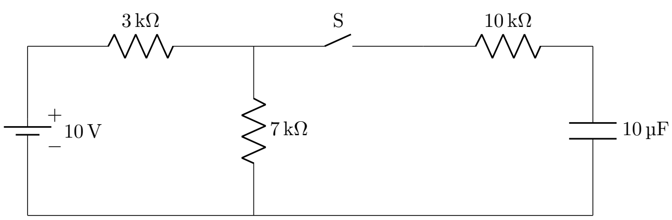
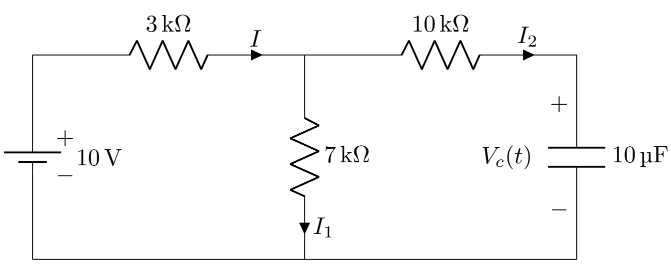
GATE 2023 BM 30

EE23BTECH11007 - Aneesh Kadiyala*

Question: In the following circuit, the switch S is open for $t < 0$ and closed for $t \geq 0$. What is the steady state voltage (in Volts) across the capacitor when the switch is closed?



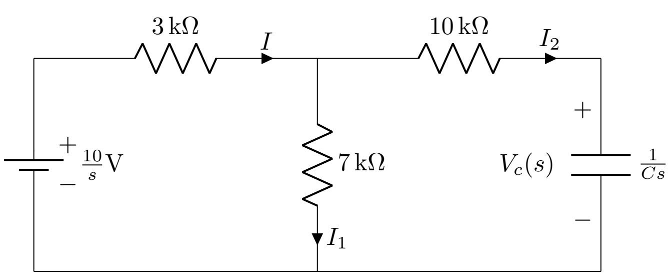
Solution:



In steady state, no current flows through the capacitor.

$$I_2 = 0 \tag{1}$$
$$V_c = (7\text{k}\Omega)\, I_1 \tag{2}$$
$$= (7\text{k}\Omega)\, I \tag{3}$$
$$= (7\text{k}\Omega)\, \frac{10\text{V}}{10\text{k}\Omega} \tag{4}$$
$$\implies V_c = 7\text{V} \tag{5}$$

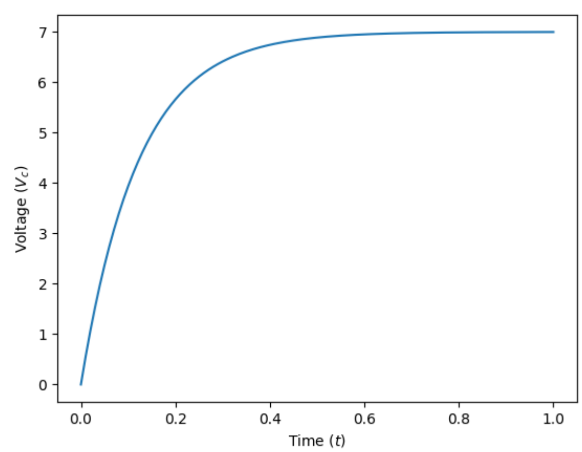
In s-domain:



$$\implies I(s) = \frac{\frac{10}{s}\text{V}}{3\text{k}\Omega + \frac{(7\text{k}\Omega)(10\text{k}\Omega + \frac{1}{sC})}{17\text{k}\Omega + \frac{1}{sC}}} \tag{6}$$
$$I = I_1 + I_2 \tag{7}$$
$$I_1\,(7\text{k}\Omega) = I_2 \left(10\text{k}\Omega + \frac{1}{sC}\right) \tag{8}$$
$$I_2(s) = \frac{7\text{k}\Omega}{17\text{k}\Omega + \frac{1}{sC}} I(s) \tag{9}$$
$$\implies I_2(s) = \frac{7\left(10^{-5}\right)}{0.121s + 1} \tag{10}$$
$$V_c(s) = I_2(s)\, \frac{1}{sC} \tag{11}$$
$$= \frac{7}{s\,(0.121s + 1)} \tag{12}$$
$$= 7\left(\frac{1}{s} - \frac{1}{s + \frac{1}{0.121}}\right) \tag{13}$$

Taking inverse Laplace transform:

$$V_c(t) = 7u(t)\left(1 - e^{-\frac{t}{-0.121}}\right) \tag{14}$$



In steady state $t \to \infty$. From (14):

$$\lim_{t\to\infty} V_c(t) = 7\text{V} \tag{15}$$