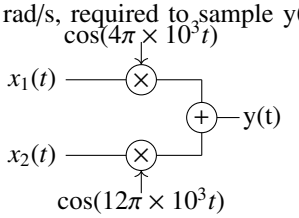
GATE 2023 - EC 50

EE23BTECH11220 - R.V.S.S Varun

Question

Let $x_1(t)$ and $x_2(t)$ be two band-limited signals having bandwidth B = $4\pi\times10^3$ rad/s each. In the figure below, the Nyquist sampling frequency, in rad/s, required to sample y(t), is

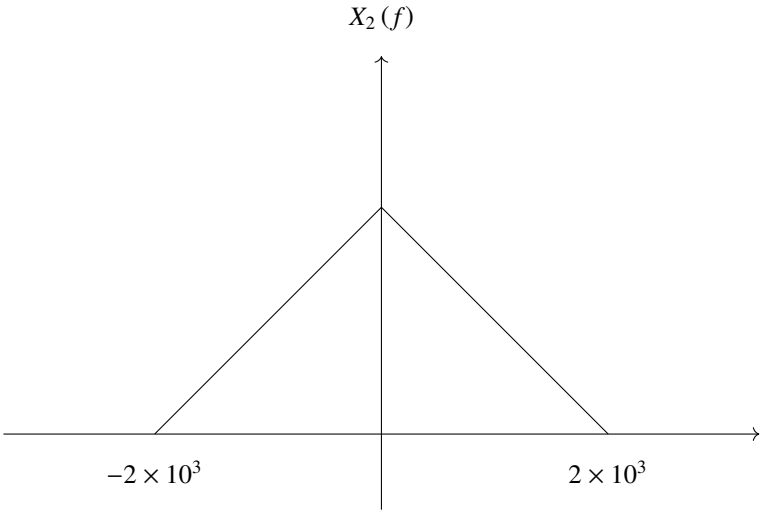


(a) $20\pi\times10^3$
(b) $40\pi\times10^3$
(c) $8\pi\times10^3$
(d) $32\pi\times10^3$ (GATE EC 50)

Solution

Symbol	Description	Value
f_1	Frequency of $\cos\left(4\pi\times10^3\right)$	2×10^3
f_2	Frequency of $\cos\left(12\pi\times10^3\right)$	6×10^3
f_m	Maximum frequency of the output signal	-
ω_m	-	$2\pi f_m$
ω_s	Nyquist sampling rate	$2\omega_m$

TABLE 0
Table of parameters



From question figure ,

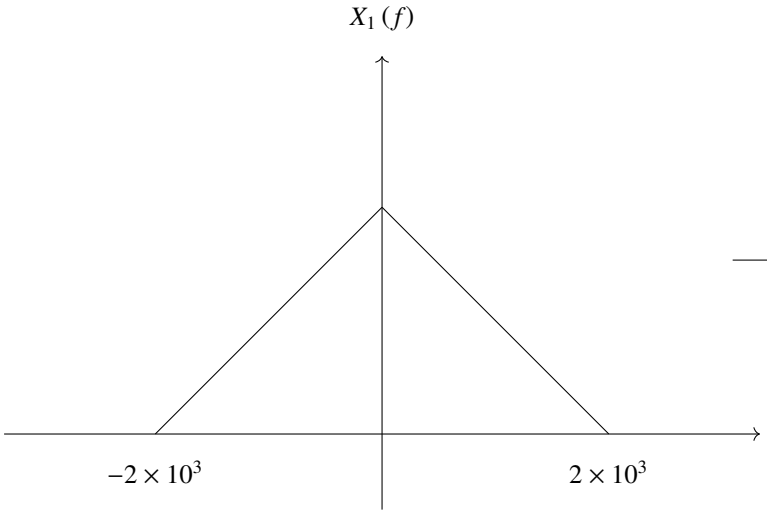
$$y(t) = x_1(t)\times cos\left(4\pi\times10^3t\right) + x_2(t)\times cos\left(12\pi\times10^3t\right) \quad (1)$$

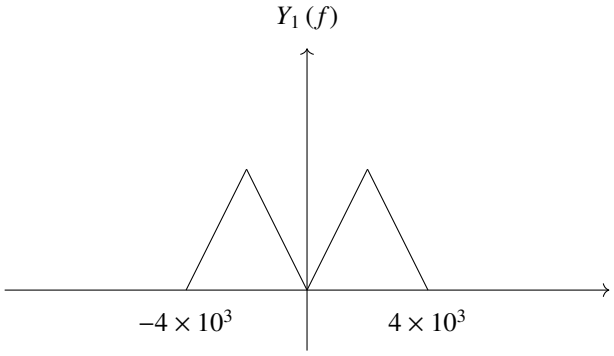
$$y(t) = y_1(t) + y_2(t) \quad (2)$$

$$Y(f) = Y_1(f) + Y_2(f) \quad (3)$$

$$Y_1(f) = X_1(f) * \frac{1}{2}[\delta(f-f_1) + \delta(f+f_1)] \quad (4)$$

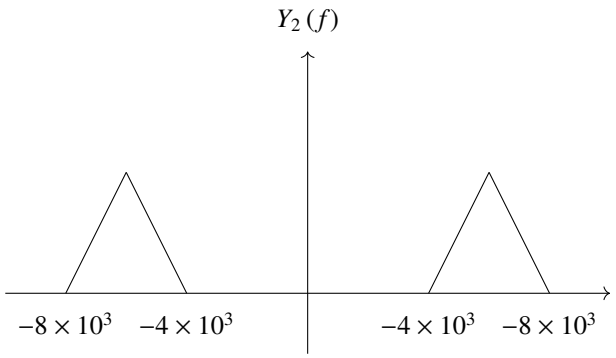
$$= \frac{1}{2}[X_1(f-f_1) + X_1(f+f_1)] \quad (5)$$



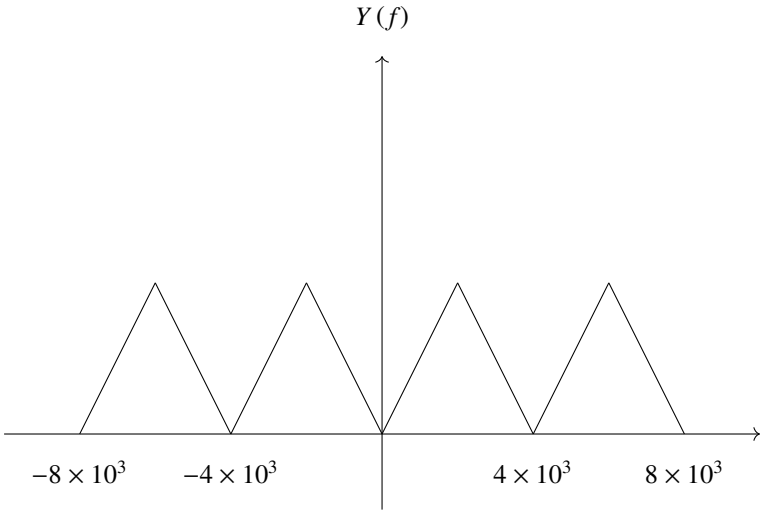


$$Y_2(f) = X_2(f) * \frac{1}{2}[\delta(f-f_2) + \delta(f+f_2)] \quad (6)$$

$$= \frac{1}{2}[X_2(f-f_2) + X_2(f+f_2)] \quad (7)$$



From (3):



From table,

$$\omega_m = 16\pi \times 10^3 rad/sec. \tag{8}$$

$$\omega_s = 2\omega_m = 32\pi \times 10^3 rad/sec. \tag{9}$$