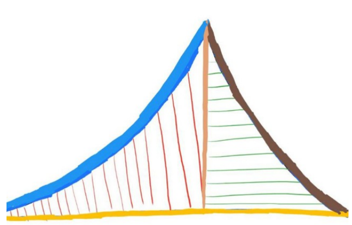
# SIGNAL PROCESSING

# Through GATE

G. V. V. Sharma



# Contents

# Introduction

This book provides solutions to signal processing problems in GATE.

# Chapter 1

# Harmonics

# Chapter 2

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# Chapter 4

# Systems

4.1 Consider a unity-gain negative feedback system consisting of the plant $G(s)$ and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{(s-1)}$$

**Solution:**

| **Parameter** | **Description** | **Value** |
|---|---|---|
| $K_p$ | Proportional Gain | 3 |
| $K_i$ | Integral Gain | 1 |
| $r(t)$ | Reference Input | $u(t)$ |
| $w(t)$ | Controller Output | ? |
| $y(t)$ | Plant Output | ? |
| $e(t)$ | Error Input | $r(t) - y(t)$ |

Table 1: Parameter Table

From theFig. 4.1:

$$E(s) = U(s) - Y(s) \tag{4.1}$$

$$W(s) = 3E(s) + \frac{1}{s}E(s) \tag{4.2}$$

$$Y(s) = G(s)\,W(s) \tag{4.3}$$

Some results:

$$tx(t) \xleftrightarrow{\mathcal{L}} -\frac{dX(s)}{ds} \tag{4.4}$$

$$e^{-at}x(t) \xleftrightarrow{\mathcal{L}} X(s+a) \tag{4.5}$$

By using (4.4) and (4.5):

$$e^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+1}, Re(s) > -1 \tag{4.6}$$

$$te^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{(s+1)^2}, Re(s) > -1 \tag{4.7}$$

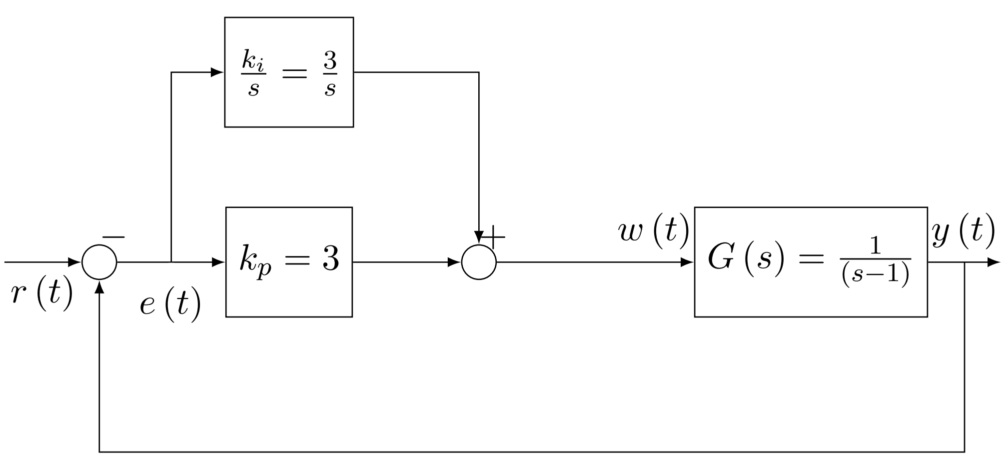



Figure 4.1: Block Diagram of System

(a) **Plant Output:**

From (4.1) , (4.2) and (4.3):

$$Y(s) = \frac{3s+1}{s(s+1)^2}, Re(s) > -1 \tag{4.8}$$

Final Value Theorem:

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) \tag{4.9}$$

Using (4.9) on Y($s$):

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{4.10}$$

$$= 1 \tag{4.11}$$

Taking partial fraction of (4.8) :

$$Y(s) = \frac{1}{s} + \frac{2}{(s+1)^2} - \frac{1}{s+1} \tag{4.12}$$

Using (4.6) and (4.7):

$$\therefore y(t) = u(t) + 2te^{-t}u(t) - e^{-t}u(t) \tag{4.13}$$

(b) **Controller Output:**

From (4.2)

$$W(s) = \frac{3}{s} + \frac{1}{s^2} - Y(s)\left(3 + \frac{1}{s}\right) \tag{4.14}$$

Substituting (4.8)

$$W\left(s\right)=\frac{\left(s-1\right)\left(3s+1\right)}{s\left(s+1\right)^{2}},Re\left(s\right)>-1 \tag{4.15}$$

Using (4.9) on W($s$)

$$\lim_{t\to\infty}w\left(t\right)=\lim_{s\to 0}sW\left(s\right) \tag{4.16}$$

$$=-1 \tag{4.17}$$

Taking partial fraction of equation(4.15) :

$$W\left(s\right)=-\frac{1}{s}-\frac{4}{\left(s+1\right)^{2}}+\frac{4}{s+1} \tag{4.18}$$

Using equations (4.6) and (4.7) and taking inverse lapalace transform:

$$w\left(t\right)=-u\left(t\right)-4te^{-t}u\left(t\right)+4e^{-t}u\left(t\right) \tag{4.19}$$

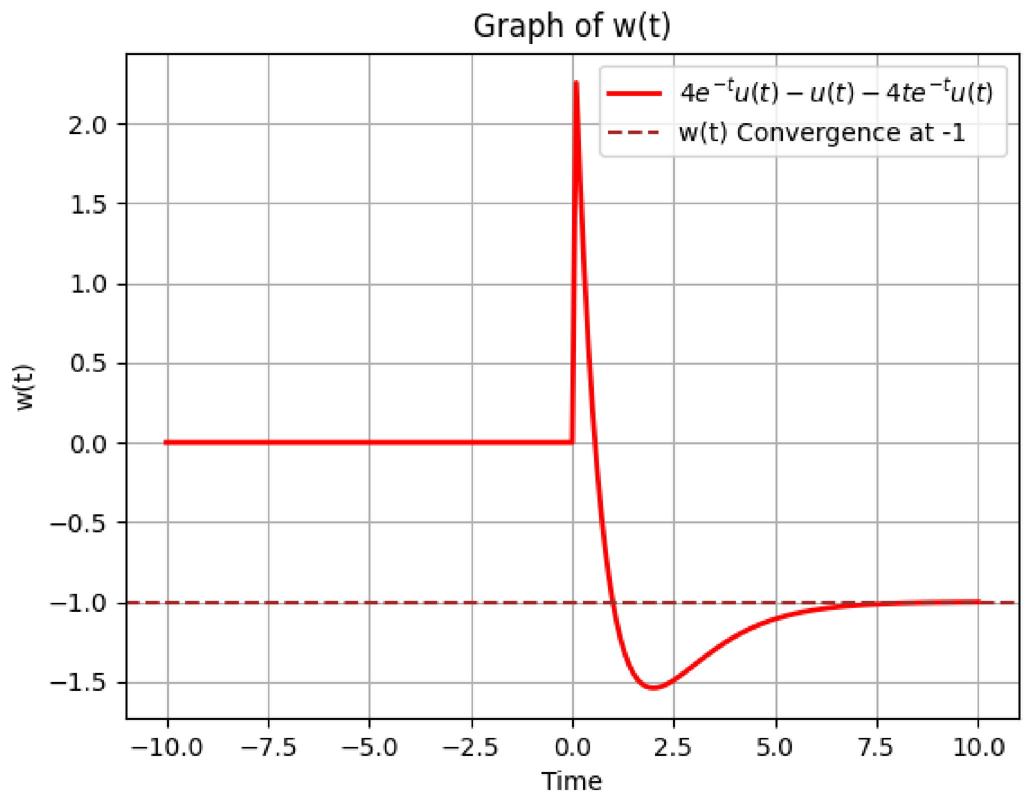



Figure 4.2: $w(t)$ converges at -1.

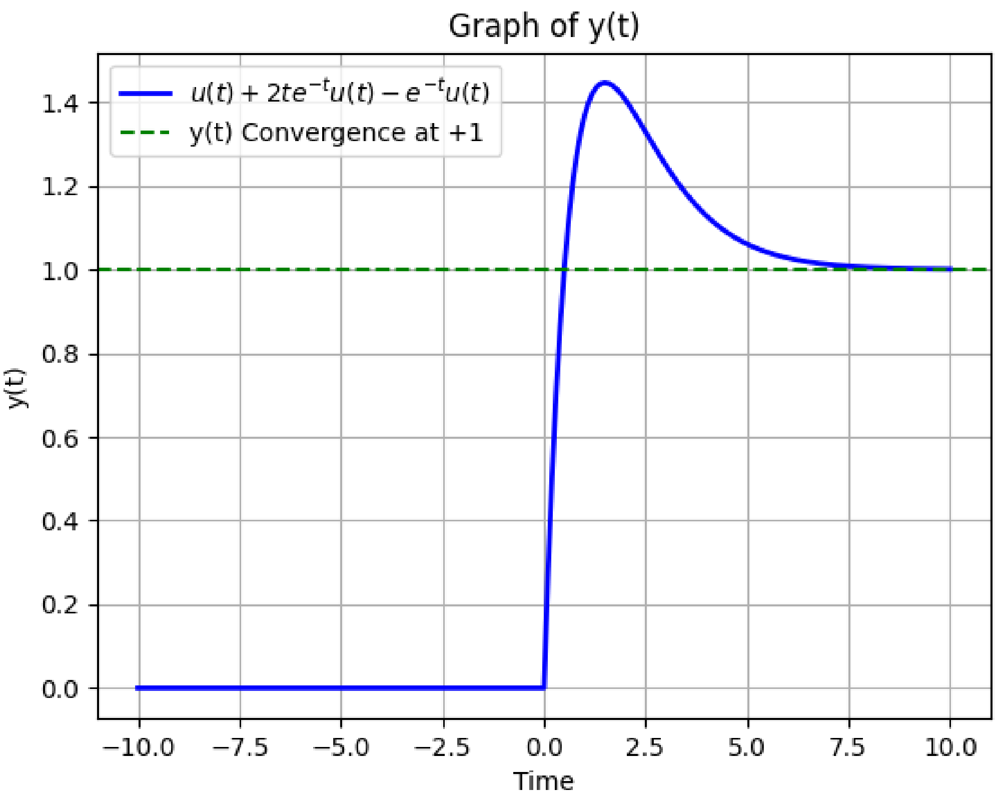



Figure 4.3: $y\,(t)$ converges at $+1$

# Chapter 5

# Sequences

5.1 Consider the discrete time signal $x[n] = u[-n+5] - u[n+3]$, where

$$u[n] = \begin{cases} 1; n \geq 0 \\ 0; n < 0 \end{cases}$$

The smallest n for which $x[n] = 0$ is?

**Solution:** From Fig. 1, the minimum value of $n$ is given as

$$n = -3 \tag{5.1}$$

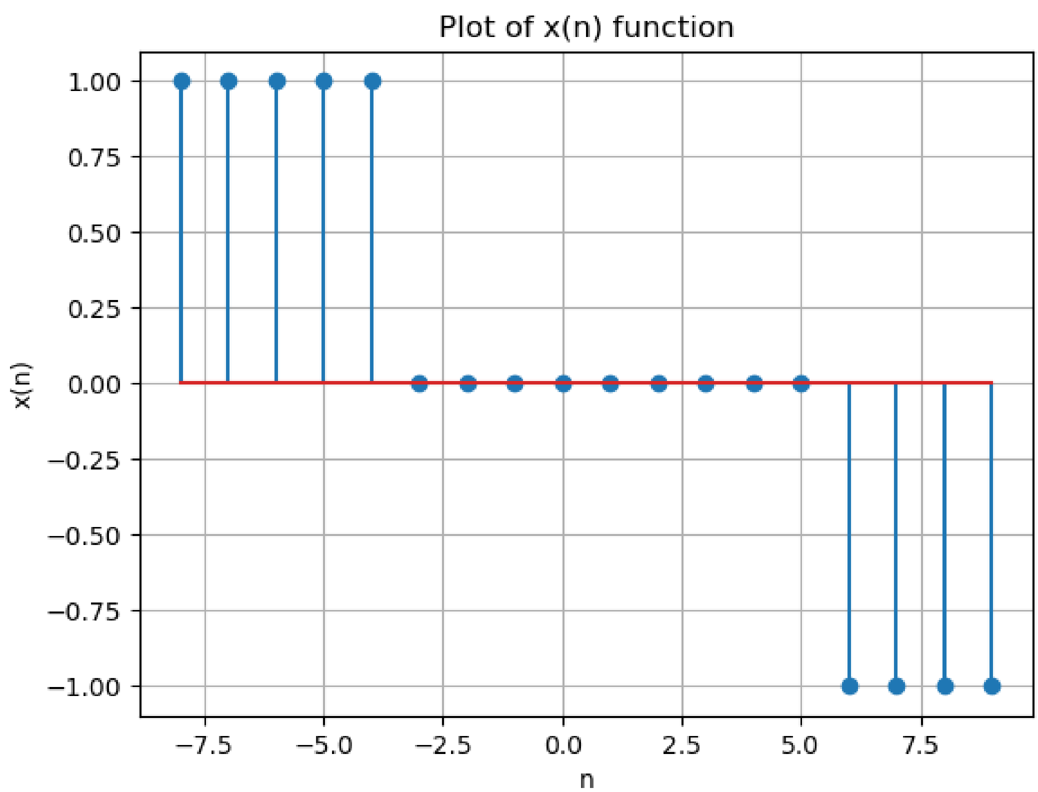



Figure 1: Plot of function $x\left(n\right)$ taken from python3

# Chapter 6

# Contour Integration

# Chapter 7

# Laplace Transform

7.1 The number of zeroes of the polynomial $P(s) = s^3 + 2s^2 + 5s + 80$ in the right side of the plane? (GATE IN 2023)

**Solution:** The table below shows the Routh array of the $n^{th}$- order characteristic polynomial :

$$a_0 s^n + a_1 s^{n-1} ...... + a_{n-1} s^1 + a_n s^0 \tag{7.1}$$

| | | | | |
|---|---|---|---|---|
| $s^n$ | $a_0$ | $a_2$ | $a_4$ | ... |
| $s^{n-1}$ | $a_1$ | $a_3$ | $a_5$ | ... |
| $s^{n-2}$ | $b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1}$ | $b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}$ | ... | .. |
| $s^{n-3}$ | $c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$ | $\vdots$ | | |
| $\vdots$ | $\vdots$ | $\vdots$ | | |
| $s^1$ | $\vdots$ | $\vdots$ | | |
| $s^0$ | $a_n$ | | | |

Table 7.1: Routh Array

Characteristic Equation:

$$s^3 + 2s^2 + 5s + 80 = 0 \tag{7.2}$$

From Table 7.1:

| | | |
|---|---|---|
| $s^3$ | 1 | 5 |
| $s^2$ | 2 | 80 |
| $s^1$ | $\dfrac{2 \times 5 - 80 \times 1}{2} = -35$ | |
| $s^0$ | $\dfrac{-35 \times 80}{-35} = 80$ | |

Table 7.2:

From Table 7.2:

Since there are 2 sign changes in the first column of the Routh tabulation. So, the number of zeros in the right half of the s-plane will be 2.

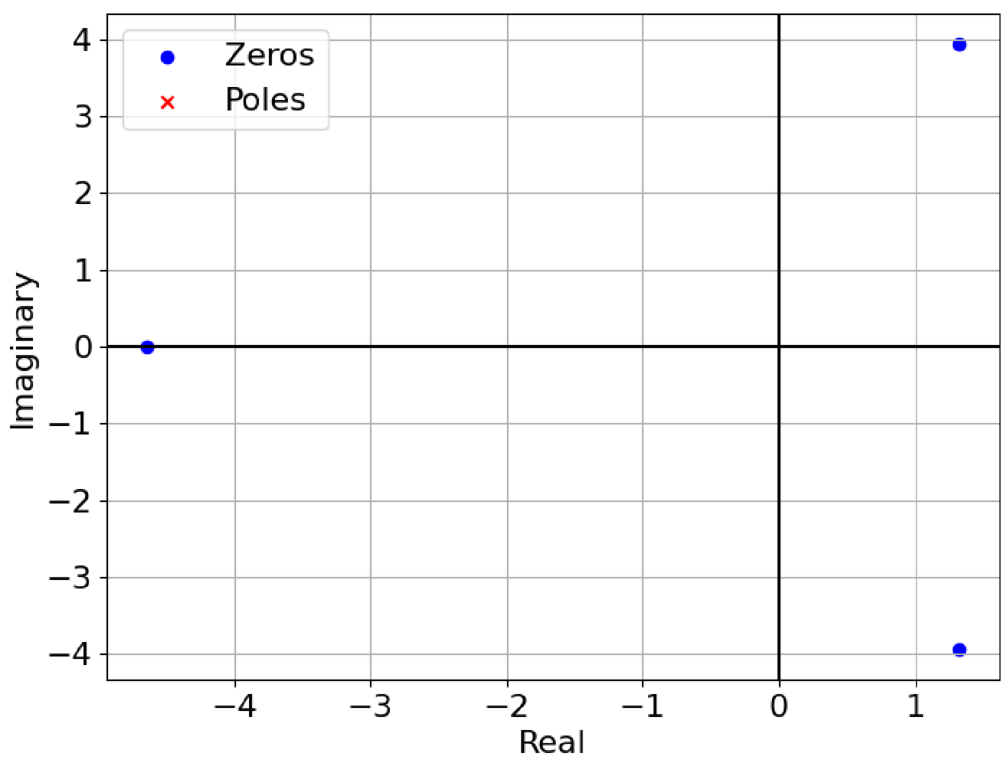
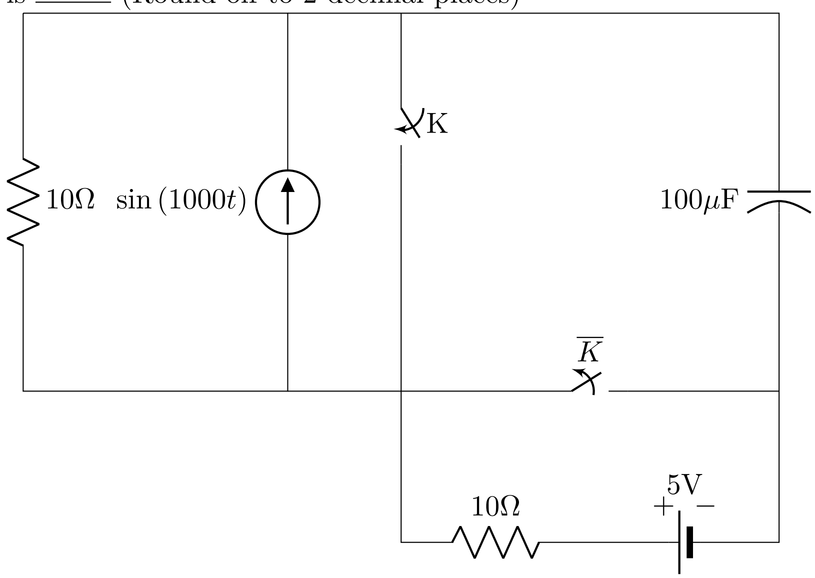



Figure 7.1:

7.2 The circuit shown in the figure is initially in the steady state with the switch K in open condition and $\overline{K}$ in closed condition. The switch K is closed and $\overline{K}$ is opened simultaneously at the instant $t = t_1$, where $t_1 > 0$. The minimum value of $t_1$ in milliseconds such that there is no transient in the voltage across the 100 $\mu F$ capacitor, is ____ (Round off to 2 decimal places) (GATE EE 2023)



7.3 $y = e^{mx} + e^{-mx}$ is the solution of which differential equation?

**1.** $\frac{dy}{dx} - my = 0$

**2.** $\frac{dy}{dx} + my = 0$

**3.** $\frac{d^2y}{dx^2} + m^2y = 0$

**4.** $\frac{d^2y}{dx^2} - m^2y = 0$

(GATE AG 2023) **Solution:**

7.4 In the differential equation $\frac{dy}{dx} + \alpha xy = 0, \alpha$ is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of $\alpha$ is (rounded off to three decimal places). (GATE CE 2023) **Solution:**