 2025]

30. Consider sorting the array $[60, 70, 80, 90, 100]$ using in-place Quicksort with the last element as pivot. The minimum number of swaps performed is _____.

    a) 0
    b) 1
    c) 2
    d) 3

[GATE EE 2025]

31. Consider the following two tables named Raider and Team in a relational database maintained by a Kabaddi league. The attribute ID in table Team references the primary key of the Raider table, ID.

    Raider

ID	Name	Raids	RaidPoints
1	Arjun	200	250
2	Ankush	190	219
3	Sunil	150	200
4	Reza	150	190
5	Pratham	175	220
6	Gopal	193	215

    Team

City	ID	BidPoints
Jaipur	2	200
Patna	3	195
Hyderabad	5	175
Jaipur	1	250
Patna	4	200
Jaipur	6	200

    The SQL query described below is executed on this database:

    ```
    SELECT *
    FROM Raider, Team
    WHERE Raider.ID=Team.ID AND City="Jaipur" AND RaidPoints > 200;
    ```

    The number of rows returned by this query is _____.

32. The fundamental operations in a double-ended queue D are: `insertFirst(e)`, `insertLast(e)`, `removeFirst()`, `removeLast()`.

    In an empty double-ended queue, the following operations are performed:

    insertFirst(10)
    insertLast(32)
    $a \leftarrow$ removeFirst()
    insertLast(28)
    insertLast(17)

$a \leftarrow$ removeFirst()
$a \leftarrow$ removeLast()
The value of a is _____.

33. Let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) = \frac{1}{1+e^{-x}}$. The value of the derivative of f at x where $f(x) = 0.4$ is _____ (rounded off to two decimal places).
    Note: $\mathbb{R}$ denotes the set of real numbers.

34. The sample average of 50 data points is 40. The updated sample average after including a new data point taking the value of 142 is _____.

35. Consider the 3×3 matrix

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \\ 4 & 3 & 6 \end{pmatrix}$$

The determinant of $(\mathbf{M}^2 + 12\mathbf{M})$ is _____.

36. A fair six-sided die (with faces numbered 1, 2, 3, 4, 5, 6) is repeatedly thrown independently.
    What is the expected number of times the die is thrown until two consecutive throws of even numbers are seen?

    (A) 2   (C) 6
    (B) 4   (D) 8

    [GATE EE 2025]

37. Let $f : \mathbb{R} \to \mathbb{R}$ be a function.

$$f(x) = \begin{cases} -x, & \text{if } x < -2 \\ ax^2 + bx + c, & \text{if } x \in [-2, 2] \\ x, & \text{if } x > 2 \end{cases}$$

    Which ONE of the following choices gives the values of a, b, c that make the function f continuous and differentiable?

    (A) $a = \frac{1}{4}$, $b = 0$, $c = 1$ (C) $a = 0$, $b = 0$, $c = 0$
    (B) $a = \frac{1}{2}$, $b = 0$, $c = 0$ (D) $a = 1$, $b = 1$, $c = -4$

    [GATE EE 2025]

38. Consider the following Python code:

```
def count(child_dict, i):
    if i not in child_dict.keys():
        return 1
    ans = 1
    for j in child_dict[i]:
        ans += count(child_dict, j)
    return ans

child_dict = dict()
```

```
child_dict[0] = [1,2]
child_dict[1] = [3,4,5]
child_dict[2] = [6,7,8]
print(count(child_dict,0))
```

Which ONE of the following is the output of this code?

(A) 6   (C) 8
(B) 1   (D) 9

39. Consider the function computeS(X) whose pseudocode is given below:

```
computeS(X):
    S[1] ← 1
    for i ← 2 to length(X):
        S[i] ← 1
        if X[i − 1] ≤ X[i]:
            S[i] ← S[i] + S[i-1]
        end if
    end for
    return S
```

Which ONE of the following values is returned by the function computeS(X) for $X = [6, 3, 5, 4, 10]$?

(A) 1, 1, 2, 3, 4   (C) 1, 1, 2, 1, 2
(B) 1, 1, 2, 3, 3   (D) 1, 1, 2, 1, 5

40. Let $F(n)$ denote the maximum number of comparisons made while searching for an entry in a sorted array of size n using binary search. Which ONE of the following options is TRUE?

(A) $F(n) = F(\lfloor n/2 \rfloor) + 1$ (C) $F(n) = F(\lfloor n/2 \rfloor)$
(B) $F(n) = F(\lfloor n/2 \rfloor) + F(\lceil n/2 \rceil)$ (D) $F(n) = F(n − 1) + 1$

41. Consider the following Python function:

```
def fun(D, s1, s2):
    if s1 < s2:
        D[s1], D[s2] = D[s2], D[s1]
        fun(D, s1+1, s2−1)
```

What does this Python function fun() do? Select the ONE appropriate option below.

(A) It finds the smallest element in D from index s1 to s2, both inclusive.

(B) It performs a merge sort in-place on this list D between indices s1 and s2, both inclusive.

(C) It reverses the list D between indices s1 and s2, both inclusive.

(D) It swaps the elements in D at indices s1 and s2, and leaves the remaining elements unchanged.

[GATE EE 2025]

42. Consider the table below, where the $(i, j)^{th}$ element of the table is the distance between points x_i and x_j. Single linkage clustering is performed on data points x_1, x_2, x_3, x_4, x_5.

	x_1	x_2	x_3	x_4	x_5
x_1	0	1	4	3	6
x_2	1	0	3	5	3
x_3	4	3	0	2	5
x_4	3	5	2	0	1
x_5	6	3	5	1	0

Which ONE of the following is the correct representation of the clusters produced?

a)

b)

c)

d)
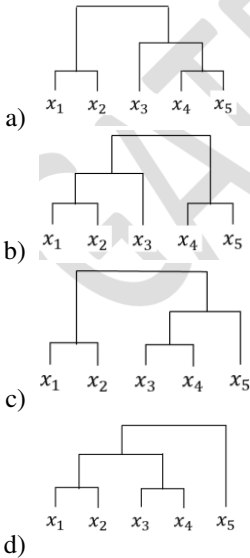
[GATE EE 2025]

[GATE EE 2025]

43. Consider the two neural networks (NNs) shown in Fig. 1 and Fig. 2, with ReLU activation ReLU$(z) = \max\{0, z\}$. The connections and weights are shown; all biases = 0. For what values of p, q, r in Fig. 2 are the two NNs equivalent when x_1, x_2, x_3 are positive?

a) $p = 36$, $q = 24$, $r = 24$

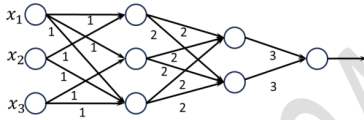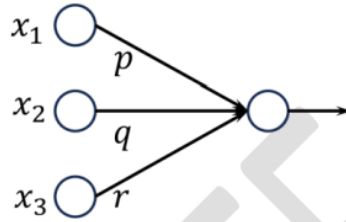## Figure 2

Figure 1

b) $p = 24$, $q = 24$, $r = 36$
c) $p = 18$, $q = 36$, $r = 24$
d) $p = 36$, $q = 36$, $r = 36$

[GATE EE 2025]

[GATE EE 2025]

44. Consider a state space where the start state is number 1. The successor function for the state numbered n returns two states numbered $n + 1$ and $n + 2$. Assume that the states in the unexpanded state list are expanded in the ascending order of numbers and the previously expanded states are not added to the unexpanded state list. Which ONE of the following statements about breadth-first search (BFS) and depth-first search (DFS) is true, when reaching the goal state number 6?

(A) BFS expands more states than DFS.
(B) DFS expands more states than BFS.
(C) Both BFS and DFS expand equal num-
ber of states.
(D) Both BFS and DFS do not reach the goal state number 6.

[GATE EE 2025]

45. Consider the following sorting algorithms: (i) Bubble sort (ii) Insertion sort (iii) Selection sort
Which ONE among the following choices of sorting algorithms sorts the numbers in the array [4, 3, 2, 1, 5] in increasing order after exactly two passes over the array?

(A) (i) only
(B) (iii) only
(C) (i) and (iii) only
(D) (ii) and (iii) only

[GATE EE 2025]

46. Given the relational schema $R = (U, V, W, X, Y, Z)$ and the set of functional dependencies: $\{U \rightarrow V, U \rightarrow W, WX \rightarrow Y, WX \rightarrow Z, V \rightarrow X\}$. Which of the following functional dependencies can be derived from the above set?

(A) $VW \rightarrow YZ$ (C) $VW \rightarrow U$

(B) $WX \rightarrow YZ$ (D) $VW \rightarrow Y$

[GATE EE 2025]

47. Select all choices that are subspaces of $\mathbb{R}^3$.
    Note: $\mathbb{R}$ denotes the set of real numbers.

(A) $\left\{ \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : \mathbf{x} = \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \alpha, \beta \in \mathbb{R} \right\}$

(B) $\left\{ \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : \mathbf{x} = \alpha^2 \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \beta^2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \alpha, \beta \in \mathbb{R} \right\}$

(C) $\left\{ \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : 5x_1 + 2x_3 = 0, 4x_1 - 2x_2 + 3x_3 = 0 \right\}$

(D) $\left\{ \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : 5x_1 + 2x_3 + 4 = 0 \right\}$

[GATE EE 2025]

48. Which of the following statements is/are TRUE?
    Note: $\mathbb{R}$ denotes the set of real numbers.

(A) There exist $M \in \mathbb{R}^{3\times3}$, $p \in \mathbb{R}^3$, and $q \in \mathbb{R}^3$ such that $M\mathbf{x} = p$ has a unique solution and $M\mathbf{x} = q$ has infinite solutions.
(B) There exist $M \in \mathbb{R}^{3\times3}$, $p \in \mathbb{R}^3$, and $q \in \mathbb{R}^3$ such that $M\mathbf{x} = p$ has no solutions and $M\mathbf{x} = q$ has infinite solutions.
(C) There exist $M \in \mathbb{R}^{2\times3}$, $p \in \mathbb{R}^2$, and $q \in \mathbb{R}^2$ such that $M\mathbf{x} = p$ has a unique solution and $M\mathbf{x} = q$ has infinite solutions.
(D) There exist $M \in \mathbb{R}^{3\times2}$, $p \in \mathbb{R}^3$, and $q \in \mathbb{R}^3$ such that $M\mathbf{x} = p$ has a unique solution and $M\mathbf{x} = q$ has no solutions.

[GATE EE 2025]

49. Let $\mathbb{R}$ be the set of real numbers, U be a subspace of $\mathbb{R}^3$ and $M \in \mathbb{R}^{3\times3}$ be the matrix corresponding to the projection onto the subspace U. Which of the following statements is/are TRUE?

(A) If U is a 1-dimensional subspace of $\mathbb{R}^3$, then the null space of M is a 1-dimensional subspace.
(B) If U is a 2-dimensional subspace of $\mathbb{R}^3$, then the null space of M is a 1-dimensional subspace.
(C) $M^2 = M$
(D) $M^3 = M$

[GATE EE 2025]

50. Consider the function $f : \mathbb{R} \to \mathbb{R}$, where $\mathbb{R}$ is the set of all real numbers,
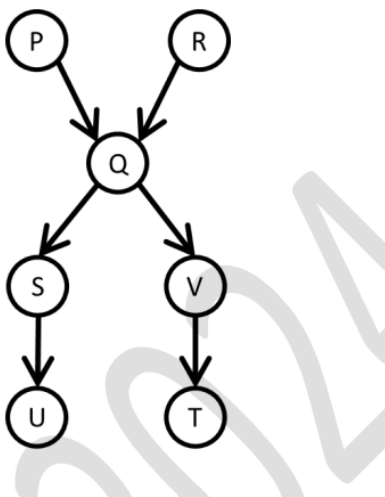
$$f(x) = \frac{x^4}{4} - \frac{2x^3}{3} - \frac{3x^2}{2} + 1$$

Which of the following statements is/are TRUE?

(A) $x = 0$ is a local maximum of f (C) $x = -1$ is a local maximum of f
(B) $x = 3$ is a local minimum of f (D) $x = 0$ is a local minimum of f

[GATE EE 2025]

51. Consider the directed acyclic graph (DAG) below:

Which of the following is/are valid vertex orderings that can be obtained from a topological sort of the DAG?

a) P Q R S T U V
b) P R Q V S U T
c) P Q R S V U T
d) P R Q S V T U

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52. Let H, I, L, N represent height, number of internal nodes, number of leaf nodes, and the total number of nodes respectively in a rooted binary tree. Which of the following statements is/are always TRUE?
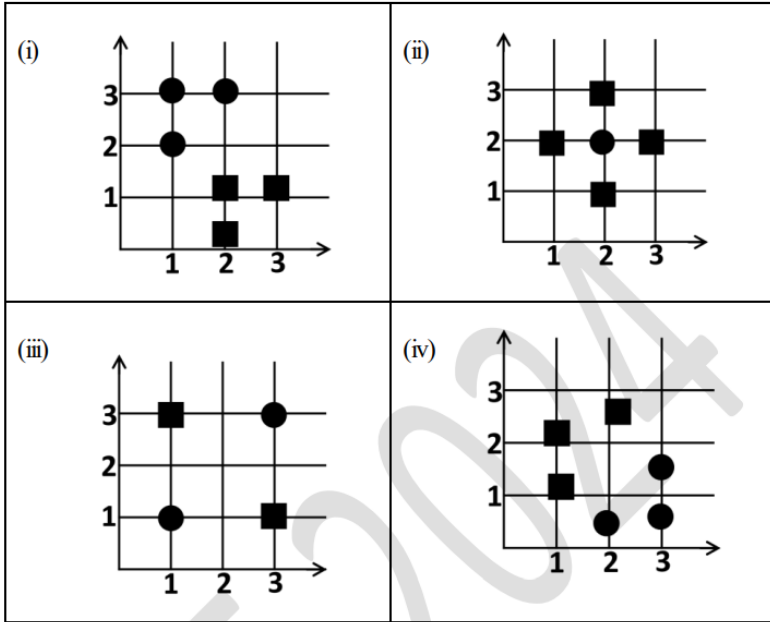
(A) $L \le I + 1$

(C) $H \le I \le 2^H - 1$

(B) $H + 1 \le N \le 2^{H+1} - 1$

(D) $H \le L \le 2^H - 1$

53. Consider the following figures representing datasets consisting of two-dimensional features with two classes denoted by circles and squares:

Which of the following is/are TRUE?

a) (i) is linearly separable.
b) (ii) is linearly separable.
c) (iii) is linearly separable.
d) (iv) is linearly separable.

54. Let `game(ball, rugby)` be true if the ball is used in rugby and false otherwise. Let `shape(ball, round)` be true if the ball is round and false otherwise. Consider the following logical sentences:

s_1: ∀ ball ¬game(ball, rugby) $\implies$ shape(ball, round)

s_2: ∀ ball ¬shape(ball, round) $\implies$ game(ball, rugby)

s_3: ∀ ball game(ball, rugby) $\implies$ ¬shape(ball, round)

s_4: ∀ ball shape(ball, round) $\implies$ ¬game(ball, rugby)

Which of the following choices is/are logical representations of the assertion, "All balls are round except balls used in rugby"?

(A) $s_1 \wedge s_3$
(B) $s_1 \wedge s_2$

(C) $s_2 \wedge s_3$
(D) $s_3 \wedge s_4$

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55. An OTT company is maintaining a large disk-based relational database of different movies with the following schema:

Movie(ID, CustomerRating)
Genre(ID, Name)
Movie_Genre(MovieID, GenreID)

Consider the following SQL query on the relational database above:

```
SELECT *
FROM Movie, Genre, Movie_Genre
WHERE Movie.CustomerRating > 3.4
  AND Genre.Name = "Comedy"
  AND Movie_Genre.MovieID = Movie.ID
  AND Movie_Genre.GenreID = Genre.ID;
```

This SQL query can be sped up using which of the following indexing options?

a) B+ tree on all the attributes.
b) Hash index on Genre.Name and B+ tree on the remaining attributes.
c) Hash index on Movie.CustomerRating and B+ tree on the remaining attributes.
d) Hash index on all the attributes.

[GATE EE 2025]

56. Let X be a random variable uniformly distributed in the interval $[1, 3]$ and Y be a random variable uniformly distributed in the interval $[2, 4]$. If X and Y are independent of each other, the probability $P(X \geq Y)$ is _____ (rounded off to three decimal places).

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57. Let X be a random variable exponentially distributed with parameter $\lambda > 0$. The probability density function of X is given by:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

If $5E(X) = Var(X)$, where $E(X)$ and $Var(X)$ indicate the expectation and variance of X, respectively, the value of λ is _____ (rounded off to one decimal place).

[GATE EE 2025]

58. Consider two events T and S. Let $\bar{T}$ denote the complement of the event T. The probability associated with different events are given as follows:

$$P(\bar{T}) = 0.6, \ P(S|T) = 0.3, \ P(S|\bar{T}) = 0.6$$

Then, $P(T|S)$ is _____ (rounded off to two decimal places).

[GATE EE 2025]

59. Consider a joint probability density function of two random variables X and Y:

$$f_{X,Y}(x, y) = \begin{cases} 2xy, & 0 < x < 2, \ 0 < y < x \\ 0, & \text{otherwise} \end{cases}$$

Then, $E[Y|X = 1.5]$ is ——————.

[GATE EE 2025]

60. Evaluate the following limit: $\lim\limits_{x \to 0} \dfrac{\ln((x^2 + 1)\cos x)}{x^2} =$ ——————

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61. Let $\mathbf{u} = [1 \ 2 \ 3 \ 4 \ 5]^T$, and let $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$ be the singular values of the matrix $M = \mathbf{uu}^T$ (where $\mathbf{u}^T$ is the transpose of $\mathbf{u}$). The value of $\sum\limits_{i=1}^{5} \sigma_i$ is ——————.

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62. Details of ten international cricket games between two teams "Green" and "Blue" are given in Table C. This table lists matches played on different pitches, across formats, along with their winners. Pitch can be spin-friendly (S) or pace-friendly (F). Format can be one-day (O) or test (T).

A cricket organization would like to use this data to develop a decision-tree model to predict outcomes. The computed InformationGain(C, Pitch) with respect to the Target is ——————— (rounded off to two decimal places).

| Match No.	Pitch	Format	Winner (Target)
1	S	T	Green
2	S	T	Blue
3	F	O	Blue
4	S	O	Blue
5	F	T	Green
6	F	O	Blue
7	S	O	Green
8	F	T	Blue
9	F	O	Blue
10	S	O	Green

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63. Given the two-dimensional dataset consisting of 5 data points from two classes (circles and squares) and using Euclidean distance as the metric, the minimum odd value of k in the k-nearest neighbour algorithm for which the diamond ($\lozenge$) point is assigned the label "square" is ——————.
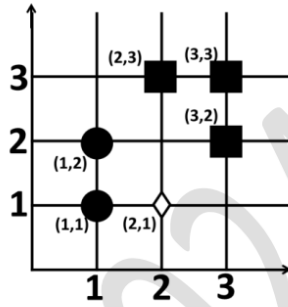
64. Given the following Bayesian Network of four Bernoulli random variables and the associated conditional probability tables:
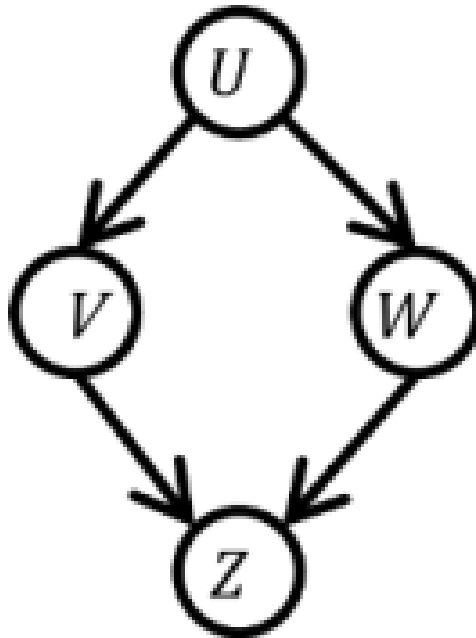
The value of $P(U = 1, V = 1, W = 1, Z = 1)$ is _____ (rounded off to three decimal places).

65. Two fair coins are tossed independently. X is a random variable that takes a value of 1 if both tosses are heads and 0 otherwise. Y is a random variable that takes a value of 1 if at least one of the tosses is heads and 0 otherwise. The value of the covariance of X and Y is _____ (rounded off to three decimal places).

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