Assignment 5

Adarsh Sai - AI20BTECH11001

Download all python codes from

```
https://github.com/Adarsh541/AI1103−prob−and−ranvar/blob/main/Assignment5/codes/Assignment5.py
```

Download latex-tikz codes from

```
https://github.com/Adarsh541/AI1103−prob−and−ranvar/blob/main/Assignment5/Assignment5.tex
```

1 Problem(GATE 2020(ST) Q16)

The characteristic function of a random variable X is given by

$$\phi_X(t) = \begin{cases} \frac{\sin t \cos t}{t} & t \neq 0 \\ 1 & t = 0 \end{cases} \tag{1.0.1}$$

Then $P\left(|X| \leq \frac{3}{2}\right) =$

2 Solution(GATE 2020(ST) Q16)

The pdf is given by

$$f_X(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_X(t)\, e^{-jxt} dt \tag{2.0.1}$$

If

$$g(x) \overset{\mathcal{H}}{\longleftrightarrow} FG(t) \tag{2.0.2}$$

$$\implies G(t) \overset{\mathcal{H}}{\longleftrightarrow} Fg(-x) \tag{2.0.3}$$

where $\left(\overset{\mathcal{H}}{\longleftrightarrow} F\right)$ represents Fourier transform and

$$G(t) = \int_{-\infty}^{\infty} g(x)\, e^{-j2\pi xt} dx \tag{2.0.4}$$

we know that the Fourier transform of rectangular function is sinc function

$$rect\left(\frac{x}{\tau}\right) \overset{\mathcal{H}}{\longleftrightarrow} F\tau sinc(t\tau) \tag{2.0.5}$$

from (2.0.3) we get

$$\tau sinc(t\tau) \overset{\mathcal{H}}{\longleftrightarrow} Frect\left(-\frac{x}{\tau}\right) \tag{2.0.6}$$

$$\implies rect\left(-\frac{x}{\tau}\right) = \int_{-\infty}^{\infty} \tau \frac{\sin \pi t\tau}{\pi t\tau} e^{-j2\pi xt} dt \tag{2.0.7}$$

substituting $\tau = \frac{2}{\pi}$ and changing $2\pi x \to x$ we get

$$\frac{1}{4} rect\left(\frac{-x}{4}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\frac{\sin 2t}{2t}\right) e^{-jxt} dt \tag{2.0.8}$$

So

$$f_X(x) = \frac{1}{4} rect\left(\frac{-x}{4}\right) \tag{2.0.9}$$

$$P\left(|X| \leq \frac{3}{2}\right) = \int_{\frac{-3}{2}}^{\frac{3}{2}} \frac{1}{4} dx \tag{2.0.10}$$

$$= \frac{3}{4} \tag{2.0.11}$$

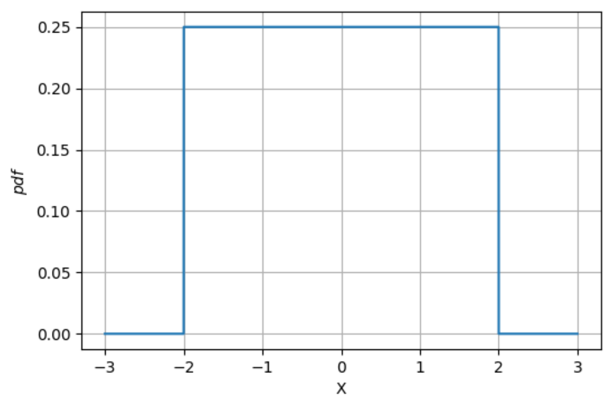

Fig. 0: $f_X(x)$