GATE-2023 EC Q.25

EE23BTECH11214 - Harsha Vardhan Kumar

Question: In the context of signals and systems, determine the phase cross-over frequency of the open-loop transfer function

$$G(s) = \frac{k \cdot s \cdot (1 + sT_1) \cdot (1 + sT_2)}{s}$$

with positive constants k, T_1, T_2 are positive constants.The phase crossover fequency,in rad/s,is [EC,GATE-2023]

(a) $\frac{1}{\sqrt{T_1 T_2}}$
(b) $\frac{1}{T_1 T_2}$
(c) $\frac{1}{T_1 \sqrt{T_2}}$
(d) $\frac{1}{\sqrt{T_2} T_1}$

Solution: The phase of $G(s)$

$$\angle G(s) = \angle(ks(1 + sT_1)(1 + sT_2)) - \angle s \tag{1}$$
$$= \angle ks + \angle(1 + sT_1) + \angle(1 + sT_2) - \angle s \tag{2}$$

The phase contribution of each term

$$\angle ks = \angle k + \angle s = 0 + \frac{\pi}{2} \tag{3}$$
$$= \frac{\pi}{2} \text{ radians} \tag{4}$$
$$\angle(1 + sT_1) = \tan^{-1}(0) + \tan^{-1}(sT_1) \tag{5}$$
$$= \tan^{-1}(sT_1) \tag{6}$$
$$\angle(1 + sT_2) = \tan^{-1}(0) + \tan^{-1}(sT_2) \tag{7}$$
$$= \tan^{-1}(sT_2) \tag{8}$$
$$\angle s = \frac{\pi}{2} \text{ radians} \tag{9}$$

So, the total phase of $G(s)$ becomes:

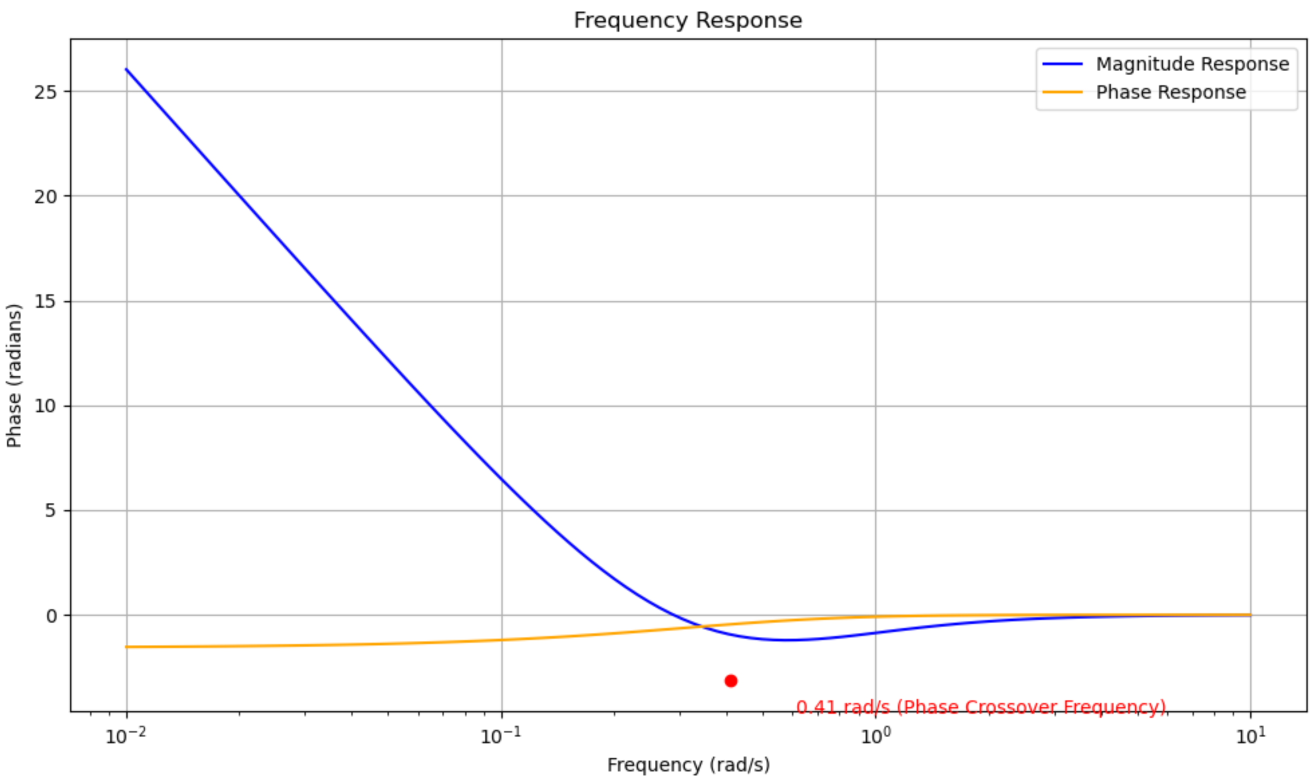
$$\angle G(s) = \frac{\pi}{2} + \tan^{-1}(sT_1) + \tan^{-1}(sT_2) - \frac{\pi}{2} \tag{10}$$
$$= \tan^{-1}(sT_1) + \tan^{-1}(sT_2) \tag{11}$$

the frequency at which the phase angle $\angle G(s)$ equals $-\pi$ radians.

$$\tan^{-1}(j\omega T_1) + \tan^{-1}(j\omega T_2) = -\pi \tag{12}$$
$$\tan^{-1}(j\omega T_1) + \tan^{-1}(j\omega T_2) = -\frac{\pi}{2} \tag{13}$$
$$\tan^{-1}(j\omega T_1) = -\frac{\pi}{2} - \tan^{-1}(j\omega T_2) \tag{14}$$
$$j\omega T_1 = \tan\left(-\frac{\pi}{2} - \tan^{-1}(j\omega T_2)\right) \tag{15}$$
$$j\omega T_1 = -\frac{1}{\tan(\tan^{-1}(j\omega T_2))} \tag{16}$$
$$j\omega T_1 = -\frac{1}{j\omega T_2} \tag{17}$$
$$\omega T_1 = \frac{1}{\omega T_2} \tag{18}$$
$$\omega^2 = \frac{1}{T_1 T_2} \tag{19}$$
$$\omega = \frac{1}{\sqrt{T_1 T_2}} \tag{20}$$

the phase cross-over frequency is

$$\frac{1}{\sqrt{T_1 T_2}}$$


Frequency Response
Magnitude Response
Phase Response
25
20
15
10
5
0
Phase (radians)
0.41 rad/s (Phase Crossover Frequency)
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/s)