SIGNAL PROCESSING
Through GATE

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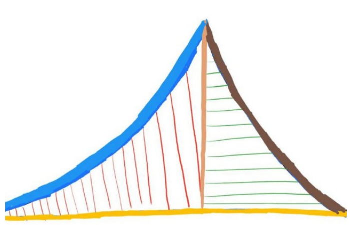


Contents

Introduction

This book provides solutions to signal processing problems in GATE.

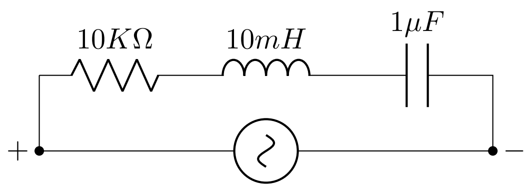
Chapter 1

Harmonics

1.1 Let y(t)=x($4t$),where x(t) is a continous-time periodic signal of 100s.the fundamental period of y(t) is (**rounded off to the nearest integer**) (GATE IN 2023)

Solution:

1.2 In the circuit shown below, it is observed that the amplitude of voltage across the resistor is the same as the amplitude of the sorce voltage. What is the angular frequency ω_0(in rad/s)?



(GATE BM 2023) **Solution:**

Chapter 2

Filters

2.1 For the circuit given below, choose the angular frequency ω_0 at which voltage across capacitor has maximum amplitude?

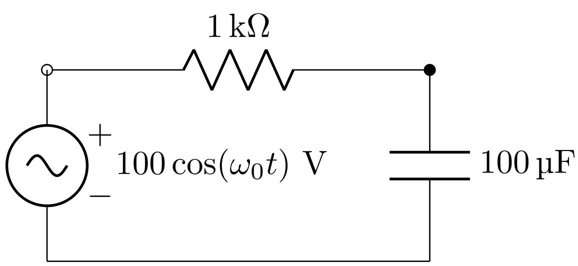


Figure 2.1: circuit

(A) 1000

(B) 100

(C) 1

(D) 0

(GATE BM 2023 Question 16)

Solution:

Parameter	Description	Value
$V_i(j\omega)$	Input voltage	100
$v_c(t)$	Potential difference across Capacitor	?
$V_c(s)$	Potential difference across Capacitor	$V_c(s)$
$H(s)$	Transfer function	$\frac{V_c(s)}{V_i(s)}$
V_o	Amplitude of input voltage	$100\,\mathrm{V}$
R	Resistance in circuit	$1\,\mathrm{k\Omega}$
C	Capacitance in circuit	$100\,\mu\mathrm{F}$
ω_o	Angular frequency of input voltage	ω_o

Table 2.1: input values

$$V_c\left(s\right) = \frac{V_1\left(s\right)\frac{1}{sC}}{R+\frac{1}{sC}} \tag{2.1}$$

$$\implies H\left(s\right) = \frac{1}{1+sRC} \tag{2.2}$$

$$\implies H\left(j\omega\right) = \frac{1}{1+j\omega RC} \tag{2.3}$$

$$\left|H\left(j\omega\right)\right| = \frac{1}{\sqrt{1+\left(\omega RC\right)^2}} \tag{2.4}$$

$$v_c\left(t\right) = \frac{100}{\sqrt{1+\left(\omega_o RC\right)^2}}\left(\cos\omega_o t+\arctan\left(\frac{1}{\omega_o RC}\right)\right) \tag{2.5}$$

Maximum amplitude of $v_c\left(t\right)$ occurs at $\omega_o = 0$

$$\therefore \omega_o = 0 \tag{2.6}$$

$\therefore$ maximum value of $v_c\left(t\right)$ at steady state is 100 Volts.

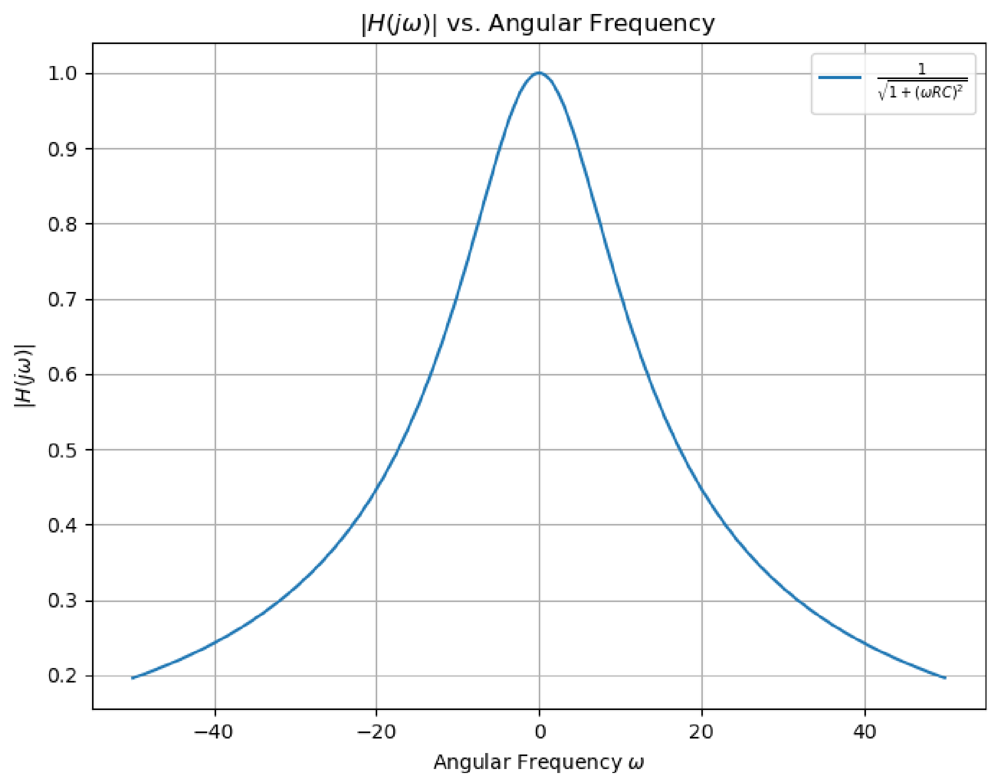


Figure 2.2: $|H\left(j\omega\right)|$

2.2 In the following circuit, the switch S is open for $t < 0$ and closed for $t \geq 0$. What is the steady state voltage (in Volts) across the capacitor when the switch is closed?

(GATE BM 2023 Question 30)

2.3 A finite impulse response (FIR) filter has only two non-zero samples in its impulse response $h[n]$, namely $h[0] = h[1] = 1$. The Discrete Time Fourier Transform (DTFT) of $h[n]$ equals $H(e^{j\omega})$, as a function of the normalized angular frequency ω. For the range $|\omega| \leq \pi$, $\left|H(e^{j\omega})\right|$ is equal to

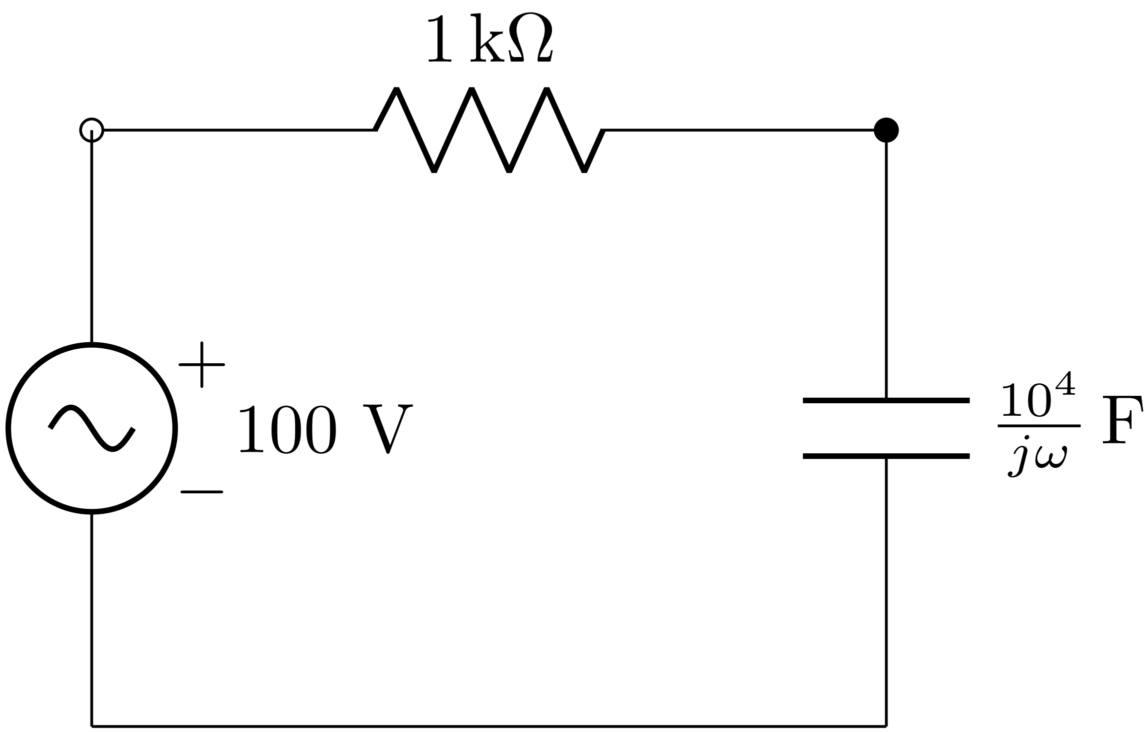


Figure 2.3: circuit in ω-domain

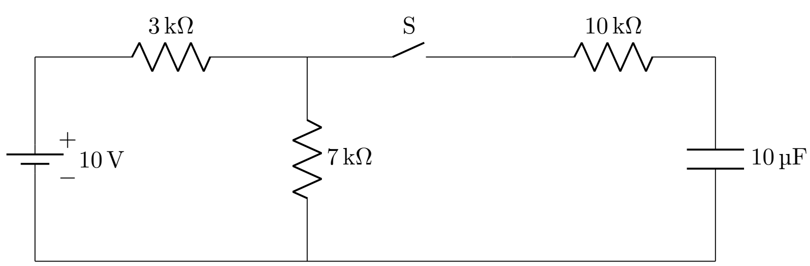


Figure 2.4: circuit

(A) $2\left|\cos(\omega)\right|$

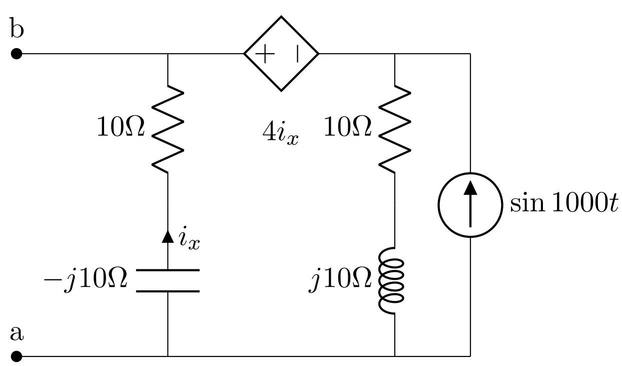
(B) $2\left|\sin(\omega)\right|$

(C) $2\left|\cos(\frac{\omega}{2})\right|$

(D) $2\left|\sin(\frac{\omega}{2})\right|$

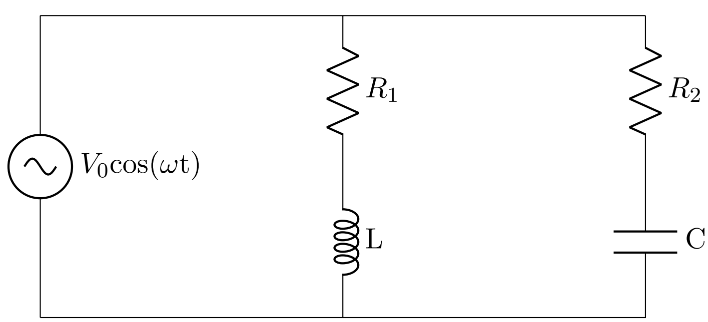
(GATE BM 2023 Question 17)

2.4 For the circuit shown,if $i = \sin 1000t$, the instantaneous value of the Thevenin's voltage(in volts) across the terminals a anb b at time t=5ms is



(GATE EE 2023 Question 51)

2.5 In the circuit shown ,$\omega = 100\pi$rads/s, R1=R2=2.2Ω and L=7mH. the capacitance C for which Y_{in} is purely real is mF



(GATE IN 2023 Q46)

Solution:

2.6 An input voltage in the form of a square wave of frequency 1 kHz is given to a circuit, which results in the output shown schematically below. Which one of the following options is the CORRECT representation of the circuit? (GATE PH 2023 Q37)

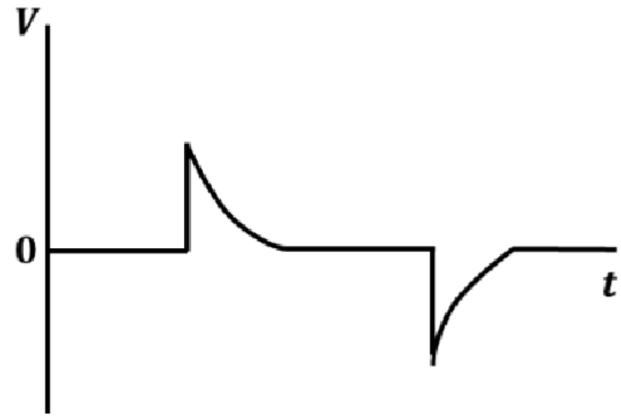
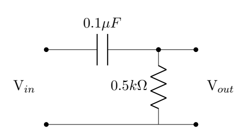
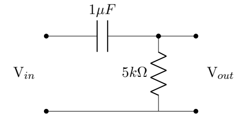


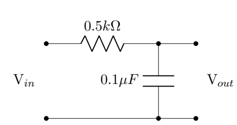
Figure 2.5:



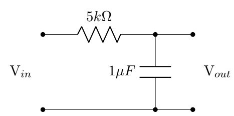
(a)



(b)



(c)



(d)

(GATE 2023 PH 37) **Solution:**

2.7 In the circuit shown below, switch S was closed for long time. If the switch is opened at $t = 0$, the maximum magnitude of the voltage V_R , in volts is (rounded off to the nearest integer) (GATE 2023 EC 35)

Solution:

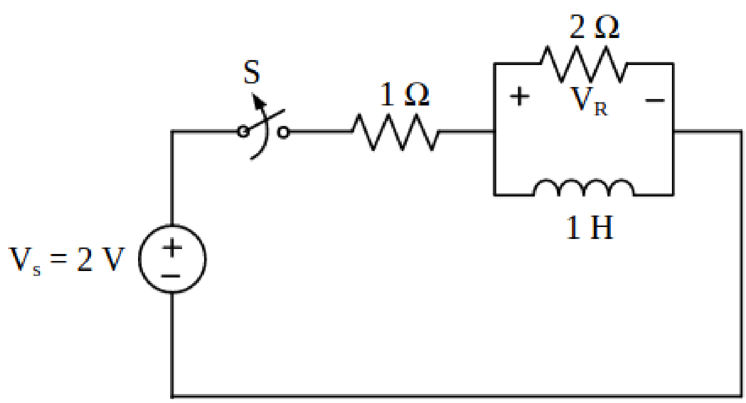


Figure 2.6:

Chapter 3

Z-transform

Chapter 4

Systems

4.1 Consider a unity-gain negative feedback system consisting of the plant $G(s)$ and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{(s-1)}$$

(GATE EE 2023)

Solution:

Parameter	**Description**	**Value**
K_p	Proportional Gain	3
K_i	Integral Gain	1
$r(t)$	Reference Input	$u(t)$
$w(t)$	Controller Output	?
$y(t)$	Plant Output	?
$e(t)$	Error Input	$r(t) - y(t)$

Table 1: Parameter Table

From theFig. 4.1:

$$E(s) = U(s) - Y(s) \tag{4.1}$$

$$W(s) = 3E(s) + \frac{1}{s}E(s) \tag{4.2}$$

$$Y(s) = G(s)W(s) \tag{4.3}$$

Some results:

$$tx(t) \xleftrightarrow{\mathcal{L}} -\frac{dX(s)}{ds} \tag{4.4}$$

$$e^{-at}x(t) \xleftrightarrow{\mathcal{L}} X(s+a) \tag{4.5}$$

By using (4.4) and (4.5):

$$e^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+1}, Re(s) > -1 \tag{4.6}$$

$$te^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{(s+1)^2}, Re(s) > -1 \tag{4.7}$$

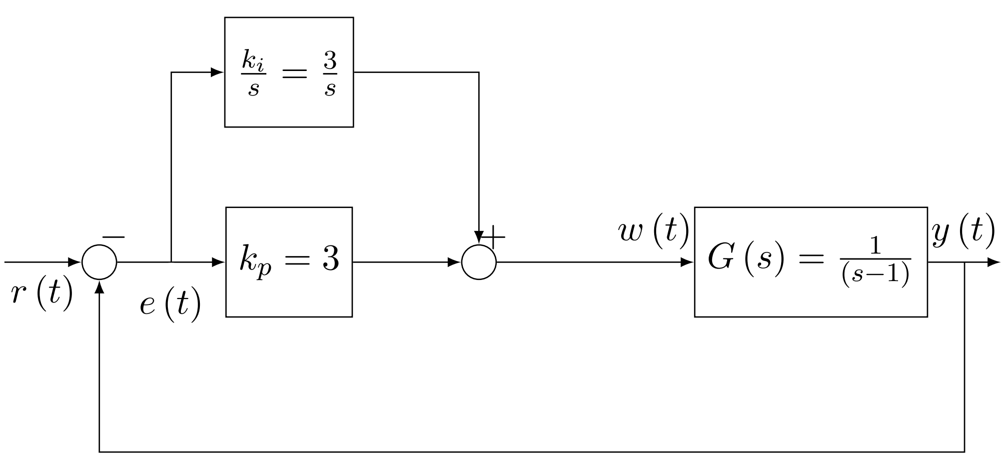


Figure 4.1: Block Diagram of System

(a) **Plant Output:**

From (4.1) , (4.2) and (4.3):

$$Y(s) = \frac{3s+1}{s(s+1)^2}, Re(s) > -1 \tag{4.8}$$

Final Value Theorem:

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) \tag{4.9}$$

Using (4.9) on Y(s):

$$\begin{align} \lim_{t\to\infty} y(t) &= \lim_{s\to 0} sY(s) \tag{4.10} \\ &= 1 \tag{4.11} \end{align}$$

Taking partial fraction of (4.8) :

$$Y(s) = \frac{1}{s} + \frac{2}{(s+1)^2} - \frac{1}{s+1} \tag{4.12}$$

Using (4.6) and (4.7):

$$\therefore y(t) = u(t) + 2te^{-t}u(t) - e^{-t}u(t) \tag{4.13}$$

(b) **Controller Output:**

From (4.2)

$$W(s) = \frac{3}{s} + \frac{1}{s^2} - Y(s)\left(3 + \frac{1}{s}\right) \tag{4.14}$$

Substituting (4.8)

$$W(s) = \frac{(s-1)(3s+1)}{s(s+1)^2}, Re(s) > -1 \tag{4.15}$$

Using (4.9) on W(s)

$$\lim_{t\to\infty} w(t) = \lim_{s\to 0} sW(s) \tag{4.16}$$

$$= -1 \tag{4.17}$$

Taking partial fraction of equation(4.15) :

$$W(s) = -\frac{1}{s} - \frac{4}{(s+1)^2} + \frac{4}{s+1} \tag{4.18}$$

Using equations (4.6) and (4.7) and taking inverse lapalace transform:

$$w(t) = -u(t) - 4te^{-t}u(t) + 4e^{-t}u(t) \tag{4.19}$$

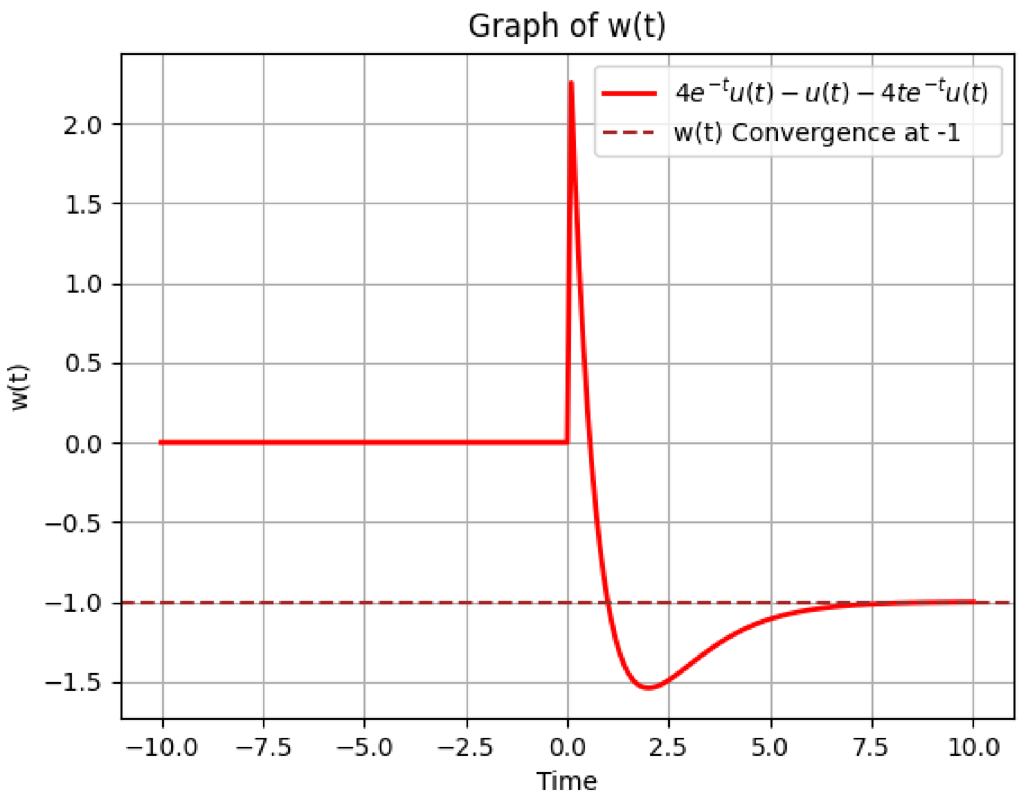


Figure 4.2: $w\,(t)$ converges at -1.

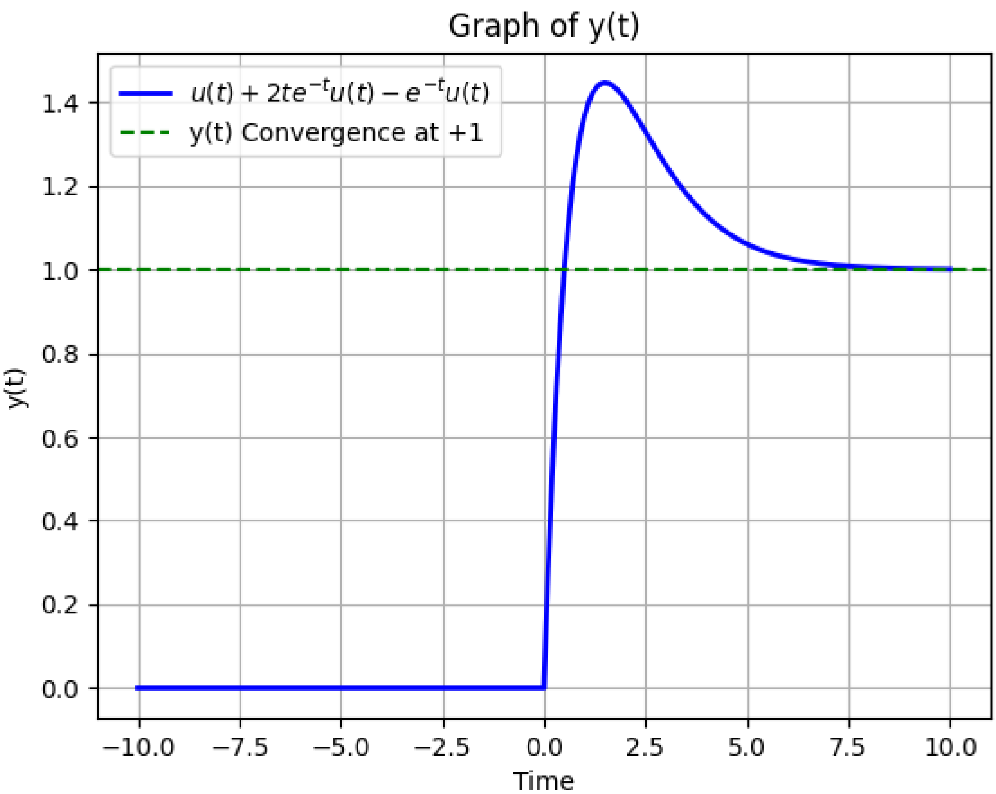


Figure 4.3: $y\left(t\right)$ converges at $+1$

4.2 Level (h) in a steam boiler is controlled by manipulating the flow rate (F) of the break-up(fresh) water using a proportional (P) controller. The transfer function between the output and the manipulated input is

$$\frac{h(s)}{F(s)} = \frac{0.25(1-s)}{s(2s+1)}$$

The measurement and the valve transfer functions are both equal to 1. A process engineer wants to tune the controller so that the closed loop response gives the decaying oscillations under the servo mode. Which one of the following is the CORRECT value of the controller gain to be used by the engineer?

(a) 0.25

(b) 2

(c) 4

(d) 6

GATE CH 2023

Solution:

4.3 The figure shows a block of mass m = 20 kg attached to a pair of identical linear springs, each having a spring constant k = 1000 N/m. The block oscillates on a frictionless horizontal surface. Assuming free vibrations, the time taken by the block to complete ten oscillations is ____ seconds . (Rounded off to two decimal places) Take $\pi = 3.14$. (GATE ME 2023)

4.4 A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s-\pi}{s+\pi}$$

let $u(t)$ be the unit step function.The input $x(t)$ that results in a steady-state output $y(t) = sin(\pi t)$ is __. (GATE IN 2023)

Solution:

4.5 The state equation of a second order system is

$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t), \quad \boldsymbol{x}(0)$ is the initial condition.

Suppose λ_1 and λ_2 are two distinct eigenvalues of A, and ν_1 and ν_2 are the corresponding eigenvectors. For constants α_1 and α_2, the solution, $\boldsymbol{x}(t)$, of the state equation is

(A) $\sum_{i=1}^{2} \alpha_i e^{\lambda_i t} \nu_{\mathbf{i}}$

(B) $\sum_{i=1}^{2} \alpha_i e^{2\lambda_i t} \nu_{\mathbf{i}}$

(C) $\sum_{i=1}^{2} \alpha_i e^{3\lambda_i t} \nu_{\mathbf{i}}$

(D) $\sum_{i=1}^{2} \alpha_i e^{4\lambda_i t} \nu_{\mathbf{i}}$

GATE 2023 EC Question 43

4.6 Consider the complex function

$$f(z) = \frac{z^2 \sin z}{(z-\pi)^4}$$

At $z = \pi$, which of the following options is (are) correct?

(A) The order of the pole is 4

(B) The order of the pole is 3

(C) The residue at the pole is $\frac{\pi}{6}$

(D) The residue at the pole is $\frac{2\pi}{3}$

(GATE PH 2023)

4.7 A buoy of virtual mass 30 kg oscillates in a fluid medium as a single degree of freedom system. If the total damping in the system is set as 188.5 N-s/m, such that the oscillation just ceases to occur, then the natural period of the system is ____ s (round off to one decimal place) (GATE MN 2023 question 63)

4.8 Which of the following statement(s) is/are true?

(a) If an LTI system is causal, it is stable.

(b) A discrete time LTI system is causal if and only if its response to a step input $u[n]$ is 0 for $n < 0$.

(c) If a discrete time LTI system has an impulse response $h[n]$ of finite duration the system is stable.

(d) If the impulse response $0 < |h[n]| < 1$ for all n, then the LTI system is stable.

(GATE EE 2023 question 27)

Solution:

4.9 The outlet concentration C_A of a plug flow reactor (PFR) is controlled by manipulating the inlet concentration C_{A0}.The following transfer function describes the dynamics of this PFR.

$$\frac{C_A(s)}{C_{A0}(s)} = e^{-(\frac{V}{F})(k+s)}$$

In the above question, V=1m^3,F=0.1$m^3 min^{-1}$ and k=0.5min^{-1}.The measurement and valve transfer functions are both equal to 1.The ultimate gain, defined as the proportional controller gain that produces sustained oscillations, for this system is (GATE 2023 CH 61)

Solution:

4.10 For the block diagram shown in the figure, the transfer function $\frac{Y(s)}{R(s)}$ is

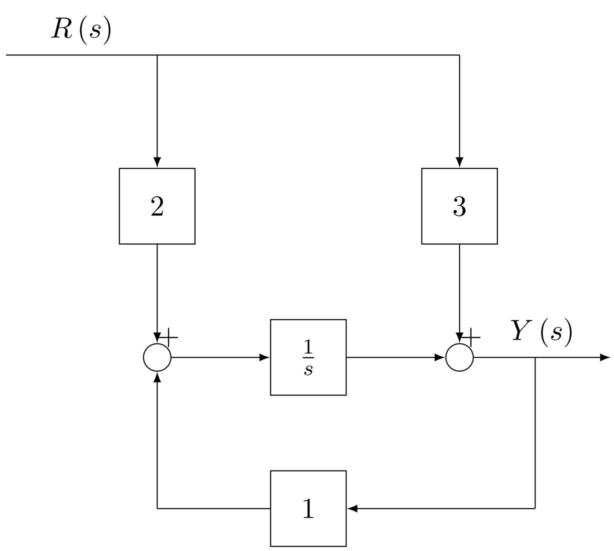


Figure 4.4: Block diagram

(GATE EE 2023)

Solution:

Chapter 5

Sequences

5.1 Consider the discrete time signal $x[n] = u[-n+5] - u[n+3]$, where

$$u[n] = \begin{cases} 1; n \geq 0 \\ 0; n < 0 \end{cases}$$

The smallest n for which $x[n] = 0$ is?

Solution: From Fig. 1, the minimum value of n is given as

$$n = -3 \tag{5.1}$$

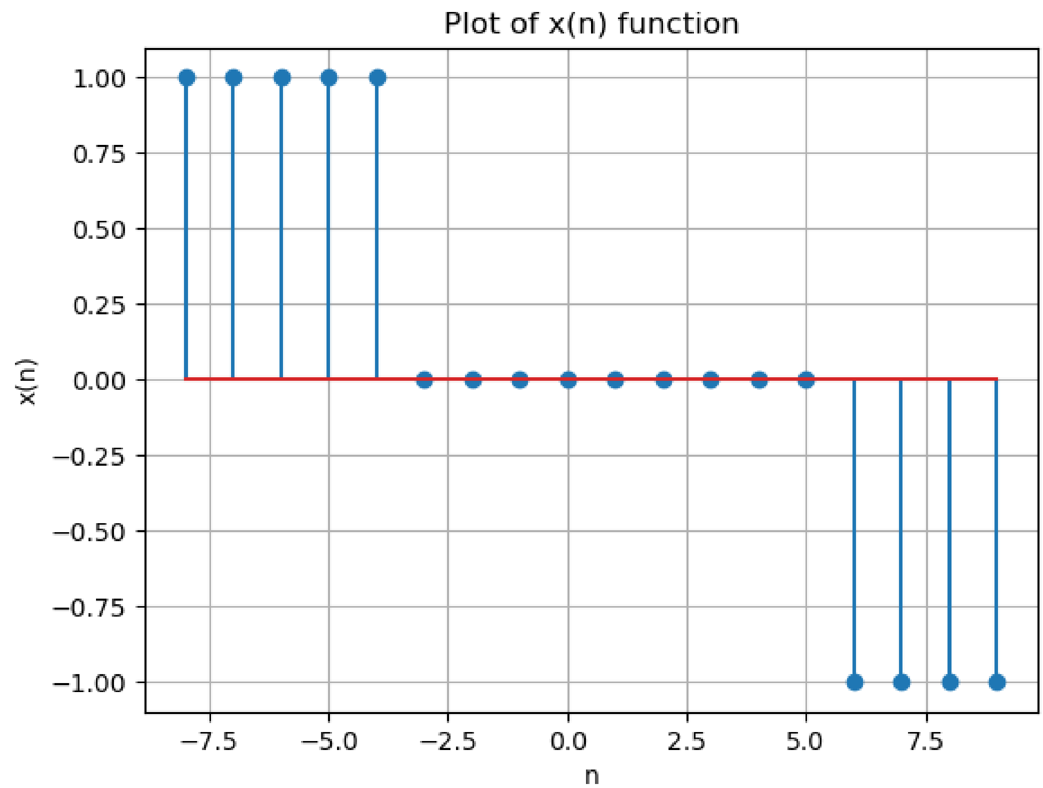


Figure 1: Plot of function $x(n)$ taken from python3

5.2 Two sequences $x_1[n]$ and $x_2[n]$ are described as follows:

$$x_1[0] = x_2[0] = 1 \tag{5.2}$$

$$x_1[1] = x_2[2] = 2 \tag{5.3}$$

$$x_1[2] = x_2[1] = 1 \tag{5.4}$$

$x_1[n] = x_2[n] = 0$ for all $n < 0$ and $n > 2$

If $x[n]$ is obtained by convoluting $x_1[n]$ with $x_2[n]$, which of the following equations is/are TRUE?

(A) $x[2] = x[3]$

(B) $x[1] = 2$

(C) $x[4] = 3$

(D) $x[2] = 5$

Solution:

5.3 A series (S) is given as S=1+3+5+7+9+..... The sum of the first 50 terms of S is ____________ (GATE 2023 BT 32) **Solution:**

Chapter 6

Sampling

6.1 An 8 bit ADC converts analog voltage in the range of 0 to $+5\,V$ to the corresponding digital code as per the conversion characteristics shown in figure. For $V_{in} = 1.9922\,V$, which of the following digital output, given in hex, is true?

(a) $64H$

(b) $65H$

(c) $66H$

(d) $67H$

(GATE EE 40)

Solution:

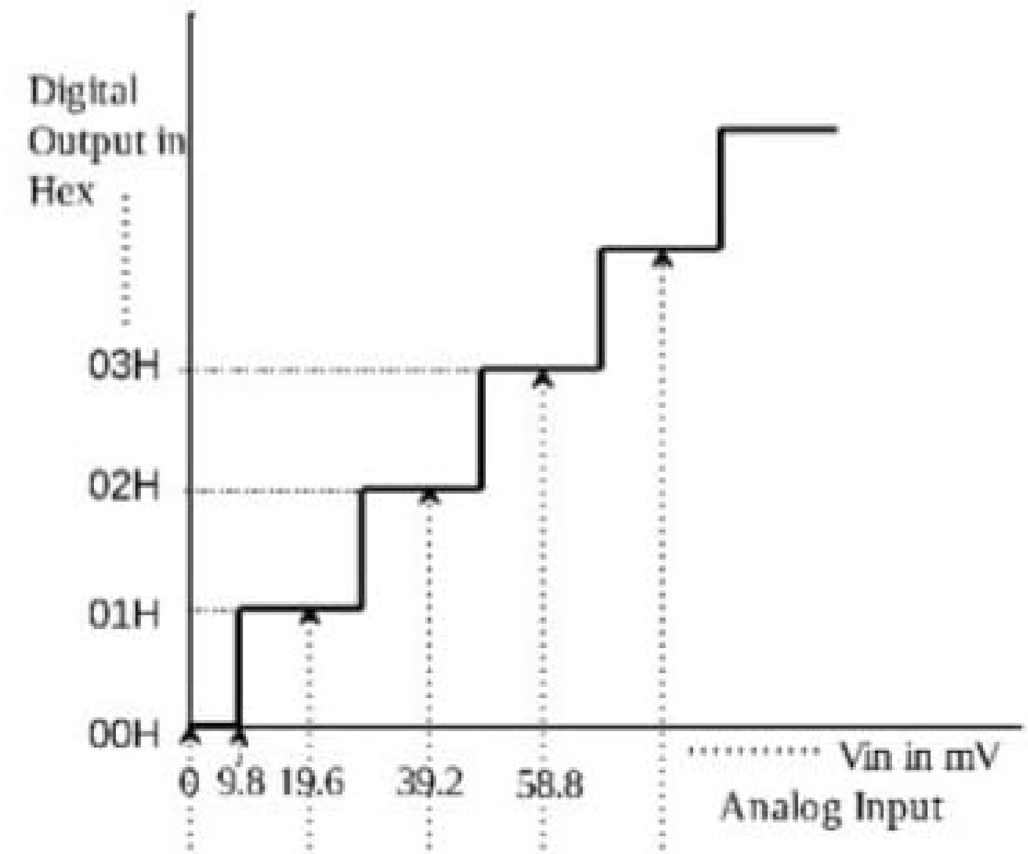


Figure 6.1:

Calculating the step-size:

$$
\begin{aligned}
\Delta V_{in} &= \frac{V_{max} - V_{min}}{2^n - 1} && (6.1)\\
&= \frac{5 - 0}{2^8 - 1} && (6.2)\\
&= \frac{5}{255} && (6.3)\\
\implies V_{out} &= \frac{V_{in}}{\Delta V_{in}} && (6.4)\\
&= \frac{1.9922 \times 255}{5} && (6.5)\\
&= 101.59 && (6.6)\\
&\approx 102_{10} && (6.7)
\end{aligned}
$$

Symbol	**Value**	**Description**
n	8	Number of bits of ADC
V_{min}	$0V$	Minimum Analog Voltage
V_{max}	$5V$	Maximum Analog Voltage
V_{in}	$1.9922V$	Input Voltage
V_{out}		Output Voltage

Table 6.1: Given Parameters

$\therefore$ correct answer is option (c).

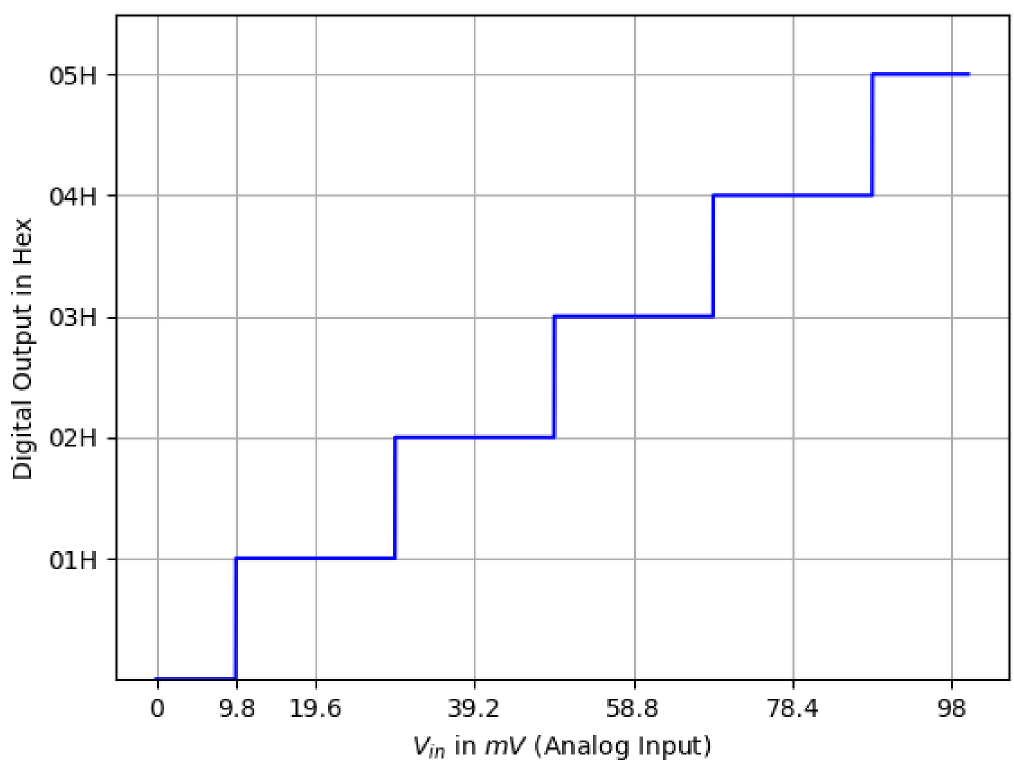

05H
04H
03H
02H
01H
Digital Output in Hex
0
9.8
19.6
39.2
58.8
78.4
98
V_{in} in mV (Analog Input)

Figure 6.2:

Chapter 7

Contour Integration

Chapter 8

Laplace Transform

8.1 The number of zeroes of the polynomial $P(s) = s^3 + 2s^2 + 5s + 80$ in the right side of the plane? (GATE IN 2023)

Solution: The table below shows the Routh array of the n^{th}- order characteristic polynomial :

$$a_0 s^n + a_1 s^{n-1} ...... + a_{n-1} s^1 + a_n s^0 \tag{8.1}$$

s^n	a_0	a_2	a_4	...
s^{n-1}	a_1	a_3	a_5	...
s^{n-2}	$b_1 = \dfrac{a_1 a_2 - a_3 a_0}{a_1}$	$b_2 = \dfrac{a_1 a_4 - a_5 a_0}{a_1}$	...	..
s^{n-3}	$c_1 = \dfrac{b_1 a_3 - b_2 a_1}{b_1}$	$\vdots$		
$\vdots$	$\vdots$	$\vdots$		
s^1	$\vdots$	$\vdots$		
s^0	a_n			

Table 8.1: Routh Array

Characteristic Equation:

$$s^3 + 2s^2 + 5s + 80 = 0 \tag{8.2}$$

From Table 8.1:

s^3	1	5
s^2	2	80
s^1	$\frac{2 \times 5 - 80 \times 1}{2} = -35$	
s^0	$\frac{-35 \times 80}{-35} = 80$	

Table 8.2:

From Table 8.2:

Since there are 2 sign changes in the first column of the Routh tabulation. So, the number of zeros in the right half of the s-plane will be 2.

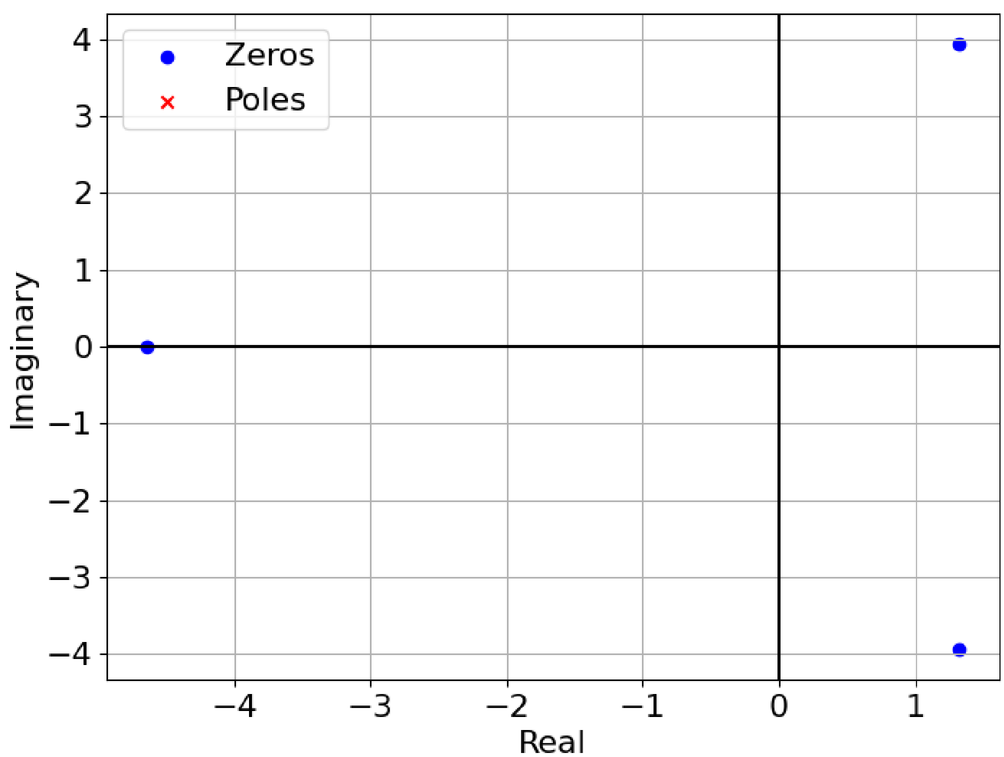
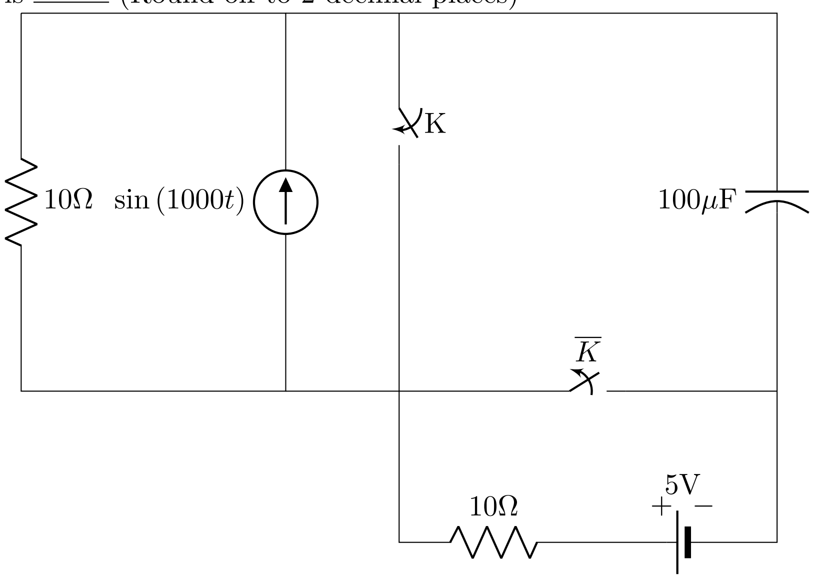


Figure 8.1:

8.2 The circuit shown in the figure is initially in the steady state with the switch K in open condition and $\overline{K}$ in closed condition. The switch K is closed and $\overline{K}$ is opened simultaneously at the instant $t = t_1$, where $t_1 > 0$. The minimum value of t_1 in milliseconds such that there is no transient in the voltage across the 100 μF capacitor, is ____ (Round off to 2 decimal places) (GATE EE 2023)



8.3 $y = e^{mx} + e^{-mx}$ is the solution of which differential equation?

1. $\frac{dy}{dx} - my = 0$

2. $\frac{dy}{dx} + my = 0$

3. $\frac{d^2y}{dx^2} + m^2y = 0$

4. $\frac{d^2y}{dx^2} - m^2y = 0$

(GATE AG 2023) **Solution:**

8.4 A cascade control strategy is shown in the figure below. The transfer function between the output (y) and the secondary disturbance (d_2) is defined as

$$G_{d2}(s) = \frac{y(s)}{d_2(s)}$$

. Which one of the following is the CORRECT expression for the transfer function $G_{d2}(s)$?

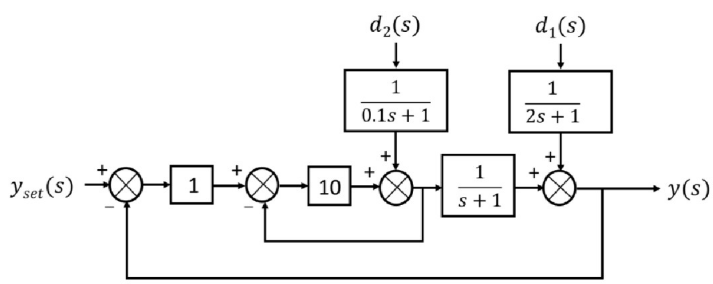


Figure 8.2:

A. $\frac{1}{(11s+21)(0.1s+1)}$

B. $\frac{1}{(s+1)(0.1s+1)}$

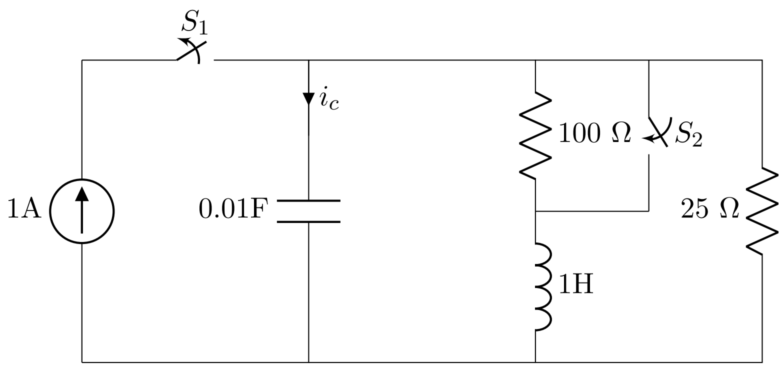
C. $\frac{(s+1)}{(s+2)(0.1s+1)}$

D. $\frac{(s+1)}{(s+1)(0.1s+1)}$

(GATE CH 2023) **Solution:**

8.5 In the differential equation $\frac{dy}{dx} + \alpha xy = 0$, α is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of α is (rounded off to three decimal places). (GATE CE 2023) **Solution:**

8.6 The switch S_1 was closed and S_2 was open for a long time. At t=0,switch S_1 is opened and S_2 is closed,simultaneously. The value of $i_c(0^+)$, in amperes, is . (GATE EC 2023)



8.7 The continuous time signal $x(t)$ is described by:

$$x(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq 1 \\ 0, & \text{elsewhere} \end{cases} \tag{8.3}$$

If $y(t)$ represents $x(t)$ convolved with itself, which of the following options is/are TRUE?

A $y(t) = 0$ for all $t < 0$

B $y(t) = 0$ for all $t > 1$

C $y(t) = 0$ for all $t > 3$

D $\int_{0.1}^{0.75} \frac{dy(t)}{dt}$ dt $\neq 0$

Solution:

8.8 The Z-transform of a discrete signal $x(n)$ is

$$X(z) = \frac{4z}{\left(z-\frac{1}{5}\right)\left(z-\frac{2}{3}\right)(z-3)} \text{ with ROC} = R \tag{8.4}$$

Which one of the following statements is TRUE?

(a) Discrete time Fourier transform of $x[n]$ converges if R is $|z| > 3$

(b) Discrete time Fourier transform of $x[n]$ converges if R is $\frac{2}{3} < |z| < 3$

(c) Discrete time Fourier transform of $x[n]$ converges if R is such that $x[n]$ is a left-sided sequence.

(d) Discrete time Fourier transform of $x[n]$ converges if R is such that $x[n]$is a right-sided sequence.

GATE EE 2023 **Solution:**

8.9 The phase margin of the transfer function $G(s) = \frac{2(1-s)}{(1+s)^2}$ is ______ degrees. (rounded off to the nearest integer). (GATE IN 2023)

Solution:

8.10 Consider the second-order linear differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0,\ x \geq 1$$

with the initial conditions

$$y(x = 1) = 6, \quad \frac{dy}{dx}\Big|_{x=1} = 2.$$

Then the value of y at $x = 2$ is __________.

GATE ME 2023

Solution:

8.11 The transfer function of a measuring instrument is

$$G_m(s) = \frac{1.05}{2s+1} exp(-s)$$

At time $t = 0$, a step change of $+1$ unit is introduced in the input of this instrument.The time taken by the instrument to show an increase of 1 unit in its output is(rounded off to two decimal places).

(GATE CH 2023) **Solution:**

8.12 The laplace transform of $x_1(t) = e^{-t}u(t)$ is $X_1(s)$, where $u(t)$ is the unit step function. The laplace transform of $x_2(t) = e^{t}u(-t)$ is $X_2(s)$. Which one of the following statements is TRUE?

(a) The region of convergence of $X_1(s)$ is $Re(s) \geq 0$

(b) The region of convergence of $X_2(s)$ is confined to the left half-plane of s.

(c) The region of convergence of $X_1(s)$ is confined to the right half-plane of s.

(d) the imaginary axis in the s-plane is included in both the region of convergence of $X_1(s)$ and the region of convergence of $X_2(s)$.

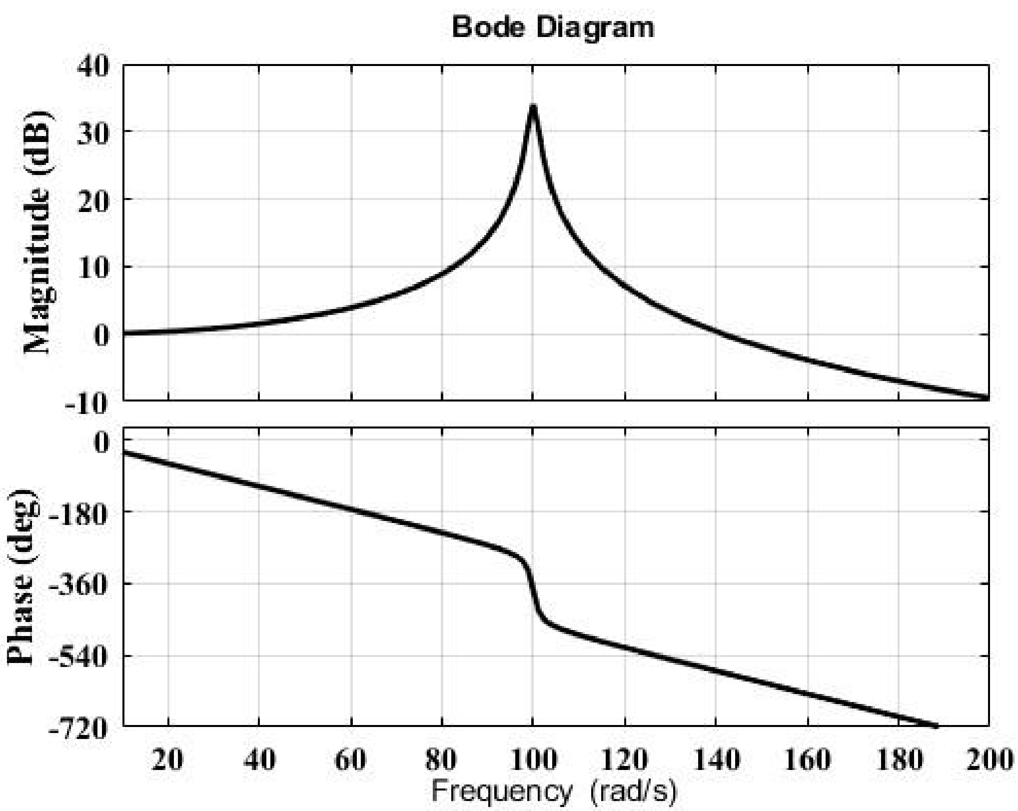
(GATE BM 2023)

Solution:

8.13 Given that $\frac{dy}{dx} = 2x + y$ and $y = 1$,when $x = 0$ Using Runge-Kutta fourth order method,the value of y at $x = 0.2$ is (GATE 2023 AG 50)

Solution:

8.14 The magnitude and phase plots shown in the figure match with the transfer- function



(a) $\frac{10000}{s^2+2s+10000}$

(b) $\frac{10000}{s^2+2s+10000}e^{-0.05s}$

(c) $\frac{10000}{s^2+2s+10000}e^{-0.5\times10^{-12}s}$

(d) $\frac{100}{s^2+2s+100}$

(GATE IN 2023) **Solution:**

8.15 The Laplace transform of the continuous-time signal $x(t) = e^{-3t}u(t-5)$ is ______,

where $u(t)$ denotes the continuous-time unit step signal.

A) $\frac{e^{-5s}}{s+3}$, Real$\{s\} > -3$

B) $\frac{e^{-5(s-3)}}{s-3}$, Real$\{s\} > 3$

C) $\frac{e^{-5(s+3)}}{s+3}$, Real$\{s\} > -3$

D) $\frac{e^{-5(s-3)}}{s+3}$, Real$\{s\} > -3$

Solution:

Chapter 9

Fourier transform

9.1 The discrete-time Fourier transform of a signal x$[n]$ is $X(\Omega) = (1+\cos\Omega)\, e^{-j\Omega}$. Consider that $x_p[n]$ is a periodic signal of period $N = 5$ such that

$$x_p[n] = x[n], \text{for n= 0, 1, 2} \tag{9.1}$$

$$= 0, \text{for n= 3, 4} \tag{9.2}$$

Note that $x_p[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$. The magnitude of the Fourier series coefficient a_3 is ________________ (Round off to 3 decimal places). (GATE EE 2023) **Solution:**

9.2 Let a frequency modulated (FM) signal : $x(t) = A\cos(\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda)$, where $m(t)$is a message signal of bandwidth W. It is passed through a non-linear system with output $y(t) = 2x(t) + 5(x(t))^2$. Let $B_T$denote the FM bandwidth. The minimum value of ω_c required to recover $x(t)$ from $y(t)$ is:

(A) $B_T + W$

(B) $\frac{3}{2}B_T$

(C) $2B_T + W$

(D) $\frac{5}{2}B_T$

Solution:

9.3 Let an input $x[n]$ having discrete-time Fourier transform $X(e^{j\Omega}) = 1 - e^{-j\Omega} + 2e^{-3j\Omega}$ be passed through an LTI system. The frequency response of the LTI system is $H(e^{j\Omega}) = 1 - \frac{1}{2}e^{-2j\Omega}$ The output $y[n]$ of the system is

(GATE EC 2023) **Solution:**

9.4 The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note:$\int_{-\infty}^{\infty} e^{-y^2}\, dy = \sqrt{\pi}$

A) $\sqrt{\pi} e^{\frac{\omega^2}{2}}$

B) $\frac{e^{\frac{-\omega^2}{4}}}{2\sqrt{\pi}}$

C) $\sqrt{\pi} e^{\frac{-\omega^2}{4}}$

D) $\sqrt{\pi} e^{\frac{-\omega^2}{2}}$

Gate 2023 EC Question 28

9.5 Let $x(t) = 10\cos(10.5\omega t)$ be passed through an LTI system with impulse response $h(t) = \pi\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \cos(10\omega t)$. The output of the system is:

(GATE EC 2023) **Solution:**

9.6 Q27) Let m(t) be a strictly band-limited signal with bandwidth B and energy E. Assuming $\omega_0 = 10\text{B}$, the energy in the signal $\text{m}\,(t)\cos(\omega_0 \text{t})$

(A) $\frac{E}{4}$

(B) $\frac{E}{2}$

(C) E

(D) $2E$ (GATE EC 2023)

Solution:

9.7 The following function is defined over the integral $[-L, L]$:

$$f(x) = px^4 + qx^5$$

It is expressed as a Fourier series,

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left\{ a_n \sin\left(\frac{\pi x}{L}\right) + b_n \cos\left(\frac{\pi x}{L}\right) \right\}$$

which options amongst the following are true?

(a) a_n, $n = 1, 2, .., \infty$ depend on p

(b) a_n, $n = 1, 2, .., \infty$ depend on q

(c) b_n, $n = 1, 2, .., \infty$ depend on p

(d) b_n, $n = 1, 2, .., \infty$ depend on q

(GATE 2023 CE Question 25)

Solution:

9.8 A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0, \forall\, t < 0$. From the list given below, select ALL the signals whose continuous-time Fourier transform is purely imaginary.

(a) $x(t) + x(-t)$

(b) $x(t) - x(-t)$

(c) $j(x(t) + x(-t))$

(d) $j(x(t) - x(-t))$

(GATE IN 2023)

Solution:

9.9 Let $x_1(t) = u(t + 1.5) - u(t - 1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t)\, dt$ is __________.

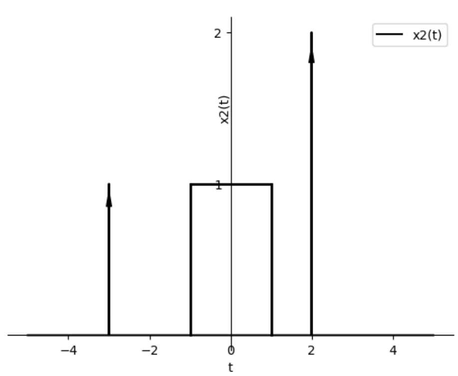


Figure 9.1: Figure

(GATE IN 2023)

Solution:

9.10 Consider a discrete-time signal with period $N = 5$. Let the discrete-time Fourier series (DTFS) representation be $x[n] = \sum_{k=0}^{4} a_k e^{\frac{jk2\pi n}{5}}$ where $a_0 = 1$, $a_1 = 3j$, $a_2 = 2j$, $a_3 = -2j$, $a_4 = -3j$. The value of the sum $\sum_{n=0}^{4} x[n] \sin\left(\frac{4\pi n}{5}\right)$ is

(A) -10

(B) 10

(C) -2

(D) 2

Gate 2023 EC 47 **Solution:**