Gate Assignment 2

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Download all python codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/GateAssignment2/codes

and latex-tikz codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/GateAssignment2/GateAssignment2.tex

1 Problem (EC-2010 Q15)

Two discrete time systems with impulse responses $h_1[n] = \delta[n-1]$ and $h_2[n] = \delta[n-2]$ are connected in cascade. The overall impulse response of the cascaded system is

1) $\delta[n-1] + \delta[n-2]$
2) $\delta[n-4]$
3) $\delta[n-3]$
4) $\delta[n-1]\delta[n-2]$

2 Solution

Definition 2.1 (Discrete Time Fourier Transform)**.** *It is the member of the Fourier transform family that operates on aperiodic, discrete signals. If $x[n]$ is the input signal in time domain, its DTFT is $X(\Omega)$, a complex function in frequency domain.*

$$X(\Omega) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\Omega k} \tag{2.0.1}$$

Definition 2.2 (Inverse DTFT)**.** *If $X(\Omega)$ is the DTFT of $x[n]$, then $x[n]$ is the inverse DTFT of $X(\Omega)$.*

Lemma 2.1. *For any real c, if $x[n] = \delta[n-c]$, then*

$$X(\Omega) = e^{-j\Omega c} \tag{2.0.2}$$

Proof.

$$X(\Omega) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\Omega k} = \sum_{k=-\infty}^{\infty} \delta[k-c]e^{-j\Omega k} \tag{2.0.3}$$

$$= \delta[0]e^{-j\Omega c} = e^{-j\Omega c} \tag{2.0.4}$$

□

Corollary 2.1.1. *For any real c, if $X(\Omega) = e^{-j\Omega c}$, then $x[n] = \delta[n-c]$*

Theorem 2.2 (Convolution theorem)**.** *If $x[n] = x_1[n] * x_2[n]$, then the DTFT of $x[n]$ can be given by*

$$X(\Omega) = X_1(\Omega)X_2(\Omega) \tag{2.0.5}$$

Theorem 2.3. *For a cascade system, the overall impulse response is given by*

$$h[n] = h_1[n] * h_2[n] \tag{2.0.6}$$

Proof. Given, two impulse responses $h_1[n], h_2[n]$ in cascade. The output signal $y[n]$ for the input signal $x[n]$ is given by

$$y[n] = h_2[n] * (h_1[n] * x) \tag{2.0.7}$$

$$= (h_2[n] * h_1[n]) * x[n] \tag{2.0.8}$$

$$= (h_1[n] * h_2[n]) * x[n] \tag{2.0.9}$$

$$= h[n] * x[n] \tag{2.0.10}$$

$h[n] = h_1[n] * h_2[n]$ is overall impulse response. (Convolution is associative and commutative) □

Given,

$$h_1[n] = \delta[n-1] \tag{2.0.11}$$

$$h_2[n] = \delta[n-2] \tag{2.0.12}$$

To find: $h(t)$. We know,

$$h[n] = h_1[n] * h_2[n] \tag{2.0.13}$$

From (2.0.5)

$$H(\Omega) = H_1(\Omega)H_2(\Omega) \tag{2.0.14}$$

Using (2.0.2),

$$H(\Omega) = e^{-j\Omega}e^{-2j\Omega} = e^{-3j\Omega} \tag{2.0.15}$$

$$\Rightarrow h[n] = \delta[n-3] \tag{2.0.16}$$

Option 3 is the correct answer.

Theorem 2.4. *If $y[n] = x[n] * \delta[n-n_0]$, then $y[n] = x[n-n_0]$*

Proof.

$$y[n] = x[n] * \delta[n - n_0] \tag{2.0.17}$$

$$= \sum_{k=-\infty}^{\infty} x[k]\delta[n - n_0 - k] \tag{2.0.18}$$

$$= x[n - n_0] \tag{2.0.19}$$

□

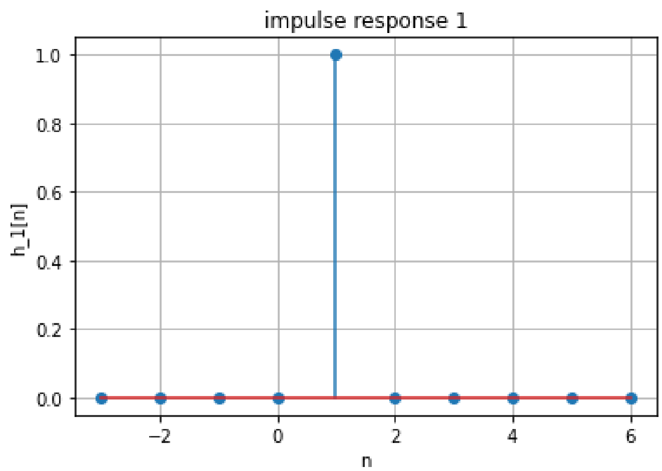

Fig. 4: Plot of $h_1[n] = \delta[n - 1]$

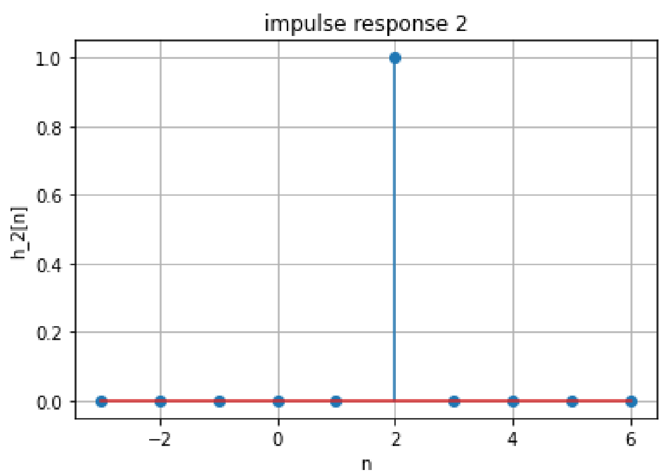

Fig. 4: Plot of $h_2[n] = \delta[n - 2]$

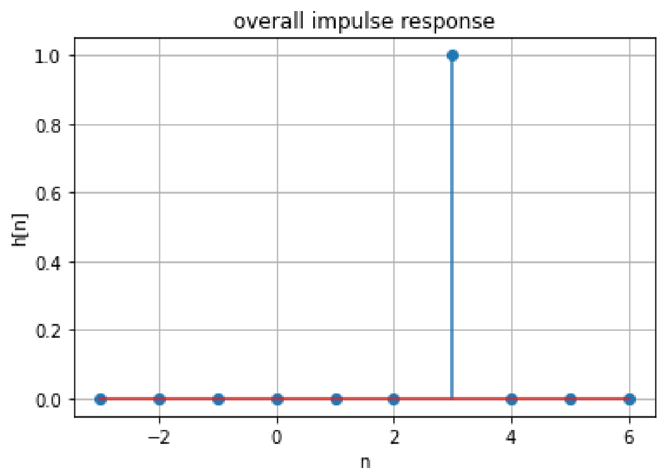

Fig. 4: Plot of $h[n] = \delta[n - 3]$