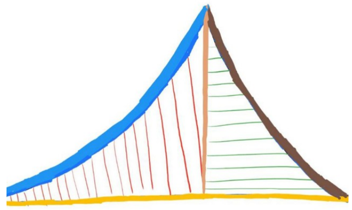
# SIGNAL PROCESSING

# Through GATE


**G. V. V. Sharma**

# Contents

# Introduction

This book provides solutions to signal processing problems in GATE.

# Chapter 1

# Harmonics

# Chapter 2

# Filters

# Chapter 3

# Z-transform

# Chapter 4

# Systems

4.1 Consider a unity-gain negative feedback system consisting of the plant $G(s)$ and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{(s-1)}$$

(GATE EE 2023)

**Solution:**

| **Parameter** | **Description** | **Value** |
|---|---|---|
| $K_p$ | Proportional Gain | 3 |
| $K_i$ | Integral Gain | 1 |
| $r(t)$ | Reference Input | $u(t)$ |
| $w(t)$ | Controller Output | ? |
| $y(t)$ | Plant Output | ? |
| $e(t)$ | Error Input | $r(t) - y(t)$ |

Table 1: Parameter Table

From theFig. 4.1:

$$E(s) = U(s) - Y(s) \tag{4.1}$$

$$W(s) = 3E(s) + \frac{1}{s}E(s) \tag{4.2}$$

$$Y(s) = G(s)\,W(s) \tag{4.3}$$

Some results:

$$tx(t) \overset{\mathcal{L}}{\longleftrightarrow} -\frac{dX(s)}{ds} \tag{4.4}$$

$$e^{-at}x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s+a) \tag{4.5}$$

By using (4.4) and (4.5):

$$e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+1}, Re(s) > -1 \tag{4.6}$$

$$te^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{(s+1)^2}, Re(s) > -1 \tag{4.7}$$

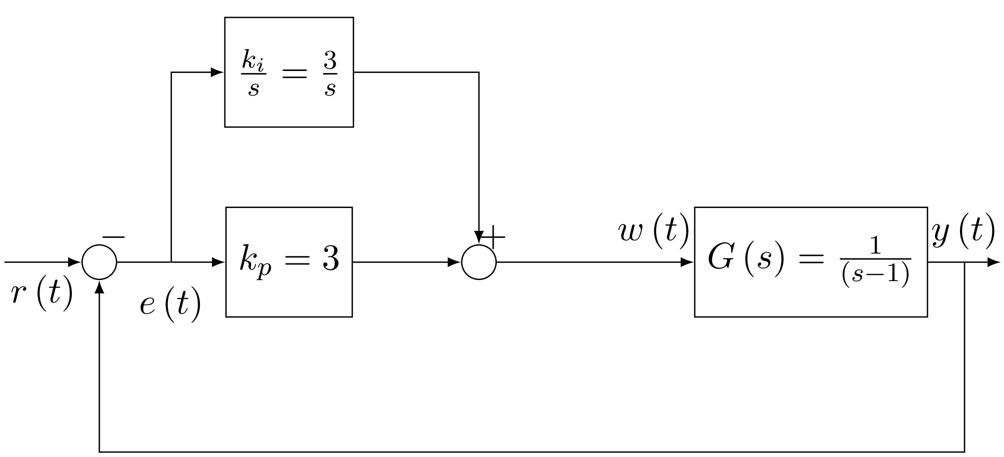



Figure 4.1: Block Diagram of System

(a) **Plant Output:**

From (4.1) , (4.2) and (4.3):

$$Y(s) = \frac{3s+1}{s(s+1)^2}, Re(s) > -1 \tag{4.8}$$

Final Value Theorem:

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) \tag{4.9}$$

Using (4.9) on Y($s$):

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{4.10}$$

$$= 1 \tag{4.11}$$

Taking partial fraction of (4.8) :

$$Y(s) = \frac{1}{s} + \frac{2}{(s+1)^2} - \frac{1}{s+1} \tag{4.12}$$

Using (4.6) and (4.7):

$$\therefore y(t) = u(t) + 2te^{-t}u(t) - e^{-t}u(t) \tag{4.13}$$

(b) **Controller Output:**

From (4.2)

$$W(s) = \frac{3}{s} + \frac{1}{s^2} - Y(s)\left(3 + \frac{1}{s}\right) \tag{4.14}$$

Substituting (4.8)

$$W\left(s\right) = \frac{\left(s-1\right)\left(3s+1\right)}{s\left(s+1\right)^2}, Re\left(s\right) > -1 \tag{4.15}$$

Using (4.9) on W($s$)

$$\lim_{t\to\infty} w\left(t\right) = \lim_{s\to 0} sW\left(s\right) \tag{4.16}$$

$$= -1 \tag{4.17}$$

Taking partial fraction of equation(4.15) :

$$W\left(s\right) = -\frac{1}{s} - \frac{4}{\left(s+1\right)^2} + \frac{4}{s+1} \tag{4.18}$$

Using equations (4.6) and (4.7) and taking inverse lapalace transform:

$$w\left(t\right) = -u\left(t\right) - 4te^{-t}u\left(t\right) + 4e^{-t}u\left(t\right) \tag{4.19}$$

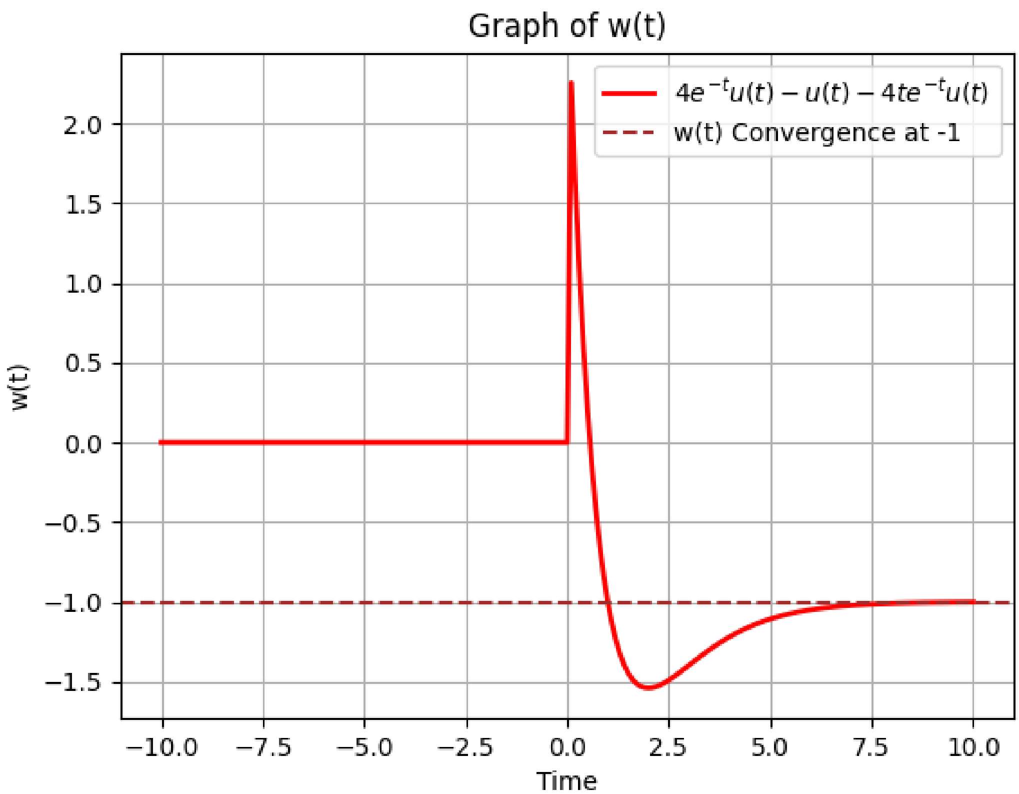



Figure 4.2: $w\left(t\right)$ converges at -1.

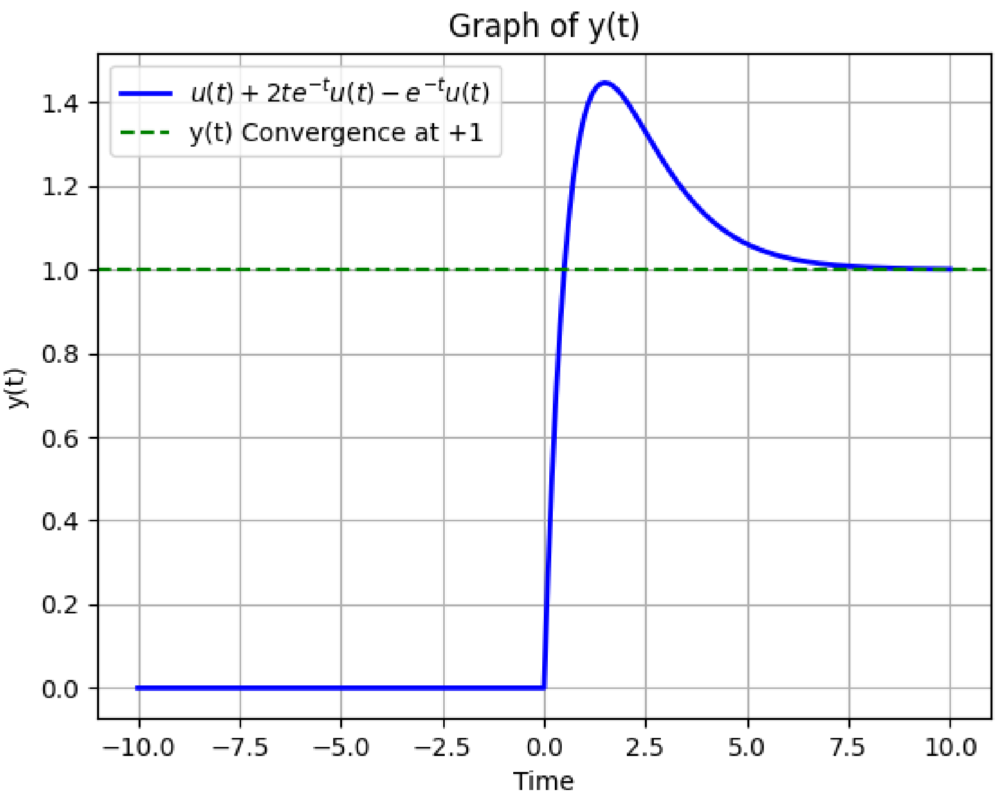



Figure 4.3: $y(t)$ converges at $+1$

4.2 Level ($h$) in a steam boiler is controlled by manipulating the flow rate ($F$) of the break-up(fresh) water using a proportional ($P$) controller. The transfer function between the output and the manipulated input is

$$\frac{h(s)}{F(s)} = \frac{0.25(1-s)}{s(2s+1)}$$

The measurement and the valve transfer functions are both equal to 1. A process engineer wants to tune the controller so that the closed loop response gives the decaying oscillations under the servo mode. Which one of the following is the CORRECT value of the controller gain to be used by the engineer?

(a) 0.25

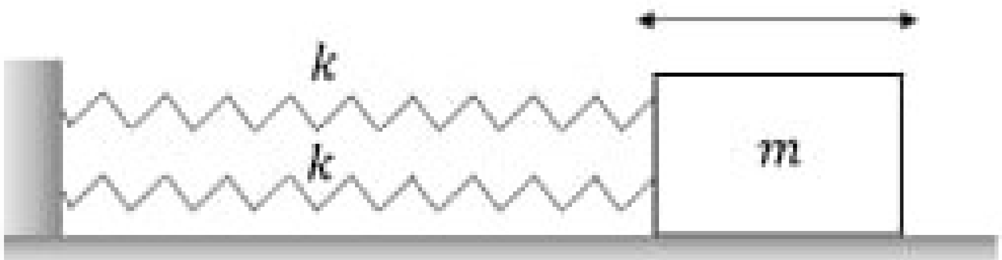
(b) 2

(c) 4

(d) 6

**Solution:**

4.3 The figure shows a block of mass m = 20 kg attached to a pair of identical linear springs, each having a spring constant k = 1000 N/m. The block oscillates on a frictionless horizontal surface. Assuming free vibrations, the time taken by the block to complete ten oscillations is ____ seconds . (Rounded off to two decimal places) Take $\pi = 3.14$. GATE ME 30




4.4 A system has transfer function

$$\frac{Y(s)}{X(s)} = \frac{s - \pi}{s + \pi}$$

let $u(t)$ be the unit step function.The input $x(t)$ that results in a steady-state output $y(t) = sin(\pi t)$ is __. (GATE IN 2023)

**Solution:**

# Chapter 5

# Sequences

5.1 Consider the discrete time signal $x[n] = u[-n+5] - u[n+3]$, where

$$u[n] = \begin{cases} 1; n \geq 0 \\ 0; n < 0 \end{cases}$$

The smallest n for which $x[n] = 0$ is?

**Solution:** From Fig. 1, the minimum value of $n$ is given as

$$n = -3 \tag{5.1}$$

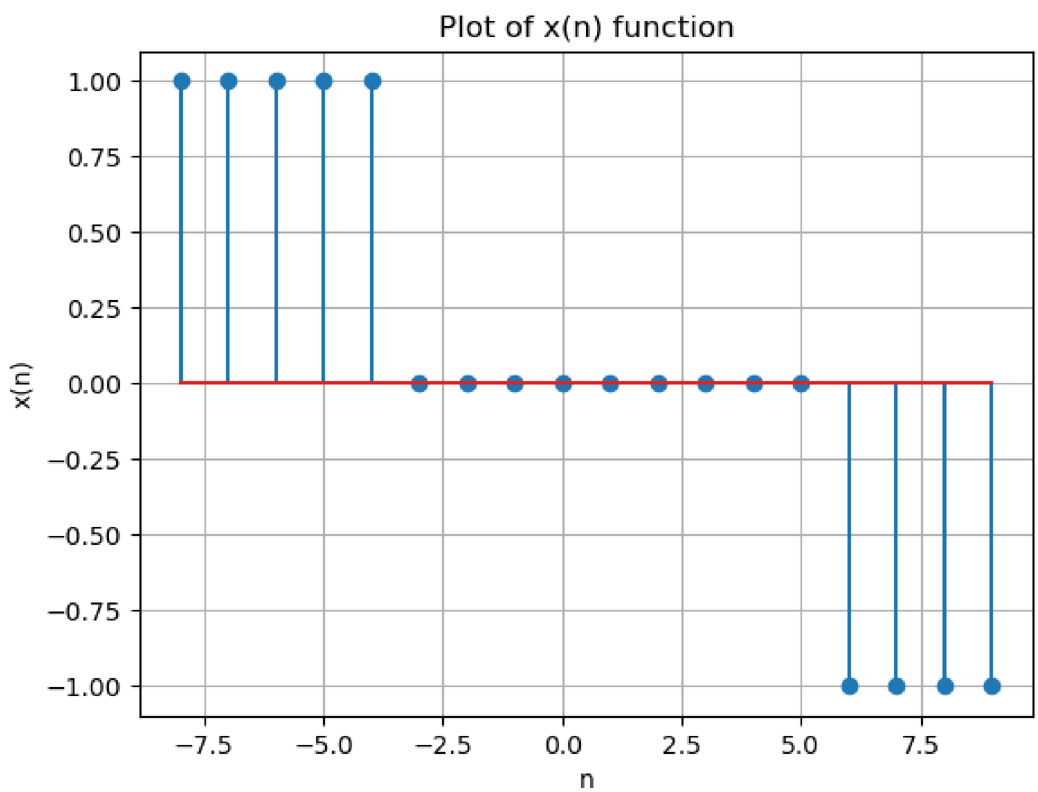



Figure 1: Plot of function $x\left(n\right)$ taken from python3

# Chapter 6

# Sampling

6.1 An 8 bit ADC converts analog voltage in the range of 0 to $+5\,V$ to the corresponding digital code as per the conversion characteristics shown in figure. For $V_{in} = 1.9922\,V$, which of the following digital output, given in hex, is true?

(a) $64H$

(b) $65H$

(c) $66H$

(d) $67H$

(GATE EE 40)

**Solution:**

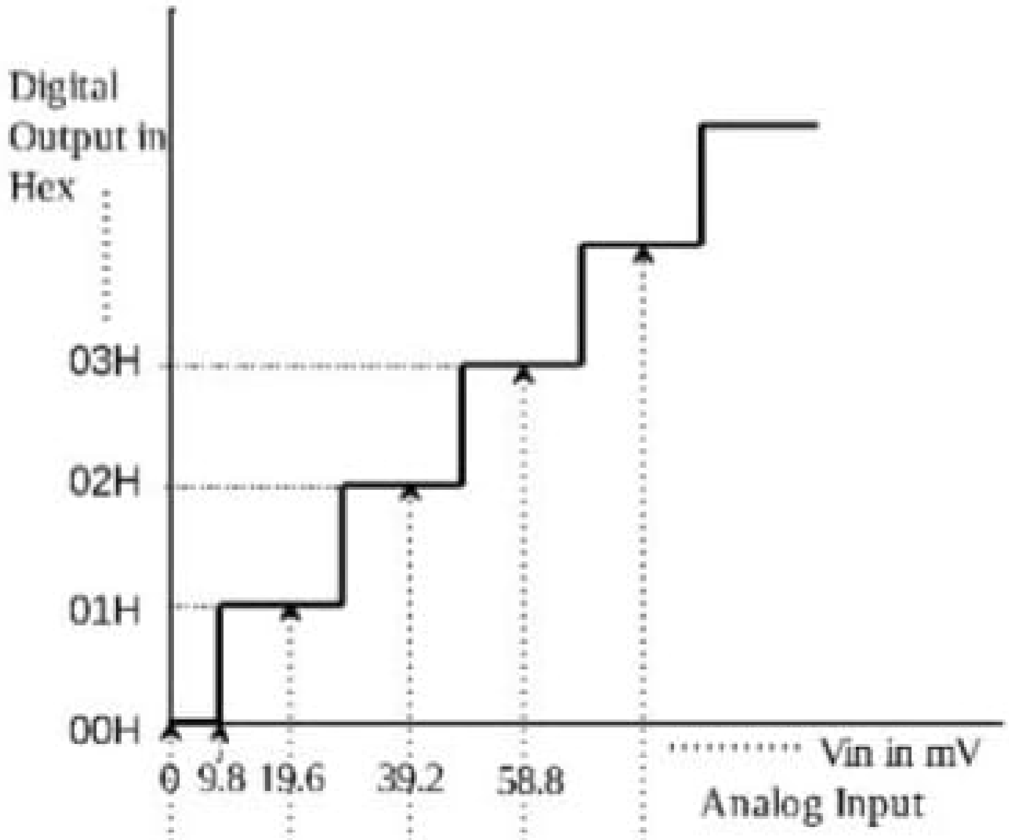



Figure 6.1:

Calculating the step-size:

$$\begin{aligned}
\Delta V_{in} &= \frac{V_{max} - V_{min}}{2^n - 1} && (6.1)\\
&= \frac{5-0}{2^8 - 1} && (6.2)\\
&= \frac{5}{255} && (6.3)\\
\implies V_{out} &= \frac{V_{in}}{\Delta V_{in}} && (6.4)\\
&= \frac{1.9922 \times 255}{5} && (6.5)\\
&= 101.59 && (6.6)\\
&\approx 102_{10} && (6.7)
\end{aligned}$$

| **Symbol** | **Value** | **Description** |
|---|---|---|
| $n$ | 8 | Number of bits of ADC |
| $V_{min}$ | $0V$ | Minimum Analog Voltage |
| $V_{max}$ | $5V$ | Maximum Analog Voltage |
| $V_{in}$ | $1.9922V$ | Input Voltage |
| $V_{out}$ | | Output Voltage |

Table 6.1: Given Parameters

$\therefore$ correct answer is option (c).

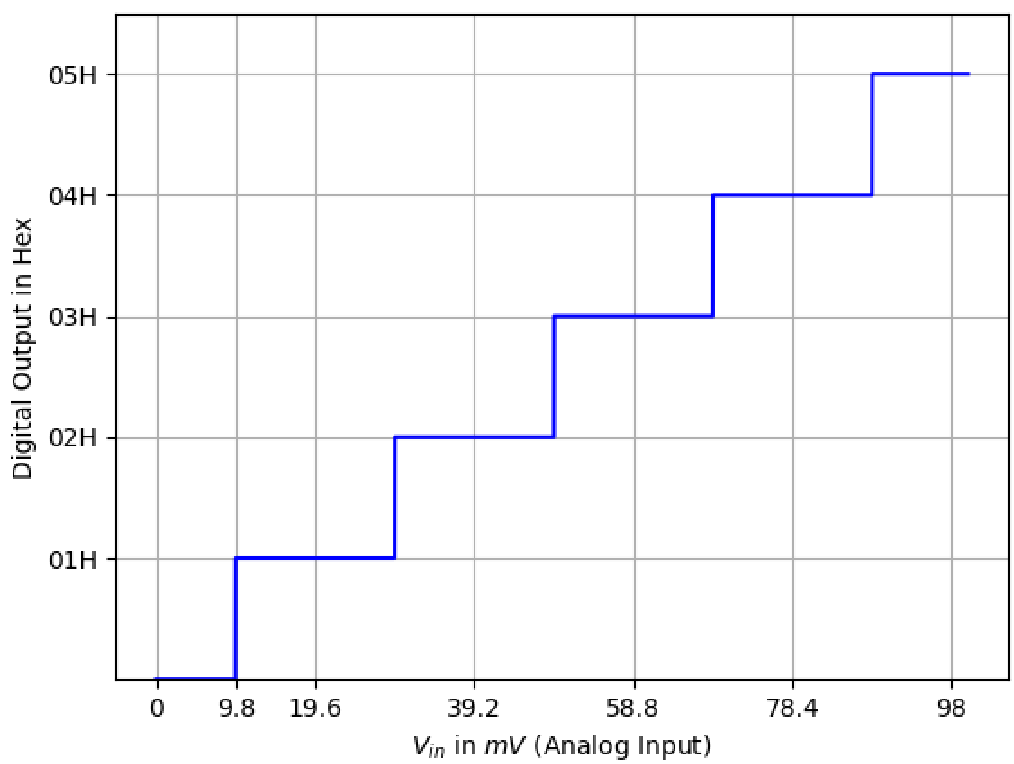

05H
04H
03H
02H
01H
Digital Output in Hex
0
9.8
19.6
39.2
58.8
78.4
98
$V_{in}$ in $mV$ (Analog Input)

Figure 6.2:

Chapter 7

# Contour Integration

# Chapter 8

# Laplace Transform

8.1 The number of zeroes of the polynomial $P(s) = s^3 + 2s^2 + 5s + 80$ in the right side of the plane? (GATE IN 2023)

**Solution:** The table below shows the Routh array of the $n^{th}$- order characteristic polynomial :

$$a_0 s^n + a_1 s^{n-1} ...... + a_{n-1} s^1 + a_n s^0 \tag{8.1}$$

| | | | | |
|---|---|---|---|---|
| $s^n$ | $a_0$ | $a_2$ | $a_4$ | ... |
| $s^{n-1}$ | $a_1$ | $a_3$ | $a_5$ | ... |
| $s^{n-2}$ | $b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1}$ | $b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}$ | ... | .. |
| $s^{n-3}$ | $c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$ | $\vdots$ | | |
| $\vdots$ | $\vdots$ | $\vdots$ | | |
| $s^1$ | $\vdots$ | $\vdots$ | | |
| $s^0$ | $a_n$ | | | |

Table 8.1: Routh Array

Characteristic Equation:

$$s^3 + 2s^2 + 5s + 80 = 0 \tag{8.2}$$

From Table 8.1:

| | | |
|---|---|---|
| $s^3$ | 1 | 5 |
| $s^2$ | 2 | 80 |
| $s^1$ | $\frac{2 \times 5 - 80 \times 1}{2} = -35$ | |
| $s^0$ | $\frac{-35 \times 80}{-35} = 80$ | |

Table 8.2:

From Table 8.2:

Since there are 2 sign changes in the first column of the Routh tabulation. So, the number of zeros in the right half of the s-plane will be 2.

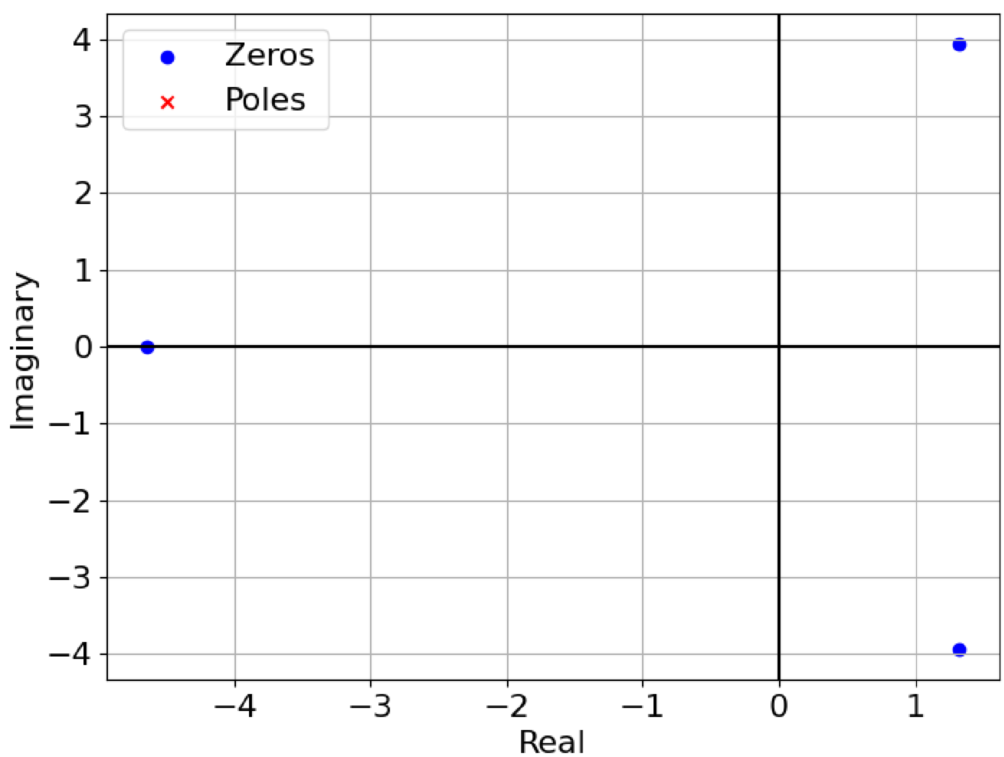
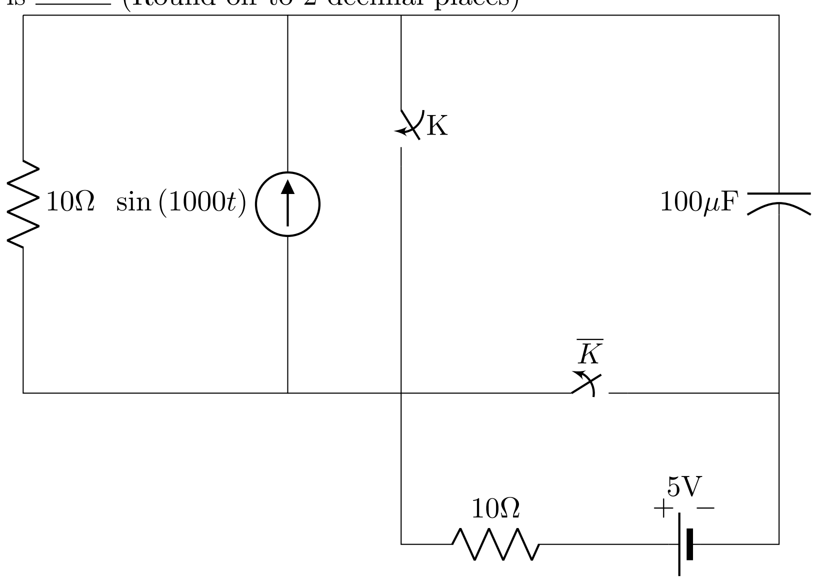



Figure 8.1:

8.2 The circuit shown in the figure is initially in the steady state with the switch K in open condition and $\overline{K}$ in closed condition. The switch K is closed and $\overline{K}$ is opened simultaneously at the instant $t = t_1$, where $t_1 > 0$. The minimum value of $t_1$ in milliseconds such that there is no transient in the voltage across the 100 $\mu F$ capacitor, is ______ (Round off to 2 decimal places) (GATE EE 2023)



8.3 $y = e^{mx} + e^{-mx}$ is the solution of which differential equation?

**1.** $\frac{dy}{dx} - my = 0$

**2.** $\frac{dy}{dx} + my = 0$

**3.** $\frac{d^2y}{dx^2} + m^2y = 0$

**4.** $\frac{d^2y}{dx^2} - m^2y = 0$

(GATE AG 2023) **Solution:**

8.4 A cascade control strategy is shown in the figure below. The transfer function between the output ($y$) and the secondary disturbance ($d_2$) is defined as

$$G_{d2}(s) = \frac{y(s)}{d_2(s)}$$

. Which one of the following is the CORRECT expression for the transfer function $G_{d2}(s)$?

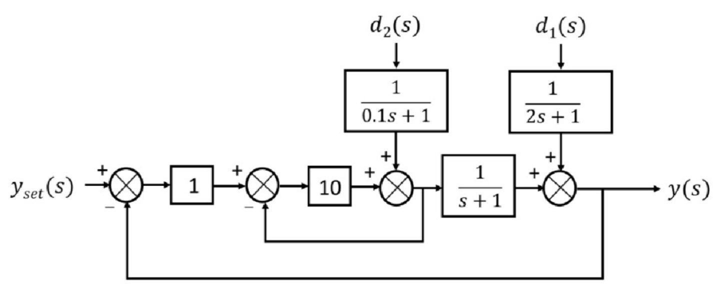



Figure 8.2:

A. $\frac{1}{(11s+21)(0.1s+1)}$

B. $\frac{1}{(s+1)(0.1s+1)}$

C. $\frac{(s+1)}{(s+2)(0.1s+1)}$

D. $\frac{(s+1)}{(s+1)(0.1s+1)}$

(GATE CH 2023) **Solution:**

8.5 In the differential equation $\frac{dy}{dx}+\alpha xy = 0, \alpha$ is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of $\alpha$ is (rounded off to three decimal places). (GATE CE 2023) **Solution:**

# Chapter 9

# Fourier Transfrom

9.1 The discrete-time Fourier transform of a signal x $[n]$ is $X(\Omega) = (1+\cos\Omega)\, e^{-j\Omega}$. Consider that $x_p[n]$ is a periodic signal of period $N = 5$ such that

$$x_p[n] = x[n], \text{for n= 0, 1, 2} \tag{9.1}$$

$$= 0, \text{for n= 3, 4} \tag{9.2}$$

Note that $x_p[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$. The magnitude of the Fourier series coefficient $a_3$ is ______________ (Round off to 3 decimal places). (GATE EE 2023) **Solution:**