

# NCERT DISCRETE

EE23BTECH11020 - Raghava Ganji\*

**GATE 2023 BM.48:** The function  $f(z) = \frac{1}{z-1}$  of a complex variable  $z$  on a closed contour in an anti-clockwise direction. For which of the following contours, does this integral have a non-zero value?

(A)  $|z - 2| = 0.01$

(B)  $|z - 1| = 0.1$

(C)  $|z - 3| = 5$

(D)  $|z| = 2$

**Solution:**

Cauchy's Integral Formula and Residue Theorem.

$$\oint_c f(z) = 2\pi j \text{Res}[f(z), z_0] \quad (1)$$

$$\text{Res}[f(z), z_0] = \lim_{z \rightarrow z_0} [(z - z_0) f(z)] \quad (2)$$

Here  $z_0$  is pole of the  $f(z)$

Using (1)

$$\oint_c \frac{1}{z-1} dz = 2\pi j \text{Res}\left[\frac{1}{z-1}, 1\right] \quad (3)$$

- 1) For option A the pole is outside the contour, then Residue is zero.

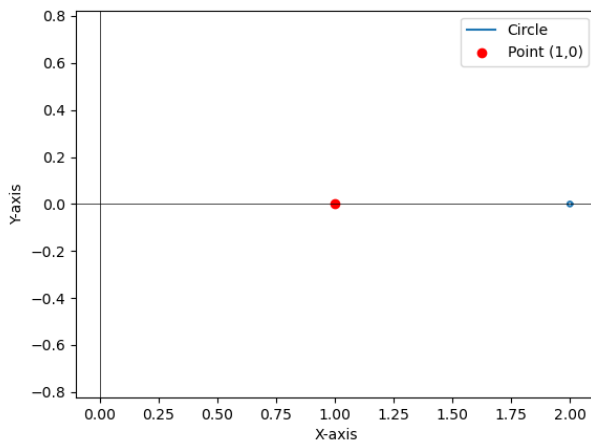


Fig. 1. graph of option A

$$\Rightarrow \oint_c \frac{1}{z-1} dz = 2\pi j(0) \quad (4)$$

$$\Rightarrow 0 \quad (5)$$

- 2) For option B the pole is inside the contour. Then, using (2)

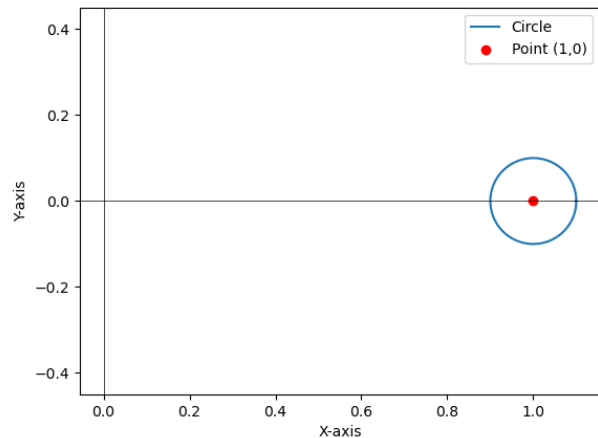


Fig. 2. graph of option B

$$\text{Res}\left[\frac{1}{z-1}, 1\right] = \lim_{z \rightarrow 1} (z-1) \frac{1}{z-1} \quad (6)$$

$$= 1 \quad (7)$$

$$\Rightarrow \oint_c \frac{1}{z-1} dz = 2\pi j(1) \quad (8)$$

$$\Rightarrow 2\pi j \quad (9)$$

- 3) For option C the pole is inside the contour. Then, using (2)

$$\text{Res}\left[\frac{1}{z-1}, 1\right] = \lim_{z \rightarrow 1} (z-1) \frac{1}{z-1} \quad (10)$$

$$= 1 \quad (11)$$

$$\Rightarrow \oint_c \frac{1}{z-1} dz = 2\pi j(1) \quad (12)$$

$$\Rightarrow 2\pi j \quad (13)$$

- 4) For option D the pole is inside the contour. Then, using (2)

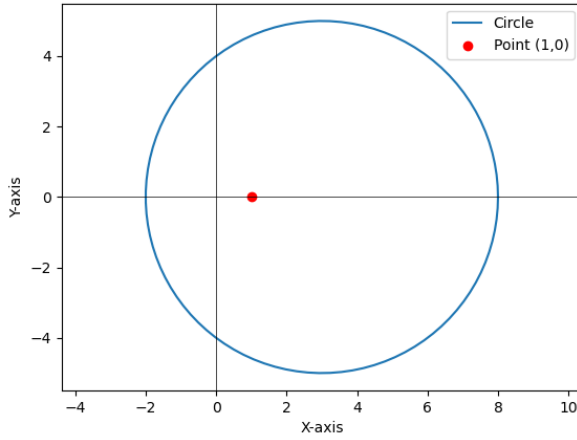


Fig. 3. graph of option C

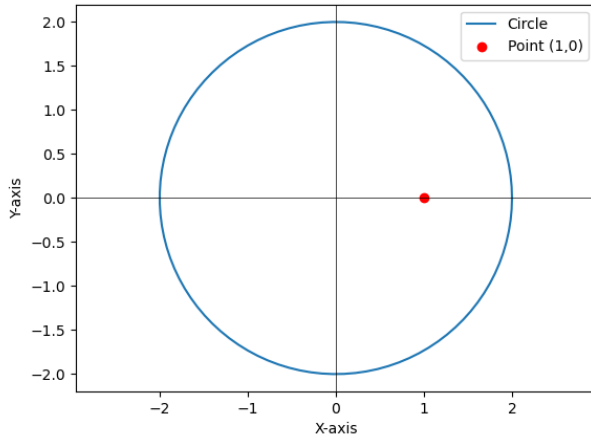


Fig. 4. graph of option D

$$\operatorname{Res}\left[\frac{1}{z-1}, 1\right] = \lim_{z \rightarrow 1} (z-1) \frac{1}{z-1} \quad (14)$$

$$= 1 \quad (15)$$

$$\Rightarrow \oint_c \frac{1}{z-1} dz = 2\pi j(1) \quad (16)$$

$$\Rightarrow 2\pi j \quad (17)$$

We can conclude that for options B,C,D contours have the non-zero value for this integral.