

GATE -BM 16

EE23BTECH11057 - Shakunaveti Sai Sri Ram Varun

Question: A buoy of virtual mass 30 kg oscillates in a fluid medium as a single degree of freedom system. If the total damping in the system is set as 188.5 N-s/m, such that the oscillation just ceases to occur, then the natural period of the system is _____ s (round off to one decimal place) (GATE MN 2023 question 63)

Solution:

Parameter	Description	Value
$X(s)$	position in laplace domain	$X(s)$
$x(t)$	position of buoy w.r.t time	$x(t)$
m	mass of buoy	30kg
λ	damping coefficient of the system	$188.5 \approx 60\pi$
ω_o	natural angular frequency of the system	?
ω_d	damping frequency of the system	0 rad s^{-1}

TABLE I
INPUT VALUES

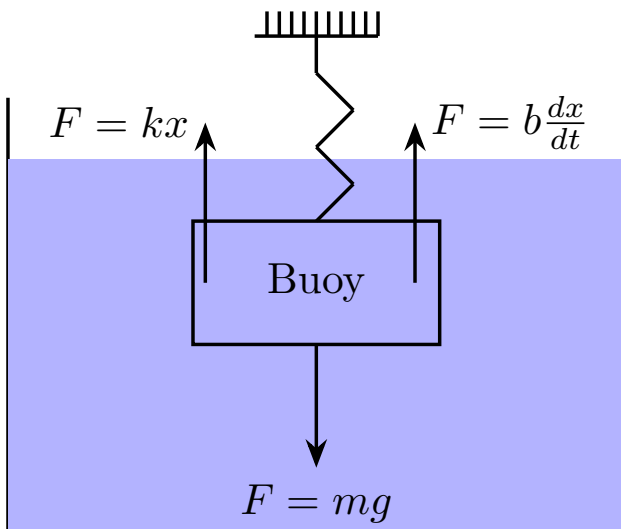


Fig. 1. Figure given in question

The differential equation of the system is:

$$m \frac{d^2 x(t)}{dt^2} + \lambda \frac{dx(t)}{dt} + m\omega_0^2 x(t) = 0 \quad (1)$$

Taking laplace transform:

$$ms^2 X(s) + \lambda s X(s) + m\omega_0^2 X(s) = 0 \quad (2)$$

$$\Rightarrow ms^2 + s\lambda + m\omega_0^2 = 0 \quad (3)$$

$$\therefore s = \frac{-\lambda \pm \sqrt{\lambda^2 - 4m^2\omega_0^2}}{2m} \quad (4)$$

where $\sqrt{\lambda^2 - 4m^2\omega_0^2}$ is ω_d .

From Table I,

$$\omega_d = 0 \quad (5)$$

$$\Rightarrow \lambda = 2\omega_o m \quad (6)$$

$$\Rightarrow \omega_o \approx \pi \quad (7)$$

$$T_i = \frac{2\pi}{\omega_o} \quad (8)$$

$$\therefore t_i = 2 \text{ seconds} \quad (9)$$

To find $x(t)$, we assume the initial amplitude of oscillations to be 1 meter and it is situated at extreme position.

$$s^2 X(s) + \lambda s X(s) + \omega_o^2 X(s) = sx(0) + x(0) \quad (10)$$

$$\Rightarrow X(s) = \frac{1 + s}{s^2 + s\frac{\lambda}{m} + \omega_o^2} \quad (11)$$

substituting values from Table I and ω_o ,

$$X(s) = \frac{1 + s}{(s + \pi)^2} \quad (12)$$

Taking inverse laplace transform by method of partial fractions,

$$X(s) = \frac{1}{s + \pi} + \frac{1 - \pi}{(s + \pi)^2} \quad (13)$$

$$\therefore x(t) = (1 + (1 - \pi)t) e^{-\pi t} \quad (14)$$

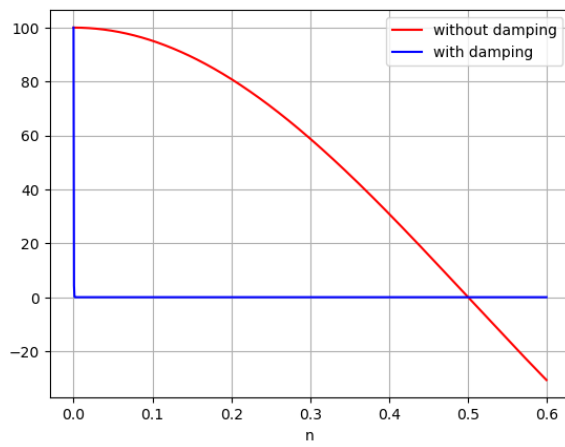


Fig. 2. $x(t)$ with and with out damping