

1.5.9

EE25BTECH11021 - Dhanush Sagar

Problem (1.5.9). Find the ratio in which the Y-axis divides the line segment joining

$$A = \begin{pmatrix} 5 \\ -6 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -1 \\ -4 \end{pmatrix}. \quad (0.1)$$

Also, find the coordinates of the point of intersection.

Solution:

given points are A and B

$$\mathbf{A} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

Let the Y-axis divide the $\overline{\mathbf{AB}}$ at point \mathbf{P} in the ratio $k : 1$. Since \mathbf{P} lies on Y-axis, let intersection point P be

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix}$$

The point \mathbf{A} , \mathbf{B} , \mathbf{P} are collinear.

$$\implies \text{rank}(\mathbf{B} - \mathbf{A} \quad \mathbf{P} - \mathbf{A}) = 1 \quad (0.2)$$

$$\begin{pmatrix} -6 & -5 \\ 2 & y+6 \end{pmatrix} \xrightarrow{R_2 \rightarrow \frac{1}{3}R_1 + R_2} \begin{pmatrix} -6 & -5 \\ 0 & y + \frac{13}{3} \end{pmatrix} \quad (0.3)$$

The number of nonzero rows in the row reduced matrix is defined as the rank. For above matrix to be of rank 1,

$$y + \frac{13}{3} = 0 \quad (0.4)$$

$$y = -\frac{13}{3} \quad (0.5)$$

\therefore The coordinates of the point of intersection are

$$\mathbf{P} = \begin{pmatrix} 0 \\ -\frac{13}{3} \end{pmatrix}$$

Substituting the values of \mathbf{A} , \mathbf{B} and \mathbf{P} ,

$$k = \frac{\begin{pmatrix} 5 & -5 \\ 2 & y+6 \end{pmatrix} \begin{pmatrix} 1 \\ -\frac{1}{3} \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ -\frac{1}{3} \end{pmatrix} \right\|^2} = 5 \quad (0.6)$$

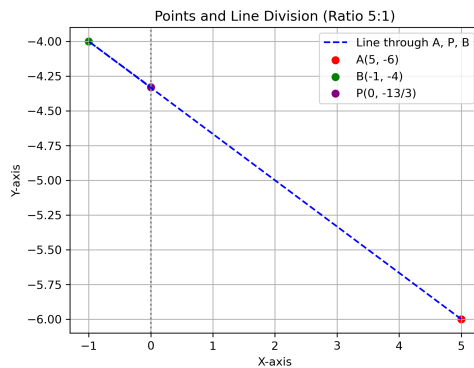


Fig. 0.1

Answer: The Y-axis divides \overline{AB} in the ratio 5 : 1 (internally), and the intersection point is

$$P = \begin{pmatrix} 0 \\ 13 \\ 3 \end{pmatrix} \quad (0.7)$$