2.4.29

EE25BTECH11033 - Kavin

Question:
The points $\mathbf{A}(2,9), \mathbf{B}(a,5)$ and $\mathbf{C}(5,5)$ are the vertices of a triangle **ABC** right angled at **B**. Find the values of a and hence the area of $\triangle$**ABC**.

Solution:
Given the points,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 9 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} a \\ 5 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} \tag{1}$$

Also it is given that the triangle **ABC** right angled at **B**.

$\therefore$ The vectors $(\mathbf{A} - \mathbf{B})$ and $(\mathbf{C} - \mathbf{B})$ are perpendicular.

The angle θ between vectors $(\mathbf{A} - \mathbf{B}), (\mathbf{C} - \mathbf{B})$, is given by

$$\cos\theta = \frac{(\mathbf{A} - \mathbf{B})^\top (\mathbf{C} - \mathbf{B})}{\|\mathbf{A} - \mathbf{B}\| \, \|\mathbf{C} - \mathbf{B}\|} \tag{2}$$

Here $\theta = 90°$.

$$\implies (\mathbf{A} - \mathbf{B})^\top (\mathbf{C} - \mathbf{B}) = 0 \tag{3}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 - a \\ 4 \end{pmatrix}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 5 - a \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 - a \\ 4 \end{pmatrix}^\top \begin{pmatrix} 5 - a \\ 0 \end{pmatrix} = 0 \tag{4}$$

$$\begin{pmatrix} 2 - a & 4 \end{pmatrix} \begin{pmatrix} 5 - a \\ 0 \end{pmatrix} = 0 \tag{5}$$

$$\implies (2 - a)(5 - a) + (4 \times 0) = 0 \tag{6}$$

$$\implies (2 - a)(5 - a) = 0 \tag{7}$$

$$\implies a = 2 \tag{8}$$

Here $a = 5$ is not considered because when $a = 5$, the points **B** and **C** will be the same and hence a triangle cannot be formed.

$$\mathbf{B} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

The area of $\triangle ABC$ is given by

$$Area = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C})\| \tag{9}$$

$$(\mathbf{A} - \mathbf{B}) = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$
$$(\mathbf{A} - \mathbf{C}) = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$$\implies Area = \frac{1}{2} \left\| \begin{pmatrix} 0 \\ 4 \end{pmatrix} \times \begin{pmatrix} -3 \\ 4 \end{pmatrix} \right\| \tag{10}$$

$$\implies Area = \frac{1}{2} \|0 + 12\| \tag{11}$$

$$\implies Area = 6 \tag{12}$$

Hence the area of $\triangle$**ABC** is 6 sq.units.
See Fig. 0 ,

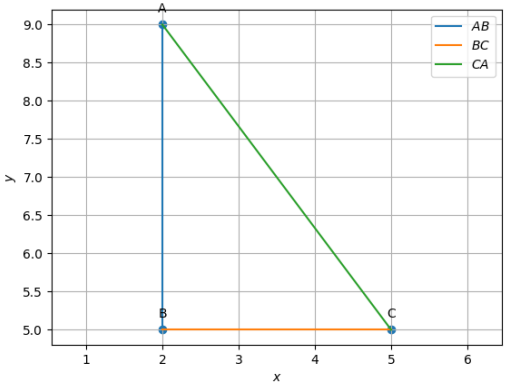

Fig. 0