# GATE 2008 MA

## EE25BTECH11001 - AARUSH DILAWRI

### Q.1-Q.20 carry one mark each.

1) Consider the subspace $W = \{[a] : a = 0 \text{ if } i \text{ is even}\}$ of all $10 \times 10$ real matrices. Then the dimension of $W$ is GATE MA 2008

   a) 25
   b) 50
   c) 75
   d) 100

2) Let $S$ be the open unit disk and $f : S \to \mathbb{C}$ be a real-valued analytic function with $f(0) = 1$. Then the set $\{z \in S : f(z) \neq 1\}$ is GATE MA 2008

   a) empty
   b) nonempty finite
   c) countably infinite
   d) uncountable

3) Let $E = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq x\}$. Then $\iint_E f(x + y)\, dx\, dy$ is equal to GATE MA 2008

   a) -1
   b) 0
   c) 1
   d) 2

4) For $(x, y) \in \mathbb{R}^2$, let

$$f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

   Then GATE MA 2008

   a) $f_x, f_y$ exist at $(0, 0)$ and $f$ is continuous at $(0, 0)$
   b) $f_x, f_y$ exist at $(0, 0)$ and $f$ is discontinuous at $(0, 0)$
   c) $f_x, f_y$ do not exist at $(0, 0)$ and $f$ is continuous at $(0, 0)$
   d) $f_x, f_y$ do not exist at $(0, 0)$ and $f$ is discontinuous at $(0, 0)$

5) Let $y$ be a solution of $y' = e^{2x} - 1$ on $[0, 1]$ with $y(0) = 0$. Then GATE MA 2008

   a) $y(x) > 0$ for $x > 0$
   b) $y(x) < 0$ for $x > 0$
   c) $y$ changes sign in $[0, 1]$
   d) $y = 0$ for $x > 0$

6) For the equation

$$x(x - 1)y'' + \sin x\, y' + 2x(x - 1)y = 0,$$

   consider the statements:

   - $P$: $x = 0$ is a regular singular point.
   - $Q$: $x = 1$ is a regular singular point.

   Then GATE MA 2008

a) both $P$ and $Q$ are true
b) $P$ is false but $Q$ is true
c) $P$ is true but $Q$ is false
d) both $P$ and $Q$ are false

7) Let $G = \mathbb{R} \setminus \{0\}$ and $H = \{-1, 1\}$ be groups under multiplication. The map $\phi : G \to H$ defined by $\phi(x) = \text{sgn}(x)$ is GATE MA 2008
   a) not a homomorphism
   b) a one-one homomorphism, which is not onto
   c) an onto homomorphism, which is not one-one
   d) an isomorphism
8) For $1 \le p \le \infty$, let $\|\cdot\|_p$ denote the $p$-norm on $\mathbb{R}^2$. If $\|\cdot\|_p$ satisfies the parallelogram law, then $p$ equals GATE MA 2008

   a) 1
   b) 2
   c) 3
   d) 4

9) The number of maximal ideals in $\mathbb{Z}_{27}$ is GATE MA 2008

   a) 0
   b) 1
   c) 2
   d) 3

10) Consider the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x) = y_0. \tag{10.1}$$

To compute the value $y_1 = y(x + h)$, $h > 0$, equate $y_1$ to the value of the straight line passing through $(x, y)$ with slope equal to the slope of the curve $y(x)$ at $x$, resulting in the method called GATE MA 2008

   a) Euler's method
   b) Improved Euler's method
   c) Backward Euler's method
   d) Taylor series method of order 2

11) The solution of $xu_x + yu_y = 0$ is of the form GATE MA 2008

   a) $f(y/x)$
   b) $f(x + y)$
   c) $f(x - y)$
   d) $f(xy)$

12) If the partial differential equation $(x - 1)^2 u_x + (y - 2)^2 u_y + 2x + 2yu_x + 2xyu = 0$ is parabolic in $S \subset \mathbb{R}^2$ but not in $\mathbb{R}^2 \setminus S$, then $S$ is GATE MA 2008

   a) $\{(x, y) \in \mathbb{R}^2 : x = 1 \text{ or } y = 2\}$
   b) $\{(x, y) \in \mathbb{R}^2 : x = 1 \text{ and } y = 2\}$
   c) $\{(x, y) \in \mathbb{R}^2 : x = 1\}$
   d) $\{(x, y) \in \mathbb{R}^2 : y = 2\}$

13) Let $E$ be a connected subset of $\mathbb{R}$ with at least two elements. Then the number of elements in $E$ is GATE MA 2008

a) exactly two
b) more than two but finite
c) countably infinite
d) uncountable

14) Let $X$ be a non-empty set. Let $\mathcal{I}_1$ and $\mathcal{I}_2$ be two topologies on $X$ such that $\mathcal{I}_1$ is strictly contained in $\mathcal{I}_2$. If $I : (X, \mathcal{I}_1) \rightarrow (X, \mathcal{I}_2)$ is the identity map, then GATE MA 2008
    a) both $I$ and $I^{-1}$ are continuous
    b) both $I$ and $I^{-1}$ are not continuous
    c) $I$ is continuous but $I^{-1}$ is not continuous
    d) $I$ is not continuous but $I^{-1}$ is continuous
15) Let $X_1, X_2, \ldots, X_{10}$ be a random sample from $N(80, 3^2)$ distribution. Define

$$S = \sum_{i=1}^{10} X_i, \quad T = \sum_{i=1}^{10} \frac{X_i - 80}{3}. \tag{15.1}$$

Then the value of $E(ST)$, the expectation of the product, is GATE MA 2008

a) 0
b) 1
c) 10
d) 3

16) Two distinguishable fair coins are tossed simultaneously. Given that one of them lands heads, the probability that the other lands tails is GATE MA 2008

a) $\frac{1}{3}$
b) $\frac{1}{2}$
c) $\frac{2}{3}$
d) 1

17) Let $c \geq 2$ be the cost of the $(i, j)$-th cell of an assignment problem. If a new cost matrix is generated by the elements $c' = 2 + c$, then GATE MA 2008
    a) the optimal assignment plan remains unchanged and cost of assignment decreases
    b) the optimal assignment plan changes and cost of assignment decreases
    c) the optimal assignment plan remains unchanged and cost of assignment increases
    d) the optimal assignment plan changes and cost of assignment increases
18) Let a primal linear programming problem admit an optimal solution. Then the corresponding dual problem GATE MA 2008
    a) does not have a feasible solution
    b) has a feasible solution but does not have any optimal solution
    c) does not have a convex feasible region
    d) has an optimal solution
19) In any system of particles, if internal forces are not assumed to come in pairs, the fact that the sum of internal forces is zero follows from GATE MA 2008
    a) Newton's second law
    b) conservation of angular momentum
    c) conservation of energy
    d) principle of virtual displacement
20) Let $q_1, q_2, \ldots, q_n$ be the generalized coordinates and $\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_n$ be the generalized velocities in a conservative force field. Under a transformation, the new coordinate

system has generalized coordinates $Q_1, Q_2, \ldots$ and velocities $\dot{Q}_1, \dot{Q}_2, \ldots$. Then the equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = 0 \tag{20.1}$$

takes the form GATE MA 2008

a) $\frac{d}{dt}\frac{\partial L'}{\partial \dot{Q}_k} - \frac{\partial L'}{\partial Q_k} = 0$ b) $\frac{d}{dt}\frac{\partial L'}{\partial \dot{Q}_k} + \frac{\partial L'}{\partial Q_k} = 0$ c) $-\frac{d}{dt}\frac{\partial L'}{\partial \dot{Q}_k} + \frac{\partial L'}{\partial Q_k} = 0$ d) $\frac{\partial L'}{\partial \dot{Q}_k} - \frac{d}{dt}\frac{\partial L'}{\partial Q_k} = 0$

21) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear map satisfying

$$T(e_1) = e_2, \quad T(e_2) = e_3, \quad T(e_3) = 0, \quad T(e_4) = e_3, \tag{21.1}$$

where $\{e_1, e_2, e_3, e_4\}$ is the standard basis of $\mathbb{R}^4$. Then GATE MA 2008

a) $T$ is idempotent
b) $T$ is invertible
c) Rank $T = 3$
d) $T$ is nilpotent

22) Let

$$M = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \tag{22.1}$$

and $V = \{Mx' : x \in \mathbb{R}^3\}$. Then an orthonormal basis for $V$ is GATE MA 2008

a) $\left\{(1,0,0)', \begin{pmatrix} 0 \\ \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}, \begin{pmatrix} \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}\right\}$

b) $\left\{(1,0,0)', \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}\right\}$

c) $\left\{(1,0,0)', \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}, \begin{pmatrix} \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}\right\}$

d) $\{(1,0,0)', (0,0,1)'\}$

23) For any $n \in \mathbb{N}$, let $P_n$ denote the vector space of all polynomials with real coefficients and of degree at most $n$. Define $T : P_n \to P_{n+1}$ by

$$T(p)(x) = p'(x) - \int_0^x p(t)\, dt. \tag{23.1}$$

Then the dimension of the null space of $T$ is GATE MA 2008

a) 0 b) 1 c) $n$ d) $n+1$

24) Let

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \tag{24.1}$$

where $0 < \theta < \frac{\pi}{2}$. Let $V = \{u \in \mathbb{R}^3 : Mu^2 = u'\}$. Then the dimension of $V$ is GATE MA 2008

a) 0
b) 1
c) 2
d) 3

25) The number of linearly independent eigenvectors of the matrix

$$\begin{pmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} \tag{25.1}$$

is GATE MA 2008

a) 1
b) 2
c) 3
d) 4

26) Let $f$ be a bilinear transformation that maps $-1$ to 1, $i$ to $\infty$, and $i$ to 0. Then $f(1)$ is equal to GATE MA 2008

a) $-2$
b) $-1$
c) $i$
d) $-i$

27) Which one of the following does NOT hold for all continuous functions $f : [-\pi, \pi] \to \mathbb{C}$? GATE MA 2008

a) If $f(t) = f(-t)$ for each $t \in [-\pi, \pi]$, then $\int_{-\pi}^{\pi} f(t)\,dt = 2\int_0^{\pi} f(t)\,dt$
b) If $f(t) = -f(-t)$ for each $t \in [-\pi, \pi]$, then $\int_{-\pi}^{\pi} f(t)\,dt = 0$
c) $\int_{-\pi}^{\pi} f(-t)\,dt = -\int_{-\pi}^{\pi} f(t)\,dt$
d) There exists an $a$ with $-\pi < a < \pi$ such that $\int_{-\pi}^{\pi} f(t)\,dt = 2\pi f(a)$

28) Let $S$ be the positively oriented circle $|z - 3i| = 2$. Then the value of

$$\int_S \frac{dz}{z^2 + 4} \tag{28.1}$$

is GATE MA 2008

a) $-\pi$
b) $2\pi$
c) $-i\pi$
d) $i\pi$

29) Let $T$ be the closed unit disk and $\partial T$ be the unit circle. Then which one of the following holds for every analytic function $f : T \to \mathbb{C}$? GATE MA 2008

a) $f$ attains its minimum and its maximum on $\partial T$
b) $f$ attains its minimum on $\partial T$ but need not attain its maximum on $\partial T$
c) $f$ attains its maximum on $\partial T$ but need not attain its minimum on $T$
d) $f$ need not attain its maximum on $\partial T$ and also need not attain its minimum on $T$

30) Let $S$ be the disk $|z| < 3$ in the complex plane and let $f : S \to \mathbb{C}$ be an analytic function such that

$$f\left(\frac{\sqrt{2n}}{n^2+1}\right) = \frac{1+\sqrt{2n}}{n^2} \tag{30.1}$$

for each natural number $n$. Then $f(\sqrt{2})$ is equal to GATE MA 2008

a) $3 - 2\sqrt{2}$  b) $3 + 2\sqrt{2}$  c) $2 - 3\sqrt{2}$  d) $2 + 3\sqrt{2}$

31) Which one of the following statements holds? GATE MA 2008

a) The series $\sum_{n=0}^{\infty} x^n$ converges for each $x \in [-1, 1]$
b) The series $\sum_{n=0}^{\infty} x^n$ converges uniformly in $(-1, 1)$
c) The series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges for each $x \in [-1, 1]$
d) The series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges uniformly in $(-1, 1)$

32) For $x \in [-\pi, \pi]$, let

$$f(x) = (\pi + x)(\pi - x) \quad \text{and} \quad g(x) = \begin{cases} \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}. \tag{32.1}$$

Consider the statements
$P$: The Fourier series of $f$ converges uniformly to $f$ on $[-\pi, \pi]$.
$Q$: The Fourier series of $g$ converges uniformly to $g$ on $[-\pi, \pi]$. Then GATE MA 2008

a) $P$ and $Q$ are true
b) $P$ is true but $Q$ is false
c) $P$ is false but $Q$ is true
d) both $P$ and $Q$ are false

33) Let $W = \{(x, y, z) \in \mathbb{R}^3 : 1 \leq x^2 + y^2 + z^2 \leq 4\}$ and $F : W \to \mathbb{R}^3$ be defined by

$$F(x, y, z) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} [x^2 + y^2 + z^2]^{3/2} \tag{33.1}$$

for $(x, y, z) \in W$. If $\partial W$ denotes the boundary of $W$ oriented by the outward normal $n$ to $W$, then

$$\iint_{\partial W} F \cdot n \, dS \tag{33.2}$$

is equal to GATE MA 2008

a) 0  b) $4\pi$  c) $8\pi$  d) $12\pi$

34) For each $n \in \mathbb{N}$, let $f_n : [0, 1] \to \mathbb{R}$ be a measurable function such that $|f_n(t)| \leq \frac{1}{t}$ for all $t \in (0, 1]$. Let $f : [0, 1] \to \mathbb{R}$ be defined by $f(t) = 1$ if $t$ is irrational and $f(t) = -1$ if $t$ is rational. Assume that $f_n(t) \to f(t)$ as $n \to \infty$ for all $t \in [0, 1]$. Then GATE MA 2008

a) $f$ is not measurable
b) $f$ is measurable and $\int_{[0,1]} f_n \, d\mu \to 1$ as $n \to \infty$

c) $f$ is measurable and $\int_{[0,1]} f_n \, d\mu \to 0$ as $n \to \infty$
d) $f$ is measurable and $\int_{[0,1]} f_n \, d\mu \to -1$ as $n \to \infty$

35) Let $y_1$ and $y_2$ be two linearly independent solutions of $y'' + (\sin x)y = 0$, $0 \le x \le 1$. Let $g(x) = W(y_1, y_2)(x)$ be the Wronskian of $y_1$ and $y_2$. Then GATE MA 2008

a) $g' > 0$ on $[0,1]$
b) $g' < 0$ on $[0,1]$
c) $g'$ vanishes at only one point of $[0,1]$
d) $g'$ vanishes at all points of $[0,1]$

36) One particular solution of $y^{(4)} - y'' - y' + y = -e^x$ is a constant multiple of GATE MA 2008

a) $xe^x$
b) $xe^{-x}$
c) $x^2e^x$
d) $x^2e^{-x}$

37) Let $a, b \in \mathbb{R}$. Let $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ be a solution of the system of equations

$$y_1' = y_2, \quad y_2' = ay_1 + by_2. \tag{37.1}$$

Every solution $y(x) \to 0$ as $x \to \infty$ if GATE MA 2008

a) $a < 0$, $b < 0$
b) $a < 0$, $b > 0$
c) $a > 0$, $b > 0$
d) $a > 0$, $b < 0$

38) Let $G$ be a group of order 45. Let $H$ be a 3-Sylow subgroup of $G$ and $K$ be a 5-Sylow subgroup of $G$. Then GATE MA 2008

a) both $H$ and $K$ are normal in $G$
b) $H$ is normal in $G$ but $K$ is not normal in $G$
c) $H$ is not normal in $G$ but $K$ is normal in $G$
d) both $H$ and $K$ are not normal in $G$

39) The ring $\mathbb{Z}[\sqrt{-11}]$ is GATE MA 2008

a) a Euclidean Domain
b) a Principal Ideal Domain, but not a Euclidean Domain
c) a Unique Factorization Domain, but not a Principal Ideal Domain
d) not a Unique Factorization Domain

40) Let $R$ be a Principal Ideal Domain and $a, b$ any two non-unit elements of $R$. Then the ideal generated by $a$ and $b$ is also generated by GATE MA 2008

a) $a + b$
b) $ab$
c) $\gcd(a, b)$
d) $\operatorname{lcm}(a, b)$

41) Consider the action of $S_4$, the symmetric group of order 4, on $\mathbb{Z}[X_1, X_2, X_3, X_4]$ given by

$$\sigma p(X_1, X_2, X_3, X_4) = p(X_{\sigma(1)}, X_{\sigma(2)}, X_{\sigma(3)}, X_{\sigma(4)}) \quad \text{for } \sigma \in S_4. \tag{41.1}$$

Let $H_S$ denote the cyclic subgroup generated by (1423). Then the cardinality of the orbit $O_H(X_1X_3 + X_2X_4)$ of $H$ on the polynomial $X_1X_3 + X_2X_4$ is GATE MA 2008

a) 1 b) 2 c) 3 d) 4

42) Let $f : l^2 \to \mathbb{R}$ be defined by $f(x_1, x_2, \dots) = \sum_{n=1}^{\infty} \frac{x_n^2}{n^2}$. Then $\|f\|$ is equal to GATE MA 2008

a) 1 b) $\frac{1}{2}$ c) 2 d) $\sqrt{2} - 1$

43) Consider $\mathbb{R}^3$ with norm $\|\cdot\|$ and the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by the matrix

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 3 \\ 1 & 3 & 3 \end{pmatrix}. \tag{43.1}$$

Then the operator norm $\|T\|$ of $T$ is equal to GATE MA 2008

a) 6 b) 7 c) 8 d) $\sqrt{42}$

44) Consider $\mathbb{R}^2$ with norm $\|\cdot\|$, and let $Y = \{(y_1, y_2) \in \mathbb{R}^2 : y_1 + y_2 = 0\}$. If $g : Y \to \mathbb{R}$ is defined by $g(y_1, y_2) = y_2$ for $(y_1, y_2) \in Y$, then GATE MA 2008

a) $g$ has no Hahn-Banach extension to $\mathbb{R}^2$
b) $g$ has a unique Hahn-Banach extension to $\mathbb{R}^2$
c) Every linear functional $f : \mathbb{R}^2 \to \mathbb{R}$ satisfying $f(-1, 1) = 1$ is a Hahn-Banach extension of $g$ to $\mathbb{R}^2$
d) The functionals $f_1(x_1, x_2) = x_2$ and $f_2(x_1, x_2) = -x_1$ are both Hahn-Banach extensions of $g$ to $\mathbb{R}^2$

45) Let $X$ be a Banach space and $Y$ be a normed linear space. Consider a sequence $(F_n)$ of bounded linear maps from $X$ to $Y$ such that for each fixed $x \in X$, the sequence $(F_n(x))$ is bounded in $Y$. Then GATE MA 2008

a) For each fixed $x \in X$, the sequence $(F_n(x))$ is convergent in $Y$
b) For each fixed $n \in \mathbb{N}$, the set $\{F_n(x) : x \in X\}$ is bounded in $Y$
c) The sequence $(\|F_n\|)$ is bounded in $\mathbb{R}$
d) The sequence $(F_n)$ is uniformly bounded on $X$

46) Let $H = L^2([0, \pi])$ with the usual inner product. For $n \in \mathbb{N}$, let

$$u_n(t) = \sqrt{\frac{2}{\pi}} \sin(nt), \quad t \in [0, \pi], \tag{46.1}$$

and $E = \{u_n : n \in \mathbb{N}\}$. Then GATE MA 2008

a) $E$ is not a linearly independent subset of $H$
b) $E$ is a linearly independent subset of $H$, but is not an orthonormal subset of $H$
c) $E$ is an orthonormal subset of $H$, but is not an orthonormal basis for $H$
d) $E$ is an orthonormal basis for $H$

47) Let $X = \mathbb{R}$ and let $\mathcal{I} = \{U \subseteq X : X \setminus U \text{ is finite}\} \cup \{\emptyset, X\}$. The sequence $(1/n)_{n=1}^{\infty}$ in $(X, \mathcal{I})$ GATE MA 2008

a) Converges to 0 and not to any other point of $X$

b) Does not converge to 0
c) Converges to each point of $X$
d) Is not convergent in $X$

48) Let

$$E = \{(x, y) \in \mathbb{R}^2 : |x| \leq 1, |y| \leq 1\}, \quad \text{and define } f : E \to \mathbb{R} \text{ by } f(x, y) = 1 + x^2 + y^2. \tag{48.1}$$

Then the range of $f$ is a GATE MA 2008

a) Connected open set
b) Connected closed set
c) Bounded open set
d) Closed and unbounded set

49) Let $X = \{1, 2, 3\}$ and $\mathcal{I} = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, X\}$. The topological space $(X, \mathcal{I})$ has the property $P$ if for any two proper disjoint closed subsets $Y$ and $Z$ of $X$, there exist disjoint open sets $U, V$ such that $Y \subseteq U$ and $Z \subseteq V$. Then the space $(X, \mathcal{I})$ GATE MA 2008

a) Is $T_1$ and satisfies $P$
b) Is $T_1$ and does not satisfy $P$
c) Is not $T_1$ and satisfies $P$
d) Is not $T_1$ and does not satisfy $P$

50) Which one of the following subsets of $\mathbb{R}$ (with the usual metric) is NOT complete? GATE MA 2008

a) $[1, 2] \cup [3, 4]$
b) $[0, \infty)$
c) $[0, 1)$
d) $\{0\} \cup \mathbb{N}$

51) Consider the function

$$f(x) = \begin{cases} k(x - \lfloor x \rfloor) & 0 \leq x < 2 \\ 0 & \text{otherwise} \end{cases} \tag{51.1}$$

where $\lfloor x \rfloor$ is the integral part of $x$. The value of $k$ for which $f$ is a probability density function of some random variable is GATE MA 2008

a) $\frac{1}{4}$
b) $\frac{1}{2}$
c) 1
d) 2

52) For two random variables $X$ and $Y$, the regression lines are given by $Y = 5X - 15$ and $X = 10Y - 35$. Then the regression coefficient of $X$ on $Y$ is GATE MA 2008

a) 0.1
b) 0.2
c) 5
d) 10

53) In an examination there are 80 questions each having four choices. Exactly one choice is correct and the other three are wrong. A student is awarded 1 mark for each correct answer, and -0.25 for each wrong answer. If a student ticks the answer of each question randomly, then the expected value of the total marks in the examination is GATE MA 2008

a) $-15$ b) $0$ c) $5$ d) $20$

54) Let $X_1, X_2, \dots, X_n$ be a random sample from a uniform distribution on $[0, \theta]$. Then the maximum likelihood estimator (MLE) of $\theta$ based on the sample is GATE MA 2008

a) $X_1$
b) $\frac{1}{n}\sum_{i=1}^{n} X_i$
c) $\min\{X_1, X_2, \dots, X_n\}$
d) $\max\{X_1, X_2, \dots, X_n\}$

55) The cost matrix of a transportation problem is given by

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 0 \\ 0 & 2 & 2 & 1 \end{pmatrix} \tag{55.1}$$

A feasible solution has $X_{12} = 6$, $X_{23} = 2$, $X_{24} = 6$, $X_{31} = 4$, $X_{33} = 6$. Then the solution is GATE MA 2008

a) degenerate and basic
b) non-degenerate and basic
c) degenerate and non-basic
d) non-degenerate and non-basic

56) The maximum value of $z = 3x_1 - x_2$ subject to $2x_1 - x_2 \le 1$, $x_1 \le 3$, and $x_1, x_2 \ge 0$ is GATE MA 2008

a) $0$ b) $4$ c) $6$ d) $9$

57) Consider the problem of maximizing $z = 2x_1 + 3x_2 - 4x_3 + x_4$ subject to

$$\begin{cases} x_1 + x_2 + x_3 = 2, \\ x_1 - x_2 + x_3 = 2, \\ 2x_1 + 3x_2 + 2x_3 - x_4 = 0, \\ x_i \ge 0, \quad i = 1, 2, 3, 4. \end{cases} \tag{57.1}$$

Then GATE MA 2008

a) $(1, 0, 1, 4)$ is a basic feasible solution but $(2, 0, 0, 4)$ is not
b) $(1, 0, 1, 4)$ is not a basic feasible solution but $(2, 0, 0, 4)$ is
c) Neither $(1, 0, 1, 4)$ nor $(2, 0, 0, 4)$ is a basic feasible solution
d) Both $(1, 0, 1, 4)$ and $(2, 0, 0, 4)$ are basic feasible solutions

58) In the closed system of a simple harmonic motion of a pendulum, let $H$ denote the Hamiltonian and $E$ be the total energy. Then GATE MA 2008

a) $H$ is a constant and $H = E$
b) $H$ is a constant but $H \ne E$
c) $H$ is not constant but $H = E$
d) $H$ is not constant and $H \ne E$

59) The possible values of $a$ for which the variational problem

$$J[y(x)] = \int_0^1 (3y^2 + 2x^2y')\,dx, \quad y(a) = 1, \tag{59.1}$$

has extremals are GATE MA 2008

a) $-1, 0$ b) $0, 1$ c) $-1, 1$ d) $-1, 0, 1$

60) The functional

$$\int_0^1 (y^2 + x)\,dx, \tag{60.1}$$

given $y(1) = 1$, achieves its GATE MA 2008

a) weak maximum on all its extremals
b) weak minimum on all its extremals
c) weak maximum on some, but not on all of its extremals
d) weak minimum on some, but not on all of its extremals

61) The integral equation

$$x(t) = \sin t + \lambda \int_0^1 (s^2t^3 + e^{s^2+t^2})x(s)\,ds, \quad 0 \le t \le 1, \lambda \in \mathbb{R}, \lambda \ne 0, \tag{61.1}$$

has a solution for GATE MA 2008

a) all non-zero values of $\lambda$
b) no value of $\lambda$
c) only countably many positive values of $\lambda$
d) only countably many negative values of $\lambda$

62) The integral equation

$$x(t) - \int_0^1 \cos t\, x(s)\,ds = \sinh t, \quad 0 < t \le 1, \tag{62.1}$$

has GATE MA 2008

a) no solution
b) a unique solution
c) more than one but finitely many solutions
d) infinitely many solutions

63) If

$$y_{i+1} = y_i + hp(f, x, y, h), \quad i = 1, 2, \dots, \tag{63.1}$$

where

$$p(f, x, y, h) = af(x, y) + bf(x + h, y + hf(x, y)), \tag{63.2}$$

is a second order accurate scheme to solve the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x) = y_0, \tag{63.3}$$

then $a$ and $b$, respectively, are GATE MA 2008

a) 2, 2 b) 1, $-1$ c) 2, 1/2 d) $h$, $-h$

64) If a quadrature formula

$$\int_{-1}^{1} f(x)\,dx \approx -3f(-1) + Kf(0) + f(1), \tag{64.1}$$

that approximates $\int_{-1}^{1} f(x)\,dx$, is found to be exact for quadratic polynomials, then the value of $K$ is GATE MA 2008

a) 2 b) 1 c) 0 d) -1

65) If

$$\frac{279}{58} = a, \tag{65.1}$$

then the value of $a$ is GATE MA 2008

a) -2 b) -1 c) 1 d) 2

66) Using the least squares method, if a curve

$$y = ax^2 + bx + c \tag{66.1}$$

is fitted to the collinear data points $(-1,-3)$, $(1,1)$, $(3,5)$, and $(7,13)$, then the triplet $(a,b,c)$ is equal to GATE MA 2008

a) $(-1,2,0)$ b) $(0,2,-1)$ c) $(2,-1,0)$ d) $(0,-1,2)$

67) A quadratic polynomial $p(x)$ is constructed by interpolating the data points $(0,1)$, $(1,e)$, and $(2,e^2)$. If $\sqrt{e}$ is approximated by using $p(x)$, then its approximate value is GATE MA 2008

a) $(3 + 6e - e^2)$
b) $(3 - 6e + 2e^2)$
c) $(3 - 6e - e^2)$
d) $(3 + 6e - 2e^2)$

68) The characteristic curve of

$$2yu_x + (2x + y^2)u_y = 0 \tag{68.1}$$

passing through $(0,0)$ is GATE MA 2008

a) $y^2 = 2(e^x + x - 1)$
b) $y^2 = 2(e^x - x + 1)$
c) $y^2 = 2(e^x - x - 1)$
d) $y^2 = 2(e^x + x + 1)$

69) The initial value problem

$$u_t + u_x = 1, \quad u(s,s) = \sin s, \quad 0 \le s \le 1, \tag{69.1}$$

has GATE MA 2008

a) two solutions
b) a unique solution
c) no solution
d) infinitely many solutions

70) Let $u(x,t)$ be the solution of

$$u_{tt} - u_{xx} = 1, \quad x \in \mathbb{R}, t > 0, \tag{70.1}$$

with initial conditions

$$u(x,0) = 0, \quad u_t(x,0) = 0, \quad x \in \mathbb{R}. \tag{70.2}$$

Then $u\left(\frac{1}{2}, \frac{1}{2}\right)$ is equal to GATE MA 2008

a) $\frac{1}{8}$
b) $-\frac{1}{8}$
c) $\frac{1}{4}$
d) $-\frac{1}{4}$

71) Let $X = C([0,1])$ with sup norm $\|\cdot\|$.
Let $S = \{x \in X : \|x\| \le 1\}$. Then GATE MA 2008

a) $S$ is convex and compact
b) $S$ is not convex but compact
c) $S$ is convex but not compact
d) $S$ is neither convex nor compact

72) Which one of the following is true? GATE MA 2008

a) $C([0,1])$ is dense in $X$
b) $X$ is dense in $L^0([0,1])$
c) $X$ has a countable basis
d) There is a sequence in $X$ which is uniformly Cauchy on $[0,1]$ but does not converge uniformly on $[0,1]$

73) Let $I = \{x \in X : x(0) = 0\}$. Then GATE MA 2008

a) $I$ is not an ideal of $X$
b) $I$ is an ideal, but not a prime ideal of $X$
c) $I$ is a prime ideal, but not a maximal ideal of $X$
d) $I$ is a maximal ideal of $X$

74) Let $X = C'([0,1])$ and $Y = C([0,1])$, both with the sup norm. Define $F : X \to Y$ by $F(x) = x + x'$ and $f(x) = x(1) + x'(1)$ for $x \in X$. Then GATE MA 2008

a) $F$ and $f$ are continuous
b) $F$ is continuous and $f$ is discontinuous
c) $F$ is discontinuous and $f$ is continuous
d) $F$ and $f$ are discontinuous

75) Then GATE MA 2008

a) $F$ and $f$ are closed maps
b) $F$ is a closed map and $f$ is not a closed map
c) $F$ is not a closed map and $f$ is a closed map
d) Neither $F$ nor $f$ is a closed map

76) Let

$$N = \begin{pmatrix} \frac{4}{5} & \frac{3}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{76.1}$$

Then $N$ is GATE MA 2008

a) non-invertible
b) skew-symmetric
c) symmetric
d) orthogonal

77) If $M$ is any $3 \times 3$ real matrix, then trace$(NMN')$ is equal to GATE MA 2008

a) $(\text{trace}(N))^2\,\text{trace}(M)$
b) $2\,\text{trace}(N) + \text{trace}(M)$
c) $\text{trace}(M)$
d) $(\text{trace}(N))^2 + \text{trace}(M)$

78) Let $f(z) = \frac{\cos z - 1}{z}$ for non-zero $z \in \mathbb{C}$ and $f(0) = 0$. Also, let $g(z) = \sinh z$ for $z \in \mathbb{C}$. Then $f(z)$ has a zero at $z = 0$ of order GATE MA 2008

a) 0
b) 1
c) 2
d) greater than 2

79) Then $g(z)/(zf(z))$ has a pole at $z = 0$ of order GATE MA 2008

a) 1
b) 2
c) 3
d) greater than 3

80) Let $n \geq 3$ be an integer. Let $y$ be the polynomial solution of

$$(1 - x^2)y'' - 2xy' + n(n-1)y = 0 \tag{80.1}$$

satisfying $y(1) = 1$. Then the degree of $y$ is GATE MA 2008

a) $n$
b) $n - 1$
c) Less than $n - 1$
d) Greater than $n + 1$

81) If

$$I = \int y(x)\,x\,dx \quad \text{and} \quad J = \int y(x)\,x^2\,dx, \tag{81.1}$$

then GATE MA 2008

a) $I \neq 0,\ J \neq 0$
b) $I \neq 0,\ J = 0$
c) $I = 0,\ J \neq 0$
d) $I = 0,\ J = 0$

82) Consider the boundary value problem

$$u_{xx} + u_{yy} = 0, \quad x \in (0, \pi), \quad y \in (0, \pi), \tag{82.1}$$

with boundary conditions

$$u(x,0) = u(x,\pi) = u(0,y) = 0. \tag{82.2}$$

Any solution of this boundary value problem is of the form GATE MA 2008

a) $\sum_{n=1}^{\infty} a_n \sinh nx \sin ny$
b) $\sum_{n=1}^{\infty} a_n \cosh nx \sin ny$
c) $\sum_{n=1}^{\infty} a_n \sinh nx \cos ny$
d) $\sum_{n=1}^{\infty} a_n \cosh nx \cos ny$

83) If an additional boundary condition

$$u_x(\pi, y) = \sin y \tag{83.1}$$

is satisfied, then

$$u\left(x, \frac{\pi}{2}\right) \tag{83.2}$$

is equal to GATE MA 2008

a) $\frac{\pi}{2}\frac{e^x-e^{-x}}{e^\pi+e^{-\pi}}$ b) $\frac{\pi(e^x+e^{-x})}{e^\pi-e^{-\pi}}$ c) $\frac{\pi(e^x-e^{-x})}{e^\pi+e^{-\pi}}$ d) $\frac{\pi}{2}\frac{e^x+e^{-x}}{e^\pi+e^{-\pi}}$

84) Let a random variable $X$ follow the exponential distribution with mean 2. Define

$$Y = [X - 2 \mid X > 2]. \tag{84.1}$$

The value of $P(Y \geq t)$ is GATE MA 2008

a) $e^{-t/2}$ b) $e^{-2t}$ c) $\frac{1}{2}e^{-t/2}$ d) $\frac{1}{2}e^{-t}$

85) The value of $E(Y)$ is equal to GATE MA 2008

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) 1 d) 2