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Matrix 1.7.1

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Question

Show that the points $(0, 0)$, $(2m, -4)$, and $(3, 6)$ are collinear, and hence find m, using the rank method.

Solution

Let the given points be

$$A = (0, 0), \quad B = (2m, -4), \quad C = (3, 6).$$

Step 1: Form vectors

$$\mathbf{AB} = B - A = \begin{pmatrix} 2m \\ -4 \end{pmatrix}, \quad \mathbf{AC} = C - A = \begin{pmatrix} 3 \\ 6 \end{pmatrix}.$$

Step 2: Matrix form

Construct the matrix

$$M = \begin{pmatrix} 2m & 3 \\ -4 & 6 \end{pmatrix}.$$

For the points to be collinear, the two vectors $\mathbf{AB}$ and $\mathbf{AC}$ must be linearly dependent. This means

$$\text{rank}(M) = 1 \quad \Leftrightarrow \quad \det(M) = 0.$$

Step 3: Row-reduction

$$\begin{pmatrix} 2m & 3 \\ -4 & 6 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} -4 & 6 \\ 2m & 3 \end{pmatrix} \xrightarrow{R_1 \leftarrow -\frac{1}{4} R_1} \begin{pmatrix} 1 & -\frac{3}{2} \\ 2m & 3 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 - 2m\, R_1} \begin{pmatrix} 1 & -\frac{3}{2} \\ 0 & 3(m+1) \end{pmatrix}.$$

If $m \neq -1$, the second row has a pivot, so the RREF is I_2 and $\text{rank}(M) = 2$. For the rank to drop, we require

$$3(m+1) = 0 \quad \Rightarrow \quad m = -1.$$

When $m = -1$,

$$\begin{pmatrix} 1 & -\frac{3}{2} \\ 0 & 0 \end{pmatrix}$$

is the reduced row-echelon form (rank = 1).

Final Answer

The given points are collinear when

$$\boxed{m = -1}$$

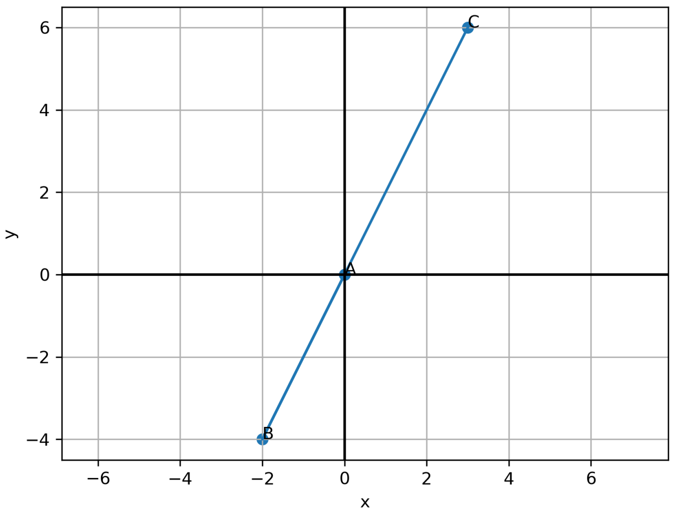

Figure 1: Graph