Signal Processing

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Contents

***Abstract*—This manual provides solved problems in signal processing from GATE exam papers.**

1. Let the state-space representation on an LTI system be $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + du(t)$ where A,B,C are matrices, d is a scalar, u(t) is the input to the system, and y(t) is its output. Let $B = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^\top$ and $d = 0$. Which one of the following options for A and C will ensure that the transfer function of this LTI system is

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{1.1}$$

(A) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

(B) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

(C) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

(D) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

Solution: From the given information,

$$\begin{pmatrix} \dot{x}(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} A & B \\ C & d \end{pmatrix} \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} \tag{1.2}$$

Taking Laplace transform on both sides,

$$\begin{pmatrix} sX(s) \\ Y(s) \end{pmatrix} = \begin{pmatrix} A & B \\ C & d \end{pmatrix} \begin{pmatrix} X(s) \\ U(s) \end{pmatrix} \tag{1.3}$$

$$\implies sX(s) = AX(s) + BU(s) \tag{1.4}$$

$$\implies X(s) = (sI - A)^{-1} BU(s) \tag{1.5}$$

$$\implies Y(s) = CX(s) + dU(s) \tag{1.6}$$

$$= C(sI - A)^{-1} BU(s) + dU(s) \tag{1.7}$$

By definition,

$$Y(s) = H(s)U(s) \tag{1.8}$$

$$\implies H(s) = C(sI - A)^{-1} B + d \tag{1.9}$$

$$= \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{1.10}$$

$$\implies C(sI - A)^{-1} B + d = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{1.11}$$

Now we cross verify the options with eq 1.11. By using a python script,

(A)

$$C(sI-A)^{-1}B+d = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{1.12}$$

(B)

$$C(sI-A)^{-1}B+d = \frac{1}{s^3 + 1s^2 + 2s + 3} \tag{1.13}$$

(C)

$$C(sI-A)^{-1}B+d = \frac{s^2}{s^3 + 3s^2 + 2s + 1} \tag{1.14}$$

(D)

$$C(sI-A)^{-1}B+d = \frac{s^2}{s^3 + 1s^2 + 2s + 3} \tag{1.15}$$

Hence A is the correct option.

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