1.4.25

EE25BTECH11009-Anshu kumar ram

Question:
Find the position vector of a point **R** which divides the line joining two points **P** and **Q** whose position vectors are $2\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - 3\mathbf{b}$ externally in the ratio 1 : 2.
Solution:

The given position vectors are

$$\mathbf{P} = 2\mathbf{a} + \mathbf{b}, \tag{0.1}$$

$$\mathbf{Q} = \mathbf{a} - 3\mathbf{b}. \tag{0.2}$$

$$\begin{pmatrix}\mathbf{P} & \mathbf{Q}\end{pmatrix} = \begin{pmatrix}\mathbf{a} & \mathbf{b}\end{pmatrix}\begin{pmatrix}2 & 1\\ 1 & -3\end{pmatrix} \tag{0.3}$$

Now, for external division of **PQ** in the ratio 1 : 2, the point **R** is given by

$$\mathbf{R} = \frac{1\mathbf{Q} - 2\mathbf{P}}{1 - 2} \tag{0.4}$$

$$\mathbf{R} = \begin{pmatrix}\mathbf{P} & \mathbf{Q}\end{pmatrix}\begin{pmatrix}-2\\ 1\end{pmatrix} \cdot \frac{1}{-1} \tag{0.5}$$

Substituting $\begin{pmatrix}\mathbf{P} & \mathbf{Q}\end{pmatrix}$ in terms of $\mathbf{a}, \mathbf{b}$:

$$\mathbf{R} = \begin{pmatrix}\mathbf{a} & \mathbf{b}\end{pmatrix}\begin{pmatrix}2 & 1\\ 1 & -3\end{pmatrix}\begin{pmatrix}-2\\ 1\end{pmatrix} \cdot \frac{1}{-1} \tag{0.6}$$

$$\therefore \mathbf{R} = \frac{1}{-1}\begin{pmatrix}\mathbf{a} & \mathbf{b}\end{pmatrix}\begin{pmatrix}-3\\ -5\end{pmatrix} \tag{0.7}$$

$$= \begin{pmatrix}\mathbf{a} & \mathbf{b}\end{pmatrix}\begin{pmatrix}3\\ 5\end{pmatrix} \tag{0.8}$$

From the figure, it is verified that the theoretical result matches the computational solution.