Assignment 5

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Download all python codes from

https://github.com/AmulyaTallamraju/Assignment-5/blob/main/Assignment5/codes/Assignment-5.py

and latex-tikz codes from

https://github.com/AmulyaTallamraju/Assignment-5/blob/main/Assignment5/Assignment-5.tex

GATE 2017 MA - Q.46

Let X be a random variable with probability mass function $p(n) = \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{n-1}$ $n = 1, 2\dots$
Then $E[X - 3|X > 3]$

Solution

Given

$$\Pr(X = n) = \begin{cases} \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{n-1} & n = 1, 2\dots \\ 0 & otherwise \end{cases} \tag{0.0.1}$$

Using the linearity of the expectation operator:

$$E[X - 3 \mid X > 3] = E[X \mid X > 3] - 3 \tag{0.0.2}$$

Now ,

$$E[X \mid X > 3] = \sum_{x=1}^{\infty} x \Pr(X = x \mid X > 3) \tag{0.0.3}$$

$$= \sum_{x=1}^{\infty} x \frac{\Pr(X = x, X > 3)}{\Pr(X > 3)} \tag{0.0.4}$$

Calculating $\Pr(X > 3)$

$$\Pr(X > 3) = 1 - \Pr(X \leq 3) \tag{0.0.5}$$

$$= 1 - \sum_{x'=1}^{3} \Pr(X = x') \tag{0.0.6}$$

$$= 1 - \sum_{x'=1}^{3} \left(\frac{3}{4}\right)^{x'-1} \left(\frac{1}{4}\right) \tag{0.0.7}$$

$$= \frac{27}{64} \tag{0.0.8}$$

Also,

$$\Pr(X = x, X > 3) = \begin{cases} \Pr(X = x) & x > 3 \\ 0 & x \leq 3 \end{cases} \tag{0.0.9}$$

Substituting (0.0.8) and (0.0.9) in (0.0.4) we get

$$E[X \mid X > 3] = \sum_{x=1}^{3} 0 + \sum_{x=4}^{\infty} \left[x \frac{\Pr(X = x)}{\frac{27}{64}} \right] \tag{0.0.10}$$

$$= \frac{64}{27} \sum_{x=4}^{\infty} \left[x \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{x-1} \right] \tag{0.0.11}$$

$$= \frac{16}{27} \sum_{x=4}^{\infty} \left[x \left(\frac{3}{4}\right)^{x-1} \right] \tag{0.0.12}$$

Let

$$S = \sum_{x=4}^{\infty} \left[x \left(\frac{3}{4}\right)^{x-1} \right] \tag{0.0.13}$$

Multiplying ((0.0.13)) with $\frac{3}{4}$ on both sides gives

$$\frac{3}{4} S = \sum_{x=4}^{\infty} x \frac{1}{4} \left(\frac{3}{4}\right)^{x} \tag{0.0.14}$$

From (0.0.14) and(0.0.13)

$$S = 4\left(\frac{3}{4}\right)^3 + 5\left(\frac{3}{4}\right)^4 + 6\left(\frac{3}{4}\right)^5 \dots \tag{0.0.15}$$

$$\frac{3}{4} S = 0\left(\frac{3}{4}\right)^3 + 4\left(\frac{3}{4}\right)^4 + 5\left(\frac{3}{4}\right)^5 + \dots \tag{0.0.16}$$

subtracting (0.0.14) from (0.0.13) we get

$$\frac{S}{4} = 4\left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^4 + \left(\frac{3}{4}\right)^5 + \left(\frac{3}{4}\right)^6 + \dots \tag{0.0.17}$$

$$= 4\left(\frac{3}{4}\right)^3 + \sum_{x=4}^{\infty} \left(\frac{3}{4}\right)^x \tag{0.0.18}$$

$$= \frac{189}{64} \tag{0.0.19}$$

Substituting vale of S in (0.0.12) we get

$$E[X|X > 3] = 7 \tag{0.0.20}$$

Thus putting this in (0.0.2)

$$E[X-3|X>3]=4 \tag{0.0.21}$$

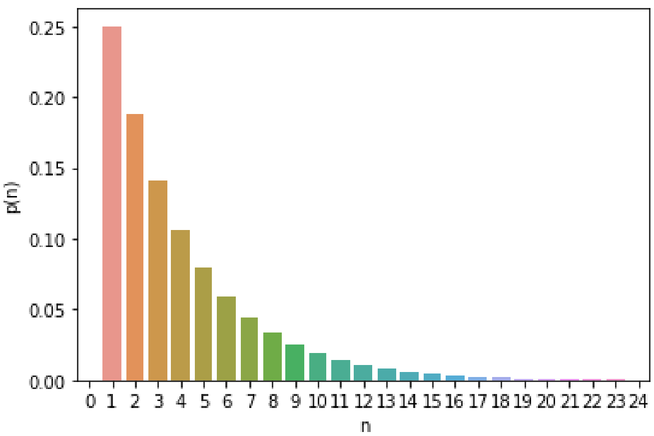

Fig. 0: PMF of X