GATE-ES.47

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Question: Second order ordinary differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ has values $y = 2$ and $\frac{dy}{dx} = 1$ at $x = 0$.The value of y at $x = 1$ is?(*round off to three decimal places*)
[GATE-ES 2023]

Solution:
We convert given second order differential equation to s domain using Laplace transform and solve for $Y(s)$ and take inversion to get $y(x)$.

Symbol	Values	Description
$Y(s)$	$\frac{2s-1}{s^2-s-2}$	y in s domain
$y(x)$	$e^{2x} + e^{-x}$	y in x domain
$y(0)$	2	y at $x = 0$
$y'(0)$	1	$y'(x)$ at $x = 0$
$u(x)$	$u(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & o.w \end{cases}$	unit step function

TABLE I
PARAMETERS

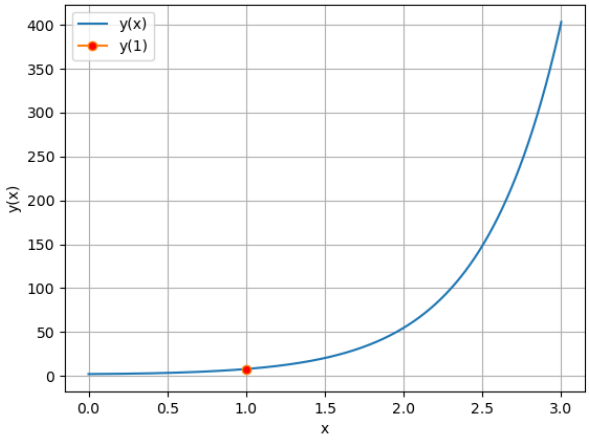

Fig. 1. Plot of y(x)

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y \overset{\mathcal{L}}{\longleftrightarrow} s^2Y(s) - sy(0) - y'(0) - sY(s) + y(0) - 2Y(s) \tag{1}$$

$$Y(s)\left(s^2 - s - 2\right) = 2s - 1 \tag{2}$$

$$\Rightarrow Y(s) = \frac{2s-1}{s^2-s-2} \tag{3}$$

$$\Rightarrow Y(s) = \frac{1}{s-2} + \frac{1}{s+1} \tag{4}$$

For inversion of $Y(s)$ in partial fractions-

$$\frac{b}{s+a} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} be^{-ax}u(x) \tag{5}$$

Where b, a are real numbers, we invert $Y(s)$ to get $y(x)$:-
From (5)

$$Y(s) \overset{\mathcal{L}^{-1}}{\longleftrightarrow} y(x)u(x) \tag{6}$$

$$y(x) = \left(e^{2x} + e^{-x}\right)u(x) \tag{7}$$

$$\Rightarrow y(1) = 7.757 \tag{8}$$