Q.1 Rajiv Gandhi Khel Ratna Award was conferred _____ Mary Kom, a six-time world champion in boxing, recently in a ceremony _____ the Rashtrapati Bhawan (the President's official residence) in New Delhi.

a) with, at
b) on, in
c) on, at
d) to, at

(GATE MA 2020)

Q.2 Despite a string of poor performances, the chances of K. L. Rahul's selection in the team are __________.

a) slim
b) bright
c) obvious
d) uncertain

(GATE MA 2020)

Q.3 Select the word that fits the analogy: Cover : Uncover :: Associate : __________

a) Unassociate
b) Inassociate
c) Misassociate
d) Dissociate

(GATE MA 2020)

Q.4 Hit by floods, the kharif (summer sown) crops in various parts of the country have been affected. Officials believe that the loss in production of the kharif crops can be recovered in the output of the rabi (winter sown) crops so that the country can achieve its food-grain production target of 291 million tons in the crop year 2019–20 (July–June). They are hopeful that good rains in July–August will help the soil retain moisture for a longer period, helping winter sown crops such as wheat and pulses during the November–February period.
Which of the following statements can be inferred from the given passage?

a) Officials declared that the food-grain production target will be met due to good rains.
b) Officials want the food-grain production target to be met by the November–February period.
c) Officials feel that the food-grain production target cannot be met due to floods.
d) Officials hope that the food-grain production target will be met due to a good rabi produce.

(GATE MA 2020)

Q.5 The difference between the sum of the first $2n$ natural numbers and the sum of the first n odd natural numbers is __________.

a) $n^2 - n$
b) $n^2 + n$
c) $2n^2 - n$
d) $2n^2 + n$

(GATE MA 2020)

Q.6 Repo rate is the rate at which Reserve Bank of India (RBI) lends commercial banks, and reverse repo rate is the rate at which RBI borrows money from commercial banks.
Which of the following statements can be inferred from the above passage?

a) Decrease in repo rate will increase cost of borrowing and decrease lending by commercial banks.
b) Increase in repo rate will decrease cost of borrowing and increase lending by commercial banks.
c) Increase in repo rate will increase cost of borrowing and decrease lending by commercial banks.
d) Decrease in repo rate will decrease cost of borrowing and increase lending by commercial banks.

(GATE MA 2020)

Q.7 P, Q, R, S, T, U, V and W are seated around a circular table. I. S is seated second place to the right of R. II. V is seated at the third place to the left of R. III. Q is a neighbour of V. IV. R is a neighbour of U.

Which of the following must be true?

a) Q is a neighbour of R.
b) U is a neighbour of S.
c) P is not a neighbour of R.
d) P is the left neighbour of R.

(GATE MA 2020)

Q.8 The distance between Delhi and Agra is 233 km. A car P started travelling from Delhi to Agra and another car Q started from Agra to Delhi along the same road 1 hour after the car P started. The two cars crossed each other 75 minutes after the car Q started. Both cars were travelling at constant speed. The speed of car P was 10 km/h more than the speed of car Q. How many kilometers the car Q had travelled when the cars crossed each other?

a) 66.2
b) 75.2
c) 85.8
d) 116.2

(GATE MA 2020)

Q.9 For a matrix $M = \left(m_{ij}\right), i, j = 1, 2, 3, 4$, the diagonal elements are all zero and $m_{ij} = -m_{ji}$. The minimum number of elements required to fully specify the matrix is __________.

a) 0
b) 6
c) 12
d) 16

(GATE MA 2020)

Q.10 The profit shares of two companies P and Q are shown in the figure. If the two companies have invested a fixed equal amount every year, then the ratio of the total revenue of company P to total revenue of company Q, during 2013–2018, is __________.

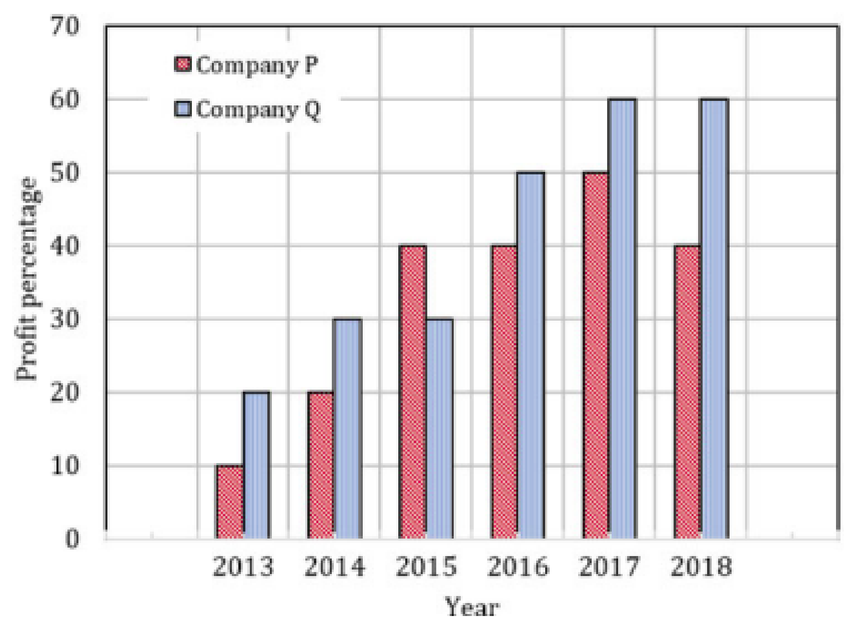

Fig. 1.

a) 15 : 17
b) 17 : 15
c) 17 : 13
d) 17 : 11

(GATE MA 2020)

Q.11 Suppose that d_1, d_2 and d_3 are topologies on X induced by metrics d_1, d_2 and d_3, respectively, such that $S_{d_1} \subseteq S_{d_2} \subseteq S_{d_3}$. Then which of the following statements is TRUE?

a) If a sequence converges in (X, d_2) then it converges in (X, d_1)
b) If a sequence converges in (X, d_3) then it converges in (X, d_2)
c) Every open ball in (X, d_1) is also an open ball in (X, d_2)
d) The map $x \mapsto x$ from (X, d_1) to (X, d_3) is continuous

(GATE MA 2020)

Q.12 Let $D = (-1, 1) \times (-1, 1)$. If the function $F : D \to \mathbb{R}$ is defined by

$$F(x, y) = \begin{cases} \dfrac{xy^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

then I. F is continuous at $(0, 0)$ II. both the first order partial derivatives of F exist at $(0, 0)$ III. $\iint_D |F(x, y)|\, dx\, dy$ is finite IV. $\iint_D F(x, y)\, dx\, dy$ is finite

a) Only I and II are correct
b) Only III and IV are correct
c) I, II and III are correct
d) All of I, II, III and IV are correct

(GATE MA 2020)

Q.13 The initial value problem

$$y' = y^2, \quad y(0) = b$$

has

a) a unique solution if $b = 0$
b) no solution if $b = 1$
c) infinitely many solutions if $b = 2$
d) a unique solution if $b = 1$

(GATE MA 2020)

Q.14 Consider the following statements: I. $\log(z)$ is harmonic on $\mathbb{C} \setminus \{0\}$ II. $\log(z)$ has a harmonic conjugate on $\mathbb{C} \setminus \{0\}$

Then

a) Both I and II are true
b) I is true but II is false
c) I is false but II is true
d) Both I and II are false

(GATE MA 2020)

Q.15 Let G and H be defined by

$$G = \{z = x + iy \in \mathbb{C} : x \cdot y = 0\}, \quad H = \{z = x + iy \in \mathbb{C} : x = 0,\ y \neq 0\}$$

Suppose $f : G \to \mathbb{C}$ and $g : H \to \mathbb{C}$ are analytic functions. Consider the following statements:
I. $\int_\gamma f(z)\, dz$ is independent of path γ in G joining $-i$ and i II. $\int_\gamma g(z)\, dz$ is independent of path γ in H joining $-i$ and i

Then

a) Both I and II are true
b) I is true but II is false
c) I is false but II is true
d) Both I and II are false

(GATE MA 2020)

Q.16 Let $f(n) = n^2$, $n \in \mathbb{Z}_{>0}$ and let, for $n \in \mathbb{N}$,

$$R_n = \{x + y\sqrt{2} : x, y \in \mathbb{Z},\ |x| \leq n^2,\ |y| \leq n^2\} \subseteq \mathbb{Q}(\sqrt{2}).$$

If for a subset $S \subseteq \mathbb{R}$, $\overline{S}$ denotes the closure of S in $\mathbb{R}$, then
a) $\overline{T(\mathbb{Q})} = T(\mathbb{Q})$
b) $\overline{T(R_n)} = T(R_n)$
c) $\overline{T(\mathbb{Q})} = \overline{T(R_n)}$
d) $\overline{T(\mathbb{Q})} = T(R_n)$

(GATE MA 2020)

Q.17 Suppose that

$$U = \mathbb{R}^2 \setminus \{(x, y) \in \mathbb{R}^2 : x \cdot y = 0\}, \quad V = \mathbb{R}^2 \setminus \{(x, y) \in \mathbb{R}^2 : x \cdot y = 1\}.$$

Then which one of the following statements is TRUE?
a) Both U and V are bounded
b) Both U and V are connected
c) U is connected but V is disconnected
d) U is disconnected but V is connected

(GATE MA 2020)

Q.18 Consider the two–dimensional dual problems of the linear programming problem

$$\text{(P1)} \quad \min z = x_1 + 2x_2, \quad \text{subject to } x_1 + x_2 \geq 1,\ x_1 \geq 0,\ x_2 \geq 0,$$

$$\text{(P2)} \quad \max w = y_1, \quad \text{subject to } y_1 \leq 1,\ y_1 + y_2 \leq 2,\ y_1, y_2 \geq 0.$$

Then
a) Both (P1) and (P2) are infeasible
b) (P1) is infeasible and (P2) is feasible
c) (P1) is feasible and bounded but (P2) is infeasible
d) (P1) is feasible and unbounded but (P2) is feasible

(GATE MA 2020)

Q.19 If $f(x, y) = 5x + 6y - 6x^2 - 7xy - 2y^2 + 18y + x^3 + y^3$, where $(x, y) \in \mathbb{R}^2$, then
a) $(0, 0)$ is a point of local maximum of f
b) $(0, 0)$ is a saddle point of f
c) $(0, 0)$ is a point of local minimum of f
d) $(0, 0)$ is neither a local minimum nor a local maximum of f

(GATE MA 2020)

Q.20 Consider the iterative scheme

$$x_{n+1} = \frac{2}{3}x_n + \frac{1}{x_n^2}, \quad n \geq 1,$$

with initial point $x_1 > 0$. Then the sequence $\{x_n\}$
a) converges to 1 if $x_1 > 0$

b) converges to 2 if $x_1 > 0$
c) converges to 3 if $x_1 > 0$
d) does not converge for any $x_1 > 0$

(GATE MA 2020)

Q.21 Let $C[0,1]$ denote the space of all real–valued continuous functions on $(0,1)$ equipped with the supremum norm $\|\cdot\|_\infty$. Let $T : C[0,1] \to C[0,1]$ be the linear operator defined by

$$T(f)(x) = \int_0^1 e^{xy} f(y)\,dy.$$

Then
a) $\|T\| = 1$
b) T^{-1} is not invertible
c) T is surjective
d) $1 + \|T\| = 1 + \|T\|^2$

(GATE MA 2020)

Q.22 Suppose that M is a 5×5 matrix with real entries and $p(x) = \det(xI - M)$. Then $p(x) = \det(M)$ if and only if
a) Every eigenvalue of M is real
b) If $p(x) = 0$ then $p(x) + p(2) = 0 = p(2) + p(3)$
c) M^n is necessarily a polynomial in M of degree ≤ 4
d) M is invertible

(GATE MA 2020)

Q.23 Let $C[0,1]$ denote the space of all real–valued continuous functions on $(0,1)$ equipped with the supremum norm $\|\cdot\|_\infty$. Let $f \in C[0,1]$ be such that

$$|f(x) - f(y)| \leq M|x - y|, \quad \forall\, x, y \in [0,1] \text{ and for some } M > 0.$$

For $n \in \mathbb{N}$, let $S_n(f) = \{f\left(\frac{k}{n}\right) : k \in \mathbb{N}, k \leq n\}$. Then the closure of $S = \bigcup_{n\in\mathbb{N}} S_n(f)$
a) is closed and bounded
b) is bounded but not totally bounded
c) is compact
d) is closed but not bounded

(GATE MA 2020)

Q.24 Let $K : \mathbb{R} \times (0,\infty) \to \mathbb{R}$ be a function such that the solution of the initial value problem

$$\frac{du(x)}{dx} = \int_0^\infty K(x-y) f(y)\,dy, \quad u(0) = f(x), \quad x \in \mathbb{R},\ t \in (0,\infty),$$

is given by

$$u(x,t) = \int_0^\infty K(x-y) f(y)\,dy$$

for all bounded continuous functions f. Then the value of $\int_0^\infty K(x,y)\,dx$ is
a) 0
b) 1
c) 2
d) 3

(GATE MA 2020)

Q.25 The number of cyclic subgroups of the quaternion group

$$Q_8 = \{a, b : a^4 = 1,\ a^2 = b^2,\ ba = a^{-1}b\}$$

is __________.

a) 5
b) 6
c) 7
d) 8

(GATE MA 2020)

Q.26 The number of elements of order 3 in the symmetric group S_6 is __________.

a) 20
b) 40
c) 60
d) 80

(GATE MA 2020)

Q.27 Let F be the field with 4096 elements. The number of proper subfields of F is __________.

a) 2
b) 3
c) 4
d) 5

(GATE MA 2020)

Q.28 If (x_1, x_2^*) is an optimal solution of the linear programming problem,

$$\text{minimize} \quad x_1 + 2x_2$$

subject to

$$\begin{aligned} 4x_1 - x_2 &\geq 8 \\ 2x_1 + x_2 &\geq 10 \\ -x_1 + x_2 &\leq 7 \\ x_1, x_2 &\geq 0 \end{aligned}$$

and $(\lambda_1^*, \lambda_2^*, \lambda_3^*)$ is an optimal solution of its dual problem, then

$$\sum_{i=1}^{2} x_i^{*2} + \sum_{j=1}^{3} \lambda_j^{*2}$$

is equal to __________ (correct up to one decimal place).

a) 20.2
b) 21.6
c) 22.3
d) 23.8

(GATE MA 2020)

Q.29 Let $a, b, c \in \mathbb{R}$ be such that the quadrature rule

$$\int_{-1}^{1} f(x)\, dx \approx af(-1) + bf(0) + cf(1)$$

is exact for all polynomials of degree less than or equal to 2. Then b is equal to __________ (rounded off to two decimal places).

a) 1.33
b) 1.67
c) 2.00
d) 2.33

(GATE MA 2020)

Q.30 Let $f(x) = x^4$ and let $p(x)$ be the interpolating polynomial of f at nodes $1, 2$ and 3. Then $p(0)$ is equal to __________.

a) -2
b) -4
c) -6
d) -8

(GATE MA 2020)

Q.31 For $n \geq 2$, define the sequence $\{x_n\}$ by

$$x_n = \frac{1}{2\pi} \int_0^{\pi/2} \tan^n t \, dt.$$

Then the sequence $\{x_n\}$ converges to __________ (correct up to two decimal places).

a) 0.50
b) 0.75
c) 1.00
d) 1.25

(GATE MA 2020)

Q.32 Let

$$L^2[0,10] = \{f : [0,10] \to \mathbb{R} : f \text{ is Lebesgue measurable and } \int_0^{10} f^2 dx < \infty\}$$

equipped with the norm

$$|f| = \left(\int_0^{10} f^2 dx\right)^{\frac{1}{2}}$$

and let T be the linear functional on $L^2[0,10]$ given by

$$T(f) = \int_0^2 f(x)dx - \int_3^{10} f(x)dx.$$

Then $|T|$ is equal to __________

(GATE MA 2020)

Q.33 If $\{x_{13}, x_{22}, x_{23} = 10, x_{31}, x_{32}, x_{34}\}$ is the set of basic variables of a balanced transportation problem seeking to minimize cost of transportation from origins to destinations, where the cost matrix is,

	D_1	D_2	D_3	D_4	Availability
O_1	6	2	-1	0	10
O_2	4	2	2	3	$\lambda + 5$
O_3	3	1	2	1	3λ
Demand	10	$\mu - 5$	$\mu + 5$	15	

TABLE I
*

and $\lambda, \mu \in \mathbb{R}$, then x_{32} is equal to __________

(GATE MA 2020)

Q.34 Let $\mathbb{Z}_{225}$ be the ring of integers modulo 225. If x is the number of prime ideals and y is the number of nontrivial units in $\mathbb{Z}_{225}$, then $x + y$ is equal to __________.

(GATE MA 2020)

Q.35 Let $u(x,t)$ be the solution of

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0,$$
$$u(x,0) = f(x),$$
$$\frac{\partial u}{\partial t}(x,0) = 0, \quad x \in \mathbb{R},\ t > 0,$$

where f is a twice continuously differentiable function. If $f(-2) = 4$, $f(0) = 0$, and $u(2,2) = 8$, then the value of $u(1,3)$ is _________.

(GATE MA 2020)

Q.36 Let $\{e_n\}_{n=1}^{\infty}$ be an orthonormal basis for a separable Hilbert space H with the inner product $\langle \cdot, \cdot \rangle$. Define

$$f_n = e_n - \frac{1}{n+1} e_{n+1} \quad \text{for } n \in \mathbb{N}.$$

Then

a) the closure of the span $\{f_n : n \in \mathbb{N}\}$ equals H
b) $f = 0$ if $\langle f, f_n \rangle = \langle f, e_n \rangle$ for all $n \in \mathbb{N}$
c) $\{f_n\}_{n=1}^{\infty}$ is an orthogonal subset of H
d) there does not exist nonzero $f \in H$ such that $\langle f, e_2 \rangle = \langle f, f_2 \rangle$

(GATE MA 2020)

Q.37 Suppose V is a finite dimensional nonzero vector space over $\mathbb{C}$ and $T : V \to V$ is a linear transformation such that $\mathrm{Range}(T) = \mathrm{Nullspace}(T)$. Then which of the following statements is FALSE?

a) the dimension of V is even
b) 0 is the only eigenvalue of T
c) both 0 and 1 are eigenvalues of T
d) $T^2 = 0$

(GATE MA 2020)

Q.38 Let $P \in M_{m\times n}(\mathbb{R})$. Consider the following statements:
I : If $XPY = 0$ for all $X \in M_{1\times m}(\mathbb{R})$ and $Y \in M_{n\times 1}(\mathbb{R})$, then $P = 0$.
II : If $m = n$, P is symmetric and $P^2 = 0$, then $P = 0$.
Then

a) both I and II are true
b) I is true but II is false
c) I is false but II is true
d) both I and II are false

(GATE MA 2020)

Q.39 For $n \in \mathbb{N}$, let $T_n : (\ell^1, |\cdot|_1) \to (\ell^\infty, |\cdot|_\infty)$ and $T : (\ell^1, |\cdot|_1) \to (\ell^\infty, |\cdot|_\infty)$ be the bounded linear operators defined by

$$T_n(x_1, x_2, \ldots) = (y_1, y_2, \ldots), \quad \text{where } y_j = \begin{cases} x_j, & j \le n \\ x_n, & j > n \end{cases}$$

and

$$T(x_1, x_2, \ldots) = (x_1, x_2, \ldots).$$

Then

a) $|T_n|$ does not converge to $|T|$ as $n \to \infty$
b) $T_n - T$ converges to zero as $n \to \infty$
c) for all $x \in \ell^1$, $|T_n(x) - T(x)|$ converges to zero as $n \to \infty$

d) for each nonzero $x \in \ell^1$, there exists a continuous linear functional g on ℓ^∞ such that $g(T_n(x))$ does not converge to $g(T(x))$ as $n \to \infty$

(GATE MA 2020)

Q.40 Let $\mathcal{P}(\mathbb{R})$ denote the power set of $\mathbb{R}$, equipped with the metric

$$d(U, V) = \sup_{x \in \mathbb{R}} |\chi_U(x) - \chi_V(x)|,$$

where χ_U and χ_V denote the characteristic functions of the subsets U and V, respectively, of $\mathbb{R}$. The set $\{\{m\} : m \in \mathbb{Z}\}$ in the metric space $(\mathcal{P}(\mathbb{R}), d)$ is

a) bounded but not totally bounded
b) totally bounded but not compact
c) compact
d) not bounded

(GATE MA 2020)

Q.41 Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} \chi_{(n,n+1]}(x),$$

where $\chi_{(n,n+1]}$ is the characteristic function of the interval $(n, n+1)$. For $\alpha \in \mathbb{R}$, let $S_\alpha = \{x \in \mathbb{R} : f(x) > \alpha\}$. Then

a) $S_{\frac{1}{2}}$ is open
b) $S_{\sqrt{2}}$ is not measurable
c) S_0 is closed
d) $S_{\frac{1}{\sqrt{2}}}$ is measurable

(GATE MA 2020)

Q.42 For $n \in \mathbb{N}$, let $f_n, g_n : (0, 1) \to \mathbb{R}$ be functions defined by

$$f_n(x) = x^n \quad \text{and} \quad g_n(x) = x^n(1 - x).$$

Then

a) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly
b) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly
c) both $\{f_n\}$ and $\{g_n\}$ converge uniformly
d) neither $\{f_n\}$ nor $\{g_n\}$ converge uniformly

(GATE MA 2020)

Q.43 Let u be a solution of the differential equation $y' + xy = 0$ and let $\phi = u\psi$ be a solution of the differential equation

$$y'' + 2xy' + (x^2 + 2)y = 0$$

satisfying $\phi(0) = 1$ and $\phi'(0) = 0$. Then $\phi(x)$ is

a) $(\cos x)e^{-\frac{x^2}{2}}$
b) $(\cos x)e^{-\frac{x^2}{2}}$
c) $(1 + x^2)e^{-\frac{x^2}{2}}$
d) $(\cos x)e^{-x^2}$

(GATE MA 2020)

Q.44 For $n \in \mathbb{N} \cup \{0\}$, let y_n be a solution of the differential equation

$$xy'' + (1 - x)\, y' + ny = 0$$

satisfying $y_n(0) = 1$. For which of the following functions $w(x)$, the integral

$$\int_0^\infty y_p(x)\, y_q(x)\, w(x)\, dx, \quad (p \neq q)$$

is equal to zero?

a) e^{-x^2}
b) e^{-x}
c) xe^{-x^2}
d) xe^{-x}

(GATE MA 2020)

Q.45 Suppose that

$$X = \{(0,0)\} \cup \left\{ \left(x, \sin \frac{1}{x} \right) : x \in \mathbb{R} \setminus \{0\} \right\}$$

and

$$Y = \{(0,0)\} \cup \left\{ \left(x, x \sin \frac{1}{x} \right) : x \in \mathbb{R} \setminus \{0\} \right\}$$

are metric spaces with metrics induced by the Euclidean metric of $\mathbb{R}^2$. Let B_X and B_Y be the open unit balls around $(0,0)$ in X and Y, respectively. Consider the following statements:
I : The closure of B_X in X is compact.
II : The closure of B_Y in Y is compact.
Then

a) both I and II are true
b) I is true but II is false
c) I is false but II is true
d) both I and II are false

(GATE MA 2020)

Q.46 If $f : \mathbb{C} \setminus \{0\} \to \mathbb{C}$ is a function such that $f(z) = f\left(\frac{z}{|z|}\right)$ and its restriction to the unit circle is continuous, then

a) f is continuous but not necessarily analytic
b) f is analytic but not necessarily a constant function
c) f is a constant function
d) $\lim_{z \to 0} f(z)$ exists

(GATE MA 2020)

Q.47 For a subset S of a topological space, let $\text{Int}(S)$ and $\bar{S}$ denote the interior and closure of S, respectively. Then which of the following statements is TRUE?

a) If S is open, then $S = \text{Int}(\bar{S})$
b) If the boundary of S is empty, then S is open
c) If the boundary of S is empty, then S is not closed
d) If $\bar{S} \setminus S$ is a proper subset of the boundary of S, then S is open

(GATE MA 2020)

Q.48 Suppose $\mathscr{T}_1$, $\mathscr{T}_2$ and $\mathscr{T}_3$ are the smallest topologies on $\mathbb{R}$ containing S_1, S_2 and S_3, respectively, where

$$S_1 = \left\{ \left(a, a + \tfrac{\pi}{n} \right) : a \in \mathbb{Q}, n \in \mathbb{N} \right\},$$

$$S_2 = \{(a,b) : a < b,\ a, b \in \mathbb{Q}\},$$

$$S_3 = \{(a,b) : a < b,\ a, b \in \mathbb{R}\}.$$

Then

a) $\mathscr{T}_3 \supsetneq \mathscr{T}_1$

b) $\mathscr{T}_3 \supsetneq \mathscr{T}_2$
c) $\mathscr{T}_1 = \mathscr{T}_2$
d) $\mathscr{T}_1 \supsetneq \mathscr{T}_2$

(GATE MA 2020)

Q.49 Let

$$M = \begin{pmatrix} \alpha & 3 & 0 \\ \beta & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}.$$

Consider the following statements:
I : There exists a lower triangular matrix L such that $M = LL^t$, where L^t denotes the transpose of L.
II : Gauss-Seidel method for $Mx = b$ ($b \in \mathbb{R}^3$) converges for any initial choice $x_0 \in \mathbb{R}^3$.
Then

a) I is not true when $\alpha > \frac{9}{2}$, $\beta = 3$
b) II is not true when $\alpha > \frac{9}{2}$, $\beta = -1$
c) II is not true when $\alpha = 4$, $\beta = \frac{3}{2}$
d) I is true when $\alpha = 5$, $\beta = 3$

(GATE MA 2020)

Q.50 Let I and J be the ideals generated by $\{5, \sqrt{10}\}$ and $\{4, \sqrt{10}\}$ in the ring $\mathbb{Z}[\sqrt{10}] = \{a + b\sqrt{10} \mid a, b \in \mathbb{Z}\}$, respectively. Then

a) both I and J are maximal ideals
b) I is a maximal ideal but J is not a prime ideal
c) I is not a maximal ideal but J is a prime ideal
d) neither I nor J is a maximal ideal

(GATE MA 2020)

Q.51 Suppose V is a finite dimensional vector space over $\mathbb{R}$. If W_1, W_2 and W_3 are subspaces of V, then which of the following statements is TRUE?

a) If $W_1 + W_2 + W_3 = V$ then $\operatorname{span}(W_1 \cup W_2) \cup \operatorname{span}(W_2 \cup W_3) \cup \operatorname{span}(W_3 \cup W_1) = V$
b) If $W_1 \cap W_2 = \{0\}$ and $W_1 \cap W_3 = \{0\}$, then $W_1 \cap (W_2 + W_3) = \{0\}$
c) If $W_1 + W_2 = W_1 + W_3$, then $W_2 = W_3$
d) If $W_1 \neq V$, then $\operatorname{span}(V \setminus W_1) = V$

(GATE MA 2020)

Q.52 Let $\alpha, \beta \in \mathbb{R}$, $\alpha \neq 0$. The system

$$\begin{aligned} x_1 - 2x_2 + \alpha x_3 &= 8 \\ x_1 - x_2 + x_4 &= \beta \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

has NO basic feasible solution if

a) $\alpha < 0$, $\beta > 8$
b) $\alpha > 0$, $0 < \beta < 8$
c) $\alpha > 0$, $\beta < 0$
d) $\alpha < 0$, $\beta < 8$

(GATE MA 2020)

Q.53 Let $0 < p < 1$ and let

$$X = \left\{ f : \mathbb{R} \to \mathbb{R} \text{ is continuous and } \int_{\mathbb{R}} |f(x)|^p \, dx < \infty \right\}.$$

For $f \in X$, define

$$|f|_p = \left(\int_{\mathbb{R}} |f(x)|^p \, dx \right)^{\frac{1}{p}}.$$

Then

a) $|\cdot|_p$ defines a norm on X
b) $|f+g|_p \leq |f|_p + |g|_p$ for all $f, g \in X$
c) $|f+g|_p^p \leq |f|_p^p + |g|_p^p$ for all $f, g \in X$
d) if f_n converges to f pointwise on $\mathbb{R}$, then $\lim_{n\to\infty} |f_n|_p = |f|_p$

(GATE MA 2020)

Q.54 Suppose that ϕ_1 and ϕ_2 are linearly independent solutions of the differential equation

$$2x^2 y'' - \left(x + x^2\right) y' + \left(x^2 - 2\right) y = 0,$$

and $\phi_1(0) = 0$. Then the smallest positive integer n such that

$$\lim_{x \to 0} x^n \frac{\phi_2(x)}{\phi_1(x)} = 0$$

is __________.

(GATE MA 2020)

Q.55 Suppose that $f(z) = \prod_{n=1}^{17} \left(z - \frac{\pi}{n}\right)$, $z \in \mathbb{C}$ and $\gamma(t) = e^{2it}$, $t \in [0, 2\pi]$. If

$$\int_\gamma \frac{f'(z)}{f(z)} \, dz = \alpha \pi i,$$

then the value of α is equal to __________.

(GATE MA 2020)

Q.56 If $\gamma(t) = \frac{1}{2} e^{3\pi i t}$, $t \in [0, 2]$ and

$$\int_\gamma \frac{1}{z^2 \left(e^z - 1\right)} \, dz = \beta \pi i,$$

then β is equal to __________ (correct up to one decimal place).

(GATE MA 2020)

Q.57 Let $K = \mathbb{Q}\left(\sqrt{3} + 2\sqrt{2}, \omega\right)$, where ω is a primitive cube root of unity. Then the degree of extension of K over $\mathbb{Q}$ is __________.

(GATE MA 2020)

Q.58 Let $\alpha \in \mathbb{R}$. If $(3, 0, 0, \beta)$ is an optimal solution of the linear programming problem

$$\text{minimize} \quad x_1 + x_2 + x_3 - \alpha x_4$$

subject to

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 6, \\ -x_1 + x_2 + x_4 &= \beta, \\ x_1, x_2, x_3, x_4 &\geq 0, \end{aligned}$$

then the maximum value of $\beta - \alpha$ is __________.

(GATE MA 2020)

Q.59 Suppose that $T : \mathbb{R}^4 \to \mathbb{R}[x]$ is a linear transformation over $\mathbb{R}$ satisfying

$$T(-1, 1, 1, 1) = x^2 + 2x^4, \qquad T(1, 2, 3, 4) = 1 - x^2, \qquad T(2, -1, -1, 0) = x^3 - x^4.$$

Then the coefficient of x^4 in $T(-3, 5, 6, 6)$ is __________.

(GATE MA 2020)

Q.60 Let $\mathbf{F}(x, y, z) = (2x - 2y\cos x)\,\hat{i} + \left(2y - y^2 \sin x\right)\hat{j} + 4z\,\hat{k}$ and let S be the surface of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.
If $\hat{n}$ is the unit outward normal to the tetrahedron, then the value of

$$\iint_S \mathbf{F} \cdot \hat{n}\, dS$$

is __________ (rounded off to two decimal places).

(GATE MA 2020)

Q.61 Let $\mathbf{F} = (x + 2y)\, e^{z^2}\hat{i} + \left(ye^z + x^2\right)\hat{j} + y^2 z\,\hat{k}$ and let S be the surface

$$x^2 + y^2 + z = 1, \quad z \geq 0.$$

If $\hat{n}$ is a unit normal to S and

$$\left| \iint_S (\nabla \times \mathbf{F}) \cdot \hat{n}\, dS \right| = \alpha\pi,$$

then α is equal to __________.

(GATE MA 2020)

Q.62 Let G be a non-cyclic group of order 57. Then the number of elements of order 3 in G is __________.

(GATE MA 2020)

Q.63 The coefficient of $(x - 1)^5$ in the Taylor expansion about $x = 1$ of the function

$$F(x) = \int_1^x \frac{\ln t}{t - 1}\, dt, \qquad 0 < x < 2$$

is __________ (correct up to two decimal places).

(GATE MA 2020)

Q.64 Let $u(x, y)$ be the solution of the initial value problem

$$\frac{\partial u}{\partial x} + \left(\sqrt{u}\right)\frac{\partial u}{\partial y} = 0,$$
$$u(x, 0) = 1 + x^2.$$

Then the value of $u(0, 1)$ is __________ (rounded off to three decimal places).

(GATE MA 2020)

Q.65 The value of

$$\lim_{n \to \infty} \int_0^1 n\, x^n e^{x^2}\, dx$$

is __________ (rounded off to three decimal places).

(GATE MA 2020)